Relative measures of economic insecurity*

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Abstract. We characterize a new class of individual measures of economic insecurity in a setting where there is a single relevant variable that can be interpreted as income or consumption. Insecurity is intended to capture the difficulties faced by an economic agent when confronted with adverse events. We work with an intertemporal model and base our measures on the changes in the variable when moving from one period to the next. Our approach is axiomatic and differs from the existing literature in two respects. First, we adopt a relative (scale-invariant) concept of insecurity and, second, we restrict attention to a relatively small set of requirements that we consider plausible and intuitively appealing. As a result, we identify a large class of measures that can be thought of as providing a tool box to empirical researchers who can select those members of our class that they consider suitable for the application in question. In addition, we present a dominance criterion based on our new insecurity measures. *Journal of Economic Literature* Classification Nos.: D63, D84.

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1 Introduction

The notion of economic insecurity has become increasingly important in policy debates. Economic insecurity is the anxiety produced by the possible exposure to adverse economic events and by the anticipation of the difficulty to recover from them. The Commission on the Measurement of Economic Performance and Social Progress (see Stiglitz, Sen, and Fitoussi, 2009) argued in its report that economic insecurity should be one of the dimensions according to which individual well-being is to be analyzed. There are many factors shaping economic insecurity which are reflected in the variety of approaches used to measure them (see, for example, Osberg, 2018, and Rohde and Tang, 2018, for excellent surveys). Economic insecurity strongly impacts numerous facets of life such as the quality of health (Catalano, 1991), the incidence of family break-ups (Larson, Wilson, and Beley, 1994), consumption patterns (Linz and Semykina, 2010), and fertility choices (Clark and Lepinteur, 2022).

Our approach to measuring the insecurity experienced by an individual is based on the variations in the values of a single economic variable—such as aggregate consumption or income—over time. What we see as the crucial question is how well an economic agent can deal with a future loss, and our fundamental hypothesis is that past gains and losses determine the confidence an individual has today.

This paper provides an alternative perspective on the measurement of economic insecurity that differs from the analyses carried out in Bossert and D'Ambrosio (2013) and in Bossert, Clark, D'Ambrosio, and Lepinteur (2023). While absolute (in the sense of translation-invariant) indexes were the focus of these earlier contributions, we examine relative (that is, scale-invariant) measures instead. However, rather than merely reformulating the previous axioms and results in a scale-invariant setting, we follow a considerably more comprehensive approach in that we allow for a large class of measures that are compatible with our requirements. This is achieved by restricting attention to a small set of axioms that, we believe, have strong intuitive appeal but are still powerful enough to impose some structure on the resulting class of measures. Thus, there are two aspects in which the current proposal differs from those of the above-mentioned earlier contributions.

First, we replace translation invariance with scale invariance, thereby focusing on a relative rather than an absolute notion of insecurity. This is analogous to the field of inequality measurement, where both relative and absolute indexes have been employed for many decades. As is the case for inequality measures, we do not think of either of these two underlying invariance principles to be superior to the other; rather, the objective as we see it is to provide empirical researchers with a large tool box from which they can select a measure (or a class of measures) they find suitable for the application at hand. In this vein, we do not think that any of the properties that were employed in earlier work should be abandoned in favor of the axioms presented here; again, this viewpoint is completely in line with the motivation that underlies the desire to provide more than just a single measure of inequality to be used in all applications. We do not think that there is universal agreement regarding what is the 'best' index in all situations and, on the basis of this lack of unanimity, it seems only natural to provide alternatives, as long as they are based on assumptions that are ethically appealing.

The second important deviation manifests itself in the considerably broader vantage point adopted here. Rather than identifying a relatively narrow class of measures the members of which satisfy a set of requirements, we restrict attention to a small set of axioms that we consider essential for a relative measure of individual insecurity. This results in a multitude of possible choices that are available for applied studies. We argue that our axioms are very natural indeed for the issue at hand and, therefore, the class of indexes they characterize provides a solid basis for future explorations; more specific measures (or families of measures) can be obtained by adding further properties that are deemed desirable.

Much of the earlier literature that examined an absolute approach to insecurity measurement is motivated by the observation that the variable(s) relevant for the phenomenon may take on non-positive values; this is the case, for instance, when net wealth is being considered. However, there are circumstances under which the variable in question can reasonably be assumed to be positive-valued. If the variable being considered represents individual consumption, it is plausible to require its values to be positive. Likewise, the income or budget available to a consumer is typically assumed to be positive in economic models. For convenience, we refer to the variable in question as income in this paper but note that our observations are applicable to any framework in which positive-valuedness is a suitable assumption.

We employ an intertemporal setup based on streams of positive individual incomes that may be of varying length. An individual measure (or index) of economic insecurity is then defined as a sequence of functions, one for each possible length of a stream, each of which assigns an insecurity value to each stream in its domain. That we allow the lengths of these streams to be different from one individual to another is essential; if only streams of a fixed length were permitted, this restriction would amount to the assumption that all members of a society become economic agents at the same time—which, to us, seems overly demanding. To be clear, if we consider a stream that involves two time periods, for example, we do not think of these periods representing the chronological age of an agent; rather, the two-period span can be thought of as the economic age of the individual. The interpretation thus is that the agent started making independent economic choices at the beginning point of the stream—anything that happened before was determined by her or his parents or guardians. A concrete (and natural) method of defining the starting point of an agent's economic lifetime is the time at which he or she enters the labor market for the first time.

Our result identifies the class of individual measures of economic insecurity that satisfy three axioms we consider essential, in addition to being scale invariant. The first of these is a natural monotonicity condition that requires the index to respond appropriately to specific changes in one of the components of an income stream. As should become clear once it has been defined formally, the axiom is intuitively appealing and suitably captures the notion of insecurity. In the context of discussing this property, it is important to emphasize that our approach links an agent's current sentiment of insecurity to the gains and losses experienced in the past. Thus, loosely speaking, a past loss contributes to a higher level of insecurity, whereas experiencing a gain in an earlier period reduces insecurity. This observation highlights that our notion of insecurity is not based on some (ex-ante) notion of variation: losses and gains have opposite effects on the value of the insecurity measure, which is in clear conflict with the properties of a dispersion measure, such as the variance.

Our second axiom is extension invariance. Suppose that an income stream is extended by appending one additional past period in which income is the same as in the most remote period prior to the extension. Because there is neither an additional gain nor an additional loss in this case, it seems very natural to require that an extension of this nature has no effect on economic insecurity. In contrast to the above-described monotonicity axiom that restricts attention to income streams of the same length, the property of extension invariance imposes restrictions on the comparison of streams that differ in the number of past periods taken into consideration.

The third axiom is a property that we refer to as temporal decomposability. As is the case for extension invariance, it provides a link between the economic insecurity assigned to streams of different lengths and, moreover, it is responsible for endowing the index with an additively separable structure. Because of the appealing independence properties associated with additively separable measures, this appears to be a fundamental and essential property as well. We also note that virtually all measures that are commonly used in the context of social index numbers are, indeed, additively separable.

Section 2 of the paper introduces our notation and definitions, followed by the statement and proof of our main result. In particular, we characterize the class of relative individual measures of economic insecurity that satisfy the monotonicity property, extension invariance, and the temporal decomposability requirement alluded to above. In Section 3, we define a dominance criterion that specifies the circumstances under which all of our measures agree. Section 4 provides some guidance regarding the choice of a member of our class to be used in applied studies. The section concludes with a few thoughts on possible future work.

2 Measuring relative insecurity

We examine the measurement of individual insecurity in a context that involves a single positive-valued variable. For concreteness, we refer to this variable as income but note that our approach is applicable under numerous alternative interpretations, such as thinking of the requisite values as indicators of consumption.

For any $T \in \mathbb{N}$, let $\mathbb{R}^{T+1}_{++}$ be the positive orthant of the $(T+1)$-dimensional Euclidean space with components labeled $(-T, \ldots, 0)$. Zero is interpreted as the current period and T is the number of past periods taken into consideration. The notion of insecurity that we consider is based on changes in the variable from one period to the next and, therefore, we assume that there is at least one past period. A measure of insecurity is a sequence of functions $I = \left\langle I^T : \mathbb{R}^{T+1}_{++} \to \mathbb{R} \right\rangle_{T \in \mathbb{N}}$. Thus, each I^T assigns an insecurity value to each stream $x = (x_{-T}, \ldots, x_0) \in \mathbb{R}^{T+1}_{++}$. We allow T to vary; this seems to be a reasonable assumption because it would be rather restrictive to suppose that every income recipient or consumer in an economy (or in a subgroup thereof) has become an economic agent in the same past period. Thus, we can think of T as the economic rather than the chronological age of an individual.

We employ an axiomatic approach to identify a class of relative (that is, scale-invariant) insecurity measures. Our focus is on axioms that we consider plausible and intuitively appealing and, as a consequence, the resulting class is relatively rich. This allows us to present applied researchers with a plethora of possible measures, thereby enabling them to choose members of this class according to their comparative appeal.

The first property captures our fundamental position: insecurity is determined by the gains and losses experienced over time, with smaller gains leading to higher insecurity and smaller losses to diminished insecurity. Of course, gains and losses are two sides of the same coin because a loss can be viewed as a negative gain.

Monotonicity. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{T+1}_{++}$, and for all $x'_{-T} \in \mathbb{R}_{++}$, if $x'_{-T} > x_{-T}$, then

$$I^T(x'_{-T}, x_{-(T-1)}, \ldots, x_0) > I^T(x_{-T}, x_{-(T-1)}, \ldots, x_0).$$

The appeal of this monotonicity property is immediate. Consider two streams of length $T+1$ that differ only in the earliest period $-T$. The two values of the variable in that period are x_{-T} and x'_{-T}, and we assume that, without loss of generality, x'_{-T} is greater than x_{-T}. From period $-(T-1)$ up to the present period 0, the two streams are identical and given by $(x_{-(T-1)}, \ldots, x_0)$. Because x'_{-T} is greater than x_{-T}, the gain when moving from period $-T$ to period $-(T-1)$ becomes smaller when replacing x_{-T} with the larger number x'_{-T}. There are no further differences between the two streams and, therefore, the smaller gain (which could be a larger loss) is assumed to be unambiguously associated with higher insecurity. We emphasize that the value of the variable in period $-(T-1)$ is the same for the two streams under consideration so that the comparison of the gains for the two values x_{-T} and x'_{-T} is entirely determined by the comparison of the two values x_{-T} and x'_{-T} themselves. Monotonicity is similar in spirit to the axiom of difference monotonicity employed by Bossert and D'Ambrosio (2013) but the two are independent. Difference monotonicity involves a comparison of two streams of different lengths (one of length T and one of length $T+1$), whereas both streams in the monotonicity property defined above are of length $T+1$. See Bossert and D'Ambrosio (2013, p. 1020) for a detailed discussion of difference monotonicity.

It is important to note that, in our setting, measuring economic insecurity differs substantially from other tasks, such as measuring the variability associated with an income stream. What matters when assessing insecurity is the evolution of an income stream in the past: an experienced gain reduces the sentiment of insecurity because it increases the confidence of an economic agent, whereas an experienced loss increases insecurity—the outlook of an agent cannot but be adversely affected by a negative event of this nature. This is in stark contrast with the measurement of variability—in which case all variations are considered undesirable, no matter whether they are positive or negative. To provide a concrete example, consider the income streams $x = (5, 10, 15)$ and $x' = (10, 10, 10)$. There is less insecurity in x because the agent experiences two encouraging (insecurity-reducing) events—namely, income gains from 5 to 10 and from 10 to 15. In contrast, there are neither positive nor negative experiences inherent in the stream x' because income has been stagnating throughout. Clearly, a measure of variation such as the variance reverses this ranking—there is a positive amount of variance associated with x but there is none in x'. To us, the notion of insecurity is adequately described by our monotonicity axiom, and it differs fundamentally from other phenomena such as variability. Observe that the first T components of a stream $(x_{-T}, \ldots, x_{-1}, x_0)$ are experienced income levels in the past that determine the extent to which an agent is insecure in the present period. They do not represent future data points to be assessed with an ex-ante variability criterion.

Whereas monotonicity is a property that applies to income streams of the same length, our second axiom of extension invariance provides a condition on specific extensions of a stream by one past period. Suppose that we start out with a stream $(x_{-T}, \ldots, x_0) \in \mathbb{R}^{T+1}_{++}$ and extend the stream by appending a period $-(T+1)$ with an income level of x_{-T} so that the new stream is given by $(x_{-T}, x_{-T}, \ldots, x_0) \in \mathbb{R}^{T+2}_{++}$. Because neither a gain nor a loss is added by this extension, it is natural to demand that the level of insecurity in the extended stream is equal to the level of insecurity in the original.

Extension invariance. For all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$,

$$I^{T+1}(x_{-T}, x) = I^T(x).$$

The third axiom that we use in this paper is temporal decomposability, a requirement that establishes another link between streams of different lengths. The axiom is not new; a version of it also appears in Bossert, Clark, D'Ambrosio, and Lepinteur (2023) under the label of quasi-linearity. In our setting, temporal decomposability requires that the insecurity $I^T(x)$ associated with an income stream $x \in \mathbb{R}^{T+1}_{++}$ can be decomposed into two terms—namely, the insecurity generated by the T income levels $(x_{-(T-1)}, \ldots, x_0)$ and a function of the incomes x_{-T} and $x_{-(T-1)}$ experienced in the most distant past. Clearly, the axiom encompasses an additivity requirement. See Bossert, Clark, D'Ambrosio, and Lepinteur (2023) for further remarks on this axiom.

Temporal decomposability. For all $T \in \mathbb{N} \setminus \{1\}$, there exists a function $\Phi^T \colon \mathbb{R}^2_{++} \to \mathbb{R}$ such that, for all $x \in \mathbb{R}^{T+1}_{++}$,

$$I^T(x_{-T}, \ldots, x_0) = I^{T-1}(x_{-(T-1)}, \ldots, x_0) + \Phi^T(x_{-T}, x_{-(T-1)}).$$

Because we employ a relative approach, we require the individual measure of insecurity to be scale invariant. This is a standard property in the literature on social index numbers, particularly when issues such as inequality and poverty are addressed.

Scale invariance. For all $T \in \mathbb{N}$, for all $x \in \mathbb{R}^{T+1}_{++}$, and for all $\lambda \in \mathbb{R}_{++}$,

$$I^T(\lambda x) = I^T(x).$$

As usual, scale invariance demands that if the incomes of all periods are multiplied by the same positive constant, insecurity remains unchanged.

The following result characterizes all individual measures of insecurity that satisfy our four axioms.

Theorem 1. *A measure of insecurity* $I = \langle I^T : \mathbb{R}^{T+1}_{++} \to \mathbb{R} \rangle_{T \in \mathbb{N}}$ *satisfies monotonicity, extension invariance, temporal decomposability, and scale invariance if and only if there exists a sequence of increasing functions* $\langle f^t : \mathbb{R}_{++} \to \mathbb{R} \rangle_{t \in \mathbb{N}}$ *such that* $f^t(1) = 0$ *for all* $t \in \mathbb{N}$ *and*

$$I^T(x) = \sum_{t=1}^{T} f^t \left(\frac{x_{-t}}{x_{-(t-1)}} \right) \tag{1}$$

for all $T \in \mathbb{N}$ *and for all* $x \in \mathbb{R}^{T+1}_{++}$.

Proof. 'If.' That the measures identified in the theorem statement satisfy monotonicity follows from the increasingness of the functions f^t.

Extension invariance follows because $f^t(1) = 0$ for all $t \in \mathbb{N}$.

To prove that temporal decomposability is satisfied, define $\Phi^T(y, z) = f^T(y/z)$ for all $T \in \mathbb{N} \setminus \{1\}$ and for all $(y, z) \in \mathbb{R}^2_{++}$.

Scale invariance follows immediately because any positive multiplicative constant cancels out by definition.

'Only if.' To prove the reverse implication, suppose that $I = \langle I^T : \mathbb{R}^{T+1}_{++} \to \mathbb{R} \rangle_{T \in \mathbb{N}}$ satisfies the axioms of the theorem statement. Defining $\Phi^1(x_{-1}, x_0) = I^1(x_{-1}, x_0)$ for all $(x_{-1}, x_0) \in \mathbb{R}^2_{++}$, it follows from temporal decomposability that

$$\Phi^T(x_{-T}, x_{-(T-1)}) = I^T(x_{-T}, \ldots, x_0) - I^{T-1}(x_{-(T-1)}, \ldots, x_0)$$

for all $T \in \mathbb{N} \setminus \{1\}$ and for all $x \in \mathbb{R}^{T+1}_{++}$. By scale invariance, each Φ^t can be expressed as a function of the ratio $x_{-t}/x_{-(t-1)}$. Denoting this function by $f^t : \mathbb{R}_{++} \to \mathbb{R}$ and substituting back, it follows that (1) is satisfied for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$. By monotonicity, each f^t is increasing. By extension invariance, $f^t(1) = 0$ for all $t \in \mathbb{N}$. ■

The following examples establish the independence of our axioms.

Example 1. *Let*

$$I^T(x) = \sum_{t=1}^{T} \left(\frac{x_{-(t-1)}}{x_{-t}} - 1 \right)$$

for all $T \in \mathbb{N}$ *and for all* $x \in \mathbb{R}^{T+1}_{++}$.

Extension invariance is satisfied because

$$I^{T+1}(x_{-T}, x) = \sum_{t=1}^{T} \left(\frac{x_{-(t-1)}}{x_{-t}} - 1 \right) + \left(\frac{x_{-T}}{x_{-T}} - 1 \right) = I^T(x)$$

for all $T \in \mathbb{N}$ *and for all* $x \in \mathbb{R}^{T+1}_{++}$.

Defining $\Phi^T(y, z) = z/y - 1$ *for all* $T \in \mathbb{N} \setminus \{1\}$ *and for all* $(y, z) \in \mathbb{R}^2_{++}$, *it follows that temporal decomposability is satisfied.*

That the measure satisfies scale invariance is immediate.

Monotonicity is violated because I^T *is decreasing rather than increasing in* x_{-T} *for all* $T \in \mathbb{N}$.

Example 2. *Let*

$$I^T(x) = \sum_{t=1}^{T} \frac{x_{-t}}{x_{-(t-1)}}$$

for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$.

This measure satisfies monotonicity because I^T is increasing in x_{-T} for all $T \in \mathbb{N}$.

That temporal decomposability is satisfied follows from defining $\Phi^T(y,z) = y/z$ for all $T \in \mathbb{N} \setminus \{1\}$ and for all $(y,z) \in \mathbb{R}^2_{++}$.

Clearly, this measure satisfies scale invariance.

Extension invariance is violated because, for instance, $I^2(1,1,1) = 2 \neq 1 = I^1(1,1)$.

Example 3. *Define, for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$,*

$$I^T(x) = \sum_{t=1}^{T} \frac{x_{-t} - x_{-(t-1)}}{x_0}.$$

The measure satisfies monotonicity because I^T is increasing in x_{-T} for all $T \in \mathbb{N}$.

Extension invariance is satisfied because

$$I^{T+1}(x_{-T}, x) = \sum_{t=1}^{T} \frac{x_{-t} - x_{-(t-1)}}{x_0} + \frac{x_{-T} - x_{-T}}{x_0} = I^T(x)$$

for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$.

Scale invariance follows immediately from the definition of the measure.

For each $T \in \mathbb{N} \setminus \{1\}$, the function I^T can be written as

$$I^T(x_{-T}, \ldots, x_0) = I^{T-1}(x_{-(T-1)}, \ldots, x_0) + \Psi^T(x_{-T}, x_{-(T-1)}, x_0)$$

for all $x \in \mathbb{R}^{T+1}_{++}$, where

$$\Psi^T(x_{-T}, x_{-(T-1)}, x_0) = (x_{-T} - x_{-(T-1)})/x_0$$

for all $(x_{-T}, x_{-(T-1)}, x_0) \in \mathbb{R}^3_{++}$. Because Ψ^T cannot be expressed as a function of x_{-T} and $x_{-(T-1)}$ alone, temporal decomposability is violated.

Example 4. *Define*

$$I^T(x) = x_{-T}$$

for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$.

The measure satisfies monotonicity because I^T is increasing in x_{-T} for all $T \in \mathbb{N}$.

Extension invariance is satisfied because $I^{T+1}(x_{-T}, x) = x_{-T} = I^T(x)$ for all $T \in \mathbb{N}$ and for all $x \in \mathbb{R}^{T+1}_{++}$.

That temporal decomposability is satisfied follows from defining $\Phi^T(y,z) = y - z$ for all $T \in \mathbb{N} \setminus \{1\}$ and for all $(y,z) \in \mathbb{R}^2_{++}$.

Scale invariance is violated because, for instance, $I^1(2 \cdot 1, 2 \cdot 1) = 2 \neq 1 = I^1(1,1)$.

3 Dominance

There is a natural way of defining a dominance criterion based on the class of insecurity measures characterized in the previous section. This is accomplished by defining a dominance quasi-ordering $\succsim$ (that is, a reflexive and transitive but not necessarily complete relation) on the set of income streams with the property that $x \succsim x'$ if the insecurity associated with x is greater than or equal to the insecurity associated with x' for all possible choices of an insecurity measure within our class, where x and x' are income streams that may be of different lengths.

To be precise, define the quasi-ordering $\succsim$ on $\cup_{T \in \mathbb{N}} \mathbb{R}^{T+1}_{++}$ by letting, for all $T, T' \in \mathbb{N}$, for all $x \in \mathbb{R}^{T+1}_{++}$, and for all $x' \in \mathbb{R}^{T'+1}_{++}$,

$$x \succsim x' \;\Leftrightarrow\; \begin{cases} \frac{x_{-t}}{x_{-(t-1)}} \geq \frac{x'_{-t}}{x'_{-(t-1)}} & \text{for all } t \in \{1, \ldots, \min\{T, T'\}\} \text{ and} \\ \frac{x_{-t}}{x_{-(t-1)}} \geq 1 & \text{for all } t \in \{T'+1, \ldots, T\} \text{ if } T > T' \text{ and} \\ 1 \geq \frac{x'_{-t}}{x'_{-(t-1)}} & \text{for all } t \in \{T+1, \ldots, T'\} \text{ if } T < T'. \end{cases}$$

Clearly, the relation $\succsim$ is reflexive and transitive.

This relation indeed captures the dominance relationship induced by the class of measures that are generated by the functions f^t. If two streams of the same length are to be compared, the requisite income ratios comprise all the information that is required because the f^t are increasing as a consequence of the monotonicity axiom. Comparisons across different stream lengths can be accommodated by adding the required number of income levels of one to the shorter stream; this follows because the values of the f^t are equal to zero at one, a property that follows from extension invariance.

A somewhat related issue is whether the proposed measures themselves (not just the income distributions that belong to their domains) can be ranked. While such an analysis may indeed be interesting and informative, it seems to us that it does not necessarily have to be included in the current paper; our primary objective is to introduce and analyze some fundamental axioms (and the resulting measures) that may form a foundation for possible extensions to be addressed in future work.

4 Discussion

The class of measures axiomatized in our result allows for a broad choice when it comes to assessing real-world data with respect to the notion of economic insecurity. It is straightforward to identify examples for measures within our class by choosing the requisite functions f^t applied to the income ratios; to ensure that monotonicity is satisfied, any sequence of increasing functions will do, and to comply with extension invariance, the functions must assign a value of zero to the income level one. For instance, these functions may be linear or logarithmic functions. A linear example is given by $f^t(y) = y - 1$ for all $t \in \mathbb{N}$ and for all $y \in \mathbb{R}_{++}$. Clearly, these functions are increasing and satisfy $f^t(1) = 0$. A logarithmic variant is defined by $f^t(y) = \ln(y)$ for all $t \in \mathbb{N}$ and for all $y \in \mathbb{R}_{++}$. Again, it is clear that these functions are increasing and assume the value zero at income level one. Note that, in the latter case, the corresponding insecurity index simplifies because all income levels between those in periods $-T$ and 0 cancel out.

There is no restriction on how each of these per-period functions treat gains relative to losses. If a move from period $-t$ to period $-(t-1)$ entails a gain, the ratio to which the function is applied is between zero and one; if the move represents a loss, this ratio is greater than one. The values that are assigned by f^t to ratios below one need not bear any relationship to the values that are assigned to ratios above one—gains and losses do not have to be treated symmetrically in any sense. This makes it possible to choose measures that, in a sense, assign more weight to losses than to gains. Specifically, suppose that an individual experiences a gain by moving from a given income level in period $-t$ to a larger income level in period $-(t-1)$. This means that the requisite ratio y of the incomes before and after the increase is less than one. Now suppose that the roles of the two income levels are reversed so that a loss is experienced when moving from period $-t$ to period $-(t-1)$. The resulting ratio is $1/y$ which, because y is less than one, must be greater than one. If it is intended to assign more importance to losses than to gains, this means the absolute value of f^t at y must be less than the value of f^t at $1/y$ (recall that $f^t(y)$ is negative because y is less than one, and $f^t(1/y)$ is positive because $1/y$ is greater than one). Thus, giving more weight to losses than to gains means that the inequality $-f^t(y) < f^t(1/y)$ must be satisfied for all $t \in \mathbb{N}$ and for all y less than one. An example of a sequence of functions that satisfies this requirement is obtained by defining, for all $t \in \mathbb{N}$, $f^t(y) = y - 1$ for all y less than one, and $f^t(y) = 2 - 2/y$ for all y greater than or equal to one. That the requisite inequality is satisfied can be verified by substitution.

There is also a considerable amount of flexibility regarding how the functions f^t that apply to different periods are related to each other. The assessment of a change from period $-t$ to period $-(t-1)$ may be independent of the specific value of t, or it may be such that more recent experiences are given higher weight than those that occurred further back in time. The period-independent variants are consistent with Allais's (1966, p. 1128) view that, in some circumstances, "...forgetfulness per unit of time is constant." A concrete example is given by the class of measures generated by the power functions $f^t(y) = (y-1)^\alpha$ for all $t \in \mathbb{N}$ and for all $y \in \mathbb{R}_{++}$, where $\alpha \in \mathbb{R}_{++}$ is a parameter. Because α is period-independent, this class conforms to Allais's position as stated above. If the parameter is allowed to vary with t, it is possible to include measures that are time-sensitive in the sense that the importance they assign to past experiences may depend on how far removed from the present these experiences occur. Geometric discounting is one particular method that entails time sensitivity; in that case, one possibility is to employ the functions given by $f^t(y) = \delta^t g(y)$ for all $t \in \mathbb{N}$ and for all $y \in \mathbb{R}_{++}$, where $g\colon \mathbb{R}_{++} \to \mathbb{R}$ is an increasing function such that $g(1) = 0$, and $\delta \in (0,1)$ is a discount factor. The latter example also illustrates that gains and losses may be treated differently as well as time periods. Clearly, the values of the function g at income levels below one need not bear any relationship to the values attained at or above one; for instance, g may be such that $g(y) = (y-1)$ for all values of y less than one, and $g(y) = \ln(y)$ for all values of y greater than or equal to one.

An issue to be addressed in future work is how to deal with insecurity in the presence of several variables. For instance, an agent's insecurity may be influenced not only by income fluctuations but also by variations in the employment rate—an increase in the relevant employment rate is likely to decrease an agent's sentiment of insecurity. In order to arrive at an index of economic insecurity in such cases, a change in an agent's income must be traded off against a change in the employment rate—and this trade-off will, in general, depend on the values of the requisite variables. If these trade-offs turn out to be difficult to pin down, however, one may have to settle for a dominance criterion rather than an index that allows for all possible comparisons. Whatever the case may be, the extension to multiple variables seems to constitute an essential task on the research agenda.

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