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A Financial Contracting-Based Capital Asset Pricing Model

Roberto Steri
University of Luxembourg

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Abstract

I show that an asset pricing model for the equity claims of a value-maximizing firm can be constructed from its optimal financial contracting behavior. Deals between firms and financiers reveal the importance of contractible states for firm's equity value, namely the stochastic discount factor the firm responds to. I empirically evaluate the model in the cross section of expected equity returns. I find that the financial contracting approach goes a long way in rationalizing observed cross-sectional differences in average returns, also in comparison to leading asset pricing models.

Keywords: Cross Section of Returns, Production-Based Asset Pricing, Capital Asset Pricing Model, Stochastic Discount Factor, Dynamic Contracting.

JEL Classification Numbers: C61, C63, D21, D24, G10, G12, G31, G32, G35.

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†Luxembourg School of Finance, University of Luxembourg. E-mail: roberto.steri@uni.lu.

1 Introduction

Stochastic discount factors are the cornerstone of modern asset pricing. They allow to compute asset prices as the expected discounted value of future cashflows. Asset pricing theory ordinarily focuses on the demand of securities, and derives a stochastic discount factor from the optimizing behavior of an investor who decides over consumption and portfolio allocations. This paper instead focuses on the supply of securities. I build upon corporate finance theory to show that, under some assumptions, a valid stochastic discount factor to price the equity claims of a firm can be backed out from its optimal financial contracting behavior. This leads to an asset pricing model, which I refer to as the Contracting Asset Pricing Model. I take a step in empirically evaluating the model in the cross section of expected equity returns. My results suggest that the financial contracting approach goes a long way in rationalizing observed cross-sectional differences in average returns.

The empirical appeal of the contracting approach to cross-sectional asset pricing lies in its ability to link unobservable state-contingent prices to observable firms' quantity choices. In this perspective, this work mainly relates to the papers of Jermann (2010) and Belo (2010), who recover a stochastic discount factor from firm's real investment decisions on the production side of the economy. Instead, I recover a stochastic discount factor from firms' contracting behavior. In corporate finance, financial contracting is not only a pivotal economic mechanism to raise external financing, but also a fundamental channel through which firms transfer resources across states of the world. As the very large literature that has developed over the last fifty years shows, financial contracting is first order to study corporate policies. External financing contracts are practically implemented with common financial instruments including credit lines, financial derivatives, equity issuance, and portfolios of bonds with different maturities.[1]

Hart (2001) describes financial contracting as "the theory of what kinds of deals are made between financiers and those who need financing". A firm that optimally arranges a financing contract with an external lender issues promises to pay in some states of the world. I interpret firms broadly, as production entities that may represent individual establishments, entire industries,

[1]In particular, credit lines appear to be a prominent implementation of state-contingent debt contracts. Sufi (2009) reports that credit lines constitute more than 80 percent of bank debt for public firms in the US. Colla, Ippolito, and Li (2013) report that the drawn part of credit lines accounts for 22 percent of their total debt.

market segments, or sectors. I show that the state-contingent nature of the contract is informative of the stochastic discount factor the firm responds to. By trading off the costs and benefits of a given promise to pay in a specific state, a firm reveals information on the importance of that state for its own value. The value of each state is also measured by the stochastic discount factor the firm responds to. Therefore, the stochastic discount factor can be identified through observed firms' decisions, and used to price firms' equity claims. In the model, the key friction that restricts firms' access to external financing and firms' promises to repay is collateral constraints. Collateral constraints that arise from limited enforcement restrict credible promises to repay, and hence the amount of resources firms can effectively transfer across states.[2]

The Contracting Asset Pricing Model relates the stochastic discount factor a certain firm responds to the growth rate of the marginal value of its net worth.[3] The economic mechanism driving this result can be interpreted through the lens of firms' optimal contracting decisions. Firms have a motive to engage in financial contracting to transfer resources (net worth) to states that are most important to maximize their shareholder market value. The optimal lending contract increases net worth and lowers its marginal value in states in which the stochastic discount factor is presumably high, such as — as conventional wisdom suggests — bad times. Importantly, collateral constraints limit firms' ability to achieve this goal. The more financially constrained firms are, the less their effective ability to transfer resources to most important states, in spite of their motives.[4] Thus, the contracting model does not predict that one should empirically observe net worth growing in high-value states. Rather, high-value states can be identified as those to which firms, coping with their borrowing constraints, actively transfer net worth and lower its marginal value more than in low-value future states.

Any contracting model relies on a series of possibly restrictive assumptions. To empirically assess how far the contracting approach goes in pricing equity claims in the cross section of expected returns, I show that, while the growth of the marginal value of net worth is inherently unobservable,

[2]See also Rampini and Viswanathan (2013). The quantitative implications of contracting models with collateralized financing have been examined by Li, Whited, and Wu (2016), and Nikolov, Schmid, and Steri (2019).

[3]As standard in the dynamic contracting literature, net worth is the firm's counterpart of household's wealth, and captures how constrained a company is with respect to funds to allocate to investment, and distributions.

[4]Rampini and Viswanathan (2013), Li, Whited, and Wu (2016), and Nikolov, Schmid, and Steri (2019) show that financially constrained firms engage less in risk management because their immediate financing needs override their hedging concerns.

the model structure imposes restrictions on its dependence on the dynamics of observable variables. In the model, firms make optimal investment, financing, and payout decisions to maximize their equity market value. Firms have valuable investment opportunities that arise stochastically over time. However, they have limited funds, and they sign contracts with external lenders to aid external financing of profitable investments. Contracts have limited enforcement. This friction captures the difficulties of enforcing contracts that involve transfers of resources backed only by promises to repay. In equilibrium, the limited commitment problem endogenously imposes a collateral constraint, and firms implicitly borrow constrained against their equity value. The riskfree borrowing rate reflects variations in aggregate funding costs that external lenders are exposed to. In this context, value maximization provides a rationale to transfer resources to more valuable states, in a tradeoff with their funding needs for current investment and distributions. Firms' external financing capacity is limited, and firms can preserve it for specific future states by optimally contracting state-contingent repayments with the lender. A firm can therefore transfer net worth to any future state by promising a low repayment in the case that state occurs. Hence, firms can in effect transfer resources (net worth) across states. In this setting, the stochastic discount factor reflects which state must have led a firm to optimally make its observed decisions, and can be backed out from the firms' state-by-state first-order conditions with respect to contractual repayments. Conditional on how financially constrained they are, firms implement investment and financing policies to transfer resources to most important states, where the stochastic discount factor is high.

Given the model structure described above, the Contracting Asset Pricing Model ultimately relates the stochastic discount factor firms respond to, to three observable variables. In the model, every firm takes actions as best responses to all the relevant information required about its value maximization problem to make the current decisions. This information is summarized by a collection of state variables, whose dynamics in turn affect the growth of the marginal value of net worth and the stochastic discount factor. It is important to notice that all the state variables of the problem — net worth, profitability, and the interest rate — determine firm policies, and in turn affect firms' contracting behavior. The dynamics of the state variables therefore drive the dynamics of the stochastic discount factor, which depends on the growth rate of net worth, the growth rate of profitability, and the growth rate of the market riskfree rate. The three variables have an intuitive interpretation. They measure the amount of resources available to the firm in a certain future state.

These resources (net worth) can be either actively transferred by arranging financial contracts (net worth growth), originate from existing profitable investments (profitability growth), or be raised at a cost through external financing (interest rate growth).

Table 1
Contracting Model: Predictions and Evidence.

	Model Predictions		
	Net Worth Growth	Profitability Growth	Interest Rate Growth
Most Important States	High	Low/High	Low/High
Expected Return Spreads	Negative	Positive if Low Negative if High	NA

	Empirical Evidence		
	Net Worth Growth	Profitability Growth	Interest Rate Growth
Most Important States	High	Low	High
Expected Return Spreads	Negative	Positive	NA

The top panel of Table 1 summarizes the key testable predictions of the Contracting Asset Pricing Model. First, as the model predicts that most important states for equity prices of any firm are inversely related to the growth rate of the marginal value of net worth given the severity of its financial constraints, net worth growth is expected to load positively on the stochastic discount factor. Instead, for both profitability growth and interest rate growth, two contrasting forces determine the ultimate dominating effect on the backed-out stochastic discount factor. This is due to the persistence in the stochastic processes governing the profitability of investment opportunities and borrowing costs.[5] The contracting model also imposes restrictions on the observed spreads in cross-sectional average returns in accordance to the states that are most important for firms. As in

[5]Empirically, there is ample evidence that productivity and interest rates exhibit positive persistence. As I discuss in Section 5, when productivity is high, persistent shocks reduce the marginal value of net worth because the expected profitability and net worth going forward are high too. However, high current profitability increases investment needs because of the higher expected conditional profitability of investment. The second effect increases the marginal value of net worth. Similarly, persistence in interest rates raises the borrowing cost for the firm and its marginal value of net worth when interest rates are high. Vice versa, high interest rates increase the conditional expected cost of borrowing, and contribute to reduce the demand for debt to be otherwise optimally repaid. This increases preserved future net worth and reduces its marginal value.

textbook consumption-based models, securities that pay out in most important states are a hedge against sustained downturns and earn higher expected returns. The only difference is that most important states are not directly linked to aggregate consumption risk, but are now backed out from the optimal firm responses.[6] Recall that the most important states can be backed out as those in which, in the presence of borrowing constraints, firm's net worth increases the most. Thus, shares of firms whose net worth grows are a hedge in that they pay out more in most important states, in which the stochastic discount factor is high. Accordingly, high net worth growth stocks provide a hedge in more important states and require lower expected returns. Similarly, if reductions in profitability identify most important states, shares of firms whose profits grow provide higher payoffs in less important states and earn higher expected returns. Finally, if most important states are those that occur in times when interest rates spike, interest rate increases are associated with lower stock prices, and higher expected equity returns.

I test the asset pricing implications of the model in the cross section of average equity returns. The results are summarized in the bottom panel of Table 1. First, I document that the two aforementioned firm characteristics, namely the growth rate of firm's net worth and the growth rate of firm's profitability, generate sizeable spreads in cross-sectional equity returns. Second, I implement asset pricing tests with the Generalized Method of Moments (GMM) to assess the empirical performance of the model. As the recent empirical literature recommends (Lewellen, Nagel, and Shanken (2010), Daniel and Titman (2012)), I consider different test assets in empirical tests, namely the 25 Fama-French portfolios sorted by size and book-to-market equity, the 30 Fama-French industry portfolios, and 25 portfolios sorted by the growth rates of net worth and profitability. Overall, the Contracting Asset Pricing Model finds support in the data. The model prices the test assets well, and delivers low pricing errors even in comparison to leading asset pricing models, as the CAPM, the Consumption CAPM, and the Fama and French three-factor model. Historically, asset pricing models obtained from consumption-based stochastic discount factors have not succeeded in accounting for the variation of expected returns across stocks. One important reason for their empirical failure is the smoothness of consumption data. This prevents expected returns to line up with covariances with consumption aggregates, as these models predict. On the contrary, the

[6]Observe that, even in the case of complete markets, firms in general respond differently to the same aggregate state because they are heterogeneous, for example with respect to size or financial constraints.

Contracting Asset Pricing Model gets traction because it links the relevant stochastic discount factor for firms' equity prices responds to the firm's characteristics, which exhibit larger fluctuations.

The Contracting Asset Pricing Model is not a multifactor model, as it is derived at the firm level. Thus, it is not suited to derive direct aggregate implications. However, this does not necessarily imply an inconsistency with the existence of aggregate risk factors that price the equity of large public firms, such as reduced-form factor models, or with the consumption-based paradigm. To empirically explore this relationship, I construct two ad-hoc empirical factors using net worth growth and profitability as sorting variables, following the approach in Fama and French (2016), to obtain a projection on the space of traded stocks. The results indicate that the variables that the Contracting Asset Pricing Model predicts to generate cross-sectional spreads in average returns can be used to construct factors the relate to those that both Hou, Xue, and Zhang (2015) and Fama and French (2016) show to be essential for pricing the equity of large listed firms. In particular, the aggregate factor constructed using net worth growth as a sorting variable very closely relates to both the investment and the value factors in the two aforementioned studies. In this context, the contracting approach offers an alternative interpretation of the anomalies that arise in the cross section of equity returns. Through the lens of the Contracting Asset Pricing Model, anomaly variables can be regarded as possibly omitted determinants of the growth of the marginal value of net worth that are not captured by differences across firms in loadings on macroeconomic factors in benchmark asset pricing models.

Related Literature. This paper lies at the intersection of three lines of research. First, this work is closely related to Jermann (2010), Belo (2010), Jermann (2013), and Chen, Cooper, Ehling, and Xiouros (2017), who propose models in which the pricing kernels that drive firms' decisions can be recovered from observed investment and capital choices. The key difference with these works is the economic mechanism that allows to recover the stochastic discount factor from firms' decisions. Belo (2010) relies on a representation of production sets in which firms can affect idiosyncratic productivity shocks, while Jermann (2010), Jermann (2013) and Chen, Cooper, Ehling, and Xiouros (2017) investigate the equity premium, the term structure of interest rates, and the comovements between consumption and investment, by taking advantage of state-contingent production technologies. Here, the relevant state-contingent action that allows to identify the stochastic discount factor is based on the corporate finance theory, in the context of dynamic contracting. Unlike these studies, which rely on reduced-form for state-contingent production functions and parameterizations of firms' production possibility frontiers, in this study I rely on microfoundations based on the financial contracting literature. Second, this paper is linked to studies that analyze the asset pricing implications of endogenous borrowing and solvency constraints. These papers include Alvarez and Jermann (2000), Alvarez and Jermann (2001), Lustig and Van Nieuwerburgh (2005), Krueger, Lustig, and Perri (2008), Chien and Lustig (2010), Krueger and Lustig (2010), Lustig and Van Nieuwerburgh (2010), Chien, Cole, and Lustig (2011), Chien, Cole, and Lustig (2012). While the previous papers focus on the aggregate implications of households' borrowing constraints, the present work considers the implications of firms' collateral constraints for the cross-sectional differences in the risk and return tradeoff of securities. Finally, more broadly, this work also expands on the large literature that develops quantitative production models to investigate the cross section of equity returns. Recent contributions include Zhang (2005), Livdan, Sapriza, and Zhang (2009), Gomes and Schmid (2010), Garlappi and Yan (2011), and Bazdrech, Belo, and Lin (2013). With respect to these papers, my focus is to obtain a stochastic discount factor, instead of rationalize observed spreads in returns with respect to specific firms' characteristics.

The paper is organized as follows. Section 2 introduces the key insights of the paper in a stylized general equilibrium economy. This illustration highlights that the stochastic discount factors backed out from firms' optimal financing contracts bypasses, under some conditions, structural considerations associated with the consumption-based asset pricing, which poses challenges related to preferences (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004), intermediaries (e.g., He and Krishnamurthy, 2013), measurement issues (e.g., Bansal and Yaron, 2004; Barro, 2006; Savov, 2011; Kroencke, 2017), heterogeneity and constraints (e.g., Alvarez and Jermann, 2000; Koijen and Yogo, 2019). Section 3 builds on this insights and lays out the dynamic contracting model. Section 4 derives the Contracting Asset Pricing Model and its properties. Section 5 presents the empirical asset pricing tests and analyses. Section 6 concludes.

2 An Illustration in General Equilibrium

This section conveys the main insights of this work and sets the groundwork for the subsequent analysis. I build upon the exchange economy with borrowing constraints by Alvarez and Jermann (2000), in which I introduce production and financial contracting. I show that asset prices can be computed using stochastic discount factors backed out from firms' optimal financing contracts. This approach independent from structural considerations associated with the consumption side of the economy. In contrast, the consumption-based asset pricing approach within this context is significantly more intricate. The complexity arises from the necessity to keep track for the state-contingent intertemporal rates of substitution among heterogeneous investors.

Time and Uncertainty. There are two dates, $t = 0$ (today) and $t = 1$ (tomorrow). Uncertainty is represented by a finite number S of distinct states of nature, indexed by $\omega \in \Omega$, with probability density $\pi(\omega)$. Uncertainty is realized at the beginning of $t = 1$, when the state ω is revealed. Investors trade securities at $t = 0$. Their payoffs are realized at $t = 1$. Firms invest and arrange external financing contracts at $t = 0$, while firms' cash flows realize at $t = 1$.

2.1 Consumption Side: Investors

The economy is populated by a set $\mathcal{I} = \{1, 2, ..., I\}$ of investors, indexed by $i \in \mathcal{I}$. Investors consume one homogeneous good. We denote their exogenously-given endowments as $e_i = \{e_{i,0};\ e_{i,1}(\omega)\} \in \mathcal{R}_+^{S+1}$, at $t = 0$ and in each state at $t = 1$. This formulation enables endowments to be a generic function of the state, providing a versatile representation that can accommodate both idiosyncratic and aggregate shocks.

Investor preferences are identical and have a time-separable expected utility representation over their consumption choices $c_i = \{c_{i,0};\ c_{i,1}(\omega)\} \in \mathcal{R}_+^{S+1}$. The expected utility of an investor is

$$U(c_i) \equiv u(c_{i,0}) + \beta \sum_{\omega \in \Omega} \pi(\omega) \cdot u(c_{i,1}(\omega)),$$

where the period utility $u : \mathcal{R}^+ \rightarrow \mathcal{R}$ is strictly increasing, concave, and continuously differentiable, and $\beta \in (0, 1)$ is a time discount factor.

Investors have access to a complete set of S Arrow-Debreu securities that pay off one unit of consumption exclusively if state ω occurs. I denote the portfolio holdings of investor i in these securities at $t = 0$ as $a_{i,1}(\omega)$, and their prices as $p_{a,1}(\omega)$. Arrow-Debreu securities exist in zero net supply, that is, $\sum_{i \in \mathcal{I}} a_{i,1}(\omega) = 0$. Investors can also trade the claims to the cash flows of J firms, indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$. I denote the stock holdings by investor i as $n^F_{i,j,0}$, and their prices as $p^F_{j,0}$. Firms' shares distribute dividends $d^F_{j,0} \geq 0$ at $t = 0$, and $d^F_{j,1}(\omega) \geq 0$ in each state at $t = 1$. Dividend payouts are the result of firms' investment and contracting decisions, as described below. Firms' shares are in unit net supply, that is $\sum_{i \in \mathcal{I}} n^F_{i,j,0} = 1$. Investors have initial endowments of firms' shares $\overline{n}^F_{i,j,0} \geq 0$.

Investors are subject to the following budget constraints:

$$e_{i,0} + \sum_{j \in \mathcal{J}} p^F_{j,0} \overline{n}^F_{i,j,0} = c_{i,0} + \sum_{\omega \in \Omega} p_{a,1}(\omega) a_{i,1}(\omega) + \sum_{j \in \mathcal{J}} (p^F_{j,0} + d^F_{j,0}) n^F_{i,j,0}, \tag{1}$$

$$c_{i,1}(\omega) = a_{i,1}(\omega) + e_{i,1}(\omega) + \sum_{j \in \mathcal{J}} d^F_{j,1}(\omega) n^F_{i,j,0}, \forall \omega \in \Omega, \tag{2}$$

which equalize sources and uses of funding in each date and state.

Investors also face state-contingent borrowing constraints as follows:

$$a_{i,1}(\omega) + \sum_{j \in \mathcal{J}} d^F_{j,1}(\omega) n^F_{i,j,0} \geq B_{i,1}(\omega), \forall \omega \in \Omega. \tag{3}$$

In Equation (3), $B_{i,1}(\omega) \in [-\infty, 0]$ represents the borrowing limit for investor i in state ω. This constraint limits negative positions in Arrow-Debreu securities, with the inclusion of $-\infty$ allowing for the possibility of unrestricted borrowing in certain states.

In their infinite-horizon economies, Alvarez and Jermann (2000) and Alvarez and Jermann (2001) endogenize the borrowing limits in (3) as those that render investors with financial wealth precisely equal to the limit indifferent between remaining active in the economy or defaulting on their repayment obligations, with the latter decision leading to their permanent exclusion from asset markets in the future. Formally, in this context, $B_{i,1}(\omega)$ satisfies the participation constraint $u(c_{i,1}(\omega)) \geq u(e_{i,1}(\omega))$ with equality.

While not critical to the central theme of this illustration, it is worth noting that, with a finite horizon, "not too tight" constraints imply autarchy if all securities are in zero net supply. This is because, in the final period, investors have no incentives to fulfill their commitments to close out short positions. Formally, this implies $a_{i,1}(\omega) \geq 0$, $\forall\ i \in \mathcal{I}$. However, if only Arrow-Debreu securities are traded, market clearing can only be attained when $a_{i,1}(\omega) = 0, \forall\, i \in \mathcal{I}$. Thus, although firms' shares are redundant for investors' risk sharing when Arrow-Debreu claims are complete, they are useful to support the endogenous determination of borrowing limits of Alvarez and Jermann (2000).[7]

Investors solve the following portfolio and consumption problem:

$$\max_{c_{i,0},c_{i,1}(\omega),a_{i,1}(\omega),n^F_{i,j,0}} U(c_i),$$

subject to (1), (2), (3), and the following market clearing conditions in the good and security markets:

$$\begin{aligned}
\sum_{i\in\mathcal{I}} e_{i,0} &= \sum_{i\in\mathcal{I}} c_{i,0}, \\
e_1(\omega) &= \sum_{i\in\mathcal{I}} c_{i,1}(\omega), \forall\omega \in \Omega\,, \\
\sum_{i\in\mathcal{I}} a_{i,1}(\omega) &= 0, \\
\sum_{i\in\mathcal{I}} n^F_{i,j,0} &= 1, \forall j \in \mathcal{J}, \\
\sum_{i\in\mathcal{I}} \overline{n}^F_{i,j,0} &= 1, \forall j \in \mathcal{J}.
\end{aligned}$$

Let $\beta\pi(\omega)\lambda_{i,1}(\omega) \geq 0$, $\forall\omega \in \Omega$, be the Lagrange multipliers on the borrowing constraints (3). The following results hold:

[7]"Not-too-tight" borrowing constraints have been introduced by Kehoe and Levine (1993). Despite their elegance, several studies have documented less appealing properties, such as the indeterminacy of equilibria (e.g., Bloise, Reichlin, and Tirelli, 2013; Bethune, Hu, and Rocheteau, 2018), and the existence of bubbles (Kocherlakota, 2008).

Proposition 1 (Marginal Investor(s) and Asset Prices) *i) The prices of Arrow-Debreu securities and of firms' shares are given by*

$$p_{a,1}(\omega) = \pi(\omega) m_1(\omega), \tag{4}$$

and

$$p^F_{j,0} = E_0[m_1(\omega) d^F_{j,1}(\omega)], \tag{5}$$

where

$$m_1(\omega) \equiv \frac{\beta(u'(c_{i,1}(\omega)) + \lambda_{i,1}(\omega))}{u'(c_{i,0})}, \forall i \in \mathcal{I}. \tag{6}$$

ii) A unique marginal investor does not exist, that is, the stochastic discount factor $m_{i,1}(\omega)$ does not, in general, correspond to any individual investor's marginal rate of substitution. In particular:

$$MRS_{i,1}(\omega) \equiv \frac{\beta u'(c_{i,1}(\omega))}{u'(c_{i,0})} \leq m_1(\omega), \ \forall i \in \mathcal{I}. \tag{7}$$

iii) Unconstrained investors equalize their marginal rates of substitutions, i.e., $\forall l \in \mathcal{I}$ such that $a_{l,1}(\omega) + \sum_{j \in \mathcal{J}} d^F_{j,1}(\omega) n^F_{l,j,0} > B_{l,1}(\omega)$,

$$MRS_{l,1}(\omega) = \max_{i \in \mathcal{I}} MRS_{i,1}(\omega) = m_1(\omega).$$

iv) Prices are "too high for everyone", that is, $\forall i \in \mathcal{I}$

$$p_{a,1}(\omega) \geq \frac{MRS_{i,1}(\omega)}{\pi(\omega)},$$

and

$$p^F_{j,0} \geq E_0[MRS_{i,1}(\omega) d^F_{j,1}(\omega)].$$

Part i) of Proposition 1 establishes that state prices $p_{a,1}(\omega)$ and stock prices $p^F_{j,0}$ can be computed using a stochastic discount factor $m_1(\omega)$, which is equalized across investors who need to agree on market prices. The term $\lambda_{i,1}(\omega)$ in the numerator of $m_1(\omega)$ indicates that payoffs in a given state are more valuable for investors that are more constrained in that state. However, Part ii) shows that, departing from textbook complete-market economies without borrowing constraints, the stochastic discount factor does not generally coincide with the marginal rate of substitution $MRS_{i,1}(\omega)$ of any investor. Part iii) asserts that this is the case only when investors are unconstrained in a certain state. When investors are unconstrained, they engage in risk-sharing and harmonize their marginal rates of substitution. A critical observation is the identification of the relevant pricing kernel for asset pricing as the stochastic discount factor $m_1(\omega)$, which embodies complexity due to its dependence on various factors, including agents' preferences, consumption patterns, and borrowing restrictions. Even in scenarios where an unconstrained investor is present in every state, there remains no assurance that the same investor $l \in \mathcal{I}$ remains unconstrained across all possible states $\omega \in \Omega$. Thus, the notion of a singular "marginal investor" dissolves, as the marginal rates of substitutions from various investors across different states are required to compute asset prices. Part iv) formalizes an asset pricing implication of this property. Since borrowing constraints increase the stochastic discount factor $m_1(\omega)$, asset prices generally attain levels exceeding the marginal valuation $E_0[MRS_{i,1}(\omega)d^F_{j,1}(\omega)]$ of any investor.

Although asset prices are greater or equal than all investors' marginal valuations, the market is free of arbitrage opportunities, as the following proposition establishes.

Proposition 2 (No Arbitrage) *The market is arbitrage free, that is, for any feasible* $i \in \mathcal{I}$, $\nexists$ $\theta_i \equiv (a_{i,1}(\omega), n^F_{i,j,0}) \in \mathcal{R}^{S+J}$ *such that one of the following holds:*

- $p\theta_i \leq 0$, *and* $x\theta_i \geq 0$ *with at least one strict inequality; or*
- $p\theta_i < 0$, *and* $x\theta_i \geq 0$,

where $p \equiv (p_{a,1}(\omega), p^F_{j,0}) \in \mathcal{R}^{S+J}$ *is a price vector, and* x *is a* $S \times (S+J)$ *payoff matrix defined as* $x = [I_S|d^F]$, *where* I_S *is a* $S \times S$ *identity matrix,* d^F *is a* $S \times S$ *matrix with elements* $d^F_{j,1}(\omega)$, *and the operator* $|$ *stands for horizontal matrix concatenation.*

Proposition 2 establishes that investors are unable to devise a trading strategy, denoted as θ_i, involving Arrow-Debreu securities and firm stocks such that a positive payoff is attainable with non-zero probability through a self-financing strategy, or a non-negative future payoff is feasible via a cash inflow originating from short sales.[8] The absence of arbitrage opportunities can be intuitively understood by agents being unable to short sell securities in quantities that are sufficiently large. In a manner analogous to the foundational work by Harrison and Kreps (1979), the absence of arbitrage in this context is congruent with a linear pricing rule. However, the presence of borrowing constraints implies that the stochastic-discount factor $m_1(\omega)$ does not coincide with investors' marginal rate of substitution as in standard consumption-based models.[9]

2.2 Production Side: Firms

I define production entities — henceforth referred to as firms — as units that may represent individual establishments, entire industries, market segments, or sectors. Firms can operate a production technology that delivers a stochastic output in terms of the consumption good. This consumption good, besides fulfilling consumption demands, is used for investment. In each state, realized output in state ω at $t = 1$ is $A_{j,1}(\omega)\dot{}f(k_{j,0})$, where $k_{j,0} \geq 0$ is capital investment at $t = 0$, $A_{j,1} : \Omega \to \mathcal{R}^+$ is a productivity shock, and $f : \mathcal{R}^+ \to \mathcal{R}^+$ is a strictly increasing, strictly concave, differentiable production function, which satisfies the Inada conditions (i.e., $f(0) = 0$, $f'(0) = \infty$, $f(\infty) = \infty$, $f'(\infty) = 0$).

Firms begin period 0 with a wealth endowment $w_{j,0}$ in cash. Firms can obtain external financing through financing contracts with specialized external lenders, to which I refer equivalently as "lenders", "bankers", or "financiers". Lenders are perfectly competitive and have deep pockets. Financing contracts include state-contingent transfers $b_{j,1}(\omega)$ from the borrower to the lender in state ω at $t = 1$, and an upfront transfer $p^B_{j,0}$ at $t = 0$ from the lender to the borrower, which is determined given the promised transfers $b_{j,1}(\omega)$.

Firms maximize their market value, that is the expected discounted value at $t = 0$ of their cash flows to equityholders, taking the stochastic discount factor $m_1(\omega)$ as given.

[8]It is straightforward to extend the investor's problem by allowing trading of additional "complex" securities, which can be valued as portfolios of Arrow-Debreu securities.

[9]For example, Jouini and Kallal (1995), Jouini and Kallal (1999), and Jouini and Kallal (2001) study arbitrage and equilibrium in markets with frictions.

$$V(w_{j,0}) \equiv \max_{k_{j,0}, b_{j,1}(\omega)} d^F_{j,0} + \sum_{\omega \in \Omega} \pi(\omega) m_1(\omega) d^F_{j,1}(\omega),$$

subject to

$$\begin{aligned} d^F_{j,0} &= w_{j,0} - k_{j,0} + p^B_{j,0}, && (8) \\ d^F_{j,1}(\omega) &= A_{j,1}(\omega)\dot{} f(k_{j,0}) - b_{j,1}(\omega), \ \forall \omega \in \Omega, && (9) \\ d^F_{j,0} &\geq 0, && (10) \\ d^F_{j,1}(\omega) &\geq 0, \ \forall \omega \in \Omega. && (11) \end{aligned}$$

Equations (8) and (9) are budget constraints, which equalize sources and uses of funds. Constraints (10) and (11) restrict dividends to be positive and rule out that firms have access to additional funding. Thus, firms are financially constrained. Observe that (10) and (11) can be rearranged as follows

$$\begin{aligned} p^B_{j,0} &\geq k_{j,0} - w_{j,0}, && (12) \\ b_{j,1}(\omega) &\leq A_{j,1}(\omega)\dot{} f(k_{j,0}), \ \forall \omega \in \Omega, && (13) \end{aligned}$$

to underline the borrowing constraints that the firms face. In particular, the promised repayments $b_{j,1}(\omega)$ are limited and fully secured by realized output in each state, while the current funds raised $p^B_{j,0}$ must suffice to cover external financing needs $k_{j,0} - w_{j,0}$. Notice that the constraint $k_{j,0} \geq 0$ is never active because of the properties of the production function.

2.3 Lenders

The initial contractual transfer $p^B_{j,0}$ depends on the structure of the primary lending markets. Lenders have a large endowment of funds in all dates and states, as for example in Dávila and Korinek (2018), in which lenders' borrowing constraints are never binding. For simplicity, and following the literature on collateralized financing (e.g., Rampini and Viswanathan, 2010; Rampini and Viswanathan, 2013), I assume lenders are risk neutral with time discount factor β. Under this assumption:

$$p_{j,0}^{B} = \beta \sum_{\omega \in \Omega} \pi(\omega) b_{j,1}(\omega). \tag{14}$$

The risk neutrality assumption is convenient as it does not require to impose additional structure on the lenders' possible stochastic discount factor. One possible rationalization of risk-neutral financing is competition among lenders with heterogeneous risk aversion.

Suppose a continuum of deep-pocket lenders, indexed by their risk aversion $b \in \mathcal{B} = [0, \infty)$, that compete for offering a financing contract with promised transfers $b_{j,1}(\omega)$ to firm j. Let $u_b : \mathcal{R} \to \mathcal{R}$ be a strictly increasing, weakly concave, and continuously differentiable utility function, with an Arrow-Pratt coefficient of risk aversion increasing in b, and $p_{j,b,0}^{B}$ the initial transfer from lender b to firm j. Profit-maximizing lenders solve

$$\max_{p_{j,b,0}^{B}} u_b \left(e_{b,0} - \sum_{j \in \mathcal{J}} p_{j,b,0}^{B} \right) - \sum_{\omega \in \Omega} \beta \pi(\omega) u_b \left(e_{b,1}(\omega) + \sum_{j \in \mathcal{J}} b_{j,1}(\omega) \right),$$

where $e_{b,0}$ and $e_{b,1}(\omega)$ are large endowments, subject to the following participation constraints:

$$\sum_{\omega \in \Omega} \beta \pi(\omega) u_b(b_{j,1}(\omega)) \geq u_b(p_{j,b,0}^{B}), \ \forall j \in \mathcal{J},$$

which state that the utility of promised repayments exceeds those of the funds provided. Because of competition in credit markets, participation constraints always bind. Any firm j always chooses to arrange external financing contracts from risk-neutral lenders as $p_{j,0,0}^{B} \geq p_{j,b,0}^{B}$, $\forall b \in \mathcal{B} = [0, \infty)$. For risk-neutral lenders, $\sum_{\omega \in \Omega} \beta \pi(\omega) u_0(b_{j,1}(\omega)) = u_0 \left(\beta \sum_{\omega \in \Omega} \pi(\omega) b_{j,1}(\omega) \right)$,which pins down $p_{j,0,0}^{B} = p_{j,0}^{B}$ as in (14) from the participation constraint. For any other risk-averse lender b, as u_b is increasing and weakly concave,

$$u_b(p_{j,b,0}^{B}) = \sum_{\omega \in \Omega} \beta \pi(\omega) u_b(b_{j,1}(\omega)) \leq u_b \left(\sum_{\omega \in \Omega} \beta \pi(\omega) b_{j,1}(\omega) \right)$$

which implies $p_{j,b,0}^{B} \leq \sum_{\omega \in \Omega} \beta \pi(\omega) b_{j,1}(\omega) = p_{j,0,0}^{B}$.[10] Expected utility theory also suggests that

[10] The same conclusions hold with a discrete set of bankers, $\mathcal{B} = \{1, ...B\}$, as long as at least one of them is risk-neutral. This is because lenders compete to give external financing to firms in a first-price auction.

risk neutrality can capture the evidence for which wealthy individuals behave as if they were risk neutral with respect to small risks (Rabin, 2000). In models with large investors, the latter are often modeled as risk neutral, or are endogenously risk neutral when they have deep pockets. Several utility functions, such as log utility, imply wealth-dependent absolute (Arrow-Pratt) risk aversion, which converges to risk neutrality for sufficiently high wealth.

2.4 Production Side: Asset Prices and Corporate Policies

Let $\lambda_{j,0} \geq 0$ and $\pi(\omega)m_1(\omega)\lambda_{j,1}(\omega) \geq 0$ be, respectively, the Lagrange multipliers on firm j's borrowing constraints (10) and (11). In this setup, the following result holds.

Proposition 3 (Asset Prices: SDF Recovery) *The stochastic discount factor $m_1(\omega)$, $\forall \omega \in \Omega$, can be recovered from firm j's optimal policies as*

$$m_1(\omega) = \beta \frac{1 + \lambda_{j,0}}{1 + \lambda_{j,1}(\omega)}. \tag{15}$$

Equation (15) recovers the stochastic discount factor from firm policies, irrespective of the structure of the consumption side of the economy, such as preferences, endowments, and short sale constraints. The multipliers $\lambda_{j,0}$ and $\lambda_{j,1}(\omega)$ are endogenous entities that depend on firm investment and contracting decisions. They reflect the adjustments of firm policies to the value of each state ω, as gauged by $m_1(\omega)$.[11] The economic interpretation of the representation in (15) is the following. Firm maximize their market value by arranging state-contingent financing contracts. These contract trade off dividend distributions today (i.e., thereby raising $\lambda_{j,0}$), against resource transfers to most valuable states (i.e., thereby reducing $\lambda_{j,1}(\omega)$). At the optimum, the tradeoff between today's marginal values $(1 + \lambda_{j,0})$ and those of state ω tomorrow $(1 + \lambda_{j,1}(\omega))$ equates the ratio between the marginal investors' stochastic discount factor $m_1(\omega)$ and lenders' time discount factor β. Therefore, states with high $m_1(\omega)$ can be identified as those to which firms allocate the most resources, relative to their current financial constraints, which are captured by their current marginal value of funds $\lambda_{j,0}$. Given that the solution to firm j's problem is contingent on its wealth, two firms with

[11] Clearly, one can equivalently recover state prices, as Equation (4) shows.

different initial wealth in general implement different policies. In the dynamic model of Section 3, I operationalize the theory by mapping unobservable Lagrange multipliers to an econometrician's information set via the state variables of the firm's problem.

The following corollary establishes that, in the special case where contracts are incomplete, meaning lenders only offer state-uncontingent financing, i.e., $b_{j,1}(\omega) = b_{j,1}$, $\forall \omega \in \Omega$, as in corporate investment and financing models with exogenous contracts (e.g., Hennessy and Whited, 2005; Hennessy and Whited, 2007), the optimal financing choices of firms would not allow the recovery of the stochastic discount factor $m_1(\omega)$ in each state. Instead, they would only impose an ex-ante restriction in expectation.

Corollary 1 (Special Case: No Contracting) *Assume $b_{j,1}(\omega) = b_{j,1}$. Then the stochastic discount factor $m_1(\omega)$, $\forall \omega \in \Omega$ cannot be recovered from firms' optimal policies if $S > 2$. Then, the following holds*

$$E_0 \left[m_1(\omega)(1 + \lambda_{j,1}(\omega)) \right] = \beta(1 + \lambda_{j,0}). \tag{16}$$

Intuitively, firms are unable to redistribute resources across all states tomorrow, given that they only have two independent decision variables at their disposal: capital $k_{j,0}$ and uncontingent financing $b_{j,1}$. Consequently, firms' decision cannot span dividend payouts across all states, unless only two states of nature exist ($S = 2$). This limitation arises because, by adjusting investment and financing decisions within the feasible set, firms influence the joint payoffs in future states, as opposed to the individual ones, as would be the case with state-contingent contracts. The optimality condition (16) equalizes the expected discounted cost of an additional dollar to be repaid in all future states (on the left-hand side) with the marginal benefit of an additional dollar raised today (on the right-hand side).

Finally, I characterize optimal corporate policies.

Proposition 4 (Corporate Policies) *i) The optimal capital stock is*

$$k_{j,0} = f_k^{-1} \left(\frac{1}{\beta \sum_{\omega \in \Omega} \pi(\omega) A_{j,1}(\omega)} \right).$$

ii) Define the most valuable state ω_M as $\omega_M = \max_{\omega \in \Omega} m_1(\omega)$. If $m_1(\omega_M) > \beta$, the firm never pays dividends at $t = 0$, i.e., $d^F_{j,0} = 0$.

iii) Assume $m_1(\omega_M) > \beta$. Firms exhaust their financing capacity at $t = 0$ and at $t = 1$, $\forall \omega \in \Omega \backslash \omega_M$, i.e.

$$\begin{aligned} \lambda_{j,0} &> 0, \\ \lambda_{j,1}(\omega) &> 0, \ \forall \omega \in \Omega \backslash \omega_M, \\ \lambda_{j,1}(\omega_M) &= 0. \end{aligned}$$

In particular

$$\begin{aligned} b_{j,1}(\omega_M) &= \frac{k_{j,0} - w_{j,0} - \sum_{\omega \in \Omega \backslash \omega_M} \pi(\omega) \beta A_{j,1}(\omega) f(k_{j,0})}{\beta \pi(\omega_M)}, \\ b_{j,1}(\omega) &= A_{j,1}(\omega) f(k_{j,0}), \forall \omega \in \Omega \backslash \omega_M, \end{aligned}$$

with

$$\begin{aligned} \lambda_{j,0} &= \frac{m_1(\omega_M) - \beta}{\beta}, \\ \lambda_{j,1}(\omega) &= \frac{m_1(\omega_M) - m_1(\omega)}{m_1(\omega)}, \ \forall \omega \in \Omega \backslash \omega_M, \\ \lambda_{j,1}(\omega_M) &= 0. \end{aligned}$$

Part i) shows that, within this economy, firms are unrestricted in their investment choices, and are always capable of financing $k_{j,0}$, which is increasing in expected discounted productivity. Borrowing constraints do not restrict investment decisions because the state-contingent borrowing limits in (13) bound promised repayments precisely by the output $A_{j,1}(\omega) f(k_{j,0})$. Part ii) reveals that firms forfeit dividend payments at $t = 0$ to reallocate resources to the "most valuable" state ω_M, the one with the highest stochastic factor $m_1(\omega_M)$. This holds true under the mild condition that the highest stochastic discount factor exceeds the time discount rate under risk neutrality, that is $m_1(\omega_M) > \beta$. Part iii) describes the optimal contract and the distribution of state-contingent repayments across future states $\omega \in \Omega$. Firm j pays out as much as possible in the most valuable state ω_M, in compliance with the budget constraint (8), while borrowing constraints are binding in all other states $\Omega \backslash \omega_M$. The multipliers $\lambda_{j,1}(\omega)$ have intuitive expressions. They are equal to the relative importance of the most valuable state compared to other states, measured as the fractional increase in the stochastic discount factor between state ω_M and ω. Similarly, $\lambda_{j,0}$ represents the fractional increase between $m_1(\omega_M)$ and the time preference parameter β. Overall, in this stylized setting, firms follow a stark policy by arranging contracts to transfer as many resources as possible to most valuable state for value maximization.

3 The Dynamic Contracting Model

While the stylized economy in the previous section conveniently conveys the main theoretical insights of this study, its practical applicability is limited. First, firm policies are notably stark, especially as borrowing constraints do not limit corporate investment. Second, it does not provide a clear mapping between the Lagrange multipliers on borrowing constraints and observable variables. Third, for the sake of illustration, it does not fully leverage one of the primary advantages of the contracting approach, which is the ability to abstract from the comprehensive modeling of the consumption side of the economy.

This section develops a discrete-time dynamic limited enforcement model in a neoclassical environment. Firms make investment and financing decisions with an infinite time horizon. Thus, they take into account the expected consequences of current actions for the feasibility of their future plans. Firms can arrange lending contracts with competitive lenders to raise external financing. Lenders have limited ability to enforce credit arrangements that manifests itself in the presence of endogenous collateral constraints.[12] Specifically, lenders restrict credit supply such that, in each period and state, firms' continuation value is greater than what it would be if they did not honor their promises to repay and lenders could liquidate a fraction of their assets pledged as collateral. The state-contingent nature of the contract allows firms to transfer resources to states and times where they are more valuable. This section details the technology and the industry environment,

[12]This paper studies the asset pricing implications of related limited enforcement problems. Related problems are proposed, for example, by Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), Rampini and Viswanathan (2013), and Li, Whited, and Wu (2016). This paper considers the asset pricing implications.

and the financial contracting problem. Section 4 derives the key asset pricing results of this paper that I test empirically in Section 5.

Technology and Competitive Environment. A continuum of perfectly competitive firms operates in an industry. Each firm produces a homogeneous product, whose price is normalized to one. In period t, a fraction ϕ of new firms randomly enters the industry. Existing firms become unproductive and exit with probability ϕ, so that the total mass of operating firms is unchanged over time.

An entrant i arrives with some initial capital stock $k_{i,0}$. Entrants engage in a contract with lenders to obtain external financing. Firms have access to a production technology, which generates a stochastic stream of profits

$$\Pi(k_{i,t}, s_{i,t}) \equiv A(s_{i,t})k_{i,t}^{\alpha},$$

where $k_{i,t}$ is the capital of firm i at time t, $\alpha \in (0, 1)$ is the curvature parameter of the production function, which exhibits decreasing returns to scale, and $A(s_{i,t})$ is a stochastic process describing productivity. The idiosyncratic shock $s_{i,t}$ is the driving force of firm-level heterogeneity, and generates a nontrivial cross section of firms. $s_{i,t}$ follows a Markov process with finite support $\mathcal{S}$ and a stationary discrete transition function $Q_s(s_{i,t+1}|s_{i,t})$.

Equity Value Maximization. Drawing upon the insights in Section 2, I do not impose structure on the consumption side of the economy. Instead, I merely assume the existence of a valid stochastic discount factor of marginal investors for the pricing of firms' equity (formally, the counterpart of $m_1(\omega)$). I posit that firms maximize the expected discounted value of dividends, taking such stochastic discount factor as given.

Assumption: Marginal Investor's SDF and Value Maximization. There exists a stochastic discount factor process $\{M_{i,t}\}_{t=0}^{\infty}$, with $M_{i,t} > 0$. Firm i maximizes $E_t\left[\sum_{\tau=0}^{\infty} M_{i,t+\tau}d_{i,t+\tau}\right]$, where $\{d_{i,t}\}_{t=0}^{\infty}$ are equity payouts.

In the interest of generality, I accommodate the possibility that the stochastic discount factor $M_{i,t}$ is specific to firm i. This caters to potentially complex market structures on the consumption side of the economy, particularly certain forms of market incompleteness. It is well known that in incomplete markets different investors have different stochastic discount factors and therefore disagree on the solution to the optimal decision problem of the firm.[13] Complete markets are nested as a special case in which $M_{i,t} = M_t$, for all firms in the economy. As firms take the stochastic discount factor as given, the optimality conditions I derive and empirically test are the same that would arise in general equilibrium.[14]

Financing and Contracting. Upon arriving in the industry, firms can enter a contractual relationship with a representative lender. The contract not only provides initial funding, but also financing over the firm's life cycle. As discussed in Section 2, I assume the lender is risk neutral and has deep pockets. In this setting, both parties have motives to arrange contracts to implement, albeit imperfect, risk sharing.

Lenders are instead exposed to time-varying borrowing costs because of, for example, aggregate credit risk, and lend at the gross interest rate R_t, which is taken ask given. I impose that the stochastic discount factor is consistent with the market riskfree rate, that is $R_t = E_t[M_{i,t+1}]$.

The timing of events over a firm's life cycle is as follows. As soon as a firm enters the industry, it signs a contract with the lender to obtain initial funding. Then, at the beginning of each period, the firm first faces the exogenous exit shock, and the state $s_{i,t}$ realizes and the market rate R_t is observed. There are no information asymmetries because $s_{i,t}$ is publicly known. Second, firm's decisions and operations occur: inputs are purchased, production takes place, revenues are collected, transfers to and from lenders are made, and dividends are distributed. Third, the firm chooses either to renege the contract or to continue operations. This limited enforcement problem is discussed in more detail below. Figure 1 summarizes the intra-period timing.[15]

[Insert Figure 1 Here]

In this setup, the financing contract includes transfers $\{\tau_{i,t}\}_{t=0}^{\infty}$ which, as I detail below, can be expressed recursively as time-t decisions over transfers $R_t b(s_{i,t+1}, R_{t+1})$ contingent to each state at

[13] See, for example, Ekern and Wilson (1974) (Chapter 10), Radner (1974) (Chapter 11) Magill and Quinzii (2002) (Chapter 6), and Gomes and Michaelides (2007).

[14] Given that the model is derived at the firm level, it does not necessarily bear implications for the behavior of aggregate asset prices and consumption. However, it is immune to the aggregation critiques in Deaton (1992) and Kirman (1992) for its predictions about cross-sectional differences in returns. For a related discussion, see Cochrane (1996) (p. 618).

[15] In this setup, the contract has one side commitment. While there is a limited commitment problem on the firm's side, the lender honors the long-term contract. This feature becomes apparent in the recursive formulation in Appendix A.3, where the lender's promise-keeping constraint is part of the problem.

time $t+1$. Given this set of promised repayments, the firm receives $E_t[b(s_{i,t+1}, R_{t+1})]$ upfront at time t (the counterpart of $p^B_{j,0}$ in Section 2).

In this work I remain agnostic about the implementation of the optimal contract. Existing studies show that state-contingent debt contracts can be implemented in practice with common financial instruments such as credit lines (Nikolov, Schmid, and Steri, 2019), financial derivatives (Rampini and Viswanathan, 2013) and portfolios of bonds with different maturities (Angeletos, 2002). External equity issuance might also be part of the implementation, despite corporations do not often engage in seasoned equity offering and often do so as a result of the exercise of employees stock options (e.g., McKeon, 2015).

One interpretation of lenders in the model are bankers, who cannot purchase (substantial) shares of the firms on the secondary market. The U.S. legislation prevents banks from holding firms' equity (Eckbo, 2007, Chapter 5), as commercial banks are restricted to hold equity only through venture capital subsidiaries and through the restructuring of distressed loans.[16]

Recursive Formulation. Closely following Abreu, Pearce, and Stacchetti (1990), the contracting problem can be formulated recursively using net worth $w_{i,t}$ as a state variable, as detailed in Appendix A.3. Realized net worth in a future state determined by $s_{i,t+1}$ and R_{t+1} determines the amount of resources that are available to the firm in a certain state, net of liabilities. Intuitively, net worth is the corporate counterpart of household's wealth, and captures how constrained a company is in terms of resources to allocate to investment, and distributions. The recursive contracting problem is the following:

$$V(w_{i,t}, s_{i,t}, R_t) = \max_{\{d_{i,t}, k_{i,t+1}, b(s_{i,t+1}, R_{t+1}), w(s_{i,t+1}, R_{t+1})\}} d_{i,t} + E_t\left[M_{i,t+1} V(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \quad (17)$$

$$s.t.$$

$$\theta k_{i,t+1} \leq E_t\left[M_{i,t+1} V(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \quad (18)$$

$$d_{i,t} \geq 0 \quad (19)$$

$$w_{i,t} \geq d_{i,t} + k_{i,t+1} - E_t[b(s_{i,t+1}, R_{t+1})] \quad (20)$$

$$w(s_{i,t+1}, R_{t+1}) \leq \Pi(k_{i,t+1}, s_{i,t+1}) + (1-\delta)k_{i,t+1} - R_t b(s_{i,t+1}, R_{t+1}) \qquad \forall s_{i,t+1}, \forall R_{t+1}. \quad (21)$$

[16] In other countries, such as in Germany and Japan, banks are instead allowed to undertake equity investments. The regulatory debate about whether U.S. commercial banks should be allowed to expand their ability to hold equity falls beyond the scope of this work.

The collateral constraint (18) arises from the limited enforcement problem. The lender anticipates that the borrower has limited commitment and can renege the contract. Thus, the lender is willing to lend until the amount $\theta k_{i,t+1}$ the borrower can divert does not exceed the borrower's continuation value $E_t\left[M_{i,t+1}V(w(s_{i,t+1},R_{t+1}),s_{i,t+1},R_{t+1})\right]$. This imposes an enforcement constraint on the firm's side, and makes reneging the contract never optimal. Funding is therefore riskfree, and firms raise an amount $E_t[b(s_{i,t+1},R_{t+1})]$ at time t backed up by state-contingent promises to repay $b(s_{i,t+1},R_{t+1})$ at time $t+1$. Constraint (19) imposes that equity is in fixed supply by restricting dividend payments $d_{i,t}$ to be non-negative. Thus, the firm cannot raise additional funds by issuing costless external equity (i.e., negative dividends). Without this constraint, firms would be financially unconstrained and the contracting problem would be trivial. Constraints (20) and (21) are the budget constraints at time t and in all states $s_{i,t+1}$ at time $t+1$. Specifically, equation (20) at time t ensures that available funds, namely net worth and raised funds, suffice to cover dividend and capital expenditures. Equations (21) define net worth in all future states as the sum of cash flows and the depreciated value of capital, minus the state-contingent amount to be repaid in that state.[17]

The recursive formulation in terms of net worth not only improves the computational efficiency of the numerical solution because of the smaller state-space, but is also convenient to introduce the hedging interpretation of the model. The firm has a limited borrowing capacity because of the collateral constraint. In this formulation, the firm has the possibility to choose state-contingent promised utility (contractual repayments) $b(s_{i,t+1},R_{t+1})$ for each state. The firm can therefore choose to hedge a specific state s at time $t+1$ by choosing a lower repayment $b(s_{i,t+1},R_{t+1})$. All else equal, hedging a state has three effects. First, the firm saves borrowing capacity by relaxing the enforcement constraint. Second, as Equation (21) shows, the firm increases its net worth in state s at time $t+1$, by lowering its required repayment. As a result, more resources are available for investments and distributions in state s. Third, as Equation (20) illustrates, a lower repayment in some future state implies a lower amount of external funds raised at time t, and less net worth available for today's investment and distributions. In sum, the firm can effectively hedge a state by transferring net worth from today to specific future states tomorrow. Because the firm's external financing capacity is limited by the borrowing constraint, the company faces a tradeoff between raising funds today, and preserving them for specific states that may occur tomorrow.

[17] In the equilibrium contract, the budget constraints are always satisfied with equality.

4 An Asset Pricing Model

This section introduces the key asset pricing results of this paper. Specifically, I recover firms' stochastic discount factors in terms of firm's policies and characteristics. This leads to an asset pricing model, which I refer to as the "Contracting Asset Pricing Model" or simply as the "Contracting Model". Before stating the main result, the following proposition shows that the firm problem has a well-defined solution and establishes key properties of the value function that stem from the main economic mechanism of the model.

Proposition 5 (Properties of the Value Function) *Let T be the Bellman operator associated with the problem (17)-(18), $V^{UB}(w_{i,t}, s_{i,t}, R_t)$ the solution of the same problem without Constraint (18), and $V^{LB}(w_{i,t}, s_{i,t}, R_t)$ a function over the same domain of $V(w_{i,t}, s_{i,t}, R_t)$ such that $V^{LB}(w_{i,t}, s_{i,t}, R_t) \leq V(w_{i,t}, s_{i,t}, R_t)$. Assume the tranversality conditions*

$$\limsup_{n\uparrow\infty} E_t\left[M_{i,t+n}V^{UB}(w(s_{i,t+n}, R_{t+n}), s_{i,t+n}, R_{t+n})\right] = 0, \tag{22}$$

and

$$\liminf_{n\uparrow\infty} E_t\left[M_{i,t+n}V^{LB}(w(s_{i,t+n}, R_{t+n}), s_{i,t+n}, R_{t+n})\right] = 0. \tag{23}$$

Then

i) the value function is the unique fixed point of T in the order interval $[V^{LB}(w_{i,t}, s_{i,t}, R_t), V^{UB}(w_{i,t}, s_{i,t}, R_t)]$,

ii) the sequence of functions $\{T^n V^{LB}(w_{i,t}, s_{i,t}, R_t)\}_{n=1}^{\infty}$ converges to $V(w_{i,t}, s_{i,t}, R_t)$ pointwise,

iii) the value function is increasing, weakly concave and differentiable in net worth $w_{i,t}$,

iv) the value function is weakly increasing in $s_{i,t}$ and weakly decreasing in R_t.

The first and second part of the proposition provide a procedure to solve for the equilibrium contract. Since in the contracting problem the objective function itself appears on the right-hand side of the collateral constraint, the dynamic programming problem in (17)-(18) is not a standard convex optimization problem. In particular, verifying the discounting property of Blackwell's sufficient conditions would require the knowledge of the solution to be determined. The solution of the functional equation may therefore not be unique. However, a different approach based on Knaster-Tarski fixed-point theorem allows to establish two results. First, the value function is the unique fixed point of the Bellman operator in a restricted functional space. The lower boundary of this functional space is the zero function, while the upper boundary is the solution to a planner's problem in which the enforcement constraint is removed. Second, the sequence of functions obtained by iterating the Bellman operator from the lower bound converges pointwise to such a fixed point. Theoretically, the solution can be obtained by value function iteration from the any initial condition $V^{LB}(w_{i,t}, s_{i,t}, R_t) \leq V(w_{i,t}, s_{i,t}, R_t)$, such as the null function, as long as the transversality conditions are satisfied.[18] The third part states that the more resources the firm has available, the higher its market value, but the latter increases disproportionately less with the level of net worth. Thus, all else equal, firms that arrange financing contracts to transfer net worth to a specific future state increases the market value if that state is realized, but the marginal value of transferred net worth is decreasing. Finally, the fourth part shows that, due to the positive persistence in the Markov processes driving profitability $s_{i,t}$ and interest rates R_t (that is typically observed empirically), the value of equity is nondecreasing in $s_{i,t}$ and nonincreasing in R_t. Intuitively, high profitability states are expected to trigger good future investment prospects, while high interest rates are likely to be associated to high future borrowing costs as well.

The next proposition obtains a stochastic discount factor from firms' policies.

Proposition 6 (The Contracting Asset Pricing Model) *The stochastic discount factor firm i responds to can be backed out from the firm's optimality conditions as follows:*

$$M_{i,t+1} = \frac{1}{R_t(1+\lambda_{i,t})} \frac{1}{g^v_{i,t+1}}, \tag{24}$$

where $\lambda_{i,t}$ is the Lagrange multiplier on the collateral constraint (18) and

$$g^v_{i,t+1} \equiv \frac{V_w(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})}{V_w(w_{i,t}, s_{i,t}, R_t)} \tag{25}$$

denotes the growth of the marginal value of net worth.

Equation (24) is the counterpart of Equation (15) in the two-period example of Section 2. Its derivation is straightforward from the first-order condition with respect to the state-contingent

[18] Observe that the first-best solution $V^{UB}(w_{i,t}, s_{i,t}, R_t)$ is bounded, in that the dynamic programming problem without the collateral constraint is a standard convex Bellman problem.

repayments. This result reflects the key intuition of this study that the possibility to negotiate state-contingent repayments with the lenders allows firms to transfer resources across states. Firms have a rationale for raising external financing because of the collateral constraint, and have a motive to arrange state-contingent contracts to transfer net worth to the most important states, where the stochastic discount factor is high. In the absence of state-contingent financing, the stochastic discount factor cannot be recovered, and the first-order condition with respect to state-uncontingent financing would not deliver a stochastic discount factor for each state, but only one pricing equation containing an expectation over all future states, along the lines of (16) in Corollary 1.

Specifically, the stochastic discount factor relates to firm i's policy through the Lagrange multiplier $\lambda_{i,t}$ on the borrowing constraint, and the growth rate of the marginal value of net worth $g^v_{i,t+1}$. The right-hand side of Equation (24) illustrates how the optimal decisions of heterogeneous firms respond to the state preferences of shareholders to maximize the value for their equity holdings. Backing out the stochastic discount factor therefore amounts to investigate which state must have led a firm to optimally respond through its observed investment and financing decisions. In the absence of a state-contingent financing contract, realized net worth in individual future states could not instead be actively transferred by the firm through its decisions, but would vary across states only because of exogenous shocks. Observed firm policies would not therefore be informative of the stochastic discount factor.

The economic mechanism driving the result in Equation (24) can be interpreted in light of firms' optimal contracting. Firms have a motive to engage in financial contracting to transfer resources (net worth) to states that are most important to maximize their market value. This policy would increase net worth and lower its marginal value $g^v_{i,t+1}$ in those states. Importantly, collateral constraints limit firms' ability to achieve this goal, as the term $\frac{1}{1+\lambda_{i,t}}$ accounts for. Thus, Equation (24) does *not* imply that, empirically, one should expect to observe net worth growing in states in which the stochastic discount factor is presumably high, such as bad times. The more financially constrained firms are, the higher the shadow value $\lambda_{i,t}$ of extra borrowing, the less their effective ability to transfer resources to most important states, in spite of their motives. This is consistent with the model of Rampini and Viswanathan (2013) and the evidence in Li, Whited, and Wu (2016) and Nikolov, Schmid, and Steri (2019), according to which financially constrained firms engage less in risk management because their immediate financing needs override their hedging concerns.

It is important to notice that *all* the state variables of the problem determine firm policies, and in turn affect firms' contracting behavior. From an empirical standpoint, this result implies that firm characteristics enter the stochastic discount factor directly. This mechanism is analogous to how, on the consumption side of the economy, the state variables of the representative household's problem enter the stochastic discount factor in the intertemporal CAPM of Merton (1973).

The following proposition offers an equivalent expected return/beta representation of the Contracting Model. This formulation shows how individual securities can be priced using a stochastic discount factor derived from firms' optimal policies.

Proposition 7 (Expected Return-Beta Representation) *The expected excess return*

$$E_t[R^e_{i,t+1}] \equiv \frac{V(w_{i,t+1}, s_{i,t+1}, R_t)}{V(w_{i,t}, s_{i,t}, R_t) - d_{i,t}} - R_t \tag{26}$$

on the traded equity claims of firm i is given by the following expression:

$$E_t\left[R^e_{i,t+1}\right] = \beta_{i,t}\lambda_{i,t}, \tag{27}$$

where $\beta_{i,t} \equiv \frac{Cov_t\left[\left(g^v_{i,t+1}\right)^{-1}, R^e_{i,t+1}\right]}{Var_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}$ *where* $\lambda_{i,t} \equiv -\frac{Var_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}{E_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}$.

Proposition 7 establishes that the expected excess equity return of a generic firm i are driven by the covariance of its realized returns $R^e_{i,t+1}$ with the reciprocal of the growth of the marginal value of net worth $g^v_{i,t+1}$ which, as Proposition 6 shows, in turn depend of firm i's state variables. As standard in the asset pricing literature, $\beta_{i,t}$ can be interpreted as a quantity of risk, and $\lambda_{i,t}$ as a price of risk, with a key difference: $\lambda_{i,t}$ does not appear in Equation (27) an aggregate quantity, but rather as a firm-specific quantity. The reason is the quintessence of the Contracting Model, namely that Proposition 6 backs out a stochastic discount factor from individual firms' contracting behavior. Accordingly, the pricing of the equity claims of firm i reflects the price of risk $\lambda_{i,t}$ embedded in its marginal value of net worth, and the covariance of the firm's payoffs $R^e_{i,t+1}$ with such a marginal value, as $\beta_{i,t}$ captures. Shares of firms that are worth less in most important states, which can in turn be identified as those to which firms, coping with financial constraints, actively transfer net worth and lower its marginal value, are risky investments and require higher expected returns. Vice versa, shares of firms that provide high rewards in important states work as a hedge, are more expensive and hence require lower expected returns.

Importantly, the results in Proposition 6 and Proposition 7 *neither* require shareholders to be underdiversified *nor* imply that the Contracting Model is more suitable to price stocks of either small or private firms. Similarly, as discussed in Section 3, although the model is purely derived at the firm level and is not suited to derive direct aggregate implications, it is *not* inconsistent with the existence of aggregate risk factors that price the equity of large public firms, such as reduced-form factor models, or with the consumption-based paradigm. On the contrary, in Section 5.5, I empirically relate the Contracting Model to reduced-form multifactor models, and I document its consistency with the aggregate investment and profitability factors that both Hou, Xue, and Zhang (2015) and Fama and French (2016) show to be essential for pricing the shares of large firms.

A remark is in order about the link between the results in this section and the large literature on anomalies in the cross section of returns. Equation (27) highlights that all determinants of the growth of the marginal value of net worth $g^v_{i,t+1}$ are supposed to drive differences in cross-sectional expected equity returns. As Section 5 highlights, the model predicts that the dynamics of its state variables, which summarize all the information the firm needs at each point in time to determine its optimal policy, determine the dynamics of $g^v_{i,t+1}$. As a consequence, the pricing equation (27) suggests an interpretation of anomaly variables as possibly omitted determinants of the growth of the marginal value of net worth that are not captured by differences across firms in loadings on macroeconomic factors in benchmark asset pricing models, such as the CAPM and the Consumption CAPM.

Finally, the following corollary shows that Proposition 7 implies an approximation in which expected returns are proportional to "net worth betas" $\beta^w_{i,t}$, defined using covariances with the marginal value of net worth $g^v_{i,t+1}$ instead of its reciprocal.

Corollary 2 ("Net Worth Betas") *The expected excess return $E_t[R^e_{i,t+1}]$ on the equity of firm i can be approximated as*

$$E_t\left[R^e_{i,t+1}\right] \simeq \beta^w_{i,t}\lambda^w_{i,t},$$

where $\beta^w_{i,t} \equiv \frac{Cov_t\left[g^v_{i,t+1}-1, R^e_{i,t+1}\right]}{Var_t\left[g^v_{i,t+1}-1\right]}$ *and* $\lambda^w_{i,t} \equiv Var_t\left[g^v_{i,t+1}-1\right]$.

The difference between the expressions for expected excess returns in Proposition 7 and Corollary 2 is that the former identifies more risky stocks as those whose payoffs co-vary negatively with the stochastic discount factor, and accordingly with $\left(g^v_{i,t+1}\right)^{-1}$, while the latter shows that shares of firms whose payoffs co-vary positively with the growth rate $g^v_{i,t+1}-1$ of the marginal value of net worth have higher net worth betas and require higher expected returns. In Proposition 7, the minus sign in $\lambda_{i,t}$ implies that stocks with high $\beta_{i,t}$ are valuable in most important states and are less risky. In Corollary 2, instead, the compensation for risk is proportional to net worth betas. This distinction is also present in the CAPM within the consumption-based framework, in which expected return-beta representations can be derived both considering covariances with the stochastic discount factor and, approximately, with the market portfolio.[19] As in the case of the CAPM, with the exception of some special cases, such as normally distributed returns and quadratic utility, the formulation that associates higher betas to higher risks holds only approximately, due to the non-linearity of $M_{i,t+1}$ in $g^v_{i,t+1}$.

5 Empirical Asset Pricing Tests

In this section, I test the implications of the Contracting Model using U.S. data from 1965 to 2013. I present four sets of empirical results, which relate to (i) empirical tests of the Contracting Model by GMM, (ii) the existence of spreads in average returns associated with the model

[19]Ferson and Jagannathan (1996) show that a linear stochastic discount factor $m_{t+1} = \sum_{k=1}^{K} c^{(k)} f^{(k)}_{t+1}$ is equivalent to a "beta pricing" representation where $E[R_{t+1}] = \sum_{k=1}^{K} \beta^{(k)}\delta^{(k)}$, where $\beta^{(k)} = \frac{Cov(f^{(k)}_{t+1}, R_{t+1})}{Var(f^{(k)}_{t+1})}$, $c^{(0)} = \frac{1}{\delta^{(0)}}\left(1+\sum_{k=1}^{K}\delta^{(k)}\frac{E_t[f^{(k)}_{t+1}]}{Var_t(f^{(k)}_{t+1})}\right)$ and $c^{(k)} = -\frac{1}{\delta^{(0)}}\frac{\delta^{(k)}}{Var_t(f^{(k)}_{t+1})}$. In particular, the stochastic discount factor corresponding to the CAPM is $M_{t+1} = a_t - b_t R^W_{t+1}$, where a_t and b_t are predetermined quantities at time t and R^W_{t+1} is the gross return on the market portfolio. See, for example, Cochrane (2001), Chapter 6.

predictions, (iii) a comparison of the pricing performance of the Contracting Model and standard asset pricing models, and (iv) to the relationship between the Contracting Model and the recent multifactor models of Hou, Xue, and Zhang (2015) and Fama and French (2016).

5.1 Empirical Predictions

Proposition 6 recovers the stochastic discount factor from the growth of the marginal value of net worth $g^v_{i,t+1}$ arising from firms optimal policies. Although $g^v_{i,t+1}$ is inherently unobservable, the model structure imposes restrictions on its dependence on the dynamics of the state variables. These restrictions carry predictions for both coefficient estimates in GMM tests of the Contracting Model and for observed spreads in average returns in the cross section. The following proposition establishes that the growth of the marginal value of net worth is (weakly) decreasing in net worth growth for firm i. To simplify notation, I use the shorthald $w_{i,t+1}$ to denote $w(s_{i,t+1}, R_{t+1})$.

Proposition 8 (Net Worth Growth and its Marginal Value) *The growth of the marginal value of net worth $g^v_{i,t+1}$ is weakly decreasing in net worth growth $\frac{w_{i,t+1}}{w_{i,t}}$, for all $w_{i,t}$, $s_{i,t}$, $s_{i,t+1}$, R_{t+1}.*

This result stems from the concavity of the value function, and reflects the fact that large transfer of net worth to specific states are disproportionally less valuable than small transfers. As the stochastic discount factor in Proposition 6 predicts that most important states for equity prices of any firm i are inversely related to $g^v_{i,t+1}$ given the severity of its financial constraints $\lambda_{i,t}$, net worth growth is expected to load positively on the stochastic discount factor. I test this prediction with the GMM tests of Section 5.2.

Unlike net worth growth, the incidence of the growth rates of profitability $\frac{s_{i,t+1}}{s_{i,t}}$ and of interest rates $\frac{R_{t+1}}{R_t}$ on the stochastic discount factor depends on parametrizations, and in particular on the persistence of profitability shocks and interest rates. As in the production economy with technology shocks of Rampini and Viswanathan (2013), the positive persistence in $s_{i,t}$ and R_t typically observed in practice has two contrasting effects. First, when productivity is high (low), persistent shocks decrease (increase) the marginal value of net worth because the expected profitability and net worth going forward are high (low) too. Vice versa, persistence in interest rates raises (lowers) the borrowing cost for the firm and its marginal value of net worth when interest rates are high (low). Second, high (low) current profitability increases (decreases) investment needs because of the higher (lower) expected conditional profitability of investment, therefore increasing (decreasing) the marginal value of net worth. Similarly, high (low) interest rates increase (decrease) the conditional expected cost of borrowing, and contribute to reduce (raise) the amount of debt to be otherwise repaid, thus increasing (decreasing) preserved future net worth and reducing (increasing) its marginal value. As a consequence, for both profitability growth and interest rate growth, two contrasting forces determine the ultimate dominating effect.

Although the model structure does not restrict the dependence of the stochastic discount factor on $\frac{s_{i,t+1}}{s_{i,t}}$ and $\frac{R_{t+1}}{R_t}$, it offers predictions for their consistency with observed spreads in expected returns, as well as for the cross-sectional spread in returns associated with net worth growth $\frac{w_{i,t+1}}{w_{i,t}}$.

Proposition 9 (Realized Equity Returns) *i) Realized equity excess returns* $R^e_{i,t+1}$ *are weakly increasing in observed net worth growth* $\frac{w_{i,t+1}}{w_{i,t}}$*, ii) realized equity excess returns* $R^e_{i,t+1}$ *are weakly increasing in observed profitability growth* $\frac{s_{i,t+1}}{s_{i,t}}$*, iii) realized equity excess returns* $R^e_{i,t+1}$ *are weakly decreasing in observed interest rate growth* $\frac{R_{t+1}}{R_t}$*, for all* $w_{i,t}$*,* $s_{i,t}$*,* R_t*.*

The first part of the proposition implies a negative relationship between average returns and net worth growth in the cross section. Intuitively, shares of firm i earn higher realized excess returns $R^e_{i,t+1}$ in states in which its worth grows. Since the most important states can be backed out from firm i's policy as those in which firm i's net worth increases the most, shares of firms whose net worth grows are a hedge in that they pay out more in most important states, in which the stochastic discount factor is high. Thus, according to Corollary 2, high net worth growth stocks have low net worth betas provide a hedge in more important states and require lower expected returns. Observe that this argument is alike to the one in textbook consumption-based models, in which securities that pay out in most important states are a hedge against sustained downturns and earn higher expected returns. The only difference is that most important states are not directly linked to aggregate consumption risk, but are now backed out from the optimal response of individual firms, as Proposition 6 shows. Also notice that, even in the case of complete markets, firms in general respond differently to the same aggregate state because they are heterogeneous, for example with respect to size or financial constraints.

The intuition behind the second and the third part of Proposition 9 is similar. Expected returns are higher for those that pay out more in important states, in which the growth of the marginal value of net worth $g^v_{i,t+1}$ is lower. The second part of the proposition implies that if $g^v_{i,t+1}$ decreases with profitability growth $\frac{s_{i,t+1}}{s_{i,t}}$, shares of firms whose profits grow provide higher payoffs in less important states (low $g^v_{i,t+1}$), have high net worth betas, and earn higher expected returns. The third part implies that if $g^v_{i,t+1}$ decreases with interest rate growth, that is most important times are associated with interest rate spikes, interest rate reductions are associated with lower stock prices, high net worth betas, and higher expected equity returns. In summary, the Contracting Model imposes restrictions on the observed spreads in average returns on the basis in accordance to the states that, in the presence of borrowing constraints, are most important for individual firms. These predictions are summarized in Table 1.

5.2 GMM Tests of the Contracting Asset Pricing Model

Because the contracting model of Section 3 allows the stochastic discount factor to be firm specific, Proposition 6 provides a valid stochastic factor for the equity of an individual firm i. In this subsection, I implement GMM tests of the Contracting Model using 80 portfolios as test assets. Portfolio-level tests not only allow to assess the pricing performance of the model on several test assets, but also provide estimates of the loadings of net worth growth, profitability growth, and interest rate growth on the stochastic discount factor in Equation (24).

I examine the implications of the model for cross-sectional expected gross returns[20]. Equation (27) restricts the pricing errors for the gross return on equity of firm i to satisfy

$$E_t\left[M_{i,t+1}(1+R^e_{i,t+1})\right] = 1. \tag{28}$$

[20]It is well known that when excess returns are used to estimate stochastic discount factor models, the mean of the stochastic discount factor is not identified. Recent studies, such as Burnside (2015), find that GMM estimates are typically sensitive to the normalization of the mean of the stochastic discount factor due to weak identification. This shortcoming does not instead afflict GMM tests on gross returns.

In other words, the stochastic discount factor $M_{i,t+1}$ for the equity of an individual firm i is valid to price the equity of firm i because it is backed out from its optimal policy. The pricing error on a portfolio of securities is therefore the (weighted) average of the pricing error of its individual securities. Although, in theory, (28) could be applied to individual securities, this empirical strategy is not viable. First, accounting data to construct net worth growth and profitability growth for individual firms are only available for relatively short time series and with a limited frequency in Compustat. Second, the size of the GMM system with individual securities would be impractical to estimate.

The empirical GMM tests in this section are based on yearly data from 1965 to 2013. Because the contracting model does not provide a closed-form solution for the loadings of net worth growth, profitability growth, and interest rate growth, I adopt a conservative estimation strategy and consider a linear approximation of $M_{i,t+1}$ in which I restrict such loadings to be constant.[21] This choice avoids making specific assumptions on the functional forms that would necessarily increase the number of parameters to identify, and provides the model with less degrees of freedom for fitting observed portfolio returns. GMM estimation with constant loadings uses the test assets to capture average effects in the sample and to confront them with the model sign restrictions. Estimation is by two-step GMM, with the initial weighting matrix attaching equal weights to all assets. The Appendix provides additional details on the estimation procedure.

Table 2 presents the estimation results. Coefficient estimates for the three variables and the corresponding test statistics based on HAC standard errors are reported in Panel A. The table also reports the following goodness-of-fit measures based on first-stage inference: the mean absolute pricing error (MAE), and the cross-sectional R^2 of a regression of realized average excess returns on predicted average excess returns, computed as in Campbell and Vuolteenaho (2004). As a formal test of model mis-specification I report results from the J-test of overidentifying restrictions (Hansen and Singleton (1982))[22]. The results in Table 2 suggest that the Contracting Model finds support in the data.

[21] This estimation choice is conservative because the loadings on net worth, profits, and interest rate growth rates are allowed to be different across firms, offering several additional degrees of freedom.

[22] Although several studies, such as Ferson and Foerster (1994), Ahn and Gadarowski (2004), and Lewellen, Nagel, and Shanken (2010), document the statistical power of this test is low in the context of asset pricing tests, and their small-sample properties vary to a great extent with the sample size and the test assets, I report the results for comparability with previous studies.

The first two rows of the table report GMM estimates of the coefficients on net worth growth, profitability growth, and interest rate growth. The estimate for net worth growth is in line with the predicted sign from the model, with a loading of 0.12. The loadings of -0.16, and 0.26 on profitability growth and interest rate growth, respectively, indicate that most important states are those in which profitability lowers and borrowing costs raise. All coefficients are statistically significant ar the one percent level, with test statistics of 5.61, -9.62, and 3.04. The estimates imply that most critical states to maximize firm value are those in which firms try, in the presence of financial constraints, to transfer more net worth, those in which firms' profits decline, and those in which borrowing is more costly. Proposition 9 implies that, in accordance with these estimates, one should observe stocks of firms with low net worth growth and with high profitability growth require higher expected returns. I examine these predictions in the next section.

Panel B indicates that the Contracting Model appears to capture a substantial part of the variation in expected returns across the test assets. Mean absolute pricing errors on the tests assets are equal to 1.16% per annum, ranging from 0.78% to 1.67% per annum. Cross-sectional R^2 are also high, ranging from 0.59 for the industry portfolios to 0.81 for the net worth and profitability growth portfolios[23]. Finally, although the results of the formal test of overidentifying restrictions should be interpreted with extreme caution, the test based on the J statistic cannot statistically reject the model.

[Insert Table 2 Here]

Figure 2 provides a visual summary of the performance of the model. Panels A through D report predicted versus realized average returns for the four sets of test assets. If priced correctly, the portfolio should lie along the 45-degree line. The figure confirms that, empirically, the Contracting Model goes a long way in pricing the test assets.

[Insert Figure 2 Here]

[23]Remarkably, the model is rather successful in pricing the Fama-French 30 Industry portfolios. In fact, as Lewellen, Nagel, and Shanken (2010) document, these test assets represent a challenge for all leading asset pricing models.

5.3 The Growth Rates of Net Worth and Profitability

Table 3 considers univariate portfolio sorts involving the growth rates of net worth and profitability. The table reports average annualized returns, heteroscedasticity and autocorrelation consistent t statistics, and Sharpe ratios of 10 equally weighted and 10 value-weighted portfolios sorted on each of the two variables, and the spread between the lowest and the highest decile portfolios (L-H).

Consistent with the predictions of Proposition 6, the panel labeled "Net Worth Growth" shows that average returns decline from low-net-worth-growth to high-net-worth-growth portfolios, with an average spread of 12.29% for the equally-weighted portfolios (Sharpe ratio = 1.03) and of 3.87% for the value-weighted portfolios (Sharpe ratio = 0.28). The panel labeled "Profitability Growth" instead discloses a spread associated with the growth rate of profitability, consistent with the predictions of Proposition 6 and with the empirical evidence of the previous section. In particular, the spread in average returns between the high and the low profitability growth portfolios is 4.60% (Sharpe ratio = 0.82) when equally-weighted portfolios are considered, and 3.83% (Sharpe ratio = 0.34) when value-weighted portfolios are considered.

[Insert Table 3 Here]

Table 4 reports estimates from Fama and MacBeth (1973) cross-sectional regressions of monthly returns on the growth rates of net worth and profitability. Cross-sectional regressions are commonly used to draw inferences about the differential effect of multiple candidate determinants of average returns, and estimated regression slopes provide direct statistical estimates of these marginal effects. The first two columns of Table 4 confirm the negative spread in returns associated with the growth rate of net worth, and the premium related to the growth rate of profitability. The slope of the growth rate of net worth is -0.93, roughly 7 standard errors from zero, while the slope of the growth rate of profitability is 0.22, with a t statistic of 4.54. The third column considers the joint effect of the two variables. The estimated slopes have a similar magnitude of those in Columns 1 and 2. Indeed, the correlation between the two variables is relatively low. The fourth column adds the logarithm of market capitalization, the logarithm of book-to-market equity, and momentum as control variables. The results replicate the well-known size and value premia, and the return spread related to momentum. The latter regression specification emphasizes the economic importance of the premia related to the growth rates of net worth and profitability. In fact both variables exhibit a strong marginal explanatory power that is reflected in large average slopes and t-statistics. In particular, the t statistic of profitability growth is roughly comparable to the one of the value premium, while the t statistic of net worth growth is almost twice larger than them.

Finally, the rightmost column of Table 4 includes two additional variables, namely investment and profitability, that the empirical multifactor models of Hou, Xue, and Zhang (2015) and Fama and French (2016) identify as key determinants of cross-sectional equity returns. Perhaps not surprisingly, these two variables are correlated to net worth growth and profitability growth, although they do not appear to completely subsume them. I further investigate the relationship between the Contracting Model and the aggregate investment and profitability factors in Section 5.5.

[Insert Table 4 Here]

Overall, the results of the sorts in Table 3 and of the regressions is Table 4 consistently document that, as the Contracting Model predicts, the growth rate of net worth and the growth rate of profitability are, respectively, negatively and positively related to returns. For both variables, the marginal effects appear to be both economically sizeable and strongly statistically significant.

5.4 Comparison Among Models

Table 5 compares the pricing performance of the Contracting Model and that of the CAPM, of the consumption CAPM, and of the Fama and French three-factor model. Panel A reports estimated factor loadings for the three asset pricing models and the results of the J-test of overidentifying restrictions. Estimates involve the same 80 test considered in the previous section. Panel B reports the mean absolute pricing errors (MAE) and the cross-sectional R^2 for all the models and all the test assets. For convenience, the MAE and the R^2 for the Contracting Model are also reported.

Consistent with several previous studies, the CAPM and the Consumption CAPM are not successful in pricing the tests assets. The MAE is high, ranging from 1.44% per annum to 2.64%

per annum, and the R^2 is consistently low across all test assets. The Fama-French model instead performs better, with mean absolute pricing errors ranging from 1.18% to 1.68% per year. However, as in Lewellen, Nagel, and Shanken (2010), the three-factor model has a very low R^2 on the 30 Fama-French portfolios. In the estimates in Table 5, that refer to the joint pricing of 80 test assets, the latter R^2 is even slightly negative, with a value of -0.06. With respect to both indicators of pricing performance, the Contracting Model outperforms all models with the only exception of the 25 Fama-French portfolios sorted by size and book-to-market equity, on which the three factor model has a slightly lower MAE and a slightly higher R^2. Remarkably, unlike the other models, the Contracting Model has a satisfactory pricing performance when the 30 Fama-French industry portfolios are considered. Finally, not surprisingly, the formal tests based on the J statistics are uninformative, and are unable to reject any model.

[Insert Table 5 Here]

Figure 3 summarizes the previous comparison among models. Panels A through D depict predicted versus realized average excess returns for the CAPM, the Consumption CAPM, the Fama-French model, and the Contracting Model. The figure refers to all the test assets together. Panels A and B show that the points are far from the 45-degree line for the CAPM and the Consumption CAPM, while they line up better for the Fama and French's model (Panel C) and especially for the Contracting Model (Panel D).

[Insert Figure 3 Here]

5.5 Relation to Aggregate Multifactor Models

The Contracting Model is not a multifactor model. It is derived at the firm level and it is not suited to derive direct aggregate implications. However, this does not necessarily imply an inconsistency with the existence of aggregate risk factors that price the equity of large public firms, such as reduced-form factor models, or with the consumption-based paradigm.

The results in Section 5.3 highlight a relationship between the individual determinants of the stochastic discount factor in the Contracting Model, net worth growth and profitability growth, and the variables used to construct the investment and profitability factors in the empirical multifactor models of Hou, Xue, and Zhang (2015) and Fama and French (2016).

Table 6 further explores the relationship between these multifactor models and the Contracting Model. First, I construct two ad-hoc empirical factors using net worth growth and profitability as sorting variables, following the approach in Fama and French (2016) to obtain a projection on the space of traded stocks. Specifically, I start by forming six (2x3) value-weight portfolios formed on size and net worth growth, and six (2x3) value-weight portfolios formed on size and profitability growth. Then, I define the net worth growth factor (NWG) as the average return on the two low net worth growth portfolios minus the average return on the two high net worth growth portfolios. Similarly, I define the profitability growth factor (PRG) as the average return on the two high profitability growth portfolios minus the average return on the two low profitability growth portfolios.

Panel A reports regressions of the market (MRKRF), size (SMB), value (HML), profitability (RMW), and investment (CMA) factors on NWG and PRG. The first row in Panel A shows that the net worth growth factor is strongly positively related to the value and, especially, to the investment factor. The coefficients are large and statistically significant, with R-squared above 45 percent for the HML regression and of roughly 80 percent for the CMA regression. The second row show that, while the PRG factor is positively related to both the investment and profitability factors, its relationship to them is less pronounced in comparison to the net worth growth factor, with smaller and less significant coefficients.

Panel B instead considers "orthogonalizing" regressions, in which all factors are regressed on the remaining ones. In the regressions of Panel B, an intercept statistically indistinguishable from zero can be interpreted as the factor used as the dependent variables to be redundant after including all the others in a multifactor pricing model. Consistent with the findings of Hou, Xue, and Zhang (2015), the HML factor appears to be redundant. In addition, the net worth growth factor NWG appears to be subsumed by the others, with an intercept not significant at the 10 percent level and a high R-squared above 80 percent. Thus, the NWG ad-hoc factor defined through the Contracting

Model appears to be strongly related to the empirical factors in Hou, Xue, and Zhang (2015) and Fama and French (2016). On the contrary, the rightmost column of Panel B shows that, while the profitability growth factor is positively related to the CMA and RMW factors, it is far from being captured by them. The estimated intercept is 0.17, almost three standard errors away from zero, with an R-squared of 8.5%.

In all, the results in Table 6 suggest that, while the Contracting Model is not a multifactor model, the variables it predicts to generate cross-sectional spreads in average returns can be used to construct factors the relate to those that both Hou, Xue, and Zhang (2015) and Fama and French (2016) show to be essential for pricing the equity of large listed firms.

[Insert Table 6 Here]

6 Conclusions

I show that, under some conditions, a valid stochastic discount factor to price the equity claims of a firm can be backed out its optimal financial contracting behavior. This leads to an asset pricing model, which goes a long way in rationalizing observed cross-sectional differences in average equity returns. This approach bypasses some challenges related to the structure of the consumption side of the economy.

The Contracting Asset Pricing Model is not a multifactor model. It accommodates incomplete markets and is derived at the firm level. However, this work poses questions for future research not only for production-based asset pricing, but also for consumption-based models, and for empirical work on the cross section of expected returns. The present approach may represent a complementary tool to advance the understanding of the consumption side of the economy. The ultimate goal of asset pricing theory is to provide a general equilibrium explanation of how asset returns and consumption are jointly determined. In general equilibrium, the stochastic discount factor obtained from both the production and consumption side of the economy must have consistent properties. These additional restrictions may provide guidance in modeling the household side on the economy.

Another implication of the contracting approach is that the state variables of the firm's optimization problem, in other words the determinants of firms' decisions, drive the dynamics of the stochastic discount factor firms respond to. Although the model is not suited to derive direct aggregate implications, the contracting approach offers an alternative interpretation of the anomalies that arise in the cross section of equity returns. Through the lens of the model, anomaly variables can be regarded as possibly omitted determinants of the growth of the marginal value of net worth that are not captured by differences across firms in loadings on macroeconomic factors in benchmark asset pricing models. For empirical work, this observation may provide insights for the development of new testable hypotheses for cross-sectional differences in returns.

Table 2

GMM Tests of the Contracting Model.

The table reports the estimated loadings on the growth rate of net worth, the growth rate of profitability, and the growth rate of the riskfree interest rate for the Contracting Model. The test assets are the 25 Fama-French portfolios sorted on size and book-to-market equity, 25 portfolios sorted on the growth rates of net worth and profitability, and the 30 Fama-French industry portfolios. All returns are annual. Estimation is by two-step GMM. Test statistics (Z-Stat) based on HAC standard errors are in parentheses. The kernel is Newey-West with a lag length of 1 year. MAE denotes the percent mean absolute pricing error, and the R^2 is computed as in Campbell and Vuolteenaho (2004). The latter two statistics are based on first-stage estimates. J, df, and p(J) denote the test statistic, the degrees of freedom, and the p-value for a test of overidentifying restrictions. Data are from 1965 to 2013. All variables are defined in the Appendix.

Panel A: Coefficient Estimates				
	Net Worth Growth	Profitability Growth	Interest Rate Growth	
Loading	0.12	-0.16	0.26	
Z-Stat	(5.61)	(-9.62)	(3.04)	
J				25.29
df				76
p(J)				1.00

Panel B: Pricing Errors				
	FF 25 Size/BM	25 NW/P Growth	FF 30 Industry	All Portfolios
MAE (%)	1.67	0.78	1.04	1.16
R^2	0.70	0.81	0.59	0.70

Table 3

PORTFOLIOS SORTED ON GROWTH RATES OF NET WORTH AND PROFITABILITY.

In the table, stocks are sorted independently every June in deciles based on their values of the growth rate of net worth and on the growth rate of profitability. The table report excess returns, heteroscedasticity and autocorrelation consistent t statistics, and Sharpe ratios for the bottom decile (L), the top decile (H) and for the third, fifth and seventh decile. We also report the difference for the excess returns and the Sharpe ratio between the top decile and the bottom decile (H-L). The left panel reports equally-weighted returns, while the right panel reports value-weighted returns. T statistics are computed with a Newey-West correction with lag-length of 4. Data are from June 1965 to December 2013. All variables are defined in the Appendix.

		Average Monthly Returns											
Sorting Variable		Equally-Weighted						Value-Weighted					
		H-L	L	3	5	7	H	H-L	L	3	5	7	H
Net Worth Growth	R	-12.29	21.23	17.17	16.92	17.74	8.94	-3.87	12.88	12.01	11.77	12.33	9.01
	[t]	-6.58	4.57	5.27	5.64	5.76	2.22	-2.08	4.15	5.38	5.36	5.47	2.69
	SR	-1.03	0.77	0.87	0.90	0.90	0.34	-0.28	0.64	0.77	0.78	0.77	0.40
Profitability Growth	R	4.60	13.25	16.16	16.02	17.46	17.85	3.83	8.94	10.59	12.28	13.97	12.77
	[t]	5.56	3.13	4.81	5.17	5.71	4.39	2.19	2.66	4.07	5.34	6.21	4.21
	SR	0.82	0.51	0.77	0.83	0.90	0.71	0.34	0.41	0.60	0.77	0.88	0.62

Table 4

FAMA-MACBETH REGRESSION: GROWTH RATES OF NET WORTH AND PROFITABILITY.

The table reports coefficient estimates from Fama-MacBeth regressions. The dependent variable is the monthly stock return. Column (1) includes the growth rate of net worth. Column (2) includes the growth rate of profitability. Column (3) includes the growth rates of both net worth and profitability. Column (4) adds the logarithm of market capitalization (size), the logarithm of book-to-market equity, and momentum as controls. Column (5) adds investment and profitability. T statistics are reported in parentheses. The symbols ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. Data are from June 1965 to December 2013. All variables are defined in the Appendix.

Dependent Variable: Monthly Return					
	(1)	(2)	(3)	(4)	(5)
Net Worth Growth	-0.93		-0.93	-0.69	-0.30
	(-6.91)		(-6.82)	(-6.91)	(-2.54)
Profitability Growth		0.22	0.18	0.14	0.10
		(4.54)	(3.81)	(3.28)	(2.16)
Size				-0.13	-0.13
				(-3.35)	(-3.35)
Book-to-Market				0.22	0.24
				(3.97)	(4.04)
Momentum				0.46	0.44
				(2.81)	(2.67)
Investment					-0.38
					(-6.28)
Profitability					0.04
					(2.15)
R^2	0.004	0.001	0.005	0.035	0.037
N	1678439	1681446	1663115	1630099	1630099

Table 5
Comparison Among Models.

Panel A reports the estimated factor loading on consumption growth, the market return, the HML and the SMB factors for the CAPM, the Consumption CAPM, and the Fama-French three factor model. The test assets are the 25 Fama-French portfolios sorted on size and book-to-market equity, 25 portfolios sorted on the growth rates of net worth and profitability, and the 30 Fama-French industry portfolios. All returns are annual. Estimation is by two-step GMM. Test statistics based on HAC standard errors are in parentheses. The kernel is Newey-West with a lag length of 1 year. J and p(J) denote the test statistic, and the p-value for a test of overidentifying restrictions. Panel B reports, for all three models and for the Contracting Model, the percent mean absolute pricing error (MAE), and the R^2 is computed as in Campbell and Vuolteenaho (2004). The latter two statistics are based on first-stage estimates. Data are from 1965 to 2013. All variables are defined in the Appendix.

Panel A: Coefficient Estimates

	Cons. Gr.	Market	HML	SMB	J	p(J)
CAPM		0.35			25.48	1.00
		(1.94)				
CCAPM	0.14				25.41	1.00
	(8.59)					
Fama-French		0.67	0.81	2.15	25.45	1.00
		(2.16)	(1.77)	(4.75)		

Panel B: Pricing Errors

		FF 25 Size/BM	25 NW/P Growth	FF 30 Industry	All Portfolios
CAPM	MAE (%)	2.64	1.61	1.44	1.87
	R^2	0.23	0.20	0.18	0.22
CCAPM	MAE (%)	2.32	1.56	1.56	1.80
	R^2	0.40	0.26	-0.02	0.28
Fama-French	MAE (%)	1.34	1.18	1.68	1.42
	R^2	0.74	0.52	-0.06	0.52
Contracting Model	MAE (%)	1.67	0.78	1.04	1.16
	R^2	0.70	0.81	0.59	0.70

Table 6

Regression Involving Aggregate Factors.

Panel A reports estimates from regression of the aggregate factors in the empirical models of of Hou, Xue, and Zhang (2015) and Fama and French (2016) on the two ad-hoc factors motivated by the contracting model and described in Section 5.5. MKTRF denotes the market factor, SMB the size factor, HML the value factor, CMA the investment factor, and RMW the profitability factor. All the previous factors are constructed as in Fama and French (2016). NWG and PRG denote the factors constructed using net worth growth and profitability growth as sorting variables, using the same procedure in Fama and French (2016). Panel B reports regression estimates from orthogonalizing regressions in which all factors are regressed on the remaining ones. Heteroskedasticity and autocorrelation consistent t statistics are reported in parentheses. The symbols ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. Data are from June 1965 to December 2013. All variables are defined in the Appendix.

Panel A: Aggregate Factors vs Contracting Model Ad-Hoc Factors

	MKTRF	*SMB*	*HML*	*RMW*	*CMA*
NWG	-0.72***	-0.15**	0.91***	-0.10	0.85***
	(-8.13)	(-2.14)	(19.37)	(-1.43)	(41.29)
PRG	-0.42***	-0.10	0.13	0.32***	0.13***
	(-2.80)	(-0.88)	(1.63)	(2.73)	(4.15)
Constant	0.73***	0.32***	0.19**	0.20**	0.14***
	(4.28)	(2.58)	(2.16)	(2.36)	(3.81)
R^2	0.127	0.011	0.470	0.051	0.793

Panel B: Orthogonalizing Regressions

	MKTRF	*SMB*	*HML*	*RMW*	*CMA*	*NWG*	*PRG*
MKTRF		0.12***	0.02	-0.10***	-0.04***	-0.01	-0.02
		(3.03)	(0.74)	(-4.16)	(-3.84)	(-0.60)	(-1.23)
SMB	0.24***		0.02	-0.23***	0.00	-0.02	0.02
	(2.88)		(0.54)	(-3.44)	(0.24)	(-1.29)	(0.77)
HML	0.09	0.05		0.20**	0.10***	0.12***	0.01
	(0.75)	(0.54)		(2.28)	(4.46)	(4.96)	(0.15)
RMW	-0.38***	-0.46***	0.20**		0.00	-0.09***	0.10**
	(-3.59)	(-3.90)	(2.28)		(0.09)	(-3.25)	(2.03)
CMA	-0.83***	0.04	0.52***	0.01		0.81***	0.23***
	(-4.10)	(0.24)	(4.60)	(0.10)		(20.83)	(3.07)
NWG	-0.11	-0.18	0.51***	-0.40***	0.10***		-0.28***
	(-0.60)	(-1.26)	(4.63)	(-3.15)	(25.25)		(-3.27)
PRG	-0.18	0.09	0.01	0.22**	0.15***	-0.14***	
	(-1.21)	(0.78)	(0.15)	(2.20)	(3.21)	(-3.54)	
Constant	0.82***	0.31**	0.05	0.30***	0.15***	-0.07	0.17***
	(4.59)	(2.51)	(0.53)	(3.82)	(3.86)	(-1.48)	(2.84)
R^2	0.244	0.172	0.522	0.247	0.812	0.812	0.085

Figure 1
THE DYNAMIC LIMITED ENFORCEMENT PROBLEM.

The figure depicts the timing of events in the dynamic limited enforcement problem, as described in the text. The sequence of events occurs each period after the contract between the lender and the borrower is signed.

Intraperiod Timing

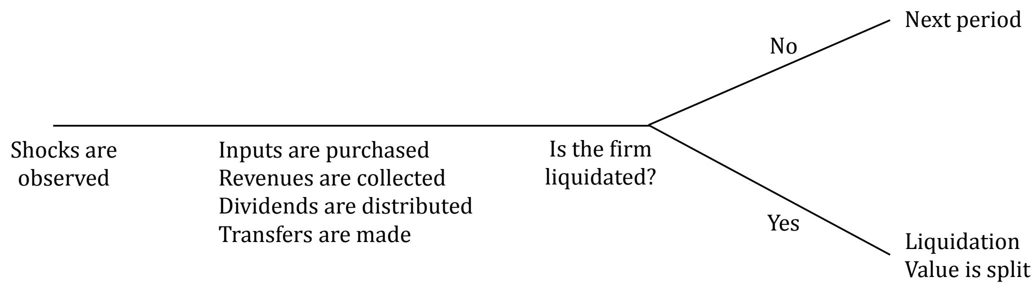


Figure 2

Predicted vs Realized Returns: Contracting Model.

The figure illustrates annual predicted and realized percent returns for the first-stage GMM estimation of the Contracting Model as in Table 2. Panels A through D refer to the following test assets: the 25 Fama-French portfolios sorted on size and book-to-market equity, 25 portfolios sorted on the growth rates of net worth and profitability, the 30 Fama-French industry portfolios, and all the previous portfolios. In panel A, the first digit of the label corresponds to the size quintile, and the second digit to the book-to-market equity quintile. In Panel B, the first digit of the label corresponds to net worth growth quintile, and the second digit to the profitability growth quintile. In Panel C, the labels are mnemonics for Fama and French 30-Industry classification as on Kenneth French's website. Data are from 1965 to 2013. All variables are defined in the Appendix.

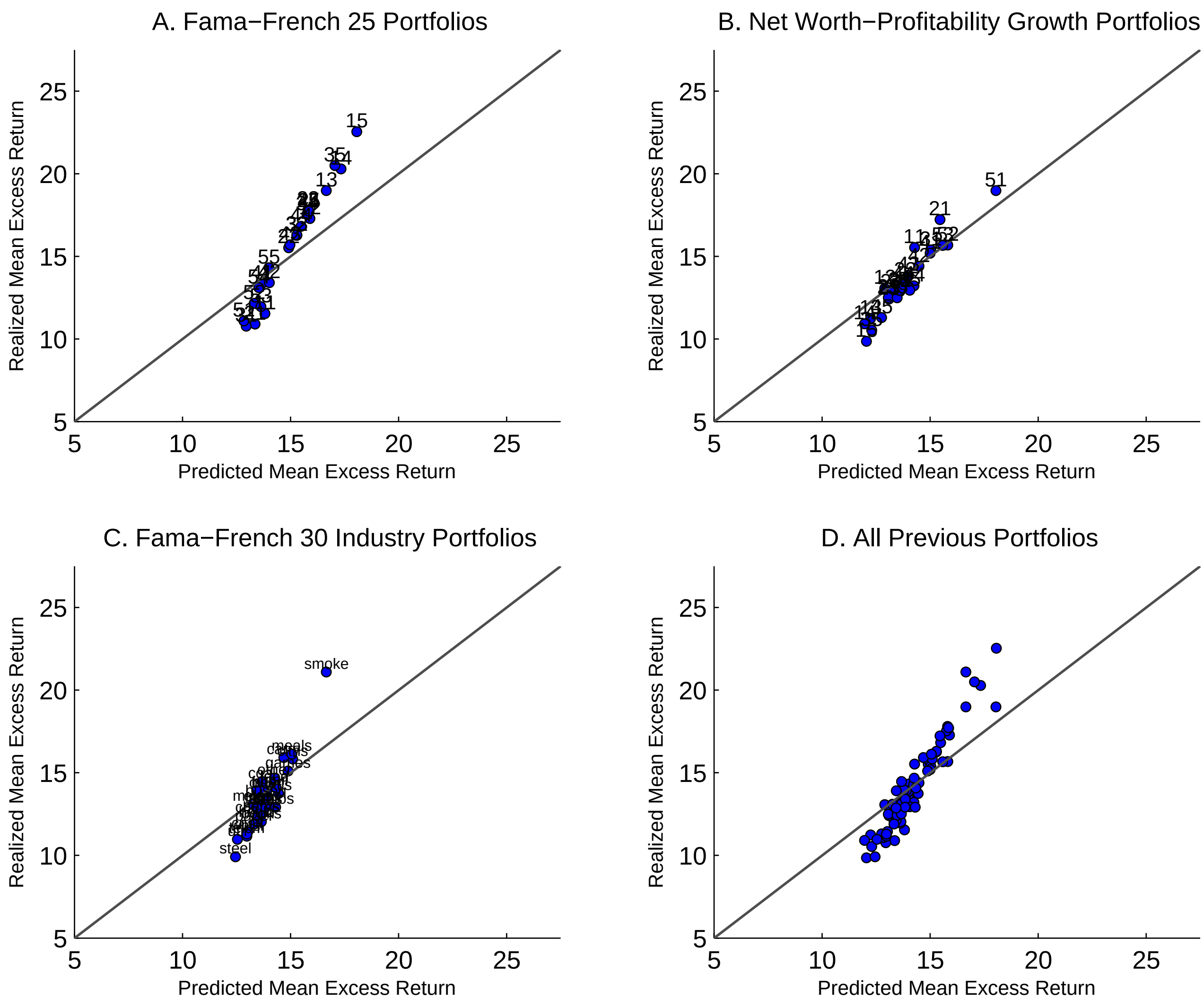
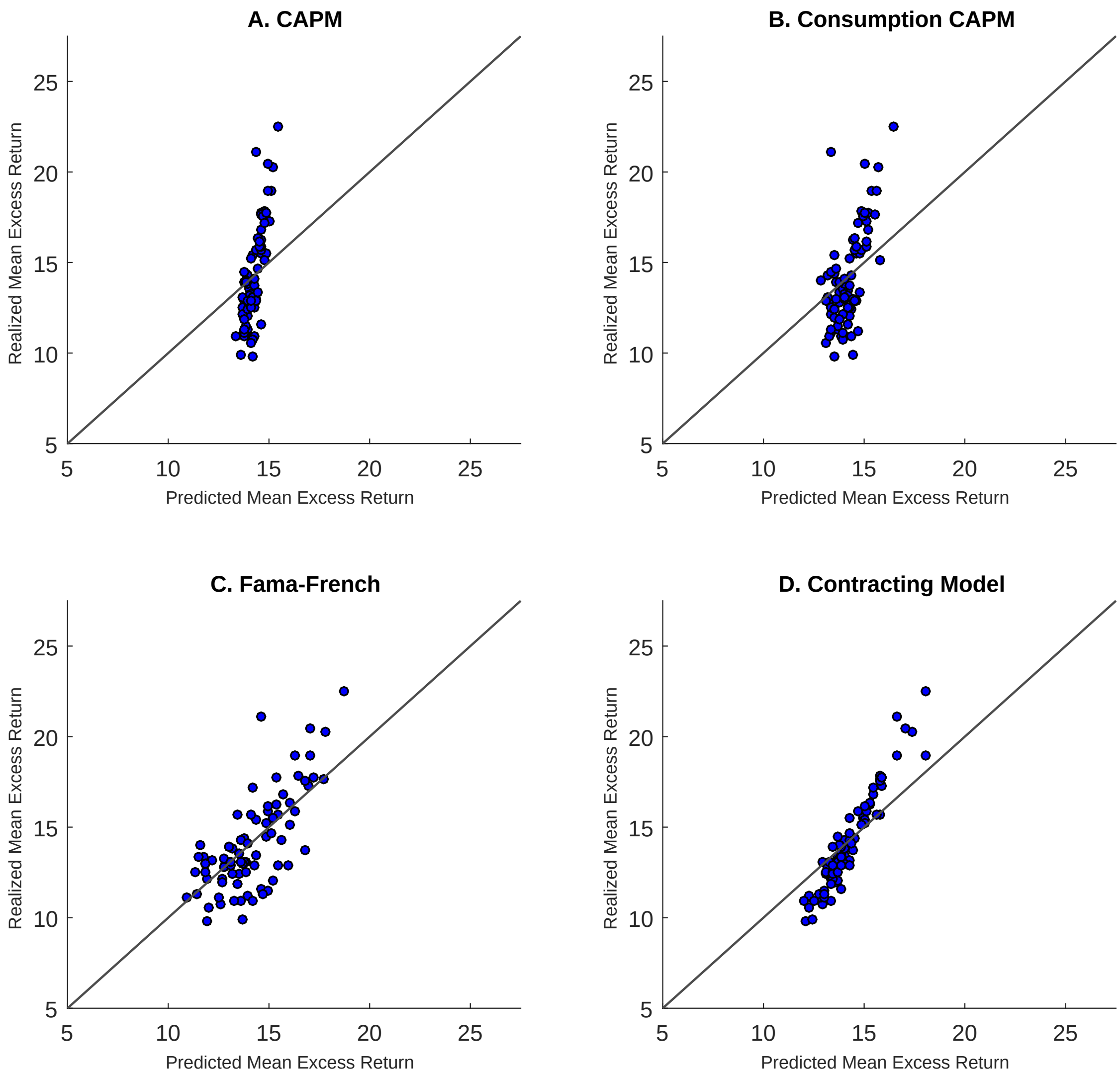


Figure 3

Predicted vs Realized Returns: Comparison Among Models.

The figure illustrates predicted and realized excess returns for the first-stage GMM estimation of different asset pricing models. The test assets are: the 25 Fama-French portfolios sorted on size and book-to-market equity, 25 portfolios sorted on the growth rates of net worth and profitability, and the 30 Fama-French industry portfolios. Panels A through D refer to the CAPM, the Consumption CAPM, the three factor model of Fama and French, and the Contracting Model. Data are from 1965 to 2013. All variables are defined in the Appendix.



References

Abreu, D., D. Pearce, and E. Stacchetti, 1990, Toward a theory of discounted repeated games with imperfect monitoring, *Econometrica*, 58(5), 1041–1063.

Ahn, S. C., and C. Gadarowski, 2004, Small sample properties of the GMM specification test based on the Hansen–Jagannathan distance, *Journal of Empirical Finance*, 11(1), 109–132.

Albuquerque, R., and H. A. Hopenhayn, 2004, Optimal lending contracts and firm dynamics, *The Review of Economic Studies*, 71(2), 285–315.

Aliprantis, C. D., and K. C. Border, 2006, *Infinite dimensional analysis: a hitchhiker's guide*. Springer.

Alvarez, F., and U. J. Jermann, 2000, Efficiency, equilibrium, and asset pricing with risk of default, *Econometrica*, 68(4), 775–797.

——— , 2001, Quantitative asset pricing implications of endogenous solvency constraints, *Review of Financial Studies*, 14(4), 1117–1151.

Angeletos, G.-M., 2002, Fiscal policy with noncontingent debt and the optimal maturity structure, *The Quarterly Journal of Economics*, 117(3), 1105–1131.

Bansal, R., and A. Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The journal of Finance*, 59(4), 1481–1509.

Barro, R. J., 2006, Rare disasters and asset markets in the twentieth century, *The Quarterly Journal of Economics*, 121(3), 823–866.

Bazdrech, S., F. Belo, and X. Lin, 2013, Labor hiring, investment and stock return predictability in the cross section, *Journal of Political Economy*, forthcoming.

Belo, F., 2010, Production-based measures of risk for asset pricing, *Journal of Monetary Economics*, 57(2), 146–163.

Bethune, Z., T.-W. Hu, and G. Rocheteau, 2018, Indeterminacy in credit economies, *Journal of Economic Theory*, 175, 556–584.

Bloise, G., P. Reichlin, and M. Tirelli, 2013, Fragility of competitive equilibrium with risk of default, *Review of Economic Dynamics*, 16(2), 271–295.

Burnside, C., 2015, Identification and inference in linear stochastic discount factor models with excess returns, *Jnl of Financial Econometrics*, 14(2), 295–330.

Campbell, J. Y., and J. H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of political Economy*, 107(2), 205–251.

Campbell, J. Y., and T. Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review*, pp. 1249–1275.

Chen, Z., I. Cooper, P. Ehling, and C. Xiouros, 2017, Risk aversion sensitive real business cycles, .

Chien, Y., H. Cole, and H. Lustig, 2011, A multiplier approach to understanding the macro implications of household finance, *The Review of Economic Studies*, 78(1), 199–234.

——— , 2012, Is the volatility of the market price of risk due to intermittent portfolio rebalancing?, *The American Economic Review*, 102(6), 2859–2896.

Chien, Y., and H. Lustig, 2010, The market price of aggregate risk and the wealth distribution, *Review of Financial Studies*, 23(4), 1596–1650.

Cochrane, J. H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy*, 104(3), 572–621.

——— , 2001, *Asset Pricing*. Princeton University Press, Princeton, NJ.

Colla, P., F. Ippolito, and K. Li, 2013, Debt specialization, *The Journal of Finance*, forthcoming.

Daniel, K., and S. Titman, 2006, Market reactions to tangible and intangible information, *The Journal of Finance*, 61(4), 1605–1643.

——— , 2012, Testing factor-model explanations of market anomalies, *Critical Finance Review*, 1(1), 103–139.

Dávila, E., and A. Korinek, 2018, Pecuniary externalities in economies with financial frictions, *The Review of Economic Studies*, 85(1), 352–395.

Deaton, A., 1992, *Understanding consumption*. Oxford University Press.

Eckbo, B. E., 2007, *Handbook of corporate finance: Empirical corporate finance*, vol. 1. Elsevier.

Ekern, S., and R. Wilson, 1974, On the theory of the firm in an economy with incomplete markets, *The Bell Journal of Economics and Management Science*, pp. 171–180.

Fama, E. F., and K. R. French, 1992, The cross-section of expected stock returns, *The Journal of Finance*, 47(2), 427–465.

Fama, E. F., and K. R. French, 2016, Dissecting anomalies with a five-factor model, *The Review of Financial Studies*, 29(1), 69–103.

Ferson, W. E., and S. R. Foerster, 1994, Finite sample properties of the Generalized Method of Moments in tests of conditional asset pricing models, *Journal of Financial Economics*, 36(1), 29–55.

Ferson, W. E., and R. Jagannathan, 1996, 1 econometric evaluation of asset pricing models, *Handbook of Statistics*, 14, 1–33.

Garlappi, L., and H. Yan, 2011, Financial distress and the cross-section of equity returns, *The Journal of Finance*, 66(3), 789–822.

Gomes, F., and A. Michaelides, 2007, Asset pricing with limited risk sharing and heterogeneous agents, *The Review of Financial Studies*, 21(1), 415–448.

Gomes, J. F., and L. Schmid, 2010, Levered returns, *The Journal of Finance*, 65(2), 467–494.

Hansen, L. P., and K. J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica: Journal of the Econometric Society*, pp. 1269–1286.

Harrison, J. M., and D. M. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic theory*, 20(3), 381–408.

Hart, O., 2001, Financial contracting, *Journal of economic Literature*, 39(4), 1079–1100.

He, Z., and A. Krishnamurthy, 2013, Intermediary asset pricing, *American Economic Review*, 103(2), 732–770.

Hennessy, C. A., and T. M. Whited, 2005, Debt dynamics, *The Journal of Finance*, 60(3), 1129–1165.

———, 2007, How costly is external financing? Evidence from a structural estimation, *The Journal of Finance*, 62(4), 1705–1745.

Hou, K., C. Xue, and L. Zhang, 2015, Digesting anomalies: An investment approach, *The Review of Financial Studies*, 28(3), 650–705.

Jagannathan, R., and Z. Wang, 1996, The conditional CAPM and the cross-section of expected returns, *The Journal of Finance*, 51(1), 3–53.

Jermann, U. J., 2010, The equity premium implied by production, *Journal of Financial Economics*, 98(2), 279–296.

Jermann, U. J., 2013, A production-based model for the term structure, *Journal of Financial Economics*, 109(2), 293–306.

Jouini, E., and H. Kallal, 1995, Martingales and arbitrage in securities markets with transaction costs, *Journal of Economic Theory*, 66(1), 178–197.

Jouini, E., and H. Kallal, 1999, Viability and equilibrium in securities markets with frictions, *Mathematical Finance*, 9(3), 275–292.

Jouini, E., and H. Kallal, 2001, Efficient trading strategies in the presence of market frictions, *The Review of Financial Studies*, 14(2), 343–369.

Kamihigashi, T., 2012, Elementary results on solutions to the Bellman equation of dynamic programming: Existence, uniqueness, and convergence, working paper.

Kehoe, T. J., and D. K. Levine, 1993, Debt-constrained asset markets, *The Review of Economic Studies*, 60(4), 865–888.

Kirman, A. P., 1992, Whom or what does the representative individual represent?, *Journal of economic perspectives*, 6(2), 117–136.

Kocherlakota, N., 2008, Injecting rational bubbles, *Journal of Economic Theory*, 142(1), 218–232.

Koijen, R. S., and M. Yogo, 2019, A demand system approach to asset pricing, *Journal of Political Economy*, 127(4), 1475–1515.

Kroencke, T. A., 2017, Asset pricing without garbage, *The Journal of Finance*, 72(1), 47–98.

Krueger, D., and H. Lustig, 2010, When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)?, *Journal of Economic Theory*, 145(1), 1–41.

Krueger, D., H. Lustig, and F. Perri, 2008, Evaluating asset pricing models with limited commitment using household consumption data, *Journal of the European Economic Association*, 6(2-3), 715–726.

Lewellen, J., S. Nagel, and J. Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial Economics*, 96(2), 175–194.

Li, S., T. M. Whited, and Y. Wu, 2016, Collateral, taxes, and leverage, *Review of Financial Studies*, p. hhw008.

Livdan, D., H. Sapriza, and L. Zhang, 2009, Financially constrained stock returns, *The Journal of Finance*, 64(4), 1827–1862.

Lorenzoni, G., and K. Walentin, 2007, Financial frictions, investment and tobin's q, working paper, National Bureau of Economic Research.

Lustig, H., and S. Van Nieuwerburgh, 2010, How much does household collateral constrain regional risk sharing?, *Review of Economic Dynamics*, 13(2), 265–294.

Lustig, H. N., and S. G. Van Nieuwerburgh, 2005, Housing collateral, consumption insurance, and risk premia: An empirical perspective, *The Journal of Finance*, 60(3), 1167–1219.

Magill, M., and M. Quinzii, 2002, *Theory of incomplete markets*, vol. 1. Mit press.

Mangasarian, O. L., 1994, *Nonlinear programming*. SIAM.

McKeon, S. B., 2015, Employee option exercise and equity issuance motives, *Available at SSRN 1920985*.

Merton, R. C., 1973, An intertemporal capital asset pricing model, *Econometrica: Journal of the Econometric Society*, pp. 867–887.

Newey, W. K., and D. McFadden, 1994, Large sample estimation and hypothesis testing, *Handbook of econometrics*, 4, 2111–2245.

Nikolov, B., L. Schmid, and R. Steri, 2019, Dynamic corporate liquidity, *Journal of Financial Economics*, 132(1), 76–102.

Rabin, M., 2000, Risk aversion and expected-utility theory: A calibration theorem, *Econometrica*, 68(5), 1281–1292.

Radner, R., 1974, A note on unanimity of stockholders' preferences among alternative production plans: A reformulation of the ekern-wilson model, *The Bell Journal of Economics and Management Science*, pp. 181–184.

Rampini, A. A., and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *The Journal of Finance*, 65(6), 2293–2322.

——— , 2013, Collateral and capital structure, *Journal of Financial Economics*, 109(2), 466–492.

Savov, A., 2011, Asset pricing with garbage, *The Journal of Finance*, 66(1), 177–201.

Spear, S. E., and S. Srivastava, 1987, On repeated moral hazard with discounting, *Review of Economic Studies*, 54(4), 599–617.

Stiemke, E., 1915, Über positive lösungen homogener linearer gleichungen, *Mathematische Annalen*, 76(2-3), 340–342.

Stokey, N. L., and R. E. Lucas, 1989, *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA.

Sufi, A., 2009, Bank lines of credit in corporate finance: An empirical analysis, *Review of Financial Studies*, 22(3), 1057–1088.

Zhang, L., 2005, The value premium, *The Journal of Finance*, 60(1), 67–103.

Appendix

A.1 Proofs of Propositions

Proof of Proposition 1. The Lagrangian of investor i's individual optimization is

$$\mathcal{L}_i \equiv u(c_{i,0}) + \beta \sum_{\omega\in\Omega} \pi(\omega) \cdot u(c_{i,1}(\omega)) - \sum_{\omega\in\Omega} \beta\pi(\omega)\lambda_{i,1}(\omega) \left(B_{i,1}(\omega) - a_{i,1}(\omega) - \sum_{j\in\mathcal{J}} d^F_{j,1}(\omega) n^F_{i,j,0} \right),$$

from which the expressions in Part i) follow immediately from the first-order conditions with respect to $a_{i,1}(\omega)$ and $n^F_{i,j,0}$. From (4), $m_1(\omega) = \frac{p_{a,1}(\omega)}{\pi(\omega)}$. Thus, the right-hand side of (6) is equalized across investors. The inequalities in Part ii) simply follow from $\lambda_{i,1}(\omega) \geq 0$. To prove Part iii), notice that $\lambda_{l,1}(\omega) = 0$ implies $m_1(\omega) = MRS_{l,1}(\omega)$. Let $h = \arg\max_{i\in\mathcal{I}} MRS_{i,1}(\omega)$ be the investor with the highest marginal rate of substitution. By contradiction, assume that h is constrained, i.e., $\lambda_{h,1}(\omega) > 0$. Then

$$MRS_{l,1}(\omega) = m_1(\omega) = \frac{\beta(u'(c_{h,1}(\omega)) + \lambda_{h,1}(\omega))}{u'(c_{h,0})} > \frac{\beta u'(c_{h,1}(\omega))}{u'(c_{h,0})} = MRS_{h,1}(\omega),$$

which contradicts h being the agent with the highest marginal rate of substitution. Thus, it must be the case that h is also unconstrained, and $MRS_{l,1}(\omega) = MRS_{h,1}(\omega)$. Part iv) follows straightforwardly from (7). ■

Proof of Proposition 2. The proof is an application of Stiemke's lemma (Stiemke, 1915), which I state for convenience.

Lemma 1 (Stiemke) *Let M be a $r \times c$ matrix. Then one and only one of the following statements is true: i) there exists a column vector in v_r in $\mathcal{R}^r$ with strictly positive coordinates, such that $v_r^T M = 0$; ii) there exists a column vector v_c in $\mathcal{R}^c$ such that $Mv_c > 0$, that is Mv_c has all non-negative elements, with at least one strictly positive element.*

Proof of Lemma 1. See Mangasarian (1994), Chapter 2.4, pag. 32. ■

Define the $(S+1) \times (S+J)$ matrix M obtained by concatenating $-p$ and x vertically, that is:

$$M \equiv \begin{bmatrix} -p_{a,1}(\omega_1) & & -p_{a,1}(\omega) & & -p_{a,1}(\omega_S) & -p^F_{1,0} & & -p^F_{j,0} & & -p^F_{J,0} \\ 1 & ... & 0 & ... & 0 & d^F_{1,1}(\omega_1) & ... & d^F_{j,1}(\omega_1) & ... & d^F_{J,1}(\omega_1) \\ 0 & ... & 1 & ... & 0 & ... & & d^F_{j,1}(\omega) & & ... \\ 0 & ... & 0 & ... & 1 & d^F_{1,1}(\omega_S) & ... & d^F_{j,1}(\omega_S) & ... & d^F_{J,1}(\omega_S) \end{bmatrix}.$$

Applying Lemma 1, statement i) is true while ii) is false, as the condition $v_r^T M = 0$ is satisfied by $v_r^T = [1, \pi(\omega_1)m_1(\omega_1), ..., \pi(\omega)m_1(\omega), ..., \pi(\omega_S)m_1(\omega_S)]$ because of (4) and (5). $\pi(\omega)m_1(\omega)$ is strictly positive as $u(\cdot)$ is increasing and $\lambda_{i,1}(\omega) \geq 0$. Thus, one cannot find any vector $\theta_i \in \mathcal{R}^{S+J}$ such that $M\theta_i > 0$, that is a trading strategy θ_i that violates the no-arbitrage condition. ■

Proof of Proposition 3. The Lagrangian of firm j's individual optimization is

$$\mathcal{L}_j \equiv d^F_{j,0} + \sum_{\omega\in\Omega} \pi(\omega)m_1(\omega)d^F_{j,1}(\omega) - \lambda_{j,0}(-w_{j,0} + k_{j,0} - p^B_{j,0}) - \sum_{\omega\in\Omega} \pi(\omega)m_1(\omega)\lambda_{j,1}(\omega)(b_{j,1}(\omega) - A_{j,1}(\omega)\dot{f}(k_{j,0})). \quad \text{(A.1)}$$

(15) immediately follows from the first-order conditions with respect to the state-contingent transfers $b_{j,1}(\omega)$. ■

Proof of Corollary 1. Equation (16) immediately follows from the first-order condition with respect to $b_{j,1}$from (A.1), in which $b_{j,1}(\omega) = b_{j,1}$. If $S > 2$, the two FOCs with respect to $b_{j,1}$ and $k_{j,0}$, that is

$$1 + \lambda_{j,0} = \sum_{\omega \in \Omega} \pi(\omega) m_1(\omega) A_{j,1}(\omega) f_k(k_{j,0}) \left(1 + \lambda_{j,1}(\omega)\right), \tag{A.2}$$

do not allow to solve for the S values $m_1(\omega)$. ■

Proof of Proposition 4. The Karush-Kuhn-Tucker conditions for the firm's problem are

$$\begin{aligned}
1 + \lambda_{j,0} &= \sum_{\omega \in \Omega} \pi(\omega) m_1(\omega) A_{j,1}(\omega) f_k(k_{j,0}) \left(1 + \lambda_{j,1}(\omega)\right), \\
m_1(\omega) &= \beta \frac{1 + \lambda_{j,0}}{1 + \lambda_{j,1}(\omega)}, \\
\lambda_{j,0} \left(w_{j,0} - k_{j,0} + p^B_{j,0}\right) &= 0, \\
\pi(\omega) m_1(\omega) \lambda_{j,1}(\omega)(b_{j,1}(\omega) - A_{j,1}(\omega) f(k_{j,0})) &= 0, \ \forall \omega \in \Omega, \\
\lambda_{j,0} &\geq 0, \\
\pi(\omega) m_1(\omega) \lambda_{j,1}(\omega) &\geq 0, \ \forall \omega \in \Omega, \\
w_{j,0} - k_{j,0} + p^B_{j,0} &\geq 0, \\
A_{j,1}(\omega) f(k_{j,0}) - b_{j,1}(\omega) &\geq 0, \ \forall \omega \in \Omega.
\end{aligned}$$

Part i) follows from combining (A.2) with (15). $f_k(\cdot)$ is invertible because of the properties of the production function. Notice that The capital stock is always financeable because, from the budget and borrowing constraints $k_{j,0} \leq p^B_{j,0} \leq \sum_{\omega \in \Omega} \pi(\omega) \beta A_{j,1}(\omega) f(k_{j,0})$. To prove Part ii), by contradiction, suppose the firm increases its dividend $d^F_{j,0}$ by the amount $s_{j,0} = w_{j,0} - k_{j,0} + p^B_{j,0} > 0$ under the contract $(k_{j,0}, \{b_{j,1}(\omega)\}_{\omega \in \Omega})$ and attain value $V(w_{j,0})$. Instead, the firm could arrange the a contract with identical capital expenditure and repayments, but with a transfer $\widehat{b}_{j,1}(\omega) = b_{j,1}(\omega) - \frac{s_{j,0}}{\pi(\omega_L)}$. This would result in $\widehat{p}^B_{j,0} = \sum_{\omega \in \Omega} \pi(\omega) \beta b_{j,1}(\omega) - \beta s_{j,0}$ and attain $\widehat{V}(w_{j,0})$. The difference in the objective functions $\widehat{V}(w_{j,0}) - V(w_{j,0})$ is $\pi(\omega_L) m_1(\omega_L) \frac{s_{j,0}}{\pi(\omega_L)} - \beta s_{j,0} = s_{j,0}(m_1(\omega_L) - \beta)$, which is positive if and only if $m_1(\omega_M) > \beta$. To prove Part iii), notice that the multipliers $\lambda_{j,1}(\omega)$ are decreasing with respect to $m_1(\omega)$. Take any two states ω_L and ω_H such that $m_1(\omega_L) \leq m_1(\omega_H)$. The optimality condition $m_1(\omega) = \beta \frac{1+\lambda_{j,0}}{1+\lambda_{j,1}(\omega)}$ implies that

$$\lambda_{j,1}(\omega_L) \geq \lambda_{j,1}(\omega_H),$$

with equality if $m_1(\omega_L) = m_1(\omega_H)$. The objective function evaluated in correspondence of the optimal capital choice is

$$\begin{aligned}
V(w_{j,0}) = &\max_{b_{j,1}(\omega)} w_{j,0} - f_k^{-1}\left(\frac{1}{\beta \sum_{\omega \in \Omega} \pi(\omega) A_{j,1}(\omega)}\right) + \sum_{\omega \in \Omega} \beta \pi(\omega) b_{j,1}(\omega) \\
&+ \sum_{\omega \in \Omega} \pi(\omega) m_1 \left(A_{j,1}(\omega) f\left(f_k^{-1}\left(\frac{1}{\beta \sum_{\omega \in \Omega} \pi(\omega) A_{j,1}(\omega)}\right)\right) - b_{j,1}(\omega)\right).
\end{aligned}$$

The objective function is linear in $b_{j,1}(\omega)$, which load with coefficients $\pi(\omega)(\beta - m_1(\omega))$. The lowest coefficient is the one of ω_M. Firms transfer resources to ω_M to set the lowest possible feasible $b_{j,1}(\omega_M)$ (i.e., they repay as much as they can $\forall \omega \in \Omega \backslash \omega_M$). From the budget constraint at $t = 0$, one obtains

$$\sum_{\omega \in \Omega \backslash \omega_M} \pi(\omega) \beta A_{j,1}(\omega) f(k_{j,0}) + \pi(\omega) \beta b_{j,1}(\omega_M) = k_{j,0} - w_{j,0},$$

which yields

$$\begin{aligned}
b_{j,1}(\omega_M) &= \frac{k_{j,0} - w_{j,0} - \sum_{\omega \in \Omega \backslash \omega_M} \pi(\omega) \beta A_{j,1}(\omega) f(k_{j,0})}{\beta \pi(\omega_M)}, \\
b_{j,1}(\omega) &= A_{j,1}(\omega) f(k_{j,0}), \forall \omega \in \Omega \backslash \omega_M.
\end{aligned}$$

This implies $\lambda_{j,1}(\omega_M) = 0$. The multipliers therefore solve the following system:

$$\begin{aligned}
m_1(\omega) &= \beta\frac{1+\lambda_{j,0}}{1+\lambda_{j,1}(\omega)},\ \forall\omega\in\Omega\backslash\omega_M,\\
1+\lambda_{j,0} &= \sum_{\omega\in\Omega}\pi(\omega)m_1(\omega)A_{j,1}(\omega)f_k(k_{j,0})\left(1+\lambda_{j,1}(\omega)\right),\\
\lambda_{j,1}(\omega_M) &= 0,
\end{aligned}$$

which is solved by

$$\begin{aligned}
\lambda_{j,0} &= \frac{m_1(\omega_M)-\beta}{\beta},\\
\lambda_{j,1}(\omega) &= \frac{m_1(\omega_M)-m_1(\omega)}{m_1(\omega)},\ \forall\omega\in\Omega\backslash\omega_M,\\
\lambda_{j,1}(\omega_M) &= 0.
\end{aligned}$$

■

Proof of Proposition 5. Denote as Y the set of the possible values for the state variables $w_{i,t}$, $s_{i,t}$, and R_t, as $\Gamma(y)$ the set of feasible actions $d_{i,t}$, $k_{i,t+1}$, $b(s_{i,t+1}, R_{t+1})$ and $w(s_{i,t+1}, R_{t+1})$ for each $y \in Y$, as y' the values of the state variables at time $t+1$ in correspondence of a feasible action $a \in \Gamma(y)$, and as $d(y,a)$ the dividend $d_{i,t}$ corresponding to the state y when the action a is chosen. Let V be the set of functions from Y to $(-\infty, \infty)$. In the remainder of the proof, I use the shorthands V^{LB} for $V^{LB}(w_{i,t}, s_{i,t}, R_t)$, V^{UB} for $V^{UB}(w_{i,t}, s_{i,t}, R_t)$, and V^* for $V(w_{i,t}, s_{i,t}, R_t)$. Denote by $\leq$ be partial order operator for the functions on V, and by T the Bellman operator defined by

$$(Tv)(y) = \sup_{a\in\Gamma(y)} \left(d(y,a) + E_t\left[M_{i,t+1}v(y')\right]\right), \qquad y, y' \in Y, v \in V. \tag{A.3}$$

In this setting, the number of states is assumed to be finite, and by no arbitrage we have $M_{i,t+1} > 0$. Therefore, from the definition of T, it follows that T is monotone. Furthermore, $T(V^{UB}) \leq V^{UB}$, and $T(V^{LB}) \geq V^{LB}$. Under these conditions, the Knaster-Tarski fixed-point theorem (Aliprantis and Border (2006), Theorem 1.10) guarantees that the Bellman operator has at least one fixed point V^{FP} in $[V^{LB}, V^{UB}]$. Define the sequence V_n^{LB}, with $n = 0, 1, 2, ...$ such that $V_0^{LB} = V^{LB}$, and $V_{n+1}^{LB} = TV_n^{LB}$. Since any fixed point of T in $[V^{LB}, V^{UB}]$ is bounded above by V^{UB}, the increasing sequence V_n^{LB} must converge to a fixed point $\hat{V}^{LB}$ in $[V^{LB}, V^{UB}]$. By definition of fixed point, $V^{FP} = TV^{FP}$, and, by construction, $V_n^{LB} \leq V^{FP}$, for all n. Thus, $\hat{V}^{LB} \leq V^{FP}$. Using the transversality conditions and since the number of states is finite, the conclusion of Theorem 4.3 in Stokey and Lucas (1989) goes through. Therefore $V^* = V^{FP}$. Finally, the transversality conditions ensure the assumptions for Lemma 4.3 in Kamihigashi (2012) are satisfied, and this guarantees that $V^* \leq \hat{V}^{LB}$. As a consequence, the following chain of inequalities holds:

$$V^* \leq \hat{V}^{LB} \leq V^{FP} = V^* \tag{A.4}$$

This establishes that the uniqueness result in part (i), and the convergence results in part (ii). To prove that the value function is increasing in net worth in part (iii), while the contraction mapping theorem does not hold, the Knaster-Tarski theorem guarantees that iterating from any feasible lower bound of the solution $\underline{V}$ allows to converge to the solution. Consider as a lower bound $\underline{V}$ of the solution the function $f = w_{i,t} \in V$, corresponding to the policy in which the firms pays out all its net worth as dividends. This initial condition is feasible but non necessarily optimal, thus is a lower bound for the solution. Consider the set A of bounded, continuous, and weakly increasing functions, and the set A_0 of bounded, continuous, and strictly increasing functions. A is closed under the sup norm, and $A_0 \subseteq A$. Since $\underline{V} \in A$, to show that the fixed point is strictly increasing in $w_{i,t}$, it suffices to show that $T(f) \in A_0$. Consider two values of net worth $w_H > w_L$, for any $s_{i,t}$ and R_t. To save notation, denote the state $(w_L, s_{i,t}, R_t)$ as

y_L and the state $(w_H, s_{i,t}, R_t)$ as y_H. Then:

$$\begin{aligned}(Tf)(y_L) &= \sup_{a\in\Gamma(y_L)} \left(d(y_L,a) + E_t\left[M_{i,t+1}f(y')\right]\right)\\ &\leq \sup_{a\in\Gamma(y_H)} \left(d(y_L,a) + E_t\left[M_{i,t+1}f(y')\right]\right)\\ &< \sup_{a\in\Gamma(y_H)} \left(d(y_H,a) + E_t\left[M_{i,t+1}f(y')\right]\right)\\ &= (Tf)(y_H),\end{aligned}$$

where the second inequality follows from the fact that $\Gamma(y_L) \subseteq \Gamma(y_H)$ and the second inequality from the fact that $d(y_H,a) > d(y_L,a)$. T then maps bounded, continuous, and weakly increasing functions into bounded, continuous, and increasing functions and, iterating from $f = w_{i,t}$, one can conclude that the value function is increasing. To prove weak concavity, denote as a'_L and a'_H feasible choices from states y_L and y_H, respectively, i.e. $a_L \in \Gamma(y_L)$ and $a_H \in \Gamma(y_H)$, and let a_L attain $T(f)(y_L)$ and a_H attain $T(f)(y_H)$. Consider a τ_{t+1} combination of the maximizers with $\alpha \in (0,1)$, $a_\alpha \equiv \alpha a_L + (1-\alpha)a_H$. Consider a weakly concave function $f \in V$. Then the constraint set is τ_{t+1}, that is $a_\alpha \in \Gamma(y_\alpha)$, because the constraint (18) is evaluated at f, and the hypograph of a concave function is a convex set. Consider the initial state $y_\alpha \equiv \alpha y_L + (1-\alpha)y_H$, and let y'_α be the values of the state variables at time $t+1$ in correspondence of a_α.Then:

$$\begin{gathered}T(f)(y_\alpha) \geq d(y_\alpha, a_\alpha) + E_t\left[M_{i,t+1}f(y'_\alpha)\right]\\ \geq \alpha d(y_L,a_L) + (1-\alpha)d(y_H,a_H) + E_t\left[M_{i,t+1}\left(\alpha f(y'_L) + (1-\alpha)f(y'_H)\right)\right]\\ = \alpha T(f)(y_L) + (1-\alpha)T(f)(y_H)\end{gathered}$$

where the first inequality follows from the absence of the sup operator on the right-hand side, the second from the weak concavity of dividends in net worth and of f, and the last equality from a_L attaining $T(f)(y_L)$ and a_H attaining $T(f)(y_H)$. Then for any y_L, y_H and any concave f, $T(f)(y_\alpha) > \alpha T(f)(y_L) + (1-\alpha)T(f)(y_H)$, i.e. T maps concave functions into concave functions. Because there exists at least a weakly concave lower bound ($f = w_{i,t}$), the Knaster-Tarski theorem implies the value function in weakly concave in net worth. It immediately follows from the concavity of the value function that the constraint set is convex, and Lemma 1 in Benveniste and Scheinkman (1979) applies, implying differentiability in net worth.

To establish part (iv), I follow the approach in Lemma 5 of Rampini and Viswanathan (2013), in that the return function $d_{i,t}$ is not necessarily increasing in $s_{i,t}$ and decreasing in R_t, and the assumptions of Theorem 9.7 in Stokey and Lucas (1989) do not hold. To prove that the value function in non-decreasing in $s_{i,t}$ for a fixed R_t, consider the ordered set $S = \{s'_1, ..., s'_S\}$ of possible realizations s'_j of $s_{i,t+1}$,with $s'_{j-1} \leq s'_j$, $j = 2, ..., S$, and two initial states s_L and s_H, with $s_H \geq s_L$. For any threshold $\omega \in [0,1]$, define the step function $\beta : [0,1] \to R$ as $\beta(\omega) = \sum_{j=1}^{S} b(s'_j, R_{t+1})1_{B_j^L}(\omega)$, where

$$\begin{aligned}B_1^L &= [0, Q_s(s_L, s'_1)],\\ B_j^L &= \left(\sum_{l=1}^{j-1} Q_s(s_L, s'_l), \sum_{l=1}^{j} Q_s(s_L, s'_l)\right],\ j = 2, ..., S.\end{aligned}$$

Given the initial state s_L, returns the repayment corresponding to the realization of the highest state s'_j whose c.d.f. does not exceed ω. The probability to observe the payment $b(s'_j, R_{t+1})$ if the current state is s_L conditional on R_{t+1} is instead the Lebesgue measure $\lambda(B_j^L)$. Analogously, for the initial state s_H, define B_j^H, $j = 1, ..., S$, and $B_{jk} = B_j^L \cap B_k^H, \forall j,k \in S$. Define the step function $\widehat{\beta} : [0,1] \to \mathcal{R}$ as $\widehat{\beta}(\omega) = \sum_{j=1}^{N}\sum_{k=1}^{N} b(s'_j, R_{t+1})1_{B_{jk}}(\omega)$, $\forall \omega \in [0,1]$, and the stochastic financing policy for $B_k^H, \forall k = 1, ..., S$, as $b^H(s'_j|s'_k, R_{t+1}) = b(s'_j, R_{t+1})$ with positive Lebesgue measure $\lambda(B_k^H > 0)$ and conditional probability $q(s'_j|s'_k, R_{t+1}) = \frac{\lambda(B_{jk})}{\lambda(B_k^H)}$. This implies a stochastic net worth

$$\begin{aligned} w(s'_j|s'_k, R_{t+1}) &= \Pi(k_{i,t+1}, s'_k) + (1-\delta)k_{i,t+1} - R_t b^H(s'_j|s'_k, R_{t+1}) \\ &\geq \Pi(k_{i,t+1}, s'_j) + (1-\delta)k_{i,t+1} - R_t b(s'_j, R_{t+1}) = w(s'_j, R_{t+1}) \text{ almost everywhere,} \end{aligned}$$

where the inequality follows from the monotonicity of the Markov chain $Q_s(s_{i,t+1}|s_{i,t})$, that is $\sum_{s' \leq \overline{s}'} Q_s(s_H, s') \leq \sum_{s' \leq \overline{s}'} Q_s(s_L, s')$, for all $s_H, s_L, \overline{s}'$, that in turn implies $\lambda(B_{jk}) = 0$ if $j > k$.

For all $w_{i,t}$ and R_t, assume $f(y_H) \geq f(y_L)$, $y_H = (w_{i,t}, s_H, R_t)$, $y_L = (w_{i,t}, s_L, R_t)$. Let $a_L \in \Gamma(y_L)$ attain $T(f)(y_L)$ and $a_H \in \Gamma(y_H)$ attain $(Tf)(y_H)$. Then:

$$\begin{aligned} (Tf)(y_L) &= d(y_L, a_L) + \sum_{R' \in \tilde{\tilde{\mathcal{R}}}} M_{i,t+1} Q_R(R_t, R') \sum_{s'_j \in \mathcal{S}} Q_s(s_L, s'_j) f(w_L(s'_j, R'), s'_j, R') \\ &\leq d(y_L, a_L) + \sum_{R' \in \tilde{\tilde{\mathcal{R}}}} M_{i,t+1} Q_R(R_t, R') \sum_{s'_k \in \mathcal{S}} Q_s(s_L, s'_k) \sum_{s'_j \in \mathcal{S}} q(s'_j|s'_k, R') f(w_L(s'_j|s'_k, R'), s'_k, R') \\ &\leq d(y_H, a_L) + \sum_{R' \in \tilde{\tilde{\mathcal{R}}}} M_{i,t+1} Q_R(R_t, R') \sum_{s'_k \in \mathcal{S}} Q_s(s_H, s'_k) \sum_{s'_j \in \mathcal{S}} q(s'_j|s'_k, R') f(w_L(s'_j|s'_k, R'), s'_k, R') \\ &\leq d(y_H, a_H) + \sum_{R' \in \tilde{\tilde{\mathcal{R}}}} M_{i,t+1} Q_R(R_t, R') \sum_{s'_j \in \mathcal{S}} Q_s(s_H, s'_j) f(w_H(s'_j, R'), s'_j, R') \\ &= (Tf)(y_H) \end{aligned}$$

where the three inequalities follow from $w(s'_j|s'_k, R_{t+1}) \geq w(s'_j, R_{t+1})$ almost everywhere, from the monotonicity of the Markov chain process followed by $s_{i,t}$ given the feasibility of a_L if the initial state is y_H (f is non-decreasing), and from the optimality of a_H. Thus, T maps increasing functions into increasing functions, hence the fixed point is strictly increasing because there exists a lower bound which is (weakly) increasing ($f = w_{i,t}$) and the Knaster-Tarski theorem applies. Finally, the proof that the value function is non-increasing in R_t for a fixed $s_{i,t}$ follows the same steps applied to the Markov process for R_t, with the difference that the stochastic net worth $w(s_{i,t+1}, R'_j|R'_k)$ is smaller or equal than $w(s_{i,t+1}, R'_j)$ almost everywhere, which immediately implies that T maps decreasing functions of R_t into decreasing functions of R_t since

$$\begin{aligned} (Tf)(y_H) &= d(y_H, a_H) + \sum_{s' \in \mathcal{S}} M_{i,t+1} Q_s(s_{i,t}, s') \sum_{R'_j \in \tilde{\tilde{\mathcal{R}}}} Q_R(R_H, R'_j) f(w_H(s', R'_j), s', R'_j) \\ &\leq d(y_H, a_H) + \sum_{s' \in \mathcal{S}} M_{i,t+1} Q_s(s_{i,t}, s') \sum_{R'_k \in \tilde{\tilde{\mathcal{R}}}} Q_R(R_H, R'_k) \sum_{R'_j \in \tilde{\tilde{\mathcal{R}}}} q(s', R'_j|R'_k) f(w_H(s', R'_j|R'_k), s', R'_k) \\ &\leq d(y_L, a_H) + \sum_{s' \in \mathcal{S}} M_{i,t+1} Q_s(s_{i,t}, s') \sum_{R'_k \in \tilde{\tilde{\mathcal{R}}}} Q_R(R_L, R'_k) \sum_{R'_j \in \tilde{\tilde{\mathcal{R}}}} q(s', R'_j|R'_k) f(w_H(s', R'_j|R'_k), s', R'_k) \\ &\leq d(y_L, a_L) + \sum_{s' \in \mathcal{S}} M_{i,t+1} Q_s(s_{i,t}, s') \sum_{R'_j \in \tilde{\tilde{\mathcal{R}}}} Q_R(R_L, R'_j) f(w_L(s', R'_j), s', R'_j) \\ &= (Tf)(y_L). \end{aligned}$$

■

Proof of Proposition 6. The first-order conditions of problem (17)-(18) with respect to $b(s_{i,t+1}, R_{t+1})$ are:

$$R_t \nu(s_{i,t+1}) M_{i,t+1} = \frac{\nu_{i,t}}{1 + \lambda_{i,t}}. \tag{A.5}$$

Solving the previous equation for $M_{i,t+1}$, the stochastic discount factor can be obtained as

$$M_{i,t+1} = \frac{1}{R_t(1+\lambda_{i,t})} \frac{\nu_{i,t}}{V_w(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})}. \tag{A.6}$$

The envelope condition with respect to the state variable $w_{i,t}$ is $\nu_{i,t} = V_w(w_{i,t}, s_{i,t}, R_t)$. Plugging the expression of the multiplier $\nu_{i,t}$ from the envelope condition into (A.6) yields $M_{i,t+1} = \mu^M_{i,t} \frac{V_w(w_{i,t}, s_{i,t}, R_t)}{V_w(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})}$. ■

Proof of Proposition 7. The excess returns on individual securities can be then priced as $E_t\left[M_{i,t+1}R^e_{i,t+1}\right] = 0$, or equivalently as

$$E_t\left[\frac{M_{i,t+1}}{E_t[M_{i,t+1}]} R^e_{i,t+1}\right] = 0. \tag{A.7}$$

Writing the previous expression in terms of moments yields

$$Cov_t\left[\frac{M_{i,t+1}}{E_t[M_{i,t+1}]}, R^e_{i,t+1}\right] + \overbrace{E_t\left[\frac{M_{i,t+1}}{E_t[M_{i,t+1}]}\right]}^{1} E_t\left[R^e_{i,t+1}\right] = 0,$$

that is

$$E_t\left[R^e_{i,t+1}\right] = -Cov_t\left[\frac{M_{i,t+1}}{E_t[M_{i,t+1}]}, R^e_{i,t+1}\right] = -\frac{Cov_t\left[\frac{1}{g^v_{i,t+1}}, R^e_{i,t+1}\right]}{E_t\left[\frac{1}{g^v_{i,t+1}}\right]}. \tag{A.8}$$

Defining $\beta_{i,t} \equiv \frac{Cov_t\left[\left(g^v_{i,t+1}\right)^{-1}, R^e_{i,t+1}\right]}{Var_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}$ and $\lambda_{i,t} \equiv -\frac{Var_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}{E_t\left[\left(g^v_{i,t+1}\right)^{-1}\right]}$ yields $E_t\left[R^e_{i,t+1}\right] = \beta_{i,t}\lambda_{i,t}$. ■

Proof of Corollary 2. The pricing kernel can be log-linearized taking a Taylor expansion of $\log M_{i,t+1}$ around $E_t[M_{i,t+1}]$ as

$$\begin{aligned}\log M_{i,t+1} &\simeq \log E_t[M_{i,t+1}] + \frac{1}{E_t[M_{i,t+1}]}\left(M_{i,t+1} - E_t[M_{i,t+1}]\right) \\ &= \log E_t[M_{i,t+1}] + \frac{M_{i,t+1}}{E_t[M_{i,t+1}]} - 1,\end{aligned}$$

which implies

$$\frac{M_{i,t+1}}{E_t[M_{i,t+1}]} \simeq 1 + \log M_{i,t+1} - \log E_t[M_{i,t+1}].$$

Plugging the expression of $M_{i,t+1}$ in Proposition 6 in the previous equation yields

$$\frac{M_{i,t+1}}{E_t[M_{i,t+1}]} \simeq 1 - \log g^v_{i,t+1} + \log E_t\left[g^v_{i,t+1}\right].$$

Rearranging (A.7) yields

$$E_t\left[R^e_{i,t+1}\right] \simeq \beta^w_{i,t}\lambda^w_{i,t}$$

using the fact that $g^v_{i,t+1}$ is a gross growth rate and $\log(1 + (g^v_{i,t+1} - 1)) \simeq g^v_{i,t+1} - 1$. ■

Proof of Proposition 8. Consider the following monotone transformation of $g^v_{i,t+1}$:

$$\log g^v_{i,t+1} = \log V_w(w_{i,t+1}, s_{i,t+1}, R_{t+1}) - \log V_w(w_{i,t}, s_{i,t}, R_t).$$

The first term on the right-hand side is non-increasing in $w_{i,t+1}$, for all $s_{i,t+1}$, R_{t+1}, because of the concavity of $V_w(w_{i,t+1}, s_{i,t+1}, R_{t+1})$. The term $-\log V_w(w_{i,t}, s_{i,t}, R_t)$ is non-decreasing in $w_{i,t}$, for all $s_{i,t}$, R_t, and by the envelope theorem and the envelope theorem $V_w(w_{i,t+1}, s_{i,t+1}, R_{t+1})$ does not depend on $w_{i,t}$ at the optimum. Thus $\log g^v_{i,t+1}$ is non-increasing in $w_{i,t+1}$ and non-decreasing in $w_{i,t}$, implying that $g^v_{i,t+1}$ is non-increasing in $\frac{w_{i,t+1}}{w_{i,t}}$. ■

Before proving Proposition 9, I shall prove the following lemma.

Lemma 2 (Payout Policy) *There exists a net worth cutoff $\overline{w}(s_{i,t}, R_t)$ such that $d_{i,t} = 0$ if $w_{i,t} < \overline{w}(s_{i,t}, R_t)$ and $d_{i,t} = w_{i,t} - \overline{w}(s_{i,t}, R_t)$ if $w_{i,t} \geq \overline{w}(s_{i,t}, R_t)$.*

Proof of Lemma 2. The envelope theorem implies that $V_w(w_{i,t}, s_{i,t}, R_t) = 1 + \upsilon_D$, where $\upsilon_D \geq 0$ denotes the Lagrange multiplier on dividend non-negativity constraint. The concavity of the value function implies that υ_D is decreasing in net worth $w_{i,t+1}$ Thus, $V_w(w_{i,t}, s_{i,t}, R_t) > 1$ if $d_{i,t} = 0$ and $V_w(w_{i,t}, s_{i,t}, R_t) = 1$ if $d_{i,t} > 0$. Define the cutoff $\overline{w}(s_{i,t}, R_t) \equiv \inf\{w_{i,t} : d_{i,t} > 0\}$. Let the optimal choice $d_{i,t} = 0$, $k_{i,t+1} = \overline{k}(s_{i,t}, R_t)$, $b(s_{i,t+1}, R_{t+1}) = \overline{b}(s_{i,t}, R_t)$, $w(s_{i,t+1}, R_{t+1}) = \overline{w}(s_{i,t+1}, R_{t+1})$ attain $V(\overline{w}(s_{i,t}, R_t), s_{i,t}, R_t)$. Then, for all $w_{i,t} > \overline{w}(s_{i,t}, R_t)$, $V_w(w_{i,t}, s_{i,t}, R_t) = 1$ yields $V_w(w_{i,t}, s_{i,t}, R_t) = V(\overline{w}(s_{i,t}, R_t), s_{i,t}, R_t) + \int_{\overline{w}(s_{i,t}, R_t)}^{w_{i,t}} 1 dv$, which is attained by the policy $d_{i,t} = w_{i,t} - \overline{w}(s_{i,t}, R_t)$, $k_{i,t+1} = \overline{k}(s_{i,t}, R_t)$, $b(s_{i,t+1}, R_{t+1}) = \overline{b}(s_{i,t}, R_t)$, $w(s_{i,t+1}, R_{t+1}) = \overline{w}(s_{i,t+1}, R_{t+1})$. ■

Proof of Proposition 9. Consider the following monotonic transformation of the realized equity return $\widetilde{R}_{i,t+1}$:

$$\log \widetilde{R}_{i,t+1} = \log V(w_{i,t+1}, s_{i,t+1}, R_t) - \log \left(V(w_{i,t}, s_{i,t}, R_t) - d_{i,t}\right). \tag{A.9}$$

To establish the result in part i), observe that the first term on the right-hand side increases in $w_{i,t+1}$ because the value function is weakly increasing in net worth, and is not affected by $w_{i,t}$ because of the envelope theorem. From lemma 2, for $w_{i,t} < \overline{w}(s_{i,t}, R_t)$, $d_{i,t} = 0$ which implies that the term $\log(V(w_{i,t}, s_{i,t}, R_t) - d_{i,t})$ is weakly decreasing in $w^{-1}_{i,t}$ because the value function is weakly increasing in net worth. For $w_{i,t} \geq \overline{w}(s_{i,t}, R_t)$, $d_{i,t} = w_{i,t} - \overline{w}(s_{i,t}, R_t)$ and $V_w(w_{i,t}, s_{i,t}, R_t)$ from lemma 2, which implies that $\log(V(w_{i,t}, s_{i,t}, R_t) - d_{i,t})$ is insensitive to $w^{-1}_{i,t}$ for dividend payers. Then $\log \widetilde{R}_{i,t+1}$ is increasing in net worth growth $\frac{w_{i,t+1}}{w_{i,t}}$, and so is $R^e_{i,t+1} = \widetilde{R}_{i,t+1} - R_t$. To establish part ii), observe that, in (A.9), $\log V(w_{i,t+1}, s_{i,t+1}, R_t)$ in weakly increasing in $s_{i,t+1}$, while $\log(V(w_{i,t}, s_{i,t}, R_t) - d_{i,t})$ is weakly decreasing in $s^{-1}_{i,t}$ for $w_{i,t} < \overline{w}(s_{i,t}, R_t)$ because the value function is weakly increasing in $s_{i,t}$, and is insensitive to $s^{-1}_{i,t}$ for $w_{i,t} \geq \overline{w}(s_{i,t}, R_t)$ for the same argument in part i). Thus, $\log \widetilde{R}_{i,t+1}$ is increasing in profitability growth $\frac{s_{i,t+1}}{s_{i,t}}$, and so is $R^e_{i,t+1} = \widetilde{R}_{i,t+1} - R_t$. Finally, the proof of part iii) follows the same steps of the one of part ii), using the fact that the value function is weakly decreasing in R_t, and so is $R^e_{i,t+1} = \widetilde{R}_{i,t+1} - R_t$. ■

A.2 Implementation with Credit Lines

The following proposition shows than the recursive contract in Section 3 can be implemented with a secured line of credit. In this implementation, the line of credit allows the firm to draw or restore right after observing the shocks $(s_{i,t+1}, R_{t+1})$. Thus, it provides the firm with liquidity contingent to the realization of the state.[24] While alternative implementations of the contract exist, for example based on forward contracts[25], credit lines are especially relevant empirically given their widespread use in US public corporations, as I discuss in Section 3.

[24]See also Nikolov, Schmid, and Steri (2019).

[25]See, for example, Rampini and Viswanathan (2013).

Proposition 10 (Credit Lines Implementation) *The one-period contingent claims* $b(s_{i,t+1}, R_{t+1})$ *in (17)-(21) can be equivalently implemented with a line of credit with drawn part* $c(s_{i,t+1}, R_{t+1})$ *and limit* $c_L(s_{i,t+1}, R_{t+1})$ *defined as the maximum drawn amount that exhausts the firm i's borrowing capacity, that is as the solution of* $\theta k_{i,t+1} = E_t\left[M_{i,t+1}V(w_L(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right]$*, where* $w_L(s_{i,t+1}, R_{t+1}) = \Pi(k_{i,t+1}, s_{i,t+1}) + (1-\delta)k_{i,t+1} - R_t c_L(s_{i,t+1}, R_{t+1})$*.*

Proof of Proposition 10. Consider any sequence of net repayments (restorations) on the credit line $\{c_R(h_{i,t})\}_{t=\tau}^{\infty}$ after any history $h_{i,t} \equiv \{k_{i,j}, c_{Ri,j-1}, d_{i,j-1}, s_{i,j}, R_j\}_{j=1}^{t}$ of previous policies and both aggregate and idiosyncratic shocks, where $c_{Ri,t}$ is the observed restoration of firm i at time t.Negative restorations represent additional draws from the line of credit. Define the drawn part of the line of credit $c_D(h_{i,\tau})$ at any time τ such that the present value of future restorations is equal to the amount to be eventually repaid to the lender, that is

$$R_{\tau-1,\tau}c_D(h_{i,\tau}) \equiv E_\tau\left[\sum_{t=\tau}^{\infty}\frac{1}{R_{\tau,t}}c_R(h_{i,t})\right], \tag{A.10}$$

where $R_{p,p+l}$ denotes the discount rate of the lenders between periods p and $p+l$. Rewriting (A.10) one period ahead yields

$$R_{\tau,\tau+1}c_D(h_{i,\tau+1}) = E_{\tau+1}\left[\sum_{t=\tau+1}^{\infty}\frac{1}{R_{\tau+1,t}}c_R(h_{i,t})\right]. \tag{A.11}$$

Using $R_{\tau,t} = R_{\tau,\tau+1}R_{\tau+1,t}$, one obtains

$$R_{\tau-1,\tau}c_D(h_{i,\tau}) = E_\tau\left[\sum_{t=\tau}^{\infty}\frac{1}{R_{\tau,\tau+1}R_{\tau+1,t}}c_R(h_{i,t})\right],$$

and, by the law of iterated expectations

$$R_{\tau-1,\tau}c_D(h_{i,\tau}) = c_R(h_{i,\tau}) + E_\tau\left[\frac{1}{R_{\tau,\tau+1}}E_{\tau+1}\left[c_R(h_{i,\tau+1}) + \frac{1}{R_{\tau+1,\tau+2}}c_R(h_{i,\tau+2}) + \frac{1}{R_{\tau+1,\tau+3}}c_R(h_{i,\tau+3}) + ...\right]\right]. \tag{A.12}$$

Combining (A.12) with (A.10) implies

$$R_{\tau-1,\tau}c_D(h_{i,\tau}) = c_R(h_{i,\tau}) + E_\tau\left[c_D(h_{i,\tau+1})\right]. \tag{A.13}$$

At any time t, the restoration can be recovered using (A.13) as $c_R(h_{i,\tau}) = R_{\tau-1}c_D(h_{i,\tau}) - E_\tau\left[c_D(h_{i,\tau+1})\right]$ in which the shorthand notation $R_{\tau-1}$ has been used, as in the recursive problem in Section 3, for the one-period riskfree rate $R_{\tau-1,\tau}$. Thus, the problem can be expressed with the recursive representation in Appendix A.3, where the flow transfers $\tau_{i,t} = c_R(h_{i,\tau})$ can be recovered from the one-period contingent claims $b(s_{i,t+1}, R_{t+1}) = c_D(h_{i,\tau})$, and the lender promise-keeping constraint is (A.13). Finally, by Lemma 3 in the Appendix A.3, the contracting problem admits the net worth representation in Section 3. ■

Proposition 10 can be extended in a straightforward way to allow for both straight debt and credit lines. For example, the presence of a monitoring cost or fee on the credit line would lead the firm to privilege straight debt for the state-uncontingent part of state-contingent debt $b(s_{i,t+1}, R_{t+1})$ (i.e. the common part across states), and to use the credit line to implement the state-contingent repayment profile implied by $b(s_{i,t+1}, R_{t+1})$.

A.3 Recursive Formulation of the Contracting Problem

In this section, I show that the contract in Section 3 can be formulated as a recursive dynamic programming problem. First, following Albuquerque and Hopenhayn (2004), I define feasible and enforceable contracts. Then, I specify the firm's optimization problem by introducing equilibrium contracts. Equilibrium contracts define the Pareto frontier between the value for the firm (which I interpret as internal equity) and the value for the lender (which I interpret as external financing), and impose restrictions the realizations of corporate policies that can be observed in the data.

A contract for a firm i that enters the industry specifies a sequence of capital advancements $\{k_{i,t+1}\}_{t=0}^{\infty}$, a sequence of transfers $\{\tau_{i,t}\}_{t=0}^{\infty}$ from the firm to the lender, and a sequence of dividend payments $\{d_{i,t}\}_{t=0}^{\infty}$ to the firm's shareholders.[26] The aforementioned investment, financing, and dividend policies are fully state-contingent, and depend on the entire history $h_{i,t} \equiv \{k_{i,j}, \tau_{i,j-1}, d_{i,j-1}, s_{i,j}, R_j\}_{j=1}^{t}$ of previous policies and both aggregate and idiosyncratic shocks. The current shock is part of the history, consistent with the timing described in Section 3. Importantly, the contract jointly specifies financing, dividend, and investment policies, in close analogy with the covenants that are routinely found in loan agreements. On the firm's perspective, a contract must be budget feasible, that is the firm's internally generated profits must suffice to cover investment expenses, repayments, and dividend distributions. In addition, the firm cannot raise additional funds by issuing equity, that is $d_{i,t} \geq 0$ for all t. The latter condition prevents the firm from raising costless external equity (i.e. to have negative distributions). Without this constraint, the contracting problem would be trivial. Finally, the contract must be consistent with the firm's limited liability, that is the value of the firm must be non-negative to prevent non-strategic default.

Definition 1 (Feasible Contract) *Let $\mathcal{H}_i$ be the set of all possible histories for firm i. A feasible contract is a mapping $\mathcal{C}_i : \mathcal{H}_i \to \mathbb{R}^3$ such that for all $h_{i,t} \in \mathcal{H}_i$, $(k_{i,t}, \tau_{i,t}, d_{i,t}) = \mathcal{C}_i(h_{i,t})$, and, for all t:*

$$d_{i,t} \geq 0, \tag{A.14a}$$

$$d_{i,t} + \tau_{i,t} + [k_{i,t+1} - (1-\delta)k_{i,t}] \leq \Pi(k_{i,t}, s_{i,t}), \tag{A.14b}$$

$$E_t\left[\sum_{\tau=0}^{\infty} M_{i,t+\tau} d_{i,t+\tau}\right] \geq 0. \tag{A.14c}$$

The contract has limited enforcement. The firm's incentive problem is illustrated in the extensive form game in Figure A.1.

[Insert Figure A.1 Here]

Each period t, after observing the shocks and choosing investment, financing, and payout policies, the firm faces an outside opportunity of total value $O(k_{i,t+1}, s_{i,t}, R_t)$. The value of the outside opportunity is common knowledge to both parties, and depends on the newly purchased capital stock, and on the current state of the economy. Different interpretations of the outside opportunity

[26] By convention, positive transfers represent repayments to the lender, while negative transfer are inflows for the firm.

can be entertained. For instance, the firm may liquidate the capital and disappear. The firm can choose either to renege the contract, divert the capital stock, and use it to pursue an outside opportunity, or to continue operations. In the former scenario, lenders liquidate the firm, and the liquidation value is split between the two parties. In particular, the firm is left with $\theta k_{i,t+1}$, while the lender expropriates $(1-\theta)k_{i,t+1}$. In equilibrium, the firm therefore compares the value of continuing running the firm with its share of the liquidation value.[27] Incentive compatibility requires the diversion value not to exceed the value of staying in the contractual relationship. In Figure A.1, this corresponds to the subgame perfect equilibrium $\{\overline{R}, L\}$ in which the firm never reneges the contract because of the threat by the lender to liquidate the firm. This leads to the following definition of enforceable contract (or self-enforcing contract).

Definition 2 (Enforceable Contract) *A feasible contract $\mathcal{C}_i(\cdot)$ is enforceable if after any history $h_{i,t}$ and for all t, the following enforcement constraint is satisfied:*

$$\theta k_{i,t+1} \leq E_t\left[\sum_{\tau=1}^{\infty} M_{i,t+\tau} d_{i,t+\tau}\right] \tag{A.15}$$

In equilibrium, contracts must be consistent with both the firm and the lender maximizing their lifetime utility. Since lenders are competitive, equilibrium contracts attain the maximum initial value for the borrower with the lender breaking even. The lender's participation constraint therefore states that the expected discounted value of repayments is non-negative.

Definition 3 (Equilibrium Contract) *An equilibrium contract $\mathcal{C}_i(\cdot)$ is an enforceable contract such that the borrower maximizes*

$$E_0\left[\sum_{t=0}^{\infty} M_{i,t} d_{i,t}\right] \tag{A.16}$$

subject to the lender's participation constraint

$$E_0\left[\sum_{t=0}^{\infty} R_{0,t} \tau_{i,t}\right] \geq 0. \tag{A.17}$$

where $R_{0,t}$ is the lender's discount rate between time 0 and time t.

Dealing with equilibrium contracts specified as sequence problems would require to keep track of an infinite sequence of occasionally binding constraints. This is due to the enforcement constraints in Equation (A.15), which must be satisfied in all future periods t. In this section, I formulate the problem recursively, so that dynamic programming techniques can be applied. I propose two recursive formulations. First, following Spear and Srivastava (1987) and Abreu, Pearce, and Stacchetti (1990), I formulate the dynamic limited enforcement model in recursive form with firm's capital and

[27] Notice that the value of the outside opportunity is irrelevant in this setup, because the lender always chooses to liquidate the firm. The strategy $\overline{L}$ in fact delivers a null payoff to the lender, and is therefore dominated by L. This is equivalent to assume $O(k_{i,t+1}, s_{i,t}, R_t) = \theta k_{i,t+1}$ in the case of liquidation.

promised utility to the lender as endogenous state variables. This formulation allows to interpret optimal contracts as internal equity/external financing pairs on a Pareto frontier.

I define promised utility $b_{i,t}$ at time t as the value of future transfers to the lender, that is:

$$b_{i,t} \equiv \sum_{j=0}^{\infty} \tau_{i,t}. \tag{A.18}$$

With this definition, Spear and Srivastava (1987) show that the equilibrium contracting problem defined in (A.16), subject to (A.14a), (A.14b), (A.14c), (A.15), and (A.17), has a stationary representation as a dynamic programming problem. This leads to the following formulation:

$$V(k_{i,t}, b_{i,t}, s_{i,t}, R_t) = \max_{\{d_{i,t}, k_{i,t+1}, b(s_{i,t+1}, R_{t+1})\}} d_{i,t} + E_t\left[M_{i,t+1} V(k_{i,t+1}, b(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \tag{A.19}$$

$$s.t.$$

$$d_{i,t} \geq 0 \tag{A.20}$$

$$d_{i,t} \leq \Pi(k_{i,t}, s_{i,t}) - I_{i,t} - \tau_{i,t} \tag{A.21}$$

$$I_{i,t} = k_{i,t+1} - (1-\delta) k_{i,t} \tag{A.22}$$

$$\tau_{i,t} = R_{t-1} b_{i,t} - E_t[b(s_{i,t+1}, R_{t+1})] \tag{A.23}$$

$$V(k_{i,t}, b_{i,t}, s_{i,t}, R_t) \geq 0 \tag{A.24}$$

$$\theta k_{i,t+1} \leq E_t\left[M_{i,t+1} V(k_{i,t+1}, b(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \tag{A.25}$$

$$b_{i,0} \geq 0 \tag{A.26}$$

In this formulation, equilibrium contracts maximize the firm's equity value, using promised utility and the capital stock as endogenous state variables. In analogy with the sequential formulation of the contract, Constraint (A.20) is the dividend non-negativity constraint, Constraint (A.21) is the budget constraint, where the auxiliary variables $I_{i,t}$ and $\tau_{i,t}$ define the current investment expense and transfer to the lender. The law of motions of $I_{i,t}$ and $\tau_{i,t}$ are specified in Constraints (A.22) and (A.23). Constraint (A.23) can be interpreted as a promise-keeping constraint for the lender. Constraint (A.24) is the limited-liability constraint for the borrower. Constraint (A.25) is the enforcement constraint, which states that the diversion value cannot exceed the continuation value. Thus, reneging the contract is never optimal. Finally, contracts are initialized such that the participation constraint (A.26) for the lender is satisfied.

The problem can be further simplified by reducing the dimension of the state space. This can be achieved using net worth as a state variable, in line with Abreu, Pearce, and Stacchetti (1990), Rampini and Viswanathan (2010), and Rampini and Viswanathan (2013). Realized net worth in state $s_{i,t+1}$ and R_{t+1} is defined as $w(s_{i,t+1}, R_{t+1}) \equiv \Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t+1} - R_t b(s_{i,t+1}, R_{t+1})$, and determines the amount of resources that are available to the firm in a certain state, net of liabilities. Intuitively, net worth is the corporate counterpart of household's wealth, and captures how constrained a company is in terms of resources to allocate to investment, and distributions. This leads to the following lemma.

Lemma 3 (Recursive Problem) *The constrained optimization problem in (A.19)-(A.26) is equivalent to:*

$$V(w_{i,t}, s_{i,t}, R_t) = \max_{\{d_{i,t}, k_{i,t+1}, b(s_{i,t+1}, R_{t+1})\}} d_{i,t} + E_t\left[M_{i,t+1} V(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \tag{A.27}$$

$$s.t.$$

$$d_{i,t} \geq 0 \tag{A.28}$$

$$w_{i,t} \geq d_{i,t} + k_{i,t+1} - E_t[b(s_{i,t+1}, R_{t+1})] \tag{A.29}$$

$$w(s_{i,t+1}, R_{t+1}) \leq \Pi(k_{i,t+1}, s_{i,t+1}) + (1-\delta)k_{i,t+1} - R_t b(s_{i,t+1}, R_{t+1}) \qquad \forall s_{it+1}, R_{t+1} \tag{A.30}$$

$$\theta k_{i,t+1} \leq E_t\left[M_{i,t+1} V(w(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right] \tag{A.31}$$

$$b_{i,0} \geq 0 \tag{A.32}$$

Proof of Lemma 3. By the definition of net worth, Equation (21) must also hold for the current state, which is measurable respect to the information set at time t. Hence

$$w_{i,t} \leq \Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t} - R_{t-1} b_{i,t}. \tag{A.33}$$

Because free disposal is never optimal, Equations (A.21) , (20) and (21) are always binding. This yields:

$$\Pi(k_{i,t}, s_{i,t}) + (1-\delta)k_{i,t} - R_{t-1} b_{i,t} = d_{i,t} + k_{i,t+1} - E_t[b(s_{i,t+1}, R_{t+1})]. \tag{A.34}$$

Equations (20) and (21), and Equations (A.21), (A.22) and (A.23) are therefore equivalent. The enforcement constraint in conjunction with the dividend non-negativity constraint imply that the limited liability constraint is always satisfied. This constraint is therefore redundant, and can be omitted from the problem. In fact, because at the optimum $V(k_{i,t}, b_{i,t}, s_{i,t}, R_t) = d_{i,t} + E_t\left[M_{i,t+1} V(k_{i,t+1}, b(s_{i,t+1}, R_{t+1}), s_{i,t+1}, R_{t+1})\right]$, Equation (A.25) can be rewritten as

$$V(k_{i,t}, b_{i,t}, s_{i,t}, R_t) \geq \theta k_{i,t+1} + d_{i,t}. \tag{A.35}$$

By (A.20), $d_{i,t} \geq 0$. Thus:

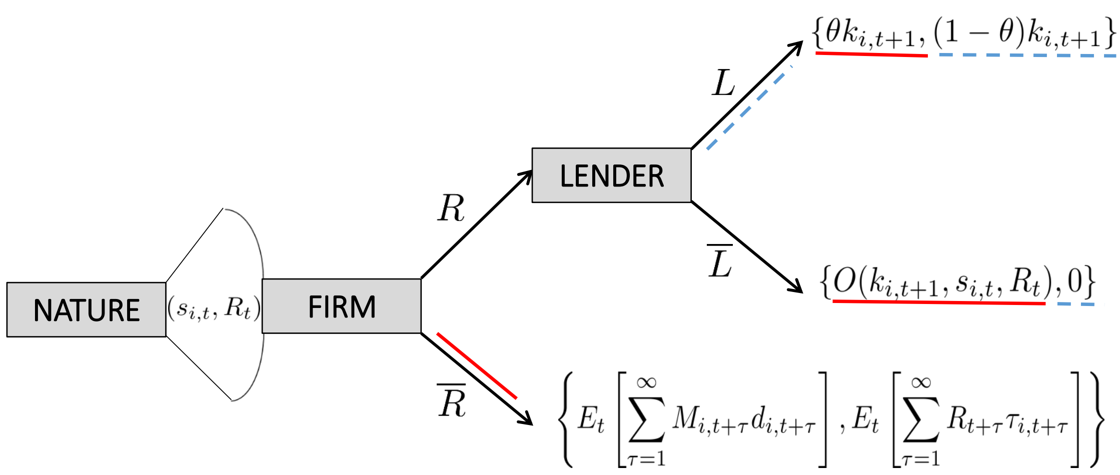
$$V(k_{i,t}, b_{i,t}, s_{i,t}, R_t) \geq \theta k_{i,t+1} + d_{i,t} \geq \theta k_{i,t+1}, \tag{A.36}$$

which implies (A.24) because the fact that $\lim_{k_{i,t} \downarrow 0} \Pi(k_{i,t}, s_{i,t}) = \infty$ makes optimal capital always strictly positive. Because only $w_{i,t}$, and not its individual components predetermined at time t, affect the return function $d_{i,t}$, the two formulations are equivalent. ■

Figure A.1

The Enforcement Problem: Extensive Form Game.

The figure shows the extensive form of the game from which enforcement constraints arise as an equilibrium outcome. Red solid lines and blue dashed lines represent optimal strategies and payoffs for the firm and the lender respectively. The possible strategies for the borrower are either to renege the contract ($\overline{R}$), or to continue running the firm (R). If the borrower decides to renege the contract, The possible strategies for the lender are either to liquidate the firm (L), or to not liquidate the firm ($\overline{L}$). At time t and for firm i, $M_{i,t+1}$ denotes the stochastic discount factor, R_t is the lender's discount rate, $d_{i,t}$ the dividend payment, $\tau_{i,t}$ the repayment to the lender, $k_{i,t}$ the firm's capital stock, $O(k_{i,t+1}, s_{i,t}, R_t)$ the value of the outside opportunity for the entrepreneur, and $1-\theta$ the fraction of capital the lender can expropriate upon liquidation. $s_{i,t}$ is a firm-specific productivity shock.



A.4 GMM Testing Procedure

Empirical tests for all the asset pricing models in Section 5 are implemented in stochastic discount factor form along the lines of Cochrane (2001), to which I refer for a textbook treatment of such tests. The pricing condition above for a portfolio with weight ω_i in the equity of firm i can be restated as

$$E_t \left[\begin{array}{c} \sum_{i=1}^{N} \overline{\kappa}\omega_i R^e_{i,t+1} + \sum_{i=1}^{N} \omega_i \overline{a} \frac{w(s_{i,t+1},R_{t+1})-w_{i,t}}{w_{i,t}} R^e_{i,t+1} + \sum_{i=1}^{N} \omega_i \overline{b} \frac{s_{i,t+1}-s_{i,t}}{s_{i,t}} R^e_{i,t+1} \\ + \sum_{i=1}^{N} \omega_i \overline{c} \frac{R_{t+1}-R_t}{R_t} R^e_{i,t+1} - \sum_{i=1}^{N} \omega_i e^{\overline{\kappa}-1} \end{array} \right] = 0,$$

using a linear approximation for $M_{i,t+1}$, where $\overline{a}$, $\overline{b}$ and $\overline{c}$ denote the constant loadings on net worth growth, profitability growth, and interest rate growth respectively, $\overline{\kappa} \equiv 1 - \log E_t[M_{i,t+1}]$, and N denotes the number of securities in the portfolio.

Denote by $y_t \equiv (R^e_t, w_t, s_t, R_t)$ the vector of data used in the estimation, where the i index has been omitted to simplify notation. Then the set of moments conditions is:

$$g(\hat{\theta}, y_t) \equiv [M_{t+1}(1 + R^e_t) - 1]\,. \tag{A.37}$$

A sufficient condition for local identification is that the covariance matrix of returns has full rank (Newey and McFadden (1994)). The objective function for the GMM estimation is:

$$\min_{\hat{\theta}\in\Theta} E^T[g'(\hat{\theta}, y_t)] W E^T[g(\hat{\theta}, y_t)], \tag{A.38}$$

where the operator $E^T(\cdot)$ denotes the sample mean for a time series of length T, and W is the positive definite weighting matrix. Estimation is by two-step GMM, with HAC standard errors. The kernel is Newey-West with a lag length of 1 year. The first-stage weighting matrix puts an equal weight on the moment conditions. The R^2 and MAE reported in the text are from first-stage estimations. The R^2 measure is computed as in Campbell and Vuolteenaho (2004). The J-test of overidentifying restrictions is performed as in Hansen and Singleton (1982), and the Hansen-Jagannathan distance, its test-statistics, and its p-value are derived as in Appendix C of Jagannathan and Wang (1996).

A.5 Data and Variables

The empirical analyses in Section 5 use data about portfolios and factors to test the Contracting Model, the CAPM, the Consumption CAPM, and the Fama-French three-factor model. The sample period is from 1965 to 2013.

The Contracting Model variable are constructed from common shares (share codes [shrcd] equal to 10 or 11) from the Compustat/CRSP merged dataset. In order to prevent look-ahead bias, fiscal years are matched to calendar years with the procedure in Fama and French (1992). Specifically, returns on the test assets formed in June of year t are matched to accounting data from the last fiscal year ending in calendar year $t-1$. This guarantees a gap of at least six months between accounting data and the date of portfolio formation. In constructing the factors for the Contracting CAPM, net worth is measured as the book value of equity, consistent with the accounting definition in the contracting model. Following Daniel and Titman (2006), the book value of equity is computed using redemption, liquidation, and par value of preferred shares, and accounting for investment tax credits and postretirement benefits. The data items used are obtained from merging the Compustat/CRSP merged with CRSP. Data on the market return, HML, SMB, CMA, RMW, the riskfree rate, and industry classification are from Kenneth French's website. Data on consumption growth in nondurable and services are from the US national accounts.

The following table summarizes variable definitions with reference to Compustat and CRSP items.

Variable	Construction
Value of Preferred Stocks (PS)	If available, in this order: $PSTKRV$, $PSTKL$, $PSTK$.
Book Equity (BE)	$CEQ + TXDITC$ (if available) $-PS$
Market Capitalization	$\log \frac{\lvert PRC \rvert \cdot SHROUT}{1000}$ (in June)
Book-to-Market Equity (B/M)	$\log \frac{BE}{\lvert PRC \rvert \cdot SHROUT/1000}$
Net Worth	$AT - DLC - DLTT$
Profitability	$\frac{REVT-COGS}{AT}$
Momentum	$\sum_{m=2}^{12} \log(1+ret_{t-m})$, t :month of forecasted return
Investment (Asset Pricing Tests)	$\frac{AT-L.AT}{L.AT}$
Investment (Corporate Policies)	$\frac{CAPX-SPPE}{PPEGT}$
Operating Income	$\frac{OIBDP}{AT}$
Leverage	$\frac{DLCC+DLT}{\lvert PRC \rvert \cdot SHROUT/1000+DLCC+DLT}$
Distributions	$\frac{DVT}{AT}$
Tobin's Q	$\frac{DLTT+DLC+PRCC_F \cdot CSHO}{AT}$

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c/o University of Geneva, Bd. Du Pont d'Arve 42, CH-1211 Geneva 4
T +41 22 379 84 71, rps@sfi.ch, www.sfi.ch