On the Ergodic Capacity of the Wideband MIMO Channel

Symeon Chatzinotas, Muhammad Ali Imran, Reza Hoshyar
Centre for Communication Systems Research
University of Surrey, United Kingdom, GU2 7XH
Email: {S.Chatzinotas, M.Imran, R.Hoshyar}@surrey.ac.uk

***Abstract*—In the ergodic capacity literature, the majority of results preserve the assumption of a flat-fading Gaussian channel. However, the actual wireless channels in current communication systems are often wideband and therefore time dispersive and frequency selective. In this direction, we study the ergodic capacity of a wideband MIMO channel. Starting from a tapped delay line model for the time domain, the frequency domain model is derived by employing the Fourier transform. However, in both cases the resulting channel matrix is characterised by non-separable correlation amongst the Gaussian blocks. The asymptotic eigenvalue probability density function and the channel capacity are calculated by specializing a theorem originating in operator-valued free probability. Finally, numerical results are presented to verify the validity of the approach and study the effect of the channel model parameters.**

I. Introduction

Across the information-theoretic literature, it has been widely proven that multiple antennas are able to increase the link throughput by providing a multiplexing gain which scales with the number of antennas [1]. Similarly, in the context of multiuser channels, the sum-rate capacity of a MIMO multiple-access channel was investigated in [2], [3]. In both cases, it was shown that the capacity scales linearly with $\min(n_R, n_T)$, where n_R and n_T is the total number of receive and transmit antennas respectively. However, the majority of results focus on flat-fading channels, which is a valid assumption for narrowband channels. Nevertheless, current wireless technologies have adopted wideband channels, which are characterized by time dispersion and frequency selectivity. Time dispersive channels can be modelled by tapped delay line models and subsequently can be converted in the frequency domain by employing the Fourier transform. However, as shown in the channel modelling section this approach induces a non-separable correlation in the channel matrix. This kind of correlation is characterized "non-separable" in contradiction to the "separable" case, which can be modelled as the product of a receive correlation matrix, a Gaussian matrix and a transmit correlation matrix [4], [5], [6]. The capacity scaling of the wideband MIMO channel was also studied in [7] based on the principles of virtual representation. In our approach, we specialize a theorem originating in operator-valued free probability in order to calculate the asymptotic eigenvalue probability density function of the channel matrix and consequently the ergodic channel capacity.

The remainder of this paper is organized as follows. Section II defines the time- and frequency domain representation of the wideband MIMO channel. Subsequently, section III focuses on the analysis of the non-separable correlation model and section IV presents some numerical results in order to verify the validity of the approach and study the effect of the channel model parameters. Section V concludes the paper.

II. Channel Model

The current model considers a MIMO channel with n_T transmit and n_R receive antennas. The communication channel is wideband and thus it is characterized by time dispersion and frequency selectivity. Starting from a tapped delay line model for the time domain, the frequency domain model is derived by employing the Fourier transform. In this context, N_c denotes the number of flat-faded narrowband subcarriers in the frequency domain and the number of frame symbols in the time domain. It should be noted that the main part of the analysis focuses on equal number of transmit and receive antennas $n_T = n_R$, while the case of $n_T \neq n_R$ is considered in subsection III-A3.

A. Time-Domain Wideband Single-Link Channel

Let us first consider a single-antenna link, which transmits in wideband mode. Due to the time dispersion, the received signal at a time index m is given by:

$$y[m] = \sum_{l=1}^{L} h_l^t[m]x[m-(l-1)] + n[m], \tag{1}$$

where L is a finite number of independent Rayleigh faded channel taps with independent complex circularly symmetric (c.c.s.) gains $h_l^t,\ \forall l = 1 \ldots L$, $x[m]$ denotes the symbol transmitted at time index m and n represents the Additive White Gaussian Noise (AWGN). The variance v_l of h_l^t decays as $l \to L$ under the normalization

$$\sum_{l=1}^{L} \mathbb{E}\left[h_l^t \left(h_l^t\right)^{\dagger}\right] = \sum_{l=1}^{L} v_l^2 = 1. \tag{2}$$

Assuming that L is smaller than N_c, then the time-domain channel matrix $\mathbf{H}_t$ over a frame size N_c can be written as a

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square matrix with dimensions $N_c \times N_c$:

$$\mathbf{H}_t = \begin{bmatrix} h^t_L & h^t_{L-1} & \cdots & h^t_1 & 0 & \cdots & 0 \\ 0 & h^t_L & \cdots & h^t_2 & h^t_1 & \cdots & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ h^t_{L-1} & h^t_{L-2} & \cdots & 0 & 0 & \cdots & h^t_L \end{bmatrix} \quad (3)$$

It should be noted that the channel taps' gains are preserved for the duration of the frame. Furthermore, the matrix $\mathbf{H}_t$ appears as circulant, after employing a cyclic prefix on the transmitted symbols [8]. The channel model can be compactly expressed as:

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{n}_t, \quad (4)$$

where $\mathbf{x}_t = [x[2-L] \ldots x[N_c - L + 1]]^T$ and $\mathbf{y}_t = [y[1] \ldots y[N_c]]^T$ are the transmitted and received symbols respectively and $\mathbf{n}_t$ is the AWGN vector. It is assumed that no input optimization is performed and thus the available transmit power P is uniformly allocated across the N_c transmitted frame symbols.

B. Frequency-Domain Wideband Single-Link Channel

Let us now consider a single antenna link, which transmits over N_c flat-faded narrowband subcarriers with bandwidth $B_c = B/N_c$,where B is the total communication bandwidth. The frequency-selective channel model can be compactly expressed as:

$$\mathbf{y}_f = \mathbf{H}_f \mathbf{x}_f + \mathbf{n}_f, \quad (5)$$

where $\mathbf{x}_f$ and $\mathbf{y}_f$ are the transmitted and received subcarriers symbols respectively and $\mathbf{n}_f$ is the AWGN vector. The $N_c \times N_c$ frequency-domain channel matrix $\mathbf{H}_f$ is defined in accordance with the time-domain channel as follows

$$\mathbf{H}_f = \begin{bmatrix} h^f_1 & 0 & \cdots & 0 \\ 0 & h^f_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h^f_{N_c} \end{bmatrix}, \quad (6)$$

where the i^{th} diagonal entry is given by the Fourier transform:

$$h^f_i = \sum_{l=1}^{L} h^t_l \exp\left(\frac{-\mathrm{j}2\pi (i-1)(l-1)}{N_c} \right). \quad (7)$$

As it can be seen, the subcarrier flat-fading coefficients h^f_i are correlated due to (7). It is assumed that no input optimization is performed and thus the available power P is uniformly allocated across the N_c transmitted subcarrier symbols.

C. Wideband MIMO Channel

Considering $n_R = n_T$ multiple receive and transmit antennas, the wideband MIMO channel can be written in the form:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \quad (8)$$

where the vector $\mathbf{Y} = [\mathbf{y}^{(1)} \ldots\ \mathbf{y}^{(n_R)}]^T$ with $\mathbf{y}^{(n)} = [y^1 \ldots\ y^{N_c}]$ represent the received signal vectors, the vector $\mathbf{X} = [\mathbf{x}^{(1)} \ldots\ \mathbf{x}^{(n_T)}]^T$ with $\mathbf{x}^{(n)} = [x^1 \ldots\ x^{N_c}]$ represents transmitted signal vectors and the components of vector $\mathbf{Z}$=$[\mathbf{z}^{(1)} \ldots\ \mathbf{z}^{(n_R)}]^T$ with $\mathbf{z}^{(n)} = [z^1 \ldots\ z^{N_c}]$ contain independent identically distributed (i.i.d) c.c.s. random variables representing AWGN. Considering that N_c, L are preserved for all transmit-receive antenna pairs, the $n_R N_c \times n_T N_c$ channel matrix can be expressed as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,n_T} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{n_R,1} & \cdots & \cdots & \mathbf{H}_{n_R,n_T} \end{bmatrix}, \quad (9)$$

where $\mathbf{H}_{i,j}$ is the channel matrix of the link between the ith receive antenna and the jth transmit antenna. For the time-domain analysis $\mathbf{H}_{i,j}$ is considered to be an instance of $\mathbf{H}_t$, while for the frequency-domain analysis an instance of $\mathbf{H}_f$. As mentioned before, the available power P is allocated uniformly across the n_T transmit antennas and across the N_c transmitted symbols and therefore $\mathbb{E}\left[\mathbf{x}_t \mathbf{x}_t^\dagger\right] = \mathbb{E}\left[\mathbf{x}_f \mathbf{x}_f^\dagger\right] = \frac{\gamma}{n_T N_c}\mathbf{I}$.

III. Capacity Analysis

In this section, we study the ergodic capacity normalized by the number of receive antennas n_R and the frame size N_c. The normalized capacity assuming uniform power allocation across the transmission dimensions and $n_R = n_T = n$ is given by [9]:

$$\begin{aligned} C_{\text{opt}} &= \lim_{n \to \infty} \frac{1}{n_R N_c} \mathcal{I}(\mathbf{x}; \mathbf{y} \mid \mathbf{H}) \\ &= \lim_{n \to \infty} \frac{1}{n_R N_c} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{\gamma}{n_T N_c} \mathbf{H}\mathbf{H}^\dagger \right) \right] \\ &= \lim_{n \to \infty} \mathbb{E}\left[\frac{1}{n_R N_c} \sum_{i=1}^{n_R N_c} \log\left(1 + 2\gamma\lambda_i \left(\frac{1}{2 n_T N_c} \mathbf{H}\mathbf{H}^\dagger \right) \right) \right] \\ &= \int_0^\infty \log(1 + 2\gamma x)\, f^\infty_{\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger}(x) dx, \end{aligned} \quad (10)$$

where $\mathbb{E}$ denotes expectation, $\lambda_i(\mathbf{X})$ denotes the eigenvalues of matrix $\mathbf{X}$ and $f^\infty_{\mathbf{X}}$ denotes the asymptotic eigenvalue probability density function (a.e.p.d.f.) of matrix $\mathbf{X}$. Thus, it is requisite to calculate the a.e.p.d.f. of matrix $\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger$, which can be obtained by determining the imaginary part of the Cauchy transform G for real arguments [9]:

$$f^\infty_{\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger}(x) = \lim_{y \to 0^+} \frac{1}{\pi} \Im\left\{ G_{\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger}(x + \mathrm{j}y) \right\}. \quad (11)$$

A. Time-Domain Wideband MIMO Channel

By reordering the rows and columns in the time domain, the $\mathbf{H}$ matrix can be written as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}^t_1 & \mathbf{H}^t_2 & \cdots & \mathbf{H}^t_L & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^t_1 & \cdots & \mathbf{H}^t_{L-1} & \mathbf{H}^t_L & \cdots & \mathbf{0} \\ \vdots & \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ \mathbf{H}^t_2 & \mathbf{H}^t_3 & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}^t_1 \end{bmatrix}, \quad (12)$$

where $\mathbf{H}_l^t,\ \forall l = 1 \dots L$ are $n_R \times n_T$ channel matrices, describing the MIMO fading coefficients for each symbol of the frame. The entries in each matrix $\mathbf{H}_l^t$ are independent and jointly Gaussian, while correlation appears between the square blocks of matrix $\mathbf{H}$ in the form of repetition. This kind of correlation is non-separable, since it cannot be expressed as a product of a correlation matrix and a Gaussian matrix, as in [10].

1) Non-Separable Correlation: According to [11, Th.1], if $\mathbf{H}$ is a square $n_R N_c \times n_T N_c$ block matrix containing correlated $n_R \times n_T$ Gaussian blocks, the Cauchy transform of the a.e.p.d.f. ($n_R, n_T \to \infty$) for $\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger$ can be calculated as follows:

$$G_{\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger}(z) = \frac{1}{N_c}\mathrm{Tr}\left(\mathcal{G}_1(z)\right), \tag{13}$$

where $\mathcal{G}_1(z)$ is the upper left block of

$$\mathcal{H}(z) = \begin{bmatrix} \mathcal{G}_1(z) & \mathbf{0} \\ \mathbf{0} & \mathcal{G}_2(z) \end{bmatrix} \tag{14}$$

and $\mathcal{H}(z)$ satisfies the following equation $\forall z \in \mathbb{C}^+$

$$z\mathcal{H}(z) = \mathbf{I} + z\eta(\mathcal{H}(z)) \cdot \mathcal{H}(z), \tag{15}$$

where η is the mapping

$$\eta : \begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_3 \\ \mathbf{D}_4 & \mathbf{D}_2 \end{bmatrix} \longmapsto \begin{bmatrix} \eta_1(\mathbf{D}_2) & \mathbf{0} \\ \mathbf{0} & \eta_2(\mathbf{D}_1) \end{bmatrix} \tag{16}$$

with

$$[\eta_1(\mathbf{D})]_{i,j} := \frac{1}{2N_c}\sum_{k,l=1}^{N_c} \tau(i,k;j,l)\cdot[\mathbf{D}]_{k,l} \tag{17}$$

$$[\eta_2(\mathbf{D})]_{k,l} := \frac{1}{2N_c}\sum_{i,j=1}^{N_c} \tau(i,k;j,l)\cdot[\mathbf{D}]_{j,i}. \tag{18}$$

The real-valued function τ depends on the cross correlation between identically-indexed elements of the matrix blocks and is defined as:

$$\mathbb{E}\left[h_{pq}^{(i,k)}\left(h_{pq}^{(j,l)}\right)^\dagger\right] = \frac{1}{2n_T N_c}\cdot\tau(i,k;j,l) \tag{19}$$

where $h_{pq}^{(i,k)}$ is the (p,q)th element of the (i,k)th block.

In this direction, we observe that the block cross-correlation function for the time-domain wideband MIMO channel is given by:

$$\tau(i,j;k,\mathrm{mod}\,(j+k-i-1,N_c)+1) = \begin{cases} 1 & \text{if } 1 \le i,k \le N_c \\ & i \le j \le i+L-1 \\ & i+L-1 \le N_c \\ 1 & \text{if } 1 \le i,k \le N_c \\ & i \le j \le K \\ & 1 \le j \le i+L-1-K \\ & i+L-1 > N_c \\ 0 & \text{elsewhere} \end{cases} \tag{20}$$

2) Iterative Mapping Solution: Equation (15) for a specific z running along the real axis can be solved by iterating over the mapping [12]:

$$\mathcal{W} \longmapsto \mathcal{M}_z(\mathcal{W}) = \frac{1}{2}\left(\mathcal{W} + (-\mathrm{j}z\mathbf{I} + z\eta(\mathcal{W}))^{-1}\right), \tag{21}$$

where $\mathcal{H} = -\mathrm{j}\mathcal{W}$. It should be noted that in some cases a simpler set of equations can be derived by observing the effect of mapping η. For example, in [11] the fact that η maps diagonal matrices to diagonal matrices reduces the problem to a set of quadratic equations. The same applies to the frequency-domain case (c.f. III-B), whereas in the time-domain the structure is more complex, since there are off-diagonal zero-values as well.

3) Rectangular Gaussian Blocks, $n_R \neq n_T$: So far we have considered an equal number of receive and transmit antennas. In case that $n_R \neq n_T$ and more specifically the ratio of transmit and receive antennas is $\alpha = n_T/n_R$, the Cauchy transform of the a.e.p.d.f. ($n_R, n_T \to \infty$) for $\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger$ can be calculated as follows [11]:

$$G_{\frac{1}{2n_T N_c}\mathbf{H}\mathbf{H}^\dagger}(z) = \frac{1}{4N_c}(a+1)\,\mathrm{Tr}\left(\mathcal{H}(z)\right) - \frac{(\alpha-1)}{2z}. \tag{22}$$

B. Frequency-Domain Wideband MIMO Channel

By reordering the rows and columns in the frequency domain, the $n_R N_c \times n_T N_c$ $\mathbf{H}$ matrix can be written as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^f & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2^f & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_{N_c}^f \end{bmatrix}, \tag{23}$$

where $\mathbf{H}_i^f,\ \forall i = 1 \dots N_c$ are $n_R \times n_T$ channel matrices, describing the MIMO flat-fading coefficients for each frequency subcarrier. The entries in each matrix $\mathbf{H}_l^f$ are independent and jointly Gaussian, while correlation appears between the blocks of matrix $\mathbf{H}$ according to Equation (7). This kind of correlation is also non-separable.

In the case of the frequency-domain Wideband MIMO channel the block cross-correlation function is given by:

$$\tau(i,i;j,j) = \begin{cases} \mathbb{E}\left[w_i w_j\right] & \text{for} \quad 1 \le i,j \le N_c \\ 0 & \text{elsewhere} \end{cases}, \tag{24}$$

where w_i is the ith element of the $N_c \times 1$ vector

$$\mathbf{w} = \operatorname{diag}\left(\mathbf{H}_{f_{(i,j)}}\right) = \mathbf{F}\mathbf{h}_{t_{(i,j)}} \tag{25}$$

and $\mathbf{F}$ is a $N_c \times N_c$ Fourier matrix, $\mathbf{H}_{f_{(i,j)}}$ is the frequency domain channel matrix and $\mathbf{h}_{t_{(i,j)}} = [h_1^t \ldots h_L^t \;\; 0 \ldots 0]^T$ contains the channel tap gains for the link between the ith receive antenna and the jth transmit antenna. Using Equation (7), the block cross correlation function can be calculated as:

$$\mathbb{E}\left[w_i w_j\right] = \sum_{l=1}^{L} v_l^2 \exp\left(\frac{-\mathrm{j}2\pi\left(i+j-2\right)\left(l-1\right)}{N_c}\right) \tag{26}$$

This channel modelling may result in complex values of τ, which cannot be handled by [11, Th.1]. However, if the employed channel model produces real subcarrier cross correlation coefficients, the a.e.p.d.f. can be calculated in the frequency-domain using the Theorem in III-A1 and Equation (24).

Alternatively, the special parallel structure of the frequency-domain wideband MIMO channel can be exploited to simplify the capacity calculation. In this direction, Equation (10) can be written as:

$$\begin{aligned} C_{\text{opt}} &= \lim_{n\to\infty} \frac{1}{n_R N_c} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{\gamma}{n_T N_c}\mathbf{H}\mathbf{H}^\dagger\right)\right] \\ &= \lim_{n\to\infty} \frac{1}{n_R N_c} \sum_{i=1}^{N_c} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{\gamma}{n_T N_c}\mathbf{H}_i^f\left(\mathbf{H}_i^f\right)^\dagger\right)\right]. \end{aligned} \tag{27}$$

As a result, the block-correlation effect is removed and the wideband ergodic capacity can be calculated as the sum of the well-studied capacities of the uncorrelated narrowband MIMO subcarriers.

IV. Numerical Results

This section is dedicated on comparing the analytical results with Monte Carlo simulations and evaluating the effect of channel model parameters, such as frame size and tap delay line power profile. The analytical results can be acquired by employing the appropriate block cross-correlation function τ and numerically integrating over the resulting a.e.p.d.f. On the other hand, Monte Carlo simulations have to be run and averaged over a large number of channel realizations. The advantage of the analytical approach is that it is independent of the MIMO system size and it can provide accurate capacity results even for small number of antennas.

For the a.e.p.d.f. results presented in this section, we have used a 4-tap delay line power profile defined as follows:

$$v_1 = 0\ dB,\ v_2 = -2\ dB,\ v_3 = -10\ dB,\ v_4 = -20\ dB \tag{28}$$

For the capacity results, γ was set to 30 dB.

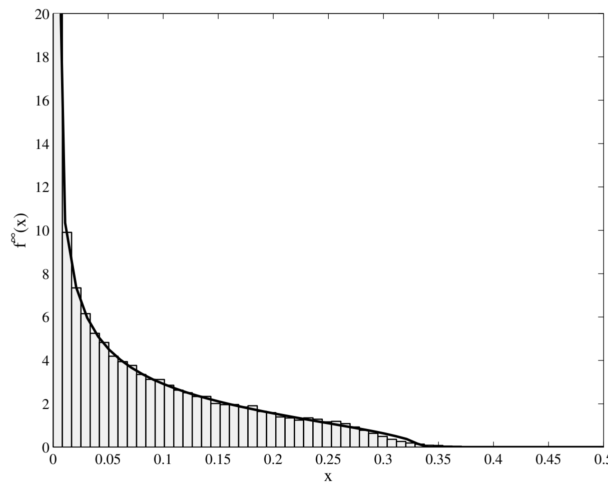

Fig. 1. Agreement between analytical (solid line) and simulated (bar plot) a.e.p.d.f. for $N_c = 6$

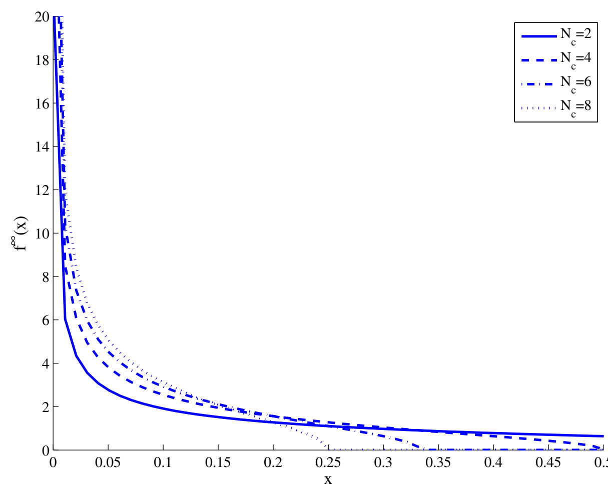

Fig. 2. The effect of frame size N_c on the a.e.p.d.f. of the time-domain wideband MIMO channel

Figure 1 depicts the close agreement between the analytical and simulated e.p.d.f. The analytical curve was calculated using the block cross-correlation function in Equation (20) , whereas for the simulated curve we used $n = 10$ receive and transmit antennas over 1000 Monte Carlo iterations. Figure 2 illustrates the effect of frame size N_c on the a.e.p.d.f. of the time-domain wideband MIMO channel, considering that the available power P does not scale with the frame size N_c. In the same direction, Figure 3 shows the decay of the normalized capacity, which occurs by increasing N_c. It should be noted that if the transmitter power P scales linearly with N_c, the produced capacity remains at the same level. In the previous numerical results, the tap delay power profile does not affect the capacity or the a.e.p.d.f. due to the normalization in Equation (2). In order to study this effect, we employ the tap delay power profile in Equation (28) without normalizing the power levels. In this context, Figure 4 depicts the normalized capacity versus the number of considered taps L. As expected,

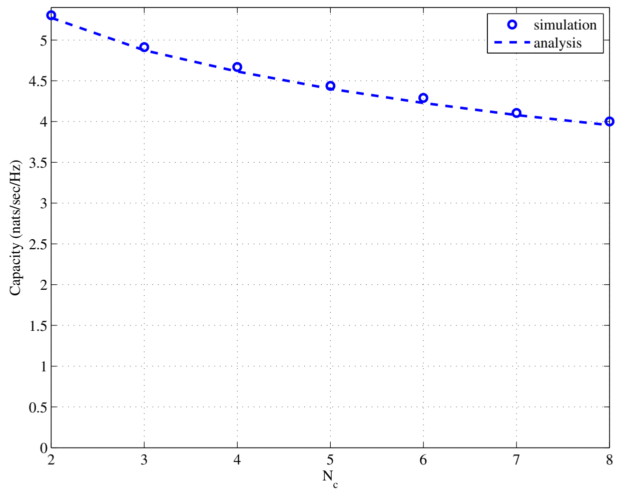

Fig. 3. Normalized Capacity in nats/sec/Hz vs the frame size N_c

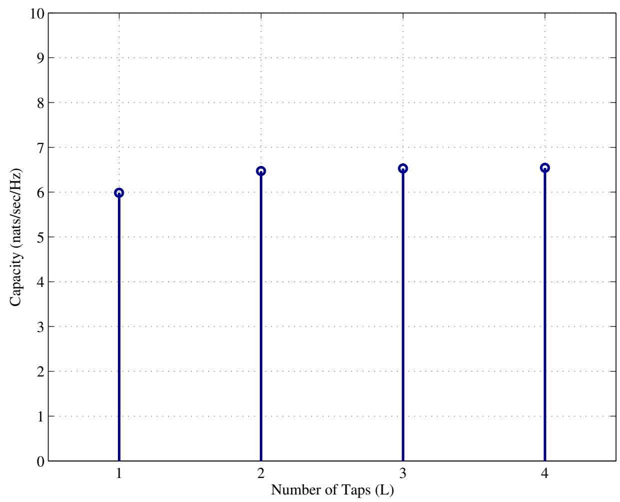

Fig. 4. Increase of normalized capacity (nats/sec/Hz) by increasing the considered number of taps L.

by including the high-power tap v_2 we can observe an increase of 0.5 nats/sec/Hz, whereas taps v_3 and v_4 have a small effect on capacity due to their low power level.

V. Conclusion

In this paper, we have studied the ergodic capacity of the wideband MIMO channel in both time- and frequency domain. The main characteristic of the channel matrix in the wideband case is that non-separable correlation appears between the considered MIMO blocks and therefore the classic narrowband flat-fading approaches cannot be applied. In this direction, we have specialized the a.e.p.d.f. Theorem in [11] by defining appropriate block cross-correlation functions, which can be utilized in combination with an iterative mapping in order to calculate the ergodic channel capacity. Finally, a number of numerical results have been presented to verify the validity of the approach and study the effect of the channel model parameters.

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