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JEL Classification : C02, C61, G11

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Practical weight-constrained conditioned portfolio optimisation using risk aversion indicator signals

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Abstract

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This paper reports empirical results obtained when backtesting conditioned portfolio strategies, parameterised in various ways, using a real-world data set. These are the first published results illustrating the performance of conditioned portfolio optimisation in the single signal, constrained-weight case; for details on the optimal control formulation that allows solving this type of problem, see [2].
The structure of the paper is as follows. Section 1 introduces the problem of conditioned mean-variance portfolio optimisation and provides a brief survey of the existing literature. Risk aversion indicators are introduced and the data set used for the present study is described. Section 2 then presents and discusses the various backtesting results obtained. Section 3 concludes.

1 Context

1.1 The conditioned portfolio problem

Following a function analytic argument in [4], the type of portfolio problem nowadays often referred to as conditioned was formulated and solved by Ferson and Siegel in [1]. The authors consider the problem of mean-variance portfolio optimisation within a discrete-time myopic investment world, such that only two time instants are considered - an initial time t, at which the investment choice is made, and a final time $t+1$, at which the investment returns are examined. The authors introduce a vector of signals s; what makes these signals interesting to the portfolio manager is that there is assumed to exist some measurable relationship $\mu(s)$ between the signal as observed at the initial time and the return as revealed at the final time.[1] The fundamental signal-return relationship is then

$$r_{t+1} = \mu(s_t) + \epsilon, \tag{1}$$

where the time indices on r and on s will be suppressed in what follows. Here ϵ is a noise term whose conditional mean given s is assumed to be zero. There is no specific a priori requirement on the functional form of $\mu(s)$. Note that, with this model,

$$E[r|s] = \mu(s) + E[\epsilon|s] = \mu(s)$$

and that $\mu(s)$ is regarded as deterministic once s is known i.e. contributes nothing to the conditional variance of returns.
The Ferson and Siegel paper derives expressions for unconditional portfolio mean and variance given the signal-return relation (1) holds, and shows that the optimal portfolio weights in the presence of n risky assets and a risk-free asset with return r_f equal

$$u'(s) = \frac{\mu_P - r_f}{E[(\mu(s) - r_f e)'\Lambda(s)(\mu(s) - r_f e)]}(\mu(s) - r_f e)'\Lambda(s), \tag{2}$$

where μ_P is the required expected unconditional portfolio return, e is an n-vector of ones and $\Lambda(s) = [(\mu(s) - r_f e)(\mu(s) - r_f e)' + \Sigma^2]^{-1}$: here Σ^2 (which may in its greatest generality be a function of s) is the conditional covariance matrix $E[\epsilon\epsilon'|s]$. A similar closed-form expression is obtained for the case where no risk-free asset is available.
Further closed-form solutions covering the benchmark tracking error minimisation versions of the basic problem are derived in [6]. Empirical illustrations respectively studies of the solutions to these problems are provided in papers such as [6], [7], [8]. [9] reports on an optimisation approach comparable, but not identical, to that proposed by Ferson and Siegel, in which quadratic utility is directly maximised, and the index value is used in a quadratic term that implements a penalty for leverage.
Except for [9], all of the above papers cover the basic version of the conditioned portfolio problem for which admissible weights are not constrained. As in the case of the classic Markowitz problem, the addition of a weights constraint leads to problem variants that can only be solved generally using a numerical scheme. [2] describes how to express conditioned optimisation problems in optimal control terms and proves extended versions of the relevant classical necessity and sufficiency results which are the Pontryagin Minimum Principle and the Mangasarian sufficiency theorem. This implies that generic variants of the basic problem, for which the signal support can correspond to the full real line in the general case, may be analysed using these

[1] Note it is this lagged relationship, with the optimisation opportunities it entails, which constitutes the principal fundamental difference between the present setup and market factor models such as the APT ([5]).

standard tools of optimal control theory, as well as solved numerically using any of the available algorithms. We note in this context that the typical conditioned problem involves optimisation of the *unconditional* mean with respect to the *unconditional* variance even though the manager by assumption has access to the signalling information. Indeed the optimality of that approach has been confirmed by a generic function theoretical argument (see [4]), by financial intuition (the manager is evaluated by generally uninformed investors, who judge manager performance based on their observation of the unconditional moments, see [1]) and by empirical comparisons with the alternative strategy where conditional moments are optimised (see [6]). We thus follow this problem setting and this entails that, in each case, the full optimal control problem has to be solved: solutions then correspond to optimal weight functionals parameterised by the signal value. Only once this full solution has been obtained, ensuring that the unconditional expected variance is minimised and the expected return constraint is respected over the full signal support, may the portfolio manager apply their privileged information by using the observed signal value with the postulated signal-return relationship when evaluating a portfolio strategy.

The specific type of conditioned optimisation problem this paper focuses on is the basic problem as analysed in [1], described above, but with the addition of portfolio weights constraints. A particularly interesting constraint for real-world applications is obtained when we exclude the possibility of negative investment amounts i.e. short positions. It is generally unrealistic to assume that these can be entered at no extra cost or risk; this is especially true for naked short positions which correspond to a short sale of a security undertaken without borrowing the asset at the same time. Additionally, a significant proportion of investors fundamentally want to avoid the unlimited downside risk associated with shorting, the increased leverage often introduced by optimal positions involving shorts or indeed, as is the case with pension funds in many countries, may be prohibited by regulators from entering short investments.

Another realistic constraint on invested weights would limit them to a certain interval often centred on zero, such as $[-a, a]$ for $a \in \mathbb{R}$, so as to avoid entering greatly leveraged positions in particular assets and thus incurring excessive undiversified risk. We will focus on the short sales constraint in what follows since it is more frequently used in a practical context, but provide a performance check for this second amplitude constraint as well.

1.2 Data

The data set used collects eleven years of daily returns data chosen to represent a market relevant to investors with domestic currency EUR. This market is made up of ten different funds[2] chosen across both equity and fixed income markets as well as Morningstar style classifications. All funds involved provide EUR return quotes and manage at most a proportion of 30% in non-EUR assets so as to manage the impact of currency risk on the choice of investments. The data covers business days from January 1999 to February 2010: in total, each series contains 2891 returns. Funds rather than individual assets were chosen given they provide a level of built-in diversification and a ten-asset market composed of funds is thus seen as more realistic as an equivalent equity market; additionally, interest-rate exposure is easily achieved through funds. Investment strategies involving funds and requiring frequent portfolio rebalancings, such as the ones being examined, have become realistically achievable even for small investors with the advent of exchange traded funds (ETF). Although the funds listed above are not ETFs, this choice was made purely because of the need to obtain sufficiently exhaustive historical data series: funds comparable to those used are nowadays accessible in an ETF format. Table 2 shows the summed log-returns of each asset over the backtesting period, while table 3 shows correlations between asset returns, and confirms the diversification point made above. The time interval covered by the data is especially interesting as it encompasses two major crises as far as market returns are concerned: the bursting of the dot-com bubble (roughly from spring 2000 to spring 2003) and the initial bear market triggered by the outbreak of the currently ongoing financial crisis (roughly spanning summer 2007 to spring 2009).

As a proxy to an idealised risk-free asset, the EURIBOR interbank rate was chosen. We use the 1 week maturity since it is thought to offer a good compromise between the conflicting requirements of high liquidity and exact correspondence to the daily frequency of the investment returns considered.

[2] AXA L Fund Equity Europe (AXA), Credit Suisse Bond Fund Management Company Luxembourg Small (CSU), Dekalux Midcap TF (DEK), Dexia Luxpart C (DEX), DWS Euro Bonds Long (DWS), Fidelity Funds Euro Bond Fund A Global Certificate (FIB), Fortis L Fund Equity Socially Responsible Europe (FOB), Invesco Pan European Small Cap Equity Fund Lux (INV), KBC Money Euro Medium Cap (KBC) and Morgan Stanley European Currencies High Yield Bond (MSE). In every case the reinvesting variant of the fund was picked.

Possible candidates for signals on which to condition span the entire spectrum from macroeconomic indicators (e.g. inflation rates or currency forward premia as in [6]) to purely behavioural signals (e.g. University of Michigan consumer sentiment data, [7]), with possible explanatory power on future returns being derived from the candidate time series' ability to reflect market risk in some form, investor attitude or some combination of the two. Under the overarching designation of risk aversion (or risk appetite) indicators, such series have also been constructed either for their own sake or in order to identify market crises and trends independently of any possible investment application, both by individual banks and in academic research. A helpful survey discussing the different types of risk aversion indicators is [3]. Intuition is perhaps the main guide as to which potential signals seem the most promising in the context of investment management, but the idea's appeal is clear and has resulted in a number of market indices that informally implement conditioned trading strategies, e.g. the Nomura Macro Pulse index, quoted on Bloomberg sub NMIRPU1E, or the JPMorgan Helix index with ticker JHLXHEU. We will now briefly expand on the subset of indicators used for signalling purposes in the present paper.

Initially, the VDAX index represents a DAX index equivalent of the CBOE volatility index VIX (see e.g. [10]). These indices (in their more recent form, available alongside the original one and used in this paper) use quoted implied volatilities on current options on their target index, across the range of quoted strikes, to obtain a single volatility level which gives an indication of the level of risk currently linked to that index. Since the DAX index can be seen as a meaningful proxy to the entire German or indeed European stock market, the VDAX index thus quantifies equity risk. Given that implied volatilities, which are observed in and set by the market as opposed to derived from historical data, are used in its elaboration, it is also understood to carry an indication of investor perceptions with respect to risk, which justifies grouping it with the family of risk aversion indicators. The intuition that the VDAX may contain information on future equity returns is then quite attractive.

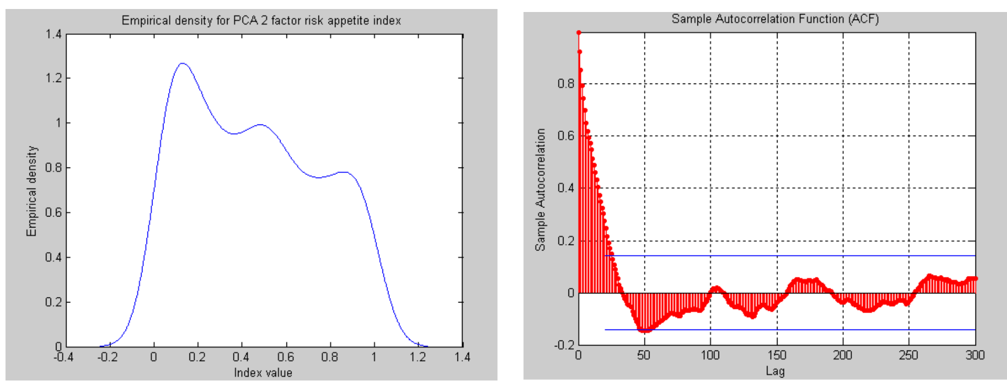
The set of funds used also covers the fixed income markets. As such, the annualised volatility figures for the Barclays Euro Aggregate bond index are used as a fixed income analogue of the VDAX. Both these indices thus represent the 'pure' market risk (perception) associated with the two main markets applicable.

With respect to the construction of risk aversion indicators, the standard ad hoc approach involves selecting several quantities which may intuitively be taken to depict various kinds of either risk or attitude toward risk, and then averaging them in some way. This may either be through directly taking an average, or through reducing the initial factors using a PCA (or similar) approach. Following the approach suggested in [9], we normalise each factor by replacing its value with its quantile over the previous length of sample window. We then estimate the sample covariance matrix between the normalised factors, set any negative covariances to zero and carry out a PCA on the resulting matrix. The non-negative first principal component obtained is then again normalised and used to calculate a weighted sum of the original factors to give each index value. The factors used are

- the previously mentioned VDAX index;
- the above Barclays Euro Aggregate bond index volatility;
- EUR corporate credit spreads between AAA and A 7 to 10 year yields as provided by Markit iBoxx;
- EUR yield curve spreads between German 5 year government bond yields and the German 3 month interbank rate.

While the first two factors represent risk observed in the equity and interest rate markets central to our chosen set of tradable assets, the latter two factors add information about credit and liquidity risk. We do not use any further factors because of the possibility that the averaging of too many sources of information may be detrimental to the predictive value of the resulting indicator - indeed, to specifically investigate the impact of averaging, we create both versions using the above four factors and alternative versions using only the two factors, VDAX and bond volatility index, which directly reflect the market risks linked to the funds in the given portfolio. We also create index variants using window sizes of 50, 100 and 200 points for further elucidation, leading to six PCA indices in all. Here the four-factor 200 point window index is (intuitively) "furthest" from the pure VDAX benchmark index, while the two-factor 50 point window index is "closest" to it.

Finally, we consider a type of risk aversion index introduced by Kumar and Persaud ([11]) and based on an earlier industry paper by Persaud, which was subsequently labelled as *global risk aversion index*, or GRAI. The intuition behind this is that it is possible to distinguish between price changes due to changes in "global" (overall market) risk and price changes due to changes in investor risk aversion, the first being directly proportional to the change in overall risk while the second is proportional to the risk of each individual asset. Accordingly, the rank correlation between (current) price changes and (previous) asset risk is expected to be significant when risk aversion has changed, but negligible when the root cause of the changes is a change in overall market risk. Misina (see [12], [13]) formalises the theoretical motivation given in the original paper by attempting to fit the developments within the framework of a CAPM market model: as such the Kumar and Persaud GRAI is seen as a theoretically, rather than intuitively, motivated indicator. We use a GRAI built on currency exchange rates and using the currency pairs of AUD, CAD, CZK, EUR, HKD, JPY, NZD, NKR, PLN, SGD, ZAR, SKR, SFR and GBP with respect to USD. Two versions are constructed, the first using 1 month forward rates (representing the price changes mentioned above), the second using 3 month forwards. Although this index does not represent an ideal choice with respect to a EUR investment universe made up of equity and interest rate investments, the selection (based on available data) is still thought a reasonable approach to representing risk as visible throughout the major global currency markets. The extent to which this indicator performs worse than other indicators closer to the investment market under consideration will provide information as to what extent risk visible in different markets can be regarded as shared between them.



(a) Kernel density estimate.

(b) SACF.

Figure 1: Nonparametric (kernel) density estimate for PCA 2 factor index with a 50 point estimation window (left) and Sample autocorrelation function for PCA 2 factor index with a 50 point estimation window with confidence bounds at lag 20 (right).

To illustrate the type of index described, the two halves of figure 1 give the kernel density estimate and the sample autocorrelation function for the two factor variant of the PCA index using a 50 point estimation window. We note that the signal density is (weakly) trimodal; this is not the case for most other indices examined in the present but does correspond to the index described in [9]. The autocorrelation function plot incorporates confidence bounds for the white noise hypothesis beyond lag 20 in order to show that the autocorrelation function decays slowly and the index contains a significant amount of dependence; this observation does apply to all signals considered, including the VDAX and bond volatility indices.

2 Results

This section presents the various results obtained. We define a benchmark problem and then vary individual settings with respect to this problem to check both whether significant backtesting improvements can be achieved and whether the presented investment approach is sensitive to its various parameters, without at the same time generating an unmanageable number of backtests.

2.1 The benchmark problem

For the benchmark problem, we specify the VDAX index as signal. Unlike the other risk indicators described, it is publicly quoted, such that there is no ambiguity with respect to the manner of its derivation. As mentioned, it also represents risk observed in the equity market only and thus avoids the information averaging

issues inherent in the PCA-based indicators, while leaving aside the interest rate information they contain: it will be interesting to check whether this leads to an advantageous or disadvantageous tradeoff.
The relationship between returns and signal is represented by a linear regression with intercept, such that we fit $\mu_i(s) = a_i + b_i s$ for every asset i. Although this relationship constitutes a very simple dependency model, we choose to use it throughout this paper so as to facilitate focus on the other possible strategies presented. In each case, we backtest the strategy by working through the full data set, using historical estimation windows of a certain size, determining the optimal portfolio weights and checking at the next date what returns they would have yielded. Portfolio rebalancing is carried out each business day. The window size used for the base case equals 60 points, which corresponds to three calendar months. We estimate the signal densities nonparametrically using kernel density estimates with Gaussian kernel. The conditional covariance matrix of asset returns is obtained as the sample covariance matrix of the regression residuals. The equivalent optimal control formulation of the portfolio optimisation problem (see [2] for details) is then solved using what is often called a *direct collocation* scheme, in which both state and control variables appearing in the optimal control are discretised and the resulting finite-dimensional optimisation problem solved using a nonlinear optimisation algorithm: a helpful discussion of such algorithms can be found in Betts ([14]). This optimal control problem is solved for 21 different values of the portfolio expected return. Optimal points on the efficient frontier thus generated are then chosen by calculating quadratic utilities for a number of possible risk aversion coefficient values and selecting the utility maximising weights for each of them.[3] The coefficient values used cover the interval from 0 (corresponding to risk neutrality) to 10; this range can be used to represent any quadratic utility function usually regarded as realistic. We compare different strategies using the Sharpe ratio as both an ex ante and an ex post metric, along with the additional ex post metrics of cumulative returns achieved, achieved standard deviations of returns, and maximum drawdowns respectively maximum drawdown durations observed. The Sharpe ratio expression used for the ex ante figures is

$$SR_P = \frac{E[P] - r_f}{\sigma_P},$$

where $E[P]$ denotes the unconditional expected portfolio return, r_f the risk-free rate of return (0 if no risk-free asset is available) and σ_P the unconditional ex ante portfolio return standard deviation. For the ex post Sharpe ratio values, the expectations were simply replaced by the sample averages of the backtest observations.

2.1.1 With risk-free asset

The first problem variant considered is that of a market in which a risk-free asset is available. Using the EURIBOR 1W rate as proxy for a risk-free asset, the previously described base problem with unconstrained weights can be solved either using the weights expression given in [1] or using a control discretisation scheme. Working through the entire data set in the way described above yields the average ex ante Sharpe ratio given for the problem 'VDAX 60 UNC RF' in the left half of table 4.[4] It can be seen that the Sharpe ratio shows a significant relative improvement of 35.52% over Markowitz; a glance at the table shows that comparable ex ante improvement levels can be observed for most problems. This implies that, within the models implied by the two strategies, the efficient frontier made accessible using conditioning information is sensibly more attractive than that generated using classical portfolio optimisation. Regarding ex post performance, ex post Sharpe ratios averaged across observed strategy performance for the whole of the efficient frontier are shown in the right half of table 4. Backtesting results covering the moments of returns individually are given in table 5, whose top third gives (absolute) additive log-returns for both the unconstrained Markowitz ('M') and conditioned ('C') strategies, along with the (relative) return improvements obtained through the use of conditioning information.[5] Two things are of note. First, it can be seen that the relative performance improvements seen remain substantial and average some 20% across the range of λ, while the observed

[3] The use of quadratic utilities makes sense if investors are concerned only with the first two moments of returns, which any mean-variance optimisation process implicitly assumes.

[4] Note all Sharpe ratios given correspond to a business daily, rather than e.g. annualised, investment period. Thus a Sharpe ratio value of 0.19, for instance, corresponds to an annualised value of $0.19\sqrt{252} = 3.02$.

[5] Such absolute figures will not be provided for later results as these are strongly dependent on the particular segment of the efficient frontier computed for each backtest, such that comparisons of these figures between backtests using different problem parameters would be misleading. Ex post standard deviations will similarly be reported as the ratios of the observed deviations for the conditioned strategy divided by those for the Markowitz strategy, rather than as absolute figures for either individual strategy.

standard deviations of returns rise by a maximum of 13%, and do so only for risk aversion levels where the variance risk accepted ex ante is small in absolute terms. This pattern was observed for a significant proportion of the problem variants discussed in the remainder of this section; more generally, the observation for low values of λ is that the Markowitz approach generates a much higher ex post risk level than the conditioned one, and that results thus differ strongly between the two strategies. Clearly and given that a finite arc of the efficient frontier is computed for each rebalancing point, a value of λ close to zero does not in this case mean that expected variance is entirely ignored in the exclusive pursuit of expected return. Rather, it means a point further to the right on the available efficient frontier arc will be chosen - for the risk neutral investor, this will be the rightmost point in every case which, however, still represents the outcome of a risk minimisation problem for a finite level of expected return. The general backtesting observation given above thus suggests that the use of information allows the conditioned strategy to implement a given ex ante level of expected portfolio return using a greater range of positions, and that this increased freedom remains applicable as the required return comes to dominate the ex ante trade-off. This contention will be discussed further in the context of the centrally relevant problem where only risky assets are available and weights are constrained not to involve any short positions. The second point of note is that the ex post improvements observed are not as large as the ex ante enhancement of the accessible Sharpe ratio. It seems reasonable to argue that the difference gives an idea of the misspecification cost of the lagged relationship between return and signal, given that the relationship is both weak and poorly approximated by a linear regression.

This suggested explanation is supported in particular by the ex post Sharpe ratio values in table 4. While the ex post values have fallen severely with respect to the ex ante figures, this is true for both classical and conditioned optimisation strategies. Even so, the relative improvement obtained through the use of conditioning information also decreases for the majority of problems considered, more or less severely depending on the particular problem variant. Clearly, the model used by conditioned optimisation is more involved than that assumed by classical mean-variance optimisation, and as such more quantities have to be estimated, leading to a greater level of estimation risk. Thus, the Markowitz strategy requires estimates of unconditional moments, while the conditioned strategy uses estimates of conditional moments as well as of the signal-returns relationship applied. By default, it is then reasonable to expect that the ex post excess performance of the conditioned strategy will decrease with respect to the ex ante situation to some extent. The varying extent of this seen when looking through table 4, which will be discussed for the individual problem variants considered, would then reflect larger or smaller estimation errors tied to the signal relationship assumed in each problem. For the currently considered unconstrained problem, the Sharpe ratio improvement thus degrades from 35.52% to 24.45%, which seems reasonable given the larger estimation errors expected in the conditioned case.

The final metric of interest, again observed ex post, is given by the maximum drawdown (MD) and maximum drawdown duration (MDD) levels. Here values (for a risk aversion coefficient value of $\lambda = 2$, which arguably represents a reasonably average level of risk appetite) are respectively 10.11% and 327 points for Markowitz and 6.26% and 142 points for the conditioned approach. This further suggests that the conditioned approach offers a tangible advantage over Markowitz for all the metrics considered, and specifically points to a degree of crisis resilience provided through the optimal use of information, given that MD events are typically experienced during difficult market periods and, specifically in this particular case, during the 2007-2009 bear market.

Adding a weights leverage constraint to only allow weights in the interval $[-1, 1]$ means the problem now only admits a numerical solution. The corresponding results are given as the second part of table 5. The strategy performance is seen to be close to that of the unconstrained version of the problem: this is not very surprising given that, by construction, the segment of the efficient frontier computed rarely requires, or would benefit from, a large amount of leverage. However, results are shown to be consistent with the unconstrained case, with slightly smaller absolute returns and a similar outperformance by the Markowitz strategy for most of the covered range, and this is confirmed by the ex ante and ex post Sharpe ratios indicated in table 4 as well. It may be pointed out that, for ex post results, the higher absolute returns obtained for the two lowest risk aversion levels with respect to the unconstrained case are not contradictory even though the same part of the efficient frontier was calculated for both runs and the unconstrained case is by definition preferable as far as any ex ante metrics are concerned. Overall, this second test case shows that the numerical solution approach proposed for the optimal control formulation of the optimisation problem leads to performance levels consistent with those obtained for the unconstrained case.

The discussion will now concentrate on the case of the short selling weights constraint, where weights are restricted to the positive half-line $[0, \infty)$ and whose practical relevance is arguably greater than holds

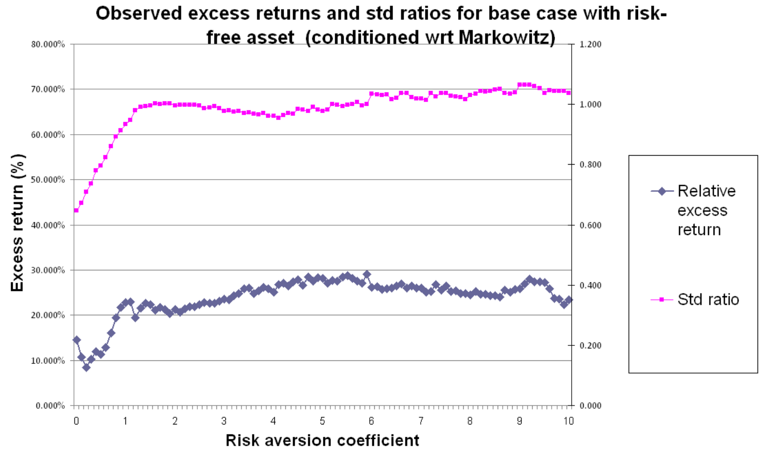


Figure 2: Relative ex post excess additive returns (left axis), standard deviation ratios (right axis) for the conditioned strategy over Markowitz, weights constrained to be positive, market with risk-free asset, 60 day estimation windows, VDAX index, for different risk aversion coefficients.

for the other two feasible weight intervals considered. As is visible in table 4, the backtest for this problem variant generates an average ex ante Sharpe ratio for the Markowitz strategy of 0.265, while that seen using the conditioned strategy is 0.343: an increase of over 29%. Interestingly, excess performance degradation of the conditioned approach is not seen in this case, with a relative increase in the Sharpe ratio of 41.55% maintained ex post. These observations are consistent with the backtesting results given in the third part of table 5. Again, a substantial performance improvement averaging about 25% can be observed over the classical Markowitz strategy - in fact, the outperformance is more attractive in this case since (as is reflected in the ex post Sharpe ratio) the increase in risk observed is either negative or marginal. Figure 2 gives a graphical representation of ex post relative excess returns (on the left-hand axis) and ratios between standard deviations (on the right-hand axis) for all risk aversion coefficients covered. Results close to investor risk neutrality preferences again show that a decrease in observed returns for the conditioned strategy relative to classical optimisation is offset by a much stronger reduction in observed risk. As can be seen, the Markowitz standard deviation for λ close to zero rises much more quickly than is the case for the conditioned strategy.

As a final form of presenting results, figure 3 shows example time paths, obtained for a typical risk aversion coefficient λ of 2, along which the additive returns of both the conditioned and classical strategies evolve. What is most noticeable in the figure is that the conditioned optimisation approach outperforms the Markowitz strategy in particular over the initial part of the current credit crisis, from June 2007 to March 2009. This period is of particular interest when considering the crisis resilience attribute of a given investment strategy as the ten funds constituting the market used lose on average 55% of their value over this period. While the Markowitz strategy is able to more or less neutralise this loss and break even, the conditioned strategy displays a small gain even though short sales are prohibited and so no profit can be derived from an expectation of falling values. This again suggests that the use of conditioning information is especially useful when extreme results are either required (e.g. in the case of very low investor risk aversion levels) or expected (e.g. in the case of market crises, for which the signal values observed are likely to be at the end of the range), and constitutes desirable behaviour for risk management as well as return maximisation purposes. It may be worth pointing out that there is no training effect involved as the window size used for estimation is held constant: thus the fact that this particular interval occurs toward the end of the data set is coincidental. The path shown also suggests that the drawdown figures generated using conditioned optimisation should be more attractive than the Markowitz ones. This is indeed the case, as the maximum drawdown for the Markowitz strategy (still for a value of $\lambda = 2$) is 4.62% for a maximum drawdown duration of 199, while the MD for the conditioned strategy is 3.80%, with an MDD of 164. Drawdown improvements of this order were observed in most experiments, such that they will not be reported in the remaining discussions.

2.1.2 With risky assets only

The other main type of market within which optimisation activities take place in practice is one in which only risky assets are available. With the risk-free asset gone, the efficient frontier no longer includes a risk-free

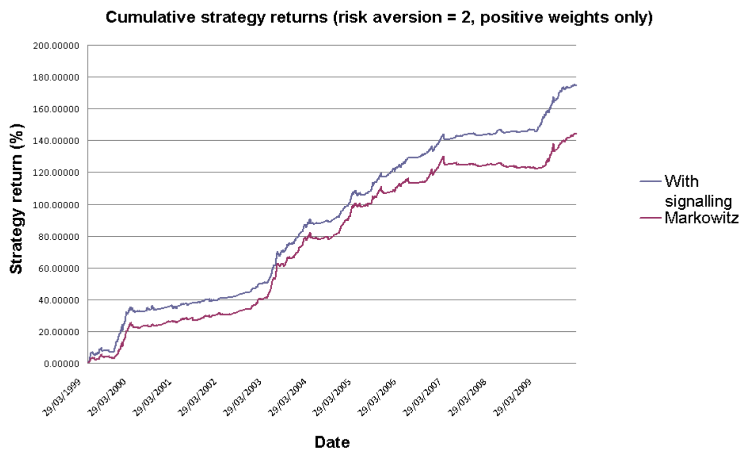


Figure 3: Time paths of additive returns observed for both the Markowitz and conditioned strategies, weights constrained to be positive, risk-free asset, 60 day estimation windows, VDAX index.

investment possibility, but starts at a minimum variance portfolio.

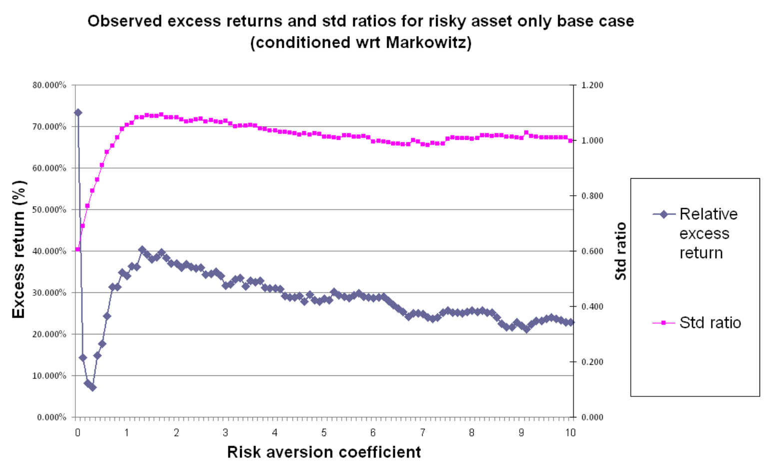


Figure 4: Relative ex post excess additive returns (left axis), standard deviation ratios (right axis) for the conditioned strategy over Markowitz, weights constrained to be positive, risky assets only, 60 day estimation windows, VDAX index, for different risk aversion coefficients.

The ex ante and ex post Sharpe ratios recorded at the end of the backtest can be consulted on the right half of table 4. Ex ante improvements across the table are in line with those seen for the problem with risk-free asset, while none of the corresponding ex post improvements deviate strongly enough from the ex ante figures as to necessitate a separate explanation. In particular, the observed risk-return trade-off for the practically most relevant problem variant for which short positions are disallowed still shows an increase in excess of 25% when the VDAX signal is used. Backtesting results for the two individual moments analogous to those discussed above in the presence of a risk-free asset are given in table 6. As suggested by the Sharpe ratio results, the performance increases seen are comparable to those seen in the presence of a risk-free asset for all three types of weight constraint, with the limiting (and unrealistic) cases of investor risk neutrality preferences and portfolio weights either unconstrained or constrained in modulus as the only ones in which the Markowitz strategy may be seen as offering the superior ex post risk-return tradeoff.

This is not the case for any value of the risk aversion coefficient given the problem variant with short sales constraint, as figure 4 shows graphically. At risk neutrality, the behaviour of the Markowitz optimum can again be seen to lose its ability to trade off expected return against risk when high levels of expected return are required. The resulting positions show high volatility, which may imply high returns (as seen for the first two weight constraint types) or low ones as seen by the 73% relative return advantage at lower volatility

	Markowitz (M)	Conditioned (C)	Difference
Entire data set	0.1295	0.2261	0.1366
01 June 2007 to 01 March 2009	0.3872	0.6334	0.2462

Table 1: Respective average optimal investment weights, using both Markowitz and conditioned optimisation, for (money market) KBC asset, benchmark problem, risky assets only available, for entire data set and across the 2007-2009 bear market.

displayed by the conditioned optimiser. As before, these observations strongly suggest that the desired resilience property of conditioned solutions with respect to low levels of investor risk aversion continues to be verified in the presence of weight constraints. To examine this hypothesis more closely, table 1 shows the respective relative investment weights allocated to the KBC asset, which corresponds to a money market fund and thus involves by far the least amount of risk of all assets constituting the market used. In the first row of the table, these weights are averaged over the entire data set spanning 1999 to 2010; for both table rows, the weights averaged are those returned as optimal for the efficient frontier point with the highest expected level of return, i.e. that uniformly chosen for an investor risk aversion coefficient of $\lambda = 0$. The intention behind examining these figures is to see how each strategy manages to limit risk when the problem parameters fail to do so.

As the first row of the table shows, the conditioned strategy ends the backtest with an average relative investment into the quasi risk-free KBC asset more than double of that generated by the classical strategy. This means that the conditioned strategy has achieved the same set of required expected returns with a significantly smaller average position in the higher-risk assets, and justifies the resilience suggestion made above.

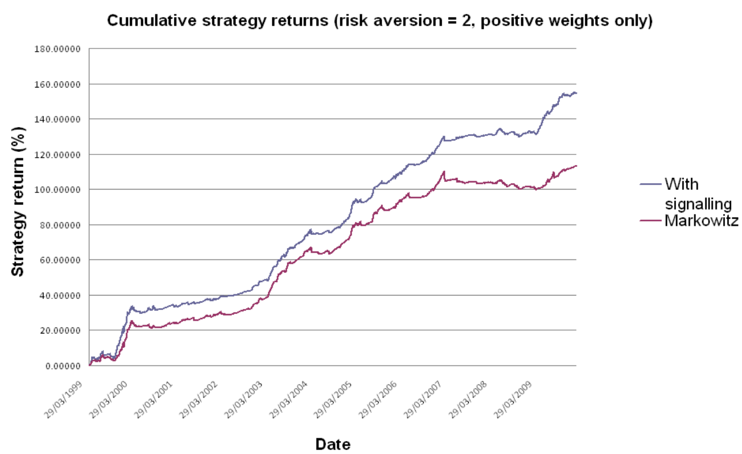


Figure 5: Time paths of additive returns observed for both the Markowitz and conditioned strategies, weights constrained to be positive, risky assets only, 60 day estimation windows, VDAX index.

As for the variant of the problem with a risk-free asset, the analogous contention with respect to *crisis* resilience of the conditioned portfolio is again supported by a returns time path shown for a representative risk aversion coefficient of 2 in figure 5. Here, it can again be confirmed that the conditioned optimiser manages the end of the dot-com bubble with gains similar to the Markowitz strategy, while it finishes the initial bear market linked to the currently ongoing financial crisis with further small gains and no significant drawdown, unlike the Markowitz optima which generate a small loss along with a two-year drawdown. The second row of table 1 further illustrates this: here, the previous respective positions in the KBC asset are averaged only over the bear market period spanning 01 June 2007 to 01 March 2009. As can be seen, the difference is again very significant, with the conditioned optimiser generating an average majority position in the low-risk asset over the entire bear period. To some extent, this is what a portfolio optimiser is expected to achieve, and it can indeed be seen that the Markowitz optimiser does reduce risk over this period with respect to the data set average, increasing as it does the KBC average from 0.1295 to 0.3872. It is still thought significant that the conditioned optimiser manages this to a larger extent as it responds to market conditions by increasing the KBC average from 0.2261 to 0.6334. Table 1 is thus felt to contain evidence not only in favour of the greater resilience of the conditioned strategy with respect to extreme (risk-neutral) investor preferences (which is visible in the positive differences in KBC investment weights between the two strategies seen in both rows), but also with respect to difficult market periods: this is seen in the fact that the relative weight of the low-risk asset is increased further with respect to the classical strategy during the recent bear market.

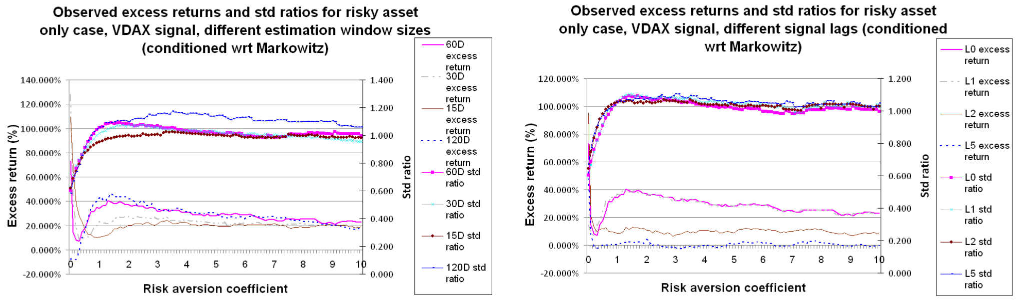
To also give a point of reference with respect to another type of investment approach that involves risky assets only, backtest results were additionally computed for a momentum strategy often used in practice. This compounds historical returns of each asset over a certain window size at certain rebalancing intervals, ranks the compounded returns and makes equal long investments in the n highest ranking assets, where n is a strategy parameter. Such a strategy can then be compared to portfolio optimisation subject to a short selling constraint if only long investments are made, and to unconstrained optimisation if equal and negative investments are also made in the n lowest ranking assets. Setting $n = 3$ and using a window size of 60 as well as daily rebalancing to remain as close as possible to the portfolio optimisation problems under discussion generates the backtest results included in table 6. These show that all portfolio optimisation variants checked deliver significantly superior results with lower overall risk than this standard investment strategy.

2.2 Different window sizes

The choice of window size used for the estimation of returns moments and the signal-return relationship involves a clear trade-off. It represents a compromise between the desire to use estimates of good statistical quality on the one hand and the need to minimise averaging across local variations in asset behaviour resulting, for instance, from the presence of different market regimes or of conditional heteroscedasticity (GARCH effects), neither of which can be captured by the basic regression model used. The present subsection checks whether the initially chosen window size represents a suitable value. Results will indicate how sensitive strategy performance is with respect to the window length. The additional cases considered in this subsection postulate 15, 30 and 120 points per estimation window, and all of them exercise the VDAX signal.

As for all problem variants, the Sharpe ratios anticipated and obtained are contained in table 4. Here, as for the following problem variants, only the short selling weights constraint is applied. Interestingly, the ex ante ratios decrease with window size, for both optimisation strategies and whether or not a risk-free asset is present, with the levels obtained for 15 day windows by far the largest obtained in the present section. This is likely a consequence of the fact that expected asset returns tend to be larger in absolute value when estimated using fewer points given the impact of averaging is less pronounced - a general observation that is confirmed, for instance, in the case where only risky assets are available, for which the mean absolute expected asset return estimated over the entire data set equals 0.1157% for the standard 60 point window, but 0.1923% for the 15 point window. The fact that these larger estimates, in spite of the wider confidence intervals they imply as a consequence of the smaller number of points used, then yield higher ex ante Sharpe ratios and more attractive efficient frontiers is a reminder that all ex ante figures reflect model assumptions whose value for an individual data set is most conclusively verified by their impact on ex post results. Consultation of the ex post Sharpe ratios directly confirms that the high ex ante values obtained for small estimation window sizes are misleading, as the levels observed using the backtesting results are in line with the average of table 4, or indeed slightly lower for the 15 point window size and a market containing risky assets only. The other interesting ex post ratio observed is that for 120 point windows in the presence of a risk-free asset: while the conditioned strategy's outperformance is large in this case (and larger than the 32.79% observed ex ante), this is due to a drop in the unconditioned ratio rather than high strategy performance. No such observation can be made in the case of risky assets only, such that the general tendency seen is that the Sharpe ratios observed for both the longest and shortest window sizes begin to drop off, suggesting that the 30 and 60 point window sizes are preferable, for the data set used at least.

Backtesting results for the individual moments are shown, in the relative form that will exclusively be used for the remaining results, in Table 7. The left half of table 6 provides a graphical representation of the data in the case of the market consisting of risky assets only. Compared to the table of averaged Sharpe ratios, this presentation of backtest results is able to show more precisely how the average performance difference seen in the Sharpe ratio table originates from performance difference seen at different levels of investor risk aversion. In particular, the high volatility of the Markowitz performance for low levels of the investor risk aversion coefficient is again visible from the erratic return figures seen for both markets. The stability, and hence resilience with respect to extreme investor preferences, of the performance delivered by the conditioned strategy can be seen in the standard deviation ratios, which show large falls as λ tends



(a) Different window sizes. (b) Different lags.

Figure 6: Relative ex post excess additive returns (left axis), standard deviation ratios (right axis) for the conditioned strategy over Markowitz, weights constrained to be positive, risky assets only, 60 day estimation windows, VDAX index, for different risk aversion coefficients, given different estimation sizes (left) and signal lags (right).

to 0 given the Markowitz risk levels quickly increase while the conditioned strategy's observed standard deviations remain largely constant. The other noticeable feature of the table data is that the relative return improvements shown in general become somewhat more pronounced as the estimation window sizes lengthen, while the observed risk increases for the conditioned strategy as well. As the fall in observed Sharpe ratios (dependent as it may be on the efficient frontier arc which is computed in each case, and which may have a lower highest expected return for large window sizes given the greater amount of returns averaging that occurs) indicates, however, this is not thought to make the case of a 120 point window size preferable in general terms. Intuition also suggests that the basic estimates used will be unable to capture conditional variations in the relationships estimated, such that overly long window sizes will cause any data features to be averaged away and thus invalidate any profitable modelling of returns moments or the signal-returns relationship. As such, the conclusion of this subsection is that the interior window sizes used, viz. 30 and 60 points, are seen as preferable given the backtesting context, and that the original benchmark choice of a 60 point window seems justified in retrospect.

2.3 Lagged signals

It was seen in section 1 that the autocorrelation function values of the different indicators used decay only slowly for increasing lags. This type of dependence may be reflected in the relationship between signal and returns and thus additional information may be present at lags larger than unity. Intuition would support this, as there is no reason why news would impact on markets for only one day at a time. However, the single signal supported by the optimal control setup of the present chapter does not allow for the simultaneous introduction of conditioning information delayed by different lags. In order to check for the informational value of an increasingly delayed signal within the linear regression, single signal context under discussion at this point, the benchmark VDAX signal was delayed by various lags and it was checked how the conditioned strategy performs using the various delayed instances of the signal as the sole source of conditioning information in each of the problem variants thus created. Here the set of lag amounts used includes a single business day (i.e. the signal value used for today's rebalancing is that which was known on the previous business day), two days, five days and ten days. Viewing the ex ante Sharpe ratios found in table 4, it can be seen that the metrics obtained for the conditioned strategies decrease very slowly as the signal lag is increased, and remain attractive for a ten day lag. Comparing this observation to the ex post metrics given as in the previous discussion reminds one of the model dependence of the ex ante figures, as the ex post Sharpe ratios are seen to decrease significantly with respect to the no-lag benchmark case as soon as the signal is lagged by two additional points, and conditioned optimisation becomes entirely unattractive for the five-lag and ten-lag cases. Note it is correct that the Sharpe ratios recorded in the Markowitz case, are not constant given a new subset of the efficient frontier, based on integration of individual assets over the signal dependent interval of non-negligible probability density, is obtained for every new problem.

Ex post figures pertaining to observed returns and standard deviations are shown in table 8 and represented graphically, for the case where only risky assets are available and except for the clearly uninteresting variant where a lag of ten points was used, in the right half of figure 6. The figure also includes the benchmark case for reference, using the label 'L0'. Apart from the (now familiar) poor performance of classical optimisation for low levels of risk aversion and using the market which contains risky assets only, these results confirm that the conditioned strategy remains attractive when run using the VDAX signal lagged by both one and two points. For the cases of five and ten lags, no improvement in return is seen, while the observed standard deviations slightly increase in what seems to be a feature characteristic of the conditioned strategy: this poor trade-off accounts for the Sharpe ratio figures seen.

On the one hand, these results suggest that the use of autoregressive relationships between signal and return might further improve the performance of conditioned strategies as information at several lags could then be used at the same time. This conclusion should not, however, be drawn without reservation. Indeed, it is those informational shocks reflected by the signal at some particular point in time which may remain visible in that same signal one or several points later, except that newer information that has arisen in the interim will also feed the newer signal values. This means it is possible that the only explanation for the satisfactory performance of the signal at one or two lags lies in the fact that an lagged version of the same signal to returns relationship that is more clearly visible between the returns and the original signal series is used, and retains some explanatory power. If that is the case, simultaneous use of the same signal at multiple lags, as might be implemented in a multiple signal setting by the use of a multiple regression for the estimation of the signal to return relationship function $\mu(s)$ in equation (1), would be unlikely to yield any of the expected added benefit, and the results obtained in the present section would be misleading. This question will be investigated further in a future paper.

2.4 Weights averaging

The implementation of the conditioned optimisation approach is subject to one additional risk, beyond those linked to the increased propensity to estimation errors, with respect to the Markowitz strategy. This is given by the use of the currently observed value of the signal in order to move from the optimal weight vector valued function as obtained through the optimisation process to a concrete set of portfolio weights to use. It is desirable to check the sensitivity of the conditioned strategy's performance to the signal value seen at rebalancing time. In order to quantify this to some extent, this subsection discusses backtesting results for the benchmark problem, where weights are averaged over a certain number of points on the discretised signal support, viz. the weight values used correspond to averages of the respective weight values that would be used if the observed signal value equalled one of a number of values adjacent to the value actually observed. Thus, an n-point average weight, where n is odd, averages the $\frac{n-1}{2}$ weight values to both the left and the right of the pointwise optimal weight, along with that weight itself. If fewer points are available at either end of the discretised signal support vector, the average only uses as many as are available. The values for n used are 3, 5, 7, 9 and 11, where the widest averaging range covers half of the efficient frontier segment computed.

For this set of problem variants, table 4 only gives the ex post Sharpe ratios observed as the ex ante values are determined by the benchmark problem with no weights averaging, i.e. no separate problem is solved to obtain the averaged strategies. Consequently, the Markowitz figures remain identical for each case. An examination of the figures in table 4 quickly indicates that the behaviour under averaging is very different for the two markets, one with risk-free asset and the other without, under consideration. It is true that both markets show a monotonic decrease in the Sharpe ratio, which is what one would expect to see as the exact solution issued from the conditioned problem constitutes a theoretical optimum. However, the rate of decrease seen for the market containing risky assets only is slight, while that encountered in the presence of a risk-free asset is much larger, such that the use of conditioning information stops appearing interesting from a 7-point average onward. This is thought to be a consequence of the constraint requiring portfolio weights to sum to unity at each signal point for the problem with risky assets only. Linearity of the sum function means that averaged weights will still sum to unity in this case, where any weights differences between assets have been scaled by the inverse of the number of assets with nonzero weights through the averaging operation. For the market with risk-free asset, the risk-free position acts as a slack variable and the weights constraint is no longer necessary. As a consequence, the risk-free weight will thus markedly change when weights are averaged, to reflect the sum total of changes made to the individual asset weights, which more often that not vary in the same direction given no constraint prevents them from doing so. In particular and for the risk-free benchmark problem, the mean of absolute differences between positions in the risk-free asset that are optimal across the range of discretised signal values for each problem equals 0.1613, while a similar mean of absolute differences between positions in the risky assets equals 0.0167. The same mean in the case of the benchmark problem with risky assets only is smaller still at 0.0095, which supports the contention that the larger variations in ex post performance seen for the problem with risk-free asset are precisely a consequence of the larger variations in the risk-free asset weight. As the averaging range is extended, the averaged portfolio is moved away from the optimal portfolio much more quickly than is the case for the problem with risky assets only, and thus performance degrades quickly. To a large extent, this is seen as an artifact of the problem specification in the presence of a risk-free asset, and the market without risk-free asset is suggested as preferable for the evaluation of the impact of averaging over the optimal weights initially computed.

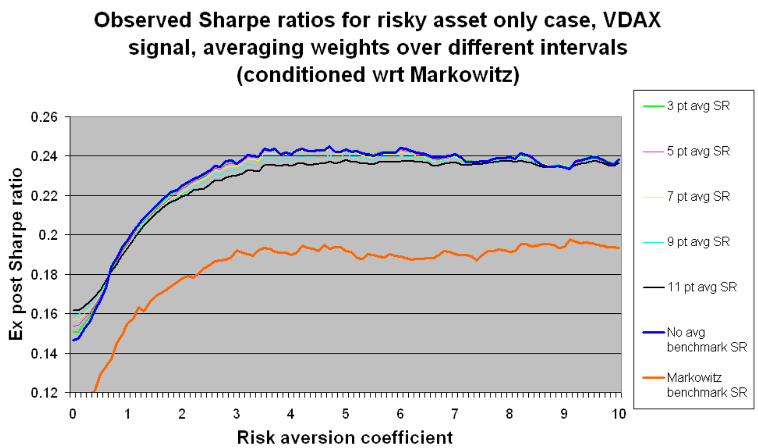
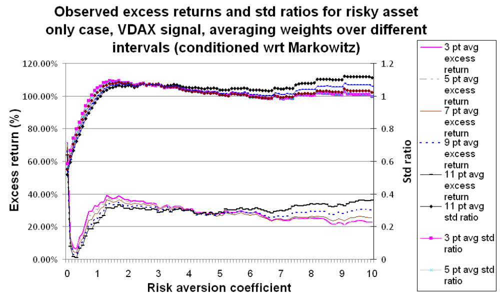


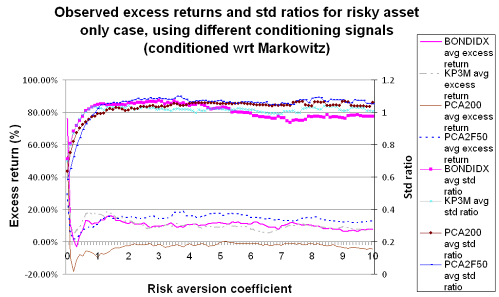
Figure 7: Ex post Sharpe ratios observed for both the Markowitz and conditioned strategies, weights constrained to be positive, risky assets only, 60 day estimation windows, VDAX index, optimal weights averaged across various intervals of the signal range.

It is of interest to examine how averaging impacts performance across the range of risk aversion coefficients. Figure 7 shows how the ex post Sharpe ratios for the different averaging ranges evolve with λ, and includes the benchmark conditioned and unconditioned problems for comparison. It can be seen that the averaged strategies obtain Sharpe ratios slightly higher than those for the benchmark problems for investors that are practically or entirely risk-neutral. A plausible intuition is that the averaging of optimal weights mitigates risk if the non-averaged weights are particularly risky (i.e. at the high expected return end of the efficient frontier as an investor with low risk aversion will tend to prefer), and increases risk if the non-averaged weights are chosen to minimise risk as far as possible, which will generally be the case for investors with high risk aversion. The ratios in figure 7 show a trade-off between this tendency and the answer to the question how increased risk translates into higher returns. Returns and standard deviation performance figures with respect to Markowitz may be hoped to further elucidate the shape of this trade-off, and are given in table 9 as well as represented in the left half of figure 8 for the market consisting of risky assets only.

It has been argued that the impact of the risk-free asset weight average represents an effect that to some extent distorts what the present subsection is attempting to examine, such that it may be beneficial to concentrate on the second half of the table. For these results covering the market consisting of risky assets only, the observed outperformance in returns with respect to the Markowitz strategy increases toward the top left hand corner of the table, for which averaging covers the smallest range and the investor risk aversion level is least. Conversely, outperformance also (broadly) increases toward the bottom right hand corner, which represents the largest averaging range and the most risk averse investor type. This is plausible: while the top left hand corner implicates a very risky, high expected return position which has not been diluted through averaging, moving downward involves just that and the resulting performance may be expected to yield a somewhat lower return coupled with a lowered level of risk. The bottom right corner point of the table through averaging introduces a larger risk and greater expected return than will be visible for the remaining points in the rightmost table column, for which averaging is less. These two tendencies are less clear-cut in the interior columns of the table, where the averaging process will introduce weights that are both more and less risky than the optimal set obtained as the problem solution. Note that the observed levels of standard deviation exactly match the previous justification, which explains why the lowest and highest



(a) Different optimal weights averages.



(b) Different signals.

Figure 8: Relative ex post excess additive returns (left axis), standard deviation ratios (right axis) for the conditioned strategy over Markowitz, weights constrained to be positive, risky assets only, 60 day estimation windows, VDAX index, for different risk aversion coefficients, given different averaging ranges (left) and signals (right).

ex post standard deviation ratios, respectively, are found for the widest averaging range at both ends of the risk aversion coefficient range. As far as the initially posed question of the sensitivity of the conditioned strategy with respect to individual signal values is concerned, the differences between averaging ranges and the benchmark problem seen using table 9 are small enough to allay any worries about the portfolio weight choice made in the conditioned strategy: for the market of risky assets only, the optimum's performance shows itself to be quite robust with respect to results obtained across the signal support. Even so, the figures in table 4 still confirm the optimal status of the problem solution compared to the various averages.

2.5 Different signals

As a final set of problem variants, the different risk aversion indicators discussed in section 1 were evaluated. These are thought to involve another tradeoff as far as conditioned optimisation is concerned, namely between the benefits of introducing heterogeneous types of risk into the conditioning relationship (and thus possibly applying additional information to the model) and the drawbacks resulting from the averaging of information which, necessarily, reduces the visibility of specific information more and more as the number of heterogeneous information sources is increased. Heterogeneity is introduced both by applying the GRAI indicators, which reflect a different type of risk to those obviously affecting the applicable market, and by using both indicators that reduce several factors using principal components analysis. The Sharpe ratios obtained for the different signals (all tested using 60 day estimation windows) are given in table 4. The absolute levels achieved both ex ante and ex post are unremarkable for the entire set of tested signals, with the exception of the 200 point PCA signals in the market with risk-free asset, for which the values shown represent the highest of the entire set of problems covered. However, this is equally the case for the unconditioned variant of either of these problems, such that this result may safely be explained by the arc of the efficient frontier used as a result of the support limits for those two particular signals observed. Looking at the relative improvements obtained, the picture is mixed, with reasonable results provided by the bond volatility, GRAI 3 month and PCA two factor, 50 point window indices, and poor performance shown by all remaining signals.

Table 10 and, for the case without a risk-free asset, the left half of figure 8 show ex post results explicitly by the moments of returns. On the one hand, the figures can be interpreted to confirm that, in particular, the significant reduction of observed risk with respect to classical optimisation obtained for low values of risk aversion seems to be a feature of the conditioned optimisation approach rather than a consequence of the particular data set, given that it has been observed for all of the problem variants covered. At the same time, it is also apparent that the performance obtained for VDAX is superior to that seen for any other of the indices tested. For the Kumar and Persaud indices, this is perhaps not particularly surprising given they attempt to capture foreign exchange risk, and the results thus suggests that, while there is some relevant information conveyed by the 3 month index variant at least, it remains important to specifically target the chosen market when selecting a signal. The bond volatility index is also seen to add some value with respect to the pure Markowitz strategy. Given that most funds in the chosen market involve equity exposure and given that bond markets are less volatile than equity markets, suggesting that an interest rate based indicator may be inherently less informative with respect to overall market states, the degradation with respect to VDAX is seen as perfectly plausible. Finally, the group of PCA indicators would appear to confirm that the averaging of information has an adverse effect on the quality of each indicator. Performance is seen to degrade as more averaging takes place (by increasing the window size, adding more factors or both) and the relatively best performance is offered by the two-factor index with 50 day windows, which remains closest to the pure VDAX.

Finally, the investigation checked for the attractive possibility of using the regression statistics in order to choose which assets, or which signal, could most advantageously be chosen as inputs to the optimiser at the current point in time. To this end, all signal series were normalised, all asset series were regressed against every signal window at every point, following which it was tried to identify exploitable relationships of any kind. However, no such systematic relationships could be found: in particular, there seems to be no correlation between either the strength of the regression relationship, or the modulus of the regression coefficient, and the performance increase obtained using conditioned optimisation. Accordingly, backtesting using a strategy of this type yields no results of significance and no such results are reported.

3 Conclusion

The present paper has reported the results of an empirical investigation into the performance of constrained-weight conditioned portfolio optimisation strategies carried out using backtesting on a realistic data set. After sketching the nature of the conditioned portfolio optimisation problem and explaining why its weight constrained case was not previously open to solution, the data set and range of signals used were introduced. Results were given and discussed for various settings of the strategy parameters modified with respect to a benchmark problem introduced at the start.

Intrinsically, the behaviour of the conditioned solution strategy is comparable to that of the Markowitz solution. In particular, optimal weights may turn out to be small, and the resulting optimal portfolio undesirable in the presence of transaction costs. Also, the sensitivity of results on the estimates used and the exact utility function chosen is often criticised. However, we emphasise that any investor who accepts these drawbacks of classical portfolio theory will see all relevant metrics improve with the use of conditioning information and given a good signal - whether they be ex ante or ex post. As such, we suggest that the type of conditioned optimisation proposed is of significant interest in any portfolio investment situation where portfolio weights have to be constrained, and where Markowitz optimisation would otherwise be used.

This is the case especially given that the outperformance displayed is fairly robust with respect to the exact configuration of the problem. Different weights constraints, estimation window sizes and signal lags all yield plausible responses, and confirm that there is some advantage to be consistently had by the systematic exploitation of conditioning information. Results reported when averaging pointwise optimal weights show that the algorithm is not unduly sensitive to the exact observed value of the signal.

At the same time, the results show that the choice of conditioning information is central to the resulting performance. It is interesting that the simplest risk aversion indicator used - the VDAX, which is a straight gauge of equity market risk - also performs best out of the selection of indicators used. Clearly, for the fund market chosen, equity market risk is the most significant risk component, and this results suggests that the pertinent information is degraded through averaging whenever other factors are added to the indicator. This deduction is statistically plausible when considering that risk aversion indicators share very little common ground as is evidenced through their correlation and crisis prediction properties, reported in [3]. It can also be seen that, across the set of six different PCA indicators, conditioning performance increases as the indicator is brought "closer" to VDAX, either by shortening the estimation window or by reducing the number of original factors.[6] Finally, the bond volatility index by itself again performs reasonably well, especially considering that it will almost by definition not reflect market risk to the same extent as indices elaborated using equity factors. Again, though, it gives direct indications of the second family of major risks - interest rate and credit risks - that affect the portfolio of funds selected for this study. Altogether, this set of observations leads us to suggest that the difficulty of simultaneously using multiple signals, which is nontrivial for the optimal control formulation used, does probably not represent as large an issue as may appear at first.

As a final point, we point out that the results reported above suggest that further improvements may be possible if a more advanced model is used for the relationship between signal and return. The linear regression

[6] We note that this conflicts with the results of [9], who find that a six-factor PCA index is more interesting informationally that the VIX. However, the paper does not strictly perform conditioned optimisation, but uses the (trimodal) index to extract market regimes which then drive a classical utility function optimisation with added index penalty term.

model is clearly unsatisfactory, and exposes the investor to two conflicting strands of motivation. On one hand, outperformance over Markowitz somewhat increases with the size of estimation window: the relationship estimates become more statistically meaningful. On the other hand, the absolute performance falls - as window sizes increase, any conditional stationarity assumption becomes more and more inappropriate, and different local behaviours of the relationship will inevitably be averaged. A more advanced model would allow the manager to increase window sizes while, ideally, adequately modelling different market regimes including several conditional mean return levels and conditional heteroscedasticity, along with arbitrary autocorrelation functions. Integrating such a model will thus be a topic of further research.

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4 Appendix

This appendix contains all tables mentioned in the paper but not included in the body of the text.

Asset	Cumulative log return
AXA	-27.91%
CSU	22.56%
DEK	3.55%
DEX	12.91%
DWS	40.43%
FIB	29.86%
FOB	-27.54%
INV	-18.07%
KBC	30.01%
MSE	-6.62%

Table 2: Cumulative log returns of individual assets over period covered by 60 point estimation windows.

Assets	AXA	CSU	DEK	DEX	DWS	FIB	FOB	INV	KBC	MSE
AXA	1	0.09	0.46	0.14	0.00	0.00	0.04	0.47	-0.03	0.21
CSU	0.09	1	0.58	0.59	0.02	-0.14	0.69	0.51	-0.20	0.15
DEK	0.46	0.58	1	0.40	0.0	-0.10	0.48	0.84	-0.20	0.16
DEX	0.14	0.59	0.40	1	0.02	-0.12	0.58	0.37	-0.14	0.10
DWS	0.00	0.02	0.00	0.02	1	0.00	0.03	-0.01	-0.02	0.01
FIB	0.00	-0.14	-0.10	-0.12	0.00	1	-0.17	-0.06	0.39	0.05
FOB	0.04	0.69	0.48	0.58	0.03	-0.17	1	0.42	-0.22	0.10
INV	0.47	0.51	0.84	0.37	-0.01	-0.06	0.42	1	-0.13	0.19
KBC	-0.03	-0.20	-0.20	-0.14	-0.02	0.39	-0.22	-0.13	1	0.03
MSE	0.21	0.15	0.16	0.10	0.01	0.05	0.10	0.19	0.03	1

Table 3: Correlations between log returns of individual assets over period covered by 60 point estimation windows.

	Ex ante			Ex post		
	Markowitz (M)	Conditioned (C)	Improvement (I)	M	C	I
VDAX 60 UNC RF	0.391	0.529	35.52%	0.153	0.190	24.45%
VDAX 60 M11 RF	0.376	0.521	38.67%	0.152	0.190	25.11%
VDAX 15 POS RF	0.551	0.712	29.16%	0.138	0.153	10.51%
VDAX 30 POS RF	0.364	0.464	27.28%	0.125	0.171	36.57%
VDAX 60 POS RF	0.265	0.343	29.34%	0.128	0.181	41.55%
VDAX 120 POS RF	0.192	0.256	32.79%	0.097	0.151	56.87%
VDAXL1 60 POS RF	0.265	0.337	27.11%	0.128	0.153	19.28%
VDAXL2 60 POS RF	0.266	0.324	22.10%	0.128	0.146	13.94%
VDAXL5 60 POS RF	0.266	0.318	19.63%	0.128	0.121	-5.43%
VDAXL10 60 POS RF	0.266	0.314	18.24%	0.127	0.128	1.03%
VDAXAVG3 60 POS RF				0.128	0.170	33.12%
VDAXAVG5 60 POS RF				0.128	0.156	22.24%
VDAXAVG7 60 POS RF				0.128	0.140	9.64%
VDAXAVG9 60 POS RF				0.128	0.124	-2.98%
VDAXAVG11 60 POS RF				0.128	0.109	-14.68%
BONDIDX 60 POS RF	0.266	0.318	19.57%	0.132	0.151	14.30%
KP1M 60 POS RF	0.265	0.311	17.31%	0.127	0.132	4.05%
KP3M 60 POS RF	0.265	0.314	18.40%	0.129	0.148	14.58%
PCA50 60 POS RF	0.268	0.316	18.21%	0.121	0.128	5.97%
PCA100 60 POS RF	0.275	0.320	16.20%	0.116	0.101	-13.24%
PCA200 60 POS RF	0.248	0.317	28.53%	0.189	0.193	2.13%
PCA2F50 60 POS RF	0.263	0.329	25.14%	0.130	0.149	14.05%
PCA2F100 60 POS RF	0.266	0.327	23.04%	0.117	0.119	2.26%
PCA2F200 60 POS RF	0.231	0.326	41.09%	0.189	0.207	9.80%
VDAX 60 UNC RISKY	0.384	0.490	27.75%	0.252	0.301	19.61%
VDAX 60 M11 RISKY	0.370	0.478	29.12%	0.186	0.215	15.36%
VDAX 15 POS RISKY	0.640	0.881	37.67%	0.167	0.203	21.85%
VDAX 30 POS RISKY	0.468	0.624	33.11%	0.185	0.225	21.61%
VDAX 60 POS RISKY	0.363	0.482	32.71%	0.181	0.229	26.46%
VDAX 120 POS RISKY	0.298	0.391	31.21%	0.178	0.208	16.70%
VDAXL1 60 POS RISKY	0.363	0.472	30.33%	0.181	0.225	24.03%
VDAXL2 60 POS RISKY	0.363	0.455	25.35%	0.182	0.194	7.05%
VDAXL5 60 POS RISKY	0.363	0.444	22.24%	0.180	0.173	-4.08%
VDAXL10 60 POS RISKY	0.363	0.439	20.88%	0.181	0.175	-2.93%
VDAXAVG3 60 POS RISKY				0.181	0.229	26.55%
VDAXAVG5 60 POS RISKY				0.181	0.229	26.40%
VDAXAVG7 60 POS RISKY				0.181	0.228	26.15%
VDAXAVG9 60 POS RISKY				0.181	0.227	25.62%
VDAXAVG11 60 POS RISKY				0.181	0.226	24.75%
BONDIDX 60 POS RISKY	0.362	0.438	20.77%	0.182	0.200	9.87%
KP1M 60 POS RISKY	0.365	0.436	19.34%	0.181	0.172	-4.68%
KP3M 60 POS RISKY	0.364	0.441	21.28%	0.182	0.197	8.72%
PCA50 60 POS RISKY	0.374	0.451	20.70%	0.177	0.186	5.45%
PCA100 60 POS RISKY	0.371	0.446	20.38%	0.173	0.154	-10.73%
PCA200 60 POS RISKY	0.373	0.450	20.56%	0.169	0.159	-5.79%
PCA2F50 60 POS RISKY	0.369	0.466	26.24%	0.178	0.194	8.82%
PCA2F100 60 POS RISKY	0.335	0.408	21.94%	0.174	0.171	-1.72%
PCA2F200 60 POS RISKY	0.374	0.458	22.43%	0.170	0.179	4.88%

Table 4: Ex ante and ex post (business daily) Sharpe ratios and relative differences, for all different problems reported, using both Markowitz and conditioned optimisation. Meaning of description: First word - index used, ('KP[1|3]M': Kumar and Persaud GRAI using respectively 1 and 3 month forwards, 'BONDIDX': Bond volatility index, 'VDAXLn': VDAX index lagged by n points, 'VDAXAVGn': VDAX index with weights averaged over n points, 'PCA[2F]n': PCA index based on either 4 or ('2F') 2 factors and constructed using window size n), number - estimation window size used, 'UNC'/'M11'/'POS' - weights constraint used (unconstrained, $[-1, 1]$ and positive weights only), 'RF' - market with risk-free asset, 'RISKY' - risky assets only available.

λ	0	1	2	3	4	5	6	7	8	9	10
M unconstrained ret	264.66%	246.01%	195.63%	162.01%	137.08%	118.93%	106.36%	96.86%	89.80%	84.20%	79.22%
C unconstrained ret	230.20%	230.84%	216.12%	192.06%	169.39%	150.14%	133.97%	122.39%	113.33%	105.57%	99.83%
Return improvement	-13.02%	-6.17%	10.47%	18.55%	23.57%	26.24%	25.96%	26.35%	26.20%	25.38%	26.01%
M unconstrained std	0.552	0.488	0.375	0.301	0.249	0.199	0.169	0.145	0.126	0.113	0.105
C unconstrained std	0.420	0.406	0.352	0.293	0.248	0.212	0.185	0.162	0.142	0.128	0.116
Std ratio	0.760	0.831	0.938	0.973	0.996	1.066	1.096	1.112	1.128	1.130	1.111
M $[-1,1]$ ret	261.62%	230.95%	178.28%	147.08%	128.01%	113.15%	102.92%	94.39%	88.25%	82.88%	78.18%
C $[-1,1]$ ret	235.12%	236.60%	213.59%	186.26%	162.72%	144.28%	129.59%	119.34%	111.69%	103.89%	98.61%
Return improvement $[-1,1]$	-10.129%	2.44%	19.80%	26.64%	27.11%	27.52%	25.91%	26.43%	26.57%	25.36%	26.142%
M $[-1,1]$ std	0.566	0.463	0.333	0.264	0.219	0.184	0.162	0.143	0.124	0.112	0.103
C $[-1,1]$ std	0.429	0.406	0.335	0.278	0.237	0.204	0.179	0.159	0.139	0.126	0.115
Std ratio	0.758	0.877	1.007	1.054	1.082	1.111	1.105	1.111	1.116	1.128	1.114
M $[0,+\infty)$ ret	185.07%	172.99%	143.93%	121.03%	103.39%	89.06%	79.89%	73.82%	68.16%	63.59%	60.68%
C $[0,+\infty)$ ret	211.85%	212.31%	174.58%	149.51%	129.35%	114.10%	100.87%	93.05%	84.84%	80.07%	74.88%
Return improvement	14.47%	22.73%	21.29%	23.54%	25.11%	28.12%	26.11%	26.04%	24.46%	25.92%	23.40%
M $[0,+\infty)$ std	0.646	0.397	0.276	0.222	0.188	0.156	0.126	0.112	0.096	0.085	0.079
C $[0,+\infty)$ std	0.419	0.371	0.274	0.217	0.181	0.152	0.130	0.114	0.099	0.09	0.082
Std ratio C / M	0.648	0.934	0.995	0.978	0.961	0.978	1.034	1.019	1.031	1.066	1.038

Table 5: Overall returns, relative improvements obtained using various strategies in the presence of a risk-free asset (M = Markowitz, C = Conditioned), with short selling constraint, estimation window sizes of 60 points, using the VDAX signal and as a function of the quadratic risk aversion coefficient λ.

λ	N/A	0	1	2	3	4	5	6	7	8	9	10
M unconstrained ret		261.90%	228.32%	172.04%	139.80%	118.40%	103.79%	94.18%	86.25%	81.88%	76.82%	73.48%
C unconstrained ret		227.09%	226.43%	203.35%	174.01%	150.75%	133.61%	120.03%	110.39%	103.02%	96.04%	90.55%
Return improvement		-13.29%	-0.83%	18.20%	24.47%	27.32%	28.73%	27.45%	27.98%	25.81%	25.01%	23.23%
M unconstrained std		0.762	0.334	0.224	0.178	0.156	0.141	0.132	0.124	0.114	0.109	0.104
C unconstrained std		0.460	0.352	0.243	0.191	0.161	0.143	0.131	0.122	0.115	0.110	0.103
Std ratio C / M		0.604	1.054	1.083	1.070	1.034	1.012	0.996	0.985	1.005	1.008	0.998
Momentum long/short ret	135.16%											
Momentum long/short std	1.055											
M $[-1,1]$ ret		261.39%	215.57%	156.18%	127.63%	110.44%	97.59%	90.00%	82.37%	78.41%	73.61%	70.07%
C $[-1,1]$ ret		224.72%	223.04%	197.22%	167.10%	143.63%	126.67%	113.98%	104.09%	97.48%	90.37%	85.91%
Return improvement $[-1,1]$		-14.029%	3.47%	19.80%	26.28%	30.05%	29.80%	26.64%	26.37%	24.32%	22.76%	22.61%
M $[-1,1]$ std		0.591	0.464	0.318	0.246	0.209	0.185	0.167	0.152	0.139	0.128	0.120
C $[-1,1]$ std		0.460	0.420	0.336	0.276	0.231	0.205	0.185	0.167	0.152	0.139	0.130
Std ratio C / M		0.779	0.905	1.057	1.124	1.106	1.109	1.105	1.097	1.097	1.088	1.079
M $[0,+\infty)$ ret		110.29%	146.76%	112.95%	96.94%	83.81%	76.51%	70.42%	66.60%	61.89%	59.51%	56.67%
C $[0,+\infty)$ ret		191.11%	196.71%	154.67%	127.67%	109.85%	98.27%	90.55%	83.10%	77.80%	72.62%	69.63%
Return improvement		73.27%	34.04%	36.94%	31.70%	31.07%	28.44%	28.59%	24.77%	25.71%	22.03%	22.87%
M $[0,+\infty)$ std		0.762	0.334	0.224	0.178	0.156	0.141	0.132	0.124	0.114	0.109	0.104
C $[0,+\infty)$ std		0.460	0.352	0.243	0.191	0.161	0.143	0.131	0.122	0.115	0.110	0.103
Std ratio C / M		0.604	1.054	1.083	1.070	1.034	1.012	0.996	0.985	1.005	1.008	0.998
Momentum long only ret	113.63%											
Momentum long only std	0.6168											

Table 6: Overall returns, relative improvements obtained using various strategies in the presence of risky assets only (M = Markowitz, C = Conditioned, Momentum with 3 assets, daily rebalancing, 60 day historical window), with short selling constraint, estimation window sizes of 60 points, using the VDAX signal and as a function of the quadratic risk aversion coefficient λ.

λ	0	1	2	3	4	5	6	7	8	9	10
RF 15D return improvement	-4.534%	-8.545%	-3.233%	0.419%	4.317%	6.829%	9.141%	11.856%	12.028%	15.118%	14.390%
RF 30D return improvement	4.381%	7.684%	13.402%	19.023%	20.879%	23.195%	23.101%	22.184%	21.237%	19.469%	22.078%
RF 120D return improvement	12.073%	25.276%	37.362%	35.631%	33.696%	34.445%	34.012%	31.451%	31.731%	35.019%	39.188%
RF 15D std ratio	0.677	0.854	0.922	0.958	0.979	1.005	1.018	1.027	1.027	1.030	1.030
RF 30D std ratio	0.634	0.882	0.942	0.953	0.956	0.949	0.934	0.917	0.942	0.972	0.973
RF 120D std ratio	0.588	0.977	1.167	1.162	1.164	1.167	1.161	1.142	1.163	1.101	1.136
Risky 15D return improvement	109.708%	10.582%	20.126%	21.544%	22.235%	19.825%	20.119%	21.395%	18.662%	19.669%	18.773%
Risky 30D return improvement	127.653%	22.950%	27.490%	25.446%	24.747%	21.287%	22.318%	20.122%	21.733%	20.631%	18.346%
Risky 120D return improvement	-7.456%	42.434%	42.034%	33.234%	31.380%	30.079%	25.906%	27.820%	23.131%	20.240%	17.663%
Risky 15D std ratio	0.619	0.951	0.999	1.008	1.014	1.001	0.985	0.984	0.997	0.991	0.980
Risky 30D std ratio	0.623	1.017	1.075	1.054	1.032	1.028	1.003	1.003	0.992	0.976	0.956
Risky 120D std ratio	0.581	1.059	1.099	1.157	1.151	1.139	1.130	1.102	1.101	1.103	1.059

Table 7: Overall return and standard deviation relative improvements obtained using various estimation window sizes, both in the presence of a risk-free asset and given risky assets only, with short selling constraint, using the VDAX signal and as a function of the quadratic risk aversion coefficient λ.

λ	0	1	2	3	4	5	6	7	8	9	10
RF lag 1 return improvement	7.90%	10.04%	8.57%	11.78%	13.85%	12.50%	14.13%	12.84%	15.87%	14.68%	18.17%
RF lag 2 return improvement	8.34%	5.40%	5.51%	8.19%	10.79%	13.38%	11.23%	9.96%	11.71%	10.59%	11.73%
RF lag 5 return improvement	4.37%	0.21%	-5.84%	-3.36%	-0.00%	0.84%	0.10%	-2.70%	1.15%	0.92%	2.48%
RF lag 10 return improvement	16.56%	2.96%	-1.70%	0.86%	2.13%	1.62%	1.83%	1.06%	2.15%	1.73%	2.22%
RF lag 1 std ratio	0.664	0.973	1.034	1.023	1.018	1.033	1.041	1.053	1.047	1.071	1.059
RF lag 2 std ratio	0.692	0.976	1.008	1.010	1.002	1.023	1.056	1.047	1.068	1.095	1.036
RF lag 5 std ratio	0.699	0.985	1.031	1.039	1.047	1.070	1.084	1.077	1.101	1.120	1.109
RF lag 10 std ratio	0.721	0.995	1.034	1.018	1.012	1.040	1.031	1.058	1.049	1.039	1.017
Risky lag 1 return improvement	73.27%	34.04%	36.94%	31.70%	31.07%	28.44%	28.597%	24.77%	25.71%	22.03%	22.87%
Risky lag 2 return improvement	95.33%	8.61%	10.66%	7.46%	10.70%	8.82%	11.01%	10,15%	11.30%	7.16%	8.97%
Risky lag 5 return improvement	52.50%	0.23%	3.55%	-1.56%	-0.078%	0.619%	1.51%	-0.424%	1.986%	-0.165%	-0.276%
Risky lag 10 return improvement	52.68%	2.47%	1.19%	-2.19%	-1.64%	-1.9%	-0.56%	-0.69%	2.93%	-0.25%	-0.813%
Risky lag 1 std ratio	0.582	1.064	1.057	1.060	1.053	1.052	1.038	1.015	1.026	1.049	1.028
Risky lag 2 std ratio	0.645	1.054	1.057	1.060	1.043	1.039	1.034	1.008	1.040	1.041	1.030
Risky lag 5 std ratio	0.670	1.054	1.077	1.105	1.076	1.045	1.052	1.052	1.041	1.055	1.046
Risky lag 10 std ratio	0.674	1.050	1.075	1.091	1.070	1.047	1.023	0.994	1.007	1.018	1.010

Table 8: Overall return and standard deviation relative improvements obtained both in the presence of a risk-free asset ('RF') and given risky assets only ('Risky'), with short selling constraint, estimation window sizes of 60 points, using the VDAX signal lagged by different numbers of points and as a function of the quadratic risk aversion coefficient λ.

λ	0	1	2	3	4	5	6	7	8	9	10
RF 3 average points return improvement	9.59%	17.76%	16.32%	18.30%	19.64%	22.43%	20.10%	20.07%	18.21%	19.87%	16.83%
RF 3 average points std ratio	0.627	0.914	0.979	0.962	0.944	0.963	1.021	1.005	1.017	1.050	1.024
RF 5 average points return improvement	5.06%	12.79%	11.12%	12.66%	13.60%	15.81%	13.06%	12.69%	10.61%	11.95%	8.74%
RF 5 average points std ratio	0.608	0.893	0.964	0.952	0.936	0.960	1.028	1.016	1.035	1.075	1.056
RF 7 average points return improvement	0.014%	7.48%	6.11%	6.64%	7.30%	9.00%	6.14%	4.73%	2.70%	3.46%	0.35%
RF 7 average points std ratio	0.589	0.875	0.953	0.945	0.934	0.967	1.044	1.047	1.083	1.137	1.132
RF 9 average points return improvement	-5.11%	1.89%	1.05%	0.52%	0.56%	1.71%	-0.99%	-2.37%	-4.20%	-4.01%	-6.54%
RF 9 average points std ratio	0.571	0.853	0.942	0.941	0.939	0.986	1.077	1.102	1.160	1.233	1.245
RF 11 average points return improvement	-10.24%	-3.90%	-4.36%	-5.55%	-5.79%	-4.72%	-7.72%	-8.83%	-10.66%	-10.47%	-12.65%
RF 11 average points std ratio	0.555	0.832	0.933	0.941	0.951	1.013	1.126	1.166	1.258	1.357	1.388
Risky 3 average points return improvement	71.68%	32.20%	36.41%	31.42%	30.94%	28.11%	28.24%	24.45%	25.43%	21.93%	22.73%
Risky 3 average points std ratio	0.584	1.050	1.082	1.069	1.032	1.018	0.995	0.982	1.002	1.006	0.995
Risky 5 average points return improvement	70.25%	30.34%	35.35%	31.26%	30.73%	27.69%	27.67%	24.02%	25.12%	21.82%	22.51%
Risky 5 average points std ratio	0.566	1.036	1.078	1.068	1.031	1.006	0.993	0.980	1.001	1.006	0.997
Risky 7 average points return improvement	68.48%	27.85%	34.06%	30.66%	30.49%	27.01%	27.02%	24.73%	26.45%	24.33%	25.48%
Risky 7 average points std ratio	0.550	1.020	1.077	1.069	1.030	1.005	0.994	0.989	1.012	1.022	1.019
Risky 9 average points return improvement	66.23%	25.14%	32.78%	29.85%	29.74%	27.50%	28.20%	26.77%	29.45%	28.18%	30.31%
Risky 9 average points std ratio	0.534	1.003	1.070	1.071	1.036	1.015	1.009	1.008	1.039	1.055	1.060
Risky 11 average points return improvement	63.67%	22.19%	31.24%	28.68%	30.10%	28.83%	30.25%	29.86%	33.90%	33.16%	36.03%
Risky 11 average points std ratio	0.519	0.982	1.064	1.073	1.052	1.038	1.037	1.042	1.078	1.100	1.113

Table 9: Overall return and standard deviation relative improvements obtained both in the presence of a risk-free asset ('RF') and given risky assets only ('Risky'), with short selling constraint, estimation window sizes of 60 points, using the VDAX signal and optimal weights averaged over different numbers of points of the signal support.

λ	0	1	2	3	4	5	6	7	8	9	10
RF BONDIDX return improvement	14.07%	10.99%	6.09%	8.41%	9.22%	9.46%	9.38%	5.10%	8.12%	6.59%	5.46%
RF BONDIDX std ratio	0.715	0.994	0.987	0.974	0.978	0.955	1.000	1.007	1.000	1.055	1.062
RF KP1M return improvement	6.80%	4.71%	-1.26%	1.56%	2.89%	2.91%	6.98%	1.57%	7.78%	7.49%	5.90%
RF KP1M std ratio	0.724	1.007	1.032	1.010	1.025	1.018	1.005	1.035	1.023	1.064	1.020
RF KP3M return improvement	8.89%	6.12%	6.17%	7.93%	10.17%	9.91%	11.42%	7.06%	11.57%	12.62%	11.88%
RF KP3M std ratio	0.715	0.976	0.989	0.970	0.981	1.010	1.024	1.035	1.054	1.053	1.050
RF PCA50 return improvement	13.42%	6.77%	4.76%	6.30%	8.57%	5.25%	4.36%	8.12%	9.67%	8.90%	6.92%
RF PCA50 std ratio	0.737	0.998	1.022	1.070	1.062	1.045	1.103	1.052	1.066	1.063	1.104
RF PCA100 return improvement	-0.01%	-14.69%	-12.41%	-10.05%	-5.64%	-4.80%	-2.43%	-2.70%	-1.38%	-3.23%	-0.79%
RF PCA100 std ratio	0.704	0.983	1.021	1.020	1.036	1.036	1.061	1.094	1.077	1.117	1.079
RF PCA200 return improvement	19.14%	0.28%	1.56%	0.53%	1.53%	2.78%	4.00%	6.82%	8.96%	8.53%	8.57%
RF PCA200 std ratio	0.666	0.894	0.990	1.018	1.017	1.049	1.053	1.063	1.058	1.069	1.058
RF PCA2F50 return improvement	20.51%	6.97%	8.02%	9.72%	15.23%	15.07%	15.50%	12.67%	12.43%	13.13%	11.67%
RF PCA2F50 std ratio	0.653	0.970	1.010	1.031	1.049	1.061	1.104	1.142	1.110	1.132	1.146
RF PCA2F100 return improvement	19.42%	-9.18%	-3.64%	-1.22%	3.91%	5.98%	3.40%	5.05%	7.47%	6.53%	6.89%
RF PCA2F100 std ratio	0.633	0.926	0.993	1.036	1.025	1.057	1.076	1.103	1.034	1.049	1.033
RF PCA2F200 return improvement	10.40%	5.46%	4.62%	7.66%	9.32%	8.37%	8.86%	9.26%	11.23%	9.07%	7.87%
RF PCA2F200 std ratio	0.544	0.900	0.965	0.976	0.972	1.011	1.020	1.028	1.033	1.026	1.029
Risky BONDIDX return improvement	76.17%	11.50%	11.20%	9.50%	10.57%	11.77%	10.89%	9.94%	10.18%	6.58%	7.73%
Risky BONDIDX std ratio	0.712	1.046	1.057	1.072	1.042	1.027	0.995	0.966	0.973	0.965	0.973
Risky KP1M return improvement	49.48%	0.77%	1.36%	-2.81%	0.90%	1.19%	2.16%	1.75%	4.33%	2.66%	2.19%
Risky KP1M std ratio	0.727	1.078	1.104	1.123	1.101	1.073	1.064	1.043	1.038	1.018	1.005
Risky KP3M return improvement	61.68%	16.64%	12.84%	9.20%	10.27%	8.78%	7.94%	8.63%	9.96%	9.20%	7.55%
Risky KP3M std ratio	0.699	1.031	1.010	1.020	1.016	1.023	1.015	0.989	1.038	1.018	1.006
Risky PCA50 return improvement	43.98%	12.05%	15.27%	12.21%	13.19%	9.88%	10.97%	11.86%	10.75%	9.92%	9.57%
Risky PCA50 std ratio	0.663	1.051	1.075	1.099	1.075	1.091	1.079	1.021	1.055	1.060	1.043
Risky PCA100 return improvement	55.08%	-12.48%	-9.62%	-10.16%	-6.90%	-7.99%	-6.38%	-7.39%	-4.70%	-6.35%	-6.96%
Risky PCA100 std ratio	0.669	1.010	1.033	1.053	1.050	1.063	1.043	1.047	1.059	1.037	1.034
Risky PCA200 return improvement	53.12%	-9.72%	-2.46%	-1.65%	-2.88%	-1.35%	-0.35%	-1.26%	-1.23%	-3.61%	-4.38%
Risky PCA200 std ratio	0.635	0.990	1.014	1.033	1.053	1.051	1.034	1.050	1.038	1.068	1.059
Risky PCA2F50 return improvement	29.54%	10.72%	15.13%	14.20%	16.00%	17.34%	14.78%	14.43%	14.69%	12.19%	13.16%
Risky PCA2F50 std ratio	0.584	1.023	1.068	1.050	1.063	1.072	1.062	1.056	1.087	1.077	1.026
Risky PCA2F100 return improvement	78.47%	-2.06%	0.70%	0.65%	2.77%	1.65%	1.40%	-0.02%	2.92%	1.26%	0.28%
Risky PCA2F100 std ratio	0.605	0.987	1.043	1.044	1.058	1.061	1.060	1.054	1.042	1.039	1.027
Risky PCA2F200 return improvement	120.32%	5.76%	5.46%	3.34%	6.52%	4.15%	5.03%	2.22%	2.67%	1.50%	2.02%
Risky PCA2F200 std ratio	0.615	0.966	1.023	1.010	1.020	1.019	0.999	1.008	0.990	0.987	0.987

Table 10: Overall return and standard deviation relative improvements obtained using various signals, estimation window sizes of 60 points, both in the presence of a risk-free asset ('RF') and given risky assets only ('Risky'), with short selling constraint, as a function of the quadratic risk aversion coefficient λ.