Endless leverage certificates

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ARTICLE INFO	ABSTRACT
	An endless leverage certificate (ELC) is a novel retail structured product that gives its holder the right to claim the difference between the value of an underlying security and an interest accruing financing level. An ELC ceases to exist if the underlying breaches a contractual knockout level, if the holder exercises, or if she/he sells it back to the issuer. We use Monte Carlo analysis to value ELCs and find that due to limited liability, a typical ELC written on a typical DAX stock can be worth 0.3% more than its intrinsic value (the difference between the value of the underlying and the financing level). Empirically, we find that in January 2007, the 5129 ELCs issued on the thirty DAX stocks traded at an average premium of 0.67% over the intrinsic value, and that the median bid-ask spread, expressed as a percentage of the underlying, was 0.18%. For covered warrants and options this spread measure was almost twice as high. Finally, we find that upon knockout, investors received on average 3.2% less than the theoretical knockout value, which is consistent with discontinuous trading of the underlying. Overall, our findings suggest that ELCs *complete the market* for leverage seeking retail investors. © 2009 Elsevier B.V. All rights reserved.

1. Introduction

An endless leverage certificate is a retail structured product that gives its holder the right to claim the realized difference between the value of an underlying and a contractual *financing level*. At the end of each trading day the financing level is multiplied by a predetermined interest rate factor and reduced by any dividends paid by the underlying. When an issuing bank sells an ELC, it hedges its exposure by taking a position in the underlying. ELCs come in longs and shorts. An ELC-long can be interpreted as a textbook levered position that is partly financed with a loan from the issuer. An ELC-short can be interpreted as a deposit with the issuer combined with a short position in the underlying. As a protection against the issuer's position, an ELC has a contractual *knockout level* that lies between the value of the underlying and the financing level.

An ELC ceases to exist in three ways. First, ELCs can be *knocked out*. If the underlying breaches the knockout level, the issuer unwinds the hedge portfolio and, if the proceeds from selling the underlying exceed the contemporaneous financing level, returns a *residual value* to the investor. Issuers contractually commit to exercising best efforts in the unwind procedure and disclose residual values on their websites so that investors can verify the fairness of the unwind procedure. Second, ELC holders are allowed to *exercise*. That is, they can require the issuer to unwind the hedge portfolio and claim the residual value before a knockout event occurs. Finally, ELC holders can sell the ELC. Because issuers post bid quotes very close to the intrinsic value, and pay residual values with a lag (e.g. three working days), this is the most common investor initiated termination.

ELCs are marketed under names such as *Turbos*, *Waves*, *Mini-Futures*, *Speeders* and *Sprinters*. In October 2008, more than 45,000 ELC contracts were outstanding in Germany and the Netherlands alone, and the total monthly trading turnover in Germany topped €1.2 billion.[1] ELCs are also popular in France, Switzerland and Austria, and are currently being introduced in the UK, Italy, Spain and Portugal, among others. In the German market, ELCs booked the highest trading turnover of all derivatives in 2007, and by the end of 2008 virtually all major banks included ELCs in their offering of retail structured products.[2] Issuers aggressively market their ELCs with internet banners and newspapers ads. The typical tagline

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[1] Source: Deutsches Derivate Institut (DDI) and Deutsche Börse. For a comparison, during the same period, the trading turnover of the 30 Xetra-Dax stocks was €172 billion.

[2] The pioneers in the ELC market were ABN-AMRO and Commerzbank. The last large bank to enter the market was ING, who started offering *Sprinters* in November 2008, at the height of the credit crisis. Although ELC trading volumes decreased during 2008, the number of outstanding contracts and their share in the structured products market has steadily increased since their first appearance in 2004.

emphasizes the leveraged returns, the limited liability, the perpetual nature, and the low sensitivity to volatility.[3]

In this paper, we hypothesize that ELCs complete the market as in Ross (1989) and Gale (1992), or Grinblatt and Longstaff (2000). The market completion hypothesis predicts that ELC market prices do not stray far from their risk adjusted values while offering investors lower round trip transaction costs than alternative financial securities.

To test this we first focus our attention on the valuation of ELCs. We observe that a lower bound for the value of a continuously exercisable ELC is its intrinsic value. However, an ELC should be worth more than its intrinsic values, because it comes with limited liability: an ELC holder has the right to leave the issuer with a loss on its hedge portfolio if the underlying precipitates through the knockout and the financing level at the same time, which may happen during a large price jump or an overnight trading halt. This means that issuers are exposed to *gap risk*. Although they are infrequent, gap crossings *do* happen. During the five years ending December 31st 2006, the thirty DAX stocks saw 101 absolute overnight returns higher than 5%, the typical distance between financing and knockout levels.

To value ELCs, we derive a valuation model that takes into account gap risk due to overnight trading halts. We show that, because the interest rate is set at a contractual and constant spread over the risk free rate, an ELC has an unobservable *exercise level*. If the underlying breaches this level, rational investors exercise (or sell) because the interest rate on the financing level no longer justifies the ELC's limited liability. An ELC will thus be alive, and worth more than its intrinsic value, as long as the underlying trades between its knockout level and its exercise level.

To illustrate this, consider an ELC-long on an underlying that, with 99.8% probability books independent overnight returns of either 0.5% or −0.5%, and with 0.2% probability *jumps* 20%, either up or down. It can be verified that, if the risk free rate is zero, the financing level €100, and the interest rate on the financing level 2%, the exercise level is €115.[4] If, moreover, the underlying returns either 0.5% or −0.5% during daytime, and trades at €110, while the knockout level is set at €105, it can be shown that the ELC's value is approximately €10.19, or 1.9% above its intrinsic value.[5]

To obtain value estimates based on more realistic return distributions, we develop a Monte Carlo algorithm that uses historical returns. We find that if the return distribution of DAX stocks over the five year period ending December 2006 is representative for the future, a typical ELC on an typical DAX stock is worth up to 0.3% more than its intrinsic value. We also find that for stocks (or time periods) that are twice as volatile, ELCs may be worth 3% more than their intrinsic value. Finally, we find that most ELCs have expected lifespans of less than two weeks.

We then study ELC quotes empirically, by analyzing a large and exhaustive sample of 5129 ELCs written on the thirty DAX stocks and 444 ELCs written on the 25 AEX stocks. We find that in January 2007, ELC-longs traded at an average premium of 0.55% above their intrinsic value while the average premium for ELC-shorts was 1.01%. Consistent with theory and our Monte Carlo simulations,

we find premiums to increase in the volatility of the underlying. Interestingly, we do not find that premiums decrease in the distance to the financing level, suggesting that high lever ELCs are more competitively priced than low lever ELCs. We also find that that the vast majority of all listed ELCs trade beyond the estimated (typical) exercise level, which suggests that at least some ELC investors exercise too late and leave money on the table for the issuers.

To gain further insight in the competitiveness of ELC markets we investigate bid and ask quotes in the secondary market. We measure a median bid-ask spread of €0.02, or 0.68% of the mid-quote. To compare bid-ask spreads across derivatives, we develop a novel statistic, the *leverage adjusted spread*, defined as the bid-ask spread expressed as a percentage of the underlying value that it claims. For ELCs we find an average leverage adjusted spread of approximately 0.25%, significantly lower than the average spreads on covered warrants (0.46%) and exchange traded options (0.98%). We attribute these narrow spreads to the ELC's first order simplicity and to the fierce competition in this growing market.

We then study the fairness of knockout terminations. An empirical analysis of 1801 residual values reveals that, on average, ELC holders received 3.19% less than the difference between the knockout level and the financing level, and that the average implied liquidation value of the underlying fell short of the knockout value by 0.24%. Part of this can be explained by discontinuous trade of the underlying: we find that average shortfalls for overnight knockouts are 2.5 times those for daytime knockouts.

Early papers on structured products include Chen and Kensinger (1990), Chen and Sears (1990) and Baubonis et al. (1993), who study index certificates, such as MICDs, SPINs and SPDRs. They report an average overpricing of 2–4% upon issue that gradually disappears over the life of the security. Chan and Pinder (2000) analyze covered warrants, which essentially are European options issued by banks. For a large sample of Australian warrants the authors find substantial overpricing and significantly lower bid-ask spreads compared to exchange originated options.[6] Stoimenov and Wilkens (2005) provide a taxonomy of retail structured products, and report that structured products are overpriced by 4%, on average. They also find that overpricing is more severe for more complicated products, and that it decreases over the life of the product.[7] Scholz et al. (2005), Wilkens and Stoimenov (2007) and Muck (2007) investigate Classic Turbos, that essentially are down-and-out barrier options. Classic Turbos are the predecessors of ELCs. They have barriers higher than or equal to the strike price, and are marketed as levered positions with the present value of the strike price as the loan of the issuer. The authors document an overpricing of approximately 5%.

The only other paper on ELCs to our knowledge is Entrop et al. (2009). They value ELCs under the assumption that the underlying is continuously traded and that investors have exogenous holding periods. Subsequently they conclude that ELCs are significantly overpriced, and that the overpricing increases in investors' holding periods. Our valuation model assumes endogenous holding periods and gives an equilibrium price that depends on ELC characteristics and the return distribution of the underlying.

The next section introduces key terminology and provides an overview of the ELC market. In Section 3 we develop a valuation model for ELCs and use Monte Carlo analysis to estimate typical ELC values. Section 4 contains our empirical analysis, of market

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[3] On the Goldman Sachs website we read "High profit potential with limited stakes. With Mini-futures investors can accelerate price developments. The volatility does not affect the value. The lifespan is unlimited." ("Grosse Chancen bei kleinem Kapitaleinsatz. Mit Mini-futures nehmen Anleger überproportional an der Kursentwicklung des Basiswerts teil. Die Volatilität hat keinen Einfluss auf den Preis. Die Laufzeit ist unbegrenzt.").

[4] This follows from solving $0.999 \times 100 \times (1 + r_{ELC}) + 0.001 \times 0.8E = 100$. With $r_{ELC} = 0.0079\%$ for the daily interest on the financing level, we find $E = 115$.

[5] In the appendix we show that the probability of a jump ending is 9.24%, so that the expected loss on the loan is $5.12\% \times (100 - 0.8 \times 110) = €0.55$. We also show that the expected life is 46 days, for an expected interest income for the bank is €0.36. The value of the ELC is then $€10 + €0.55 − €0.36 = €10.19$.

[6] Introduced in the 1990s, covered warrants are popular in almost all countries with an active stock exchange except the United States. Banks write covered warrants on indices, stocks, baskets of stocks, commodities and exchange rates, among others.

[7] Other studies on structured products include Burth et al. (2001), Grünbichler and Wohlwend (2005), Benet et al. (2006) and Hernandez et al. (2007), who find overpricing levels between 2% and 5%.

prices, bid-ask spreads and residual values. Section 5 concludes. The appendix contains an example of an ELC valuation and the algorithm of our Monte Carlo analysis.

2. Terminology and market overview

We define ELCs as securities that are open-ended and have intrinsic values in the form of max $(|S-D|, 0)$, where S is the value of the underlying and D is a financing level that increases at an interest rate that is set at a fixed markup over an interbank benchmark such as Euribor or EONIA. In the remainder of this article we denote this markup the *credit spread*, as it is similar to the interest rate spread that lenders charge as a compensation for default risk on loans against risky collaterals.[8] The ELC should also have a knockout level, denoted K, with $K > D$ for ELC-longs and $K < D$ for shorts. Any dividends paid by the underlying should accrue to the financing level.

All issuers in the German and Dutch markets offer ELCs on DAX and AEX stocks. They also offer ELCs on midcap and foreign stocks, indices, bonds, currencies, and commodities. The main ELC characteristics vary across issuers and underlyings. The average and median credit spread is 200 basispoints. For ELC-longs (-shorts) knockout levels lie 3–10% above (below) the financing level. At issuance the average and median *knockout premium* (defined as $|K - D|/D$) is 5%. To keep the knockout premium approximately constant, most issuers adjust K once a month.[9]

Issuers make their ELCs exercisable at different frequencies. Some issuers specify a monthly exercise date, e.g. every first working day of the month. Others offer daily exercise. In general, exercising is discouraged by stipulating conditions such as a written notice sent several days or weeks in advance, and then paying the exercise proceeds with a lag (e.g. three days). Because issuers quote bid prices that are very close to, and often above the intrinsic value, investors generally sell their ELC instead of exercising it.[10] Most issuers retain the right to terminate the contract unilaterally if hedging becomes difficult or impossible. The prospectus usually specifies a list of disruption events (takeovers, restructurings, or threats thereof) that can trigger such a termination. Many issuers give their ELCs a ratio, which refers to the number of underlying units per certificate. Ratios are chosen by the issuer to arrive at attractive (low) prices.

Issuers employ careful legal language in their ELC prospectuses to explain how hedge portfolios are unwound and residual values are computed. Most issuers guarantee a minimum value based on the most unfavorable price of the underlying in a short event window (e.g. one hour) around the knockout event.[11] All issuers report residual values on dedicated websites, but do so in varying detail. Some issuers only disclose the ISIN numbers, the knockout dates, and the residual values. Others also report knockout levels, financing levels, and the knockout times, precise to the second.

Issuers employ different strategies to compete in the ELC market. They make their certificates attractive by charging low credit spreads, and issuing them at high *levers* (defined as $S/|S - D|$) and low knockout premiums. The issuer who charged the lowest credit spread (140 basispoints over Euribor), and issued ELCs with relatively high levers and low knockout premiums saw its ELCs being the most actively traded during our sample period of January 2007. The issuer that offered the most aggressive levers and knockout premiums, while charging a credit spread of 200 basispoints over Euribor, also attracted considerable trading volume.

3. Valuing ELCs

3.1. A valuation model

In the following we model an ELC-long as a long position in an underlying, partially financed by a loan that is secured only by the underlying. Because the ELC holder benefits from limited liability and has the right to exercise, we decompose the value of an ELC as follows:

$$ELC = |S - D| - PV\ (credit\ spread) + Gap\ Risk\ Put + Exercise\ Put.$$
(1)

The first component is the intrinsic value. The second component is the net present value of the interest rate markup over the expected life of the loan, if it were riskless. The Gap Risk Put gives the ELC holder the right to sell the underlying to the issuer in exchange for the face value of the loan, and the Exercise Put gives the holder the right to exit.[12] Because the present value of the credit spread is also contingent on the underlying, we henceforth refer to the last three components of (1), as the ELC's option component.

In our analysis we assume that issuers can exercise their ELC at any time, so that the intrinsic value is a lower bound for the ELC value.[13] If the underlying follows a sufficiently smooth process, and is continuously traded in a market with sufficient depth, the value of the gap risk put is zero, so that the intrinsic value is also the *upper* bound for the ELC's value. This implies that exercisable ELCs on 'jump-free', continuously traded underlyings should not exist. Entrop et al. (2009) take this view and assume that ELC holders exogenously choose holding periods. They show that in the absence of gap risk the arbitrage profits for the issuers increase in the holding period without bound.[14]

We know however that underlying securities do *not* follow smooth processes and are *not* continuously traded. Gap risk events may occur during the trading day upon dramatic jumps, or more likely, overnight. To derive a value function for the option component of a continuously exercisable ELC we make the following simplifying assumptions:

Assumption 1: During daytime, the price follows a smooth process between an opening and a closing auction, and trading does not come with transaction costs or a price impact. Overnight, between the closing and opening auctions, there is no trade.

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[8] Notice that the banks' position is a loan (in the case of an ELC-long), collateralized only by the underlying.

[9] For the issuers who adjust K monthly, the knockout level is the identifying characteristic. Brokers and traders refer to, e.g., the "Speeder Porsche 806.00" or the "BMW Turbo 46.10" (pun intended).

[10] Of 23,672 quote snapshots, no bid quote was more than €0.01 below the intrinsic value. The median and mode of the bid quotes equaled the intrinsic value; the average was 0.02% *higher* than the intrinsic value.

[11] E.g. one prospectus explains: "The Knockout Disbursement Amount is the difference by which the Hedge Price exceeds (in the case of Long Certificates) or falls below (in the case of Short Certificates) the Financing Level, multiplied with the Subscription Ratio. The Hedge Price is the volume-weighted imputed average of the prices attained by the Issuer for the dissolution of the Hedge Position held by it for the Certificate. The Issuer will dissolve the Hedge Position within a maximum of 60 minutes after the occurrence of the Knockout Event."

[12] The gap risk put captures the value of having limited liability. Limited liability gives the holder the right to default on the loan. Practitioners use the term *gap risk* to describe the issuer's risk of not recovering the financing level in case of a knockout.

[13] Clearly, if there is an exercise delay, the ELC can be worth less than the intrinsic value. If the delay is t days, the lower bound value is given by $S - D \cdot e^{cs \cdot t}$, where cs is the spread over the risk free rate. For a monthly exercisable ELC with a lever of 5 and a cs of 2%, the minimum lower bound is $5-4e^{0.00167} = 99.33\%$ of the intrinsic value. Based on this one would expect ELCs that are not daily exercisable to trade at a lower prices, and their premiums over intrinsic value to increase over the time interval to the next exercise opportunity. However, we do not find any evidence for this in the data.

[14] In a subsection of their paper Entrop et al. (2009) take into account gap risk. However, due to exogenous holding periods, they find gap risk to have only a small impact on ELC values.

Assumption 2: The risk free interest rate is zero. There are no dividends and no taxes. The financing and knockout levels are constant during the trading day and are reset every morning before the opening auction. Both levels increase at the same constant rate: $D_{t+1} = D_t e^{cs}$ and $K_{t+1} = K_t e^{cs}$, where cs the contractual credit spread.

Assumption 3: Issuers do not unilaterally unwind the security and always fulfill their exercise obligation.

Assumption 4: The ELC holder exercises so as to maximize the value of the security.

The first assumption implies that the only source of gap risk derives from overnight trading halts. Notice that this assumption may negatively bias our valuation: the probability of daytime jumps, and a price impact would increase the gap risk for the issuer hence increase the value of the ELC.

The second assumption follows industry practice: all issuers in our sample use daily compounding and reset financing levels overnight. Assumption 3 implies that issuers do not opportunistically terminate their ELCs, and expend genuine *best efforts* in the unwind procedure. This assumption is backed by empirical observations: we often see that issuers pay residual values that are *higher* than $|K - D|$, and do not find any early termination near K.[15] Notice that if issuers would behave opportunistically, our model leads to a positive bias in ELC value estimates. Assumption 3 also implies that there is no counterparty risk. Although such risk has increased substantially during 2008, we conjecture it to be small for ELCs even during the credit crisis. This is because ELCs are highly levered securities that have short life spans, and because issuers are contractually obliged to hedge their exposure upon issuance. Also, counterparty risk would be lower for ELC-longs than for shorts: an ELC-long contains a loan *from* the issuer, an ELC-short contains a loan *to* the issuer. Hence, if there were counterparty risk we would expect prices to be lower for ELC-shorts. Hull and White (1995) present a model for valuing derivative securities when there is counterparty risk. Baule et al. (2008) show that for Discount Certificates (combinations of bonds and written puts), this risk is priced. However, we find, as do Wilkens and Stoimenov (2007), that short certificates trade at significantly higher premiums than longs.

Naturally, the credit spread that issuers charge on the financing level is the price that the issuers pay for their limited liability. Because the value of the gap risk put decreases in the distance between S and D, while the credit spread is constant, there exists an *exercise level*, denoted E^*, where the credit spread no longer justifies the gap risk put. In assumption 4 we conjecture that investors exercise when the underlying crosses E^*.

In the following we denote $L(S|D, K, E^*)$ the value of the ELC loan held by the issuer, so that the value of the ELC is given by:

$$\text{ELC}(S|D, K, E^*) = S - L(S|D, K, E^*). \tag{2}$$

Because K and D are reset overnight, ELC holders make their exercise decision during the closing auction, after comparing the value of the gap risk put wit the credit spread. In the following, we let S'_t and S''_t denote the value of the underlying at the opening and closing auction of trading day t respectively. Similarly, $L'(S'_t|D_t, K_t, E^*_t)$ and $L'(S'_t|D_t, K_t, E^*_t)$ denote the value of the loan during the opening and closing auction. The risk neutral probability distribution function of the next day's opening price, S'_{t+1}, conditional on S'_t, is de-

noted $\psi(S'_{t+1}) \equiv \psi(S'_{t+1}|S'_t)$, so that the value of the financing level immediately after the closing auction is given by:

$$L'(S'_t|D_t, K_t, E^*_t) = \int_0^{D_{t+1}} S'_{t+1} d\psi(S'_{t+1}) + \int_{D_{t+1}}^{K_{t+1}} D_{t+1} d\psi(S'_{t+1}) + \int_{K_{t+1}}^{\infty} L'(S'_{t+1}) d\psi(S'_{t+1}). \tag{3}$$

The first integral refers to the states where the underlying opens at a price below next day's financing level, so that the issuer recovers $S'_{t+1} < D_{t+1}$. The second integral refers to the states where the ELC is knocked out and the loan is fully recovered. The integrand in the final integral denotes the value of the loan during the next day's opening auction if the underlying opens at a price higher than K_{t+1}.

We now let $\pi \equiv \pi(S'_{t+1})$ denote the probability of a daytime knockout, and let $\vartheta(S'_{t+1}) \equiv \vartheta(S'_{t+1}|S'_{t+1})$ stand for the risk neutral probability distribution function of the closing price, conditional on the opening price of the underlying *and* on not seeing a knockout during the day, so that the value of the loan at the opening auction is:

$$L'(S'_{t+1}|D_{t+1}, K_{t+1}, E^*_{t+1}) = \pi D_{t+1} + (1 - \pi) \\ \times \left(\int_{K_{t+1}}^{E^*_{t+1}} L'(S'_{t+1}) d\vartheta(S'_{t+1}) + \int_{E^*_{t+1}}^{\infty} D_{t+1} d\vartheta(S'_{t+1}) \right). \tag{4}$$

The first term on the right hand side refers to a daytime knockout, when, due to assumptions 1 and 2, the issuer recovers D_{t+1}. The second term implies that the ELC holder keeps the ELC alive if the stock closes below E^*_{t+1}, and exercises otherwise. In the latter event, the issuer recovers D_{t+1}.

The optimal exercise level E^*_t can then be found from solving:

$$L''(E^*_t|D_t, K_t, E^*_t) = D_t. \tag{5}$$

Unfortunately, the system of Eqs. (3)–(5) cannot be solved analytically, not even if we assume Gaussian probability distributions. At the same time we recognize that the tails of the return distributions are important determinants of ELC values. We therefore conduct a Monte Carlo simulation based on historical data to estimate the optimal exercise levels and equilibrium values for several hypothetical ELCs.

3.2. Monte Carlo analysis

To best capture the tails of the return distribution of the underlying we pool historical intraday returns for all DAX stocks between January 2002 and December 2006, which we obtain from DataStream.[16] In total we compile 37,008 open-low, open-high, low-close, high-close and close-open returns. To make sure that all returns are genuine, we check the Factiva database to ascertain that all outliers pertain to real events, either market events such as 9/11 or idiosyncratic events such as earnings announcements. The annualized volatility of the pooled close-to-close returns was 33.2%. The excess kurtosis of daytime (nighttime) returns was 10.8 (26.0). To obtain a risk neutral probability distribution, we subtract the overall average return to obtain a sample mean of zero.

To value a typical ELC-long on a typical DAX stock, we set $D = €100$, $K = €105$ and $cs = 2\%$. The valuation occurs in two stages. First we search for the optimal exercise level by assuming an E^* and

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[15] We attribute the bank's commitment to conduct genuine best efforts, and to not unilaterally exercise, to the repeated game nature of the business. Clearly, an ELCs equilibrium value in a one shot game in which both the ELC holder and the issuer are allowed to liquidate at any time, equals the intrinsic value: if the value were higher, the issuer would terminate, and if the value were lower, the holder would exercise.

[16] In an earlier version of this paper we valued ELCs for each DAX-underlying separately, by using 15 years of historical returns. We found a large disparity between values. I.e., due to an overnight return of −29% in its history, we found one stock's ELC-longs to be significantly underpriced. Another stock did not see a single overnight return lower than -5%, suggesting that no ELC-long on it should not exist.
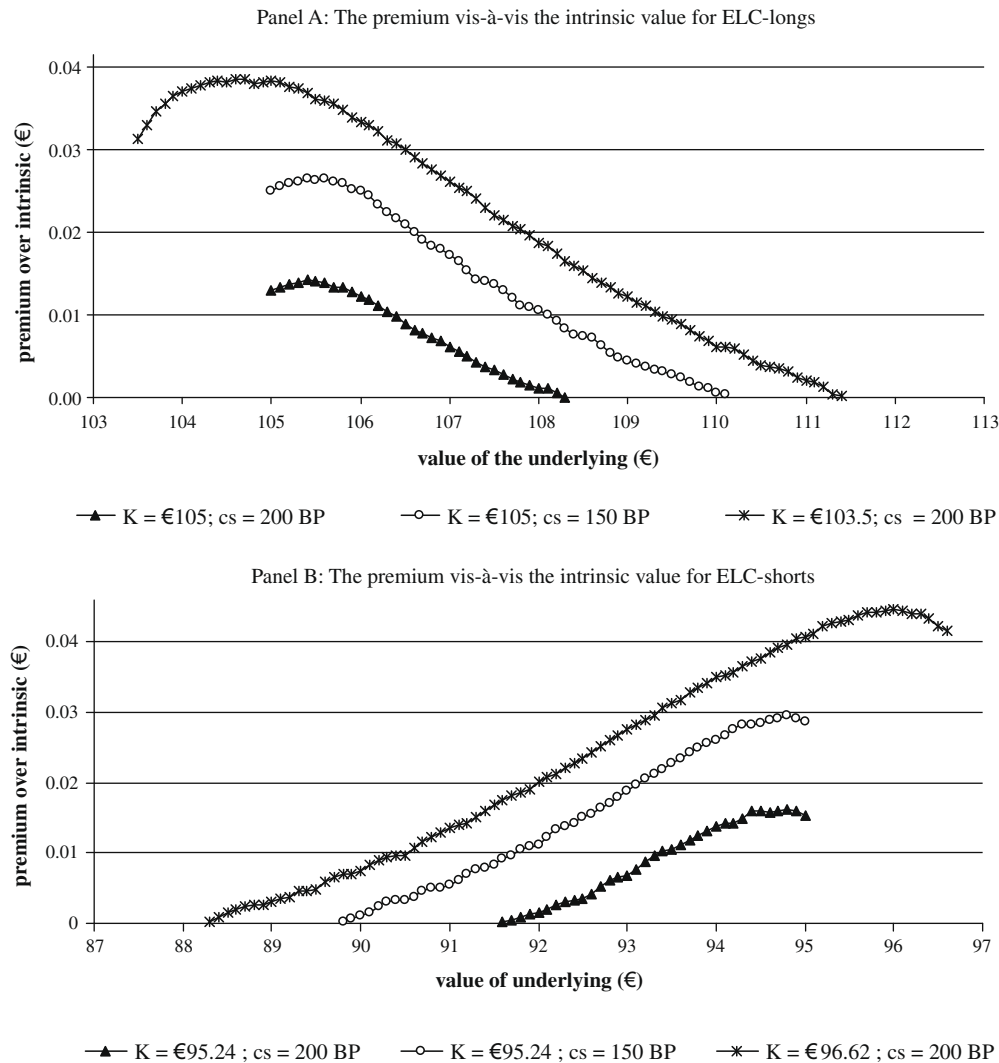
Panel A: The premium vis-à-vis the intrinsic value for ELC-longs

$$\blacktriangle \text{— } K = €105; \text{ cs} = 200 \text{ BP} \qquad \circ \text{— } K = €105; \text{ cs} = 150 \text{ BP} \qquad * \text{— } K = €103.5; \text{ cs } = 200 \text{ BP}$$

Panel B: The premium vis-à-vis the intrinsic value for ELC-shorts

$$\blacktriangle \text{— } K = €95.24 \text{ ; cs} = 200 \text{ BP} \qquad \circ \text{— } K = €95.24 \text{ ; cs} = 150 \text{ BP} \qquad * \text{— } K = €96.62 \text{ ; cs} = 200 \text{ BP}$$

Fig. 1. Monte Carlo estimated values for the ELC option value. Using pooled historical intraday returns of all Xetra-DAX stocks, we estimate the value of the option component of ELCs. We simulate the payoff to an ELC financing level of $D = €100$ that is collateralized by an underlying that is sold if it hits knockout level $K = €105$ or $€103.5$. D and K increase at a credit spread $cs = 2\%$ or 1.5%. We first search for the optimal exercise level E^*, at which the value of the option component is zero. We find optimal exercise levels E^* of $€108.40$, $€110.10$, and $€111.40$ for ELC-longs and $€91.50$, $€89.80$ and $€88.30$ for the shorts. We then use Monte Carlo simulation to evaluate the ELCs for underlying prices in the (K,E^*) intervals.

setting $S = E^*$. We then let the ELC *run*: we draw, with replacement, an overnight return and then an open-low/low-close return combination from the sample distribution. We repeat this until either a knockout or an exercise. We do this 100,000 times, and compute, for each run, the payoff to the loan. If the average payoff equals €100, the assumed E^* is the correct exercise level. The second stage consists of evaluating the ELC's option component for stock prices in the interval (K,E^*). We value ELC-shorts similarly, using the historical open-high/high-close return distribution. The appendix describes the simulation in more detail.

Fig. 1 gives the result of this analysis for several ELC-longs and shorts. Interestingly, we find that the value of the option component first increases and then decreases over the relevant range. Inspection shows that this is due to the fact that for values of S close to K, almost any adverse return will knock the ELC out, giving low probability to a gap crossing (loan default) event. As the distance to K increases, the probability of a gap crossing first increases before it decreases.

Our simulations suggest that an ELC-long with a knockout premium of 5% and credit spread of 200 basispoints written on a typical DAX stock has an optimal exercise value that lies 8.3% above

the financing level and an option component that is worth at most €0.014, or 0.28% of the intrinsic value. From our simulation we also find that the unconditional probability of a gap crossing is approximately 0.8%, the average recovery rate on loan defaults is 97.9%, and that the expected life of the typical ELC-long is 2.6 days.[17] The minor differences between the valuations of ELC-longs and ELC-shorts can be attributed to the shape of the tails of the return distribution. Fig. 1 also shows the results of a sensitivity analysis on the knockout level and the credit spread. Not surprisingly, ELC values are higher if the knockout level is set closer to the financing level and if the credit spread is lower.

Notice that our Monte Carlo valuation assumes that the observed returns of the past five years are representative for the future, and that the volatility is stationary. Because these assumptions are unlikely to be realistic, we also investigate how volatility affects ELC values. To do this we artificially increase the

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[17] The default probability, recovery rate, and life span depend on S. The default probability increases, then decreases on the interval (K,E^*). We report the average values over the (K,E^*)-interval. The maximum lifespan obtains when the stock trades at $S = €107.3$, approximately in between K and E^*.
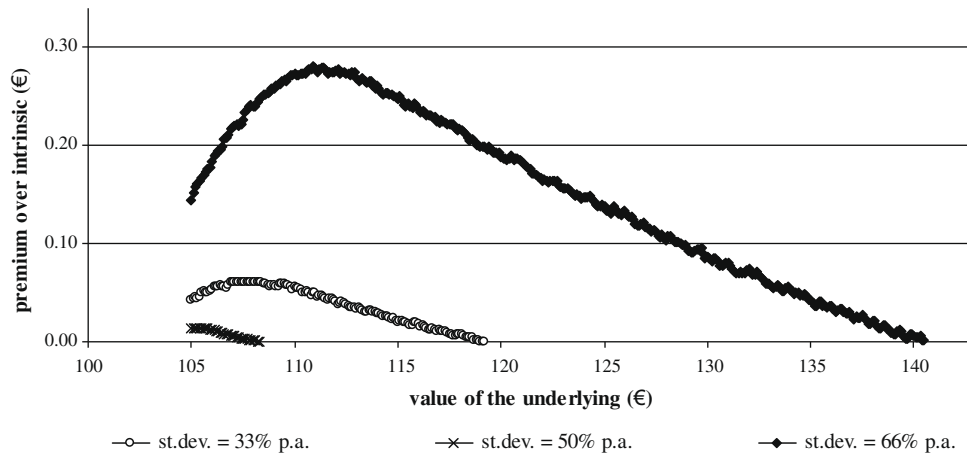
Fig. 2. ELC values and volatility. We use a Monte Carlo simulation to value an ELC-long with $D = €100$, knockout level $K = €105$ and a credit spread of 2%, for different levels of volatility of the underlying. We first search for the optimal exercise level, E^*, where the limited liability option is just offset by the credit spread, and then evaluate the ELC's value for underlying values in the (K, E^*) intervals. The base case valuation is based on sampling from the pooled historical returns of the 30 DAX stocks from January 2001 until December 2006. The annualized volatility (based on close-to-close returns) was 33%. To evaluate the ELC for different volatilities, we multiplied the sample returns by 1.5 and 2, respectively.

Table 1
ELC midquotes vis-à-vis intrinsic values.

| | Germany: 5129 ELCs on 30 DAX stocks | | | | | | Netherlands: 444 ELCs on 25 AEX-stocks | | | | | |
| | 3898 ELC-Longs | | | 1231 ELC-Shorts | | | 307 ELC-Longs | | | 137ELC-Shorts | | |
	Absolute €	%-Age of $\lvert S-D \rvert$	%-Age of underlying	Absolute €	%-Age of $\lvert S-D \rvert$	%-Age of underlying	Absolute €	%-Age of $\lvert S-D \rvert$	%-Age of underlying	Absolute €	%-Age of $\lvert S-D \rvert$	%-Age of underlying
Average	0.02	0.51	0.14	0.02	1.00	0.20	0.01	0.92	0.26	0.05	1.12	0.27
1%-Percentile	−0.56	−3.62	−1.09	−0.09	−3.65	−1.09	−0.19	−2.48	−0.65	−0.06	−1.21	−0.18
25%-Percentile	0.00	−0.10	−0.03	0.00	0.03	−0.03	0.00	0.00	0.00	0.01	0.19	0.05
Median	0.01	0.30	0.10	0.01	0.58	0.10	0.02	0.57	0.17	0.02	0.79	0.21
75%-Percentile	0.03	0.89	0.26	0.03	1.55	0.26	0.04	1.39	0.36	0.06	1.47	0.39
99%-Percentile	0.48	5.70	1.98	0.22	9.13	1.98	0.10	7.99	1.42	0.27	9.65	1.26
Standard deviation	0.20	1.57	0.41	0.08	2.23	0.41	0.06	1.87	0.50	0.08	1.68	0.30

We take five intraday snapshots of bid and ask quotes for 5129 ELCs written on German DAX stocks and 444 ELCs written on Dutch AEX stocks and match them with the contemporaneous bid and ask quotes of the underlying securities. For each snapshot, we compute the absolute overpricing as $AOP \equiv \frac{1}{2}(ELC_{ask} + ELC_{bid}) - \lvert\frac{1}{2}(S_{ask} + S_{bid}) - D\rvert$, the relative overpricing as $AOP / \lvert\frac{1}{2}(S_{ask} + S_{bid}) - D\rvert$, and the overpricing relative to the underlying as $AOP / (\frac{1}{2}(S_{ask} + S_{bid}))$.

standard deviation of our sampling distribution by multiplying all historical returns by 1.5 and 2, and repeat the simulation. Fig. 2 gives the results of this analysis.

Interestingly, the volatility has a very large effect on the value of an ELC's option component. If the underlying volatility is 66%, the ELC's option component as a proportion of the intrinsic value can be approximately 3% (this percentage holds when the underlying trades between €105 and €110, or if the lever lies between 20 and 10). The maximum expected lifespan in the high volatility case is 14.3 days.

4. Empirical analysis

For our empirical analysis we identify all ELCs written on the 30 Xetra-DAX stocks and the 25 AEX stocks offered by the ten largest issuers in January 2007. In total we find 5129 ELCs written on DAX stocks and 444 ELCs written on AEX stocks. We collect five quote snapshots during morning trading hours of different working days in January 2007, from German internet broker www.cortalcon-sors.de and its Dutch counterpart www.binck.nl, who provide simultaneous real time quotes of ELCs and their underlyings alongside the main ELC characteristics. We check the integrity of our data by checking for consistency between ELC quotes and underlying quotes. We also compare quotes with those reported by EU-WAX, Xetra, and Euronext. After deleting invalid quotes, we

arrive at a total of 23,672 ELC and underlying bid-ask quotes. All our empirical analyses are checked for robustness by computing statistics on each of the five snapshot samples separately (not reported).

4.1. Premiums over intrinsic values

To compare our Monte Carlo valuations with actual market prices, we compute the average differences between the ELC mid-quotes and their intrinsic values, based on the contemporaneous underlying midquotes. Table 1 shows the results of this analysis.

For the German market we find an average absolute premium of €0.019 and an average relative premium of 0.67%. For Dutch ELCs these numbers are €0.021 and 0.98%. Interestingly, we find premiums for ELC-shorts to be twice as high as for longs, for both markets. Our Monte Carlo analysis can explain the average premium vis-à-vis the intrinsic value but not the difference in premium between ELC-longs and shorts. A potential explanation is that retail investors, who are not allowed to short-sell, have a greater willingness to pay for ELC-shorts, while for issuers short positions are more expensive to maintain and monitor than long positions.[18]

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[18] Wilkens and Stoimenov (2007) find that also for Classical Turbos, shorts are priced higher than longs. Notice that our finding is inconsistent with counterparty risk, which would predict that ELC-shorts should be worth *less* than longs.

Table 2
Determinants of ELC premiums.

Regression model	ELC-longs		ELC-shorts	
	I	II	I	II
Lever	−0.52**	−0.58	1.22	1.15
	(0.24)	(0.39)	(1.07)	(1.17)
Knockout premium	1.52***	1.14	2.08***	2.21
	(0.23)	(1.37)	(0.45)	(1.91)
Annual volatility underlying (%)	0.57***		0.91**	
	(0.15)		(0.41)	
Issuer dummies		✔		✔
Day dummies		✔		✔
Underlying dummies		✔		✔
Observations	17,989	17,989	5683	5683
Adjusted R^2	0.023	0.365	0.010	0.192

This table gives the results of a regression analysis in which the dependent variable is the ELC premium over the intrinsic value as a proportion of the value of the underlying, expressed in basispoints. Observations were taken from five quote snapshots on different days in January 2007 of 3898 ELC-longs and 1231 ELC-shorts written on the 30 Xetra-DAX stocks, by the ten largest issuers. The lever is defined as $S/|S − D|$ and the knockout premium as $|K − D|/D$. The volatility of the underlying is the annualized volatility of daily returns computed over up to five years worth of historical data. White heteroscedasticity consistent standard errors are given in parentheses.
** Indicate significance levels of 5%, respectively.
*** Indicate significance levels of 1%, respectively.

To gauge how the main ELC characteristics affect ELC prices we regress the ELC premium, as a percentage of the underlying, on the lever, the knockout premium, and the volatility on the underlying. Due to gap risk we expect that ELC premiums increase in the lever and in the volatility of the underlying, and decrease in the knock-out premium. To control for underlying, issuer, and measurement day, we also regress on dummies for these variables. Table 2 shows the results of this regression analysis.

Consistent with option pricing theory, we find that premiums increase in the volatility of the underlying. Interestingly, we do not find significant coefficients on the lever. Part of this is due to the lack of cross-sectional variation in the relevant range: the majority of all listed ELCs had levers below ten, implying that their underlyings traded to the right of the typical E^* estimated in the previous section. Another explanation is that sell side competition is highest in the high lever segment of the market. Industry sources and trading volume observations suggest that this is indeed the case. Our finding that the coefficient on the lever is negative for longs but positive for shorts, gives a further hint that issuers are more reluctant to open levered short positions than levered long positions, possibly due to fears of insider trading.

The coefficient on the knockout premium is significantly *positive*, but only when we do not include underlying dummies in the regression. This suggests that knockout premiums are endogenous, and depend on the underlying's expected volatility. This is not very surprising as issuers often indicate that they set higher knockout levels for riskier stocks. We would however expect a negative coefficient on the knockout premium when we include underlying dummies in the regression. We attribute our failure to find negative coefficients to the limited cross-sectional variation in knockout premiums.

When we look at the range of levers in our sample of observed snapshots (not reported). In the previous subsection we saw that a typical ELC on a typical DAX stock should be exercised when its lever falls below 13. For stocks or periods with twice the volatility, typical ELCs should be exercised when their levers reach 3.5.[19] In our sample we find that 22% of all listed ELCs have levers below three (and are deep in the money). This hints that at least some

investors exit their positions too late. Notice however that we do not know how many ELC contracts of each listed series are outstanding. Industry sources tell us that, consistent with our analysis, the *open interest* dramatically decreases in the moneyness.

4.2. Bid-ask spreads

An important reason for the popularity of ELCs are the low round trip transaction costs in the secondary market. To assess the economic significance of transaction costs, we compare bid-ask spreads on ELCs with the spreads on the product's main competitors: covered warrants and exchange originated options. For the German market, the average ELC bid-ask spread for ELCs was €0.049, less than 1% of the security's midquote, and significantly lower than the average percentage spread for covered warrants, which we found to be higher than 6.5%. ELCs on Dutch stocks, were quoted with bid-ask spreads of 1.4%, while the average spread on Euronext options was 6.4%.

Part of this difference can be explained by the difference in leverage. To compare bid-ask spreads across derivatives, we define the *leverage adjusted spreads* as the euro spread divided by the value of the underlying claimed by the security. We compute the leverage adjusted spreads by multiplying the proportional spread by the instrument's ratio and hedge ratio. Table 3 shows the results of this analysis.

During our sample period, the average absolute bid-ask spread per unit of the underlying was €0.17 for German ELCs, and €0.26 for covered warrants. The average leverage adjusted spreads were 0.24% for ELCs and 0.46% for covered warrants. A comparison of medians and 25%- and 75%-percentiles shows a similar picture, and demonstrates that differences cannot be explained by particularly illiquid series in the samples. In the Netherlands we measure an average leverage adjusted spread of 0.35% for ELCs and 0.98% for Euronext Options. Our bid-ask spread analysis provides strong evidence that the secondary market for ELCs is at least as competitive as other derivative markets, and supports the market completion hypothesis.

We offer three explanations for the low bid-ask spreads on ELCs. First, we point at the contract design. Due to the right to exercise, a lower bound and first approximation for an ELC's value can be found by taking the midquote of the underlying and subtracting the observable financing level. There is no need to use complex nonlinear functions of, among others, the volatility and interest rate. Second, written ELCs are easy to hedge. Whereas options require dynamic hedging, ELCs require a single hedge transaction, which may lead to a reduction of the transaction cost component of bid-ask spreads. Third, we observe that the market is in its fast growth stage, and issuers aggressively chase market share. Industry sources confirm the strong rivalry between issuers and point out that the bid-ask spread is a key factor of competition.

Although spreads on ELCs are significantly lower than on other highly levered derivatives, ratio adjusted euro bid-ask spreads and leverage adjusted spreads for ELCs are still four times larger than the spreads on their underlying stocks. At first sight this suggests that unconstrained investors are better off trading directly in the underlying. However, due to brokerage commissions even unconstrained investors may prefer to trade ELCs instead of the underlying securities. If brokerage fees have a variable component, a round trip in an ELC will come with lower transaction costs than a round trip in the underlying. This brokerage costs advantage may well outweigh the higher bid-ask spread.

To better understand the determinants of the bid-ask spread, we regress leverage adjusted spreads on several ELC characteristics. The results of this analysis are given in Table 4.

Not surprisingly, we find that spreads increase in the contemporaneous bid-ask spread and volatility of the underlying. There is no

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[19] The critical levers can be found from Figs. 1 and 2: $108.3/8.3 \approx 13$, $140.5/40.5 \approx 3.5$.

Table 3
Bid-ask spreads of stocks, ELCs, and options.

		Germany: Xetra-DAX stocks					Netherlands: AEX-stocks				
		30 Stocks	5129 ELCs		18,003 Covered warrants		25 Stocks	444 ELCs		6141 Euronext options	
			Gross	Ratio adjusted	Gross	Ratio and delta adjusted		Gross	Ratio adjusted	Gross	Delta adjusted
Absolute spread (€-cent)	Average	6.2	4.9	17.3	2.1	25.7	2.4	6.7	8.5	11.5	18.4
	Min	1.0	1.0	1.0	0.2	1.0	1.0	1.0	1.0	5.0	5.0
	25%-Percentile	2.0	1.0	6.0	1.0	12.1	1.0	2.0	6.0	5.0	10.0
	Median	3.0	2.0	10.0	1.0	17.2	1.0	6.0	8.0	10.0	13.1
	75%-Percentile	5.0	5.0	20.0	2.0	25.4	3.0	10.0	10.0	15.0	21.0
	Max	150.0	150.0	200.0	22.0	462.9	13.0	30.0	30.0	45.0	291.5
	Stdev	16.4	10.9	25.1	2.7	45.9	2.6	5.9	4.5	6.8	17.2
			Gross	Leverage adjusted	Gross	Leverage adjusted		Gross	Leverage adjusted	Gross	Leverage adjusted
Relative spread (basispoints)	Average	6.4	97.7	24.3	655.3	46.4	8.7	143.5	35.0	644.5	97.8
	Min	1.3	2.6	1.1	7.9	2.1	1.7	25.9	2.9	26.4	2.6
	25%-Percentile	3.2	36.6	12.4	109.3	18.5	3.4	75.6	21.2	162.7	38.9
	Median	4.7	62.4	17.8	235.3	29.5	7.4	113.5	29.7	286.4	70.2
	75%-Percentile	6.8	115.1	26.2	606.1	50.8	10.5	181.8	40.4	555.6	114.0
	Max	37.9	1639.3	292.5	14782.6	4607.6	30.4	800.0	196.2	10000.0	827.2
	Stdev	6.0	112.5	24.2	1244.1	106.4	6.8	103.6	22.1	1198.2	95.0

We take 20 intraday snapshots of bid and ask quotes for 30 German DAX stocks and 25 Dutch AEX stocks, and five snapshots for all ELCs and covered warrants or options written on them. The snapshots were taken on five different days in January 2007. The *ratio adjusted* €-spread (in cents) for an ELC is the gross €-spread multiplied by the ratio. The *ratio and delta adjusted* €-spreads for the covered warrants and options are computed by multiplying the gross €-spread with the option's ratio (unity for all Euronext options) and the option's delta, which is computed with the Black–Scholes formula. In the bottom panel we list the proportional spreads. The gross proportional spreads are the spreads (ask minus bid) divided by the midprice of the derivative. The *leverage adjusted* spread is the gross spread divided by the lever. The lever for an ELC is given by $S/$

strong evidence that the leverage adjusted spread increases in the lever, suggesting that any increase in adverse selection in the high gearing segment is offset by an increase in sell side competition and a decrease in transaction costs due to economies of scale.[20] The positive coefficient on the knockout premium, and its disappearance when we include underlying dummies in the regression, provides further evidence that K is endogenous and a risk proxy.

4.3. Residual values

A key concern for investors is the fairness of ELC terminations. As mentioned, when an ELC (-long) breaches its knockout level, the issuer unwinds its hedge position and returns the proceeds of the liquidating sale minus the financing level to the ELC holders. The repeated game nature of the business, and the contractual and legal obligations regarding transparency should lead to genuine best efforts exerted by the issuer. To gauge whether knockout terminations are fair, we identify residual values for all DAX stock ELCs that were knocked out during the period January 2006 until May 2007, and for which financing and knockout values were available from their issuers' websites. In total we collect 1801 residual values from the four issuers that report D and K alongside the residual values. After confirming that all residual values concern knockout terminations (as supposed to delistings), we compute the residual value shortfalls as the differences between the reported residual values and the theoretical knockout values $|K - D|$. In Table 5 we describe the distribution of the observed residual value shortfalls.

The average residual value shortfall in our sample was 3.2%. At first sight, this seems a large loss for the ELC holders. However, ELCs are knocked out when their levers are at their highest. To put residual value shortfalls in perspective we also express them as percentage of the underlying, and divide them into daytime and overnight knockouts. We find an average shortfall as a percentage of the underlying of 0.21% for daytime knockouts and

Table 4
Determinants of leverage adjusted bid-ask spreads.

Regression model	ELC-longs		ELC-shorts	
	I	II	I	II
Bid-ask spread underlying (BP)	0.49[***]	0.07	0.45[***]	0.09
	(0.05)	(0.07)	(0.08)	(0.10)
Lever	0.36[**]	0.00	0.01	0.18
	(0.16)	(0.10)	(0.14)	(0.13)
Knockout premium	1.73[***]	−0.19	1.75[***]	0.92
	(0.12)	(0.15)	(0.19)	(0.95)
Annual volatility underlying (%)	0.52[***]		0.28[***]	
	(0.04)		(0.06)	
Issuer dummies		✔		✔
Day dummies		✔		✔
Underlying dummies		✔		✔
Observations	17,989	17,989	5683	5683
Adjusted R^2	0.121	0.701	0.315	0.777

This table gives the results of a regression analysis in which the dependent variable is the *leverage-adjusted* bid-ask spread of the ELCs. Observations are obtained from five quote snapshots on different days in January 2007 for 3898 ELC-longs and 1231 ELC-shorts written on the 30 Xetra-DAX stocks, by the ten largest issuers. The lever is given by $S/|S − D|$ and the knockout premium as $|K − D|/D$. The volatility of the underlying is the annualized standard deviation of daily returns computed over up to five years worth of historical data. White heteroscedasticity consistent standard errors are given in parentheses.
[**] Indicate a significance levels of 5%.
[***] Indicate a significance levels of 1%.

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[20] Biais and Hillion (1994) argue that highly levered derivatives attract more informed traders. Industry sources tell us that turnover is highly concentrated in the high-lever products.

Table 5
Residual value analysis.

	Entire sample			823 Longs			978 Shorts		
	Absolute shortfall €	Relative to |K − D| (%)	Relative to K (%)	Absolute shortfall €	Relative to |K − D| (%)	Relative to K (%)	Absolute shortfall €	Relative to |K − D| (%)	Relative to K (%)
Average	−0.024	−3.186	−0.237	−0.022	−2.673	−0.247	−0.024	−3.518	−0.231
Minimum	−1.86	−100.00	−11.04	−1.86	−77.01	−9.95	−0.68	−100.00	−11.04
Median	0.00	−0.52	−0.04	0.00	−0.33	−0.03	0.00	−0.64	−0.04
Maximum	0.36	48.37	3.74	0.36		3.74	0.36	48.37	2.55
Standard deviation	0.10	10.22	0.78	0.13	8.21	0.83	0.08	11.33	0.75
%-Age zero	50.47			51.03			50.00		
%-Age positive	8.66			9.84			7.67		

	143 Overnight triggers			1658 Daytime triggers		
	Absolute shortfall €	Relative to |K − D| (%)	Relative to K (%)	Absolute shortfall €	Relative to |K − D| (%)	Relative to K (%)
Average	−0.027	−8.56	−0.54	−0.024	−2.722	−0.211
Minimum	−1.86	−100.00	−9.95	−0.82	−35.15	−4.73
Median	0.00	−0.73	−0.04	0.00	−0.20	−0.02
Maximum	0.36	9.16	0.22	0.36	33.33	3.74
Standard deviation	0.13	9.42	1.06	0.10	10.29	0.76
%-Age zero	10.49			53.92		
%-Age positive	0.70			9.35		

	Residual value shortfall relative to K (%)			
	Bank 1	Bank 2	Bank 3	Bank 4
Average	−0.237	−0.238	−0.445	−0.109
Minimum	−9.95	−5.51	−11.04	−5.28
Median	−0.03	−0.08	−0.03	0.00
Maximum	3.74	2.30	2.55	0.22
Standard deviation	0.74	0.56	1.89	0.77
Percentage zero	48.98	41.23	37.63	93.53
Percentage positive	13.38	3.46	16.13	2.88
Observations	894	386	93	285

We collect 1801 residual values from the websites of four ELC issuers who report alongside the residual value, knockout level K and financing level D. We compute the differences between the reported residual value and the theoretical knockout value, |K − D| (times the ratio, if applicable). We describe the distribution of the residual value shortfalls for the entire sample and several subsamples. The "percentage zero" rows give the proportions of residual values that fell within a one €-cent rounding range around the theoretical knockout value.

0.54% for overnight knockouts. Explanations for the average daytime shortfall are the price jump that triggered the knockout, and the price impact from unwinding the hedge portfolio. The overnight shortfall can be explained by the lack of trade during this period and the strong kurtosis of overnight returns.

Our finding that in over half of all cases ELC investors received exactly the theoretical knockout value and that 8.7% of residual values were *higher* than their theoretical knockout values suggests that residual values are fair. Our finding that residual value shortfalls are slightly higher for ELC-shorts may be explained by larger upward jumps during the sample period, or a price impact that is larger for unwind buys than unwind sells, possibly due to a more aggressive unwinding of short positions.

We also check whether there are differences in residual value shortfalls between issuers. F-tests cannot reject that the issuers' average shortfalls have different means. The differences in standard deviations and percentages of zero shortfalls (lower panel of Table 5) are statistically significant, but can be readily explained by the differences in underlying securities, knockout premiums, and ratios.

It is interesting to note that our residual value data includes five gap crossings, all triggered by the same event: on May 4th 2007, pharmaceutical company Altana booked an overnight return of 33.15% and knocked out six ELC-shorts. Only one paid a residual value to the holders.[21] The ELC-long that came closest to a gap crossing was written on Deutsche Telekom. On 9 August 2006, when the stock closed at €12.05, an ELC with ratio 10 had a D of €10.83 and K of €11.70. On August 10th, Deutsche Telekom opened at €11.20, and quickly dropped to €10.90. Three days later, ELC holders received a residual value of €0.02, a mere 23% of the theoretical knockout value, which implied that the issuer unwound the position at an average price of €11.03 (€10.83 + 10 × €0.02).

5. Summary and conclusions

In this paper we theoretically and empirically analyze a novel and increasingly popular structured product, the Endless Leverage Certificate, which essentially is a levered long or short position with an attached margin account and a contractual knockout procedure. ELCs have several desirable properties. Due to the right to exercise, investors immediately observe a lower bound and a good approximation for the certificate's value: the intrinsic value. Because the value of the option component is very small, investors are easily reassured about the fairness of the pricing, and do not need to worry about implied volatilities or Greeks. An advantage for the issuer is the constant hedge ratio, which makes ELCs easy and cheap to hedge.

We derive a value function that takes into account limited liability and the associated gap risk, and use Monte Carlo simulation to value a typical ELC on a typical DAX stock. We find its value to be less than 0.3% higher than the intrinsic value. However, we also find that for periods or stocks with twice the typical DAX stock volatility, ELCs can be worth 3% more than their intrinsic value.

We then empirically investigate a large sample of ELC market quotes. We find that midquotes lie on average less than 1% above intrinsic values, and are higher for ELCs written on more volatile

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[21] The event was driven by dividend capture. On the day before the knockout event, Altana closed at €46.50, down 8.7%. The next day, the stock started trading net of a special dividend of €34.80. The stock opened at €16.99 and reached a daily high of €20.90.

stocks, which is consistent with our theoretical analysis. We find ELC bid-ask spreads in the secondary market to be very small. In fact, of all derivative products available to retail investors, ELCs offer the lowest bid-ask spreads, making ELCs attractive for leverage seeking retail investors. An analysis of residual values cannot reject that the unwinding of ELCs is fair, and shows that overnight knock-outs result in larger residual value shortfalls, which is consistent with gap risk.

Overall, our analysis offers support for the market completion hypothesis, which explains the popularity of ELCs by the competitive prices and low round trip transaction costs.

As an additional reason for their popularity we mention the certificate's desirable behavioral properties. First, the interests of the issuer and the investor are aligned: if an investor buys an ELC-long and the issuer hedges its position, both parties to the transaction hope that the underlying increases in value, as in this case the issuer holds an increasingly safe bond on which it earns a significant credit spread, while the ELC holder benefits via the equity component. Second, the limited liability feature of ELCs is attractive to investors with loss aversion. Thanks to the limited liability and the contractual knockout level, a portfolio of ELCs can wreak less havoc than a portfolio of equally levered positions with a shared margin account.

Acknowledgements

Appendix A. Illustration of valuing an ELC

Consider an ELC-long with $D = €100$ and $K = €105$, and a $cs = 0.0079\%$ (2% annualized). The underlying returns -0.5% or 0.5%, with equal probabilities, during daytime and nighttime. During nighttime the stock jumps up by 20% with 0.1% probability, or down, also with 0.1% probability. Returns are serially independent. The risk free interest rate is zero, and there are no dividends.

If we conjecture that a downward jump results in gap crossing, the limited liability option is worth $0.001 \times (100 - 0.8 \times S)$. To find E^* we can solve $€100 \times 0.0079\% = 0.001 \times (100 - 0.8 \times S)$.

To compute the value of the ELC, we make two simplifying assumptions: (1) K and E^* remain constant at €105 and €115; (2) Every day, the stock goes up or down with €1, both with probability α, does not move with probability 2α, or jumps with $1-4\alpha$, where $\alpha = 0.2495$. Define $s = S - 110$, and denote $P(s)$ the probability of a non-jump ending. We then have that:

$$P(s) = 2\alpha P(s) + \alpha P(s-1) + \alpha P(s+1).$$ (A1)

The characteristic polynomial of which is:

$$x^2 - \frac{1-2\alpha}{\alpha}x + 1,$$ (A2)

which has roots $\{r_1, r_2\} = \{0.9144, 1.0936\}$. Hence, we have:

$$P(s) = k(r_1^s + r_2^s) \quad \text{with} \quad k = 0.4538.$$ (A3)

The constant k is derived from the initial conditions $P(-5) = P(5) = 1$. Hence, we find that the probability of a jump ending if the underlying trades at €110 is $1 - P(0) = 9.24\%$.

To value the ELC, we also need the function for the expected life of the security, which we denote $E(s)$. We know that:

$$E(s) = 1 + 2\alpha E(s) + \alpha E(s-1) + \alpha E(s+1).$$ (A4)

After some algebra we find that the solution to this recurrence relation is given by:

$$E(s) = A - Cs^2 - Ks^4 \quad \text{with} \quad A = 46.2, C = 1.87, K = 0.00123.$$ (A5)

So that the expected life of the ELC when $S = €110$ is 46.2 days.

We now can compute the value of the ELC by subtracting the expected interest over these 46.2 days, €0.366, from the intrinsic value, and the expected loss on issuer's loan, €0.554, to arrive at an ELC value of €10.19.

Appendix B. Monte Carlo simulation

We estimate the option component of an ELC in two stages. First we look for the optimal exercise level E^*. Then we estimate the value of the option component, $D - L'(S'|D, K, E^*)$. To find E^*, we set $D_t = €100$ and $K_t = €105$. We try a E_t and set $S_t = E_t$. We then simulate 100,000 runs as follows:

1. Set $i = 0$.
2. Randomly pick, with replacement, an open-low return from the sample distribution, to compute S_{t+i}^{low}. If $S_{t+i}^{low} < K_{t+i}$, the loan-payoff equals D_{t+i}. Go to step 7.
3. To compute S'_{t+i} we use the low-close return that occurred on the same day for the same stock as the open-low return picked in 2). If $S'_{t+i} > E_{t+i}$, the loan payoff is D_{t+i}. Go to step 7
4. Set i to $i+1$. Let the critical levels increase: set $D_{t+i} = D_t e^{i \cdot cs}; K_{t+i} = K_t e^{i \cdot cs}; E_{t+i} = E_t e^{i \cdot cs}$.
5. Randomly pick, with replacement, an overnight return to compute S'_{t+i}. If $S'_{t+i} < K_{t+i}$, the loan payoff equals $\min(D_{t+i}, S'_{t+i})$. Go to step 7.
6. Go to step 2.
7. Record the loan payoff, the lifespan of the *ELC*, the kind of termination (overnight knockout, intraday knockout, or exercise). Go to step 1 to simulate another run.

After 100,000 'runs', we record the average loan payoff. If it is higher (lower) than €100, it means that the assumed E_t is too high (low). In this case we repeat the simulation with a lower (higher) E_t until we find the E_t for which the average discounted loan payoff is just €100. We denote this E_t the optimal exercise level E_t^*.

To find the value of the ELC option component as a function of the underlying, we run the same simulation, but with $E_t = E_t^*$, and let S_t vary between K_t and E_t^*. We also record the default probability, average life, and the recovery rates.

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