Stochastic Social Preferences and Corporate Investment Decisions *

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ABSTRACT

This paper develops a dynamic general equilibrium model with stochastic social preferences and endogenous corporate investment decisions. We find that firms' investment decisions largely undo the effects of shifts in preferences on stock prices and risk premia. Only when most firms have already switched to a green technology do further preference changes have stronger effects on stock prices. Stochastic social preferences delay the move to a greener economy, especially when preference shocks correlate positively with aggregate cash flows. Risk aversion initially helps the transition, but later slows it down. Correlations between stock returns of firms in brown and green sectors increase (decrease) following an increase (decrease) in green investors' social preferences. Small changes in social preferences can have large supply effects even when they only have negligible effects on the cost of capital wedge between green and brown firms.

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1 Introduction

The paradigm within which shareholders make portfolio decisions has shifted in recent years. Their interest in corporate social responsibility has increased and they frequently go beyond incorporating only narrowly defined risk and return characteristics when choosing their portfolios. In accordance with this shift in tastes, assets either owned or managed by investors who take into account firms' social and environmental standards have grown exponentially.[1]

The rate at which social preferences change does not seem to evolve deterministically. In fact, Pastor et al. (2021) conclude that shocks to investor tastes were the main driver of return differences in the cross section of U.S. stocks with different ESG performances. The stochastic nature of the evolution of social preferences is also evidenced by recent pushbacks by some investors and law makers against the application of Socially Responsible Investment (SRI) filters in the context of delegated portfolio management. Also, inflows to ESG investment vehicles seem to have slowed or even reversed.[2] Furthermore, there exists academic evidence that social preferences change in response to past economic performance, consistent with a stochastic evolution of social preferences.[3]

The effect of the stochastic nature of social preferences on corporate investment strategies, on firms' cost of capital, and on the transition towards a green economy more generally remains theoretically unexplored. The only exception hereby is the seminal paper by Pastor et al. (2021) who, in an extension of their base model, allow for a one-time shock to investor tastes and for an additional round of trading once the preference shock has been realized. Our paper provides a first fully dynamic stochastic general equilibrium (DSGE) model where investors make portfolio decisions and firms make investment choices at each point in time, anticipating the stochastic evolution of a representative investor's taste for social responsibility.

This framework generates several new insights that are unlikely to emerge in partial equilibrium models with fixed technology supply and deterministic social preferences. First, we show that firms' real option to switch to a green technology crucially affects price dynamics in response to preference shocks. When social preferences are sufficiently strong so that additional firms find it optimal to switch to green technologies, rationally anticipated supply effects undo much of the effects of shocks to social preferences on stock prices. Preference

[1]For example, assets of signatories of the Principles of Responsible Investment have grown more than sixfold between 2011 and 2021. Further strong evidence for changing investor tastes is that yield spreads of green bonds issued along with essentially identical non-green twins have widened almost four-fold since they were first issued in 2020, see Pastor et al. (2022).

[2]See, e.g. Bloomberg, Why ESG in America May Face a Rough Road Ahead, March 30th 2023, or The Economist, The anti-ESG industry is taking investors for a ride, March 2nd 2023.

[3]See, e.g., Exley et al. (2023).

shocks only have significant effects on share valuations when they are far below the threshold where additional firms would find it optimal to switch to green technologies, or when most firms have already switched to a green technology.

Second, we show that uncertainty about future social preferences delays the move to a greener economy. This effect becomes even stronger, if preference shocks are positively correlated with aggregate cash flow shocks, as empirical evidence suggests. Such a correlation makes green technologies less attractive, since they become more risky: when they tend to generate low cash flows, they are also exposed to a negative valuation shock, due to a negative shock in social preferences. The opposite holds true for brown technologies.

Third, we show that higher risk aversion initially helps the transition to greener technologies, but delays a complete transition of the economy away from polluting technologies. This finding highlights an important interaction between risk-sharing and corporate social responsibility. Investors demand both green and brown shares to diversify risks. At the beginning of the transition, green shares are in scarce supply. This lowers the equilibrium cost of capital for green shares, thereby accelerating the transition. However, green shares become predominant towards the end of the transition. Scarcity in the supply of brown shares lowers their cost of capital, thereby slowing the transition towards a green economy in its final phase.

Fourth, stochastic social preferences induce time-varying correlations between green and brown technologies. For low values of social preferences relative to their historical highs, preference shocks induce negative correlations between brown and green technologies. This is so, since the value of brown firms' real option to switch to a green technology is low in this case and, thus, supply effects do not offset the impact of preference shocks on stock prices. In contrast, when social preferences are close to their historical highs, shocks to social preferences induce positive correlations between brown and green technologies, as the value of brown firms' real option to switch to a green technology is high (i.e., the option is at-the-money) and positively correlated with the value of green firms.

Finally, we find that brown firms' cost of capital is generally higher than that of green firms. This cost of capital gap widens as social preferences increase from low levels: green firms exhibit decreasing cost of capital whereas brown firms' cost of capital increases with social preferences. However, once social preferences are sufficiently strong so that some brown firms switch to become green, these dynamics are reversed. The cost of capital of green firms rises as social preferences become even stronger, and the cost of capital of brown firms drops. This is due to the fact that for high values of social preferences, relative to past highs, remaining brown firms are a good hedge against future preference shocks: if social preferences become weaker, this helps brown firms, as it generates relatively more demand for their shares. Alternatively, if social preferences become even stronger, the valuation effect from the drop in demand for shares of brown firms is largely offset by the increase in the option value to switch to the green technology. This makes brown firms less risky and thus lowers their cost of capital.

The findings delineated above can be attributed to the impact of social preferences on discount rates: investors with social preferences tilt their portfolio holdings towards green firms and away from brown firms, thereby affecting their cost of capital. The analysis intentionally excludes other conduits, through which social preferences could affect firms' cash flows, as opposed to their discount rates. For instance, social preferences could introduce a preference for goods and services from green firms over those from brown firms. Furthermore, agents with social preferences may influence managers' decisions directly via voting at shareholder meetings or via other informal channels, thereby also affecting firms' cash flows. The model presented in this paper does not consider such potential additional effects of social preferences and concentrates on the discount rate channel.

Our paper is related to several strands of literature. First, there is a growing literature that analyzes the effect of social preferences on financial market equilibrium and corporate investment in a one-period, static framework. This literature includes Heinkel et al. (2001), Gollier and Pouget (2014), Hart and Zingales (2017), Pastor et al. (2021), Pedersen et al. (2021), Broccardo et al. (2022), Edmans et al. (2022), Landier and Lovo (2022), Oehmke and Opp (2022), Goldstein et al. (2022) and Berk and van Binsbergen (2022). These models differ in how they model social preferences. Some assume that social investors simply find it wrong to own shares of firms which do not accord with their ethical considerations, others assume that social investors perceive non-pecuniary dividends from their portfolio firms, depending on how well they accord with their ethical standards. Other authors assume that social investors only care about the consequences that their portfolio decisions have on firms' negative externalities on society.[4]

All of the above papers use a static, essentially one-period framework. Only few papers analyze the role of social preferences for asset pricing and corporate investment in a dynamic framework. Such exceptions are Hong et al. (2022) and Bustamante and Zucchi (2022). We differ from the above papers by providing a continuous-time framework that captures stochastic shocks to investors' future tastes and allows both investors and firms to take this into account when making portfolio and investment decisions, respectively.

Finally, our paper is also related to a growing literature that provides experimental and empirical evidence on social preferences. Overall, this literature provides convincing support

[4]See Dangl et al. (2023) for a discussion of different modelling approaches to social preferences and a comprehensive literature survey.

for the existence of social preferences that are reflected in agents' investment decisions. Studies that present such evidence include Riedl and Smeets (2017), Bauer et al. (2021), Krueger et al. (2020), Dyck et al. (2019), Bolton et al. (2020), Hartzmark and Sussman (2019), and Barber et al. (2021). Several papers within this literature provide evidence on the specific type of social preferences that investors have. Overall, these papers find that investors with social preferences do not seem to be motivated by the perceived consequences of their investment decisions, but rather by intrinsic ethical considerations (see, e.g., Ottoni-Wilhelm et al. (2017), Hart et al. (2022), Heeb et al. (2022), Bonnefon et al. (2022), Cole et al. (2023), and Humphrey et al. (2022).) Our modelling approach accords with these experimental and empirical findings and features non-pecuniary payoffs that social investors receive from their portfolio holdings, depending on firms' social responsibility. For a more detailed discussion of the experimental and empirical evidence on social preferences, see Dangl et al. (2023).

The rest of the paper is organized as follows. Section 2 develops the model. Investors' portfolio decisions are analyzed in Section 3 and Section 4 derives corporate equilibrium decisions. Numerical results are presented in Section 5 and Section 6 concludes.

2 The model

We consider a model where time is continuous and the horizon is infinite. The financial market consists of one riskless asset and two risky assets (shares). The risk-free asset is in perfectly elastic supply and yields a return of rdt over a time period $[t, t + dt)$. Equity shares are issued by firms that produce with one of two technologies, which we refer to as green and brown.

We normalize the total mass of all (i.e., green and brown) firms and the number of shares to one and denote firm type (i.e., technology) by $f \in \{G, B\}$. The supply of firms with brown technology (i.e., the supply of brown shares) is denoted by S_B, so that the supply of firms with green technology (i.e., the supply of green shares), S_G, is $1 - S_B$.

Investors and preferences. There are two types of investors $i \in \{S, F\}$ with a total measure of one. Investors are competitive, i.e., they do not act strategically and take prices as given. A fraction θ of investors has non-consequentialist utilitarian social preferences, i.e., they perceive extra, non-pecuniary dividends of g_t per green share and of $-g_t$ per brown share. The remaining fraction $1 - \theta$ of investors are referred to as financial investors and they derive zero non-pecuniary benefits from their share holdings.

To capture the fact that social preferences may change over time, g_t is assumed to follow a stochastic process:

$$dg_t = \mu_g dt + \sigma_g dz_{g,t}. \tag{1}$$

The shocks to social preferences, $dz_{t,g}$, may be correlated with shocks to firms' cash flows, as explained below.

Each investor type has CARA utility with absolute risk aversion $\gamma^S = \gamma/\theta$ for the representative social investor and $\gamma^F = \gamma/(1-\theta)$ for the representative financial investor. Investors can therefore be modelled via a single, competitive representative investor with CARA utility and absolute risk aversion γ. This representative investor derives utility from consuming both financial benefits, which we will define below, and non-pecuniary benefits

$$\theta g_t (X_{G,t} - X_{B,t}) \tag{2}$$

where $X_{G,t}$ and $X_{B,t}$ denote the representative investor's holdings of green and brown shares at time t, respectively.

Firms, production technologies, and cash flows. At time 0, all firms are endowed with the brown technology and an option to switch to the green technology. This option can be exercised by investing I, and brown firms' value-maximizing managers choose when to optimally exercise this option, rationally anticipating other brown firms' optimal investment policies and the effects of investors' stochastic social preferences. The investment is irreversible. We hereby assume that each firm behaves competitively, taking prices as given.

Firms' cash flows are subject to aggregate shocks ($dz_{A,t}$) and technology-specific shocks ($dz_{f,t}$). Over a period $[t, t+dt)$, firms' cash flows are:[5]

$$\begin{aligned} dy_{B,t} &= \mu dt + \sigma dz_{B,t} + \sigma_A dz_{A,t}, & (3)\\ dy_{G,t} &= \mu dt + \sigma dz_{G,t} + \sigma_A dz_{A,t}. & (4) \end{aligned}$$

Technology-specific and aggregate shocks to cash flows are, by definition, pairwise uncorrelated, i.e.,

$$cov_t[dz_{G,t}, dz_{B,t}] = cov_t[dz_{G,t}, dz_{A,t}] = cov_t[dz_{B,t}, dz_{A,t}] = 0.$$

However, green and brown firms' cash flows are correlated through aggregate shocks, i.e.,

$$\begin{aligned} cov_t[dy_{G,t}, dy_{B,t}] &= cov_t[\sigma dz_{G,t} + \sigma_A dz_{A,t}, \sigma dz_{B,t} + \sigma_A dz_{A,t}] \\ &= \sigma_A^2 dt = \rho(\sigma^2 + \sigma_A^2) dt \end{aligned}$$

[5]We assume that expected cash flows of both firms equal μdt. If differences arise, e.g., caused by costly abatement technologies run by green firms, the present value of future costs can, without loss of generality, be modelled as part the adjustment costs I, introduced above.

where $\rho \equiv \frac{\sigma_A^2}{\sigma^2+\sigma_A^2}$ is defined as the correlation coefficient of the two cash flows.

Since there is evidence that shocks to financial returns are related to shocks in social preferences,[6] we allow aggregate cash flow shocks and innovations to social preferences to be correlated, i.e.,

$$cov_t[dz_{A,t}, dz_{g,t}] = \rho_{Ag} dt.$$

Firm valuation. Since firms' cash flows, dy_G and dy_B, follow stationary distributions, they do not affect ex-dividend share prices as state variables. As it will become clear in Section 4, share prices depend on both social preferences, g, and the endogenously determined supply of green firms, S_G. We, thereby, denote share prices of green and brown firms by $P_G(g; S_G)$ and $P_B(g; S_B)$, respectively.

Let $r + \pi_B(g_t; S_{G,t})$ denote the expected cum-dividend return (i.e., cost of capital), where $\pi_B(g_t; S_{G,t})$ is the required risk premium for brown shares. The share price of a brown firm can then be expressed as the present value of all expected future dividends plus a real option value to switch technology:

$$P_B(g_t; S_{G,t}) = \max_{T_t^* \geq t} E_t \left[\int_t^{T_t^*} e^{-\int_t^{\tau}(r+\pi_B(g_s;S_{G,s}))ds} dy_{B,\tau} + e^{-\int_t^{T_t^*}(r+\pi_B(g_s;S_{G,s}))ds} \left(P_G(g_{T_t^*}, S_{G,T_t^*}) - I \right) \right]$$

where T_t^* is the optimal time of the investment to switch to the green technology, defined as

$$T_t^* \equiv \inf\left(s \geq t \middle| g_s = \bar{g}(S_{G,t})\right)$$

with $\bar{g}(S_{G,t})$ being the optimal investment threshold. We take the risk premium π_B and the optimal investment threshold $\bar{g}$ as given for now and endogenize them formally with the supply of green firms S_G in Section 4 where we solve for a competitive market equilibrium.

Note that θg (or $-\theta g$) does not directly enter into the valuation formula as non-pecuniary dividends. Instead, it affects share prices through the equilibrium risk premium (or equivalently, cost of capital) channel.

Applying the Feynman-Kac Theorem to the share price function $P_B(g; S_G)$, which is expressed as the expectation of a stochastic integral, yields the following Hamilton-Jacobi-Bellman (HJB) equation for $g < \bar{g}(S_G)$:[7]

$$(r + \pi_B(g; S_G))P_B(g; S_G) = \mu + \mu_g P_B'(g; S_G) + \frac{1}{2}\sigma_g^2 P_B''(g; S_G). \tag{5}$$

Since corporate social investment is irreversible, green firms do not have a real option to

[6]See, e.g., Exley et al. (2023).

[7]See, e.g., Øksendal (2003).

switch technology. The share price of a green firm is given by

$$P_G(g_t; S_{G,t}) = E_t\left[\int_t^{\infty} e^{-\int_t^{\tau}(r+\pi_G(g_s;S_{G,s}))ds} dy_{G,\tau}\right]$$

where $\pi_G(g_t; S_{G,t})$ is the required risk premium for green shares.

Applying the Feynman-Kac Theorem, we obtain the following HJB equation for P_G[8]:

$$(r + \pi_G(g; S_G))P_G(g; S_G) = \mu + \mu_g P_G'(g; S_G) + \frac{1}{2}\sigma_g^2 P_G''(g; S_G). \tag{6}$$

Equations (5) and (6) state that in equilibrium, the premia required by investors for holding brown and green stocks, respectively (the left-hand side of the equations), must equal the expected compensation provided by the green and brown stock (the right-hand of the equations). The required risk premia, π_G and π_B, are derived from investors preferences in the following section.

3 Investors' portfolio choice problem

In the previous section, we derived the valuation equations (5) and (6) for given risk premia π_G and π_B. Now we analyze investors' demand for green and brown firms, X_G and X_B for given prices, which implicitly determines the required risk premia. In Section 4 we take these two building blocks to derive the market equilibrium and to endogenize technology choice. For exposition, in this section, we suppress the argument S_G and simply write prices as $P_G(g), P_B(g)$ and risk premia as $\pi_G(g), \pi_B(g)$.

At the beginning of each period $[t, t+dt)$, the representative investor has social preferences g_t and makes portfolio decisions. At the end of the period, dividends must be consumed since firms produce perishable goods. With W_t denoting aggregate wealth at time t, the representative investor faces the budget constraint

$$\begin{aligned} W_{t+dt} &= (1 + rdt)(W_t - P_G(g_t)X_{G,t} - P_B(g_t)X_{B,t}) + P_G(g_{t+dt})X_{G,t} + P_B(g_{t+dt})X_{B,t} \\ &= (1 + rdt)W_t + (dP_G(g_t) - rP_G(g_t)dt)X_{G,t} + (dP_B(g_t) - rP_B(g_t)dt)X_{B,t}, \end{aligned} \tag{7}$$

where $X_{G,t}$ and $X_{B,t}$ denote the number of green and brown shares she decides to hold at

[8]Since green firms do not have an optimization problem, their HJB equation is reduced to an ordinary differential equation (ODE).

time t and stock price differentials are defined as

$$dP_G(g_t) \equiv P_G(g_{t+dt}) - P_G(g_t),$$
$$dP_B(g_t) \equiv P_B(g_{t+dt}) - P_B(g_t).$$

The resulting wealth dynamics are

$$dW_t - rW_t dt = (dP_G(g_t) - rP_G(g_t)dt)X_{G,t} + (dP_B(g_t) - rP_B(g_t)dt)X_{B,t}, \tag{8}$$

with $dW_t = W_{t+dt} - W_t$ being the wealth differential.

At the end of the period, the investor derives utility from three sources: (i) from consuming the resulting dividends $dy_G X_G + dy_B X_B$, (ii) from non-pecuniary benefits and costs associated with holding green and brown stocks $\theta g(X_G - X_B)dt$, and (iii) from net capital gains $dW_t - rW_t dt$ stated in (8).

While in general wealth W_t is a state variable when solving the multi-period portfolio problem, we assume in (iii) that investors derive utility directly from the resale value of the portfolio, which is determined by her future self, at time $t + dt$, with social preferences g_{t+dt}. This resale valuation of the representative investor's holdings simplifies the analysis when compared to an unconstrained optimization of an infinitely lived investor. Essentially, it implies that the investor does not consider intertemporal portfolio strategies to hedge changes in future utility due to changing preferences. In particular, financial investors do not insure social investors against variation in their non-pecuniary dividend. Note that this is trivially true in an overlapping generations interpretation, where each cohort of the representative investor cannot hedge against the preference shocks of the next generation.

This assumption is consistent in our setting where in equilibrium, the representative investor cannot consume by liquidating savings but only from holding a risky portfolio of shares. Hence, equilibrium share prices properly reflect firms' present value regarding the provision of future (pecuniary as well as non-pecuniary) consumption. Deriving utility from resale valuation simply assumes that net gains and losses affect investors' utility instantaneously.[9] It is important to point out that the resulting valuation is not myopic, since future shocks to investor preferences and the resulting price changes are rationally anticipated when determining the equilibrium resale value of the portfolio.

We denote the total utility-relevant flow to the investor from the three sources discussed

[9] In a model with infinitely lived agents, Nagel and Xu (2022) also assume resale valuation in the presence of investors with fading memory about firm fundamentals.

above by dC, which is given by

$$\begin{aligned} dC &= dy_G X_G + dy_B X_B + \theta g (X_G - X_B) dt + dW - rW dt \\ &= \Big[dy_G + dP_G - rP_G dt + \theta g dt \Big] X_G \\ &\quad + \Big[dy_B + dP_B - rP_B dt - \theta g dt \Big] X_B. \end{aligned} \tag{9}$$

The investor maximizes expected discounted utility

$$\begin{aligned} U_t &= \max_{\left(X_{G,s}, X_{B,s}\right), s \geq t} E_t \left[-\frac{1}{\gamma} \int_t^{\infty} e^{-\delta s} \left(1 - e^{-\gamma dC_s (X_{G,s}, X_{B,s}; g_s)} \right) \right] \\ &= \max_{\left(X_{G,s}, X_{B,s}\right), s \geq t} E_t \left[\int_t^{\infty} e^{-\delta s} \left(E_s[dC_s] - \frac{\gamma}{2} E_s[(dC_s)^2] \right) \right]. \end{aligned} \tag{10}$$

In the second line of (10) we use the Taylor-series expansion of the exponential function.

As we will see below, both $E_t[dC_t]$ and $E_t(dC_t^2)$ are of order dt. Under resale valuation and CARA utility, the only state variable of the system is g, so applying the theorem of Feynman-Kac yields

$$\begin{aligned} \delta U_t &= \max_{\left(X_{G,s}, X_{B,s}\right), s \geq t} \left[\frac{E_t[dC_t]}{dt} - \frac{\gamma}{2} \frac{E_t[(dC_t)^2]}{dt} + \mu_g U_g + \frac{1}{2} \sigma_g^2 U_{gg} \right] \\ &= \max_{\left(X_{G,s}, X_{B,s}\right), s \geq t} \left[\frac{E_t[dC_t]}{dt} - \frac{\gamma}{2} \frac{E_t[(dC_t)^2]}{dt} \right] + \mu_g U_g + \frac{1}{2} \sigma_g^2 U_{gg}. \end{aligned} \tag{11}$$

Since U is only a function of current preferences g, the overall utility-maximizing portfolio choice is given by

$$(X_G^*, X_B^*) \in \arg \max_{X_G, X_B} \left\{ \frac{E_t[dC_t]}{dt} - \frac{\gamma}{2} \frac{E_t[dC_t^2]}{dt} \right\}. \tag{12}$$

Standard methods of stochastic calculus (see Appendix A) yield

$$\begin{aligned} E_t[dC_t] &= \Big[\left(\pi_G(g) P_G(g) + \theta g \right) X_G + \left(\pi_B(g) P_B(g) - \theta g \right) X_B \Big] dt, \\ E_t[dC_t^2] &= \Big[\sigma^2 + \sigma_A^2 + \sigma_g^2 \left(P_G' \right)^2 + 2 \rho_{Ag} \sigma_A \sigma_g P_G' \Big] X_G^2 dt \\ &+ \Big[\sigma^2 + \sigma_A^2 + \sigma_g^2 \left(P_B' \right)^2 + 2 \rho_{Ag} \sigma_A \sigma_g P_B' \Big] X_B^2 dt \\ &+ \Big[\sigma_A^2 + \rho_{Ag} \sigma_A \sigma_g (P_G' + P_B') + \sigma_g^2 P_G' P_B' \Big] 2 X_G X_B dt, \end{aligned}$$

with

$$\pi_G = \frac{E(dy_G + dP_G)}{P_G} - r$$
$$\pi_B = \frac{E(dy_B + dP_B)}{P_B} - r.$$

The objective function (12) has a simple mean-variance form, which we can write with compact notation,

$$(X_G^*, X_B^*) \in \arg\max_{X_G, X_B} \left\{ \begin{bmatrix} X_G \\ X_B \end{bmatrix}' \begin{bmatrix} \pi_G P_G + \theta g \\ \pi_B P_B - \theta g \end{bmatrix} - \frac{\gamma}{2} \begin{bmatrix} X_G \\ X_B \end{bmatrix}' \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} X_G \\ X_B \end{bmatrix} \right\}, \quad (13)$$

with

$$\Sigma_G \equiv \sigma^2 + \sigma_A^2 + \sigma_g^2 \left(P_G'\right)^2 + 2\rho_{Ag}\sigma_A\sigma_g P_G';$$
$$\Sigma_B \equiv \sigma^2 + \sigma_A^2 + \sigma_g^2 \left(P_B'\right)^2 + 2\rho_{Ag}\sigma_A\sigma_g P_B';$$
$$\Sigma_{GB} \equiv \sigma_A^2 + \rho_{Ag}\sigma_A\sigma_g(P_G' + P_B') + \sigma_g^2 P_G' P_B'. \quad (14)$$

The investors' demand for risky assets is then given by the first order condition

$$\begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} = \frac{1}{\gamma} \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix}^{-1} \begin{bmatrix} \pi_G P_G + \theta g \\ \pi_B P_B - \theta g \end{bmatrix}. \quad (15)$$

With the solution for investors' optimal demand in (15) we are able to solve the capital market equilibrium.

4 Social preferences and endogenous technology supply in a competitive market equilibrium

In this section we take firm valuation from Section 2 and the representative investor's demand for the risky assets derived in Section 3 and combine these building blocks to derive equilibrium share prices. We first do this for given supply of green and brown firms and then derive proper boundary conditions that determine the firms' optimal technology choice, which finally endogenizes the supply of green and brown firms.

Solving Equation (15) for the required risk premia yields the investor's inverse demand function

$$\begin{bmatrix} \pi_G \\ \pi_B \end{bmatrix} = \begin{bmatrix} P_G & 0 \\ 0 & P_B \end{bmatrix}^{-1} \left(\gamma \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} X_G^* \\ X_B^* \end{bmatrix} - \begin{bmatrix} \theta g \\ -\theta g \end{bmatrix} \right). \tag{16}$$

Clearing the market for risky assets implies that equilibrium risk premia adjust to levels that make asset demand equal to asset supply

$$X_{G,t}^* = S_{G,t}, \tag{17}$$

$$X_{B,t}^* = S_{B,t} = 1 - S_{G,t}. \tag{18}$$

Hence, imposing market clearing to inverse demand (16) yields the following proposition:

Proposition 1 *The risk premium of green and brown shares is, respectively, given by*

$$\begin{bmatrix} \pi_G \\ \pi_B \end{bmatrix} = \begin{bmatrix} P_G & 0 \\ 0 & P_B \end{bmatrix}^{-1} \left(\gamma \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ S_B \end{bmatrix} - \begin{bmatrix} \theta g \\ -\theta g \end{bmatrix} \right). \tag{19}$$

Equation (19) shows that risk premia depend on social preferences, share prices, and technology supply.

Investment boundary. We conjecture that optimal technology choice is a free boundary problem, i.e., that for given S_G, brown firms have no incentive to switch to the green technology (thereby incurring the cost I per unit of firm) as long as social preferences g are below a critical threshold $\bar{g}(S_G)$, which is subject to optimal choice by brown firms. This threshold will be optimally determined in our numerical example and the conjectured optimality of the free boundary will be verified.

For now, we concentrate on the market equilibrium in the region $g < \bar{g}(S_G)$ where S_G is fixed. To indicate that prices and risk premia also depend on S_G, we express prices as $P_G(g; S_G), P_B(g; S_G)$ and risk premia as $\pi_G(g; S_G)$ and $\pi_B(g; S_G)$. Using Proposition 1, we substitute out $\pi_G(g; S_G)$ and $\pi_B(g; S_G)$ from pricing equations (5) and (6) and derive the system of Hamilton-Jacobi-Bellman equations which must be satisfied by the price functions P_G and P_B in a range where technology supply is fixed.

Proposition 2 *For $g < \bar{g}(S_G)$ the following system of ordinary differential equations (ODEs) holds:*

$$\begin{bmatrix} rP_G \\ rP_B \end{bmatrix} + \left(\gamma \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ S_B \end{bmatrix} - \begin{bmatrix} \theta g \\ -\theta g \end{bmatrix} \right) = \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \mu_g \begin{bmatrix} P_G' \\ P_B' \end{bmatrix} + \frac{1}{2}\sigma_g^2 \begin{bmatrix} P_G'' \\ P_B'' \end{bmatrix}. \tag{20}$$

In particular, $P_B(g; S_G) \geq P_G(g; S_G) - I$ for all $g \leq \bar{g}(S_G)$, thus, brown firms have no incentive to switch to the green technology.

Asymptotic behavior of share prices for $\mathbf{g} \to -\infty$. For decreasing values of g, the value of the real option that brown firms can switch to the green technology vanishes and both prices become linear in g

$$\lim_{g\to-\infty} \begin{bmatrix} rP_G(g;S_G) \\ rP_B(g;S_G) \end{bmatrix} = \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \mu_g \begin{bmatrix} \frac{\theta}{r} \\ -\frac{\theta}{r} \end{bmatrix} - \left(\gamma \begin{bmatrix} \underline{\Sigma}_G & \underline{\Sigma}_{GB} \\ \underline{\Sigma}_{GB} & \underline{\Sigma}_B \end{bmatrix} \begin{bmatrix} S_G \\ 1-S_G \end{bmatrix} - \begin{bmatrix} \theta g \\ -\theta g \end{bmatrix} \right) \tag{21}$$

with

$$\begin{aligned} \underline{\Sigma}_G &\equiv \sigma^2 + \sigma_A^2 + \sigma_g^2 \frac{\theta^2}{r^2} + 2\rho_{Ag}\sigma_A\sigma_g\frac{\theta}{r}; \\ \underline{\Sigma}_B &\equiv \sigma^2 + \sigma_A^2 + \sigma_g^2 \frac{\theta^2}{r^2} - 2\rho_{Ag}\sigma_A\sigma_g\frac{\theta}{r}; \\ \underline{\Sigma}_{GB} &\equiv \sigma_A^2 - \sigma_g^2\frac{\theta^2}{r^2}. \end{aligned}$$

Optimal investment. At the upper, free boundary $\bar{g}(S_G)$, prices satisfy the value matching condition

$$P_B(g;S_G)\big|_{g=\bar{g}(S_G)} = P_G(g;S_G)\big|_{g=\bar{g}(S_G)} - I.$$

The optimal choice of the investment threshold $\bar{g}$ is determined by the smooth pasting condition

$$\frac{\partial P_G(g;S_G)}{\partial g}\Big|_{g=\bar{g}(S_G)} = \frac{\partial P_B(g;S_G)}{\partial g}\Big|_{g=\bar{g}(S_G)}.$$

The value matching condition implies that for a given value of S_G, when social preferences g hit the investment threshold $\bar{g}(S_G)$, brown firms start switching to the green technology. Given the competitive nature of the market, brown firms are eventually indifferent between switching to the green technology and staying brown. Furthermore, the smooth pasting condition is imposed since the investment threshold $\bar{g}(S_G)$ must be chosen optimally by brown firms.

Likewise, any further positive shock dg will motivate some brown firms to switch, thereby increasing the supply of green firms by dS_G and reducing the remaining supply of brown firms by $-dS_G$ until the new supply of green firms satisfies $g + dg = \bar{g}(S_G + dS_G)$. At that point, brown firms are again indifferent between switching and remaining brown. Prices of both firms adjust to $P_G(g + dg; S_G + dS_G)$ and $P_B(g + dg; S_G + dS_G)$. Subsequent negative shocks to g push g to the interior region $g < \bar{g}(S_G + dS_G)$ where the solution is determined by Proposition 2. Consequently, the trailing maximum of the social preferences g determines S_G.

More formally, denote the trailing maximum of the g_t process as

$$g_{\max,t} = \sup_{0 \leq s \leq t} g_s.$$

Optimal investment implies that all firms are brown (i.e., $S_G = 0\%$) as long as $g_{\max,t} < \bar{g}_{S_G=0\%}$ and all firms are green (i.e., $S_G = 100\%$) as soon as $g_{\max,t} \geq \bar{g}_{S_G=100\%}$. Since the switch to the green technology is irreversible, green firms exist for all values of the trailing maximum exceeding $\bar{g}_{S_G=0\%}$ and brown firms exist as long as the trailing maximum is below $\bar{g}_{S_G=100\%}$. If $g_{\max,t}$ is outside the range $(\bar{g}_{S_G=0\%}, \bar{g}_{S_G=100\%})$, either $S_G = 0$ or $1 - S_G = 0$.

5 Numerical Results

This section provides a numerical analysis of the effect of stochastic social preferences on share prices and corporate technology choices. It also provides insights into the effects of various model parameters on the transition path of the economy towards green technologies and how this transition depends on the properties of the stochastic process of social preferences. Table 1 summarizes the parameter values for the base case of the numerical solutions. These parameter values are chosen so that their values appear reasonable in absolute and in relative terms, but are not calibrated to match specific empirical observations.[10]

5.1 Share Price Dynamics and Social Preferences

We start with the analysis of the relation between share prices and different values of social investor preferences. Figure 1 presents the case where 25% of all firms start out using the green technology. Panel (A) illustrates the prices of green firms (dark green line) and brown firms (dark brown line) as functions of the social preference parameter g. Prices in dark green and dark brown are plotted up to the critical threshold $\bar{g}_{S_G=25\%}$, indicated as the vertical dashed line, at which social preferences are high enough to incentivize additional brown firms to switch to the green technology and both price functions adjust irreversibly, as we discuss later. For this case, when social investors do not perceive any non-pecuniary dividends, i.e. $g = 0$, the share price of a firm with green technology is just above 100, as indicated by the dark green line in Panel (A) of Figure 1, and the share price of a firm with brown technology is just below 90, as indicated by the dark brown line. Thus, even though social investors do not receive any non-pecuniary dividends from holding green firms at $g = 0$, the valuation

[10] We solve the system of ODEs (20) numerically with a Chebychev collocation approach as described in Judd (1998) and Dangl and Wirl (2004). An application to environmental economics can be found in Dangl and Wirl (2007).

of the two types of shares already reflects the effects of future preference shocks. Since the process for g exhibits positive drift, this leads to a premium in the price of green shares relative to the price of brown shares. As g increases, prices of brown firms decrease further because social investors tilt their portfolios more strongly towards green shares and decrease their holdings of brown shares. This is shown by the dark brown line. Similarly, prices of green firms increase with g, as indicated by the positive slope of the dark green line.

As g approaches the critical threshold $\bar{g}_{S_G=25\%}$, the slope of the dark green line becomes flatter, i.e. less positive, and the slope of the dark brown line becomes less negative. Both phenomena reflect the increasing value of brown firms' option to switch to the green technology. This makes the value of brown firms drop less as g increases, and it makes the value of green firms increase less, since investors rationally anticipate an increased supply of firms with the green technology. At the critical value of $\bar{g}_{S_G=25\%}$, additional brown firms start to switch, thereby increasing the total supply of green firms beyond 25%. This happens at $g = 4.6$. At this critical value of g, the slopes of the dark green and the dark brown lines are equal and the distance between the two lines equals I, the cost of switching to the green technology.

As one considers even higher values of social preferences, both share prices increase in g, but only moderately so, reflecting the continued switching of initially brown firms to the green technology. These price dynamics are shown by the light green and light brown curves to the right of the threshold at $g = 4.6$. At a critical value of $g = 6.4$, all brown firms have switched, and there are only green firms left. From this point on, the value of green firms increases again linearly in g, and the brown line ceases to exist.

As mentioned above, the fraction of green firms, S_G, is determined by the historical maximum of g and switching takes place gradually when g reaches new maxima. To shed additional light on the switching behavior of brown firms and its pricing implications, Panel (A) of Figure 1 also illustrates the case in which all firms start out with the brown technology ($S_G = 0$) and shows the share prices in light brown and light green for values of g that represent historical maxima. For low values of g such that $g < \bar{g}_{S_G=0\%}$, no switching takes place and only the light brown line is shown, as no green firms exist. This line is below the dark brown line, since there are now only brown firms, and there is less risk sharing in the economy. As before, the slope of the light brown line eventually becomes less negative, reflecting the increased value of the option to switch to the green technology, but now this effect starts much sooner, i.e. for lower values of g. When g reaches new maxima above $\bar{g}_{S_G=0\%}$, brown firms gradually switch to the green technology. Thus, for $g \geq \bar{g}_{S_G=0\%} = 4.0$ a light green line also exists, reflecting the value of green shares. For increasing values of g, more brown firms switch until eventually all have switched to the green technology when g

reaches $\bar{g}_{S_G=100\%} = 6.4$ for the first time (indicated by the right dotted vertical line). Along the light brown line, brown firms are indifferent between staying brown and switching to green and, consequently, the prices differ exactly by the investment cost I. Furthermore, both price functions are almost flat, which implies that at the investment threshold, any positive shock to g, which strengthens social investors' preferences for green firms, is largely offset by the decisions of brown firms to adopt the green technology, i.e. by the associated increase in the supply of green firms and the decrease in the supply of brown firms. When g surpasses $\bar{g}_{S_G=100\%}$ for the first time, all firms have irreversibly switched to green technology and there is no light brown line in this region. The light green line represents the value of the green firms, which increases linearly in g.

Figure 2 also illustrates the relation between share prices and social preferences, but does so for the case where there already exists a large number of green firms. Specifically, Figure 2 assumes that 75% of all firms are initially using the green technology. In this case, if social preferences are such that $g = 0$, the share price of green firms in Panel A is below 100, which is lower than the corresponding value in Figure 1. This is so, since there is a larger supply of firms with this technology and therefore their contribution to systematic risk in the representative investor's portfolio is larger, which makes them less attractive. The opposite is true for firms with the brown technology. Brown firms offer attractive diversification benefits to the representative investor, and therefore their share price is only marginally below the price of green firms.

As social preferences become stronger, i.e. g increases, the share price of green firms increases and that of brown firms decreases. The options to convert from the brown to the green technology are deep out of the money, since there is already a large supply of green firms. Only when g increases substantially to approximately 5, the option value becomes visible and the slope of the green curve declines whereas the one of the brown firms turns less negative. The switching boundary, i.e. the second dashed vertical line denoted by $\bar{g}_{S_G=75\%}$, is reached when g reaches the value of $= 5.8$. We note that, slightly before that boundary is reached, even the share price of brown firms starts to increase with g, due to the growing value of the real option to switch.

If social preferences rise to even higher levels, then remaining brown firms switch to the green technology. The last dotted vertical line at $g = 6.4$ in Panel A of Figure 2, which is identical to the one in Figure 1, indicates the level of g at which all firms have eventually switched to the green technology.

We now turn to Panels B of Figures 1 and 2, which show the expected risk premia for shares of firms with green and brown technologies, respectively, defined in Equations (6) and (5). Subpanel B of Figure 1 reveals that when 25% of all firms use the green technology and social preferences are at $g = 0$, then brown shares already have a considerably higher risk premium than green shares (see dark brown and dark green lines). Note that, at $g = 0$, there are more brown than green firms in the economy and therefore, from the representative investor's perspective, they represent a larger systematic risk component. In contrast, Subpanel B of Figure 2 indicates that, at $g = 0$, green firms exhibit higher risk premia. This reflects that there are 75% of green firms in the economy, so they represent a large fraction of the market portfolio and thus contain more systematic risk than brown firms. As g increases, social investors tilt their portfolios away from brown shares, such that financial investors must hold a disproportionally large part of brown shares. They therefore require a higher risk premium for these shares. Thus, the dark brown lines increase whereas the opposite is true for the dark green line. As g increases, this effect becomes stronger.

Focusing on Figure 1, Subpanel B shows that, once the critical value of $g = 4.6$ is reached, brown firms start to switch to the green technology. This generates a kink in the required risk premia: risk premia of brown firms start to drop along the light brown curve and risk premia of green firms start to rise along the ligh-green curve. This is so since the relative supply of brown firms drops and the one of green firms increases. This supply effect implies that brown firms' required risk premia no longer increase and green firms' required risk premia no longer decrease as g increases further. Actually, we see the opposite effect: since the supply of green firms increases, this tends to increase required risk premia as g increases, since these technologies now represent more systematic risk. The opposite is true for brown firms. Their required risk premia drop. Once social preferences reach the critical value of $g = 6.4$, all firms have switched to the green technology. Therefore, from this value of g on, only the light green curve exists and it drops furher as g increases further, reflecting the increased demand by social investors due to higher non-pecuniary dividends.

Panels A of Figures 1 and 2 also suggest that the correlation between price changes of brown and green firms should vary with social preferences. The slopes of the price functions (i.e., the green and brown lines) have opposite signs for low levels of g relative to its historical high. In contrast, they have the same sign when g comes sufficiently close to its historical high. Figure 3 therefore illustrates the correlation between the two stock price changes for different levels of g. For ease of illustration, we consider a case where a large fraction of firms has already switched to the green technology. Qualitatively identical results are obtained for smaller fractions of green firms. Specifically, Figure 3 shows the correlations of cum-dividend price changes for $\bar{g}_{S_G=95\%}$.

To interpret Figure 3 we note that the correlation of the cum-dividend price changes is

$$\begin{aligned}\rho_{G,B} &= \frac{\Sigma_{GB}}{\sqrt{\Sigma_G}\sqrt{\Sigma_B}} \\ &= \frac{\sigma_A^2 + \sigma_g^2 P_G' P_B' + \rho_{Ag}\sigma_A\sigma_g(P_G' + P_B')}{\sqrt{\sigma^2 + \sigma_A^2 + \sigma_g^2 (P_G')^2 + 2\rho_{Ag}\sigma_A\sigma_g P_G'}\sqrt{\sigma^2 + \sigma_A^2 + \sigma_g^2 (P_B')^2 + 2\rho_{Ag}\sigma_A\sigma_g P_B'}}.\end{aligned}$$

According to the numerator of the above expression, the correlation between the two stock returns generally has three components: The first component is due to the volatility of the aggregate cash flow, σ_A^2, which introduces a correlation between the two dividends. The second component is due to the stochastic nature of preferences, i.e. σ_g^2, and the third component is due to the fact that preference shocks and aggregate cash flows may be correlated, i.e. that ρ_{Ag} may be different from zero. Since in the base case $\sigma_A = 0$ (see Table 1 for parameter values), only the second term in the numerator survives and the correlation simplifies to

$$\rho_{G,B} = \frac{\sigma_g^2 P_G' P_B'}{\sqrt{\sigma^2 + \sigma_g^2 (P_G')^2}\sqrt{\sigma^2 + \sigma_g^2 (P_B')^2}}.$$

Thus, the two stock returns can be positively or negatively correlated, depending on the signs of P_G' and P_B'.

For low values of g, brown firms are far from finding it optimal to exercise their option to switch to the green technology. In this range of social preferences, supply effects do not play a significant role. Therefore, over this range of g, share prices of green firms increase (decrease) and those of brown firms decrease (increase) as g increases (decreases), implying a negative correlation, as illustrated in Figure 3. However, as g increases and the option to switch technologies becomes more important, the correlation increases and turns positive for sufficiently large values of g. Thus, when brown firms are close to switching to the green technology, both share prices benefit from an increase in g and are hurt by a decrease in g, i.e. P_G' and P_B' in the above expression have the same sign.

5.2 Time-Series Dynamics: An Example

Subsection 5.1 has analyzed the effects of social preferences on share prices, considering a given initial supply of technologies and then varying g from zero to $\bar{g}$. To shed more light on the dynamics and interactions between social preferences, supply of technologies, risk premia and stock return correlations, we now draw a specific path of the stochastic process for g. Assuming that all firms start with the brown technology, we then illustrate how the number of green firms, risk premia, and correlations evolve along this path of g . These numerical results provide additional insights into the time-series of these equilibrium properties and how they relate to the past evolution of g.

Panel A of Figure 4 shows the drawn path for g (left axis) and the corresponding evolution of the fraction of green firms, S_G (right axis). The initial value of g at $t = 0$ is 4, which, for the base-case parameters, is also the threshold at which some brown firms have an incentive to switch to the green technology. Since g initially rises along the drawn path, we observe a rising supply of green firms immediately after time $t = 0$.

An important state variable in the model is the past maximum level of g, or "high watermark". Only if the g-process reaches new highs do we observe a supply response of the corporate sector. The extended flat parts of S_G occur in periods during which social preferences are below previous highs.

Panel B of Figure 4 shows the corresponding price paths, where the price of the green shares is defined on the left axis and the one of the brown shares on the right axis. We note that the price of green stocks is higher than the price of brown stocks throughout the analyzed time period, reflecting the high demand of social investors for these shares. One can also observe that share prices move in opposite directions when g drops below its historical high, but that they appear to co-move when g reaches new highs.

Panel C of Figure 4 shows the evolution of risk premia over time. One can see that shares of brown firms always have a higher cost of capital than green firms because social investors are less willing to hold them. This is true, even taking supply effects into account. Note that this is true in particular when the economy has largely moved to the green technology, say for $t > 13$, so that green firms represent more systematic risk than brown firms. This is more than offset by the stronger preferences that social investors on average have for green shares during these periods. It is interesting to observe that when social preferences drop substantially below their historical maximum, which, in our example, is the case during periods $t = 8$ and $t = 9$, the required risk premium of brown firms drops whereas the required risk premium for green firms increases. This is so, since the option to switch is no longer at the money and green firms are therefore more exposed to social preference risks.

Finally, Panel D of Figure 4 illustrates the dynamics of the correlations between prices. They are calculated both for cum-dividend prices (solid dark green line) and ex-dividend prices (dashed light green line). As can be seen, the ex-dividend prices only vary due to variations in g, and the correlations are therefore either 1 or -1.[11] In contrast, the volatilities

[11] Precisely,

$$\rho_{P_{G,B}} = \frac{\sigma_g^2 P_G' P_B'}{\sqrt{\sigma_g^2 (P_G')^2}\sqrt{\sigma_g^2 (P_B')^2}} = \frac{P_G' P_B'}{|P_G' P_B'|}.$$

of cum-dividend prices are also influenced by the stochastic dividends, and they therefore vary without jumps. Both correlation time series vary over time, as can be seen from Panel D. While there are several periods when prices move in close lockstep (e.g., around time periods 12 and 13), there are also periods when the opposite is the case (e.g., around periods 8 to 10). As seen in this Panel, the variation in correlation is determined by the evolution of social preferences. Correlations are low when social preferences are substantially below past highs, and are high when social preferences approach new highs.

5.3 Green Transition and Model Parameters

In this subsection we analyze the effect of the main model parameters on the dynamics of the simulated economy's transition towards the green technology. We start with the effect of uncertainty of future preference shocks (i.e., σ_g) on the supply of green firms and focus on Figure 5. Panel A shows that an increase in the uncertainty about future preference shocks delays corporate decisions to become green. If there is no preference uncertainty, i.e. $\sigma_g = 0$, then the first brown firms already switch to the green technology when g is slightly below 4. For this case we also observe a fast rate at which the transition to the green technology occurs (i.e. a steep slope of the dark green line in Panel A). Once g has increased to approximately 6, the transition is complete. By contrast, for a high value of preference uncertainty, i.e. $\sigma_g = 0.5$, the switching to the green technology starts at substantially higher values. For this parameter value, g must reach a value of approximately 4.5 before the first brown firm switches to the green technology. Further transition to the green technology also occurs at a lower rate (i.e. slope of the light green line is lower). Complete transition is only reached for a value of g that is approximately 7. This is intuitive, as the option value to delay switching is more valuable in this case.

Panel B of Figure 5 shows the impact of risk aversion on the supply of green firms. The higher the risk aversion of investors, the earlier some firms start to switch to the green technology. However, the transition happens at a slower rate, i.e. the S_G-line becomes flatter as investors' risk aversion increases.Thus, for high degrees of risk aversion, even moderate social preferences induce some firms to switch to the green technology. This is so, since the green technology is initially a good diversifyer for the representative investor's portfolio, since she only holds brown shares initially. This induces some firms to switch at even low social preference values. However, the diversification benefits decrease as the number of green firms grows. In the latter case, brown firms become more valuable diversifyers, as more firms have already switched to green. This explains the lower adoption rate for the green technology for a given rise in g.

In the case of low risk aversion, it is interesting to point out that the full transition happens over a narrow range of g values. In this case, investors are not very sensitive to changes in systematic risk or to risk sharing arguments. Thus, as soon as preference levels are high enough to induce switching from brown to green technologies, the transition starts and progresses very quickly.

Finally we illustrate the effect of the fraction of social investors. Figure 5, Panel C, illustrates the transition from an economy where all firms use the brown technology to a green economy for three different values of the fraction of investors with social preferences: 25, 50 and 75%. The case of $\theta = 50\%$ represents the base-case parameter value underlying the previous analysis. In this case we see, as discussed before, that the transition towards a green economy starts around $g = 4$ and is complete for a social preference parameter of around $g = 6$. If there is a larger fraction of investors with social preferences, the transition happens earlier,i.e. starting at lower values of g, and faster, i.e., the slope of the S_G-line is steeper. This finding accords with intuition, as any change in preferences of social investors, g, is amplified by the fraction of investors with social preferences. The same intuition applies to the case when the fraction of social investors is only 25%. In this case, the transition starts later and is slower.

5.4 Correlation between Firms' Cash Flows

So far, we have considered numerical examples, where the two technologies' cash flows are uncorrelated. In this subsection we extend the numerical analysis to allow for such a correlation. To this end, we allow σ_A in equations (3) and (4) to be strictly positive, which introduces a correlation between the two technologies' cash flows via the common shock $dz_{A,t}$. To make the results comparable to the ones obtained without correlation, we keep the total variance of each technology's cash flows constant. This is achieved by setting $\sigma = \sigma_A = 2\sqrt{2}$, which implies a cash flow variance for each technology given by $\sigma^2 + \sigma_A^2 = 16$, as in the base case above. At this parameterization the two technologies' cash flows exhibit a correlation of $\frac{\sigma_A^2}{\sigma^2+\sigma_A^2} = 0.5$. To isolate the effect of cash flow correlations, we still assume that common cash flow shocks are uncorrelated with preference shocks, i.e. $\rho_{Ag} = 0$.

The results are summarized in Panel A of Figure 6. Comparing Panel A of Figure 1, which assumes uncorrelated cash flows, to Panel A of Figure 6 reveals that share prices are generally lower for both brown and green firms in the latter Figure. This is so since, compared to the base case, there is now more non-diversifiable risk in the economy, which implies that the representative investor generally requires higher risk premia.

However, the common cash-flow component also has important implications for brown firms' decisions to switch to the green technology: compared to the base case, the decision to switch is substantially delayed. While brown firms start switching at a g of around 4 in the base-case, they now only start switching when g is 4.6. This is due to the fact that now green firms are less effective diversifiers than in the base case. Thus, it requires stronger preferences by social investors, i.e. larger non-pecuniary benefits to achieve stock prices that incentivize brown firms to switch. However, the full transformation of the economy to the green technology is complete at a lower g than in the base-case. This is so, since the lower diversification resulting from the brown technologies makes retaining some brown firms in the economy less important.

5.5 Interactions between Aggregate Cash Flows and Social Preferences

There is empirical evidence that social preferences are affected by the state of the economy. In this subsection we therefore extend the numerical analysis and consider different levels of correlation between shocks to social preferences and common cash flow shocks. To isolate the effect of this correlation, we again keep the total variance of each technology's cash flows constant and set $\sigma = \sigma_A = 2\sqrt{2}$. In the discussion below, we compare $\rho_{Ag} = \{-0.9; 0; 0.9\}$.

Panels B and C of Figure 6 illustrate the effects of correlation between cash-flow and preference shocks on prices of brown and green firms. Comparing the two graphs, one first observes that a positive correlation of social preferences with the aggregate cash flow component makes brown firms relatively more, and green firms relatively less valuable. This is so since a positive correlation effectively makes green firms riskier compared to brown firms. To understand this, consider a negative shock to the common cash flow component, dz_A, which coincides with a negative shock to social preferences, dz_g. Since both types of technologies are exposed to this negative common cash flow shock, both firms' dividends tend to be below expectations in such a state. However, holders of brown shares are partly hedged against such a negative cash flow shock via the negative preference shock, that occurs at the same time. Such a preference shock which makes brown shares more attractive (less unattractive) to social investors which tends to help the share prices of brown firms. The opposite is true for green shares. Holders of green shares not only face a low dividend due to the negative cash flow shock, but these shares now also become less attractive to social investors, leading to a negative effect on prices. Thus, when the green firms' cash flows are low or negative, then social preferences also tend to become weaker, i.e. g tends to drop and this hurts green firms' valuation.

This intuition can be confirmed when comparing share prices for the case where $S_G = 25\%$

and $\rho = -0.9$ with the share prices when the fraction of green firms is the same ($S_G = 25\%$) but $\rho = 0.9$. At $g = 0$, for example, we observe that green firms' share price is approximately 96.9 when the correlation is positive (Panel C), and approximately 99.0 when the correlation is negative (Panel B). In contrast, brown firms' share price is 86.5 when the correlation is positive (Panel C), and 82.7 when $\rho = -0.9$ (Panel B). Also consistent with the above intuition is the fact that the critical value of social preferences, $\bar{g}_{S_G=25\%}$, at which additional brown firms wish to switch to the green technology is higher when $\rho = 0.9$ compared to the case of negative correlation. Also $\bar{g}$, i.e. the critical value of social preferences at which all firms have switched to the green technology is substantially higher when the correlation is positive than in the case of a negative correlation.

To summarize, if investors' social preferences are countercyclical, i.e. if g tends to increase in bad economic times, then they have a stronger effect on corporate behavior than in the case of procyclical preferences.

5.6 Social preferences, firms' cost of capital and technology supply

This final subsection analyzes the effect of social preferences on the difference between green and brown firms' cost of capital and discusses whether this difference is a relevant measure of the importance of social preferences on the overall supply of brown versus green firms. To this end, we calculate the wedge between the cost of capital of brown and green firms that obtains in the absence of social preferences and compare it to the one that is obtained when a fraction θ of investors exhibits social preferences. Thus, in the spirit of Berk and van Binsbergen (2022), we calculate a diff-in-diff, where the first diff refers to the cost of capital difference without social preferences, and the latter to the one with social preferences. Figure 7 illustrates this diff-in-diff measure for different levels of S_G. Importantly, we hereby always assume that social preferences are such that $g = \bar{g}$, i.e. firms are at the threshold where the marginal brown firm is indifferent between switching and not switching to the green technology.

Figure 7 displays the cost of capital diff-in-diff for two different fractions of green investors, $\theta \in \{10\%, 50\%\}$ as solid lines. We note that for small numbers of green firms, the cost of capital diff-in-diff is very small, i.e. below 50 bps for both levels of θ. As S_G increases, so does the cost of capital diff-in-diff. This increase is almost exclusively driven by the difference in green and brown firms' cost-of-capital in the absence of social preferences, as in this case the cost-of-capital of green firms increases noticeably when S_G increases.

However, to assess the relevance of a small shock in social preferences in each of these two cases, the cost of capital diff-in-diff is not the relevant characteristic. Instead, we would like to understand to what extent a small change in preferences, i.e. a small change in $\bar{g}$ conditional on a given fraction θ of investors, would affect the supply of green firms. This amounts to the first derivative of S_G w.r.t. $\bar{g}$. We note that this derivative is not related to the cost of capital diff-in-diff at all. As indicated by the dashed lines in Figure 7, it is constant for each θ. Specifically, this derivative is 0.112 for $\theta = 0.1$ and 0.615 for $\theta = 0.5$. It is intuitive that a given preference shock has stronger effects on the supply of firms when there are many green investors than when there are only few of them, i.e. when θ is low.

Importantly, however, while the difference between $dS_G/d\bar{g}$ is sizable across values of θ, the difference between the two solid lines is very small. That indicates that even though for a given level of S_G the cost-of-capital diff-in-diff does not vary much with θ, the sensitivity of S_G with respect to $\bar{g}$ is still large.

6 Conclusion

This paper develops a model where investor tastes towards firms' SRI policies evolve stochastically and firms have a real option to respond to changing investor tastes by switching from brown to green technologies. The interaction between preference shocks and supply effects of the corporate sector generates several new insights. In general, firms' responses to changing investor tastes mitigate or fully undo the effects that preference shocks would have in a world where firms' investment decisions are fixed exogenously. For example, when investors' tastes for green technologies become more pronounced, this can have positive valuation effects for brown firms, as their option to switch technologies becomes more valuable. Depending on the past evolution of tastes, preference shocks can therefore increase or decrease the correlation between brown and green firms.

We find that uncertainty about the future evolution of investor tastes delays the transition to greener technologies. When preference shocks are positively correlated with the state of the economy, i.e. social preferences become stronger in good states, as indicated by empirical evidence, then this delays the switch to green technologies even further. The analysis also documents the effect of risk aversion and firms' risk characteristics on the economy's transition to a greener economy. A higher correlation between the two technologies' cash flows always increases the speed of the transition, whereas a higher risk aversion of investors induces some firms to switch earlier, but delays a complete transition towards a green economy.

The analysis also documents that in a world with stochastic investor tastes and endogenous corporate decisions, the effect of green investors on the difference between brown and green firms' cost of capital is not a good indication of the impact that social preferences have on corporate decisions. Especially when brown and green technologies' returns are positively correlated, significant supply shifts in response to investor tastes occur even when the difference in the observed cost of capital is small.

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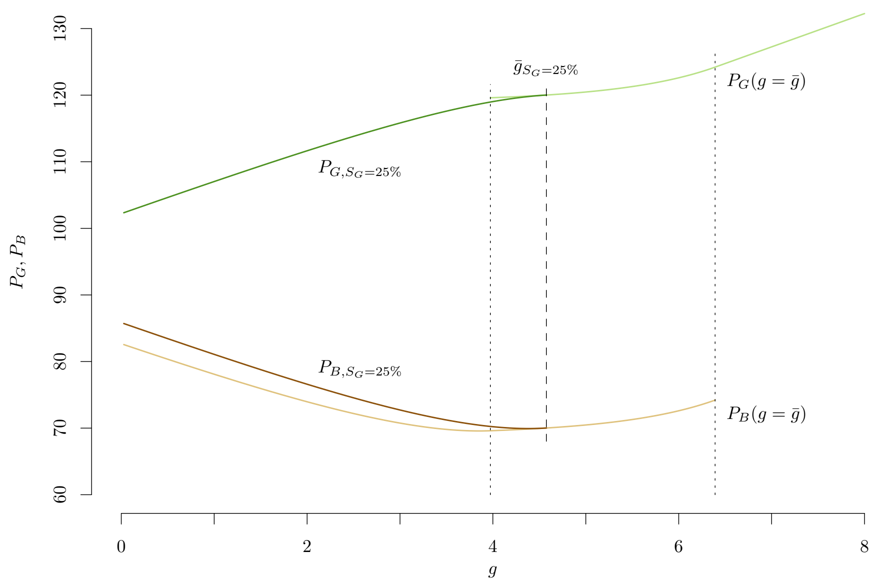
Table 1 Parameter Values

Fraction of social investors	$\theta = 0.5$
Aggregate absolute risk aversion of all investors	$\gamma = 0.075$
Expected cash-flows	$\mu = 10$
Volatility of technology-specific cash-flow shocks	$\sigma = 4$
Volatility of aggregate cash-flow shocks	$\sigma_A = 0$
Risk-free rate of return	$r = 0.1$
Expected non-pecuniary dividends	$\mu_g = 0.1$
Volatility of non-pecuniary dividend shocks	$\sigma_g = 0.2$
Correlation between preference shocks and aggregate cash-flow shocks	$\rho_{Ag} = 0.0$
Costs to switch technology	$I = 50$

Figure 1: **Share prices with dynamic technology choices**
The figure shows prices of brown and green firms for different levels of social investor preferences. Low $\bar{g}$

(A) Share prices



(B) Risk premia

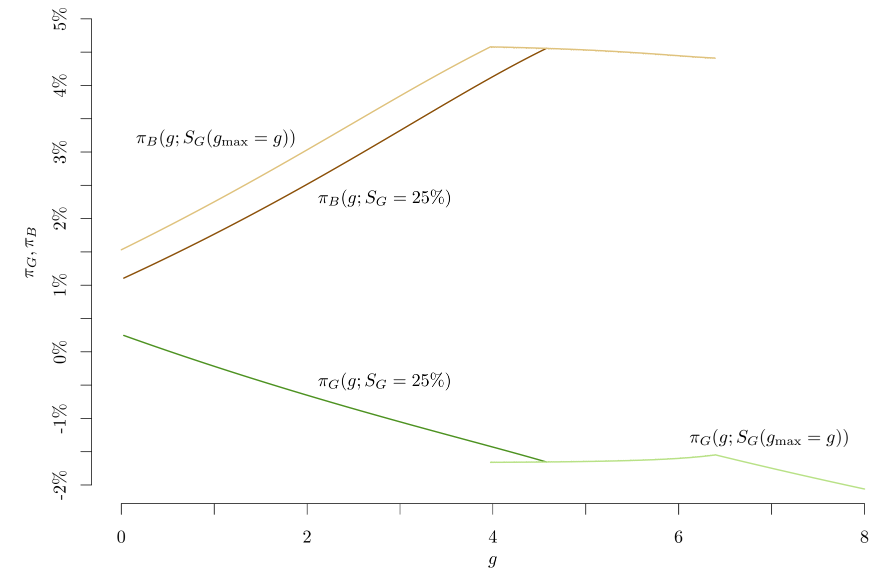
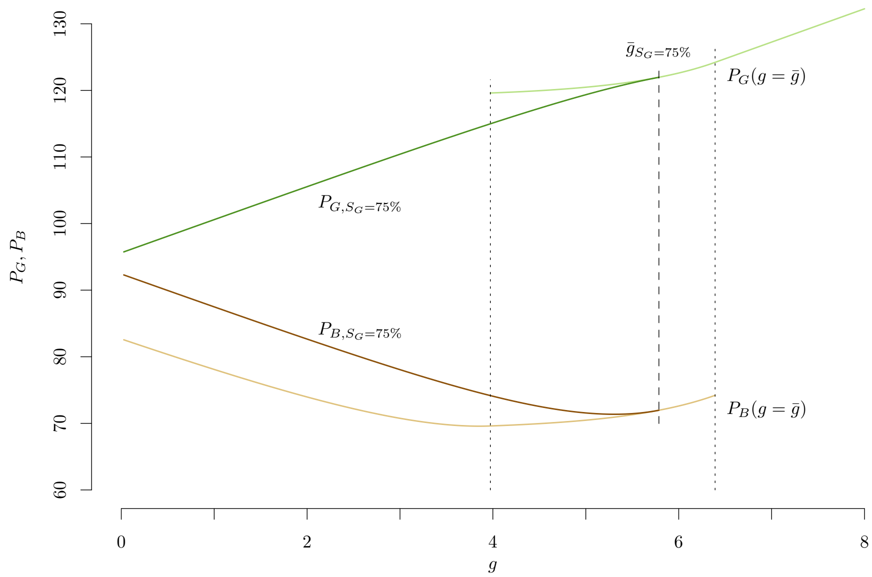


Figure 2: High $\bar{g}$

(A) Share prices



(B) Risk premia

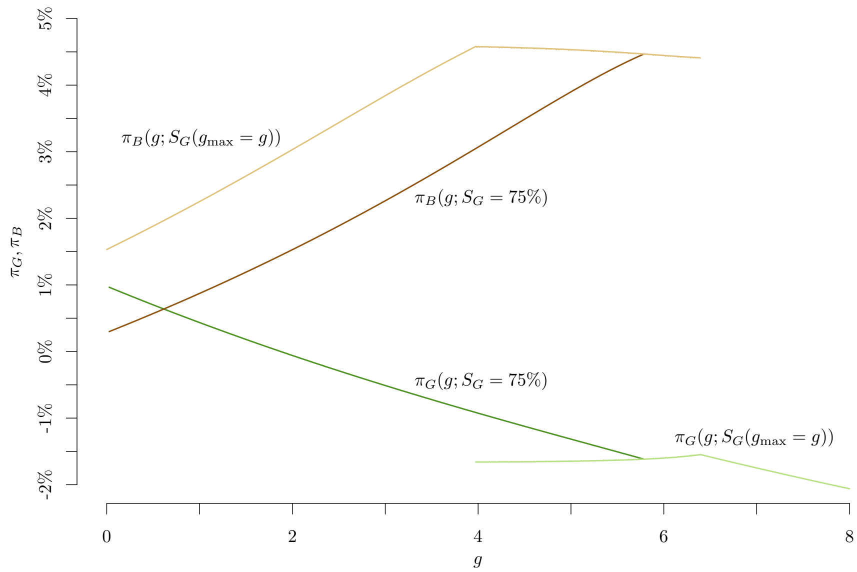


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Figure 3: **Social preferences and correlations between green and brown firms**
The figure shows correlations between price changes of brown and green firms for different levels of social investor preferences.

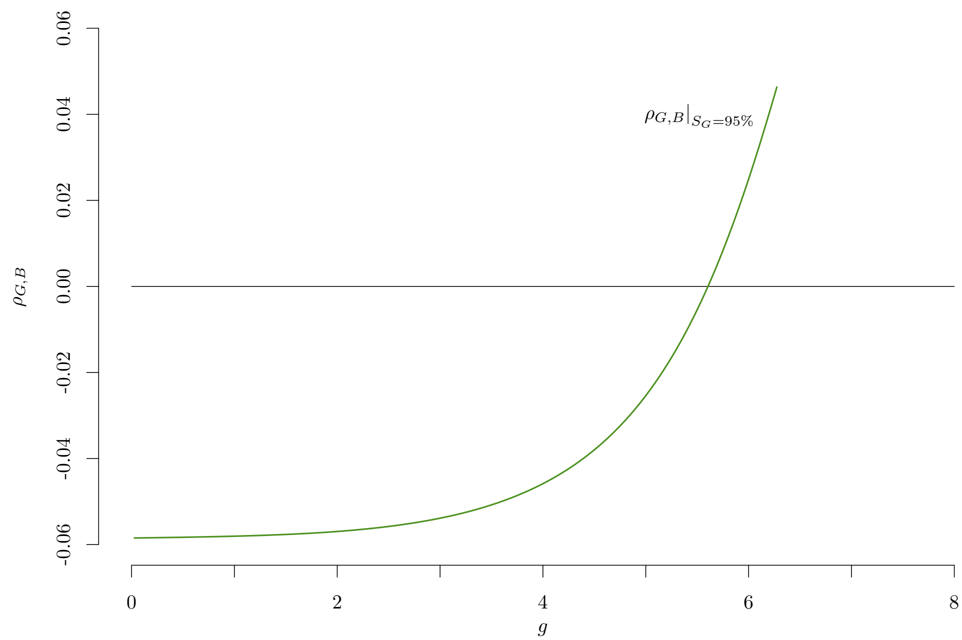
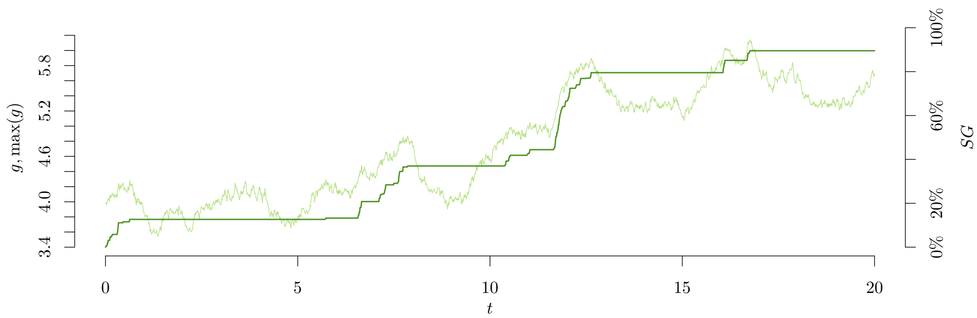
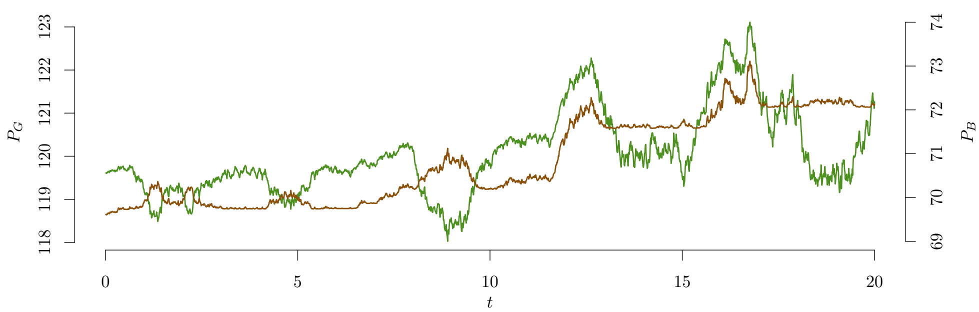


Figure 4: **A given path of social preferences.**
Panel (A) shows the sample path together with the supply of green firms; Panel (B) shows the share prices of green and brown firms for this path; Panel (C) plots risk premia of green and brown firms over the sample path; Panel (D) shows the correlation of cash flows and share price changes of green and brown firms.

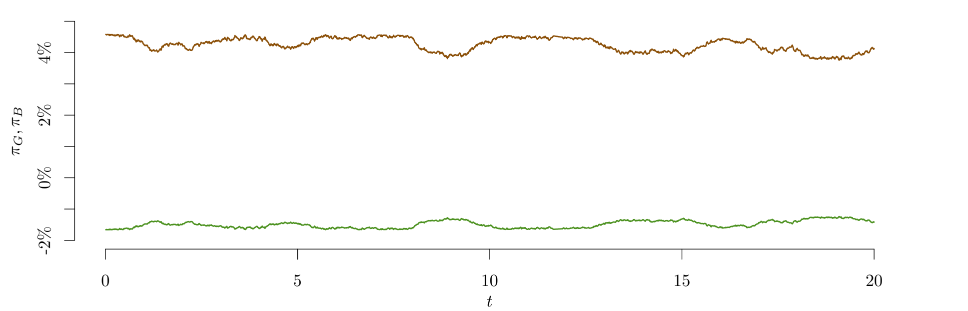
(A) Supply of green firms.



(B) Price of green and brown firms.



(C) Risk premia of green and brown firms.



(D) Correlation of cash flows and share price changes

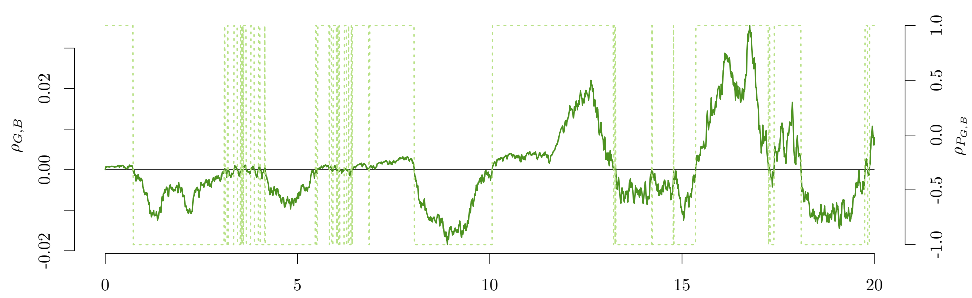
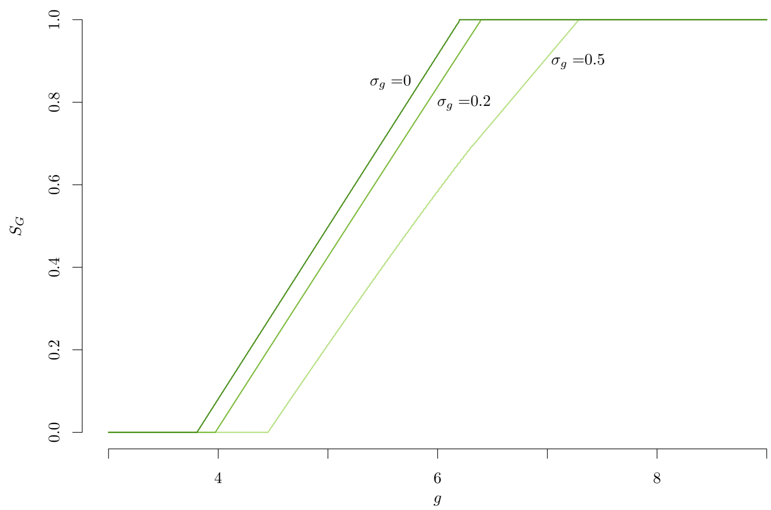
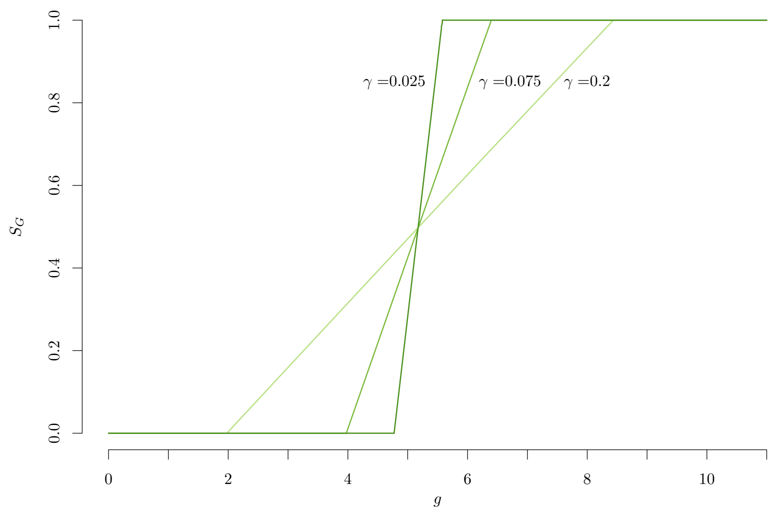


Figure 5: **Supply of green firms and comparative statics**
The figure illustrates how the supply of green firms depends on uncertainty about future social preferences (Panel A), risk aversion (Panel B) and the fraction of social investors (Panel C).

(A) Uncertainty about future social preferences



(B) Risk aversion



(C) Fraction of social investors

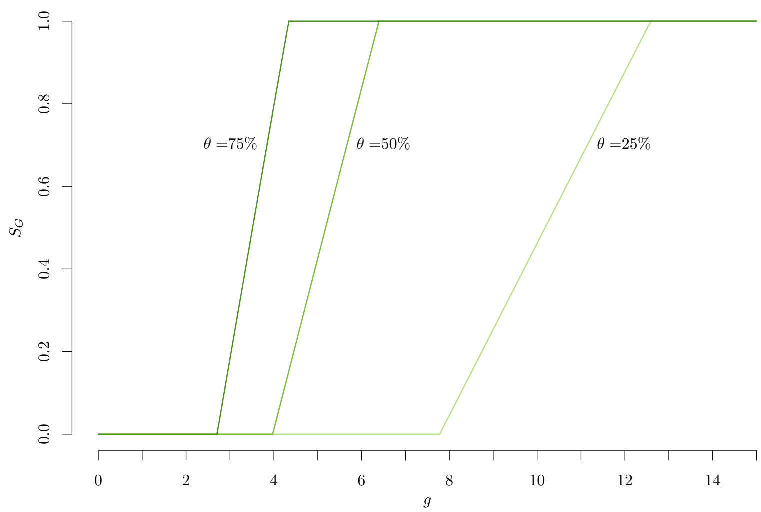
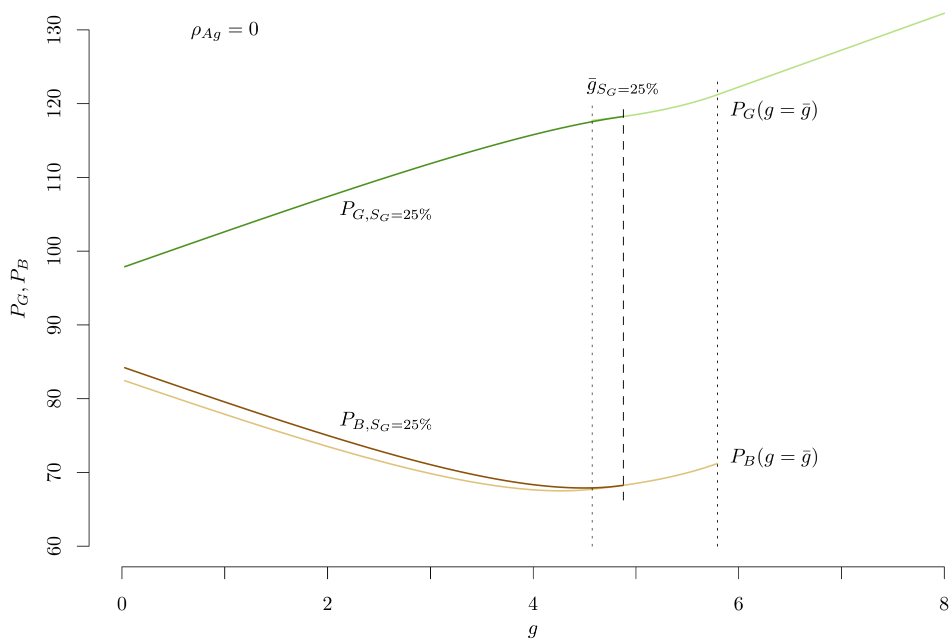


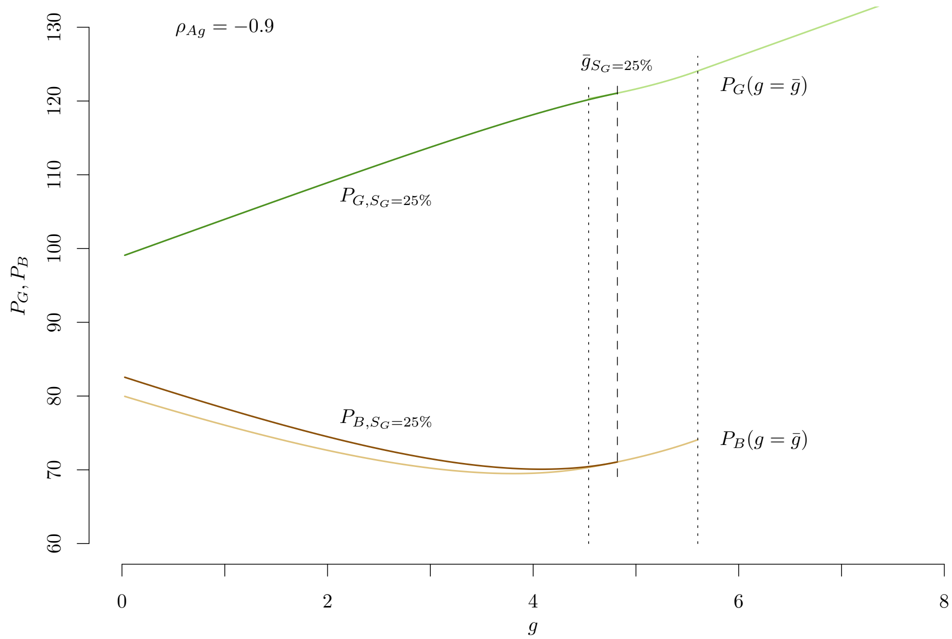
Figure 6: **Share prices with dynamic technology choices and correlations between preference and cash-flow shocks**

The figure shows prices of brown and green firms for different levels of social investor preferences for different correlations between preference shocks and aggregate cash-flow shocks. In all panels, the correlation between cash flow shocks of green and brown firms is equal to 0.5, while the total variance of cash flows is equal to one in the base case. This is done by setting $\sigma = \sigma_A = 2\sqrt{2}$, which implies $\sigma^2 + \sigma_A^2 = 16$ and $\frac{\sigma_A^2}{\sigma^2+\sigma_A^2} = 0.5$. Cash flow shocks and shocks to preferences are assumed uncorrelated in Panel (A). Panels (B) and (C) show the cases of $\rho_{Ag} = -0.9$ and $\rho_{Ag} = 0.9$, respectively.

(A) Basecase: $\rho_{Ag} = 0$



(B) Negative correlation $\rho_{Ag} = -0.9$



(C) Positive correlation $\rho_{Ag} = 0.9$

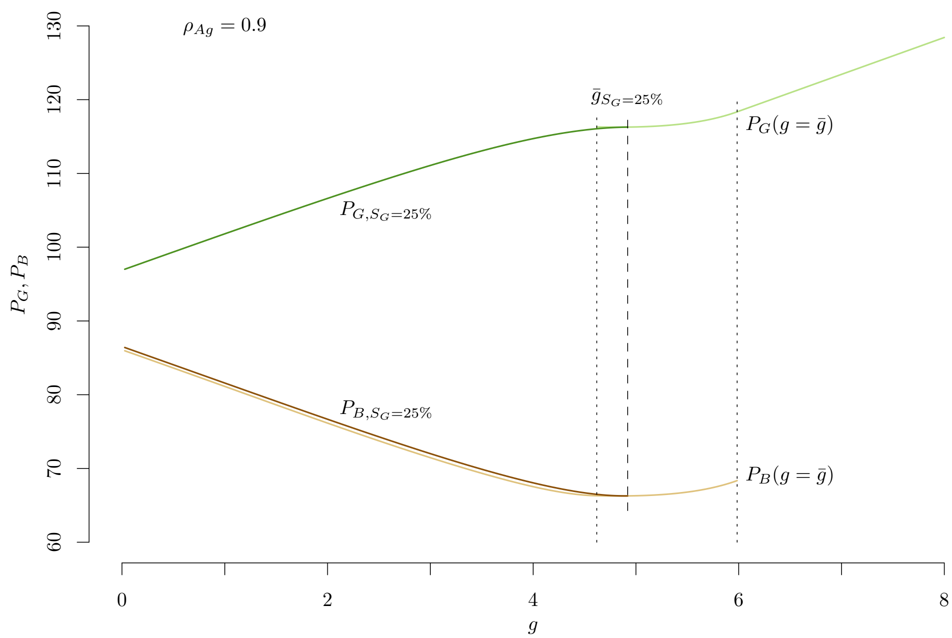
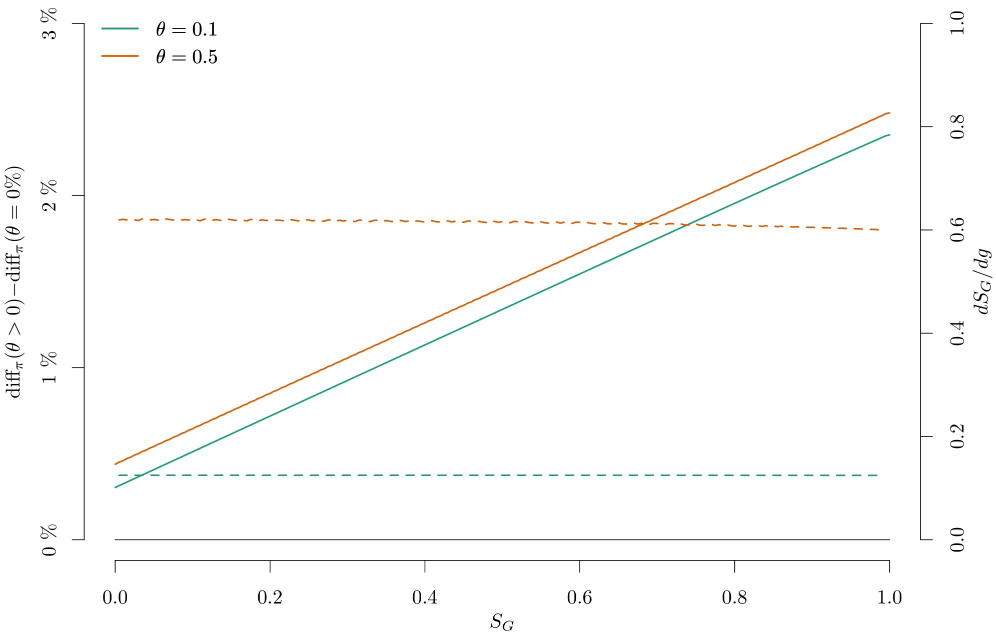


Figure 7: **Diff-in-diffs of risk premia and the sensitivity of the supply of green firms to shocks in**
g for different levels of θ

Solid lines show the diff-in-diff of risk premia (costs of capital) of green versus brown firms between a model with a fraction of θ green investors and a model with no green investors. Dashed lines show the sensitivity of the fraction of green firms to shifts in preferences S_G/dg at the investment threshold.



Appendix

A Calculating $E_t[dC_t]$ and $E_t[dC_t^2]$

For the expected value of consumption, note that

$$\begin{aligned} E_t[dC_t] &= \Big[E_t[dy_G + dP_G(g)] - rP_G(g)dt + \theta g dt\Big] X_G + \Big[E_t[dy_B + dP_B(g)] - rP_B(g)dt - \theta g dt\Big] X_B \\ &= \Big[(r + \pi_G(g))\, P_G(g)dt - rP_G(g)dt + \theta g dt\Big] X_G + \Big[(r + \pi_B(g))\, P_B(g)dt - rP_B(g)dt - \theta g dt\Big] X_G \\ &= \Big[(\pi_G(g)P_G(g) + \theta g)\, X_G + (\pi_B(g)P_B(g) - \theta g)\, X_B\Big] dt \end{aligned}$$

where the second equality follows from the definition of $\pi_G(g)$ and $\pi_B(g)$, respectively.

Next, ignoring terms that are of higher order than dt, it is easy to show

$$\begin{aligned} dC_t^2 &= \left(dy_G^2 + 2dy_G dP_G(g) + dP_G^2(g)\right) X_G^2 \\ &+ \left(dy_B^2 + 2dy_B dP_B(g) + dP_B^2(g)\right) X_B^2 \\ &+ \left(dy_G dy_B + dy_G dP_B(g) + dy_B dP_G(g) + dP_G(g) dP_B(g)\right) 2X_G X_B \end{aligned}$$

Itô's lemma implies that the dynamics of share prices depend on those of social preferences as follows:

$$\begin{aligned} dP_G(g) = P'_G dg + \frac{1}{2}P''_G (dg)^2 &= \left(\mu_g P'_G + \frac{1}{2}\sigma_g^2 P''_G\right) dt + \sigma_g P'_G dz_g, \\ dP_B(g) = P'_B dg + \frac{1}{2}P''_B (dg)^2 &= \left(\mu_g P'_B + \frac{1}{2}\sigma_g^2 P''_B\right) dt + \sigma_g P'_B dz_g. \end{aligned}$$

Using the two price dynamics we calculate the various components of $E_t[dC^2]$ as follows:

$$\begin{aligned} E_t[dy_G^2] = E_t[dy_B^2] &= \left(\sigma^2 + \sigma_A^2\right) dt, \\ E_t[dP_G^2(g)] &= \sigma_g^2 \left(P'_G\right)^2 dt, \\ E_t[dP_B^2(g)] &= \sigma_g^2 \left(P'_B\right)^2 dt, \\ E_t[dy_G dy_B] &= \sigma_A^2 dt, \\ E_t[dy_G dP_G(g)] = E_t[dy_B dP_G(g)] &= \rho_{Ag}\sigma_A\sigma_g P'_G dt, \\ E_t[dy_G dP_B(g)] = E_t[dy_B dP_B(g)] &= \rho_{Ag}\sigma_A\sigma_g P'_B dt, \\ E_t[dP_G(g) dP_B(g)] &= \sigma_g^2 P'_G P'_B dt. \end{aligned}$$

Finally, putting everything together yields the expression of $E_t[dC_t^2]$ as is stated in the main text.

It may also be worthwhile noting that

$$var_t[dC_t] = E_t[dC_t^2] - (E_t[dC_t])^2 = E_t[dC_t^2]$$

since $(E_t[dC_t])^2$ is of order $(dt)^2$ and is therefore ignored. Thus, our specification of the utility function in Equation (10) is essentially a continuous-time limit of the certainty equivalent approach in discrete time models.

B Numerical method for solving the system of Hamilton-Jacobi-Bellman equations

We start from two candidate solutions $\hat{P}_G$ and $\hat{P}_B$ that are elements of the polynomial space spanned by the first n Chebyshev polynomials $T_0, \ldots, T_{n-1}$,

$$T_i(g) = \cos(i \cos^{-1}(g)), \quad i = 0, \ldots n-1.$$

So the $2n$ coefficients $c_{G,i}$ and $c_{B,i}$ determine the functions $\hat{P}_G$ and $\hat{P}_B$

$$\begin{aligned} \hat{P}_G(g) &= \frac{1}{2}c_{G,0} + \sum_{i=1}^{n-1} c_{G,i}T_i(g), \\ \hat{P}_B(g) &= \frac{1}{2}c_{B,0} + \sum_{i=1}^{n-1} c_{B,i}T_i(g). \end{aligned}$$

Referring to the system of HJB equation (20), define the differential operator

$$\begin{aligned} \begin{bmatrix} \mathcal{D}_G(\hat{P}_G, \hat{P}_B) \\ \mathcal{D}_B(\hat{P}_G, \hat{P}_B) \end{bmatrix} &= \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \mu_g \begin{bmatrix} \hat{P}'_G \\ \hat{P}'_B \end{bmatrix} + \frac{1}{2}\sigma_g^2 \begin{bmatrix} \hat{P}''_G \\ \hat{P}''_B. \end{bmatrix} \\ &\quad - \begin{bmatrix} r\hat{P}_G \\ r\hat{P}_B \end{bmatrix} - \left(\gamma \begin{bmatrix} \Sigma_G & \Sigma_{GB} \\ \Sigma_{GB} & \Sigma_B \end{bmatrix} \begin{bmatrix} S_G \\ S_B \end{bmatrix} - \begin{bmatrix} \theta g \\ -\theta g \end{bmatrix} \right), \end{aligned}$$

where Σ_S, Σ_B and Σ_{SB} are also evaluated at $\hat{P}_\cdot$.

If $\hat{P}_G$, $\hat{P}_B$ are solutions of (20), then

$$\begin{bmatrix} \mathcal{D}_G(\hat{P}_G, \hat{P}_B) \\ \mathcal{D}_B(\hat{P}_G, \hat{P}_B) \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{22}$$

The collocation method relaxes condition (22) such that it requires $[\mathcal{D}_G(\hat{P}_G, \hat{P}_B), \mathcal{D}_B(\hat{P}_G, \hat{P}_B)]'$ to vanish only on the space spanned by the first n Chebychev polynomials (and not everywhere).

This means, that after projecting $[\mathcal{D}_G(\hat{P}_G, \hat{P}_B), \mathcal{D}_B(\hat{P}_G, \hat{P}_B)]'$ onto the Chebychev basis

$$
\begin{aligned}
\hat{\mathcal{D}}_G(\hat{P}_G, \hat{P}_B)(g) &= \frac{1}{2}k_{G,0} + \sum_{i=1}^{n-1} k_{G,i} T_i(g), \\
\hat{\mathcal{D}}_B(\hat{P}_G, \hat{P}_B)(g) &= \frac{1}{2}k_{B,0} + \sum_{i=1}^{n-1} k_{B,i} T_i(g),
\end{aligned}
$$

all projection coefficients $k_{.,i}$ must vanish,

$$
\begin{aligned}
k_{G,i} &= \langle \mathcal{D}_G(\hat{P}_G, \hat{P}_B) | T_i \rangle \\
&= \frac{2}{n} \sum_{j=0}^{n-1} \hat{\mathcal{D}}_G(\hat{P}_G, \hat{P}_B)(z_j^n) T_i(z_j^n) \\
&= 0, \\
k_{B,i} &= \langle \mathcal{D}_B(\hat{P}_G, \hat{P}_B) | T_i \rangle \\
&= \frac{2}{n} \sum_{j=0}^{n-1} \hat{\mathcal{D}}_B(\hat{P}_G, \hat{P}_B)(z_j^n) T_i(z_j^n) \\
&= 0, \\
i &= 0, \ldots, n-1,
\end{aligned} \tag{23}
$$

and z_j^n, $j = 0, \ldots, n-1$ are the n roots of T_n.

The relaxation translates the system of HJB equations (20) into a system of $2n$ nonlinear equations (23), i.e., we have to find the set of $2n$ coefficients c of $\hat{P}_.$ that make the $2n$ coefficients k of $\mathcal{D}_.$ vanish.

For the treatment of the boundary conditions, we treat $c_{.,0}$ as a function of the boundary conditions and drop two projections restrictions from the set (23).