Information Theoretic Uplink Capacity of the Linear Cellular Array

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Abstract

In the information-theoretic literature, Wyner's model has been the starting point for studying the capacity limits of cellular systems. This simple cellular model was adopted and extended by researchers in order to incorporate flat fading and path loss. However, the majority of these extensions preserved a fundamental assumption of Wyner's model, namely the collocation of User Terminals (UTs). In this paper, we alleviate this assumption and we evaluate the effect of user distribution on the sum-rate capacity. In this context, we show that the effect of user distribution is only considerable in low cell-density systems and we argue that "collocated" models can be utilized to approximate the "distributed" ones in the high cell-density regime. Subsequently, we provide closed forms for the calculation of the interference factors of "collocated" models given the system parameters. In addition, the asymptotics of the per-cell sum-rate capacity are investigated. Finally, the presented results are interpreted in the context of practical cellular systems using appropriate figures of merit.

1. Introduction

The first concrete result for the information-theoretic capacity of the Gaussian Cellular Multiple Access Channel (GCMAC) was presented by Wyner in [7]. Using a very simple but tractable model for the cellular uplink channel, Wyner showed the importance of joint decoding at the BS receivers (hyper-receiver) and found the closed forms of the maximum system capacity under the assumption of multicell processing. This model triggered the interest of the research community in the cellular capacity limits and was extended in [5] to include flat fading environments. One major assumption shared in these two models was that the cell density is fixed and only physically adjacent cells interfere. Letzepis in [3] extended the model by assuming multiple-tier interference and incorporating a distance-dependent path loss factor in order to study the effect of cell density. However, the assumption of collocation of all UTs in a single cell was maintained to keep the model tractable.

In this paper, we extend these models in order to incorporate the effect of user distribution. Instead of assuming collocated UTs, we consider that UTs are spatially distributed within the cell and each channel gain is affected by a distance-dependent path loss factor. To relate the models involving collocated UTs with our model, we propose an approach to calculate (rather than arbitrarily vary) the equivalent interference factors to be used in models assuming collocated UTs. The calculation of the interference factors is based on the cellular system parameters, such as cell density, path loss exponent and user distribution. Finally, the presented results are interpreted in the context of practical cellular systems using appropriate figures of merit.

The rest of the paper is organised as follows. In the next section, we present the proposed model and we describe the derivation of the information theoretic capacity of the cellular system. In section 3, we show how the equivalent interference factors can be calculated, followed by the study of the asymptotic capacity of the system. In section 4, the presented results are interpreted in the context of practical cellular systems. The last section concludes the paper.

2 Channel Model and Analysis

In the following formulations, D is the coverage range of the linear cellular system, N is the number of BSs, K is the number of UTs per cell and η is the power-law path loss exponent. Under these assumptions, $\Pi = N/D$ represents the cell density of the cellular system, $\varnothing = \Pi^{-1} = D/N$ represents the cell diameter of the cellular system and $R = \varnothing/2$ represents the cell radius. Throughout this paper, $\mathbb{E}[\cdot]$ denotes the expectation, $(\cdot)^*$ denotes the complex conjugate, $(\cdot)^\dagger$ denotes the Hermitian matrix and $\odot$ denotes the Hadamard product. $\sqcap(t/T)$ is the rect function, where T is the width of the pulse. The figure of merit studied in this paper is the per-cell sum-rate capacity achieved with multicell joint decoding and it is denoted by C_{opt}.

The model under consideration is a linear cellular array

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under power-law path loss and flat fading. The analysis of the planar cellular array can be found in [1]. In this context, K UTs are uniformly distributed in each cell of a system comprising N base stations distributed in a linear segment of length D. The received signal at cell n, at time index i, is given by:

$$y^n[i] = \sum_{k=1}^{K} b_k^n[i] x_k^n[i] \quad (1)$$
$$+ \sum_{j=1}^{N/2} \sum_{k=1}^{K} \alpha_{kj}^n \left(c_{kj}^n[i] x_k^{n-j}[i] + d_{kj}^n[i] x_k^{n+j}[i] \right) + z^n[i]$$

where $x_k^n[i]$ is the ith complex channel symbol of the kth UT in the nth cell and $\{b_k^n\}, \{c_{kj}^n\}, \{d_{kj}^n\}$ are independent, strictly stationary and ergodic complex random processes in the time index i, which represent the flat fading processes experienced by the UTs. The fading coefficients are assumed to have unit power, i.e. $\mathbb{E}[\|b_k^n[i]\|^2] = \mathbb{E}[\|c_{kj}^n[i]\|^2] = \mathbb{E}[\|d_{kj}^n[i]\|^2] = 1$ and all UTs are subject to an average power constraint, i.e. $\mathbb{E}[\|x_k^n[i]\|^2] \leq P$ for all (n,k). The interference factors α_{kj}^n of the k^{th} UT in cell indexed by $n-j$ and $n+j$, are calculated according to the modified power-law path loss model [3, 4]:

$$\alpha_{kj}^n = \left(1 + d_{kj}^n\right)^{-\eta/2} \quad (2)$$

where d_{kj}^n is the distance between the nth BS and the k^{th} UT in cell indexed by $n-j$ or $n+j$. In the context of mathematical analysis, the distance d_{kj}^n can be calculated assuming that the UTs are distributed on a regular grid. Dropping the time index i, the aforementioned model can be more compactly expressed as a vector memoryless channel of the form $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$, where the vector $\mathbf{y} = [y^1 ... y^N]^T$ represents received signals by the BSs, the vector $\mathbf{x} = [x_1^1 \dots x_K^N]^T$ represents transmit signals by all the UTs of the cellular system and the components of vector $\mathbf{z}$=$[z^1 ... z^N]^T$ are i.i.d c.c.s. random variables representing AWGN with $\mathbb{E}[z^n] = 0$, $\mathbb{E}[\|z^n\|^2] = \sigma^2$. The channel Matrix $\mathbf{H}$ can be written as $\mathbf{H} = \mathbf{\Sigma} \odot \mathbf{G}$, where $\mathbf{\Sigma}$ is a $N \times KN$ deterministic matrix and $\mathbf{G} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is a complex Gaussian $N \times KN$ matrix, comprising the corresponding fading coefficients. The entries of the $\mathbf{\Sigma}$ matrix are defined by the variance profile function

$$\varsigma(u,t) = \left(1 + d(u,t)\right)^{-\eta/2} \quad (3)$$

where $u \in [0,1]$ and $t \in [0,K]$ are the normalized indexes for the BSs and the UTs respectively and $d(u,t)$ is the normalized distance between BS u and UT t.

According to [6], the asymptotic sum-rate capacity C_{opt} for this model, is given by

$$\begin{aligned}\lim_{N\to\infty} C_{\text{opt}} &= \lim_{N\to\infty} \frac{1}{N} \mathcal{I}(\mathbf{x}; \mathbf{y} \mid \mathbf{H}) \\ &= \lim_{N\to\infty} \mathbb{E}\left[\frac{1}{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{N}} \log\left(1 + \frac{\tilde{\gamma}}{K} \lambda_i \left(\frac{1}{N}\mathbf{H}\mathbf{H}^\dagger\right)\right)\right] \\ &= \int_0^\infty \log\left(1 + \frac{\tilde{\gamma}}{K} x\right) d\mathrm{F}_{\frac{1}{N}\mathbf{H}\mathbf{H}^\dagger}(x) \\ &= \mathcal{V}_{\frac{1}{N}\mathbf{H}\mathbf{H}^\dagger}\left(\frac{\tilde{\gamma}}{K}\right) = K \mathcal{V}_{\frac{1}{N}\mathbf{H}^\dagger\mathbf{H}}\left(\frac{\tilde{\gamma}}{K}\right) \quad (4)\end{aligned}$$

where $\tilde{\gamma} = KN\gamma$ and $\gamma = P/\sigma^2$ are the system- and UT-transmit power normalized by the receiver noise power respectively, $\lambda_i(\mathbf{X})$ denotes the eigenvalues of matrix $\mathbf{X}$ and

$$\begin{aligned}\mathcal{V}_{\mathbf{X}}(y) &\triangleq \mathbb{E}[\log(1 + y\lambda_i(\mathbf{X}))] \\ &= \int_0^\infty \log(1 + y\lambda_i(\mathbf{X})) \, d\mathrm{F}_{\mathbf{X}}(x) \quad (5)\end{aligned}$$

is the Shannon transform [6] of a random square Hermitian matrix $\mathbf{X}$, whose limiting eigenvalue distribution has a cumulative function denoted by $F_{\mathbf{X}}(x)$. For a Gaussian matrix $\mathbf{G} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, the empirical eigenvalue distribution of $\frac{1}{N}\mathbf{G}^\dagger\mathbf{G}$ converges almost surely (a.s.) to the nonrandom limiting eigenvalue distribution of the Marčenko-Pastur law, whose Shannon transform is given by

$$\mathcal{V}_{\mathbf{H}^\dagger\mathbf{H}}(y) \xrightarrow{a.s.} \mathcal{V}_{\text{MP}}(y, K) \quad (6)$$

where $\mathcal{V}_{\text{MP}}(y,K) = \log\left(1 + y - \frac{1}{4}\phi(y,K)\right)$

$$+ \frac{1}{K}\log\left(1 + yK - \frac{1}{4}\phi(y,K)\right) - \frac{1}{4Ky}\phi(y,K) \quad (7)$$

and $\phi(y,K) =$

$$\left(\sqrt{y\left(1+\sqrt{K}\right)^2 + 1} - \sqrt{y\left(1-\sqrt{K}\right)^2 + 1}\right)^2. \quad (8)$$

2.1 Marčenko-Pastur Law Approximation

According to the Marčenco-Pastur Law approximation in [3], if $\mathbf{\Sigma}$ is a path loss dependent $N \times KN$ deterministic matrix, the limiting eigenvalue distribution of $\mathbf{H}^\dagger\mathbf{H}$ and its Shannon transform can be approximated by a scaled version of the Marčenko-Pastur law

$$\mathcal{V}_{\mathbf{H}^\dagger\mathbf{H}}(\gamma) \simeq \mathcal{V}_{\text{MP}}\left(q_K(\mathbf{\Sigma})\frac{\tilde{\gamma}}{K}\right) \quad (9)$$

where $q_K(\mathbf{\Sigma}) \triangleq \|\mathbf{\Sigma}\|^2 / KN^2$ with $\|\mathbf{\Sigma}\| \triangleq \sqrt{tr\{\mathbf{\Sigma}^\dagger\mathbf{\Sigma}\}}$ being the Frobenius norm of the $\mathbf{\Sigma}$ matrix. In the asymptotic case and noticing the row-regularity [6, Def. 2.10] of

$$\sum_{j=1}^{M}\alpha_j^2=\frac{N}{K}\int_0^{K/2}\varsigma^2\left(\sum_{j=1}^{M}\left(\frac{1}{2\Pi}F_u^{-1}\left(N\frac{2}{K}t_j\right)+\frac{1}{\Pi}j\right)\sqcap\left(\frac{t_j}{K/N}\right)\right)dt-\frac{1}{2},\text{ where } t_j=t-j\frac{K}{N} \tag{A}$$

$\mathbf{\Sigma}$ matrix, $q_K(\mathbf{\Sigma})$ is given by

$$\lim_{N\to\infty} q_K(\mathbf{\Sigma})=\frac{1}{K}\int_0^K\varsigma^2(t)dt,\forall r\in[0,1]. \tag{10}$$

According to [3], this approximation holds for UTs collocated with the BS. In [1] we show that this approximation also holds for the case where the UTs are distributed within the cells. In this paper, we show how this model can be used to calculate the appropriate values of the path loss factors used in the collocated-UTs models. We also study the asymptotic behaviour of the cellular capacity using this model and we interpret the presented results in the context of practical cellular systems.

3 Results

Without loss of generality, the linear cellular array can be considered circular and Equation (10) can be further simplified to

$$\lim_{N\to\infty} q_K(\mathbf{\Sigma})=\frac{2}{K}\int_0^{K/2}\varsigma^2\left(t\right)dt. \tag{11}$$

User distribution effectively alters the distance d_{kj}^n and therefore it modifies the variance profile function and the resulting sum-rate capacity given by Equation (4) and (9).

Theorem 1 (from [1]). *Let us assume that the transmitters of each cell are positioned on a grid generated according to an invertible Cumulative Distribution Function (CDF) $F_u(r)$, where $r\in[0,1]$ corresponds to the normalized single-cell distance from the BS. The variance profile function $\tilde{\varsigma}(t)$ w.r.t. the normalized index t of the distributed UTs is given by*

$$\tilde{\varsigma}(t)=\varsigma\left(\sum_{i=-\frac{N}{2}}^{\frac{N}{2}}\left(\frac{1}{2\Pi}\tilde{F}_u^{-1}\left(N\frac{2}{K}t_i\right)+\frac{1}{\Pi}i\right)\sqcap\left(\frac{t_i}{K/N}\right)\right)$$
$$\tilde{F}_u^{-1}(r)=\begin{cases}F_u^{-1}(r) & r>0\\ -F_u^{-1}(-r) & r<0\end{cases}\ \textit{and}\ t_i=t-i\frac{K}{N} \tag{12}$$

where $t\in[0,K/2]$.

3.1 Interference Factors

On the grounds of Theorem 1, the interference factors α_j in the high cell density regime can be calculated based on the cellular system parameters, namely the number of BSs N , the number of UTs per cell K, the power-law path loss exponent η, the cell density Π and the user distribution CDF $F_u(d)$.

Corollary 1. *In the high cell density regime, the "distributed" cellular model can be represented by the "collocated" one, using the interference factors α_j given by Equation* (A) *at the top of the page, where $M\in[1,N/2]$ denotes the number of interfering neighboring cells taken into account.*

Proof. Based on the Marčenko-Pastur approximation in [3] of Somekh-Shamai's model [5], the sum rate capacity is given by

$$\lim_{N\to\infty} C_{opt}\simeq K\mathcal{V}_{\mathrm{MP}}\left(\left(1+2\alpha^2\right)\tilde{\gamma}/K\right). \tag{13}$$

By considering interference from M tiers and by following the same derivation as in [3], it can be easily proved that the sum rate capacity is given by

$$\lim_{N\to\infty} C_{opt}(\gamma)\simeq K\mathcal{V}_{\mathrm{MP}}\left(\left(1+2\sum_{j=1}^{M}\alpha_j^2\right)\frac{\tilde{\gamma}}{K}\right). \tag{14}$$

By combining Equations (13) and (14), the interference factors can be calculated by using Equation (A) recursively. □

As shown in [1], the sum-rate capacities for the "collocated" and the "distributed" models converge in the high cell density regime. Therefore, the "collocated" model can be used to approximate the "distributed" one, by approximating the interference factors α_j using Equation (2)

$$\alpha_j\approx(1+j/\Pi)^{-\eta/2}. \tag{15}$$

In the low-cell density regime, these approximations do not hold, since user distribution affects the produced sum-rate capacity. However, due to the fast decay of the path-loss coefficients, a simplified model with a single interfering tier can be considered.

3.2 Asymptotics of Sum-rate Capacity

In order to study the asymptotics of the per-cell sum-rate capacity, the cell density $\Pi=N/D$ and the cell diameter $\varnothing=\Pi^{-1}=D/N$ of the cellular system are kept constant,

Table 1. Value/Range of parameters for practical cellular systems

Parameter	Symbol	Value/Range
Cell Radius	R	$0.1 - 3\,km$
Reference Distance	d_0	$1\,m$
Path Loss at ref. distance	L_0	38 dB
Path Loss Exponent	η	$\{2, 3.5\}$
UTs per cell	K	20
UT Transmit Power	P_T	$100 - 200\,mW$
Thermal Noise Density	N_0	$-169\,dBm/Hz$
Channel Bandwidth	B	$5\,MHz$

while both N and D grow large. Considering the uniform user distribution, the sum-rate capacity is given by

$$\lim_{N,D\to\infty} C_{opt}(\gamma) \overset{(a)}{=} \lim_{N,D\to\infty} K\mathcal{V}_{\mathrm{MP}}\left(\frac{2}{K}\int_0^{K/2} \varsigma^2(t)\,dt \cdot \frac{\tilde{\gamma}}{K}, K\right) \overset{(b)}{=} \lim_{N\to\infty} K\mathcal{V}_{\mathrm{MP}}\left(\frac{2}{K}\int_0^{K/2}\left(1+\varnothing\frac{N}{K}t\right)^{-\eta} dt\cdot\frac{\tilde{\gamma}}{K}, K\right) \quad (16)$$

where (a) follows from Equations (4), (9) and (11) and (b) follows from Equation (12). If γ is finite, then

$$\lim_{N,D\to\infty} C_{opt}(\gamma) = K\mathcal{V}_{\mathrm{MP}}\left(\frac{\gamma}{(\eta-1)R}, K\right). \quad (17)$$

According to [6], the asymptotic of the Shannon transform for $K > 1$ is given by

$$\lim_{x\to\infty} K\mathcal{V}_{\mathrm{MP}}(y) = \log(Ky) - (K-1)\log(K-1/K) - 1. \quad (18)$$

Therefore, for large values of γ the asymptotic sum-rate capacity is given by combining Equations (16) and (18),

$$\lim_{N,D,\gamma\to\infty} C_{opt} = \log\left(\frac{\gamma K}{(\eta-1)R}\right) - (K-1)\log(K-1/K) - 1. \quad (19)$$

Furthermore, the asymptotic sum-rate capacity for a very large number of UTs per cell converges to

$$\lim_{N,D,\gamma,K\to\infty} C_{opt}(\gamma) = \log\left(\frac{\gamma K}{(\eta-1)R}\right) \quad (20)$$

since $\lim_{K\to\infty}\left(1+\frac{1}{K}\right)^K = e$.

4 Practical Considerations

The employed power-law path loss model of Equation (3) provides a variance profile coefficient as a function of the normalized distance $d(t)$. Similar path-loss models have been already utilized in the information-theoretic literature [3, 4]. However, in order to apply the aforementioned results to real-world cellular systems, a reference distance d_0 is required to interconnect the normalized distance $d(t)$ and the actual distance $\hat{d}(t)$. Assuming that the power loss at the reference distance d_0 is L_0, the scaled variance profile function is given by

$$\varsigma(d(t)) = \sqrt{L_0\left(1+\hat{d}(t)/d_0\right)^{-\eta}}. \quad (21)$$

In the context of a macro-cellular scenario, the typical parameters of Table 1 will be considered. Figures 1 and 2 depict the per-cell capacity of the linear cellular system versus the cell radius R and the UT transmit power P_T respectively.

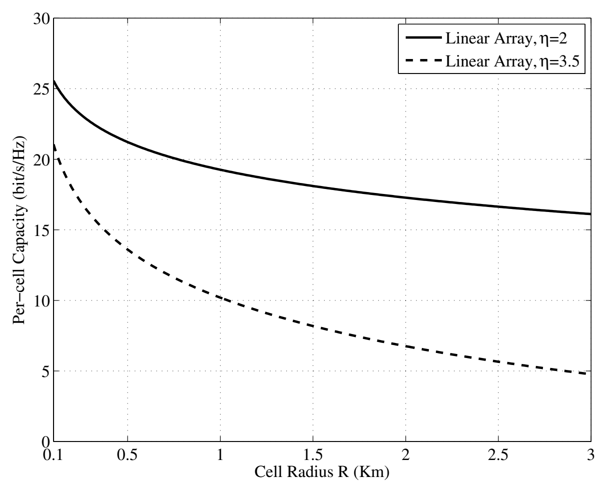

Figure 1. Per-cell capacity (bit/s/Hz) vs. cell radius R for the linear cellular system. Parameters: $\eta = \{2, 3.5\}$ and $P_T = 0.2\,W$.

4.1 Figures of Merit

In the practical engineering design of cellular systems, the main figure of merit that determines the capacity rate of a UT is the $SINR = \frac{P_R}{I+N_R}$ where P_R is the received power at the BS of interest, N_R is the thermal AWGN at the receiving BS and I is the inter-cell and intra-cell interference received from other UTs of the system. However, in the information-theoretic analysis of multicell processing systems, the main figure of merit that determines the per-cell capacity is $\gamma = \frac{P_T}{N_{HR}}$, where P_T is the transmit power of the UT and N_{HR} is the AWGN thermal noise at the hyper-receiver. The main reason that $SINR$ does not constitute

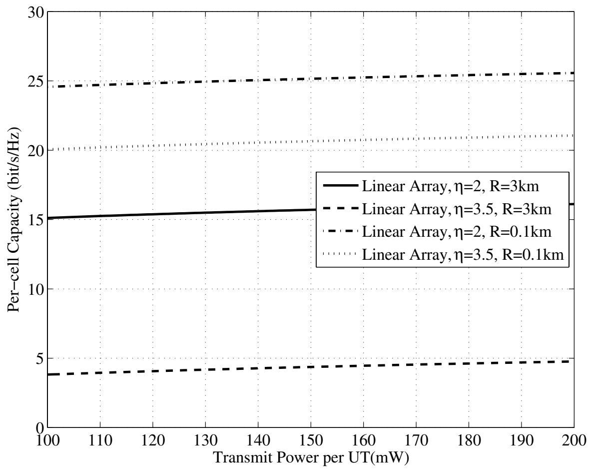

Figure 2. Per-cell sum-rate capacity (bit/s/Hz) vs. UT transmit power P_T (mW) for the linear cellular system. Parameters: $\eta = \{2, 3.5\}$ and $R = \{100m, 3Km\}$

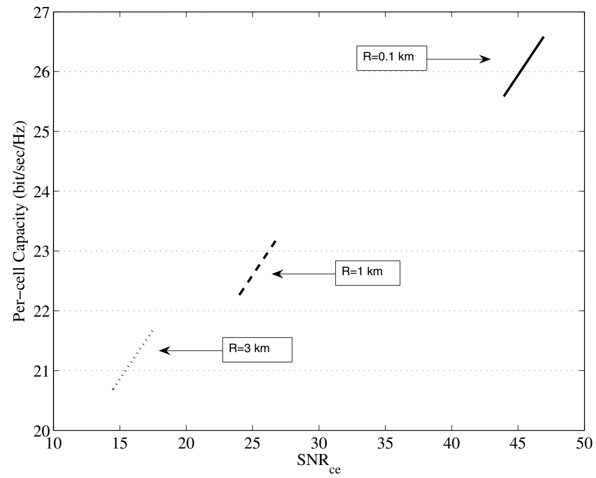

Figure 3. Per-cell sum-rate capacity (bit/s/Hz) vs. the cell-edge SNR. Parameters: $\eta = 2$ and $P_T = 100 - 200mW$.

an appropriate figure of merit for multi-cell joint processing analysis is that inter-cell interference is not harmful and thus the term I can be added to the nominator. Since there is no harmful interference, there is no need for power control and thus the UTs constantly transmit with the maximum available power P_T [2, Proposition 6]. In this context, the transmit power P_T remains fixed for all the UTs, whereas the received power differs for each UT. In addition, since the objective function is the per-cell capacity, the power variable affecting the value of this function should have a constant value throughout the whole cell. Taking this into account, it is reasonable to calculate the per-cell capacity as a function of P_T or γ, which is a fixed system parameter, common for all the UTs of a cell. In this context, three approaches which are described in the following paragraphs can be employed. For each approach, the per-cell capacity will be evaluated based on Equation (20) for the aforementioned macro-cellular scenario.

4.1.1 Cell-edge SNR

In practical engineering design of cellular systems, the objective is to provide network coverage to all the subscribers. Therefore, the cellular system has to be designed in a way that it even allows cell-edge UTs to communicate effectively with the receiving BS. Thus, it would be reasonable to consider the cell-edge SNR as the figure of merit that determines the per-cell capacity. Assuming that $N_{HR} = N_R = N_c$, the cell-edge SNR can be defined as:

$$SNR_{CE} = \gamma L_0 \left(1 + R\right)^{-\eta}. \tag{22}$$

Figure 3 depicts the per cell capacity vs. cell-edge SNR.

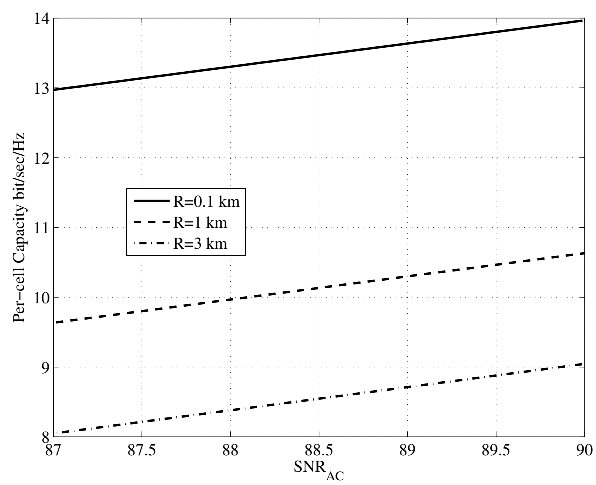

Figure 4. Per-cell sum-rate capacity (bit/s/Hz) vs. the average cell SNR.Parameters: $\eta = 2$ and $P_T = 100 - 200mW$.

4.1.2 Average cell SNR

A second figure of merit which could be used to determine the per-cell capacity is the average cell SNR, which is defined as the average of the received SNRs of all the UTs in a cell. Assuming that $N_{HR} = N_R = N_c$ and uniformly distributed UTs, the cell-edge SNR can be defined as

$$SNR_{AC} = 2\gamma L_0 \int_0^R (1+r)^{-\eta}\, dr. \qquad (23)$$

Figure 4 depicts the per cell capacity vs. average cell SNR.

4.1.3 Rise over Thermal

In multicell processing systems, the Rise over Thermal (RoT) is defined as the ratio of the total signal power received from all the UTs of the system at a single BS to the thermal AWGN. More specifically, assuming uniformly distributed UTs, RoT is given by:

$$RoT = 2\gamma L_0 \int_0^{D/2} (1+r)^{-\eta}\, dr. \qquad (24)$$

For an infinite cellular array, the coverage span D grows to infinity and therefore

$$RoT = 2\gamma L_0 \int_0^{\infty} (1+r)^{-\eta}\, dr = \frac{2\gamma L_0}{n-1}. \qquad (25)$$

Figure 5 depicts the per cell capacity vs. RoT. The RoT curves (thick lines) have been drawn on top of the $\log(1+x)$ curve (thin line).

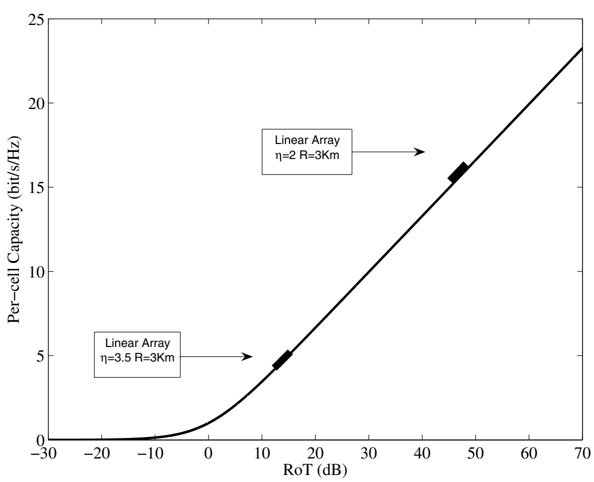

Figure 5. Per-cell sum-rate capacity (bit/s/Hz) vs. RoT (dB) for the linear cellular system. Parameters: $\eta = \{2, 3.5\}$ **and** $P_T = 100 - 200mW$**.**

5 Conclusion

The already existing information-theoretic models for cellular systems are based on the assumption that the UTs of each cell are collocated. In this paper, we have investigated the optimal information-theoretic capacity under the assumption of distributed UTs. Based on the presented results, we can conclude that a cellular model assuming distributed UTs can be approximated by a model assuming collocated UTs only in the high cell density regime. In this case, we have proposed an approach for calculating the interference factors of the "equivalent" collocated model based on the system's parameters. Furthermore, the asymptotic cellular capacity has been studied and plotted by varying the path loss exponent and the UT transmit power. Finally, the presented results were interpreted in the context of practical cellular systems using appropriate figures of merit, such as Rise over Thermal.

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