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by

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MULTIPLAYER EQUILIBRIUM IN MOBILITY-AS-A-SERVICE SYSTEMS

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Abstract

In the last decades, transportation systems have been offering an increasing number of multi-modal options thanks to the introduction of new mobility solutions (e.g., car- and bike-sharing, carpooling, e-scooter, on-demand services), often integrated into single platforms (e.g. Mobility-as-a-Service) with the purpose of improving the efficiency of last-mile connectivity to public transport (PT) and in turn to reduce private car (PC) use. The proliferation of these new services has increased the adoption of competition and cooperation strategies in the transportation market, where different mobility service providers (MSPs) operate in pursuit of both profits and business sustainability. Each MSP tries to generate revenues by applying different policies, for instance by changing prices, varying their fleet size, or cooperating with other mobility suppliers to attract a sufficient share of the market demand.

In this context, user choices assume a fundamental role in determining the longevity of mobility services within the transportation system. Therefore, it becomes imperative to develop a suitable transportation modeling approach to predict how choices are made by users and how they respond to the strategies of MSPs. The study of these interactions helps in understanding the conditions under which an MSP can maintain a profitable business, the implications of new competitors entering the market, and the potential benefits of cooperation among service providers.

The significance of competition and collaboration among MSPs in the market is evident. However, it is contended that there exists a lack of studies examining the various forms of competition and collaboration that emerge when mobility packages are introduced or when multiple service providers coexist in the same market. Additionally, observations have been made that existing studies predominantly focus on uni-modal networks with homogeneous demand characteristics, overlooking interactions between different modes considering separable cost functions. This research aims to fill these gaps, seeking to comprehend the various dynamics that may arise in the transportation network due to complex interactions between the different actors present in the systems, including different modes of transportation sharing the same infrastructure and competing for travel demand.

In this thesis, an analysis is conducted of the interactions between the

market strategies of MSPs and diverse classes of users within a multi-modal transportation network. The network supply structure is defined using a novel supernetwork approach that represents users' daily trip chains and models the mobility services used to reach each destination.

The problem formulation follows a bi-level structure that extends and generalises Multi-modal Network Design principles. At the upper level, a profit maximization formulation is introduced to describe the behavior of each MSP. At the lower level, transportation users are categorized into heterogeneous classes based on their travel characteristics and are assigned to a multi-modal supernetwork characterized by non-separable cost functions to account for mode interactions in capacitated networks. Consequently, the traffic network equilibrium conditions are formulated as a Variational Inequality (VI).

First, a Mathematical Program with Equilibrium Constraints (MPEC) is formulated to capture the relationships between a MSP and the users of a multi-modal transport network. Subsequently, to consider the interaction between multiple suppliers at the upper-level, the problem evolves into an Equilibrium Problem with Equilibrium Constraints (EPEC).

Being the EPEC an extremely complex problem to solve, an iterative solution algorithm has been developed that combines the classical Diagonalization Method (DM) with an adaptive Extragradient Method (EM) to address this complex problem. This algorithm has been tested on simple examples, demonstrating the validity of the approach in identifying the range of solutions where MSPs can potentially run profitable businesses and quantifying the Price of Anarchy resulting from MSPs competition. The results also highlight the existence of multiple equilibrium points for the same problem, which are strongly influenced by variations in pricing schemes, demand definitions, or MSP strategic decisions.

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Piergiorgio, I truly feel like the luckiest person to have you right by my side. Thank you for always supporting me in my decisions and being the first to believe in me.

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Chapter 1

Introduction

1.1 Background

It is common knowledge that the travel demand arises from users' need to engage in activities at various locations throughout the day (Bowman & Ben-Akiva, 2001). In transportation, this sequence of activities is referred to as a daily trip chain or daily tour, and typically starts and ends at the home location. Daily trip chains are closely linked to the modal choices made by individuals. For instance, the choice of transportation mode for the first trip to reach an activity location significantly influences subsequent choices (Schneider et al., 2021). As a result, individuals pre-plan the modes of transport they will use throughout their daily trip chain. Furthermore, over the past few decades, there has been a significant increase in the number of daily activities, largely attributed to improved economic and social well-being (Cartenì et al., 2010). Notably, research has shown that as these trip chains become more intricate, people tend to opt for PCs over public transport (PT)(Schneider et al., 2021) due to their flexibility, door-to-door convenience, and overall comfort (Punzo et al., 2022).

Recent data from Eurostat (2023), the statistical office of the European Union (EU), reveals a consistent increase in the number of cars per inhabitant between 2001 and 2021 across European countries, as illustrated in Figure 1.1. Unfortunately, the surge in PC usage has had negative impacts on the environment, including increased air pollution, noise, and land use, as well as on society, leading to issues such as congestion, accidents, and public health concerns (Punzo et al., 2022). Only urban areas are responsible for 23% of all transport-related greenhouse gas emissions. On this topic, TomTom (2023) conducted an assessment of CO2 emissions in various cities during rush hour while travelling for a distance of 10 km. Figure 1.2 shows that in 2022, a petrol vehicle in London, the most congested city, emitted 1133 kg of CO2, a quantity that would require 113 trees to grow for a year to offset.

Fortunately, in order to deal with the global crisis due to climate change,

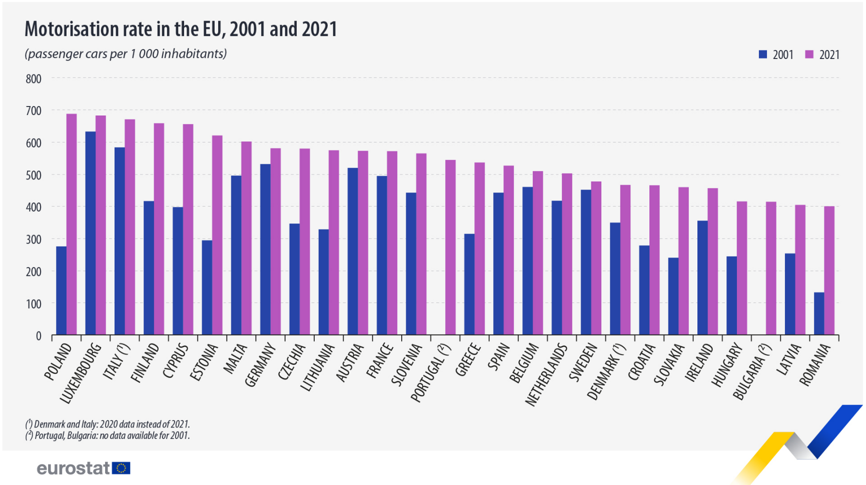


Figure 1.1: Motorisation rate in EU, 2001 and 2021 (Eurostat, 2023)

the Parliament (2023) adopted the European Climate Law, with the target of reducing the total greenhouse gas emissions by at least 55% by 2030 and by 90% by 2050.

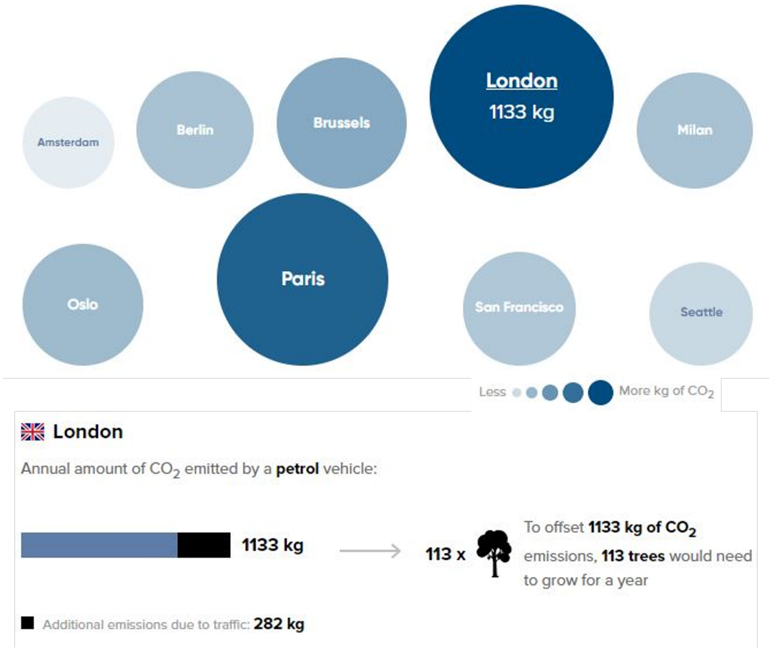


Figure 1.2: Kg of CO2 emitted during rush hour per year across different cities (TomTom, 2023)

To achieve these goals, the EU has proposed measures to encourage member states to develop urban transport systems that are "safe, accessible, inclusive, affordable, smart, resilient, and emission-free" (Commission, 2023). Following these targets, in recent years different new mobility solutions have been introduced in the transport system, with the purpose of supporting the PT in first and last mile connections, in order to reduce PC usage. To provide insights into the various modal options available within an urban context and their respective efficiency, Wong et al. (2020) defined the framework illustrated in Figure 1.3. The plane is defined by two axes that represent spatial efficiency (measured as passengers per vehicle) and temporal efficiency (considered as the amount of time a vehicle spends on the road). Through this figure it is possible to see how conventional transportation modes are integrated with new mobility solutions.

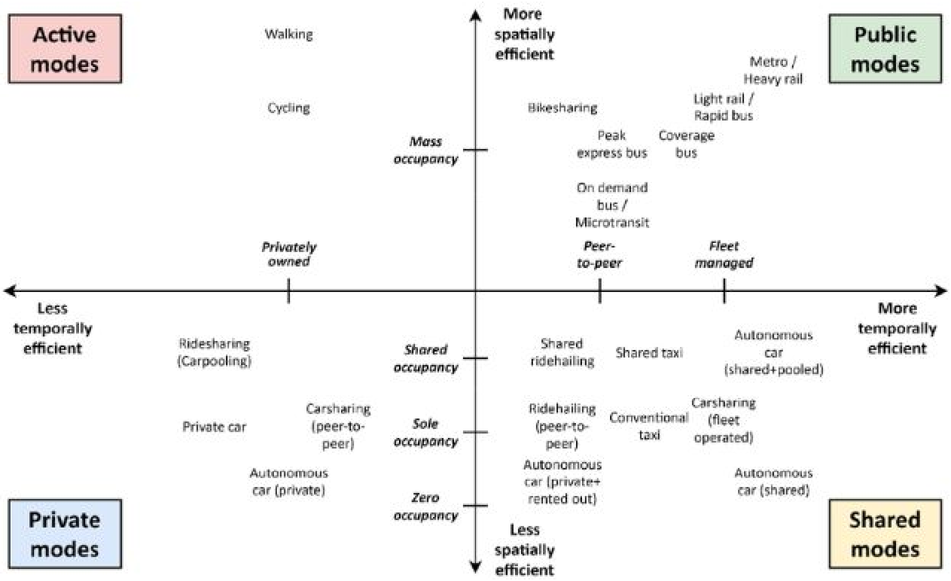


Figure 1.3: The modal efficiency framework from Wong et al. (2020)

For the purposes of this thesis, some of the displayed new mobility solutions will be taken into account. Consequently, the following sections introduce these selected transportation services, describing their main characteristics.

Car-sharing

As stated by Shaheen et al. (1999), car-sharing operates on the fundamental premise that users, instead of possessing PCs, gain access to a fleet of shared vehicles, having the advantage of car utilization without the financial burdens and obligations of ownership. Travelers must sign in through a mobile application or web platform, providing personal credentials, such as a driver's license. If accepted, they can use the service and pay directly through the app, with costs being automatically debited from their bank accounts. They have the option to pay through either a monthly subscription or a pay-as-you-go (PAYG) arrangement. Usually, fuel is included in the price, however travellers may have additional cost per hour and/or km driven.

Following the literature review of Ferrero et al. (2018), car-sharing service vehicles can be electric or powered by traditional fuels, and the system ca be divided in:

- **two-way station based**: when the car is picked up and dropped off at the same station, in which there are a number of defined parking lots.
- **one-way station based**: when users have the option to return the car to a station different from the one where they picked it up.
- **free-floating**: when the car can be used in a defined area of a city, and can be directly picked up and dropped off in public parking spaces in that area.

During the last decade car-sharing services have become very popular all over the world. Figure 1.4 shows the free-floating car-sharing service available in Europe (Friedel, 2020). The coloured dots identify distinct companies, and when the color matches, it indicates that the same company operates across multiple cities/countries. Conversely, the grey dots denote providers exclusively operating within the confines of a particular city.

Ride-sharing, ride hailing and carpooling

In a ride-sharing system, freelance drivers offer their personal vehicles to provide rides through a company (Kooti et al., 2017), such as Uber (2023). This company owns a mobile app that connects individuals who are headed in the same direction or have similar destination points, allowing them to share the same ride. The company sets a fixed price for the requested ride, and users pay directly through the app, drivers get a percentage of the total amount and additional tips.

Ride-hailing has the same principle as the aforementioned ride-sharing, however users do not share their trip with other travellers.

A carpooling system, instead, involves the practice of sharing a journey with other individuals while also splitting the associated expenses, such as fuel and tolls. This sharing of costs can take various forms, including arrangements among friends, coworkers, and neighbors, or it can be facilitated through a mobile application. In the latter scenario, the driver typically doesn't incur any costs, as the passengers who join the trip contribute money toward these expenses. The company managing the application often retains a portion of these contributions as a service fee. Additionally, in some instances, government subsidies may be in place, leading to situations where

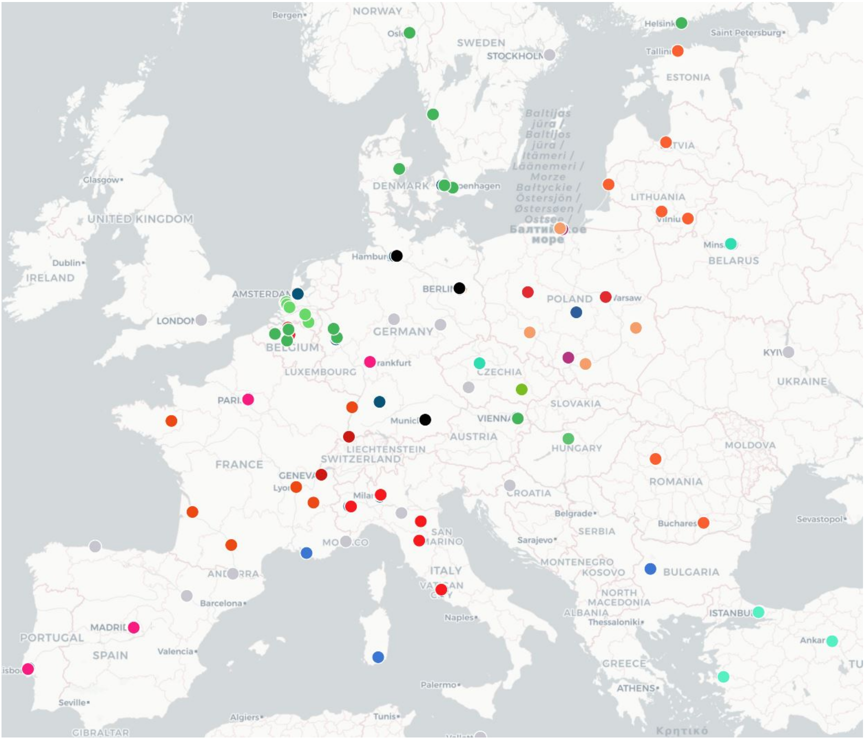


Figure 1.4: Map of free-floating Car-sharing services in Europe by Friedel (2020)

travelers don't have to make direct payments for the carpooling service (Gouvernement.lu, 2020).

Bike-sharing

The bike-sharing service operates on a similar sharing principle as the car-sharing system. This service offers users access to bicycles, which can be either electric or regular, and it comes in two primary formats: dock-less or station-based (Eren & Uz, 2020). In both cases, bike can be unlocked using a smartphone app. Dock-less bikes can be picked up and dropped off within a defined area around the city. On the other hand, station-based bikes are located at specific designated stations, typically situated near PT hubs like metro or bus stops. In this arrangement, users are required to pick up a bike from one of these stations and, after using it, return it to the nearest station that has an available spot for docking. This system promotes a more structured and organized approach to bike-sharing, ensuring that bicycles are consistently available for users at specific locations.

These services are typically accessible through a mobile application, which serves as the primary platform for users to access and utilise them. Users

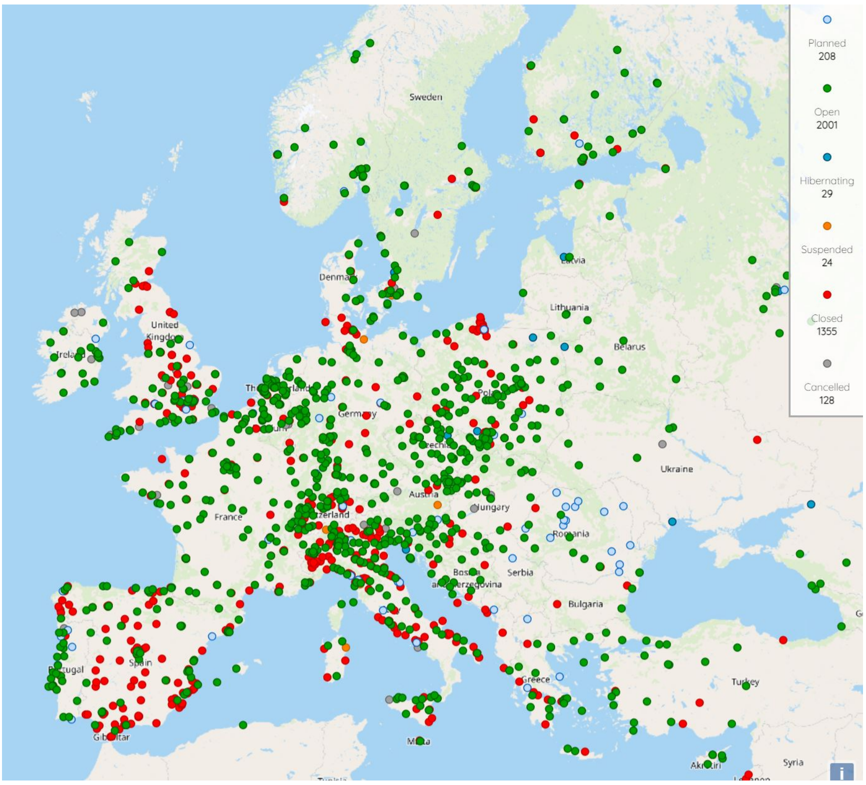


Figure 1.5: Map Bike-sharing services in Europe by O'Brien et al. (2023)

have the possibility to choose between two payment options, a monthly subscription or a PAYG fee structure. In several cases, in order to encourage the usage of bikes in urban areas, Government or local authorities provide this service for free for residents or general subscribers.

To gain insights into the evolution of this particular service over the years, Figure 1.5 provides an analysis spanning from 2001 onwards. It categorizes bike-sharing services into several distinct phases: planned (light blue), opened (green), hibernating (blue), suspended (orange), closed (red), and canceled (grey).

E-Scooter sharing

Recently, electric motorised scooters have emerged as a novel mobility solution in urban areas. The primary objective behind their introduction is to offer commuters a convenient and flexible mode of transportation that complements the existing public transit systems. This service is typically characterized by its dock-less nature, similar to other sharing services, and users have the option to choose between two primary payment models: a subscription-based approach or PAYG.

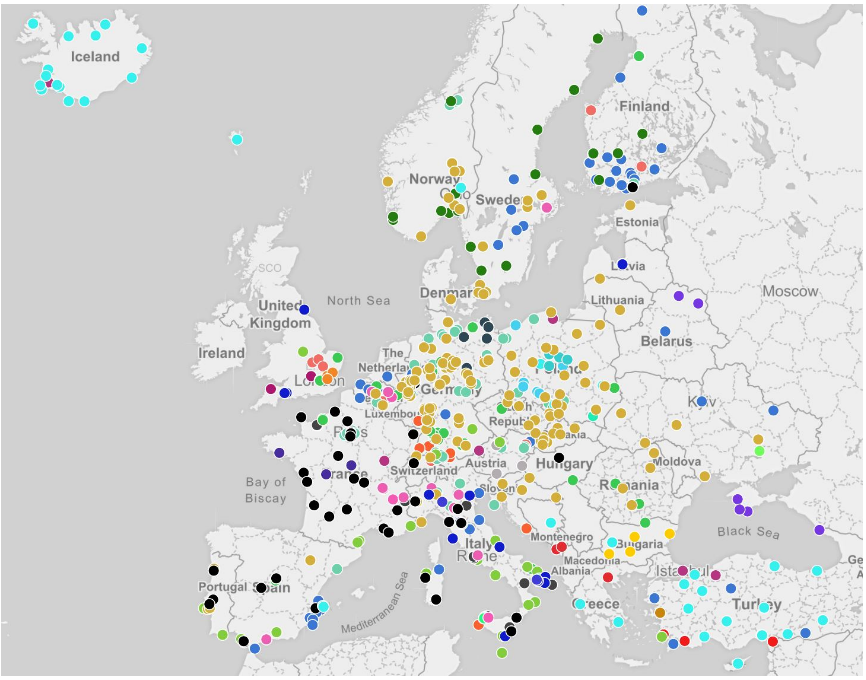


Figure 1.6: Map e-scooter sharing services in Europe by Friedel (2021)

Despite being one of the newest entrants in the transportation market, this shared mobility solution has rapidly gained traction across Europe over the past five years, as depicted in Figure 1.6. This Figure illustrates the widespread distribution of electric motorized scooters, underscoring their increasing popularity and adoption as a viable urban transportation option.

It is clear that over the last years, there has been a growing introduction of new mobility solutions, these options are often marketed as alternatives to PCs or as a means of providing first/last mile support to PT (Shaheen & Chan, 2016). However, providing users with a wider range of multi-modal transportation choices has increased competition between mobility service providers (MSPs) without a clear impact on travel behaviour (Garus et al., 2022). Generally, each MSP tries to generate revenues by applying different policies, for instance by changing prices, varying their fleet size, or cooperating with other mobility suppliers to attract a sufficient share of the market demand. In this context, the new concept of Mobility-as-a-Service (MaaS) has emerged as a way to integrate various transportation modes into a single service that can be accessed and paid for through a single platform or app. Its main purpose is to offer a portfolio of mobility solutions meeting the travellers' needs, and in turn facilitating and encouraging the use of PT and shared mobility options as alternative to privately owned vehicles.

Despite the abundance of studies on the subject, MaaS appears to be more of a theoretical concept than a practical solution (Hensher et al., 2021b). Specifically, several are the challenges on its successful implementation, including regulatory barriers, technical complexities, and the need for collaboration between multiple service providers of different sectors (Butler et al., 2021), often characterised by different and conflicting objectives. Moreover, the success of MaaS will depend on a range of factors, including user acceptance, affordability, and the availability and reliability of transportation options (Hoerler et al., 2020).

In this thesis, the focus is on a comprehensive examination of the concept and framework of MaaS. The goal is to gain a deep understanding of its intricate structure, which involves multiple actors and a diverse range of transportation modes. This understanding is crucial for effectively representing not only MaaS, but the current state of the transportation market and its ever-evolving dynamics. While these systems are characterized by the presence and interaction of various stakeholders, including regulators, investors, research institutes, IT providers, MSPs, and users (Kamargianni & Matyas, 2017), the primary focus of the proposed study lies in examining the dynamics among different MSPs offering their services and potentially collaborate with other suppliers via mobility packages, and users with their different transport needs (Figure 1.7).

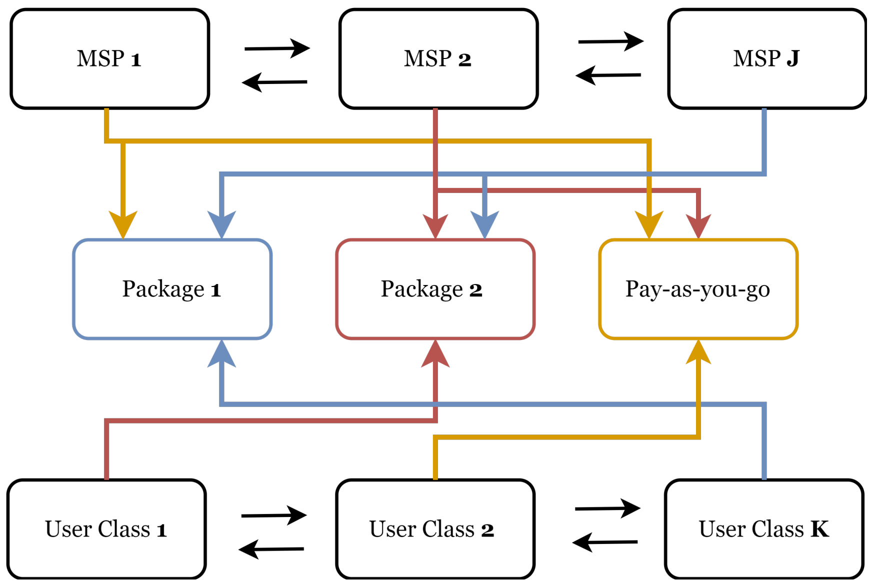


Figure 1.7: MaaS relationships analysed in this thesis

Within this context, to properly assess the feasibility and potential adoption of such a complex system, it is crucial to develop models that take into account various factors, such as users' transportation needs and budget constraints, the availability and composition of different mobility packages, and the strategies that can be adopted by different MSPs to establish a profitable business. Such models would provide a framework for evaluating the real-world applicability of MaaS and determining its impact on various actors, including users, PT operators, and private MSPss.

1.2 Research Objectives and Contribution

The central purpose of this thesis is to develop a comprehensive model that can effectively identify the necessary conditions for achieving equilibrium among various MSPs, each operating with their unique business model, while explicitly considering the choices made by users within a multi-modal transportation network. In light of this aim, the primary research question guiding this study can be succinctly framed as follows:

Main RQ: What are the conditions that allow meeting the objectives of multiple actors in a multi-modal transport system?

To address the main research question, it is essential to gain a profound understanding of the various characteristics of a complex multi-actor and multi-modal transport system, such as MaaS. This ecosystem encompasses the coexistence of multiple MSPs and heterogeneous users, defining a network where numerous actors, at times engaging in competitive interactions and occasionally collaborating, fervently pursue their individual goals. To begin tackling this challenge, it is crucial to understand:

Objective 1: How to define a network model that includes the different characteristics of users and services of a multi-modal transport system?

In understanding the characteristics associated with a multi-modal and multi-actor system, several key aspects come into play. Firstly, it entails the integration of mobility packages, where diverse mobility services are seamlessly combined thanks to the collaboration among multiple MSPs. These mobility packages are designed with the primary goal of catering to the diverse needs of users within the transport network. Consequently, it becomes crucial to develop models that accurately capture the heterogeneity in users' preferences and choices.

Furthermore, the coexistence of multiple mobility services, coupled with the growing complexity of users' daily trip chains, underscores the necessity of creating a network that allows users to seamlessly make multi-modal trips. This demands a thoughtful modeling approach to effectively accommodate several modes of transport and to simultaneously capture revenues and costs associated to each MSPs.

Moreover, it is necessary to consider the congestion effects that arise when users choose the same transportation modes, or different modes that can interfere between each other. This entails ensuring a comprehensive representation of users' decision-making processes under various conditions and scenarios. By accounting for these congestion effects, it is possible to gain deeper insights into how users interact with and adapt to multi-modal transportation systems.

In order to address this first objective, in this thesis a novel network approach is employed, incorporating the aforementioned key elements inside the proposed modelling approach. Firstly, the concept of mobility package is integrated directly into the multi-modal network by leveraging the notion and structure of supernetworks. This approach allows for the modelling of collaboration and revenue sharing strategies among different MSPs effectively. Moreover, the proposed network structure incorporates the intricate effects of congestion stemming from the presence of various transportation modes and user classes. This aspect is crucial for gaining insights into how users shift between modes, depending also on changes in service capacity and the strategies employed by MSPs.

Generally, the methodological approach used in this thesis has the purpose of evaluating the strategies employed by MSPs and the choices made by users, enabling predictions of the macroeconomic impact on the transportation market. For this reason, it seems fundamental to introduce the second main objective of this thesis, as follow:

Objective 2: Given a set of strategies from diverse actors, what is the emerging behavior of the whole multi-modal transport system?

To fulfill this objective, an Equilibrium Model is presented in Chapter 4, aiding in the understanding of how the strategies employed by multiple MSPs offering diverse mobility services influence one another. The model takes into account the potential aggregation of these services into mobility packages, targeting heterogeneous groups of users, minimizing their travel costs while performing their daily trip chains. In this context, a formulation with a bi-level structure is presented. At the upper-level, a general profit maximization formulation is proposed to represent different MSPs within the transport network. At the lower-level, due to congestion effects and non-separability of the network cost functions, a Variational Inequality (VI) formulation is introduced to capture the equilibrium conditions of multi-class users.

In Chapter 5, the proposed formulation takes the form of a Mathematical Program with Equilibrium Constraints (MPEC) when applied to understand the interactions arising between a single MSP, setting different network strategies, and users of the multi-modal network. Generally, these strategies collectively determine the transportation options available to users. Users, in turn, make travel choices aimed at minimizing their costs, while also considering the preferences of different user classes assigned to the same transportation network.

In Chapter 6, the formulation evolves in an Equilibrium Problem with Equilibrium Constraint (EPEC), due to the simultaneous evaluation at the upper-level of different MSPs' strategies. The complexity arising with this approach increases due to the fact that the strategies adopted by each MSP significantly influence the strategies of other players, all of whom aim to maximize their individual profits.

Through this methodological approach, an analysis of the strategic conditions that enable MSPs to enhance their profits based on the strategies of other MSPs is conducted, along with an exploration of the implications of new market entrants and an assessment of the consequences of increased overall travel demand. It's essential to note that the model developed in this thesis is tailored for strategic, long-term planning rather than day-to-day operational management. In summary, this study offers a comprehensive analysis of the intricate dynamics within multi-modal transportation networks, focusing on the strategic interplay among MSPs and the diverse choices made by users in this dynamic environment.

1.3 Thesis Structure

The structure of the proposed thesis is summarized in Figure 1.8. Specifically, after the introduction of the general research context with the motivation and contributions in Chapter 1, the manuscript is organised as follows:

- **Chapter 2** presents the literature review on the topic of MaaS. Within this chapter, the aim is to introduce the main concepts, delve into the various modeling approaches applied to this complex system, analyse existing research for limitations and gaps, and capture the essential characteristics of this multi-modal, multi-actor system. The main contribution of this chapter is to provide a comprehensive analysis of the literature review on MaaS. This analysis particularly emphasizes pilot projects developed since 2013. The goal is to gather valuable insights from these projects and compile a set of actionable steps that can guide the execution of successful pilot projects, drawing from the lessons learned.

- **Chapter 3** includes a literature review on multi-modal and multi-actor network design modelling. Specifically, this chapter undertakes an in-depth analysis of several key aspects, including the methodologies employed in modeling multi-modal networks, the formulation of travel demand, the exploration of interactions between service providers and demand, and the range of solution approaches proposed within these models. The main contribution of this chapter is to study conventional approaches with the aim of adapting and extending them to better accommodate the intricacies of more complex transportation networks, such as MaaS.

- **Chapter 4** presents the proposed methodological approach. This chapter describes the assumptions concerning the structure of the multi-modal network, and the formulations describing the different MSPs and multi-class users. The main contribution of this chapter is the introduction of a novel methodology that employs a supernetwork approach to seamlessly integrate multi-modal constraints. Specifically, it includes elements such as mobility packages, users daily trip chains and MSPs market strategies. A general profit maximization formulation is proposed to represent different MSPs. The transportation users are, instead, divided into heterogeneous classes based on their travel characteristics, and assigned to the multi-modal supernetwork characterised by non-separable cost functions to account for the interaction between modes in capacitated networks. Due to non-separabilty the traffic network equilibrium conditions are written as VI problem, introducing an adaptive Extragradient Method (EM) in order to solve it.

- **Chapter 5** the formulations proposed in Chapter 4 are combined in a bi-level structure. This structure is designed to explore the relationship between market strategies of a single MSP and the equilibrium flow distribution of various user classes within multi-modal supernetworks. The main contribution of this chapter is the introduction of an MPEC able to study the economic assessment of any MSP influenced by the fixed strategies of other competitors and by users' choices. An iterative solution algorithm is proposed to solve the problem using a sequential quadratic programming (SQP) method at the upper-level, while the proposed EM evaluates the network equilibrium at the lower-level.

- **Chapter 6** extends the problem addressed in Chapter 5, simultaneously considering the strategies of multiple MSPs at the upper-level. The main contribution of this chapter is the introduction of an EPEC. This framework accounts for both competitive and collaborative strategies among various MSPs, aiming to identify equilibrium conditions that allow different actors to coexist within a complex multi-modal system. To tackle the complexity of the resulting mathematical problem, an iterative solution approach based on the Diagonalization method (DM) is devised. In this chapter, different examples show the main characteristics of the methodological approach and its applicability.

- **Chapter 7** describes the main conclusion of the proposed analysis and outlines future research directions.

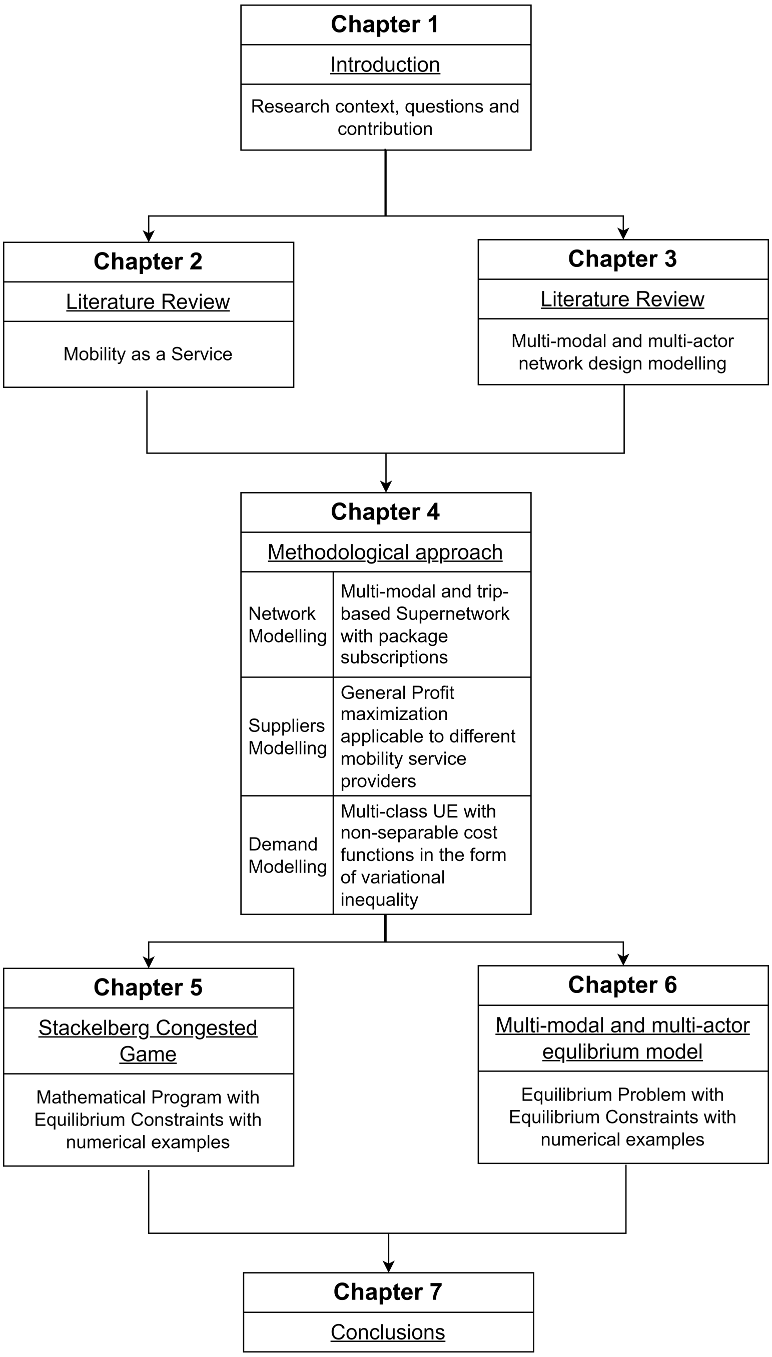


Figure 1.8: Thesis Structure

Chapter 2

Mobility as a Service: A Review

2.1 Introduction

In recent years, transportation networks have witnessed a surge in the introduction of various new mobility solutions, including car-sharing, bike-sharing, and carpooling (Cohen & Shaheen, 2018). This trend had provided a wider range of multi-modal transportation choices for users, intensifying competition among MSPs.

Within this context, various market solutions have been proposed by MSPs to increase profitability, and public authorities to optimize the utilization of these new transportation systems, and generally reduce car usage. One of the most prominent concepts in the literature addressing this issue is MaaS. This innovative concept was first introduced in Sonja Heikkilä's Master's thesis (Heikkilä, 2014), with guidance from Sampo Hietanen, considered as the pioneer of MaaS. Hietanen presented this groundbreaking idea at the Finnish Science Center Heureka in 2014 and went on to establish MaaS Finland Oy (now MaaS Global) in 2015, which developed the Whim app (Whim, 2023), widely considered the world's first MaaS application. The core idea behind MaaS was to provide users with a comprehensive range of mobility services accessible through a single mobile application. These services would be bundled together and offered at varying prices based on the combination of transportation modes included in the packages.

Since its academic introduction, manifold have been the definition of MaaS (Jittrapirom et al., 2017). In this thesis MaaS is considered as an integrated, multi-modal and multi-actor ecosystem (shown in Figure 2.1). This ecosystem is often considered as a network where the different actors are connected and their choices influence each other (Kamargianni & Matyas, 2017). Following König et al. (2016), it is possible to identify four different levels in which the MaaS actors are grouped: 1) public and regulatory level, 2) MSPs' level, 3) MaaS provider level, and 4) travellers level. The complete realization of the MaaS concept entails a collaborative effort involving at least

Government, MaaS provider and MSPs to establish a seamlessly integrated multi-modal transportation system, offered to users, and accessible through a mobile application.

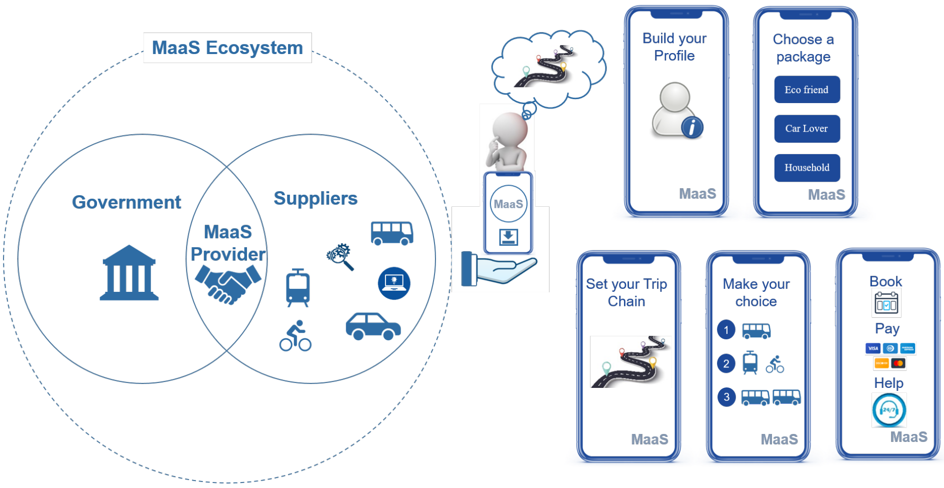


Figure 2.1: MaaS Ecosystem

In the next section, we will introduce the key actors of this ecosystem, followed by the essential levels of integration required for a comprehensive MaaS system, distinguishing it from basic uni-modal and journey planning applications. In Section 2.4 MaaS practical applications on small-scale pilot projects are analysed to understand the complications of its implementation. Finally, Section 2.5 and Section 2.6 present the simulation models and mathematical approaches designed to represent this ecosystem within a controlled environment.

2.2 The MaaS Ecosystem

The aforementioned MaaS ecosystem in its essential form includes four main actors: Government, MaaS provider, MSPs and users (Wong et al., 2020).

Government (or a regulator) has the fundamental role of creating policy frameworks for equitable competition, service accessibility and quality (Kamargianni & Matyas, 2017), and it can also use incentives such as subsidies or parking regulations (Karlsson et al., 2020) to improve MaaS diffusion. Its involvement is considered essential to protect users interests and to achieve long-term societal goals (Vij & Dühr, 2022). Smith and Hensher (2020) advised that policy-makers should define a "rulebook" with the purpose of legalizing the development of MaaS. Moreover, to help the development of MaaS in Europe, it seems essential to define new regulations at a European and national level (Pagoni et al., 2022).

Generally, the role of the Government in the MaaS system has been widely questioned in the literature. Some propose that it should act as a MaaS provider, while others favor transport operators or third-party mobility providers for this role (Jittrapirom et al., 2020; Kamargianni & Matyas, 2017). On this topic Wong et al. (2020) proposed two possible models. On the first, MaaS is developed under economic deregulation where the Government's influence is limited, and the system operates in a market-oriented environment. The different suppliers are mainly driven by commercial interests, possibly leading to monopolistic behaviours, that may be against societal goals. In the second model, MaaS is under Government contracting. Government would have a more active role, it chooses the MaaS Provider directly through a competitive tender. This contract could be renewed if different performance measures are achieved.

The MaaS provider, instead, is a new figure in transportation that gathers suppliers of the transport services as well as specialized business (i.e. digital platform, and financial enterprises) to offer customers mobility packages through a single digital platform for planning, booking, paying for their journeys (Wong et al., 2018). Following Kamargianni and Matyas (2017), the MaaS provider could be a PT authority or a private firm. In the first case, it would be easier to integrate all PT modes and secure participation from other operators. However, most of the time PT authorities are not-for-profit organisations and they may be constrained by law to develop a new services such as MaaS or not interested in participating. In the second case, under the control of a private firm MaaS market could develop faster, managing to attract more private services, such as on-demand modes. However, in this scenario, it might take more time for public services to join the new system. However, it is still unclear who should take the role of MaaS provider. Generally, it could be a PT operator, a local authority, or a private company (Esztergár-Kiss et al., 2020).

Subsequently, the presence of all the different MSPs of a transport market represents one of the main levels of integration (Cruz & Sarmento, 2020b) to consider MaaS successful. Various analysis highlight the importance of integrating PT as the backbone of the transportation system (König et al., 2016), complemented by a diverse range of mobility services that enhance the user experience in the first and last mile connections. However, it has been confirmed that one of the primary barriers in implementing MaaS is getting the various MSPs to cooperate (Meurs et al., 2020). Many MSPs are hesitant to participate in MaaS since they fear losing their identity when included in a larger platform. Each MSP involved usually has a distinct Business Model (BM) based on the "product" they are selling. The term BM here is referred as a specific modelling aspect that defines the service actors strategies, i.e. a business model represents how a company creates customer value (Eckhardt et al., 2017). When joining a MaaS system, companies need to adapt and change their BM in order to have a profitable service (Polydoropoulou et al., 2020). Understanding this adaptation, how to maintain their identity inside the MaaS market, and if it is possible to define a general BM, valid for different MSPs and scalable to multiple locations is still unclear. One of the main aspects that must be taken into account that affects the definition of a general BM is the interaction between MSPs and the MaaS provider. From an analysis of pilots and mobile applications three types of MaaS provider models have been identified: commercial, public, or public-private partnership (PPP) (Eckhardt et al., 2017). The first type can adopt a reseller model when selling some transport services on the market, or an integrator model, which includes ticketing, planning, etc. In the second case, a PT operator could integrate other transport services into the system. Lastly, the PPP model usually includes logistics services, and the public actor includes different actors and services. Moreover, an essential task of the MaaS provider is to gather the relevant MSPs from the area under analysis and create packages based on the users' needs. In order to build these packages, this actor needs to define the right business contracts with MSPs. Following Eckhardt et al. (2017), the service agreements could include re-sold services when there is a list of fares or a percentage of fixed reduction; negotiated services, instead, are considered when the fares are based on bilateral agreements. Some practical examples can be found for instance in existing mobile applications. MaaS Global, the developer of the Whim App (Whim, 2023), purchases mobility services in advance, such as bus, taxi, and bike rides, based on users' monthly trips and profiles. These rides are then combined into packages and sold for profit. In Berlin, through the Jelbi App (Jelbi, 2020), Berliner Verkehrsbetriebe (BVC) has the task of handling contracts with MSPs to have a high level of integration for users that can pay for each mobility service directly on the app. The Trafi company handles the integration. Trafi and BVG are not involved in the in the payment process, they provide only the integration in the platform. It is clear that the type of agreement adopted is based on the area analysed, regulations, and the number of MSPs willing to participate. In this context, a MaaS model must be general enough to capture the possible business agreements between MSPs and the provider.

Lastly, at the heart of this ecosystem are the users, who, after subscribing to the MaaS system, choose the mobility package that best reflects their travel needs. The research has predominantly centered around studying user preferences, defining user profiles, and understanding their willingness to pay for the services (Tsouros et al., 2021). Generally, these studies use stated-preference (Ho et al., 2018; Matyas & Kamargianni, 2017) or revealed-preference approaches (Musolino et al., 2023; Sochor et al., 2016). The first approach involves conducting surveys, wherein users express their preferences regarding various MaaS scenarios and they state their existing travel choices. This data is then utilised to validate and calibrate discrete choice models, enabling the estimation of MaaS choices. The second approach, instead, is mainly focused on users participating in pilot projects.

Throughout and upon the completion of the project, users are engaged with various questions to comprehend aspects related to their willingness to transition from PCs to MaaS subscriptions (Storme et al., 2020) or to select between different MaaS bundles (Hensher et al., 2021a). Nonetheless, the outcomes from these approaches show a strong contextual dependency, making it challenging to draw clear generalizations that can be generally applicable.

2.3 Levels of Integration

Traditionally, urban mobility has been characterized by fragmented and disjointed transportation options, each operating in isolation with separate ticketing systems and booking procedures. At its core, MaaS, instead, is intricately connected with the idea of mobility integration. However, a common strict definition of which components should be included in order to differentiate MaaS from a simple journey planner service has not been developed yet. For this reason, in this section a literature review on MaaS levels of integration is carried out in order to define a possible list of elements that need to be included in the MaaS ecosystem.

First, Kamargianni et al. (2016) defined different types of integration: i) ticket integration, when using one smart card to have access to multiple modes of transport; ii) payment integration, through one account travellers can pay all modes of transport; iii) journey planning integration, having information regarding the best modal options in between two locations; iv) booking function, to reserve one of the available modes of transport; v) mobility package integration, combination of modes of transport into packages that the customer can pre-pay, in order to use them for a specific amount of time or distance. In this scenario, a service has partial integration when including a combination of ticketing, payment, journey planning or booking. Advanced integration include all these four component. Finally, advanced integration with mobility packages is the maximum level including all components.

Sochor et al. (2017), instead, proposed a MaaS topology with five levels, where each level of integration is not automatically dependent on the others. At Level 0 there is no integration, and each service works separately; Level 1 represents the integration of information with the purpose of giving the best solution to users for a single trip; Level 2 includes the integration of booking and payment; Level 3 represents the integration of service offer where users subscribe and pay for a mobility package; Level 4 is the integration of societal goals in order to reduce car ownership and improve life in cities.

In Cruz and Sarmento (2020a), and citation therein, seven levels of MaaS have been proposed. Level 0 is the base level in which a digital interface is available for individual modes of transport. Level 1 is a one-to-one integration between some transport services. Level 2 includes the integration of ticketing and payment of a limited number of public and private providers. In Level 3 there is the presence of a unified interface for multiple modes of transport. Level 4 represents total integration of all modes of transport including routing, ticketing and payment. In Level 5 AI-driven decisions are influenced by travelers' preferences and up-to-date data to adapt to changes during a journey. Lastly, Level 6 represents the connection with internet of things, and smart cities.

Finally, Hensher et al. (2020) represent the progression from uni-modal and PAYG mobility services towards a "high level MaaS", in which users have an account with a mobility wallet, they can subscribe to bundles, paying directly trough the app, have trip planning and feedback on travel activity.

In this thesis a MaaS system should include all the following components:

- **Ticketing integration**: to have access to several modes of transport available in the area just in one platform (considering also the same mode of transport from different companies);
- **Payment integration**: to have the option to select a preferred payment method and to purchase mobility packages or single trip directly on the same platform;
- **Journey planning integration**: once the user selects a trip to take, the platform has to be able to calculate different mobility options following user's preferences and to give additional travel information;
- **Booking integration**: to have the opportunity to book directly on the App the preferred mode of transport;
- **Mobility packages integration**: to allow consumers to choose between different mobility packages (a mix of mobility services) and buy them for a fixed price. Considering also the PAYG option;
- **Responsibility and customer support**: Support the user with information and to solve issues;
- **Regulation/Subsidy**: Policies and Subsidies from the Government side to the system, in order to reduce car ownership and the impact of transport on the environment (this integration aspect is not seen directly on the platform, but it helps to run a successful ecosystem).

2.4 MaaS in Practice

Ever since the MaaS concept emerged, local authorities and academics have been curious about its potential impact on transportation, on users' modal choices (de Luca & Mascia, 2021) and willingness to subscribe (Tsouros et al., 2021), as well as on suppliers participation (Wong & Hensher, 2021). To explore these aspects, various pilot projects have been carried out worldwide. These projects involve testing the MaaS idea on a smaller scale in specific areas to see its feasibility and what effects it might have on transportation habits.

This section involves conducting an analysis of the pilot projects carried out between 2013 and 2023 to comprehend the lessons learned and determine the feasibility of establishing guidelines or common practices for the development of a MaaS platform. To conduct this study academic paper, project reports and websites have been examined.

The first and most well-known MaaS pilot took place in Gothenburg, Sweden, starting in 2013, at a time when the concept of MaaS had not yet fully developed, as part of the Go:Smart project, proposing a mobile app called Ubigo (Sochor et al., 2015b). After the success of this trial and with the growing notoriety of the topic, several other projects have been conducted as shown in Figure 2.2. The figure illustrates the development of various projects predominantly situated in Europe. It is noteworthy that several of these projects received funding from the European Commission through the EU Horizon 2020 program.

In Tables 2.1, 2.2 and 2.3 the information collected from the most relevant pilot projects found is summarised. It is clear that each pilot has different characteristics based on the purpose, in the way that it has been developed and mostly in the city in which it has been applied. It seems difficult to identify some positive or negative common aspects of these pilots based only on the information given above. For this reason, following a SWOT (Strengths - Weaknesses - Opportunities - Threats) analysis, Table 2.4 presents a compilation of suggested actions derived from the lessons learned, deemed essential for the implementation of a MaaS pilot.

In the pursuit of developing MaaS systems, various attempts have been made through pilot projects; however, notable and enduring large-scale implementations have remained scarce. This could be attributed to several factors, including inadequate financial support following the pilot phases, reluctance among MSPs to share data, challenges faced by public entities in managing payments to and from private companies, limited awareness of the MaaS concept among the majority of users, partial integration of mobility services, and the amalgamation of diverse mobility services with disparate business models. Nevertheless, in recent years, several business applications have emerged, exhibiting varying degrees of success. Among these, the renowned Whim app (Whim, 2023), launched in 2016 by MaaS Global under the leadership of CEO Sampo Hietanen, stands out prominently. Originally conceived as a MaaS app for the city of Helsinki, it subsequently expanded its presence to other urban centers such as Vienna and the Greater Tokyo area. Presently, Whim app is operational solely in Helsinki and Turku, showing that the durability of these systems is still fragile. Due to these

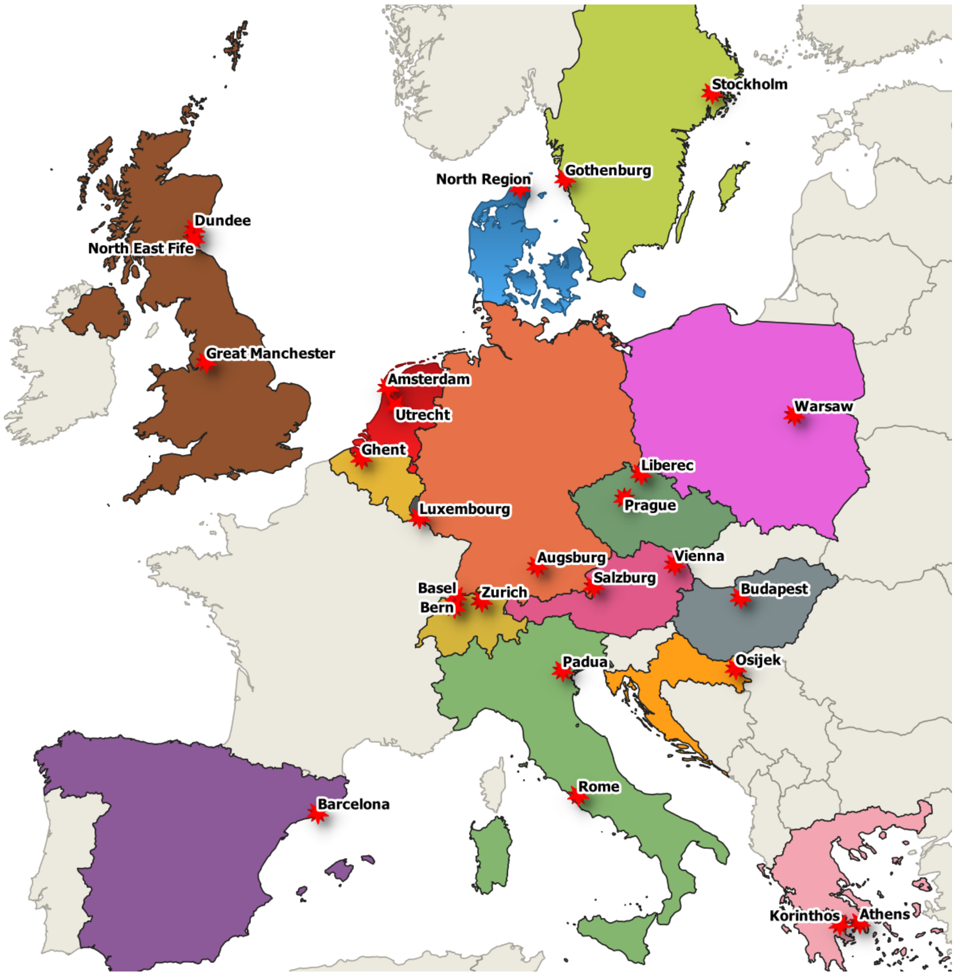


Figure 2.2: MaaS Pilots in Europe

considerations, it is essential to comprehend the consequences that emerge during the implementation of MaaS. These include examining possibilities for enhancing efficiency and service quality, as well as understanding the risks associated with the entire system becoming highly unstable or analysing what happens when a service exits/enters the market. In the transportation domain, two commonly used methods for evaluating potential scenarios are simulation and mathematical modelling.

2.5 MaaS Simulation Models

In transportation, simulation approaches refer to software tools that replicate and analyse the performances of a specific transport system (Perrone & Fujimoto, 2006). They can be grouped in four categories: macroscopic, microscopic, mesoscopic and hybrid models. Macroscopic simulation models are used to analyse and replicate the behavior of large-scale transportation systems, they usually take into account aggregated values of traffic flow, speed and density. Microscopic models, instead, focus on simulating individual vehicle movements and driver behaviors within the transportation network. They are based on "car-following models, lane-changing and gaps of the individual drivers" (Nasuha & Rohani, 2018). Mesoscopic simulation strikes a balance between macroscopic and microscopic models. Considering single agents, whose actions are influenced by aggregated traffic flow characteristics. Finally, hybrid simulations tend to overcome the limitations of all the aforementioned models combining their strengths to provide a more accurate and flexible representation of traffic behavior at different scales.

In several instances, researchers, in order to represent MaaS scenarios, have utilized MATSim (W Axhausen et al., 2016), a sophisticated agent-based and mesoscopic traffic simulation model. MATSim is an activity-based and multi-agent simulation tool implemented in Java. Built upon the daily activities performed by agents, it has been designed to model a single day in large-scale scenarios. Inside the defined network users compete between each other for time-space slots, while optimizing their daily activity trip chains (Horni et al., 2016). In this context, through an analysis of users' preferences, Cisterna et al. (2021) and Cisterna et al. (2022) conducted MATSim simulations comparing scenarios with and without the MaaS package, focusing on the synthetic network of Berlin. They explored users' choices between PT and car-sharing services. Building upon the same base scenario, Cisterna et al. (2023) further investigated the influence of total cost of ownership on MaaS demand. A more complex study by Becker et al. (2018) applied MATSim to simulate the use of MaaS services in the city of Zurich. Specifically, considering bike, PC, PT, electric bike-sharing, car-sharing and ride-hailing as available modes for the agents. Different scenarios have been studied to study how MaaS could increase the efficiency of the analysed transportation system. Focusing on competition between shared modes and their profitability, the effects at system level and their impact on low-density areas.

El-Agroudy et al. (2021) proposed, instead, a study in a multi-modal corridor in Orlando as a small-case scenario for the evaluation of MaaS implementation using VISSIM. A "microscopic, behavior-based multi-purpose traffic simulation to analyze and optimize traffic flows" (Fellendorf & Vortisch, 2010) implemented in C++. The main focus of El-Agroudy et al. (2021) was on performance impacts and interactions of multiple modes (personal vehicles, transit, ride-hailing, micro-mobility, and walking).

Although in some cases it is possible to analyse the impact of MaaS on complex scenarios involving several modes of transport, it becomes evident that agent-based simulation approaches tend to be tailored to specific cases, necessitating extensive data on users, modes of transport, and geographical locations. Being still in the phase of comprehending the underlying phenomena of this dynamic system and seeking to evaluate the potential scenarios that might emerge while adopting MaaS, the construction of mathematical models assumes paramount importance. These models could provide a valuable means to analyse and predict the possible states and outcomes of the MaaS system, to explore its behavior, interactions, and implications in a more systematic and comprehensive manner.

2.6 MaaS Models

A successful MaaS implementation relies on understanding the interaction and decision-making strategies of all actors in the MaaS ecosystem. Although in the literature aspects concerning the relevance of suppliers joining MaaS (Polydoropoulou et al., 2020), the inclusion and key role of PT in the ecosystem (König et al., 2016), and direct collaboration with the government (Wong & Hensher, 2021) have been analysed, a model that captures the complex interaction between services and actors (e.g., comparing competition vs cooperation strategies) has not been introduced yet. As a result, the aim of this section is to comprehend the process of modeling this complex system involving multiple actors and modes of transport. To achieve this, various modelling approaches that have been suggested in the existing academic literature are examined. The main purpose of this section is to identify any existing gaps and work towards the development of a more comprehensive model.

2.6.1 The two-Sided market

As already mentioned in the previous sections, the intricate dynamics and relationships that may exist between the various actors make the MaaS system exceptionally challenging to model. In addition, MaaS is not solely a multi-actor system; it also involves multiple modal choices on the users' side and various forms of implementation of mobility bundles, which result in partial cooperation between MSPs within the same package. To overcome some of these issues, in the literature some studies have been focusing only on the interaction between users and MSPs. This relationship has been represented as a two-sided market (Calderón & Miller, 2020) (or multi-sided platform). Using this approach, a platform supports the interaction between different sides and, unlike usual transportation models, it has to be attractive to MSPs and users (Meurs & Timmermans, 2017). In their discussion paper, Meurs and Timmermans (2017) define important factors to consider when modelling MaaS as a multi-sided platform. The demand can choose to use the MaaS application, where several services are offered, or directly purchase each mobility service separately. Utility functions can be defined for each service, considering classical mode choice characteristics related to the mobility service and to the users, but also taking into account new aspects connected to uncertainty and trust. MSPs, instead, might participate in the platform only if the service becomes profitable. Each MSP seeks to maximise their profit function, which depends on "the number of users of the services, price/fares of the trips, the marginal costs of the trips per traveller as well as fixed costs of the service provider and costs of the platform". The authors believe that this profit depends on three main factors: (*i*) demand, (*ii*) costs, and (*iii*) competition strategy. It seems extremely important to quantify the impact of competition between different MSPs joining the MaaS platform to understand their willingness to participate. In this context, the authors suggest game theory to study the behaviour of all MSPs at equilibrium. Albeit the interesting suggestions, this work does not include a precise modelling solution.

More practically, extending their one-side simulation-based method (Djavadian & Chow, 2017a), Djavadian and Chow (2017b) proposed an agent-based stochastic user equilibrium (SUE) under two-sided flexible transport market. Flexible transport services (FTS) (such as demand-responsive services, shared services and taxis) are modeled as sellers, users are buyers and the platform represents the built environment. This simulation process tend to adjust users' choices and FTS's operating policy. However, in the application the model assumes that travelers primarily use FTS as a first/last mile connection while relying on transit services for the main distance. Moreover, the work overlooks several key aspects of the MaaS concept. For instance, it doesn't incorporate all the diverse mobility services available in the area, and it doesn't include any type of package subscription. Xi et al. (2022) modelled the two-sided MaaS market as a single-leader multi-follower game. Specifically, the MaaS regulator (platform) is considered to be the leader with MSPs and travellers as two groups of followers. The leader aims to maximize profits by adjusting the prices for travellers and MSPs and creating MaaS bundles. Travellers seek to minimize their travel costs by selecting the most convenient means of transport within the MaaS system, whereas MSPs aim to maximize profits by choosing the proportion of their mobility resources supplied to MaaS. The number of participating MSPs and traveler requests depends on the prices set by the leader and other participants. The methodology uses a name-your-own-price auction-based mechanism, where users set their bid to have access to MaaS, and the regulator accepts or rejects them based on internal threshold price. While this approach effectively represents the dynamics of the platform, it does not account for potential interactions with modes of transportation that are not integrated into the platform.

2.6.2 Multi-modal and multi-actor system

It is clear that, in order to model a multi-modal and multi-actor system such as MaaS, classical transportation approaches have to be extended. Following this purpose, in their literature review, Pham et al. (2021) seek to identify the accessibility indicators that can influence the interaction between the different MaaS actors in order to develop a conceptual framework to model them. The main findings of this study underline the presence of several gaps in the transportation literature. In particular, current models do not consider (*i*) psychological indicators to quantify demand-supply interaction; (*ii*) dynamic pricing; (*iii*) monthly service users to optimise the offer; (*iv*) the efficiency of the entire transport system; and (*v*) MSPs' point of view when defining packages and mobility options based on users' preferences and available services.

A first step towards a more comprehensive modelling framework has been proposed by Kamargianni et al. (2019), which is divided into five components:

- The MaaS Market Model, which describes the business ecosystem. Based on which type of MaaS Broker (private company, private MSP, or PT authority) is involved, the different objectives, partnerships and commercial agreements with other MSPs, it will be able to capture which type of offer to propose to users and the different pricing strategies.

- The MaaS Integration Controller is a platform that replicates the functionalities of the MaaS platform.

- The Demand Modelling Framework is an agent-based model that can evaluate short-term and long-term users' decisions connected to MaaS subscriptions, vehicle ownership, or daily activity.

- The Multimodal Network Model simulates traffic and, in general, private and public vehicles used in the network.

- The Mobility Service Controllers manage and control the system's different services.

This general framework is combined with the simulation model SimMobility (SimMobility, 2023), an agent-based, activity-based, multi-modal simulation platform that models individual travel decision-making and transportation systems operations at different time scales. The cited work, however, proposes a framework without showing any application in a real scenario. Furthermore, the role of the government or the local authorities does not appear crucial for the development of the MaaS system.

Pantelidis et al. (2020), instead, characterized the MaaS problem as a many-to-many assignment game. In their model, users are paired with a viable path that connects their origin and destination (OD). These paths can potentially involve multiple MSPs, each of whom possesses one or more links within the network. Through the numerical example, different scenarios are analysed, such as the entry of a new competitor in the market or the acquisition of one of the two operators from a government agency. However, the MaaS structure is formulated as a matching problem in which no congestion effects derived by the usage of different modes of transport are considered.

More recently, van den Berg et al. (2022) developed an economic framework to analyse the effect on pricing, profits and welfare based on different MaaS strategies. Specifically, they defined a model in which two mobility services are represented through a supply chain structure with four links connecting an OD. In this setting they could analyse what happens with and without MaaS platform adopting different strategies. Nevertheless, congestion is not taken into account as well as the heterogeneity of users and their choices.

2.7 Conclusion

This Chapter presents a comprehensive literature review focusing on key concepts associated with MaaS. Despite being a relatively new mobility solution, both research and the transportation market have seen various initiatives in this domain. To fully analyze the MaaS system, it necessitates a seamless collaboration between governmental authorities, MaaS operators, and MSPs, all working together to deliver a fully integrated transportation system to end-users. This integrated system encompasses ticketing, payment processing, journey planning, booking services, mobility packages, responsibility allocation, customer support, and adherence to regulations and subsidies.

Numerous pilot projects have been undertaken worldwide to explore the potential of MaaS. However, achieving large-scale implementations remains a significant challenge due to several factors. These challenges encompass issues like insufficient funding post-pilot, MSPs' hesitance to share data, complex payment arrangements involving both public and private entities, limited user awareness of MaaS, partial integration of services, and the coexistence of various business models. Understanding the consequences of MaaS implementation, such as enhancing efficiency and effectively managing system stability and market dynamics, is of essential importance.

Despite extensive discussions and pilot applications, there has been a noticeable gap in studies that aim to model and assess the impact of MaaS on transportation networks. This gap primarily arises from the complexity of MaaS systems, where diverse stakeholders with varying objectives must coexist and interact within the same ecosystem. Enhancing classical transportation models becomes indispensable when aiming to realistically capture the characteristics and impacts of MaaS on the transportation network. Consequently, the following chapter will undertake a thorough examination of conventional transportation modeling within the framework of multi-modality and multi-actor systems. This analysis seeks to gain insights on how to construct a more comprehensive model that effectively accounts for the intricacies of MaaS, and generally complex hierarchical multi-modal transportation systems.

Table 2.1: MaaS Pilots analysis(1)

Project Name	Go:Smart	SMILE	-
Service Name	Ubigo	Smile	-
Location	Gothenburg, Sweden	Vienna, Austria	Ghent, Belgium
Working Period	Nov 2013 - April 2014	Nov 2014 - May 2015	April 2017 - June 2017
Integration[1]	● ● ● ● ●	● ● ● ●	● ● ●
Type of subscription	Monthly subscription	PAYG	PAYG (With monthly budgets)
MaaS Provider	Ubigo as Broker	Cooperation: Wiener Linien and Österreichische Bundesbahnen ÖBB	Touring Club Belgium
Modes of Transport involved	PT, Taxi, Carsharing, Bikesharing, Car Rentals	PT, Taxi, (e-)Carsharing, (e-)Bikesharing, Parking	PT, Bikesharing, Bike rental, Public and Private Carsharing, Car rental, Taxi
Type of customers	Urban Households	Users between 20 and 40 years old	Car-owners (i.e., Ghent University employees, 77.7% younger than 45)
Number of customers involved	70 Paying households. 195 people: 173 adults and 22 children	Over 1,000 pilot users	90
Funded by	Vinnova and Chalmers Area of Advance Transportation	The Climate and Energy Fund	Touring Club Belgium
Documents analysed	(Sochor et al., 2014) (Sochor et al., 2015a) (Karlsson et al., 2016)	(Smile, n.d.)	(Storme et al., 2020)
Project Name	Pick&Mix	MinRejseplan	CiViTAS Eccentric
Service Name	Navigogo	MinRejseplan	Ubigo
Location	Dundee and North East Fife, Scotland	Rural North Denmark Region	Stockholm, Sweden
Working Period	Oct 2017 - March 2018	May 2018 - not found	Oct 2018 - Oct 2019
Integration	● ●	●	● ● ● ● ●
Type of subscription	Young Scot card + PAYG	PAYG	PAYG billed monthly
MaaS Provider	ESP Group (lead partner)	Nordjyllands Trafikselskab (North Jutland's Traffic Company, NT) and Rejseplanen	Ubigo: MSP / project lead Fluidtime: SaaS provider / MaaS enabler
Modes of Transport involved	PT, taxis, bike schemes and car clubs	PT, city bikes, taxis, carpooling, car sharing and DRT	PT, Taxi, Carsharing, Bikesharing, Car Rentals
Type of customers	for 16 – 25 years old	Rural Areas	Households
Number of customers involved	100 young people	Free download	44 Households (50 people)
Funded by	Innovate UK	Nordjyllands Trafikselskab and Trafikstyrelsen	European Commission through EU Horizon 2020 program
Documents analysed	(Navigogo, 2018) (MaasAlliance, 2018)	(Barreto et al., 2018) (Christina Hvid, 2018) (Clienti., 2018) (Åsa Ström Hildestrand, 2018)	(MaaSAlliance, 2019a) (Fenton, 2020) (Trivector, 2020)

[1] Connected to section 2.3 (In case of partial integration the bullet points are less than seven)

Table 2.2: MaaS Pilots analysis (2)

Project Name	Mobil-Flat	MyCorridor	MaaS4EU
Service Name	Mobil-Flat	MyCorridor	MaaS4EU Budapest
Location	Augsburg, Germany	Salzburg, Athens and Korinthos, Amsterdam, Rome, Prague	Budapest, Hungary
Working Period	Nov 2018 - Oct 2019	June - Oct 2020	Sept 2020 Wave 3
Integration[2]	● ● ● ● ●	Variable	● ●
Type of subscription	Monthly package	Variable	Monthly subscription
MaaS Provider	swa Augsburg		Toll Service
Modes of Transport involved	PT, Carsharing, and Bikesharing	PT, Taxi, Carsharing, Bikesharing	PT, Taxi, Carsharing, Bikesharing, Ridesharing
Type of customers	Inhabitants of Augsburg		Residents and Tourists
Number of customers involved	45	200 (for each pilot)	338 (free download)
Funded by		European Commission through EU Horizon 2020 program	European Commission through EU Horizon 2020 program
Documents analysed	(Reck & Axhausen, 2020) (Simon-Kucher, 2018)	(MyCorridor, 2020) (Zankl et al., 2020)	(Siklósi, 2020) (Esztergár-Kiss & Aba, n.d.) (Kamargianni, 2020)

Project Name	MaaS4EU	MaaS4Eu	Sydney MaaS Trial
Service Name	MaaS4EU Manchester	MaaS4EU Luxembourg	Tripi
Location	Great Manchester, UK	Luxembourg, Luxembourg	Sydney, Australia
Working Period	Virtually (due to Covid-19)	Virtually (due to Covid-19 and free PT)	Nov 2019 - April 2020
Integration	NA	NA	●●● ● ●
Type of subscription		Monthly subscription	Monthly subscription with packages
MaaS Provider	TFGM	Sales-Lentz Autocars	Institute for Transport and Logistic Studies (ITLS),Insurance Australia Group (IAG),Skedgo and iMove CRC
Modes of Transport involved	Bus, Tram and Uber	car sharing, taxi, e-bike, airport shuttles, door to-door night bus	PT, taxi, car rental, Uber and car sharing
Type of customers	Residents, Tourists and B2B solutions	Residents and B2B solutions	IAG employees
Number of customers involved	50 (online)	-	171 participant months
Funded by	European Commission through EU Horizon 2020 program	European Commission through EU Horizon 2020 program	iMove Cooperative Research Centre (CRC) Program
Documents analysed	(Kamargianni, 2020) (Li, 2020)	(Kamargianni, 2020) (Zorn, 2020)	(Hensher, 2020) (iMove Australia, 2020) (Hensher et al., 2021a) (Hensher et al., 2020)

[1] Connected to section 2.3 (In case of partial integration the bullet points are less than seven)

Table 2.3: MaaS Pilots analysis(3)

Project Name	Mobility as a Service - regional pilots	Empirical use and Impact Analysis of MaaS	IP4MaaS
Service Name	Gaiyo	Yumuv	Travel Companion
Location	Utrecht, Netherlands	Zürich, Basel, and Berne (Switzerland)	Athens (in progress in other European cities in Fig 2.2)
Working Period	Sept 2020 - mid 2022	end 2020 - end 2021	Jul 2022 (2 weeks)
Integration	● ● ●● ●	● ●●● ●	● ● ●● ●● ●
Type of subscription	PAYG	Monthly subscription and PAYG	Monthly packages and PAYG
MaaS Provider	Goedopweg	Swiss Federal Railways (SBB CFF FFS) and the local public providers	OASA, MIRAKLIO, BRAINBOX, TAXIWAY
Modes of Transport involved	PT, Car sharing, Scooter sharing and Bike sharing	E-scooter, Bike sharing,Bike rental, and Car sharing	PT, bike-sharing, car-sharing, taxi
Type of customers	All residents of Leidsche Rijn, Vleuten and De Meern	Users with a valid PR pass	Local commuters and tourists
Number of customers involved	400 users in the pilot area	NA	140 users
Funded by	The Ministry of Infrastructure and Water Management with 7 Regions	ETH Zurich Foundation	from the Shift2Rail Joint Undertaking under the European Union's Horizon 2020 research
Documents analysed	(Gayo, 2020) (Goedopweg, 2020) (MaaSAlliance, 2019b)	(Yumuv, 2020b) (Yumuv, 2020a)	(Mitropoulos et al., 2023)

Project Name	-
Service Name	Moovit
Location	Tampa (Florida)
Working Period	Dec 2022 - Jun 2023
Integration	● ● ●● ●●
Type of subscription	PAYG
MaaS Provider	The city of Tampa's Mobility Department using Moovit app and HART mobile ticketing system
Modes of Transport involved	eBikes, scooters, and transit
Type of customers	Residents
Number of customers involved	200 users
Funded by	The city of Tampa's Mobility Department
Documents analysed	(Moovit, 2022)

[1] Connected to section 2.3 (In case of partial integration the bullet points are less than seven)

Table 2.4: Fundamental steps for a successful pilot project (from lessons learned)

How to build the Pilot	Phase 1: • Define a strong Business Plan • Identify policy barriers and try to find a connection with the Government • Define a clear Business Model (for MSPs and try to maintain their identity in the service) • Start an information campaign for users, MSPs and PT authorities Phase 2: • Recruit PT authorities and MSPs with a focus also on rural areas. It is important to have more option of the same mode of transport. Try to include services that are used also in other countries • Recruit users. It is important to have a wide and heterogeneous group of users representative of the population. They are important to define packages. Try to not include incentives because they can influence results • Develop the app Phase 3: • Run the pilot
How to build the Service	• Define different mobility packages from users information concerning modes preferences and socio-economic aspects • Include the PAYG option in order to propose to users a more inclusive service
How to develop the App	• Develop a mobile app for Androids and iOS, plus a web page • Develop an interface with multiple languages • Develop an app easy to use for customers with a single account that have direct access to all the services • Develop a direct transfer of information from MSPs side • All modes of transport included, to check tickets, with booking and payment options (also for parking spaces) • Multimodal journey planner with different option following users preferences • Offer a direct alternative in case of problems with the selected mode of transport • Validation of driving license in the app • Customer service • Presence of mobility packages and PAYG option

Chapter 3

Multi-modal and Multi-actor Modelling: A Review

3.1 Introduction

The proliferation of new mobility services in the transportation system has increased the adoption of competition and cooperation strategies in the market, where different MSPs operate in pursuit of both profits and business sustainability. Each MSP tries to generate revenues by applying different policies, for instance by changing prices, varying their fleet size, or cooperating with other mobility suppliers to attract a sufficient share of the market demand. In this context, users' modal choices play a fundamental role in determining the durability of mobility services inside the transportation system. Therefore, it is essential to design an appropriate transportation modelling approach to predict how users will make choices and react to MSPs' strategies. Studying such interactions helps to understand in which conditions an MSP can have a profitable business, what happens when a competitor enters the market, or if cooperation can improve their value in the network.

The problem of studying the interactions between transport operators and travellers has been widely studied in the literature in the context of unimodal networks (Farahani et al., 2013). When this interaction is analysed in a network where several modes of transport coexist, the problem can be defined as the Multi-modal Network Design Problem (MNDP) (Montella et al., 2000). In this system, there is the precence of multiple modes of transport operated by different MSPs. Generally, MNDPs are well known for their modelling complexity, arising from the structure of the multi-modal network, the definition of travel demand and users' modal choices, and the formulation of the interaction between suppliers and demand.

In this chapter these different aspects are examined through a literature review. In particular, Section 3.2 delves into the diverse approaches employed for defining multi-modal networks. In Section 3.3, the focus shifts towards explicating the definitions of travel demand and the distinct mathematical formulations found in the literature. Lastly, Section 3.4 describes the interaction between MSPs and travellers, encompassing different formulations proposed in the existing transportation literature.

3.2 Multi-modal Networks

Generally, the transportation network structure is often depicted using a graph. A graph is a visual representation consisting of nodes and links. Each node typically corresponds to a geographical location or a point of interest within the transportation system. Centroids are special nodes in which individual trips begin or end. On the other hand, a link serves as a connection between two nodes, symbolizing various elements such as physical roads, or even specific activities like waiting at a bus stop. A path is, instead, a collection of consecutive links connecting one centroid of origin (O) to a centroid of destination (D). Each path is exclusively associated with a single OD pair, and it is possible for multiple paths to connect the same OD (Cascetta, 2009).

As described in Fiorenzo-Calatano (2007) networks could be classified in two categories: uni-modal and multi-modal. The first type of network generally represent a single mode of transport. They could be divided in continuous and simultaneous private transport services (PC, bike and walk), and discontinuous and non-simultaneous PT services (train and bus). Usually, when representing a network with multiple modes of transport, multi-modal networks are used. These networks are more complex to represent. According to van Nes (2002) there are three options to reduce the complexity of the representation of a multi-modal network. (i) Using a combination of a reduced number of modes of transport (Liu et al., 2015b); (ii) focusing on a simple case, e.g. a transport corridor (Tirachini et al., 2014); or (iii) using a supernetwork approach (Sheffi, 1985), where the multi-modal network is divided in several uni-modal layers connected by transfer links (Carlier et al., 2003), as shown in Figure 3.1. In more traditional modeling approaches, like the one proposed by Montella et al. (2000), the network is represented through two non-connected uni-modal networks. In such a representation, even though the transportation system may consist of multiple modes of transport, users are limited to choosing only uni-modal paths. The purpose of this thesis is to represent a realistic and exhaustive multi-modal network. To achieve this, a supernetwork approach is adopted, which allows for the inclusion of a wider range of modes of transport and the future scalability of the problem.

The concept of hypernetwork (or supernetwork) was first introduced by Sheffi and Daganzo (1978) with the idea of representing the diverse choices made by users in terms of mode, route, and destination as path choices

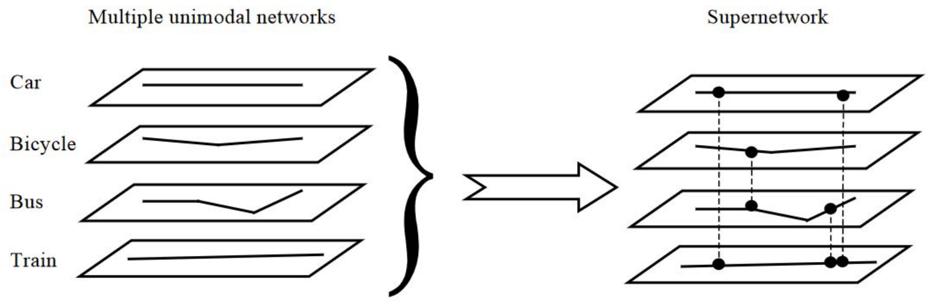


Figure 3.1: Multi-modal Supernetwork (Fiorenzo-Calatano, 2007)

within an hypothetical network (as shown in Figure 3.2). By conceptualizing these choices in a single supernetwork, it provides a powerful framework to analyse and optimize the complexities of real-world transportation systems and user decision-making processes.

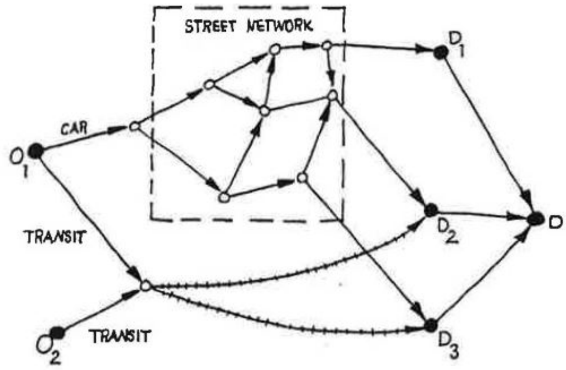


Figure 3.2: Hypernetwork with mode, route, and destination choice (Sheffi & Daganzo, 1978)

During the years the structure of these supernetworks have been used to model decision-making problems in the context of supply chain with e-commerce considering environmental criteria (Nagurney & Toyasaki, 2003), including supply and demand risks (Nagurney et al., 2005), or to reformulate financial networks as transportation networks (Liu & Nagurney, 2007). In transportation, supernetworks have evolved in order to represent more complex multi-modal trips (Carlier et al., 2003). However, as transportation theory has advanced, it has become apparent that trip-based networks have certain limitations in accurately predicting users' transportation choices. In practice, trips are part of a tour. A tour is a loop of trips that starts and ends at a home or base location. The purpose of such tours is to engage in one or more activities at different locations during the journey (Arentze & Timmermans, 2004). Therefore, modelling the travel demand considering the activities that users perform at different destinations is fundamental in MNDPs (Liu et al., 2015a). Hence to predict transport demand, it is of paramount importance to model trip chains and their impact on mode choices on a daily horizon. For this reason, supernetworks have been naturally extended to include users' daily trip chains in multi-modal contexts within activity-based models (Arentze & Timmermans, 2004; Liao et al., 2010) and in multimodal network assignment (Fu & Lam, 2014; Liu et al., 2020). These models are based on the assumption that, during a day, each travel choice made by users is influenced both by earlier decisions and by planned later trips (Scheffer et al., 2021).

In this thesis a supernetwork approach that considers sequential mode choices determined by users' trip chains is defined. In Section 4.2 the structure and the assumptions will be described. Due to the complexity of the problem caused by the combinatorial explosion of trip chain options in time and space, and since the aim of this work is to develop a strategic long-term equilibrium model, the methodology is applied to a static system in which time of departure/arrive from/to a location or duration of the activities performed at each destination are not explicitly considered.

3.3 Travel Demand and Traffic Assignment

Travel demand represents the total number of trips users want to make within a specific area, usually between a designated origin (where the trip begins) and a designated destination (where it ends). The travel demand directly derives from users' need to perform different activities in various locations (Cascetta, 2009). The choices made by users, including their preferred modes of transport, the number of activities performed in different locations, and departure times between activities, have a substantial influence on the performance of the transportation system and the potential profitability of various MSPs. Given these significant impacts, it becomes crucial to model the travel demand in a manner that closely reflects real-world behavior and aligns with the objectives of the study.

Different are the assumptions that can be employed in relation to the travel demand. It can be considered as either fixed, where the total flow remains unaffected by changes in the network, or variable, where it is influenced by congestion costs. Moreover, users can be categorized into a single class when they possess different OD pairs but have similar perceptions of costs and decision-making behaviors. In contrast, users can be divided in multiple classes. The first studies on multi-class users were developed by Dafemos in the early 1970s distinguishing passengers of PC and trucks (Dafermos, 1971, 1972). Generally, multi-class users are divided into distinct classes based on their varying cost perceptions and decision-making tendencies. In this case, within each class, individuals share similar cost perceptions and behavior patterns. Users can be divided in classes based on the modes of transport at disposal or, to represent a more realistic system, based on their socio-economic attributes and trip chains (Hasan & Dashti, 2007). Introducing multiple classes in travel forecasting models adds mathematical complexity. Single class models have simpler separable and symmetric travel costs, enabling convex optimization. However, in multi-class models, the costs of one class are influenced by other classes' decisions. This makes the cost structure non-separable, asymmetric, and unsuitable for convex optimization. Nevertheless, in multi-modal context it is fundamental to reproduce the heterogeneity of users' choices to properly determine path costs and to perform assignment procedures. These procedure have the purpose of distributing the traffic flow across a transportation network. Specifically, the traffic assignment method takes as input an OD matrix, representing the travel demand for each OD pair inside the network, giving as output the estimated traffic flow on the paths connecting each OD, usually distributed based on the corresponding travel times (or costs). Path costs could be additive or non-additive. In the first case, the cost of a path is the sum of the costs of its constituent links. On the other hand, with non-additive path costs, the total cost on a path is equal to the sum of the link travel costs plus a path-specific cost (Han & Lo, 2004). To calculate the travel time (or cost) on a specific link within the network, each link is associated with a link performance function. Commonly, in order to take into account congestion effects, the parameters of these functions change based on the number of vehicles using that particular link.

Assignment models can be classified into static and dynamic. Static traffic assignment operates under the assumption of a uniform distribution of travel demand across time, for this reason are mainly used to represent large-scale transportation planning problems. The traffic flow model relies on link performance functions that associate the flow travelling through each link with the corresponding travel time. This implies that as more travellers choose a route, the travel time for that route increases (Boyles et al., 2020). In contrast, dynamic assignment methods take into account traffic flow that evolves over time on the network, and therefore analyse congestion conditions in traffic control and management (Saw et al., 2015). Different static assignment models have been developed, here the most common methods are introduced. One of the most common is the all-or-nothing assignment; this unrealistic method assigns the flow only on the shortest path (e.g. with minimum cost or travel time) connecting one OD pair without considering traffic congestion or road capacity. The user equilibrium (UE) assignment, instead, is based on Wardrop's first equilibrium principle (Wardrop, 1952), where at equilibrium all used paths have equal minimum travel time (or cost) so that no user has any incentive to change path. Finally, the system optimum (SO) assignment finds its origin from Wardrop's second equilibrium principle, in which the entire transport system cooperate in order to minimize the total travel cost. Dynamic assignment represents an evolution of static assignment, wherein the route choice are chosen following the dynamic UE or the SO. Nonetheless, in this scenario, the travel time taken into account is not instantaneous, but the routes are chosen based on the experienced time. As congestion fluctuates over time, the paths with the lowest costs are also dynamically changing. Generally, these models can be divided into analytical and simulation-based. The former, it is characterised by mathematical programming or VIs. The simulation-based approach, instead, is developed in two steps: the dynamic network loading to compute the travel time on paths, and an algorithm to find the UE (Ameli et al., 2020).

As already mentioned, the focus of this thesis is on the canonical static assignment based on UE principle, also known as traffic network equilibrium, with a focus on multi-class formulations. Although Wardrop's first and second criterion were introduced by Wardrop (1952), the terminology of UE and SO were subsequently defined by Dafermos and Sparrow (1969). Furthermore, while Wardrop defined UE, he did not provide a mathematical formulation for the problem. The first formulation was introduced by Beckmann et al. (1956), who defined it as a convex optimization problem. Subsequently, other formulations emerged, as VI (Dafermos, 1980; Smith, 1979), and nonlinear complementary problem (Aashtiani & Magnanti, 1981). While UE can typically be expressed either in a link-based or path-based formulation, the majority of proposed solution algorithms favor the link-based approach. One significant advantage of this approach lies in its avoidance of requiring enumeration of the entire choice set of possible paths. As a result, link-based methods are notably more suitable for the efficient resolution of large-scale networks (Bekhor & Toledo, 2005). In the upcoming subsections, the different mathematical formulations of UE will be introduced and described. For each introduced formulation, consideration will be given to the existence and uniqueness of a solution. It is clearly important to establish that a given formulation does indeed have a solution; this is the concept of 'existence', that there is a least one solution that satisfies the conditions and constraints of the problem. Proving the existence of solutions often relies on the model being continuous, implying that continuous changes in the input parameters lead to continuous changes in the solution, or output, of the model e.g. the equilibrium link flows. Continuity is a desirable property in its own right. A discontinuous model would be highly problematic for forecasting, conducting sensitivity analysis, or optimization.

Once existence has been established, it is natural to consider whether the problem may have multiple solutions, which would correspond to multiple equilibria. Solution uniqueness is desirable because it simplifies the interpretation and prediction of equilibrium behavior. In the case of multiple solutions, it may be difficult (or impossible) to know which of the solutions will be obtained from the same inputs. This breaks the continuity of the model, causing problems e.g. in establishing a descent direction for optimization. Due to the problems arising from having multiple equilibrium solutions, efforts are often made to constrain the problem formulation so as to force solution uniqueness, without losing the desirable properties of the model. However, this is not always easy or possible.

3.3.1 Convex Optimization Problem

In their seminal paper, Beckmann et al. (1956) identified the relationship between Wardrop's first criterion and the Karush–Kuhn–Tucker (KKT) conditions (Boyce et al., 2005). In this section the formulations presented in the book of Sheffi (1985) are used.

Let a be a link of the network, f_a the link flow on link a and x_p^{rs} the path flow on path p, connecting the OD w. The travel cost $c(f_a)$ is function of the link flow. The link flow can be derived solving the mathematical program:

$$min\, z(\mathbf{x}) = \int_0^{f_a} c_a(\omega)\, d\omega \tag{3.1}$$

$$\text{s.t.} \sum_{p \in \mathbb{P}_w} x_p = d_w \quad \forall w \in \mathbb{W} \tag{3.2}$$

$$x_p \geq 0 \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.3}$$

$$f_a = \sum_{w \in \mathbb{W}} \sum_{p \in \mathbb{P}_w} = x_p \delta_{a,p} \quad \forall a \in \mathbb{A} \tag{3.4}$$

Constraint 3.2 ensures the flow conservation, considering that the total flow on w has to be equal to the total demand on that OD (d_w). For constraint 3.3 the link flow needs to be positive. Finally, equation 3.4 describes the relationship between link flow and path flow, where $\delta_{a,p}$ is the incidence matrix that has value equal to 1 when link a is on path p between OD w, 0 otherwise.

To demonstrate the equivalence of the minimization problem 3.1 with the equilibrium conditions, the first-order optimally conditions are computed. In constrained multidimensional minimization programs, such as this case, the first-order conditions are also known as KKT conditions, necessary to guarantee a solution in these types of problems. These conditions establish that, at the minimum, the gradient of the objective function can be expressed as a linear combination. This combination involves non-negative coefficients known as Lagrangian multipliers and the gradient of the binding constraints. Moreover, the complementary slackness condition is defined, where each Lagrangian multiplier associated to a non-binding constraint is equal zero (Sheffi, 1985). Applying these conditions, the minimization problem 3.1 can be written as:

$$x_p(C_p - u_w) = 0 \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.5}$$

$$C_p - u_w \geq 0 \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.6}$$

$$\sum_{p \in \mathbb{P}_w} x_p = d_w \quad \forall w \in \mathbb{W} \tag{3.7}$$

$$x_p \geq 0 \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.8}$$

$$u_w \geq 0 \quad \forall w \in \mathbb{W} \tag{3.9}$$

Conditions 3.5 and 3.6 represent the UE. Accordingly, it is possible to understand that if the path flow is positive the travel cost on the path, C_p, must be equal to the Lagrangian multiplier u_w connected to w. On the contrary, if x_p is equal to zero, C_p will be greater than or equal to u_w. Conditions 3.7 and 3.8 are the same as 3.2 and 3.3. For more details readers are referred to Chapter 3 in Sheffi (1985).

In order to demonstrate the uniqueness of a solution, instead, the second-order conditions have to be verified. Generally, it is necessary to demonstrate that an objective function $z(\mathbf{x})$ is strictly convex for all values of $\mathbf{x}$ (Figure 3.3). A function is considered convex if the line connecting any two points on the function always remains positioned above the function itself. To demonstrate that a function is strictly convex, the matrix of second derivatives of the analysed function, known as Hessian, has to be positive definite, meaning that at a given point it has all positive eigenvalues.

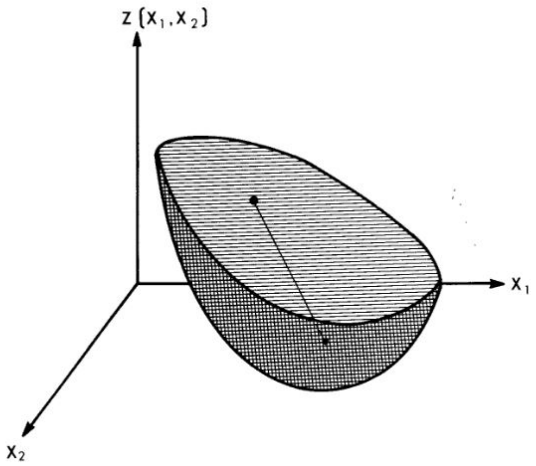


Figure 3.3: Strictly convex function (Sheffi, 1985)

A primary premise of the formulation 3.1 is that the link cost functions related to the transportation network are separable, implying that congestion on each link is solely influenced by the flow traveling through that same link. In this context, it can easily be demonstrated that the Hessian matrix, connected to the proposed mathematical program, is positive definite with respect to the link flows, since it has only values on its diagonal (due to separability) that are positive. Nevertheless, convexity and therefore uniqueness with respect to path flows is not guaranteed. This arises from the observation that within a network, it's reasonable to assume that there are more paths than links. Consequently, several path flow solutions represent the same link flow solution, all while adhering to the UE.

However, in order to model a network that reflects real-world conditions, it's essential to consider that the link flow is also impacted by the flow travelling other links. In Dafermos (1971) demonstrated that this approach can be extended in case of links interaction and for multi-class users. In this cases, it is necessary that the Jacobian of the vector of link cost functions is symmetric. The Jacobian is the matrix of first derivatives of these functions with respect of the corresponding arguments. This notion is further proved by the observation that the Hessian of the mathematical program 3.1 corresponds to the Jacobian of the link travel time vector **c** function of the vector of link flows **f**. This correspondence implies that an objective function has a unique solution when the Jacobian is positive definite.

Although Beckmann et al. (1956) provided a closed formulation to describe the traffic assignment, they did not propose a solution algorithm. The most applied algorithms are of the family of gradient descendent methods. These algorithms iteratively optimize a function in order to find its minimum (or maximum). More specifically, they work by repeatedly adjusting input parameters in the direction opposite to the gradient of the function ($-\nabla C(x)$), aiming to reach the function's minimum value. The most utilised algorithms of this type to solve UE are the Method of Successive Averages (MSA) and Frank and Worlf (F-W). In the first case, the step size decreases inversely proportional to the number of iterations (Mounce & Carey, 2015). In the F-W method, instead, is chosen a step size that minimizes the Beckmann function, through methods like bisection or the Newton's.

3.3.2 Variational Inequality Problem

An alternative formulation of the UE was provided by Smith (1979), and defined by Dafermos (1980) as VI problem applicable in case of link flow interaction, proving existence and uniqueness without the need of imposing symmetry conditions.

Introduced in the early '60s (Hartman & Stampacchia, 1966) to study partial differential equations in mechanics, during the years VIs find application in representing equilibrium problems in diverse fields like economics, physics, and transportation. VI offers a general problem formulation that can describe a wide range of mathematical problems, such as optimization problems, fixed point problems, system of equations and fixed point problems.

Following Nagurney (1998), the vector $\mathbf{x}^* \in \mathbb{X} \subset \mathbb{R}^n$ is a solution of the

VI problem VI($C, \mathbb{X}$), if it satisfies the inequality:

$$\langle C(\mathbf{x}^*)^T, (\mathbf{x} - \mathbf{x}^*)\rangle \geq 0 \quad \forall x \in \mathbb{X} \tag{3.10}$$

with $\mathbb{X}$ a feasible closed convex set with $C : \mathbb{X} \to \mathbb{R}^n$ a continuous function.

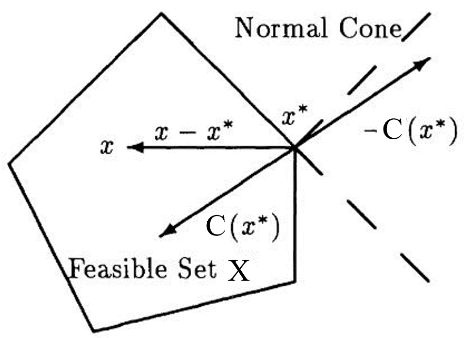


Figure 3.4: Geometric interpretation of VI (Nagurney, 1998)

Geometrically, a necessary and sufficient condition for $\mathbf{x}^*$ to be a solution of VI 3.10 is:

$$-C(x^*) \in N_{\mathbb{X}}(x^*) \tag{3.11}$$

where $N_{\mathbb{X}}$ is the normal cone associated to the convex set $\mathbb{X}$ (Figure 3.4). Given a point x on the boundary of $\mathbb{X}$, from that point a normal consists of all the vectors $\mathbf{y}$ that are orthogonal to the tangent hyperplane at that point. Therefore the normal cone at $\mathbf{x}^*$ is defined by:

$$N_{\mathbb{X}}(\mathbf{x}^*) = \{\mathbf{y} \in \mathbb{R}^n : \langle \mathbf{y}^T, \mathbf{x} - \mathbf{x}^*\rangle \geq 0 \quad \forall x \in \mathbb{X}\} \tag{3.12}$$

Moreover, $\mathbf{x}^*$ is a solution of 3.10 only if the angle between the vector $C(\mathbf{x}^*)$ and $(\mathbf{x} - \mathbf{x}^*)$ is less than or equal to 90°.

In order to prove that the solution of a VI is a Wardrop equilibria, a first connection between the VI problem and the fixed point problem is needed. Following Nagurney (1998):

Theorem 3.3.1. *Suppose that $\mathbb{X}$ is a closed and convex set.* $\mathbf{x}^*$ *is a solution of the VI($C, \mathbb{X}$) if and only if* $\mathbf{x}^*$ *is a fixed point of*

$$P_{\mathbb{X}}(\mathbf{x} - C(\mathbf{x})) : \mathbb{X} \to \mathbb{X} \tag{3.13}$$

in essence:

$$\mathbf{x}^* = P_{\mathbb{X}}(\mathbf{x}^* - C(\mathbf{x}^*)) \tag{3.14}$$

where $P_{\mathbb{X}}$ represents the projection operator. To explain what the projection operator is let $\mathbb{X} \subseteq \mathbb{R}^n$ be a closed and convex set. For each $\mathbf{x}^* \in \mathbb{R}^n \exists! \quad \eta \in \mathbb{X} \quad s.t.$

$$\eta = \underset{\mathbf{x} \in \mathbb{X}}{\operatorname{argmax}} ||\mathbf{x} - \mathbf{x}^*|| \tag{3.15}$$

where η is the projection of $\mathbf{x}^*$ into $\mathbb{X}$. Suppose that

$$\eta = proj_{\mathbb{X}}(\mathbf{x}^*) \Leftrightarrow (\eta - \mathbf{x}^*)(\mathbf{x} - \eta) \geq 0 \quad \forall \mathbf{x} \in \mathbb{X} \tag{3.16}$$

To prove theorem 3.3.1 suppose that $\mathbf{x}^*$ a solution of the VI($C, \mathbb{X}$) 3.10, therefore it is possible to write:

$$(\mathbf{x}^* - (\mathbf{x}^* - C(\mathbf{x}^*)))^T(\mathbf{x} - \mathbf{x}^*) \geq 0 \quad \forall \mathbf{x} \in \mathbb{X} \tag{3.17}$$

Using Equation 3.16, it is possible to derive that :

$$\mathbf{x}^* = P_{\mathbb{X}}(\mathbf{x}^* - C(\mathbf{x}^*)) \tag{3.18}$$

In this context, considering a continuous function C on a compact set $\mathbb{X}$, the existence of a solution of the VI($C, \mathbb{X}$) is guaranteed by Brouwer's Fixed Point Theorem.

Theorem 3.3.2 (Brouwer)**.** *Let $\mathbb{X}$ be a convex and compact set in $\mathbb{R}^n$ and a continuous function $C : \mathbb{X} \to \mathbb{X}$. There is at least one $\mathbf{x}^* \in \mathbb{X}$ such that: $C(\mathbf{x}^*) = \mathbf{x}^*$.*

If $C(\mathbf{x})$ is strictly monotone on $\mathbb{X}$, then, if there is a solution, it is unique. A function can be considered strictly monotone at the point $\mathbf{x}^*$ if the following condition is verified:

$$\langle (C(\mathbf{x}) - C(\mathbf{x}^*))^T, (\mathbf{x} - \mathbf{x}^*) \rangle \geq 0 \quad \forall \mathbf{x} \in \mathbb{X}, \mathbf{x} \neq \mathbf{x}^* \tag{3.19}$$

Finally, if $C(\mathbf{x})$ is continuously differentiable on $\mathbb{X}$ and the Jacobian matrix $\triangledown C(\mathbf{x})$ is strongly positive definite (without the need of being symmetric), then $C(\mathbf{x})$ is strongly monotone. In this conditions, it exists one solution $\mathbf{x}^*$ to the VI($C, \mathbb{X}$).

At this point, to connect the VI with the Wardrop Equilibrium, it is possible to consider that $\mathbf{x}^*$, satisfying the $VI(C, \mathbb{X})$, is a Wardrop equilibrium. Suppose that $C(\mathbf{x}^*)$ is a fixed path cost function, and $C(\mathbf{x}^*)^T\mathbf{x}$ is the system cost associated with the vector of path flows $\mathbf{x}$. It is possible to conclude that the total system cost is minimized when users are in the paths of minimum cost. Therefore, from the VI in Equation 3.10, $\mathbf{x}^*$ is a Wardrop equilibrium.

Nagurney (2000) generalised the VI problem of Equation 3.10 in order to represent the traffic network equilibrium with fixed demand considering multi-class users and multi-criteria link cost functions. Specifically, in their work the authors introduced disutility functions for network links, which consist of various cost components, each with its own weight depending on the user class.

Following the aforementioned study, let $C(\mathbf{x})$ be the path cost function, influenced by the vector of path flows $\mathbf{x}$ from different classes. A vector $\mathbf{x}^* \in \mathbb{X}$ is at equilibrium if it satisfies the inequality 3.10. If $C(\mathbf{x})$ is a continuous function in the compact set $\mathbb{X}$, then existence of a solution can be guaranteed by Theorem 3.3.2. The feasible set is compact due to traffic network equilibrium constraints. The conservation of flow constraint for each OD pair w, for a multi-class demand is:

$$d_w = \sum_{p \in P_w} \sum_{k \in K} x_p^k \quad \forall w \in W \tag{3.20}$$

where d_w is the demand connecting w that has to be equal to sum of the path flow x_p^k of all the classes k using the set of paths P_w connecting w.

The relationship between link flow f_a^k and path flow x_p^k for class k can be written as:

$$f_a^k = \sum_{p \in P} x_p^k \delta_{ap} \quad \forall k \in K, \forall a \in A \tag{3.21}$$

where the total flow on a link a for a specific class is equal to the flow of that class on all the paths containing that link. Considering the path flow vector x to be non-negative.

Following Nagurney (2000), in the context of multi-class and multi-criteria models, the proof of a unique solution, which is typically guaranteed by the strict monotonicity of the vector of class cost functions, may no longer hold. This is the case even if all the components of cost have strict monotonicity concerning the total link flow ($\mathbf{f}$). In this context, the only scenario in which a unique solution can be guaranteed is when the different cost functions associated to a link are separable with respect to the total flow on the link, and each function is strictly monotone.

Generally, if symmetry conditions hold, the problem formulated as VI is equivalent to the problem 3.1. Nonetheless, this scenario is often not met, due to the fact that the the cost on link a_1 is affected by the flow on link a_2 in the same way that the cost on link a_2 affects the flow on link a_1. Therefore the network equilibrium conditions cannot be reformulated as a solution of an optimization problem (Boyce et al., 2005), and thus, a more comprehensive formulation is required, such as that of the VI problem.

In this context, to address the UE, standard iterative techniques commonly employed for solving VI are typically utilised. These algorithms work towards achieving equilibrium by reformulating at each iteration the main problem as an optimization problem and then solving it through a nonlinear programming algorithm. According to Nagurney (1998) these solution algorithms could be grouped in general iterative schemes that include as special cases the projection, relaxation, and linearization methods, the modified projection method (or extragradient method), and decomposition algorithms.

3.3.3 Nonlinear Complementary Problem

Subsequently, Aashtiani and Magnanti (1981) formulated the traffic network equilibrium as a nonlinear complementary problem (NCP). NCPs were introduced by Cottle (1966). They are considered as a special case of VI (Noor, 1987), where a vector $\mathbf{x}^* \in \mathbb{R}^n$ satisfies the following conditions:

$$\mathbf{x}^* \geq 0 \tag{3.22}$$

$$C(\mathbf{x}^*) \geq 0 \tag{3.23}$$

$$\mathbf{x}^{*T} C(\mathbf{x}^*) = 0 \tag{3.24}$$

$C : \mathbb{X} \rightarrow \mathbb{R}^n$ is a function on a subset $\mathbb{X} \in \mathbb{R}^n$ containing at least the non-negative orthant (Yong, 2010). Geometrically, this type of problem tries to find a non-negative vector $\mathbf{x}^*$ with a non-negative function $C(\mathbf{x}^*)$ orthogonal to $\mathbf{x}^*$ (Harker & Pang, 1990).

To reformulate the traffic network equilibrium as NCP, Aashtiani and Magnanti (1981) stated that by definition Equations 3.5,3.6 and 3.9 are complementary by nature. Hence, in order to demonstrate the complementary nature of the remaining equations, they assumed:

$$\mathbf{h} = (\mathbf{x}, \mathbf{u}) \in \mathbb{R}^n \tag{3.25}$$

$$l_p(\mathbf{h}) = C_p(\mathbf{x}) - u_w \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.26}$$

$$g_w(\mathbf{h}) = \sum_{p \in \mathbb{P}_w} x_p - d_w(\mathbf{u}) \quad \forall w \in \mathbb{W} \tag{3.27}$$

As a consequence, the following NCP is similar to problem defined in Equations 3.5-3.9:

$$l_p(\mathbf{h}) x_p = 0 \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.28}$$

$$l_p(\mathbf{h}) \geq 0 \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{3.29}$$

$$g_w(\mathbf{h}) u_w = 0 \quad \forall w \in \mathbb{W} \tag{3.30}$$

$$g_w(\mathbf{h}) \geq 0 \quad \forall w \in \mathbb{W} \tag{3.31}$$

$$h \geq 0 \tag{3.32}$$

Since any traffic equilibrium solution $\mathbf{h}^*$ satisfies $g_w = 0 \quad \forall w \in \mathbb{W}$, $\mathbf{h}^*$ effectively solves the NCP given by Equations 3.28-3.32.

The principles that usually guarantee existence and uniqueness of the NCPs cannot be directly applied in the case of the reformulation of the traffic network equilibrium. This is due to the fact that these principles require the link cost function $C_p(\mathbf{x})$ to be strongly monotone in terms of path flows, whereas in practice, this function often relies on link flows (Karamardian, 1969). However, Aashtiani and Magnanti (1981) proved existence of the solution converting the NCP into a Brouwer fixed point problem. Uniqueness, instead, was proven with respect of link flows, due to the not uniqueness of path flows.

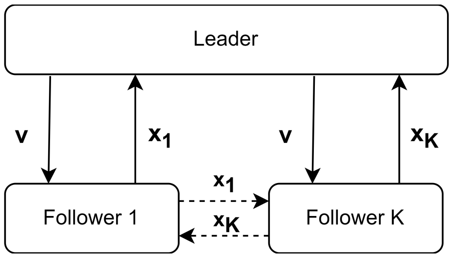
3.4 Interaction between Demand and Suppliers: a Game Theoretic Approach

Typically, in order to analyse how various actors' choices are influenced by the decisions of others (also known as players), problems related to game-based optimization are defined (Pozo et al., 2017). Developed in economic literature, the concept of game theory was first formulated by Von Neumann and Morgenstern (1947) in their book "Theory of Games and Economic Behavior". However, the first small studies of games on oligopolistic production and pricing were described by Cournot (1838) and Bertrand (1883). In economics, game theory studies multi-agent decision problems, where the outcome for a player may depend on the choices of other players. Since its first studies, in the last decades game theoretic approaches have been extended to different fields, including transportation.

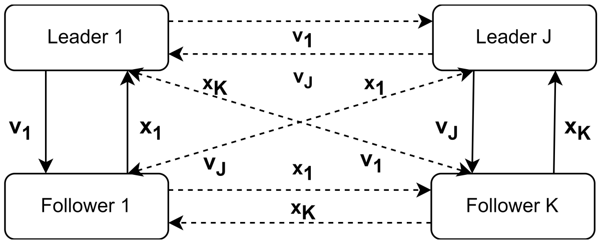
Game theory can be categorized into two domains determined by players' behavior: cooperative and non-cooperative. In cooperative games, decision-makers work together by creating coalitions, whereas non-cooperative games involve individuals who make choices without coordinating their actions with others. In both cases, it is possible to study cooperation and competition among the different players. Therefore, in cooperative game theory players can collaborate, forming coalitions to generate value, or compete to obtain value. In this section, the focus is on non-cooperative games applied to transportation.

Following Fudenberg and Tirole (1991), in 1950 Nash developed what became the "Nash Equilibrium" (Nash, 1951) in non-cooperative games as a generalization of Cournot and Bertrand specific models. In simultaneous optimization problems, a Nash equilibrium is a set of strategies in which each player's chosen strategy represents an optimal response to the strategies selected by the others. The relationship between Nash-Cournot and Wardrop equilibria was first noted by Charnes and Cooper (1958) and proved by Haurie and Marcotte (1985).

Alternatively, when players make decisions in a sequential manner, the focus shifts to bi-level games, i.e. two-level decision-making process. In this context, a Stackelberg game (Stackelberg, 1952) is a single-leader-follower game in which at the upper-level a leader sets some directives that are going to influence the decisions of other agents, called followers, situated at the lower level. This game are often modeled as a bi-level problem, i.e. an opti-



(a) Single-leader-multiple-follower game



(b) Multiple-leader-multiple-follower game

Figure 3.5: Stackelberg Games

mization problem characterised by an optimization problem as a constraint at the lower level (Sinha et al., 2017). Moreover, the Stackelberg game can be extended to a single-leader-multiple-follower game (Figure 3.5a) when the followers are competing among themselves (Pozo et al., 2017). In transportation, this problem usually considers that an operator controls some network parameters/strategies seeking to maximise profits, sometimes in combination with other objectives (e.g., ensuring basic service level to all users). As a reaction, users typically change their travel preferences, often relying on UE principles connected to congested networks. In this case, the problem can also be defined as Stackelberg congestion game. Generally, these problems can be formulated as MPEC or Mathematical Program with Complementary Constraints (MPCC) when, in a bi-level problem, the lower level is defined through equilibrium conditions in the form of VI or complementarity conditions rispectively (Luo et al., 1996). Finally, in the multi-leader-multi-follower game (Figure 3.5b) at the upper-level a Nash game is established between the different actors. Their decisions are going to impact the one's of the followers, competing in a non-cooperative manner. The natural formulation of this type of problem is through EPEC in which each leader aims to solve an MPEC.

In the next subsections the different aforementioned problem formulations are illustrated in detail with a focus on their application on transportation modelling.

3.4.1 Bi-level Programs

Bi-level problems are frequently employed to study hierarchical relationships, aiming to optimize the objectives of a decision-maker who is impacted by the reactions of other participants, optimizing their individual goals (Sinha et al., 2017). Within this framework, in the upper-level, the leader's objective function becomes subject to constraints introduced by the lower-level mathematical program (shown in Figure 3.6). In this scenario, the upper-level formulation can be written as (Colson et al., 2007):

$$\min_{\mathbf{v} \in V, \mathbf{x}} \quad Z(\mathbf{v}, \mathbf{x}) \tag{3.33}$$

$$\text{s.t.} \quad G(\mathbf{v}, \mathbf{x}) \leq 0 \tag{3.34}$$

while the lower-level formulation is:

$$\min_{\mathbf{x}} \quad C(\mathbf{v}, \mathbf{x}) \tag{3.35}$$

$$\text{s.t.} \quad g(\mathbf{v}, \mathbf{x}) \leq 0 \tag{3.36}$$

where $\mathbf{v} \in \mathbb{R}^{n_1}$ is the vector of upper-level decision variables and $\mathbf{x} \in \mathbb{R}^{n_2}$ is the vector of lower-level decison variables. The function $Z : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ is the upper-level objective function with $G : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}^{m_1}$ being its constraints functions. The function, instead, $C : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ is the lower-level objective function with $g : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}^{m_2}$ being its constraints functions.

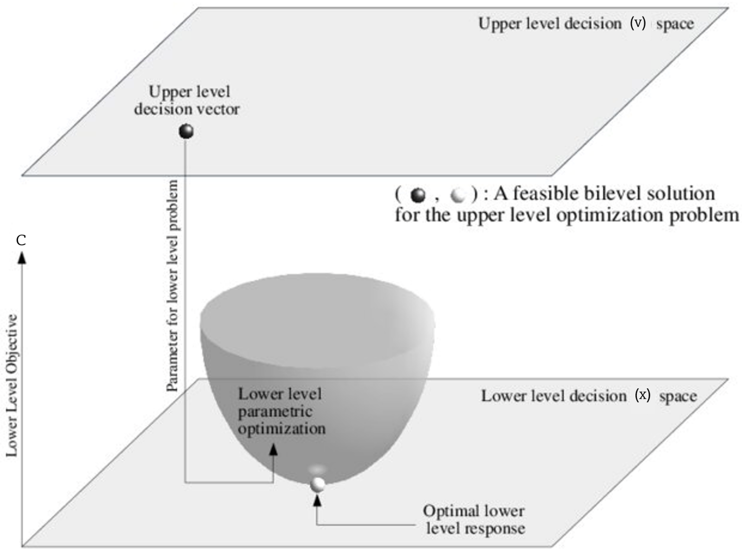


Figure 3.6: Bi-level problem (Sinha et al., 2017)

This bi-level structure is frequently applied within transportation contexts to identify equilibrium solutions between an authority and users of the transportation network. In the context of MNDP, in which there are at least two modes of transport, the upper-level decision variables can focus on a single mode of transport or combinations of decisions considering the different modes of transport under exam (Fan et al., 2014). However, bi-level linear programs are widely recognised as being strongly NP-hard (nondeterministic polynomial-time hard) (Sinha et al., 2017). This implies that the problem is at least as hard as the toughest problems in the NP. When they are applied to hierarchical problems such the network design modelling, non-convexity and discontinuity may arise (Krylatov et al., 2023). For this reason, the majority of algorithms employed to tackle this problem type opt to reformulate it into a single-level problem. In doing so, they make assumptions about the lower-level problem to reduce its complexity (Labbé & Marcotte, 2021). For example, when the lower-level is convex can be replace it with its KKT conditions.

In the last years, several studies have formulated MNDPs in the form of bi-level programs. In the context of multi-modal transit networks, Wan and Lo (2009) propose a bi-level problem in which they try to maximize the social welfare of the system while optimizing network configuration and service frequency. The authors use a State-Augmented Multi-modal (SAM) network (developed by Lo et al. (2002)) in which they could include various factors, such as road congestion for transit mode, users' multi-modal choices, in-vehicle crowdedness and non-linear fare structures. Moreover, trough the SAM they divided transport modes in classes to consider different levels of congestion. Considering Park and Ride (P+R) location, Fan et al. (2014) developed a bi-level model in which at the upper-level the authorities decide where to locate the P+R facilities while maximizing the total social welfare. At the lower-level a stochastic UE is formulated. To solve the problem, the authors introduce a heuristic algorithm that merges the genetic algorithm with the MSA. A study conducted by Rashidi et al. (2016) includes a pedestrian transportation system as an independent mode of transport together with public transit and car. They developed a bi-level mathematical programming in order to determine the optimal location of sidewalks and crosswalks, while minimizing the total transportation cost and improving pedestrians' safety. Li and Liao (2020) defined an activity-based multi-modal network in which PC, PT, and shared autonomous vehicles (SAV) are included. The bi-level formulation tries to determine hub locations, fleet size, and the initial distribution of SAVs vehicles, while the lower-level studies the scheduling behaviour of users considering a dynamic UE. Fu et al. (2020) developed an integration between Activity-Time-Space (ATS) network and the SAM transport network in the context of multi-modal transit networks, in which the individuals' accessibility to different activities and travels are considered. The model is formulated as a bi-level programming problem, in order to maximize the activity-based space-time accessibility of activity locations. At the lower-level individuals are considered as homogeneous, and their choices in terms of activities, departures, modes and routes are analysed. Ye et al. (2021) formulated a bi-level program to represent a MNDP in which determnie location and capacity of P+R facilities. The authors used a supernetwork representation to include PC, metro and bike. In this scenario, at the upper-level takes the form of a mixed-integer linear programming problem, and the lower-level the elastic demand is assigned to the network considering trip distribution and modal split. More recently, Zhou et al. (2023) consider a network including PC, bus, and rail in order to study how to locate transit-oriented development stations (TOD) while analysing users behaviour in at his multi-modal context. Formulating at the upper-level a multi-objective optimization program with a lower-level multi-modal equilibrium model to determine the travel demand for PC and PT.

3.4.2 Mathematical Programs with Equilibrium Constraints

MPECs are considered as extension of bi-level programs. Following Luo et al. (1996), an MPEC is an optimization problem with two sets of variables, $\mathbf{v} \in \mathbb{R}^{n_1}$ and $\mathbf{x} \in \mathbb{R}^{n_2}$, where some of the constraints take the form of a parametric VI or complementary system (MPCC) with $\mathbf{x}$ being the primary variable and $\mathbf{v}$ the parameter vector. Using the notation introduced in subsection 3.4.1, the MPEC can then be written as:

$$\min_{\mathbf{v} \in \mathbb{V}} \quad Z(\mathbf{v}, \mathbf{x}) \tag{3.37}$$

$$\text{s.t.} \quad (\mathbf{v}, \mathbf{x}) \in \Phi \tag{3.38}$$

$$\mathbf{x} \in \Sigma(\mathbf{v}) \tag{3.39}$$

where Φ is a nonempty closed set, representing the feasible region of the pair $(\mathbf{v}, \mathbf{x})$. Constraint 3.39 indicates that for every $\mathbf{v} \in \mathbb{V}$, where $\mathbb{V}$ is the projection of Φ onto $\mathbb{R}^{n_1}$, $\Sigma(\mathbf{v})$ represents the solution set of the VI$(C(\mathbf{v}, \cdot), \mathbb{X}(\mathbf{v}))$. Considering $\mathbb{X}(\mathbf{v})$ the restricted region of the variable $\mathbf{x}$ for every given $\mathbf{v} \in \mathbb{V}$. Meaning that constraint 3.39 is satisfied if and only if $\mathbf{x}$ is in $\mathbb{X}(\mathbf{v})$ and satisfies:

$$C(\mathbf{v}, \mathbf{x})^T(\chi - \mathbf{x}) \geq 0 \quad \forall \chi \in \mathbb{X}(\mathbf{v}) \tag{3.40}$$

Solving MPECs presents significant challenges, particularly due to the presence of disjoint constraints that give rise to combinatorial problems, posing difficulty for efficient solution algorithms. More likely, the lack of convexity and the closedness of the feasible region could be a cause of inefficiencies in finding optimal solutions for MPECs (Luo et al., 1996).

In the transportation context, MPECs have been employed to examine the characteristics of networks while assessing equilibrium solutions for users travelling in the transportation system.

In this scenario, García and Marin (2002) developed a MPEC in order to study the tactical decisions of an Authority. This involves assessing the performances of the transportation network while changing capacity and fares of P+R facilities. The aim is to minimize the overall travel costs, considering that at the lower-level a multi-modal assignment problem is formulated as a VI. The numerical examples consider the usage car and metro with the interaction of P+R facilities. The Simulated Annealing heuristic algorithm is used to solve the bi-level problem. One of the key challenges highlighted by the authors concerns the fact that the objective function cannot be precisely evaluated because the equilibrium problem can only be solved with an approximate solution. Thus, the algorithm could be non-convergent. Zhang et al. (2014) considered a multi-modal network in which the network design characteristics of PC and bus are taken into account. The authors formulated the problem as a single-level MPCC. The solution approach based on a sequential algorithm is applied to a Sioux Falls network. The algorithm first solves a relaxed MNDP and then a scheme updating problem. Nair and Miller-Hooks (2014) defined a transit network in the presence of sharing systems. In this context, they formulated a bi-level program, where at the upper level an MSP adjusts their decision variables in order to maximize profit. At the lower level, instead, users minimize their travel times and waiting times. The lower-level is a program with linear objective and linear constraints. The presence of non-linearity arises from complementarity constraints, which were transformed into linear disjunctive constraints via auxiliary binary variables. Consequently, the problem takes on the form of a mixed-integer program (MIP), amenable to solution using MIP solvers. Bingfeng et al. (2017) defined a bi-level model to study exclusive bus lane configuration in multi-modal networks, with the objective of optimizing travellers' total travel cost. The authors apply a branch-bound method to solve the formulated integer bi-level programming problem, applying it on a five link network. However, effective applications on real scenarios are not guaranteed. More recently, Nguyen et al. (2022), instead, developed a bi-level model considering a one-way car-sharing service in a multi-modal dynamic network using an activity-based approach. Travellers can be PC drivers, car-sharing drivers or transit passengers and perform three activities: go to work, shopping and return home. At the upper level the car-sharing operator tries to maximize their profit by controlling the price of the service and how to organize the vehicles at the depots. The lower-level activity-based choice model is written as a VI. Due to the complexity of the problem, the authors developed an iterative link-based method.

3.4.3 Equilibrium Problems with Equilibrium Constraints

Although the studies cited in the previous sections developed relatively complex models, there is limited research that includes multiple leaders, coexisting, competing or cooperating at the upper-level. From an economic and strategic point of view (i.e. to broadly understand if a business has potential profitability and market value), it is essential to model the interactions between MSPs and travellers of the transport network to predict the response of these actors as a consequence of the variation in strategies of the entire system. In particular, scenarios offering a new transport service, introducing new regulations/incentives, or increasing users' heterogeneity, could substantially change the equilibrium of the whole network. In the literature, few works have analysed this type of problems. Zhou et al. (2005) investigate transit competition with a bi-level equilibrium formulation presented as a Stackelberg game, considering an elastic demand assigned through a SUE model. In the context of fast charging stations for electric vehicles, Guo et al. (2016) developed a Multi-agent Optimization Problem with Equilibrium Constraints (MOPEC)-based model to study interactions between multiple competitive investors and travellers assigned to a congested transport network. Jiang et al. (2020) propose a game-theoretical model to study the market competition between two bike-sharing companies. Yang et al. (2022), instead, defined a bi-level model to optimize pricing and relocation in a competitive one-way car-sharing market.

Moreover, in order to analyse the impact of MSPs' different strategies on the transportation system different works formulate the problem as an EPEC. An EPEC can be considered as extension and generalisation of a Stackelberg game (Stackelberg, 1952). Stackelberg games are typically characterised by the presence of a (single) leader who sets strategies that impact the decisions made by a group of followers, that generally play a non-cooperative Nash game between one another (Nash, 1951). In transportation usually the followers are represented by travellers assigned to the transportation network following Wardrop's first equilibrium principle. As pointed out by (Adler et al., 2021) and citations therein, Wardrop equilibria can be considered equivalent to Nash equilibria under certain conditions. On the other end, EPECs are characterised by the presence of multiple leaders at the upper-level, who also participate in a Nash game with each other (Steffensen & Bittner, 2014). Generally, in an EPEC each leader aims to solve an MPEC (Luo et al., 1996). As a result, EPECs can be considered as a collection of MPECs that share variables and equilibrium constraints (Cottle & Su, 2005).

In this context, considering at the upper-level $\mathbb{J}$ suppliers, it is possible to write for a generic supplier j (for $j = 1, \ldots, \mathbb{J}$) an MPEC formulation, based on the independent decision variables $v^j \in \mathbb{R}^{n_j}$ and the shared decision variables

$\mathbf{x} \in \mathbb{R}^m$ of the lower-level:

$$\min_{v^j \in \mathbb{V}} \quad Z^j(v^j, \mathbf{x}; \mathbf{v}^{-j}) \tag{3.41}$$

$$\text{s.t.} \quad (v^j, \mathbf{x}; \mathbf{v}^{-j}) \in \Phi \tag{3.42}$$

$$\mathbf{x} \in \Sigma(\mathbf{v}) \tag{3.43}$$

where the notation $\mathbf{v}^{-j}$ is associated with the fixed vector of all the decision variables of all the $\mathbb{J}$ suppliers except j. Given $\mathbf{v}^{-j}$, the solution of the j-th MPEC is nonempty and can be written as SOL(MPEC($\mathbf{v}^{-j}$)). The EPEC, associated with the described $\mathbb{J}$ MPECs, corresponds on finding a Nash Equilibrium $(\mathbf{v}^*, \mathbf{x}^*)$ such that $(v^{j*}, \mathbf{x}^*) \in$ SOL(MPEC($\mathbf{v}^{-j}$)) (Cottle & Su, 2005). In Equation 3.42, Φ is a nonempty closed set, representing the feasible region of the pair $(v^j, \mathbf{x})$, given $\mathbf{v}^{-j}$. $\Sigma(\mathbf{v})$ represents the solution set of the VI$(C(\mathbf{v}, \cdot), \mathbb{X}(\mathbf{v}))$. Considering $\mathbb{X}(\mathbf{v})$ the restricted region of the variable $\mathbf{x}$ for every given $\mathbf{v} \in \mathbb{V}$. Constraint 3.43 is satisfied if and only if $\mathbf{x}$ is in $\mathbb{X}(\mathbf{v})$ and satisfies Equation 3.40.

EPECs have rarely been used in transportation problems. In the context of toll competition, Yang et al. (2009) introduced an EPEC that examines how competing firms maximize their profits by adjusting road capacity and toll charges, while homogeneous users are assigned to the network. They employed a heuristic method called the Synchronous Iterative Method to solve the problem. Koh and Shepherd (2010) investigated a similar problem by formulating an EPEC where firms generate revenues by imposing tolls on transportation network users. They proposed two heuristic solution algorithms: the Diagonalization Algorithm and the Sequential Linear Complementary Programming (SLCP) approach. Subsequently, Koh et al. (2013) analysed the competition between two city authorities. The objective was to maximize the social welfare of their respective residents while charging traffic through tolls. Results show that increasing elasticity, changing demand and congestion functions their model can shift from multiple equilibrium solutions to a single solution. Watling et al. (2015) extended the previous work casting the problem as a single level problem for each authority, in order to explore the scenario where each authority seeks a local optimum within their individual MPEC. Finally, Gu et al. (2019) structured this problem as a tri-level optimization model, considering the government at the upper-level, who seeks for the optimal toll to maximize the social welfare. Private firms try to maximize their profit based on government's decisions, charging tolls and investing in road capacities. Lastly, users are assigned to the network following a deterministic UE. The problem between private firms and travellers has the structure of an EPEC and solved through a synchronous iterative method. Wang et al. (2021) studied the competition between the government and a private firm in a public-private mixed network, while elastic demand is considered. The government's objective function maximizes social gains on public roads, instead the firm tries to maximize profit through tolls in the controlled roads. The study uses the Diagonalization method in order to solve the problem. Recently, in the context of the Crowdsourced Event Parking Market Pricing Problem, Fotouhi and Miller-Hooks (2021) formulated an EPEC to study the upper-level competition between parking space owners that want to optimize the parking price offered to an elastic demand, which can accept these prices or alternatively choose a mode of transport that doesn't require to pay for parking. They also solved the model using a Diagonalization approach.

3.5 Conclusion

In this chapter, an extensive review of transportation literature has been conducted, emphasising the fundamental modeling decisions when constructing a multi-modal and multi-actor network model. These decisions encompass the methods used to model multi-modal networks, the process of defining and formulating the travel demand, the examination of its interactions with various MSPs within the transportation network, and the development of solution algorithms.

After this analysis, it is clear that in order to understand and model the different strategies arising among firms or MSPs of the transportation market, current mathematical models have to be extended with the purpose of representing realistic dynamics. For instance, it is evident that there is a lack of studies that examine the different forms of competitions and collaborations emerging when introducing mobility packages or by the co-existence of multiple service providers in the same market. Moreover, existing studies predominantly concentrate on uni-modal networks with homogeneous demand characteristics, neglecting the interaction between different modes, i.e. considering non-separable cost functions. This research aims to fill these gaps, seeking to comprehend the various dynamics that may arise in the transportation network due to complex interactions between all actors in the systems, and the different modes sharing the same transportation infrastructure and competing for the travel demand. To pursue this objective, starting from the classical transportation models, in the next chapters the assumptions and methodological approach used in this thesis, followed by its application on small-size networks are presented, in order to understand the impact of the strategies employed by multiple MSPs that offer diverse mobility services.

Chapter 4

A MaaS Equilibrium Model: Assumptions, Formulations and Solution Algorithm

4.1 Introduction

In this chapter, the methodological approach that forms the core of this thesis is undertaken. The primary objective pursued is to delve into the research objectives delineated in Chapter 1, with a central focus on modeling an intricate system such as MaaS. This system involves various stakeholders with different and sometimes conflicting goals. These stakeholders collaborate and compete within the complex structure of a multi-modal transportation network.

After a review of existing literature in Chapter 3, assessing its strengths and limitations, within the context of multi-modal and multi-actor transportation systems, the following sections are structured around the assumptions made for key components of the methodological approach. Specifically, the notation used in the proposed methodology is listed in Table 4.1. The assumptions made concerning the network structure are addressed in Section 4.2. Section 4.3 describes the formulation able to represent different MSPs and their business models. Finally, in Section 4.4 multi-class users with their mathematical formulation, and a proposed solution algorithm are introduced. Crucially, the proposed methodology takes into account the different strategies applied by the diverse MSPs, influenced by the modal choices made by various user classes within a congested multi-modal network. While our approach is tailored to understanding MaaS dynamics, it's important to note its adaptability. The proposed method, with its intricate assumptions and interconnections, is applicable to a range of multi-actor and multi-modal transport systems. Its core purpose is to understand potential equilibrium strategies that manifest within the transportation network under diverse scenarios that will be then explored in mode detail the following Chapters 5 and 6.

4.2 Structure of the Network

In this section, a multi-modal network approach is introduced with the structure of a supernetwork. In this network, users' daily trip chains are explicitly modelled and the different mobility services available in the area are included. Since the interest is to study the long-term impacts, e.g. in terms of profitability of a certain service and how it can be optimised, the model is developed at zonal scale, without detailing performance at the route level or for each individual. To construct the proposed supernetwork, the endogenous information obtained from both users and MSPs are fundamental.

The model presented in this thesis is static, which means that its primary characteristics remain constant. Specifically, it is designed to represent typical conditions on an ordinary weekday. In this scenario, let travellers be divided into $\mathbb{K}$ classes based on their personal attributes and daily trip chains. It is assumed that users of the same class perform their activities in specific zones within the study area. Using this information it is possible to define the trip-based network where the sequence of trips between zones is modelled as a directed graph. In Figure 4.1 an example of this network is illustrated considering one class of users k. In this network a node n corresponds

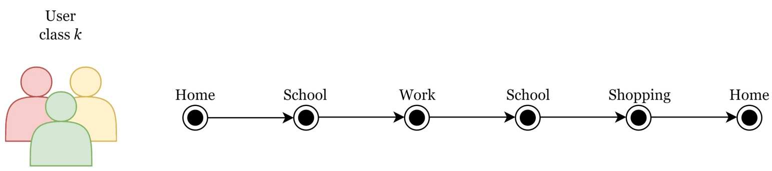


Figure 4.1: User Class k with daily trip chain

to a location (zone) and a link a indicates the trip from one location to another. In the proposed example, users in class k leave home to drop off their children at school, go to work, pick up their children for shopping, and finally return home together. For illustrative purposes, in Figure 4.1 the different visited locations of class k are explicitly identified with the activities performed at each zone. However, in the model definition and application it is not required to explicitly model activities in detail; on the contrary, it is assumed that users of different classes can perform different activities in the same location, but a user class is simply determined by the specific sequence of visited locations, which clearly reduces the problem complexity. Furthermore, this graph does not explicitly aim to replicate the intricate structure of the actual transportation infrastructure. As shown in Figure 4.2, even if the real network features various infrastructures connecting two

Table 4.1: Model Notation

Sets	Description	**Indices**	Description
$\mathbb{J}$	Set of MSPs	j	a MSP
$\mathbb{K}$	Set of user classes	k	a user class
$\mathbb{S}$	Set of mobility subscriptions	s	a mobility subscription
$\mathbb{A}$	Set of links	a	a link of the network
A^j	Subset of modal-links owned by MSP j	n	a location (node)
A_s	Subset of subscription links	w	an OD pair
A_s^j	Subset of subscription links involving MSP j	p	a path connecting an OD
$\mathbb{N}$	Set of locations (nodes)		
$\mathbb{W}$	Set of OD pairs		
$\mathbb{P}$	Set of paths in the network		
P_w	Subset of paths between $w \in W$		
$\mathbb{X}$	Set of all feasible path flows		
D	Total travel demand of the network		
Users' variables	Description	**Supplier's variables**	Description
$\mathbf{f}$	vector of link flows	$\mathbf{v}$	vector of fleet size
f_a	flow on link a	v^j	fleet size for MSP j
f_a^k	flow of class k on link a	v_a	number of vehicles on link $a \in A^j$
$\mathbf{x}$	vector of path flows		
x_p	flow on path p		
x_p^k	flow of class k on path p		
Parameters	Description	**Functions**	Description
d_w^k	demand of class k on OD w	$c^j_{lease}(\sum_{a \in A^j} v_a)$	leasing cost (€) for MSP j
γ^j	relocation factor for MSP j	$t_{a,access}(f_a, v_a)$	access time (hour)
l_a	length of link a (km)	$t_{a,main}(\mathbf{f})$	time in the main mode of transport (hour)
$c_{a,s}$	daily cost for subscription s (€/day)	$t_{a,egress}(f_a, v_a)$	egress time (hour)
$r_{a,s}$	daily subsidy based subscription s (€/day)	$t_{a,wait}(f_a, v_a)$	waiting time (hour)
$c_{a,h}$	cost per hour h travelling on link a (€/hour)	$t_{a,park}(f_a, v_a)$	parking time (hour)
$c_{a,km}$	Cost per kilometre km on link a (€/km)	$C_a^k(\mathbf{f}, v_a)$	total cost on link a for class k
$c_{a,fixed}$	ticket cost on link a (€/day)	$C^k_{a,access}(f_a, v_a)$	total access cost on link a for class k
$c_{a,fuel}$	fuel/recharge cost for vehicle (€/km)	$C^k_{a,main}(\mathbf{f}, v_a)$	total travel cost on link a for class k
$c_{a,park}$	cost to find a parking slot (€/hour)	$C^k_{a,egress}(f_a, v_a)$	total egress cost on link a for class k
$\delta_{a,p}$	incidence matrix link-path	$C_p^k(\mathbf{x}, \mathbf{v})$	total travel cost on path p for class k
$\delta_{a,s}$	incidence matrix link-subscription		

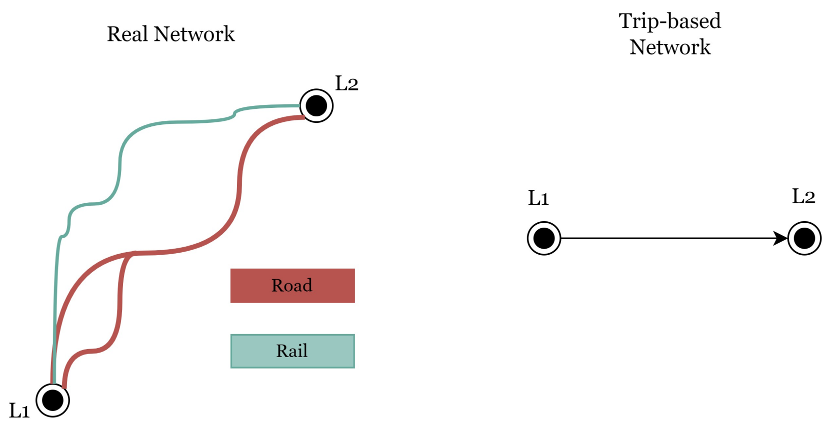


Figure 4.2: Network assumption

areas, the model simplifies this aspect, representing a single trip between two zones, without explicitly defining all the different options available in the actual network. This simplification is based on the assumption that, after the traffic assignment, the network achieves equilibrium, as described by the principle outlined by Wardrop (1952), allowing their representation into a single aggregated link (Connors & Watling, 2014).

After building the trip-chain network, the information from the different MSPs operating in each trip connection are taken into account. With the purpose of explicitly representing the different modes of transport available between two locations and eventually operated by different MSPs, the trip chain network is then expanded in several uni-modal MSP-specific, as illustrated in Figure 4.3. The first and last locations of the trip chain are

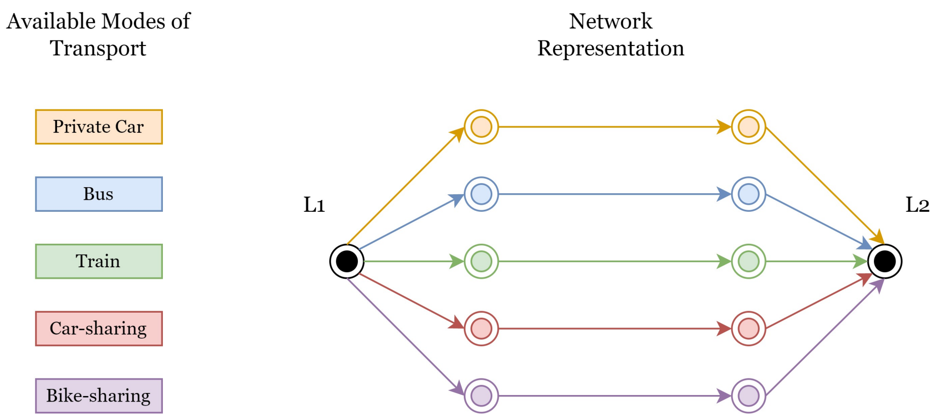


Figure 4.3: Network expansion

also considered as centroids of the network, in order to guarantee flow conservation. As a result, with this representation it is possible to take into account all the possible modes of transport active in an area, including users' private vehicles, i.e. car, bike, scooter. Combining users' trip chains with MSPs' information the network in Figure 4.1 becomes a multi-modal trip chain-based supernetwork as shown in Figure 4.4.

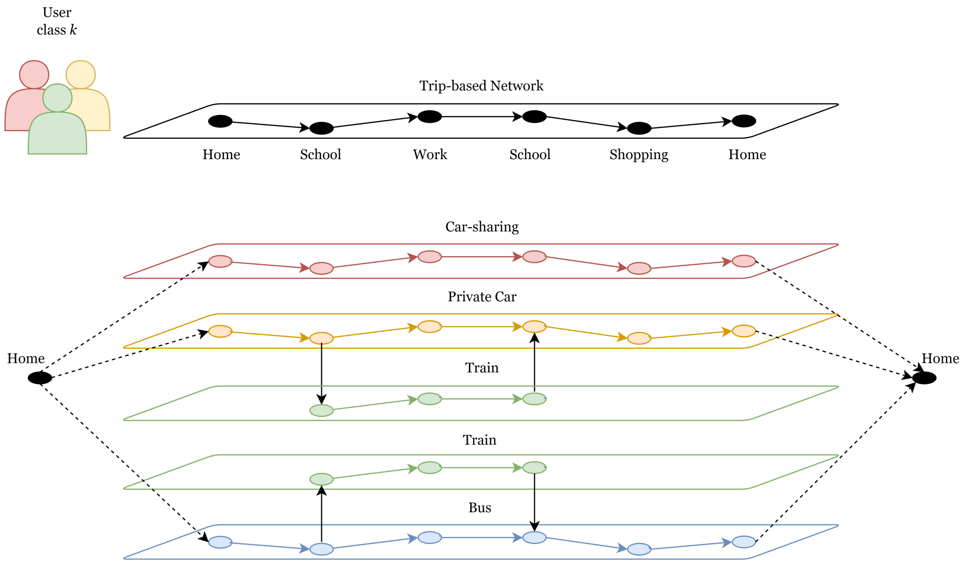


Figure 4.4: Multi-modal Trip-chain Supernetwork

The figure shows at the top the trip-based network for user class k, and below it, there is its expansion into parallel uni-modal layers. Each layer in this expansion corresponds to a specific mode of transportation. An MSP may possess one or several mobility services, and these services can be represented in the network using multiple separate layers. This arrangement is influenced by factors such as whether the service has varying prices, different offers, or if it is part of a mobility package that includes one or more other services.

In Figure 4.4 four modes of transport are included: car-sharing (red layer), PC (yellow layer), train (green layers), and bus (blue layer). According to the network representation, each traveller must choose a path p through the multi-modal network in order to perform the sequence of trips during their ordinary day. In this scenario, users have several options for their daily trip chain:

- they can exclusively use the car-sharing service (red path);
- they can rely solely on their PCs (yellow path);
- users travelling with their PC have the choice to park their vehicle at the P+R facility located in the school zone, use the train during the day,

and pick up their cars when returning to the school location (yellow and green path);

- they may opt for a package that provides access to both bus and train services. Since the train doesn't cover all locations, users can either use the bus alone (blue path) or combine both modes of transportation (blue and green path).

All the possible paths between the different ODs are previously manually enumetared, in order to capture the complex multi-modal constraints of the network (e.g. users do not have to pay two times for a package subscription or pay and use another mode of transport). A path (representing a trip chain for the specific user class k) can comprise three different types of links: 1) access links (black dashed links) allow users to access a mode of transport from their origin (Home on the left side), and egress from a mode of transport to reach their final destination (Home on the right side). Another important role of the access links is that they can capture a subscription package needed for accessing a specific mode of transport (e.g., a PT subscription) or the combination of integrated MSPs' services (a MaaS bundle subscription). 2) Mode-specific links (horizontal links) indicate trips made from one activity location to another using a specific mode of transport (designated by colour). 3) Interchange links (vertical black links) instead allow users to change to another transport mode when departing from an activity location. Each (within-layer) mode-specific link includes in turn three stages of a trip. In the first stage, the users leave the activity location and walk to reach the main mode of transport. Subsequently, travellers use the specific mode of transport in order to reach their next destination. Finally, the users leave that transport mode and walk to arrive at their next location (Arentze & Molin, 2013). Once they have performed all their trips, users return home completing their daily trip chain.

It is assumed that each MSP seeks to maximize the profit generated from their service. This profit is calculated considering that each MSP, denoted as $j \in \mathbb{J}$, manages specific and uniquely defined layers of the supernetwork, through which they collect revenues based on how many travellers use their service, while facing costs that depend on the service capacity provided e.g. number of vehicles. Hence, each MSP owns a fleet of vehicles v^j, and as part of their choice, at equilibrium, they will strategically distribute these vehicles across the links of their layer. The number of vehicles assigned at equilibrium to a specific link (v_a) will be considered available only for that trip connection. Due to the static assumption of the network, the daily (re-)location of vehicles is represented by a coefficient in the fixed costs, assuming that the optimal fleet size depends only marginally on the actual position in time and space of the vehicles. The lower-level equilibrium decision variables, instead are the vector of path flows $\mathbf{x}$.

4.3 Mobility Service Providers

As previously mentioned, in this thesis each MSP operating within the transport network, offering one or multiple mobility services, is considered as a profit maximizer. In this context, each MSP is driven by self-interest, employing distinct strategies to maximize their individual profits and secure their presence within the market. As a basic principle in economic theory, profit reaches its maximum value when the marginal revenue of a firm is equal to the marginal cost that must be sustained to offer the service (Primeaux & Stieber, 1994). Following this statement, MSP's profit (Z) is defined as the difference between total revenues (TR - comprising fixed plus variable revenues) and total costs (TC - including fixed plus variable costs):

$$Z = TR - TC \tag{4.1}$$

Starting from Equation 4.1, in this section the different revenue and cost components for a generic MSP j are defined, considering that the formulation can be applied for different mobility services such as bike sharing, PT, carpooling, car-sharing, e-scooter sharing, taxi and train. Depending on the type of service analysed some components could be omitted for the calculation, as shown in Table 4.2.

Revenues An MSP j, selling a mobility service, is considered to perceive two main revenue streams. The first component represents the fixed revenues (FR^j):

$$FR^j(\mathbf{f}) = \sum_{a \in A_s^j} (c_{a,s} + r_{a,s}) f_a \tag{4.2}$$

where a indicates respectively a link of the network that can represent a modal link or a subscription link. A modal link of the supernetwork is exclusively owned by a single MSP, and the vector of links owned by MSP j are indicated as A^j. Subscription links, instead, could be shared between different MSPs as part of the same mobility package. In this case, the vector of subscription links in which j is involved is defined as A_s^j.

In Equation 4.2, based on the number of users f_a subscribing to the service, the MSP receives a fixed revenue from the subscription fee to access the service $c_{a,s}$ and from the potential subsidy $r_{a,s}$ coming from an external actor, i.e. the government or a local authority.

The second component of income is represented by variable revenues (VR^j). This value depends on travellers' usage of the service:

$$VR^j(\mathbf{f}) = \sum_{a \in A^j} (c_{a,h} t_{a,main}(\mathbf{f}) + c_{a,km} l_a + c_{a,fixed}) f_a \tag{4.3}$$

The first term represents the time spent on board $t_{a,main}(\mathbf{f})$ using the main mode of transport. This component is non-separable, therefore depends on the vector of link flows (**f**) of the modes of transport available on the same trip connection. It is important to clarify that not all modes of transport necessarily contribute to congestion, e.g. trains have a separate infrastructure or buses can have dedicated lanes. This term is multiplied by the cost per hour $c_{a,h}$ that travellers are paying to use the mobility service, perceived as revenue for the MSP. The second term l_a indicates distance travelled, multiplied by revenue per kilometre $c_{a,km}$. The third term $c_{a,fixed}$ is the revenue from a fixed fee/ticket imposed on the user each time the service is used.

Costs An MSP j experiences costs that can be also divided into fixed and variable. The fixed costs (FC^j) are defined through a function that varies with the number of vehicles the supplier deploys on the network:

$$FC^j(v^j) = c_{lease}(\sum_{a \in A^j} v_a) \tag{4.4}$$

In the function $c_{lease}(\sum_{a \in A^j} v_a)$, it is possible to include all the costs that do not change with the number of travellers served, and that the supplier has to bear in order to operate a mobility service. More specifically, these are related to investment costs, such as purchasing/leasing the fleet of vehicles, renting parking spaces, building charging stations, paying employees, and general legal and administrative costs for the company.

The variable costs (VC^j) for MSP j are associated to the daily operations, and they are defined as:

$$VC^j(\mathbf{f}) = \sum_{a \in A^j} (c_{a,fuel} l_a)(1 + f_a)(1 + \gamma^j) \tag{4.5}$$

he cost, denoted as $c_{a,fuel}$, represents the per-unit cost borne by the supplier for fuel (or electricity) when travellers use their service, directly tied to the distance traveled l_a. When considering sharing services, these factors have to be multiplied by the number of travellers using that mode of transport f_a. Moreover, an additional component γ^j it is considered to influence this cost, connected with the relocation of vehicles or the return to a vehicle depot.

Table 4.2 shows the connection between the costs and revenues components of the profit maximization associated with the different modes of transport. The orange box indicates that a specific factor could influence the costs or revenues of an MSP based on their marketing strategies. The green box represents a component always considered for that specific mode of transport. When, instead, a component does not influence the profit of an MSP empty boxes are shown in the table.

Table 4.2: Components of cost/revenues connected to modes of transport[1]

	Mode of Transport							
Factor	Bus	Train	Car-sharing one way	Car-sharing round trip	Bike-sharing one-way	E-scooter	Taxi	Car-pooling
$c_s f_a$	✔	✔	✔	✔	✔	✔		
$r_s f_a$	✔	✔	✔	✔	✔	✔		✔
$c_{a,h} t_{a,main}(\mathbf{f}) f_a$			✔	✔	✔	✔	✔	
$c_{a,km} l_a f_a$		✔	✔	✔				
$c_{a,fixed} f_a$	✔	✔	✔	✔	✔	✔	✔	✔
$c_{lease}(\sum_{a \in A^j} v_a)$	✔	✔	✔	✔	✔	✔	✔	✔
$c_{a,fuel} l_a$	✔	✔						✔
$c_{a,fuel} l_a f_a$			✔	✔	✔	✔	✔	
$c_{a,fuel} l_a f_a y$	✔	✔	✔		✔	✔	✔	

[1] Green box: fixed cost component for that service; Orange box: a potential cost component influenced by the MSP's market strategies

4.3.1 Profit Maximization

Following Equation 4.1, an MSP of the transport network will seek to maximize the profit of the mobility service offered. The revenue generated by an MSP is directly influenced by the number of travelers using their services f_a (i.e., the network link flows), while costs primarily depend on the size of their vehicle fleet v^j (capacity), and how they are distributed among the network links v_a. Given link flows $\mathbf{f}$ the profit for MSP j is $Z^j(\mathbf{v}|\mathbf{x})$:

$$\max_{\mathbf{v_a}>0} \quad Z^j(\mathbf{v}|\mathbf{x}) = \sum_{s \in A_s^j} \sum_{a \in A^j} (C_1^j f_a + c_{a,h} t_{a,main}(\mathbf{f}) - c_{\text{lease}}(\sum_{a \in A^j} v_a) - C_2^j) \quad (4.6)$$

where the factor C_1^j includes the component of costs associated with f_a and C_2^j the constant costs that affect the MSP profit.

4.4 Users of the Multi-modal Network

As anticipated, in order to represent the heterogeneity of users' choices concerning their behaviours and travel perceptions (Cascetta, 2009), we divide the demand into $\mathbb{K}$ classes. Each class of users k is characterised by their sociodemographic characteristics, their home location and their daily trip chains, considered as a sequence of trips in a day. Users of the same class share the same origin, destination and all the locations visited in-between. At this stage, we do not explicitly consider the activities performed at each location, due to the fact that the proposed model is static and the time component is not take into account.

Undoubtedly, the choice of dividing users into classes increases the complexity of the model. However, the socio-economic characteristics only affect users' perceived costs and not the network expansion. On the other hand, defining user classes based on the combinations of daily trip chains and activity locations could become a non-trivial problem in large-scale networks. However, travel behavior literature shows that during typical weekdays the majority of travellers tend to perform home-work-home tours when using PT or add an additional activity before/after work when travelling with private vehicles (Axhausen et al., 2002; Sprumont et al., 2022). Moreover, the combination of trip chains, activity sequences and locations are spatially limited and rather repetitive (Susilo & Axhausen, 2014). Therefore, focusing on the most frequent tours, it is possible to cover most of the travel demand of an area and in turn keep the complexity of the supenetwork to a reasonable extent. However, Table 4.3 provides a comprehensive analysis of the various factors affecting network scalability, particularly focusing on the number of layers that are going to be generated when increasing the number of modes, mobility subscriptions, MSP participating in each MaaS package, and user classes.

Table 4.3: Impact on the network scalability

	single MSP subscription	**MaaS subscription**	**User classes**
No. of Layers	J	$\sum_{j=1}^{\mathbb{J}} sj$	$\sum_{n=1}^{N-1} \prod_{\tau=1}^{n-1} N(N-\tau)$

When there is an increase in the number of MSPs, and consequently an increase in the number of transportation modes offered, it is essential to understand that there will be an expansion in the number of mobility subscriptions available. MSPs may choose to either individually sell their services or collaborate to create packages that combine multiple modes of transportation. In the scenario with $\mathbb{J}$ MSPs, when each j can only offer one mobility service, the number of layers is directly proportional to the number of MSPs (first column in Table 4.3). When each j can offer a varying number in between single mobility subscription and MaaS packages (named s_j), the number of layers depends on how many suppliers are on the market and how many packages they provide. In this context, as shown in the second column of Table 4.3, the number of layers increases exponentially as the number of packages s_j provided by each of the J suppliers increases. In the last scenario (third column), the case in which the number of user classes with different daily trip chains increases has been taken into account. This was done by considering that trip chains could increase as the number of zones ($\mathbb{N}$) increases. Therefore, with an increase in the number of zones, a corresponding combinatorial increase in the variety of daily trip chains will occur, as indicated in the third column of Table 4.3. This type of variation leads to an exponential network expansion.

4.4.1 Equilibrium Assignment

After defining the structure of the supernetwork and how to divide users in classes, in this section the used UE formulation is presented. As already pointed out, inside the supernetwork, users' trip chains are represented as paths p connecting OD pairs. At this stage, it is fundamental to properly define the set of feasible paths in the supernetwork. A path encodes aspects of the multi-modal network, such as one-off subscription costs, and logical mode choice sequences. For this reason, the traffic network conditions will be expressed in a path-based approach and a priori path enumeration is taken into account. The computational expense and consequent limitations of path enumeration and difficulty to prove existence and uniqueness are well known, as already explained in Chapter 3. However, in this thesis, the analysis focuses on small networks to comprehend the characteristics of the proposed methodology. As a result, for these scenarios, the approach of enumerating paths remains applicable. The concept of existence and uniqueness of the solution, instead, will be addressed later in the next subsection.

Generally, the UE problem is complicated by the presence of multiple classes and the interdependency between flows on parallel links of the supernetwork. Concretely, some supernetwork links represent copies of the same real transport link of the underlying infrastructure network e.g. travel time on car-sharing links is influenced by travellers using PC and vice versa. Consequently, the corresponding link costs are non-separable. Given the non-separable nature of the network link cost functions, the UE is formulated as a VI (Dafermos, 1980). The users' equilibrium decision variables are the path flows, represented by the vector $\mathbf{x} \in \mathbb{X} \subseteq \mathbb{R}^p$, with p paths and $\mathbb{X}$ the set of demand-feasible flows.

In order to consider multi-class users characterised by class-specific cost perception, an extension of the multi-class and multicriteria traffic network equilibrium model proposed by Nagurney (2000) is used in this thesis. The flow conservation and the relationship between link flow and path flow have been already introduced in section 3.3.2 in Equations 3.20 and 3.21 respectively. Moreover, we could consider that the total flow on link a depends on the sum of the link flow over all classes:

$$f_a = \sum_{k \in K} f_a^k \quad \forall a \in A \tag{4.7}$$

When making their modal choices, users will encounter different costs associated with their chosen links (i.e. the mode-specific travel alternatives to reach the destination where the activity is performed). Each access link to a service (access link) will have a constant subscription cost or a fixed cost of ownership c_s, if applicable to the mode of transport. Transfer links usually have zero cost when they are part of multi-modal paths; they can have the function of the access links and thus have a constant subscription cost. Two components of unitary costs characterise, instead, each mode-specific link: real monetary costs faced by the users to use the service associated with MSP, and class-dependent perceived costs connected to access time, waiting time, congestion, etc. The latter costs are flow and capacity-dependent functions. In some cases, the congestion effects can then be influenced by the demand flow of a specific mode of transport. In others, since each layer of the supernetwork represents the same underlying physical network, congestion can also be influenced by the flow of other modes of transport and by the flow of all classes of users crossing the same links. In particular, a class k will perceive the cost on a generic mode-specific link a of the network as follows:

$$C_a^k(\mathbf{f}, v_a) = C_{a,access}^k(f_a, v_a) + C_{a,main}^k(\mathbf{f}) + C_{a,egress}^k(f_a, v_a) \qquad (4.8)$$

The first term of Equation 4.8 represents the access cost that class k will face in order to reach a mode of transport departing from an activity location. The access cost for user class k is:

$$C_{a,access}^k(f_a, v_a) = c_{a,walking}^k t_{a,walking}(f_a, v_a) + c_{a,wait}^k t_{a,wait}(f_a, v_a) \qquad (4.9)$$

where the first component considers the time needed to reach the chosen mode of transport $t_{a,walking}(f_a, v_a)$. This function could be considered as a constant value, derived for example from the average distance from a bus stop, or it can be influenced by the users choosing the same mode of transport in relation to the limited capacity of the service. In this situation the users could choose to reach the next location in which to find more available vehicles, however at the cost of an increase in travel time. This time component is then associated with a monetary value of time, a cost that has a different weight based on the users' class ($c_{a,walking}^k$). The second component in Equation 4.9 represents the time needed to wait for an available vehicle ($t_{a,walking}(f_a, v_a)$), associated with the cost $c_{a,wait}^k$ perceived by class k.

The second term in Equation 4.8, instead, represents the total cost for user class k using the main mode of transport. Specifically, this cost can be calculated as follows:

$$C_{a,main}^k(\mathbf{f}) = c_{a,fuel} l_a + c_{a,km} l_a + c_{a,h} t_{a,main}(\mathbf{f}) + c_{a,ticket} + c_{a,main}^k t_{a,main}(\mathbf{f}) \qquad (4.10)$$

The first term considers the cost for fuel (or electricity) $c_{a,fuel}$ connected to the kilometres travelled l_a. The other three components are directly connected with the service chosen by travellers, and depends on cost $c_{a,km}$ per kilometres travelled l_a, the cost $c_{a,h}$ connected to the time spent using the service $t_{a,main}(\mathbf{f})$, and an eventual fixed ticket or cost $c_{a,ticket}$. The last term is connected to the class dependent cost $c^k_{a,main}$ associated to the time spent in the mobility service with the actual time $t_{a,main}(\mathbf{f})$. Unlike other time functions, the latter is influenced not only by the flow on the specific analysed link but also by the flow on other parallel links $(\mathbf{f})$. For instance, the travel time on a link when using a PC is not solely determined by the number of PCs on that road; it is also affected by the presence of users using car-sharing services within the same trip connection. In essence, this time component takes into account the interdependencies between different modes of transportation within the same trip connection, therefore the link cost are non-separable.

Finally, the last term presented in Equation 4.8 indicates the egress cost of class k on link a:

$$C^k_{a,egress}(f_a, v_a) = c^k_{a,park} t_{a,park}(f_a, v_a) + c^k_{a,walking} t_{a,walking}(f_a, v_a) \quad (4.11)$$

These costs are calculated considering the time spent to find a parking space with a specific mode of transport $(t_{a,park}(f_a, v_a))$ and the monetary cost $(c^k_{a,park})$ that class k associates with this time. In addition, the cost of the time needed to reach the final destination is considered $c^k_{a,walking}$, once left the main mode of transport, multiplied by the time $t_{a,walking}(f_a, v_a)$.

Figure 4.5 details a section of the supernetwork of Figure 4.4, to illustrate how these costs are assigned based on the mode of transport considered.

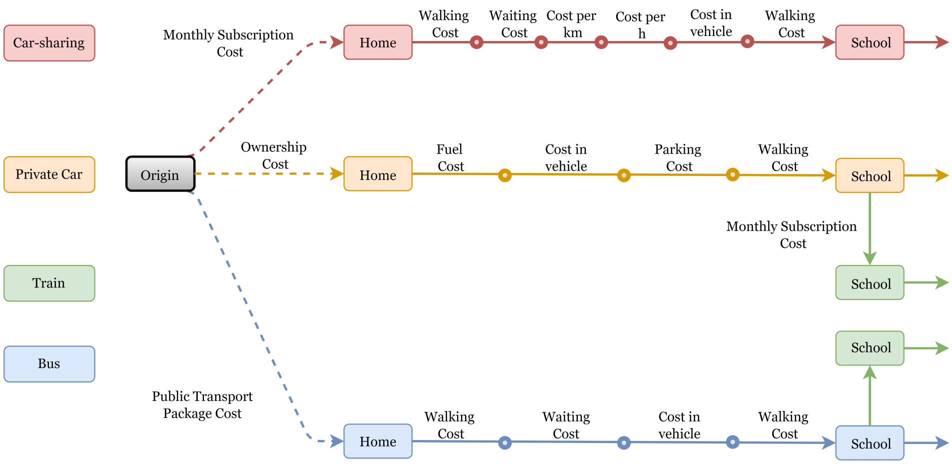


Figure 4.5: Cost details of a link from the supernetwork shown in Figure 4.4

By adopting this methodology, the supernetwork is built in a manner that allows for the incorporation of diverse multi-modal restrictions and package subscriptions directly within its framework. These components collectively contribute to the non-additive nature of path costs. Consequently, it becomes crucial to generate paths directly in order to preserve of these constraints.

It is possible to write the cost of a path connecting an OD pair w for user class k equal to the sum of its constituent link costs:

$$C_p^k(\mathbf{x},\mathbf{v}) = \sum_{s\in A_s}\sum_{a\in A} c_s\delta_{a,s}\delta_{a,p} + \sum_{a\in A} C_a^k(\mathbf{f}, v_a)\delta_{a,p} \quad \forall p \in \mathbb{P}_w, \forall w \in \mathbb{W} \tag{4.12}$$

Once the supernetwork is constructed and link costs defined, all users of each class are assigned to the network following Wardop's first equilibrium principle (Wardrop, 1952). For each class k, for all OD pairs w and for all paths p, the path flow vector $\mathbf{x}^*$ is said to be an equilibrium if the following conditions hold:

$$C_p^k(\mathbf{x},\mathbf{v}) \begin{cases} = \rho_w^k & x_p^* > 0 \\ \geq \rho_w^k & x_p^* = 0 \end{cases} \tag{4.13}$$

with

$$x_p \geq 0 \quad \forall p \in \mathbb{P}_w \tag{4.14}$$

where (4.14) is the path flow non-negativity constraint. The described model is based on non-separable link cost functions, due to the fact that the cost on a link is influenced by the flow on other links of the supernetwork.

Let $C_p^k(\mathbf{x},\mathbf{v})$ be the path cost function for a generic class $k \in \mathbb{K}$, which depends on the capacities, $\mathbf{v}$, supplied by MSPs. Then a vector of path flows $\mathbf{x}^* \in \mathbb{X}$ is a Wardrop equilibrium if and only if it satisfies the VI problem:

$$\sum_{k\in K}\sum_{w\in W}\sum_{p\in P_w} C_p^k(\mathbf{x}^*,\mathbf{v})(\xi_p - x_p) \geq 0 \quad \forall \xi_p \in \mathbb{X} \tag{4.15}$$

Given the vector of fleet sizes $\mathbf{v}$, $\mathbb{X}^*(\mathbf{v})$ represents the set of equilibrium solutions to (4.15).

As described in section 3.3.2, Nagurney (2000) demonstrates the existence of a solution for a VI having the same structure as Equation 4.15. Existence of a solution can be established when the path cost functions $C(\mathbf{x})$ are a composition of continuous functions, acting on a closed and convex feasible region $\mathbf{x} \in \mathbb{X}$. Criteria that are satisfied thanks to the network equilibrium conditions (Equation 3.20, 3.21, and 4.14). This reasoning can be extended to a path-based formulation, as is used here, following the approach of Watling (2006).

The proof of a unique solution usually relies on strict monotonicity of the vector of link cost functions. However, as pointed out by Nagurney (2000), in the context of multi-class and multi-criteria models, even if all the components of cost satisfy strict monotonicity concerning the total link flow, this may not be sufficient. In the simpler case when the link cost functions are separable with respect to the total flow on the link, uniqueness could be proved. However in this case with non-separable link cost functions, uniqueness cannot be proved.

In this thesis, an application of the solution algorithm presented in section 4.4.2 will be presented in section 4.4.3 in order to try to address the problem of existence and uniqueness numerically.

4.4.2 Solution Algorithm: The Extragradient Method

As already pointed out, VIs are used as a powerful reformulation of a wide number of problems on different fields. For this reason, several algorithms with the purpose of efficiently solving VIs have been proposed in the literature. These algorithms achieve equilibrium iteratively through equilibration procedures. Specifically, at each iteration the problems is reformulated as an optimization problem and solved using nonlinear programming algorithms (Nagurney, 1998). The algorithms employed to solve VIs can be categorized into distinct groups according to their underlying methodology: linear approximation-based methods, KKT based methods, proximal point methods, and projection-based methods (Singh, 2012).

It is important to notice that in transportation specific algorithms, such as MSA and F-W (Frank, Wolfe, et al., 1956), have been developed when formulating the traffic network equilibrium problem as a convex problem. On the contrary, when the UE is formulated as a the problem 3.10, general algorithms to commonly solve VI are usually used. In this context, the algorithm taken into account in the proposed methodology falls into the group of projection-based methods, based on an extension of Extragradient Method (EM).

The EM, often used to solve UE (Nagurney, 2000; Szeto & Jiang, 2014), is an extension of the classic projection method, which solves VIs dividing them into sub-problems that are quadratic programming problems (Nagurney, 1998). This problems consists in a special class of nonlinear programming with quadratic objective function and linear constraints that can be written as (Bazaraa et al., 2013):

$$\min \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} - \mathbf{q}\mathbf{x} \tag{4.16}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \tag{4.17}$$

$$\mathbf{x} \geq 0 \tag{4.18}$$

where $\mathbf{q}$ is a n vector, $\mathbf{b}$ is a m vector, $\mathbf{A}$ is a matrix of dimension $m \times n$, and $\mathbf{H}$ is a positive semidefinite matrix.

The EM was initially proposed by Korpelevich (1976), and it offers advantages compared to the classical projection method, such as a relaxed convergence requirement. It consists in a double projection algorithm that by sequential approximation takes a step along the gradient, and uses the value of the gradient of the new point as a direction to move to the next approximation.

In order to apply this method to the proposed UE, let's consider X as a nonempty closed convex subset of R^n, and a continuous function $C_p^k(\mathbf{x}^*, \mathbf{v})$, where $\mathbf{x}^*$ is a solution vector of the UE if it satisfies the VI problem of equation 4.15. Starting with an initial value of $\mathbf{x}^*$ for iteration i, then the EM updates $\mathbf{x}$ at each iteration i calculating a first value of $\overline{\mathbf{x}}^i$ as follows:

$$\overline{\mathbf{x}}^i = P_X(\mathbf{x}^i - \psi C_p^k(\mathbf{x}^i, \mathbf{v})) \tag{4.19}$$

with ψ is a constant positive step length between the values 0 and 1. $P_X(\bullet)$ is the orthogonal projection onto X and a solution of the quadratic problem:

$$\min \frac{1}{2}\mathbf{x}^T\mathbf{x} - (\mathbf{x}^i - \psi C_p^k(\mathbf{x}^i, \mathbf{v}))^T\mathbf{x} \tag{4.20}$$

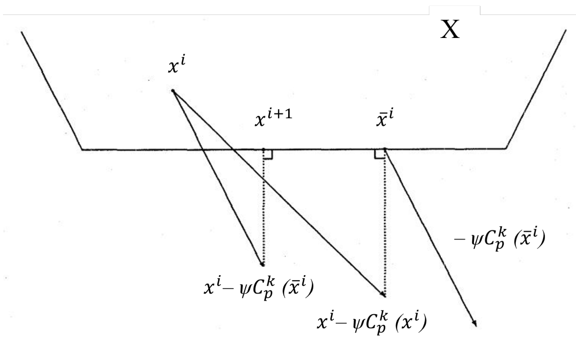


Figure 4.6: Double projection (Marcotte, 1991)

In this reformulation, the symmetric matrix $\mathbf{H}$ of Equation 4.16 becomes the identity matrix, and $\mathbf{q}$ is the value in brackets. Moreover, once the value of $\overline{\mathbf{x}}^i$ is evaluated, a second projection (as shown in Figure 4.6) is necessary in order to find the next point:

$$\mathbf{x}^{i+1} = P_X(\mathbf{x}^i - \psi C_p^k(\overline{\mathbf{x}}^i, \mathbf{v})) \tag{4.21}$$

as a solution of the quadratic problem.

However, the EM is based on a fixed ψ, and the choice of the value of this stepsize it is quite important to set. When it is too small the method tends to have slow convergence, in case of a big value of ψ, instead, it can be hard to achieve convergence. For this reason, several relaxation method have been proposed in the literature. Khobotov (1987), for example, relaxed further some assumptions made by Korpelevich (1976), such as the knowledge a priori of the Lipschitz constant associated with the cost function, and introduced a modification of the EM including an adaptive stepsize. In this thesis the improvement proposed by Panicucci et al. (2007) is incorporated, to prevent the stepsize from becoming excessively small and ensure that the algorithm converges faster. Moreover, each projection of the EM is decomposed into smaller projections, one for each OD pair and each class, using the projection algorithm proposed by Michelot (1986).

Specifically, in Algorithm 1 the different steps of the adaptive EM are listed, incorporating the projection algorithm presented in Algorithm 2. Considering μ,$\lambda \in (0, 1)$, and $\overline{\theta} > 0$.

In this application the gap function (Gap), used in Algorithm 1 as a convergence criterion, has the form of the following relative gap function (Chiu et al., 2011), often used in static traffic assignment models:

$$Gap = \sum_{k \in K} \sum_{w \in W} \frac{[\sum_{p \in P_w} (x_p^k * C_p^k(\mathbf{x}, \mathbf{v}))] - d_w^k * C^k_{p_w min}(\mathbf{x}, \mathbf{v})}{d_w^k * C^k_{p_w min}(\mathbf{x}, \mathbf{v})} < \chi^2 \tag{4.22}$$

Specifically, the numerator represents the total gap, i.e. the deviation between the current assignment solution and the ideal shortest path. This value is then divided by the total shortest path solutions. When the travel time is close to the one of the shortest path, the numerator becomes close to zero, and therefore Gap will have a small value.

Algorithm 1 Adaptive Extragradient Method with Projection Algorithm

Input Lower Level Parameters and Functions
Initialization for EM:
Set $MaxIterations, \mu, \lambda, \theta_{ii=0} = \overline{\theta}, ii = 0, Gap = \infty, \chi^2$
while $Gap > \chi^2 \quad and \quad ii < MaxIterations$ **do**
  Compute $Y = \mathbf{x}_{ii} - \theta_{ii} C(\mathbf{x}_{ii}, \mathbf{v}^j)$
  Compute First Projection using Algorithm 2
  Compute Path Flow $C(\overline{\mathbf{x}}_{ii}, \mathbf{v}^j)$
  if $\theta_{ii} > \lambda \frac{||\mathbf{x}_{ii} - \overline{\mathbf{x}}_{ii}||}{||C(\mathbf{x}_{ii}, \mathbf{v}^j) - C(\overline{\mathbf{x}}_{ii}, \mathbf{v}^j)}||$ **then**
    Reduce θ_{ii}
    $\theta_{ii} = min\{\mu \theta_{ii}, \lambda \frac{||\mathbf{x}_{ii} - \overline{\mathbf{x}}_{ii}||}{||C(\mathbf{x}_{ii}, \mathbf{v}^j) - C(\overline{\mathbf{x}}_{ii}, \mathbf{v}^j)}||\}$
    Compute Projection using Algorithm 2
  end if
  Compute $\mathbf{x}_{ii+1}$ through **Second Projection using Algorithm 2**
  Set $\theta_{ii+1} = min\{\mu \theta_{ii}, \lambda \frac{||\mathbf{x}_{ii} - \overline{\mathbf{x}}_{ii}||}{||C(\mathbf{x}_{ii}, \mathbf{v}^j) - C(\overline{\mathbf{x}}_{ii}, \mathbf{v}^j)}||\}$
  Compute Gap
  Set $it = ii + 1$
end while

Algorithm 2 Projection Algorithm

Input Lower Level Parameters and Functions
Initialization for Projection
for $w = 1 : \mathbb{W}$ **do**
  for $k = 1 : \mathbb{K}$ **do**
    $n_{paths} = length(Y(w,k))$
    $\overline{\mathbf{x}}(w,k)_{ii} = Y(w,k) + (d_w^k - sum(Y(w,k)))/n_{paths}$
    $ind = \overline{\mathbf{x}}(w,k)_{ii} > 0; \quad n_{ind} = sum(ind)$
    $\overline{\mathbf{x}}(w,k)_{new} = \overline{\mathbf{x}}(w,k)_{ii} + (d_w^k - sum(Y(w,k(ind))))/n_{paths}$
    $\overline{\mathbf{x}}(w,k(ind))_{ii} = \overline{\mathbf{x}}(w,k(ind))_{new}$
  end for
end for

4.4.3 Numerical Application

In this section, the proposed solution algorithm for the UE is applied to the network represented in top Figure 4.7 with the purpose analysing the characteristics of the different functions and their influence on the equilibrium solutions. In the proposed network a single class performs a daily tour from location L_1 to location L_2, to finally return to the first location L_1.

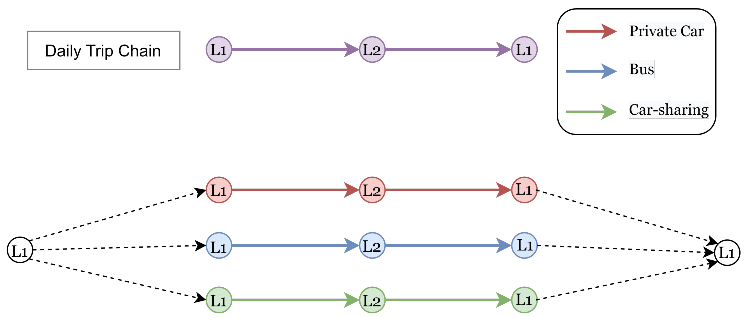


Figure 4.7: Supernetwork with PC, Bus, and Car-sharing

Three modes of transport are included: PC, bus, and a one-way car-sharing service. In the presented example it is considered that PC and the car-sharing service have non-separable cost functions due to the fact that are sharing the same infrastructure. The bus service, instead, is considered to have dedicated lanes without experiencing congestion from other modes of transportation.

Following the formulations described in Section 4.4.1, the parameters taken into account are listed in Table 4.4. In this example, the link costs functions are considered constant, or they take the form of the conventional Bureau of Public Roads (BPR) function:

$$t = t_0 \left(1 + \alpha \left(\frac{f}{C} \right)^{\beta} \right) \tag{4.23}$$

where the coefficients α and β define the shape of the function, t_0 represents the travel time in free-flow conditions. Finally the travel time will increase based on the ratio between flow (f) and capacity (C) (Maerivoet & De Moor, 2000). Table 4.5 shows the different functional parameters.

The solution algorithm described in the previous section (4.4.2) is applied using MATLAB R2019b[1]. The running time to find the equilibrium solution is < 1 second. The presented algorithm is first applied varying the algorithm parameters (μ, λ, θ) in order to understand their impact on the iterative procedure and the equilibrium solution.

Table 4.4: Parameters Scenario 1: Lower-level

Parameters	PC	Bus	Car-sharing
c_s	0.1	1.1	0.8
r_s	-	-	-
$c^1_{a,access}/c^1_{a,egress}$	8	11	9
$c^1_{a,wait}$	-	13	9
$c^1_{a,main}$	8.2	9.5	8.5
$c_{a,fuel}$	0.37	0.2	
$c_{a,h}$	-	-	0.3
$c_{a,km}$	-	-	0.3
l_a	10	10	10
$c^1_{a,park}$	11	-	-
v_a	300	300	50
d^1_1	300		

Figure 4.8 shows how the initial choice of $\overline{\theta}$ impacts the number of iterations required for convergence (as shown in Figure 4.8a). It is evident that for small values of $\overline{\theta}$ the number of iterations is extremely high with relatively high values of the resulting relative gap. The corresponding equilibrium distribution of the different modes of transport and the iterations necessary to achieve the same modal distribution are shown in Figure 4.8b. In red, PC takes almost half of the demand, where bus (blue) and car-sharing (green) have an even distribution between the two modes.

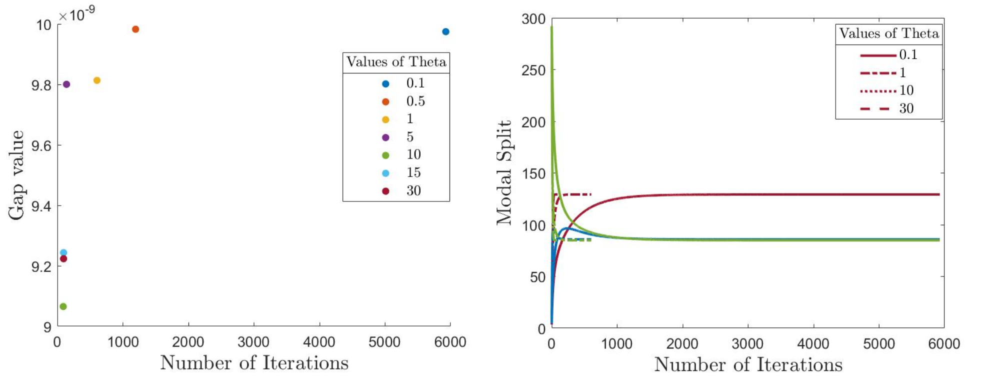
The values of parameters μ and λ can range from 0 to 1. Figure 4.9 illustrates how these values impact the number of iterations and the relative

[1]The simulations are carried out using Windows 10 laptop with an Intel(R) Core (TM) i7-8650U CPU with a base frequency of 1.90GHz and a system memory of 16.0 GB.

Table 4.5: Functional Parameters Scenario 1: Lower-level

Function	PC					Bus					Car-sharing				
	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β
$t_{a,access}(f_a, v_a)$			-			0.1		-			0.1	0.1	f_a	50	2
$t_{a,wait}(f_a, v_a)$			-			0.4	0.3	f_a	200	4	0.05	0.1	f_a	50	4
$t_{a,main}(\mathbf{f})$	0.2	2	$\mathbf{f}$	250	4	0.3	2	$\mathbf{f}$	200	4	0.2	2	$\mathbf{f}$	250	4
$t_{a,park}(f_a, v_a)$	0.1	1	f_a	300	2			-					-		
$t_{a,egress}(f_a, v_a)$	0.08		-			0.1		-			0.1	0.1	f_a	50	2

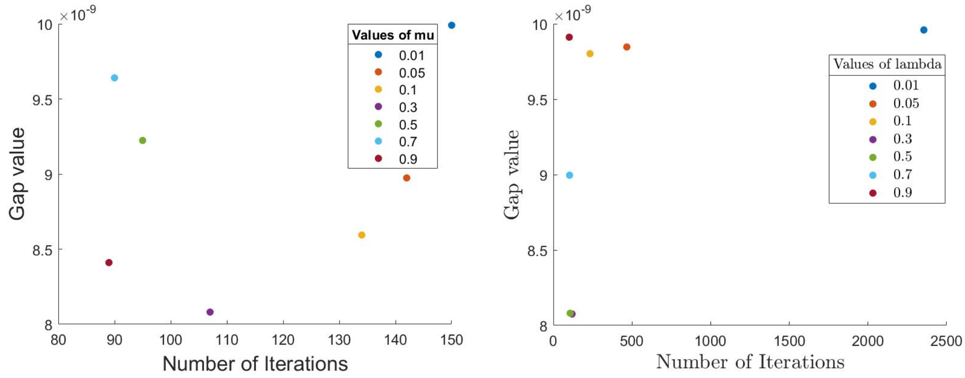
gap. It's noticeable that when the values are small, the number of iterations and the gap value tends to increase, especially for λ. In both situations, it appears that the optimal values typically fall between 0.3 and 0.5. This leads to fewer interactions and a smaller gap. After this analysis in Table 4.6 are listed the algorithm parameters used for the next results.



(a) Gap value through iterations **(b)** Equilibrium distribution through iterations

Figure 4.8: Impact of $\overline{\theta}$ variation

Subsequently, in Figure 4.10, it is shown the difference between the proposed case with non-separability between PC and car-sharing (Figure 4.10b), and a possible case in which all the modes of transport are considered to not have interaction between each other (Figure 4.10a). In the separable case, the flow on PC and car-sharing has higher values than the bus. This flow distribution arises because congestion relies solely on the flow within each mode of transportation. In general, the distribution of path flows appears comparable across iterations in both scenarios. Nonetheless, when non-separable congestion effects are introduced, the bus option begins to emerge as a more favorable solution, attracting a greater share of the flow. From this analysis, it is clear that neglecting the influence that different modes of transport have on each other could lead to an incorrect estimation of the flow, and possibly wrong estimations of fleet sizes on suppliers' side.

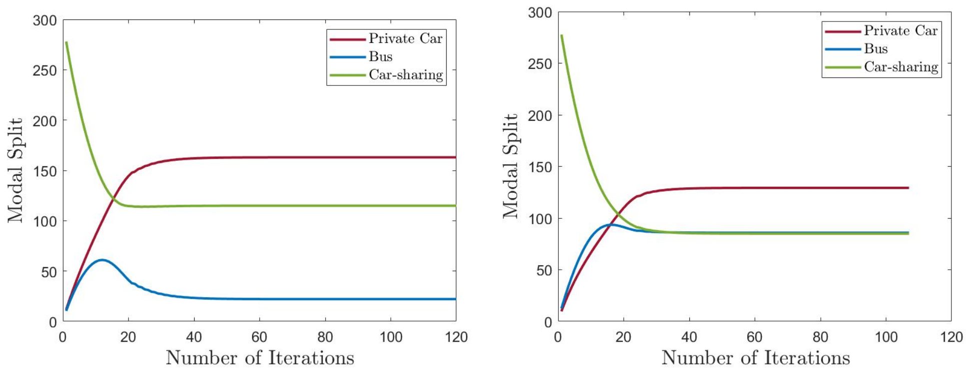


(a) μ **(b)** λ

Figure 4.9: Parameters variation

Table 4.6: Algorithms Parameters (from Algorithm 1)

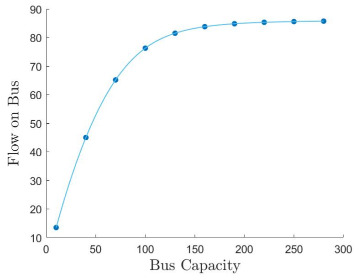
Parameters	
MaxIterations	50000
μ	0.3
λ	0.5
$\overline{\theta}$	30
χ^2	1e-8



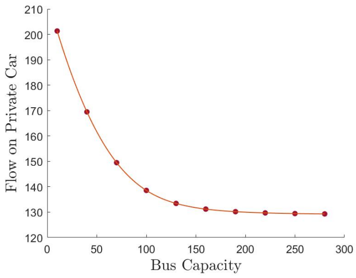
(a) separable **(b)** non-separable

Figure 4.10: Modal split variations

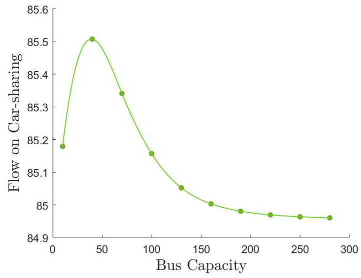
With the aim of comprehending users' modal choices within the examined scenario, a study is undertaken to assess how varying fleet sizes (service capacities) influence the outcome.



(a) Bus Flow variation

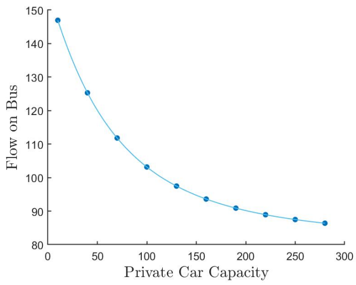


(b) PC flow variation

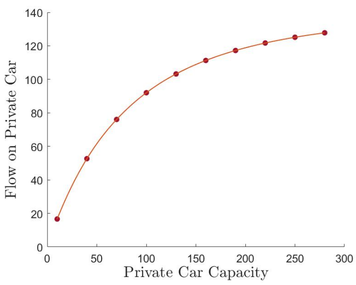


(c) Car-sharing flow variation

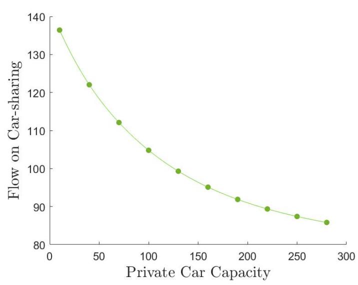
Figure 4.11: Modal Flow variations with Bus capacity



(a) Bus flow variation

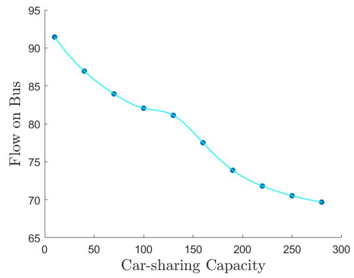


(b) PC flow variation

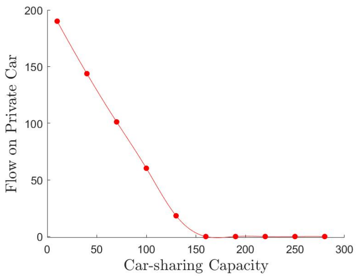


(c) Car-sharing flow variation

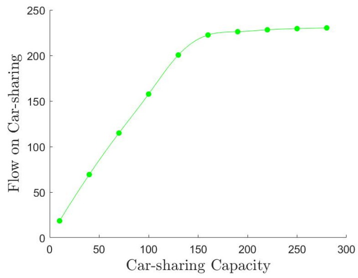
Figure 4.12: Modal Flow variations with PC capacity



(a) Bus flow variation



(b) PC flow variation



(c) Car-sharing flow variation

Figure 4.13: Modal Flow variations with Car-sharing capacity

Specifically, Figure 4.11 shows the impact of bus capacity on the different modal options. When this capacity is extremely low, PC seems to be the best options chosen by users (Figure 4.11b). Interestingly, over a capacity of 100, the equilibrium flow starts to become constant. This means that increasing the capacity doesn't seem to significantly attract more users (Figure 4.11a).

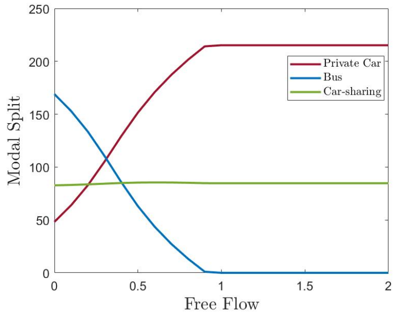
Finally, in this scenario, the car-sharing option appears to not be affected by the variation in capacity of the bus service (Figure 4.11c).

Following the same approach, Figure 4.12 shows what happens to the flow distribution when varying the available PC vehicles. Initially, when the capacity has low values, the flow splits almost evenly between bus and car-sharing (Figure 4.12a and 4.12c). However, as the number of vehicles increases, PC become the more appealing mode of transportation (Figure 4.12b), capturing the majority of the market share, while the other modes tend to exhibit consistent and gradual decreases across the different capacity values.

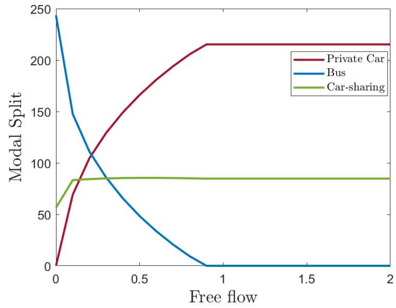
Finally, in Figure 4.13 it is possible to observe how adjustments in the car-sharing fleet size influence the various transportation modes. A noticeable correlation exists between the flow in the car-sharing mode (Figure 4.13c) and PC usage (Figure 4.13b). As the car-sharing system's capacity reaches 150 vehicles, it absorbs the entirety of the flow that was previously shared with the PC, partially capturing some of the bus service flow (Figure 4.13a).

Subsequently, a study is undertaken to investigate the influence of various parameters within the cost functions on the equilibrium flows associated with the different modal options.

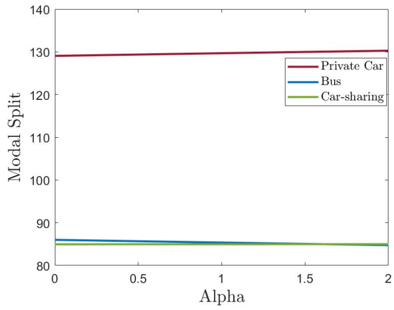
In this context, Figure 4.14 displays on the left the variation of the parameters of the used BPR function connected to the waiting time at the bus stop, and on the right what happens when varying the BPR parameters associated to the in-vehicle time. In the first case, the capacity remains fixed and equal to the vehicle capacity in Table 4.5. It is possible to notice that what impacts the most the modal flow variation on the bus service is the chosen travel time at free-flow traffic. This value is a constant for the function 4.23. However, when this constant term in the waiting time surpasses one hour, it becomes evident that users no longer perceive the bus service as a viable choice. The in-vehicle travel time, instead, is considered to be connected to the time spent in the main mode of transport to move from one location to the next one. In this case the value of β is considered fixed and equal to 4. It is clear that the behaviour of the flows variation with the free-flow travel time (Figure 4.14b) is almost the same as the waiting time, with the shift of the entire flow to PC when the free-flow reaches higher values. Another component that affects the popularity of the bus service is the capacity associated to the BPR function. As a consequence of the increasing capacity, there is a reduction of the waiting time, and more flow moves from PC to the bus service, leaving the car-sharing service flow unchanged. Lastly, the access and egress time components of the bus service are taken into account. In this case, these values are considered to be constant, representing the average walking time usually needed to reach a bus station. In Figure 4.15, the variation of both component simultaneously is displayed, proving that the maximum walking time to reach the closest bus stop should be maximum 15 minutes for both directions, to have a positive value of the traffic flow.



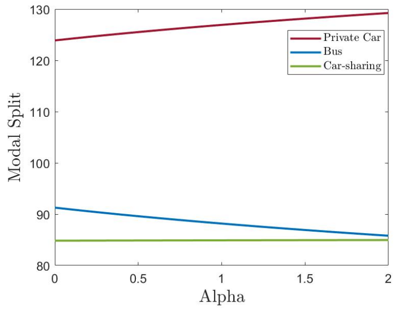
(a) Free flow variation



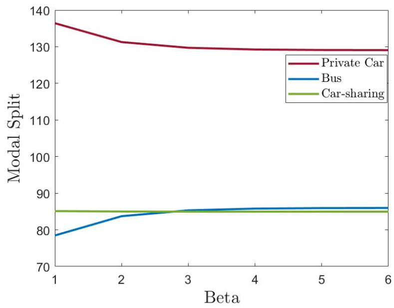
(b) Free flow variation



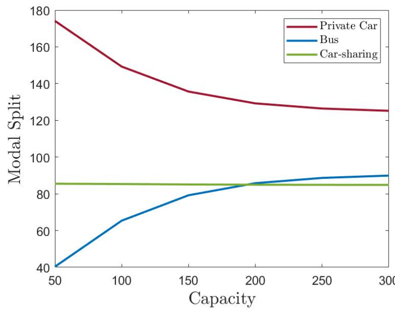
(c) Alpha variation



(d) Alpha variation



(e) Beta variation



(f) Capacity variation

Figure 4.14: Modal split variation: with Bus Waiting time parameters (left), with Bus In-Vehicle time parameters (right)

On the car side, Figure 4.16 shows the path flow variations with the variation of the egress time. In this case, the time is considered to be constant, taking into account an average time to reach the PC located in a parking. It is evident that this time has an enormous impact on users' choice, that are almost evenly distributed between bus and the one-way car-sharing service.

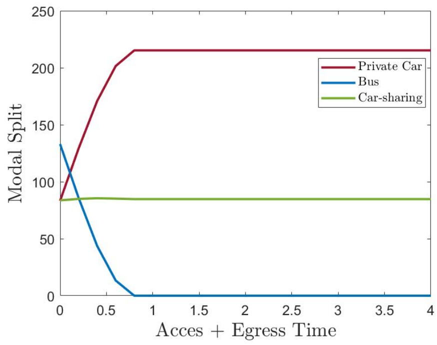


Figure 4.15: Modal split variation with Bus Access + Egress time

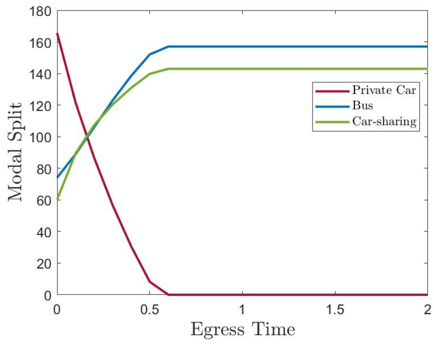
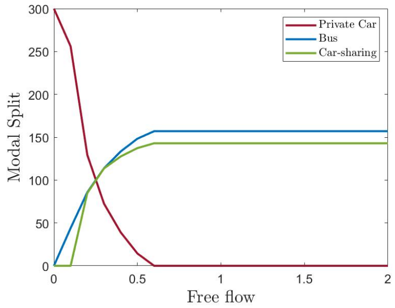


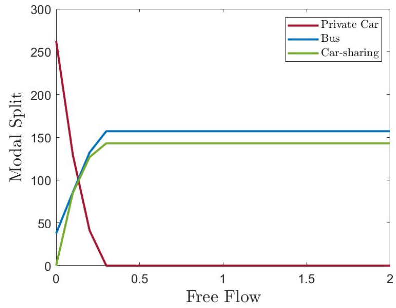
Figure 4.16: Modal split variation with Car Egress time

The variation of the parameters of the in vehicle BPR function, with respect to the PC, are illustrated on the left side of Figure 4.17. The influence of free-flow travel time (Figure 4.17a) on path flow mirrors the impact of the constant egress time. However, it's noticeable that when the free-flow component is close to the minimum, PC attracts the entire demand. The capacity factor wields a significant role within this BPR function (Figure 4.17c). As capacity increases, the flow for car-sharing experiences a sharp decline, ultimately becoming equal to zero. Whereas for lower values of α the in vehicle BPR function is close to the constant value of the free-flow travel time. When this value increases, the path flow smoothly decreases.

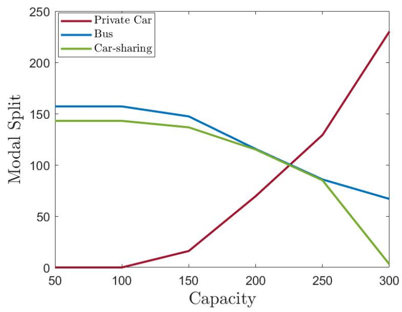
Lastly, the parking time refers to the duration required to locate an available parking space. As parking becomes increasingly congested, users must invest more time in the search for a free spot. On the right side of Figure 4.17 the functional parameters of the BPR function associated to this time are analysed. In this case, the capacity is considered to be fixed and equal to the available vehicles listed in Table 4.4.



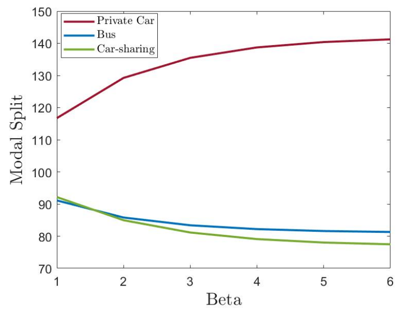
(a) Free flow variation



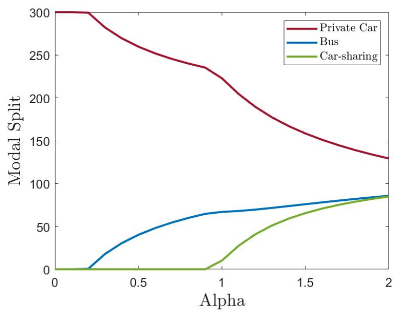
(b) Free flow variation



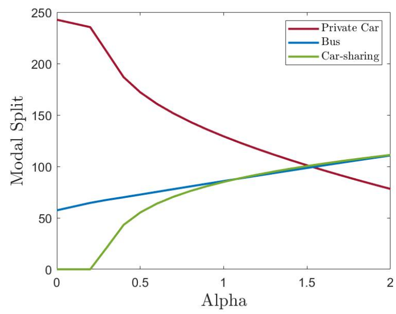
(c) Capacity variation



(d) Beta variation



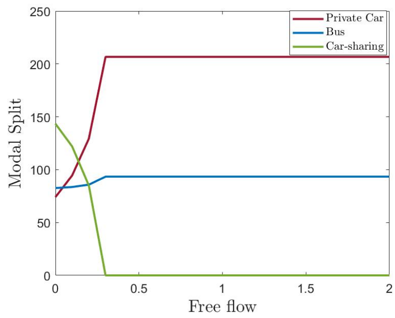
(e) Alpha variation



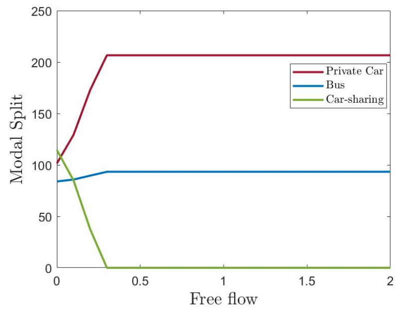
(f) Alpha variation

Figure 4.17: Modal split variation: with Car In-Vehicle time parameters (left), with Car parking time parameters (right)

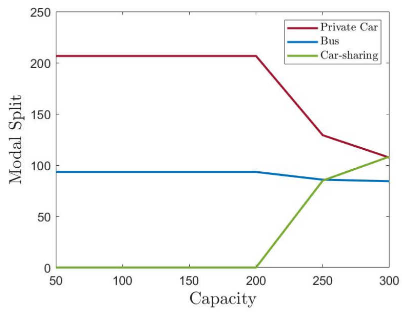
In this case, the free-flow factor is immediately affected by the increase in travel time, not even reaching the 0.5 hours (Figure 4.17b). For higher values of β there is a decrease in the total parking time that attracts more users to the PC mode of transport (Figure 4.17d). Moreover, as already pointed out for other functions and confirmed in Figure 4.17f, the increase in the value of α brings to a decrease in the usage of PC.



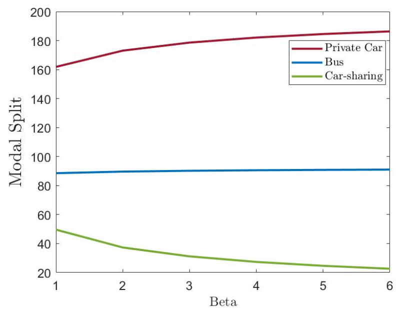
(a) Free flow variation



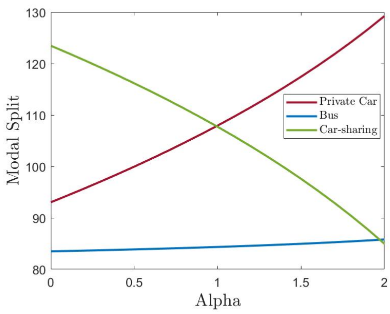
(b) Free flow variation



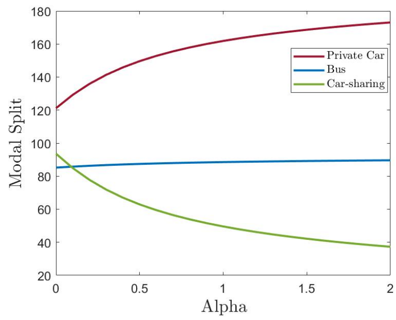
(c) Capacity variation



(d) Beta variation



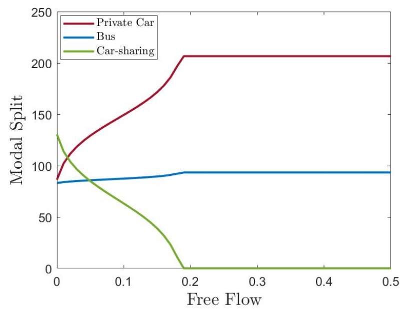
(e) Alpha variation



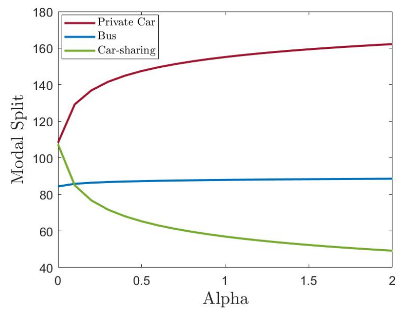
(f) Alpha variation

Figure 4.18: Modal split variation: with Ca-sharing In-Vehicle time parameters (left), with Car-sharing access time parameters (right)

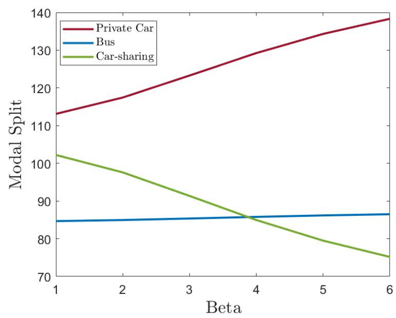
The left side of Figure 4.18 illustrates the variations of the car-sharing parameters for the in-vehicle cost function that takes the form of a BPR function. The value of β is considered to be fixed and equal to 4, as the other modes of transport. Compared to the other figures already presented, it seems really interesting to notice that this mode of transport becomes attractive to users only when the capacity is higher than the competing modes of transport (Figure 4.18c). Moreover, it is possible to notice a clear symmetric distribution of the flow with respect to the PC. The same symmetry in the distribution of flows is also evident when changing the value of α. A consistent pattern observed among all parameters is their lack of impact on the bus service. The equilibrium flow remains unchanged while the popularity of the other two transportation modes increases or decreases, driven by shifts in costs.



(a) Free Flow



(b) α



(c) β

Figure 4.19: Modal split variation with Ca-sharing Waiting time parameters (left)

The right side of Figure 4.18, instead, shows the variations of the car-sharing parameters within the BPR function linked to access time. In this instance, the egress time is omitted from display, as it shares identical attributes with the access time. Shifting the focus to Figure 4.19, the variations in waiting time parameters are shown. It's intriguing to note that in both access time in Figure 4.18 and waiting time in Figure 4.19, the observed behaviors with respect to the free flow parameters closely mirror those that have already been introduced, with a symmetric variation between PC and car-sharing and an almost constant distribution of the flow on the bus service. The factor that notably affects access time, as opposed to waiting time, is the value of β. When this parameter has lower values, waiting time doesn't significantly influence users' decisions. Furthermore, even as it increases, users tend to be more inclined to wait for an available car rather than walk a longer distance to reach the vehicle.

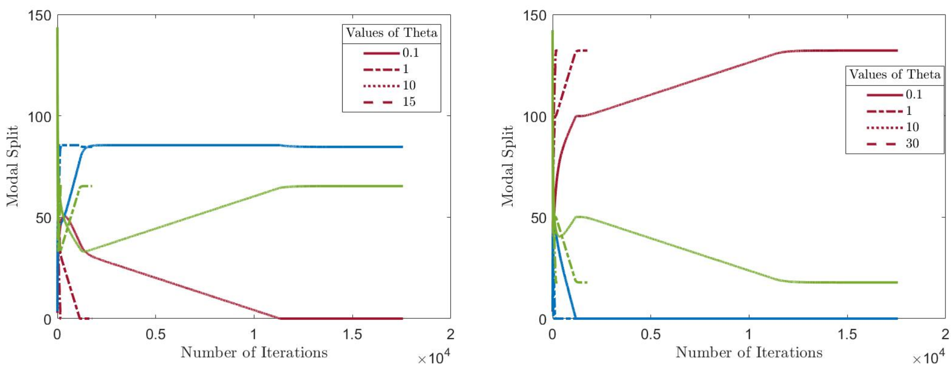
After the analysis of the functions representing the different time component that users face while choosing their mode of transport, in a second scenario a second class of users is assigned to the supernetwork introduced in Figure 4.7. The purpose of this study is to understand what happens when another class is introduced inside the network. The two different classes, named Class 1 and Class 2, have the same daily trip chain; however, they perceive the costs inside the network with different weights. Specifically, the costs perceived by the two classes are listed in Table 4.7, where Class 2 perceives higher costs than Class 1. The functional parameters used for this second scenario are the same introduced in Table 4.5.

Table 4.7: Parameters Scenario 2: Lower-level

Parameters	PC	Bus	Car-sharing
c_s	0.1	1.1	0.8
r_s	-	-	-
$c^1_{a,access}/c^1_{a,egress}$	8	11	9
$c^2_{a,access}/c^1_{a,egress}$	11.2	15.4	12.6
$c^1_{a,wait}$	-	13	9
$c^2_{a,wait}$	-	18.2	12.6
$c^1_{a,main}$	8.2	9.5	8.5
$c^2_{a,main}$	11.5	13.3	11.9
$c_{a,fuel}$	0.37	0.2	
$c_{a,h}$	-	-	0.3
$c_{a,km}$	-	-	0.3
l_a	10	10	10
$c^1_{a,park}$	11	-	-
$c^2_{a,park}$	15.4	-	-
v_a	300	300	50
d^1_1	150		
d^2_1	150		

In the first analysis, both classes are assigned with an equal demand of 150 users each. This cumulative demand aligns with the total demand in the previous scenario, a deliberate choice aimed at understanding the intricate dynamics that arise when equivalent overall demand is distributed across distinct classes.

First, the variation of $\overline{\theta}$ parameter of Algorithm 1 is presented in Figure 4.20. The Figure shows the equilibrium path flows for PC (red), bus (blue), and one-way car-sharing (green). This Figure tends to show the fact that when increasing the complexity of the problem it is essential to choose a proper value of $\overline{\theta}$ to reduce the computing time. Moreover, for different starting points of the value of $\overline{\theta}$, the equilibrium solution is the same in all cases. Finally, it is possible to notice the different behaviours of the two classes that in this case seem to be exactly the opposite. Class 1 is almost evenly distributed between bus and the car-sharing service (Figure 4.20a); Class 2, instead, uses mostly PC and with a small quota of demand on car-sharing (Figure 4.20b).

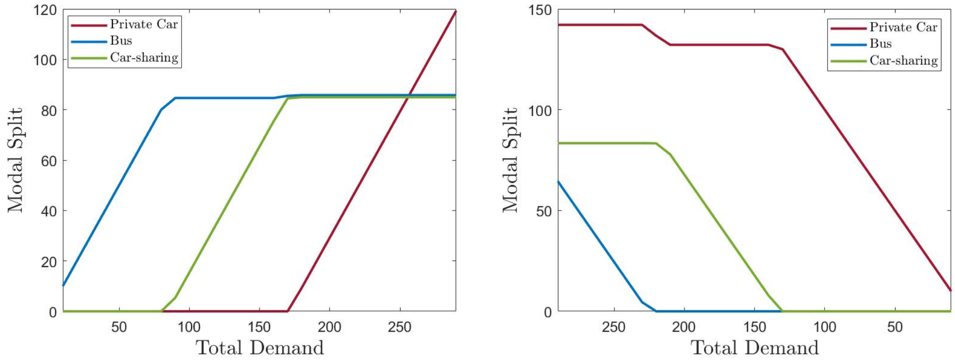


(a) Class 1 **(b)** Class 2

Figure 4.20: Modal split variations with Theta

Furthermore, this mirrored behavior becomes apparent when observing Figure 4.21, in which the total demand of the two classes is varied keeping a total demand for the OD equal to 300. Evidently, for a significant portion of the demand of Class 2, PC is more attractive. As the available capacity of the service is reached, users within this class gradually shift to other modes of transportation. On the contrary, within Class 1, the utilization of PC comes into play when the demand exceeds 150 users, aligning progressively with the decreasing demand of Class 2. This pattern can be rationalized by the principle that, as the traffic demand of Class 2 diminishes, the available capacity of PC begins to increase. As a result, users within Class 1 initially prefer the bus and car-sharing services. When these options reach their capacity, travellers gradually transition using also PC.

Finally, in Figure 4.22 is displayed the variation of the modal choices of the two classes varying the weights applied to the different cost components. In this case, is considered that Class 2 perceives higher costs (0.5 higher)

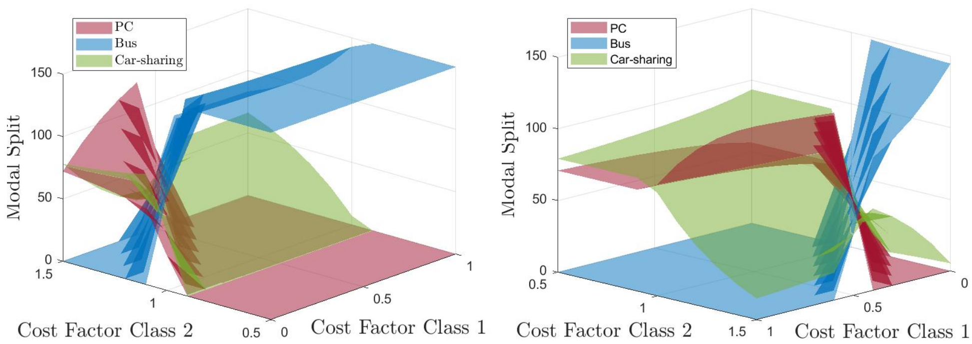


(a) Class 1

(b) Class 2

Figure 4.21: Modal split variations with Total Demand

in the network compared to Class 1. By varying the perceived costs, it is possible to observe the interplay between the different classes in influencing their modal choices within the network. When perceived costs increase, the flow distribution changes, impacting congestion effects for both classes sharing the same OD. Class 1 shows a preference for bus service as a mobility service, whereas Class 2 favors car-sharing and PC. Nevertheless, this consistent behavior alters when the perceived costs of the other class increase or decrease.



(a) Class 1

(b) Class 2

Figure 4.22: Modal split variations with Costs

4.5 Conclusion

In this chapter, the fundamental assumptions of the methodological approach used in this thesis are presented. The aim is to examine the various dynamics that may arise when different MSPs offer distinct services in the transportation market, and diverse user groups make daily travel decisions based on their socio-economic characteristics and travel patterns.

A supernetwork approach is developed to incorporate multi-modal constraints directly into the network, such as mobility package subscriptions, and the daily travel sequences of heterogeneous user classes.

Suppliers of the transport network are treated as profit maximizers, with their market presence contingent on the profitability of their mobility services. To account for the varying business strategies of different mobility services, a general profit maximization formulation is proposed.

On the user side, travel choices are determined by the network's offerings and congestion levels, with users opting for the transport mode that minimizes their network costs. To consider the influence of different transport modes on congestion and user choices at equilibrium, a path-based traffic network equilibrium in the form of a VI formulation is introduced. Additionally, a solution algorithm, based on an adaptation of the EM, is proposed to solve the UE. Numerical results demonstrate the algorithm's rapid convergence and uniqueness of the solution across different initial conditions.

Furthermore, the impact of network non-separability and the selection of different cost functional parameters on the equilibrium solution is analysed. This analysis highlights that a change in any of these parameters can result in significantly different outcomes. The choice of network non-separability underscores its importance in accurately assessing service capacity. Moreover, the available capacity of each transport mode has a substantial influence on the choices made across the entire network.

Lastly, this research emphasizes the significance of capturing user perceptions of various costs, as these perceptions directly influence their choices and, consequently, the profitability of different mobility services.

Building upon the proposed formulations and assumptions, the next chapter introduces a bi-level formulation. This formulation is designed to examine the interaction between a MSP applying various market strategies and different user classes assigned to multi-modal networks.

Chapter 5

A Stackelberg Congestion Game: An MPEC Formulation

5.1 Introduction

The focus of this Chapter is to develop a model able to study the economic assessment of any MSP influenced by the fixed strategies of other competitors and by users' choices in a congested multi-modal network. As previously described in Chapter 3, the framework for tackling such a complex problem can be termed Stackelberg Congestion Game.

The problem is formulated as an MPEC, where an MSP at the upper-level sets some strategies, changing the vehicles fleet size in the network, in order to maximize the profit. At the lower-level users are assigned to the transportation network following Wardrop's first equilibrium principle. Therefore, the decision variables of the model are the number of vehicles that the MSP can introduce in the network (their service capacity) and the path flows of users that represents the level of usage of each mobility service included in the supernetwork.

Figure 5.1 illustrates the interactions involving a MSP and user classes in relationship with the network. Specifically, variations in MSP mobility service capacities lead to corresponding alterations in network attributes. Users, with diverse perceptions of costs and engagement in different trip chains during an ordinary day, are assigned to the network with the aforementioned attributes. As a consequence of the traffic network, there will be shifts in flow distributions across the network, thereby influencing the costs and revenues experienced by the MSP. Consequently, the MSP will find it necessary to adjust their capacity strategies iteratively, aiming to optimize profits. This iterative process continues until the point where the MSP no longer finds it advantageous to modify their capacity, as any such alteration would result in reduced value of profit.

In this Chapter, a first step towards a more complex formulation is taken.

This study's fundamental goal is to grasp the complexities that emerge within a transportation network when a MSP begins changing their strategies to optimize their profit. Already with this simplified scenario with only one MSP changing their capacity, it is possible to understand the complexity of a multi-modal system operated by multiple suppliers.

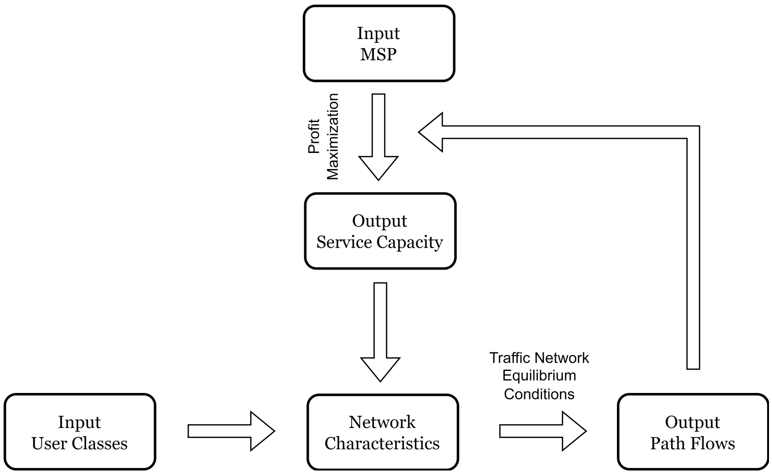


Figure 5.1: MSP-travel demand interaction

The chapter is structured as follow. The mathematical formulation is introduced in Section 5.2, followed by the proposed solution algorithm in Section 5.3. Finally, in Section 5.4 the numerical applications to show the propriety of the model and its applicability are displayed.

5.2 Mathematical Formulation

In this section, the formulations introduced in Chapter 4 are combined in a bi-level formulation in order to study the interactions between a single MSP and users of a multi-modal supernetwork. Specifically, the problem takes the form of an MPEC (Luo et al., 1996). As already pointed out in Chapter 3, an MPEC is considered as extension of a bi-level program. It is an optimization problem with two sets of variables, where some of the constraints take the form of a parametric VI. In the proposed formulation at the upper-level there is the MSP with $v \in \mathbb{R}^{n_1}$ as decision variable, representing the service capacity. At the lower-level users' variable $\mathbf{x} \in \mathbb{R}^{n_2}$, representing the vector of path flows.

In this context, let j be an MSP within the multi-modal network, offering a mobility service and guided by the objective function for profit maximization as indicated by 4.6. Assuming a single supplier at the upper level allows us to simplify the formulation by omitting the apex notation. This simplification arises because the MSP's vector J is reduced to dimension 1.

Let users of the transportation network be divided in K classes, based on their socio-economic characteristics and daily trip chains. These users, when assigned to the transportation network, encounter varying costs, and they base their choices on the UE principle by selecting the option with the minimum cost. However, it's important to note that certain cost components within the network are non-separable. Because of this, the network equilibrium conditions are defined in the form of the 4.15. Therefore, the problem of finding the optimal MSP's fleet size (v) while users are assigned to a supernetwork following Wardrop's first equilibrium principle can be written in compact form as follow:

$$\max_{v \in \mathbb{V}} \quad Z(\mathbf{x}^*, v) \tag{5.1}$$

$$\text{s.t.} \quad \mathbf{x}^* \in \mathbb{X}^*(\mathbf{v}) \tag{5.2}$$

where $Z(\mathbf{x}^*, v)$ is the profit function based on the revenues and costs deriving from the service offered by the MSP. In Equation 5.1 the profit is maximized varying MSP's fleet size (v), and depending on the equilibrium constraint stated in Equation 5.2, where x^* is the vector of the lower-level equilibrium path flows and $\mathbb{X}^*(v)$ is a subset of $\mathbb{X}$ that satisfies Equation 4.15. It is important to underline that the supplier's objective function contains the lower-level variables, and as a consequence the lower-level equilibrium strictly depends on the number of vehicles available for each trip.

5.3 Solution Algorithm

Generally, solving MPECs presents significant challenges, particularly due to the presence of disjoint constraints that give rise to combinatorial problems, posing difficulty for efficient solution algorithms. More likely, the lack of convexity and the closedness of the feasible region could be a cause of inefficiencies in finding optimal solutions for MPECs (Luo et al., 1996).

In transportation usually link-separable cost functions are considered at the lower-level in order to simplify the problem, and reducing the VI into a convex optimization problem. This allow to write the problem as a bi-level program that can be restructured into a single-level convex optimization problem. The benefit of this simplification is that it allows for the use of a wider range of existing optimization algorithms that are designed for convex optimization. However, in the proposed formulation, the lower-level multi-class UE is defined assuming that the different costs perceived by users are not-separable. This aspect increases drastically the complexity of the model, making the MPEC hard to be solved as an optimization problem using conventional solution algorithms.

The choice of algorithm depends on the specific characteristics of the transportation problem, such as the network structure, the type of users and decisions involved, and convexity properties. Several iterative methods have been proposed to solve MPECs, such as penalty interior point approach (PIPA), Implicit Programming Algorithm (IMPA), and Newton type approaches (Luo et al., 1996).

In this thesis, to solve the proposed problem 5.1, subject to constraint 5.2, an iterative solution algorithm is proposed: it searches for a local solution of the upper-level continuous objective function using a gradient-based method, based on the sequential quadratic programming (SQP) algorithm (Wright, Nocedal, et al., 1999). This algorithm is an iterative optimization algorithm commonly used for solving nonlinear constrained optimization problems. Specifically, it solves a series of quadratic sub-problems that approximate the original nonlinear problem and move iteratively towards the optimal solution. In this thesis, it is implemented using the conventional MATLAB function `fmincon` with multiple variables, including its optional settings. The optimization process starts with an initial guess for the variables of the nonlinear problem. Subsequently, the original nonlinear optimization problem is approximated into a series of of quadratic programming (QP) subproblems, based on a quadratic approximation of the Lagrangian function. Specifically, the subproblem is constructed using the Hessian (second derivative matrix) of the Lagrangian function. At each major iteration, the Hessian of the Lagrangian function is approximated using a quasi-Newton updating method. At each major iteration of the SQP method, a QP problem is solved. The solution procedure involves two phases. First, there is the evaluation of a feasible point (if one exists). Second, the generation of an iterative sequence of feasible points that converge to the solution. The QP subproblem is solved to obtain a search direction for the variables. If the current point from the SQP method is not feasible, it is possible to find the point solving a linear programming problem. A line search procedure is used in order to ensure that the step size is appropriate to guarantee convergence ((MATLAB, 2023)).

In the general iterative process, as each upper-level function evaluation occurs, a lower-level equilibrium solution is determined through the adaptive EM method described in Section 4.4.2. The sequence of steps for this procedure is detailed in Algorithm 1 and Algorithm 2. Furthermore, Section 4.4.3 provides insights into the characteristics of the lower-level solution algorithm. It also demonstrates its application on a small supernetwork with a single class of users and multiple classes of users, offering both illustration and analysis of its functionality.

In the upcoming section, the described iterative process will be applied in diverse numerical examples. The aim is to identify potential solutions, including the possibility of unique solutions. Furthermore, this application will contribute to a deeper understanding of how the methodology can be employed to assess the impact of an MSP's market strategies on user choices. It will also help comprehend how the equilibrium distribution of users within the network and the market strategies adopted by other supplier can collectively influence the MSP's final maximum profit.

5.4 Numerical Application

In this section, the algorithm proposed in Section 5.3 is initially employed with the network representation detailed in Figure 4.7. This application aims to analyse both the model's characteristics and the solution algorithm. Furthermore, an examination will be conducted on how changes in pricing strategies may influence profit variations for the studied MSP. Subsequently, a second example is introduced to explore potential economic scenarios available to an MSP when confronted with the entry of a competitor into the market.

5.4.1 Example 1: the analysis of a single MSP

In the first example the network represented in Figure 4.7 is taken into account. As already explained in Section 4.4.3, three different modes of transport are considered in the supernetwork: PC, bus and one-way car-sharing service. Two classes of users, whose perceived cost are listed in Table 5.1, will be assigned to the network. Users of both classes are performing the same trip chain, from location L_1 travelling to location L_2, and finally go back to the first location L_1. Moreover, the link costs functions take the form of the conventional Bureau of Public Roads (BPR) function (Equation 4.23), or they are considered constant. The functional parameters used are listed in Table 5.2. Congestion effects will affect users that choose car and car-sharing modes, since they are assumed to share the same links on the base network. The bus service, instead, is considered to have a dedicated lane.

In this example, the MPEC is evaluated focusing on the profit maximization goal sought by the car-sharing service to analyse different scenarios; nevertheless the same analysis can be carried out considering other suppliers at the upper level. The algorithm parameters are listed in Table 4.6.

The proposed algorithm is applied using MATLAB R2019b[1]. The algorithm solution is reach with a running time of 2.7 seconds. The results are shown in Table 5.3 with a starting point of v_a[20,20].

Regarding the upper-level profit maximization and fleet size, due to the network's symmetrical configuration and its lack of interconnection with other transportation modes, the optimal solution entails having 115 vehicles allocated to each of the two modal links owned by the car-sharing provider

[1]The simulations are carried out using Windows 10 laptop with an Intel(R) Core (TM) i7-8650U CPU with a base frequency of 1.90GHz and a system memory of 16.0 GB.

(named Link 6 and Link 9). At this stage of the research, relocation is not taken into account, therefore it is considered that the vehicles used to perform the first trip are then used to perform the second trip.

Table 5.1: Parameters Example 1

Parameters	**PC**	**Bus**	**Car-sharing**
c_s	0.1	1.1	2.2
r_s	-	-	-
$c^1_{a,access}/c^1_{a,egress}$	8.1	9.9	8.1
$c^2_{a,access}/c^1_{a,egress}$	11.7	14.3	11.7
$c^1_{a,wait}$	-	9.9	8.1
$c^2_{a,wait}$	-	14.3	11.7
$c^1_{a,main}$	7.6	8.5	7.6
$c^2_{a,main}$	11	12.3	11
$c_{a,fuel}$	0.4	0.2	
$c_{a,h}$	-	-	0.9
$c_{a,km}$	-	-	0.35
l_a	10	10	10
$c^1_{a,park}$	9.9	-	-
$c^2_{a,park}$	14.3	-	-
v_a	200	150	v
d^1_1	150		
d^2_1	150		

Table 5.2: Functional Parameters Example 1

Function	**PC**					**Bus**					**Car-sharing**				
	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β
$t_{a,access}(f_a, v_a)$	-					0.1	-				0.1	0.2	f_a	v_a	2
$t_{a,wait}(f_a, v_a)$	-					0.3	0.3	f_a	150	4	0.05	0.1	f_a	v_a	4
$t_{a,main}(\mathbf{f})$	0.2	2	$\mathbf{f}$	250	4	0.3	2	$\mathbf{f}$	200	4	0.2	2	$\mathbf{f}$	250	4
$t_{a,park}(f_a, v_a)$	0.1	1	f_a	300	2	-					-				
$t_{a,egress}(f_a, v_a)$	0.1	-				0.1	-				0.1	0.2	f_a	v_a	2
$c_{lease}(v)$	-					-					$1.6v$				

Furthermore, the table presents distinct lower-level solutions categorized by modal flow and cost for each class. Notably, the equilibrium state highlights that the paths with the lowest costs correspond to those with actual flow. Among the classes, Class 2 is the only opting for the usage of the car-sharing service, with an utilization rate of 0.88. This can be attributed to the surplus availability of vehicles compared to the number of users opting for the service. This trend is potentially linked to the minimized access time, which in turn is facilitated by the higher capacity of vehicles.

Table 5.3: Results Example 1

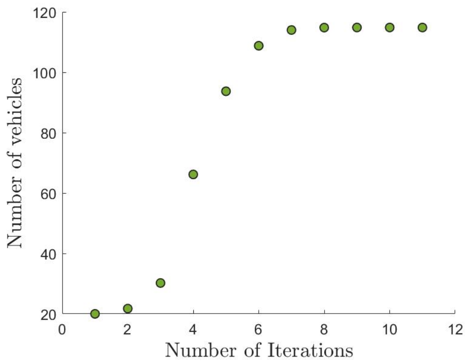
Upper-Level		
Number of iterations	11	
Function value	216.6	
v_a	115	
Lower-Level	**Class 1**	**Class2**
PC Flow	26	48
Bus Flow	126	0
Car-sharing Flow	0	102
PC Cost	18.50	23.13
Bus Cost	18.50	26.24
Car-sharing Cost	18.84	23.13

Table 5.4 effectively demonstrates that the solution remains consistent regardless of the initial points' variation, whether opting for uniform vehicle allocation on each link or distinct values. Notably, it becomes evident that an increasing number of iterations are necessary when different values of the starting points are chosen. However, this extended iterative process does not proportionately translate into a lengthier computational time to reach the optimal solution. Remarkably, the most efficient computational outcome is observed when the correspondence between the starting point and the solution is set.

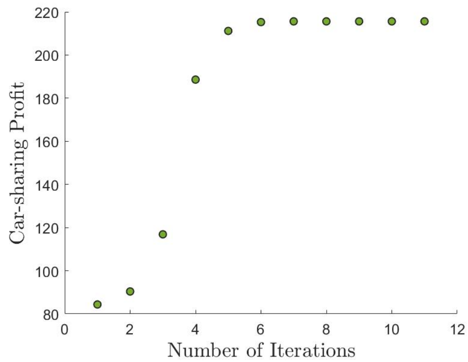
Table 5.4: Starting point variation Example 1

Starting Point		**N. iterations**	**Time (sec)**	**Solution**	
v_6	v_9			v_6	v_9
0	0	10	31.3	115	115
0	50	17	5.1	115	115
10	100	18	4.7	115	115
20	20	11	2.7	115	115
50	50	9	3.3	115	115
100	10	18	4.6	115	115
100	100	10	13.8	115	115
115	115	1	0.6	115	115
150	0	16	4.9	115	115
175	175	10	3.1	115	115

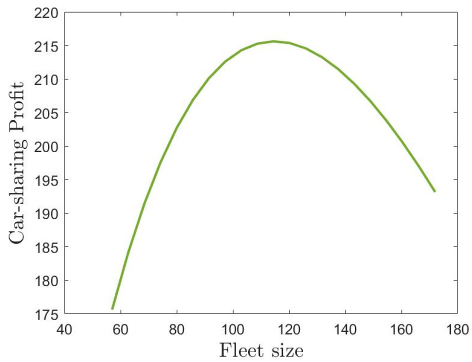
In Figure 5.2a, it is shown the upper-level fleet size solution through the different algorithm iterations starting from $v_a[20,20]$, and in Figure 5.2b it is possible to see the corresponding profit solution. Finally, in Figure 5.2c it is shown how to the optimal value of fleet size correspond to the maximum value of the profit curve for the MSP.



(a) Number of vehicle variation with iterations



(b) Profit variation with iterations



(c) Profit variation with fleet size

Figure 5.2: Example 1: Upper-level solution

Figure 5.3 illustrates the variation of the lower-level relative gap trough the different iterations when evaluating the upper-level optimal fleet size.

In the subsequent analysis, illustrated in Figure 5.4 and Figure 5.5, it is possible to see the variations of flows on each mode of transport and the relative costs for each of the two classes. The results of the modal split distribution across the different modes of transport is confirmed by the costs perceived by users. Across various iterations within Class 1, the use of the car-sharing service remains consistently unattractive. Simultaneously, within Class 2, the bus option is considered economically impractical. As the capacity of the car-sharing service is increased, a notable portion of users transition from PC to the car-sharing alternative. Consequently, a portion of the previously occupied PC spots are reallocated for utilization by Class 1. It is observable that achieving the optimal solution for the MSP necessitates a gradual augmentation of service capacity over successive iterations. In parallel, a general reduction in network costs occurs for both classes, predominantly attributed to the reduction of the network congestion due to the redistribution between the different modes of transport.

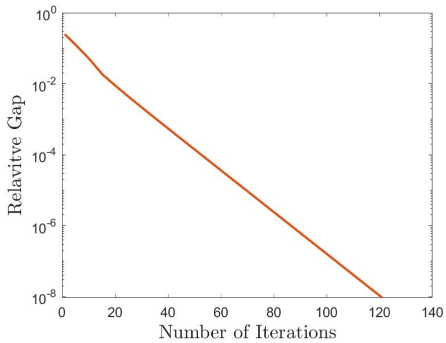
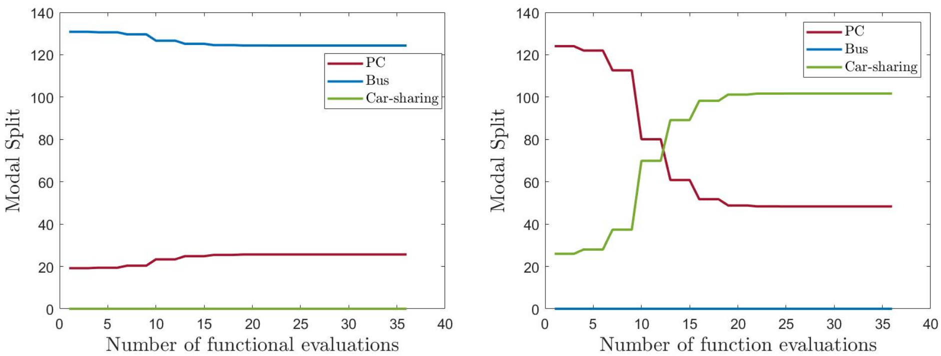


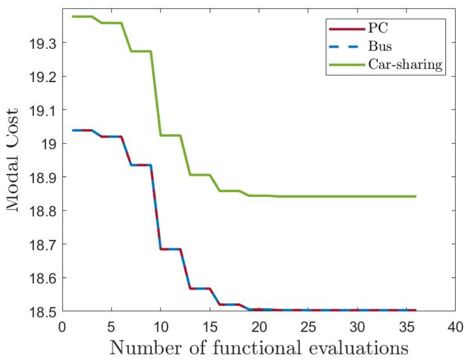
Figure 5.3: Example 1: Lower-level Relative Gap Variation (Logarithmic Scale)



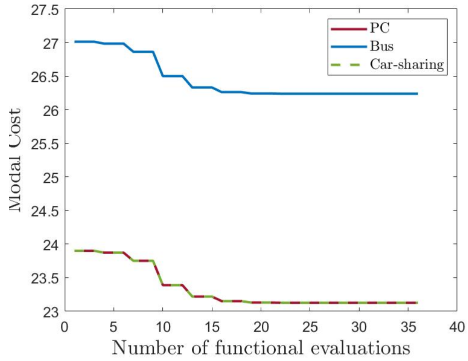
(a) Class 1 **(b)** Class 2

Figure 5.4: Example 1: Modal Split variations

Furthermore, in Figure 5.6, the various plots illustrate how the increase in link flow affects different elements of the link cost for each mode of transport. The capacity of the car-sharing service has been considered fixed and equal to the MPEC equilibrium solution. Across all transportation modes, the time spent in the main mode of transport emerges as the most influential component impacting the link costs. Notably, both car and car-sharing options exhibit analogous variation in the in-vehicle time.

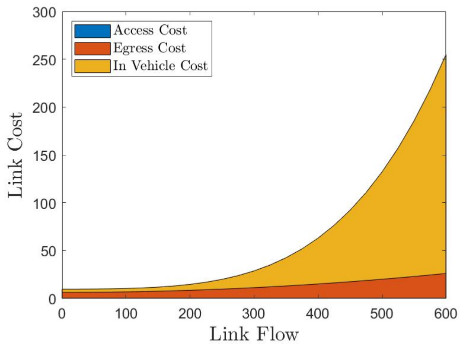


(a) Class 1

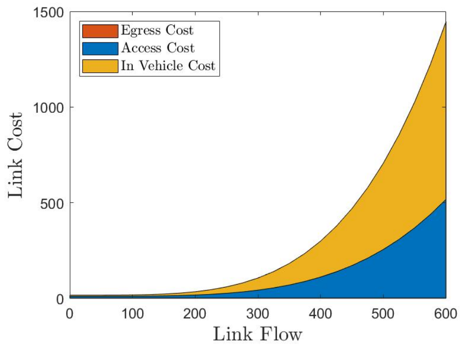


(b) Class 2

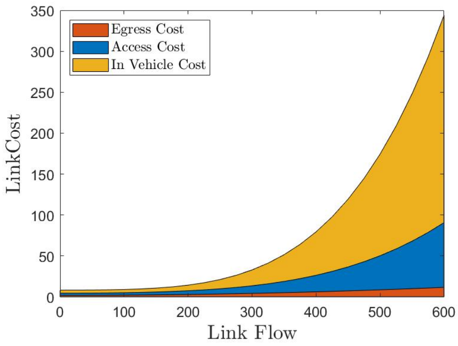
Figure 5.5: Example 1: Modal Costs variations



(a) PC Cost variation



(b) Bus cost variation



(c) Car-sharing cost variation

Figure 5.6: Example 1: Link Costs variations with total Link Flow

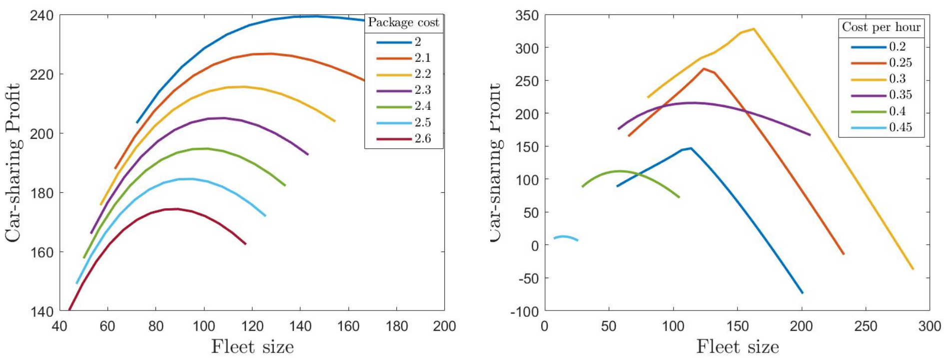
However, the car-sharing service demonstrates elevated link costs primarily due to the influence of the access cost, impacted by waiting and walking time. Conversely, the bus service showcases a notably higher total cost. This is also attributed to the access time component, wherein the increase in waiting time becomes pronounced as the link flow reaches certain volumes.

After the proposed analysis, it becomes intriguing to explore how various marketing strategies employed might impact the maximum profit of the car-sharing provider. More precisely, consideration is given to the possibility of changing subscription price, and the variable costs applied to users while travelling with the car-sharing service.

Figure 5.7a clearly illustrates that changes in the package price have a direct impact on the MSP's profit. It's intuitive to understand that increasing a fixed subscription price can discourage users from subscribing. Specifically, changing the package price s the profit Z is monotonically decreasing with a constant negative slope that can be express by equation:

$$Z = -107.2684s + 452.4479 \tag{5.3}$$

In contrast, Figure 5.7b shows a completely different pattern when we examine the change in the cost that users incur for each hour they spend using the car-sharing service. This cost component is notably significant, especially when compared to the fixed subscription cost. In this case, users are subject to a cost that increases with congestion. When this cost is at or below 0.3 euro per hour, the profit variation appears erratic, reaching a maximum value beyond which profits decline rapidly. Conversely, when the cost falls within the range of 0.35 to 0.45 euro per hour, each change in profit follows a smoother trajectory. However, even a slight increase in this hourly rate results in a substantial reduction in profit, with profits approaching zero at the hourly cost of 0.45 euro.



(a) Subscription price variation **(b)** Cost per hour variation

Figure 5.7: Example 1: car-sharing pricing strategies

5.4.2 Example 2: the entrance of a competitor in the market

In the second example it is analysed the profitability of a bike-sharing service provider operating in the network displayed on top of Figure 5.8. Due to the reduced distances often covered by bike, a simple trip chain in which two classes of users perform their daily trips is considered. Users of the two classes will travel from location L_1 to location L_2, and finally go back to the first location L_1. From the network expansion (bottom of Figure 5.8), it is possible to see that the available modes of transport are: PC, bus and an electric bike-sharing service (named bike-sharing 1). From this configuration, it is considered that an electric bike-sharing competitor (named bike-sharing 2) enters the market (Figure 5.9) subsidized by a local authority, offering a cheaper service to customers. The service characteristics of the network are defined in Tables 5.5 and 5.6.

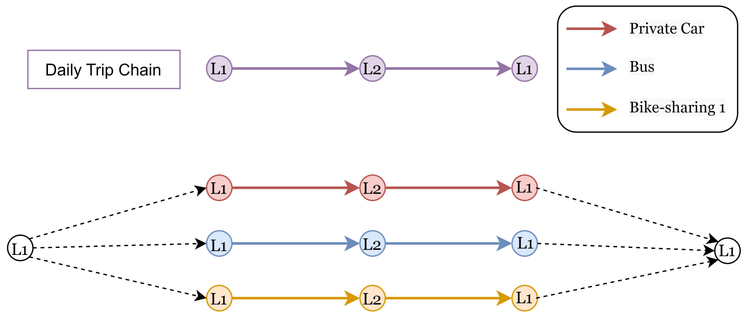


Figure 5.8: Supernetwork with PC, Bus, and Bike-sharing

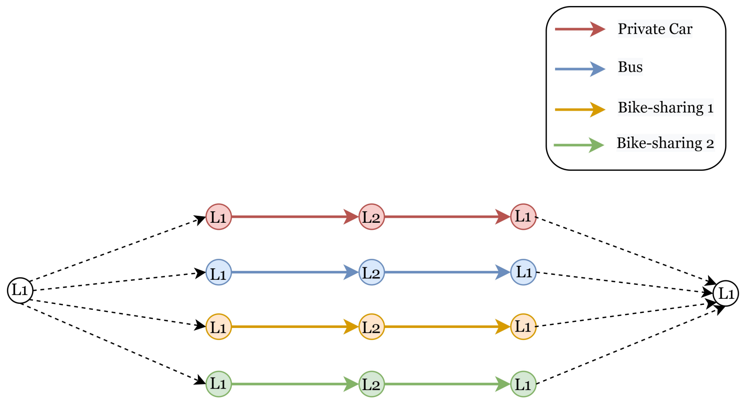


Figure 5.9: Supernetwork with PC, Bus, and two Bike-sharing services

In this example PC and bus have two different infrastructures, therefore the flows on the different modes of transport are not affecting each other. The the two bike sharing services, instead, are operating in an area in which they share the available bike lanes, and their link cost functions are non-separable.

Table 5.5: Parameters Example 2

Parameters	PC	Bus	Bike-sharing 1	Bike-sharing 2
c_s	0.3	1.2	1	0.5
r_s	-	-	-	1
$c^1_{a,access}/c^1_{a,egress}$	8	8.2	10	10
$c^2_{a,access}/c^2_{a,egress}$	5.6	5.7	7	7
$c^1_{a,wait}$	-	10	16	16
$c^2_{a,wait}$	-	7	11.2	11.2
$c^1_{a,main}$	8	10	14	14
$c^2_{a,main}$	5.6	7	9.8	9.8
$c_{a,fuel}$	0.38	-	-	-
$c_{a,h}$	-	-	0.8	1
l_a	5	5	5	5
$c^1_{a,park}$	9	-	-	-
$c^2_{a,park}$	6.3	-	-	-
v_a	200	250	v	50
d^1_1	300			
d^2_1	200			

Table 5.6: Functional Parameters Example 2

Function	PC					Bus					Bike-sharing 1/2				
	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β
$t_{a,access}(f_a, v_a)$	-					0.2	-				0.003	0.2	f_a	v_a	4
$t_{a,wait}(f_a, v_a)$	-					0.15	0.15	f_a	200	2	0.5	0.2	f_a	v_a	4
$t_{a,main}(\mathbf{f})$	0.2	4	$\mathbf{f}$	200	2	0.2	4	$\mathbf{f}$	250	2	0.3	4	$\mathbf{f}$	100	3
$t_{a,park}(f_a, v_a)$	0.15	2.5	f_a	200	2	-					-				
$t_{a,egress}(f_a, v_a)$	0.1	-				0.2	-				0.003	0.2	f_a	v_a	4
$c_{lease}(v)$	-					-					$0.3v$				

Applying the proposed methodology before and after the introduction of the competitor, it is possible to see (Figure 5.10) a reduction on the maximum value of profit that bike-sharing 1 can obtain when offering the service under the presence of a direct market competitor.

At this stage, it is of interest to identify potential strategies to increase the profitability of the bike-sharing service. For this purpose, various scenarios are tested considering different pricing strategies, displayed in Figure 5.11. In the first scenario (as shown in Figure 5.11a), a variation in the subscription price is considered. For lower values of this fixed cost, the daily profit of the MSP approaches zero, while an increase in the price results in a general profit increase with a slight decrease in the number of bicycles required. However, it is important to note that even with these adjustments, the maximum profit remains below the original scenario without competition.

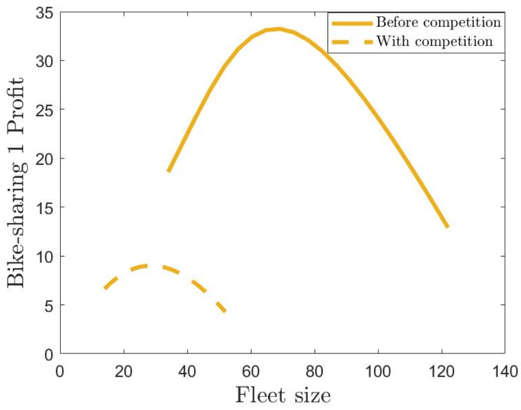
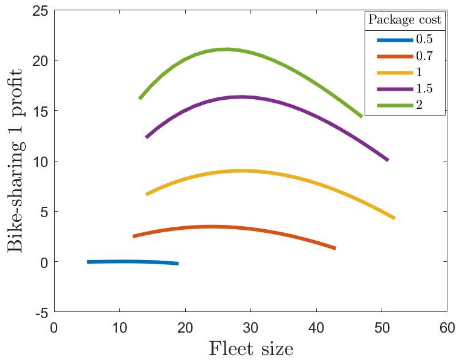
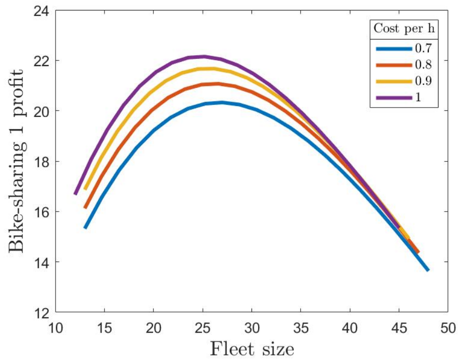


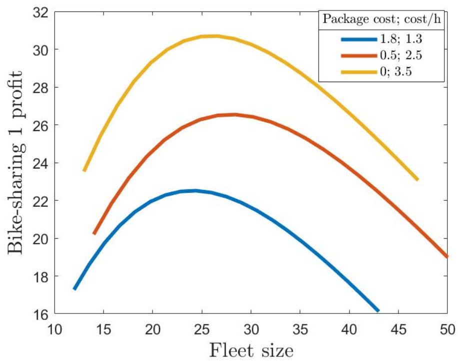
Figure 5.10: Example 2: Bike-sharing 1 profit variation with fleet size



(a) Profit variation with Subscription price



(b) Profit variation with Cost per hour



(c) Profit variation with combination subscription and cost per hour

Figure 5.11: Example 2: Bike-sharing 1 pricing strategies

Maintaining the fixed price at 2 euros, Figure 5.11b showcases a more regular behavior when increasing the variable cost per hour spent using the service. Nonetheless, in both cases (Figures 5.11a and 5.11b), the maximum profit achieved still falls short of the baseline scenario without competition. As a result, Figure 5.11c presents an analysis of a combination of changes in subscription price and cost per hour. It is evident that achieving a more comparable maximum profit is possible when the subscription cost is set to zero, while maintaining a relatively high cost per hour. Finally, with the entry of a new competitor into the market, the usage of the bike-sharing 1 service decreases. Various strategies can be considered in response to this competition. However, as displayed in Table 5.7, when comparing the scenario without a competitor to the scenario with a competitor, where the same market strategies are maintained, and to the scenario with a competitor offering a subscription-free service with a high hourly price, it becomes clear that MSP 1's overall profit decreases. Furthermore, the new mobility service gradually captures the majority of the demand that was previously served by bike-sharing 1, causing it to transition into a secondary provider within the market.

Table 5.7: Results comparison Example 2

	without competitor		**with competitor**		**new prices**	
Modal Split	Class 1	Class2	Class 1	Class2	Class 1	Class2
PC Flow	164	0	162	0	163	0
Bus Flow	136	131	138	128	137	130
Bike-sharing 1 Flow	0	69	0	24	0	16
Bike-sharing 2 Flow	-	-	0	48	0	54

5.5 Conclusion

In this chapter, the interaction between a MSP and user classes has been formulated as an MPEC. At the upper-level MSP's strategies are represented through a profit maximization formulation. At the lower-level, the model incorporates several key aspects, including the non-separability of cost functions and the non-additive nature of path costs. Consequently, the lower-level equilibrium problem is expressed as a path-based VI.

The primary objective of this Chapter was to comprehend the conditions under which MSP strategies render the service profitable, taking into account that users are assigned to a multi-modal supernetwork at equilibrium.

An iterative solution algorithm has been proposed to address this mathematical formulation. To identify the upper-level fleet size that maximizes total profit, different possible solution points are evaluated using a SQP algorithm. For each upper-level value, a corresponding lower-level equilibrium solution is assessed using an adaptation of the EM. The iterative process continues until the MSP no longer has an incentive to change the fleet size, as any change would result in reduced profit.

Two numerical examples have been presented. In the first, a car-sharing provider maximizes profit while two classes of users are assigned to a multi-modal network with three transport modes. This model consistently converges to the same solution, even when initiated from different starting points. Notably, the lower-level flow distribution follows the Wardrop's first equilibrium principle, where only paths with the minimum cost have positive flow. Additionally, the impact that of various cost components have on link flow for each transport mode have been displayed. Finally, different pricing strategies have been proposed to increase daily car-sharing profits.

In the second example, competition between two bike-sharing services has been introduced. This scenario illustrates how the profit of an MSP can diminish when a new provider enters the market with a competitive pricing strategy. The scenarios tested demonstrate the challenge faced by the bike-sharing 1 provider in achieving the same profit level as before the competitor's introduction. It becomes apparent that reducing package prices or hourly costs necessitates downsizing the business. However, it has to be clear that in the proposed example solely the profit variation of the first bike-sharing provider have been taken into account, leaving uncertainty regarding the new MSP's profitability. For this reason, it appears fundamental to study how the interaction between different MSPs' strategies and their corresponding profit variation to have a more realistic representation of the transportation market. Therefore, in the next Chapter a more complex formulation will be introduced. The main focus will be on examining the dynamics that unfold when multiple MSPs seek to maximize their profits by adapting their strategies while users are making their modal choices.

Chapter 6

A Multi-modal and Multi-actor Equilibrium Model: An EPEC Formulation

6.1 Introduction

The significance of competition among firms or MSPs in the transportation market is evident, but there is a lack of studies that examine the different forms of competitions and collaborations emerging when introducing mobility packages or by the co-existence of multiple service providers in the same market. Furthermore, as described in Section 3.4.3 existing studies predominantly concentrate on uni-modal networks with homogeneous demand characteristics, neglecting the interaction between different modes, i.e. considering non-separable cost functions. This Chapter aims to fill these gaps, seeking to comprehend the various dynamics that may arise in the transportation network due to complex interactions between the different actors present in the transportation system, and the different modes sharing the same transportation infrastructure and competing for the travel demand.

For this reason, in this Chapter, the methodology presented in Chapter 4 and partly analysed in Chapter 5 is extended considering multiple MSPs at the upper-level, while users of the transport network are assigned to a multi-modal network. In this extended formulation, each MSP seeks to maximize their individual profit. Figure 6.1 illustrates the increased complexity that arises when extending the proposed methodology from a single MSP at the upper-level (shown in Figure 5.1) to multiple competing suppliers. The general problem takes the form of an EPEC, characterized by a mutual interdependency in between the profit maximization at the upper-level and UE at the lower-level.

In this scenario, the EPEC is treated as a sum of different MPECs, in which each MSP tries to optimize the service capacity in order to maximize the profit, considering the strategies of the other MSPs, while users make their travel choices. One of the complexity of this extended version derives from the fact that from a change of one of the upper-level strategies derives a change of the lower-level equilibrium and also the strategies of other upper-level MSPs.

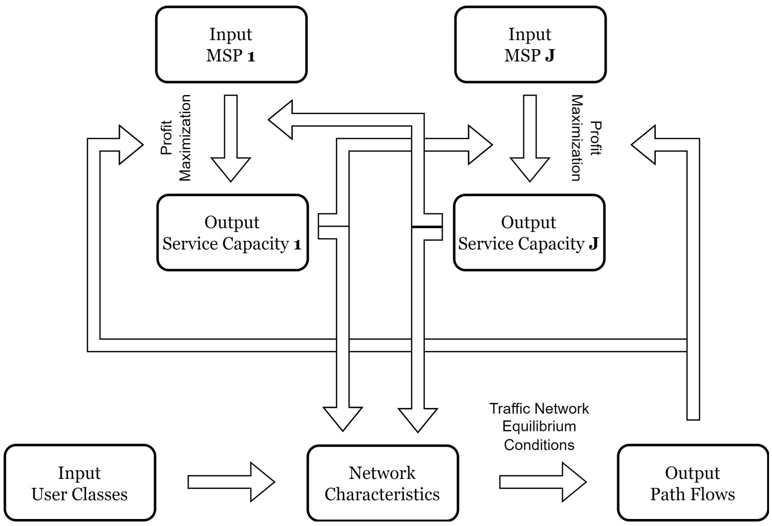


Figure 6.1: EPEC supply-demand interaction

In this Chapter, the aforementioned problem is formulated in Section 6.2. Subsequently, a solution algorithm is proposed in Section 6.3. Finally, applications to numerical examples are described in Section 6.4.

6.2 Mathematical Formulation

In this section an EPEC formulation is presented to investigate the interactions between various MSPs implementing diverse strategies aimed at optimizing their individual profits, while users are concurrently making multi-modal transport choices based on the option with the lowest cost. As discussed in Section 3.4.3, an EPEC can be viewed as an extension of a Stackelberg game, wherein, unlike the traditional setup with a single leader and followers, there are multiple decision-makers striving to reach a Nash equilibrium at the upper-level. This equilibrium implies that each participant selects their optimal strategy based on their perceptions of the choices made by others. No participant sees any advantage in changing their strategy because they believe they have already made the best decision, given the actions of others. In the context of this thesis at the the upper-level the different players are represented by the MSPs of the transport network. At the lower-level, instead, there are the travellers who are typically distributed across a network, following the Wardrop's first equilibrium principle. This principle dictates that users tend to favor routes with the minimum cost, as it aligns with their best interests.

In general, an EPEC is a problem that can be broken down into a set of MPECs. More precisely, in the problem under analysis, instead of tackling the entire complex problem as one unit, it can be divided into individual problems, with each MSP evaluating their specific MPEC, which simplifies the overall analysis and solution process.

Following this approach, a change of a MSP's fleet size determines a new equilibrium distribution of the lower-level. This new distribution, in turn, affects the profits of all MSPs at the upper level. Given that each MSP aims to maximize their profit while considering the strategies of other players, equilibrium is achieved when no MSP or users have any incentive to change their strategies, as doing so would put them in a less advantageous position. Therefore following Equation 4.6, each MSP $j \in \mathbb{J}$ selects their fleet size, v^j, to maximize their own profit, Z^j. The profit of each MSP depends on the path flows of the lower-level. The equilibrium path flows depend on the fleet sizes of all the MSPs, $\mathbf{x}^*(\mathbf{v})$. With the lower-level equilibrium constraint in place it is possible to consider that the profit of the j-th MSP is a continuously differentiable function, $Z^j(v^j|\mathbf{x}^*(\mathbf{v}))$. Naturally, each MSP can change only their own fleet size; dependence on other MSPs fleets comes through the lower-level equilibrium condition.

Equilibrium at the upper-level is reached when no MSP can unilaterally change their fleet size and increase their profit. An upper-level equilibrium is denoted $\tilde{\mathbf{v}}$, whose (scalar) j-th component is $\tilde{\mathbf{v}}^j$. When the j-th component (only) of $\tilde{\mathbf{v}}$ is set to the value y, it is possible to write $\tilde{\mathbf{v}}[\tilde{\mathbf{v}}^j = y]$. The upper-level equilibrium condition can be written for $\tilde{\mathbf{v}}$ as follows:

$$\mathrm{Z}^j(\tilde{\mathbf{v}}^j|\mathbf{x}^*(\tilde{\mathbf{v}})) - \mathrm{Z}^j(y|\mathbf{x}^*(\tilde{\mathbf{v}}[\tilde{\mathbf{v}}^j = y]) \geq 0 \quad \forall y \geq 0 \quad \text{for each } j \in \mathbb{J} \tag{6.1}$$

The lower-level equilibrium path flows $\mathbf{x}^*(\tilde{\mathbf{v}})$ are defined above (see Equation 4.15). Recall that uniqueness is not guaranteed for the lower-level equilibrium due to the non-separability of the cost functions; there might be multiple path flows solutions for a given vector of fleet sizes. Clearly, this reflects in also multiple solutions for the upper-level, as it will be shown in the numerical examples.

6.3 Solution Algorithm: Diagonalization Method

EPEC problems are well known in literature for the difficulty of finding equilibrium solutions. Moreover, there are no specifically designed algorithms to solve EPECs (Cottle & Su, 2005). As pointed out by Watling et al. (2015) and citations therein, two classes of methods are generally used to solve EPECs: the Simultaneous Methods (SM) and the Diagonalization Methods

(DM). SMs try to solve the different MPECS simultaneously, while DMs "solve a cyclic sequence of single-leader-follower games until the decision variables of all leaders reach a fixed point" (Leyffer & Munson, 2010). Specifically, a DM iteratively solves each MPEC in turn, while keeping fixed the decision variables of all other players. Common examples of DMs are the Gauss-Jacobi and Gauss-Seidel methods, initially used to solve VI problems (Harker, 1984). In the first approach, all variables are updated simultaneously using the values from the previous iteration, without updating values within the same iteration. The second approach, instead, updates each variable using the most recent values, making it more efficient in terms of convergence but less amenable to parallelization due to its sequential nature.

In this thesis a DM with a structure of a Gauss-Seidel method is used. Considering that for each MSP in turn an MPEC formulation is solved using the solution approach described in section 5.3. Considering that the order in which the MSP-specific are tackled is arbitrary in the DM. Specifically, at the upper-level a local solution of each MSP's objective function is searched using a gradient-based method, based on the sequential quadratic programming (SQP) algorithm. At the lower-level the equilibrium solution are calculated using an adaptation of the EM introduced in section 4.4.2.

Algorithm 3 shows the different steps of the proposed iterative DM, which calls for the the adaptive EM defined in Algorithm 1. Finally the projection algorithm called by the EM is detailed in Algorithm 2.

Algorithm 3 Diagonalization Method

Input Parameters and Functions $\forall J, K$
Set MaxIter, $\chi^1, \mathbf{v}_0, \mathbf{v}_{min}, \mathbf{v}_{max}$
Initialization Diagonalization:
Set $i = 1, \delta = \infty, \mathbf{v}_i = \mathbf{v}_0$
while $\delta > \chi^1$ and $i <$ MaxIter **do**
    for j = 1:J **do**
        Compute $y \in [v^j_{min}, v^j_{max}]$ **maximizing** $Z^j(y|\mathbf{x}^*(\mathbf{v}_i[\mathbf{v}^j = y]))$ **with** $\mathbf{x}^*(\mathbf{v}_i[\mathbf{v}^j = y])$ **through EM using Algorithm 1**
        Update $\mathbf{v}^j_i = y$ [j-th component]
    end for
    Compute $\delta = ||\mathbf{v}_i - \mathbf{v}_{i-1}||$
    Set $i = i + 1$
end while

6.4 Numerical Application

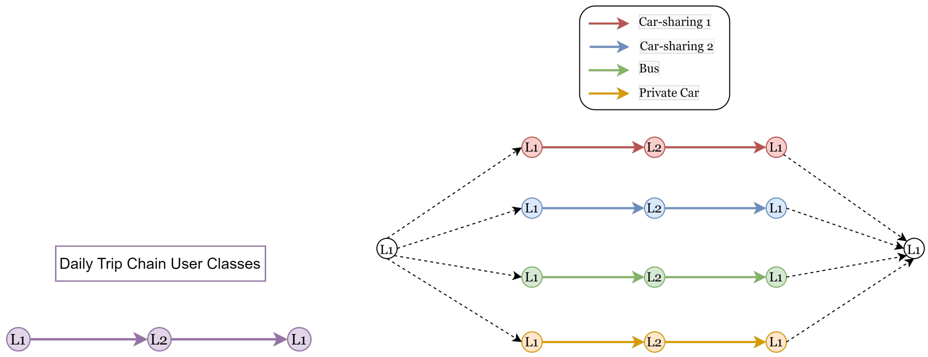
The aim of this section is to showcase the complexity of the problem under study by implementing first the proposed approach on a simple scenario

(Section 6.4.1). Through this choice it is possible to highlight the feasibility of the approach and provide initial evidence of its practicality, laying the groundwork for its potential implementation in more complex scenarios. In order to demonstrate certain features of a MaaS system, such as the inclusion of mobility packages, which can be represented using the suggested approach, the proposed solution algorithm is implemented in a more intricate scenario, illustrated in a second example (Section 6.4.2).

Algorithm 3 have been implemented using MATLAB R2019b[1]. For the upper-level solution of the profit maximization adopted is used the conventional MATLAB function `fmincon` with multiple variables, including its optional settings.

6.4.1 Example 1: Competition between two MSPs

In the first example are presented three classes of users performing the same trip chain (shown in Figure 6.2a), which consists of simply visiting two locations (L1 and L2).



(a) Daily trip chain for the two user classes

(b) Supernetwork expasion

Figure 6.2: Supernetwork with PC, Bus, and two car-sharing services

Different classes can for instance be used to distinguish users performing different activities at the two locations, which yields to a different perception of trip costs (e.g. different values of time). Each class can also perceive costs with different weights, due to their socio-economic characteristics. Four modes of transport are taken into account: two car-sharing services, a bus service, and the availability of PC. Accordingly, users are assigned to the network shown in Figure 6.2b. All modes of transport share the same trip connection, therefore link cost are non-separable. Link cost functions can

[1]The simulations are carried out using Windows 10 laptop with an Intel(R) Core (TM) i7-8650U CPU with a base frequency of 1.90GHz and a system memory of 16.0 GB.

take the form of the conventional BPR function (Equation 4.23), they can be linear or, in some cases, have a constant value.

The proposed methodology is applied to study the competition between the two car-sharing service providers, naming them car-sharing 1 ($j = 1$) and car-sharing 2 ($j = 2$). In Table 6.1, the parameters from the different algorithms used to evaluate the EPEC are listed. Table 6.2, instead, introduces the parameters associated to user classes and costs for the services provided. Finally, Table 6.3 lists the parameters associated to the different cost functions.

Table 6.1: Algorithms Parameters (see Algorithm 1 and 3)

Parameters	
MaxLoops	500
χ^1	0.5
MinFleetSize	1
MaxFleetSize	300
MaxIterations	50000
μ	0.5
λ	0.5
$\overline{\theta}$	0.7
χ^2	1e-8

The two car-sharing providers are assumed to offer similar services, but they mainly differ in terms of the offered package price. Car-sharing 1 charges a higher monthly package fee, whereas car-sharing 2 offers a service at a lower price, thanks to additional revenues received from e.g. advertising. Hence, car-sharing 2 is potentially a more attractive service than car-sharing 1 for the users, provided that they offer the same capacity (which depends on the deployed fleet size).

Figure 6.3 shows the profit surfaces of the two suppliers, obtained by computing the lower-level equilibrium flows at each combination of fleet sizes for car-sharing 1 and car-sharing 2. These plots show the non-linearity and non-convexity of the profit function surfaces, in particular revealing clear regions where the marginal profit changes at different rates with a change of fleet size(s), and regions where one or both services are not profitable (green areas). Moreover, the figure also illustrates multiple distinct EPEC solutions obtained via the solution algorithm (these equilibrium points are also reported in Table 6.4). The computational time needed to reach an equilibrium solution varies based on the starting point. The closer the starting point is to the solution, the less time is required to find it. For instance, for the tested initial points, the computational time ranged from 135 seconds to 1143 seconds.

Table 6.2: Parameters Example 1

Parameters	PC	Bus	Car-sharing 1	Car-sharing 2
c_s	0.1	1.5	0.8	0.5
r_s	-	-	-	0.5
$c^1_{a,access}/c^1_{a,egress}$	8	11	9	9
$c^2_{a,access}/c^2_{a,egress}$	7.2	9.9	8.1	8.1
$c^3_{a,access}/c^3_{a,egress}$	11.7	15.4	12.6	12.6
$c^1_{a,wait}$	-	13	9	9
$c^2_{a,wait}$	-	11.7	8.1	8.1
$c^3_{a,wait}$	-	18.2	12.6	12.6
$c^1_{a,main}$	8.2	9.9	8.5	8.5
$c^2_{a,main}$	7.4	8.9	7.6	7.6
$c^3_{a,main}$	11.5	13.8	11.9	11.9
$c_{a,fuel}$	0.37	0.2	0.2	
$c_{a,h}$	-	-	0.6	0.6
$c_{a,km}$	-	-	0.4	0.4
l_a	10	10	10	
$c^1_{a,park}$	11	-	-	-
$c^2_{a,park}$	9.9	-	-	-
$c^3_{a,park}$	15.4	-	-	-
v_a	300	400	-	-
d^1_1	300			
d^2_1	200			
d^3_1	100			

Table 6.3: Functional Parameters Example 1

Function	PC					Bus					Car-sharing 1					Car-sharing 2				
	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β
$t_{a,access}(f_a, v_a)$	-					0.1	-				0.1	0.1	f_a	v_a	2	0.1	0.15	f_a	v_a	2
$t_{a,wait}(f_a, v_a)$	-					0.1	1	f_a	400	2	0.05	0.2	f_a	v_a	4	0.05	0.2	f_a	v_a	4
$t_{a,main}(\mathbf{f})$	0.2	2	$\mathbf{f}$	600	4	0.2	3	$\mathbf{f}$	600	4	0.2	2	$\mathbf{f}$	600	4	0.2	2	$\mathbf{f}$	600	4
$t_{a,park}(f_a, v_a)$	0.1	1	f_a	300	2	-					-									
$t_{a,egress}(f_a, v_a)$	0.08	-				0.1	-				0.1	0.1	f_a	v_a	2	0.1	0.15	f_a	v_a	2
$c_{lease}(v)$	-					-					$4.2v^1$					$4.2v^2$				

Moreover, in Table 6.4 the best compromise solution for both suppliers is highlighted in red. This solution represents an equal profit point, which is achieved when offering approximately the same fleet size. Moreover, due to the symmetry of the network, the distribution of vehicles in each link is the same at equilibrium.

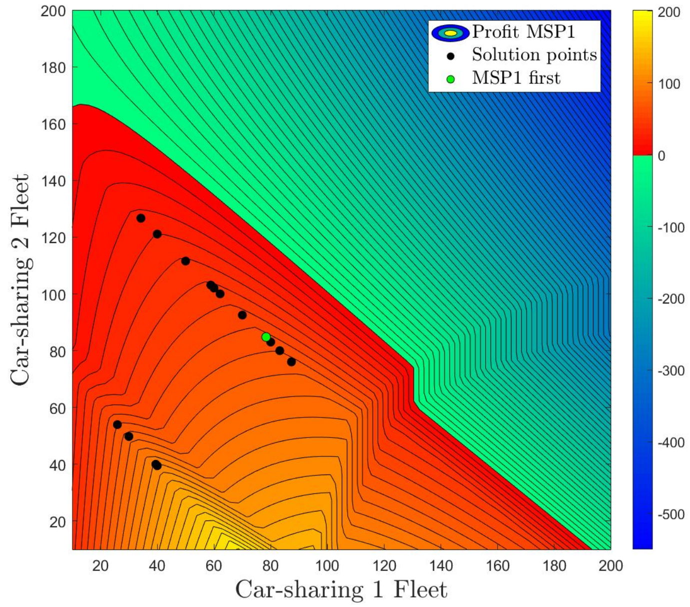
It is worth noting that all possible solution points are located within the two primary areas characterized by higher profitability for both suppliers, which can be clearly identified on Figure 6.3 (black dots). More precisely, it is clear that these points are located in two lines. Therefore it has been verified that all the point belonging to those lines are possible solution of the problem. However, each line segment has its own boundaries. For the upper line, the limits are defined by points [(34,126);(87,76)], while the lower line's limits are set by [(26,54);(43,36)]. This aspect will be further explored in the Conclusion Section, where an in-depth discussion will be provided 6.5.

Table 6.4: Solution points

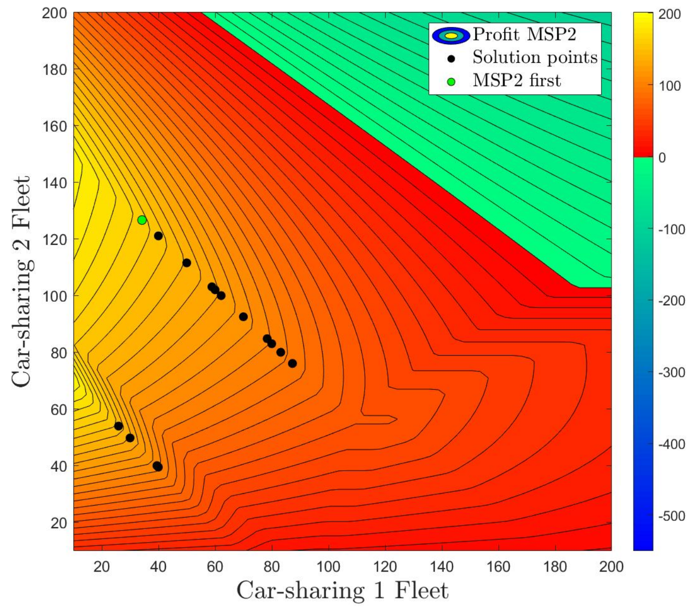
Starting MSP		**Initial Fleet Sizes**		**EPEC Solution**		**Profit**		**TTC**
MSP1	MSP 2	Fleet Size 1	Fleet Size 2	Fleet Size 1	Fleet Size 2	MSP1	MSP2	
x		0	0	78	84	68.2	112.5	14881.3
	x	0	0	34	127	29.8	168	14881.3
	x	10	30	87	76	76.2[a]	100.9[a]	14881.3
x		20	20	59	103	51.4	136.8	14881.3
	x	20	20	26	54	74	155.8	15029.1
x		20	40	87	76	76.2	100.9	14881.3
	x	20	40	34	127	29.8	168	14881.3
	x	30	30	34	126	29.8	168	14881.3
x		40	40	39	40	112.6	115.5	15029.1
	x	40	40	40	39	114[b]	114[b]	15029.1
	x	30	50	30	50	85.5	143.7	15029.1
x		40	60	87	76	76.2	100.9	14881.3
	x	40	60	40	121	34.9	160.6	14881.3
	x	50	50	50	111	43.6	148	14881.3
x		60	60	87	76	76.2	100.9	14881.3
	x	60	60	60	102	52.3	135.3	14881.3
	x	50	70	50	111	43.6	148	14881.3
x		60	80	83	80	72.6	106.1	14881.3
	x	60	80	60	102	52.3	135.3	14881.3
	x	70	70	70	92	61.1	122.7	14881.3
x		80	80	83	80	72.6	106.1	14881.3
	x	80	80	80	83	69.8	110.1	14881.3
	x	70	90	70	92	61.1	122.7	14881.3
x		80	100	62	100	54.2	132.6	14881.3
	x	80	100	80	83	69.8	110.1	14881.3
	x	90	90	87	76	76.2	100.9	14881.3
x		100	100	62	100	54.2	132.6	14881.3
	x	100	100	87	76	76.2	100.9	14881.3
	x	90	110	87	76	76.2	100.9	14881.3
	x	110	110	87	76	76.2	100.9	14881.3
	x	130	130	87	76	76.2	100.9	14881.3
	x	250	250	87	76	76.2	100.9	14881.3

[a] Numerator of Equation 6.2, [b] Best compromise solution for both suppliers

Furthermore, Figure 6.3 provides an illustration of how the order in which the MSPs' objective functions are evaluated in the Diagonalization approach influences the resulting solution points. Specifically, the green dots illustrate two scenarios solutions: one where car-sharing 1 is optimized first (left graph), and the other where car-sharing 2 is optimized first (right graph). Remarkably, starting from the same initial point (0,0) and applying the Diagonalization approach, we observe distinct solution points. This finding emphasises the critical role played by the prioritization of each car-sharing service within the Diagonalization process, as it significantly impacts the final solution.



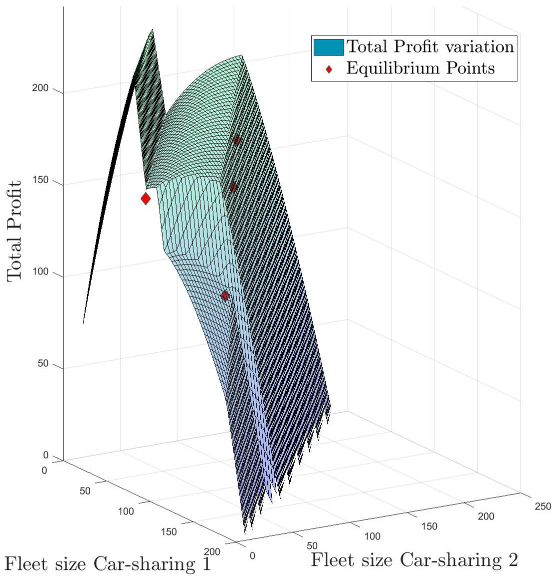
(a) Car-sharing 1



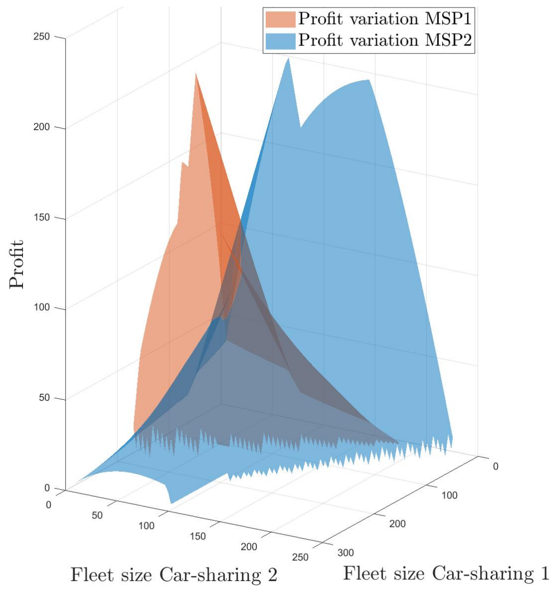
(b) Car-sharing 2

Figure 6.3: Profit variation with equilibrium points

Apart from an interpretation of these results from a computational standpoint, this solution non-uniqueness may have a practical implication; the solution found by prioritizing first car-sharing 1 could be seen as a potential equilibrium state emerging when car-sharing 2 enters the market only in a later time than car-sharing 1.



(a) Total Profit



(b) Individual MSP Profits

Figure 6.4: Profit variations

Hence, this new service provider may be able to reach the maximum achievable level of profitability only by heavily lowering the fleet with respect to the equilibrium achieved, but this strategy may not be 'observable' when being on the achieved equilibrium state as any smaller reduction or increase of the fleet would result in a potential profit loss.

When examining the total profit surface (Figure 6.4a), obtained by simply summing up the two individual profit surfaces (shown instead on Figure 6.4b), it is possible to observe that the equilibrium points deviate from the maximum achievable value that both car-sharing operators could attain if they pursued an entirely cooperative approach.

This analysis reveals that the pursuit of individual profit optimization is likely to result in a sub-optimal outcome compared to a fully collaborative strategy. This naturally prompts for evaluation of the Price of Anarchy (PoA) (Koutsoupias & Papadimitriou, 1999; Papadimitriou, 2001), which is a common measure of the inefficiency of a system in which actors are playing in a self-interested manner. Namely, it is the ratio between the worst-case Nash Equilibrium, where there is no coordination, and the optimal system performance that would be achieved if the players were compelled to coordinate their actions (Christodoulou, 2008). The PoA in this case corresponds to the worst possible profit ratio: summed profits of MSPs at equilibrium (blue in Table 6.4) divided by the maximum attainable total profit. All equilibria have not exhaustively computed and hence it is not possible to evaluate the infimum. Nevertheless, the points computed give an upper bound on the PoA:

$$PoA \leq \frac{min \sum_{NE} Z^j}{max \sum_{tot} Z^j} = \frac{177.1}{229.7} = 0.77 \tag{6.2}$$

implying that when the system exhibits non-cooperative behavior, it may diminish the maximum potential profitability by 23% (or more) compared to what could be achieved through cooperation.

Focusing on the users, it is possible to investigate the impact of capacity changes on the total travel cost (TTC) of the network, as shown in Figure 6.5. It is straightforward to observe that increasing the fleet sizes, i.e. increasing the service capacity, leads to a reduced total travel cost, since the services quality improves in terms of vehicle availability in space and time. Furthermore, Figure 6.5 reprints the position of the equilibrium solutions relative to the network travel cost. Moreover, on the last column of Table 6.4 we show for each equilibrium point the corresponding value of the TTC. We can see how this value tends to remain constant for all the equilibrium points positioned in the same diagonal, with higher values when there is higher profit for the suppliers. This means that the points corresponding to the maximum possible profit for the suppliers does not correspond to the most convenient solutions for the customers. Although the system cost shows only slight variations in this analysis, it can still provide valuable insights for government entities or public authorities seeking to assess the achievement of societal targets. By considering the relationship between equilibrium solutions and network travel cost, policymakers can gain a better understanding of the overall impact and effectiveness of their transportation policies in meeting societal objectives, e.g. seeking to push more travellers to shared mobility options.

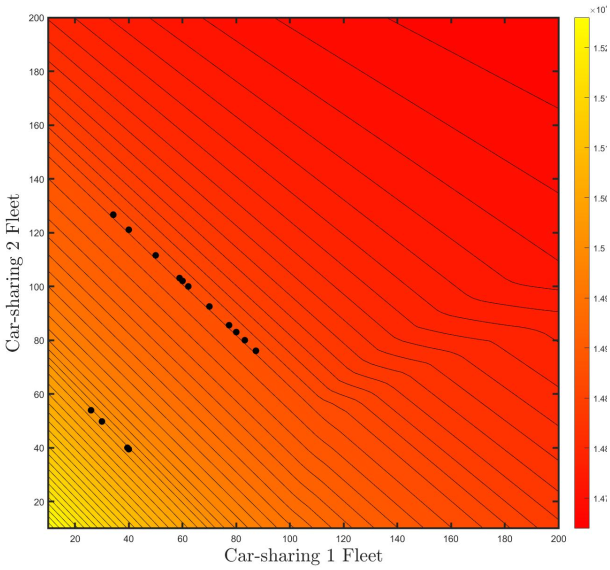


Figure 6.5: Total Travel Cost

Subsequently, it has been compared, in turn, the profit for each car-sharing service operating as a monopoly within the network. This is shown in Figure 6.6 by the two continuous lines, which have been computed assuming that the other service has no vehicles available. Several equilibrium solutions are also shown for sake of comparison, highlighting the potential trade-offs between the monopolistic scenarios and market competition. All feasible equilibrium solutions in the competitive scenario result in reduced profitability for both MSPs. Moreover, we can observe that a similar profitability could be potentially attained by the services with around 60 vehicles, whereas car-sharing 2 has also another local point of maximum when deploying around 180 vehicles.

Moreover, in order to fully understand these discontinuities in the derivatives of the profit functions, an analysis the mode shares for all four mode alternatives (car-sharing 1, car-sharing 2, car and bus) is proposed. In this context, Figure 6.7 shows the path flow variation of the two car-sharing services (Figure 6.7a), car and bus system (Figure 6.7b) with respect to the two services capacity variations. It is possible to notice, straightforwardly, how the maximum path flow on the two services is reached when the MSPs are offering the maximum fleet size. Additionally, these figures vividly illustrate how the discontinuity observed in the profit surfaces is reflected in the variations of path flow. Specially, the flow variation on car-sharing 1 presents the same behaviour as the profit of Figure 6.3 starting from a fleet size of around 100 vehicles. However, we can see on Figure 6.7b that below the same value there is an increase in path flow on both PC and bus mode alternatives. It seems also evident from the flow graphs that the second peak for the car-sharing 2 system acting as monopoly is due to attracting the PC demand, but above a certain fleet size the extra demand attracted is not resulting in sufficient revenues that would justify the additional costs.

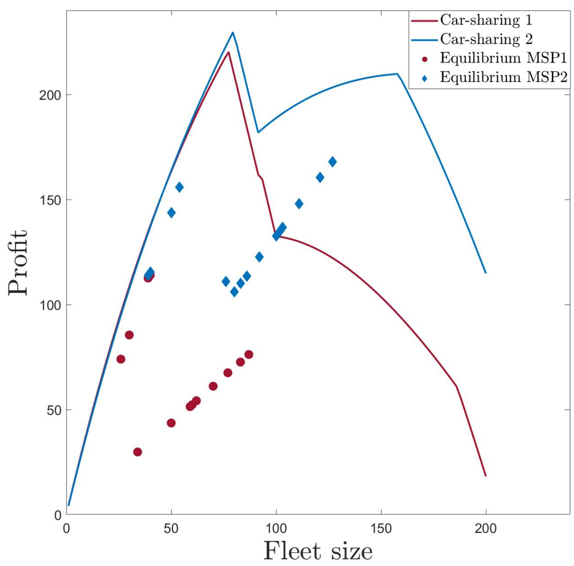
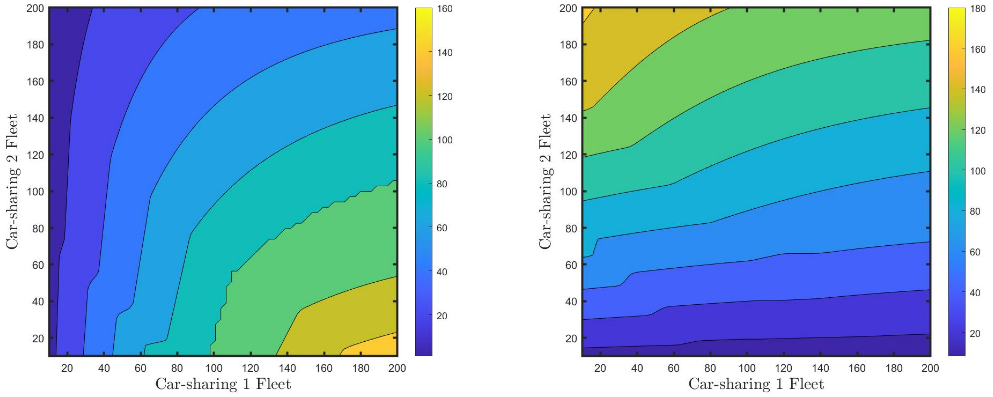


Figure 6.6: Monopoly MSPs Profit variations

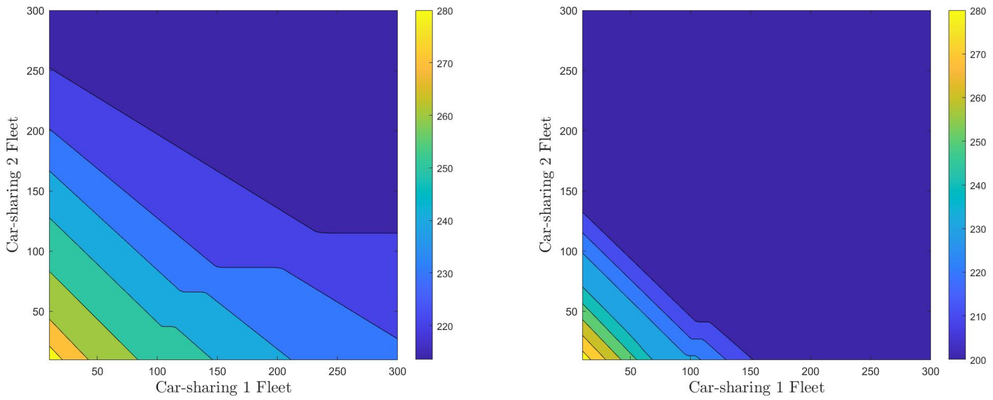
It also quite interesting to observe that the flow variation on car-sharing 2, unlike the other services, has a smoother trend. Moreover, in both car-sharing services there is a clear monotonic increase in the demand that corresponds to the first and highest peaks of MSP's profit variations. Finally, it is clear from Figure 6.7b that the maximum flow on PC and bus corresponds to the minimum fleet sizes of the car-sharing services, as expected. On the contrary, when the capacity of the sharing systems is at its maximum, the usage of PC and bus becomes constant, i.e. increasing the fleet sizes will only result in additional costs but no extra revenue will be generated.

The proposed methodology makes it also possible to analyse different scenarios, for instance in terms of supply and demand characteristics.

First, it has been investigated how car-sharing 1 can increase its own profit in the network. For this reason, it has been decreased the fixed cost of the subscription from $c_s = 0.8$ to $c_s = 0.7$ and the service cost per km from $c_{a,h} = 0.6$ to $c_{a,h} = 0.4$.



(a) Flow on Car-sharing 1 (left) Car-sharing 2 (right)



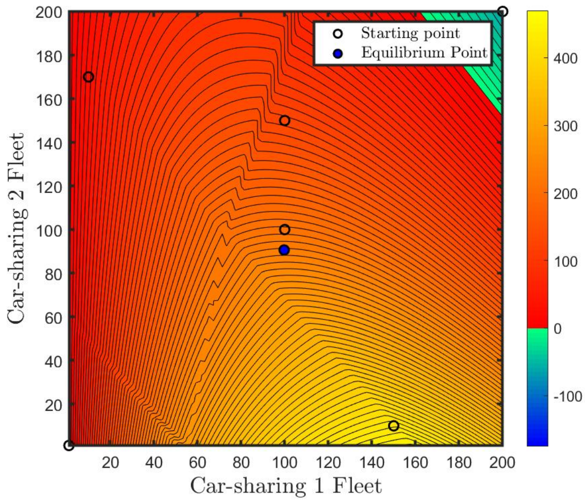
(b) Flow on PC (left) and Bus (right)

Figure 6.7: Example 1: Flow variations

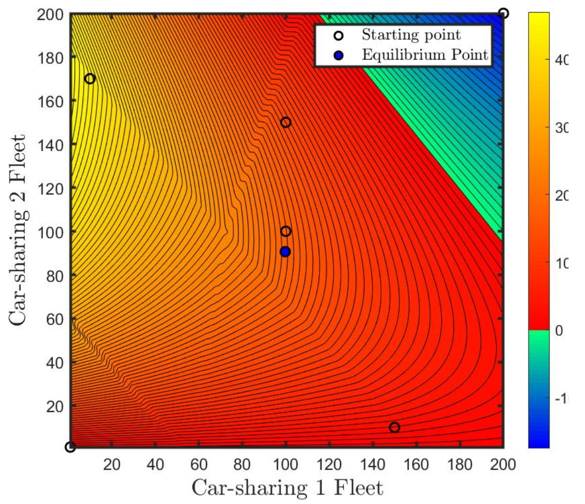
Table 6.5: Example 1: Solution with reduced prices for car-sharing 1

Initial Fleet Sizes		**EPEC Solution**		**Profit**		**PoA**	**TTC**
Fleet Size 1	Fleet Size 2	Fleet Size 1	Fleet Size 2	MSP1	MSP2		
200	200	100	91	236.6	185.8	0.89	15020.4

The results regarding the profit variation with fleet sizes are shown in Figure 6.8. It is possible to see that generally the profit has increased for both suppliers, especially for car-sharing 1. Interestingly, regardless of the initial starting point, the model consistently converged to the same equilibrium point. Notably, by merely changing some parameters specification for one of the players, the equilibrium solution has completely different characteristics from the previous scenario, that appears to be a unique solution for this problem. In Table 6.5 the results for an arbitrary starting point are explicitly displayed. The PoA in this case has a higher value, meaning that the Nash equilibrium solution has a value that is closer to the maximum profit of the system.



(a) Car-sharing 1

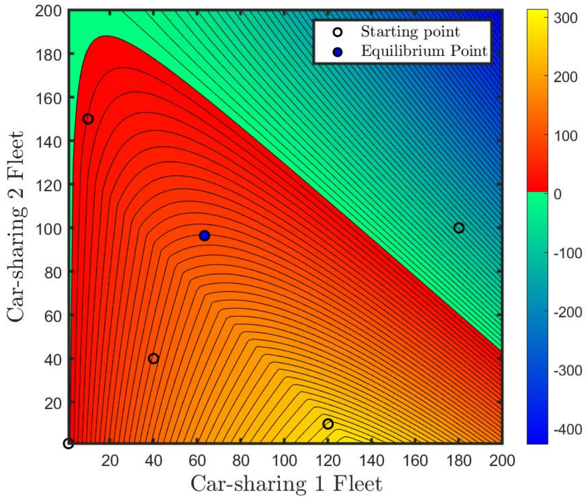


(b) Car-sharing 2

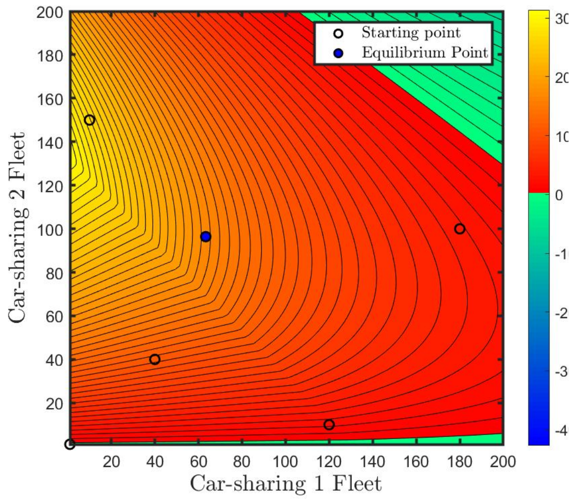
Figure 6.8: Example 1: Profit variation with equilibrium points with reduced prices for car-sharing 1

Additionally, from the main example a study on the impact of the different user classes on the model has been investigated, by keeping the same total demand and changing the value of the demand in between class 1 and class

3: $d_1^1 = 100$ and $d_1^3 = 300$.



(a) Car-sharing 1



(b) Car-sharing 2

Figure 6.9: Example 1: Profit variation with equilibrium points with demand variation

In Figure 6.9 the surfaces of the two profit variations are even smoother than the previous scenarios, suggesting that the composition of the different classes strongly influences the distribution of flow and the ultimate profit for suppliers. Looking at both Figure 6.8 and Figure 6.9 it becomes evident to observe that the equilibrium point reached in both cases is the (only) point in the explored range of fleet sizes where the profit curves are orthogonal to each other, and concurrently the partial derivative of each respective profit function is zero. Moreover, in Table 6.6 the results from an arbitrary starting point are explicitly displayed, where it is possible to see an important increase in the TTC compared to the previous cases.

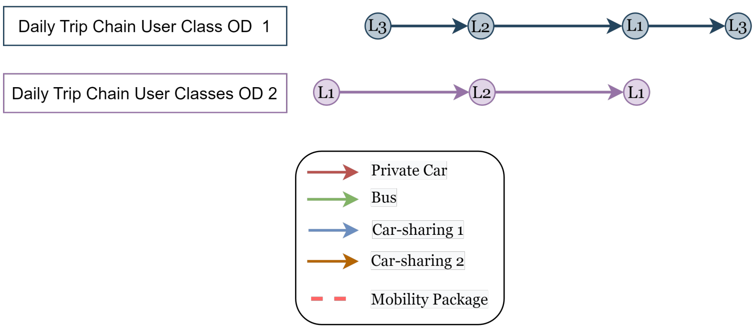
Table 6.6: Example1: Solution with demand variation

Initial Fleet Sizes		**EPEC Solution**		**Profit**		**PoA**	**TTC**
Fleet Size 1	Fleet Size 2	Fleet Size 1	Fleet Size 2	MSP1	MSP2		
180	100	63	96	97.7	175.5	0.86	15968.8

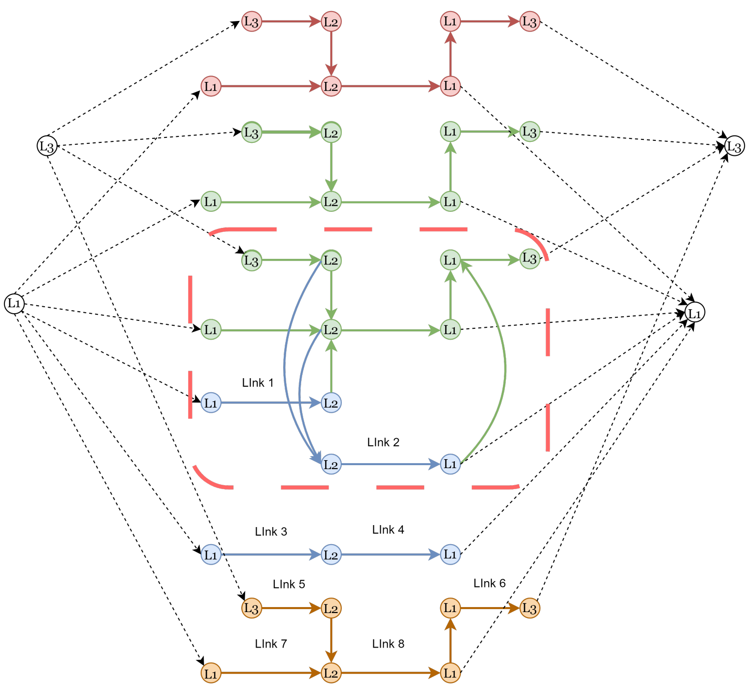
6.4.2 Example 2: Mobility package introduction and competition between MSPs

In the second example, additional characteristics outlined in the methodology are analysed to demonstrate the applicability of the proposed approach in a more complex scenario.

For the proposed example, it has been considered the transport network illustrated in Figure 6.10, which includes two ODs (Figure 6.10a). This network is composed by 39 nodes and 52 links. Three classes of users are taken into account: one on the first OD and two on the second. Four modes of transport are available: PC, bus and two car-sharing services. Car-sharing 1 service is operating only on OD2, while car-sharing 2 is available everywhere. In this particular scenario, it is explored the integration of a mobility package wherein the bus provider and car-sharing 1 supplier collaborate to offer an integrated solution. This mobility package is represented by the sub-graph inside the red-dashed box in Figure 6.10b. The transport network is designed to enable users to subscribe to a monthly package that combines bus and car-sharing 1 services, or alternatively, they can choose the bus-only option at a reduced price compared to the PAYG alternative. It is assumed that this package is subsidized by a local authority, and in order to reduce car congestion, users cannot use only car-sharing 1 when subscribing to the package, but they have to at least make a trip with the bus service. In the provided example, the competitive strategies of two car-sharing providers are considered, specifically when one of them establishes a collaboration with a third service. The algorithm parameters used are listed in Table 6.1. In Table 6.7, it is possible to find the parameters associated to user classes and costs for the services provided. It has been assumed that the daily price for using the package with only the bus service is $c_s = 0.7$ euros, whereas the package that includes both bus and car-sharing has a daily cost of $c_s = 0.9$ euros. In this case, revenue sharing is structured as follows: the bus provider receives $c_s = 0.7$ from the package cost, and $r_s = 0.5$ per day as subsidy. The car-sharing provider, instead, gets $c_s = 0.2$ euros per day from the package, and $r_s = 1$ euros of subsidy per day. This arrangement ensures that the car-sharing provider receives a total amount equivalent to their car-sharing subscription fee, which is set at $c_s = 1.2$ euros. Finally, Table 6.8 lists the parameters associated to the different cost functions. It is considered that the bus service has a dedicated lane, therefore separable cost functions with respect to the other modes of transport that, instead, are considered to influence each other in congestion.



(a) Daily trip chains and available modes of transport



(b) Supernetwork expansion

Figure 6.10: Example 2: Supernetwork with PC, BUs, Two car-sharing services and a mobility package

The proposed algorithm is applied to the provided example, obtaining a unique solution from different starting points. To assess the impact of introducing the mobility package within the network, a comparison is made between the results of the proposed example with a scenario where no mobility package was introduced between the bus and car-sharing 1. In particular, it is considered a full price for car-sharing 1 in every link, while the bus service offers two options: PAYG (c_s = 1.1) euro per day or a package (c_s = 0.7) euro per day. In the scenario without the package, car-sharing 1 has no vehicles on the network because there are not enough users traveling with this mode of transport at equilibrium. However, after introducing the package, the provider has Link 3 and Link 4 inactive, and instead, in Link 1 and Link 2, 93 and 35 vehicles respectively. On the scenario without package, car-sharing 2 has 47 vehicles on Link 7 and 8, and after the introduction of the mobility package this value decreases to 15.

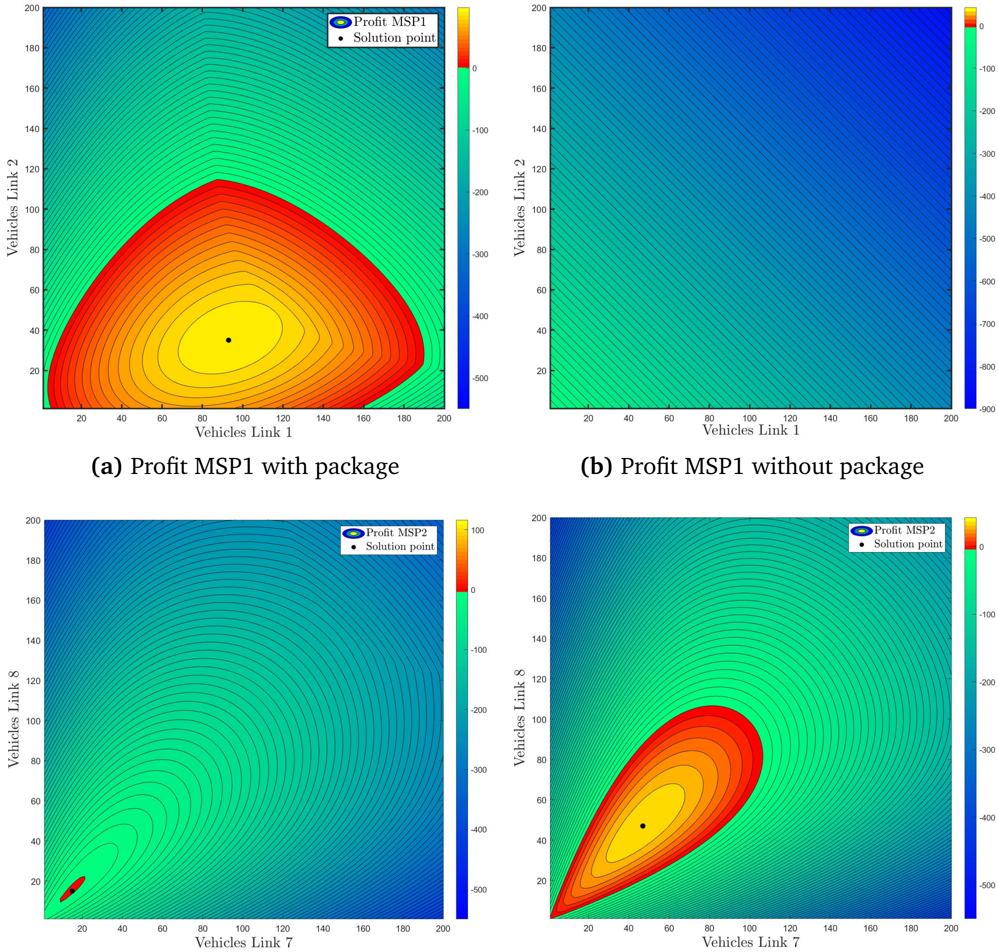
Considering the complexity in representing the profit variations on a two-dimensional plot, influenced by various variables for each supplier, the initial step involved setting the number of vehicles in the unused links to zero to display the results. Subsequently, the number of vehicles on the remaining links for each supplier has been varied, while maintaining a constant distribution of the competitor's vehicles across different links. The results are illustrated in Figure 6.11.

Table 6.7: Example 2: Parameters

Parameters	PC	Bus PAYG	Bus Package	Bus + Car-sharing 1 Package	Car-sharing 1	Car-sharing 2
c_s	0.1	1.1	0.7	0.9	1.2	1.3
r_s	-	-	0.5	0.5-1	-	-
$c^1_{a,access}/c^1_{a,egress}$	8	11	11	11-9	9	9
$c^2_{a,access}/c^2_{a,egress}$	8	11	11	11-9	9	9
$c^3_{a,access}/c^3_{a,egress}$	10.4	14.3	14.3	14.3-11.7	11.7	11.7
$c^1_{a,wait}$	-	11	11	11-9	9	9
$c^2_{a,wait}$	-	11	11	11-9	9	9
$c^3_{a,wait}$	-	14.3	14.3	14.3-11.7	11.7	11.7
$c^1_{a,main}$	8.2	9.5	9.5	9.5-8.5	8.5	8.5
$c^2_{a,main}$	8.2	9.5	9.5	9.5-8.5	8.5	8.5
$c^3_{a,main}$	10.7	12.3	12.3	12.3-11	11	11
$c_{a,fuel}$	0.37	0.2	0.2	0.2	0.2	0.2
$c_{a,h}$	-	-	-	0.4	0.9	0.7
$c_{a,km}$	-	-	-	0.4	0.5	0.4
l_a	10	10	10	10	10	10
$c^1_{a,park}$	11	-	-	-	-	-
$c^2_{a,park}$	11	-	-	-	-	-
$c^3_{a,park}$	14.3	-	-	-	-	-
v_a	300	800	800	800-	-	-
d^1_1				300		
d^2_2				400		
d^3_3				200		

Table 6.8: Example 2: Functional Parameters

Function	PC					Bus					Car-sharing 1					Car-sharing 2				
	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β	t_0	α	f	C	β
$t_{a,access}(f_a, v_a)$			-			0.1		-			0.1	0.1	f_a	v_a	2	0.1	0.15	f_a	v_a	2
$t_{a,wait}(f_a, v_a)$			-			0.1	1	f_a	800	2	0.05	0.2	f_a	v_a	4	0.05	0.2	f_a	v_a	4
$t_{a,main}(\mathbf{f})$	0.2	2	$\mathbf{f}$	600	4	0.2	3	$\mathbf{f}$	600	4	0.2	2	$\mathbf{f}$	800	4	0.2	2	$\mathbf{f}$	600	4
$t_{a,park}(f_a, v_a)$	0.1	1	f_a	300	2			-					-							
$t_{a,egress}(f_a, v_a)$	0.08		-			0.1		-			0.1	0.1	f_a	v_a	2	0.1	0.15	f_a	v_a	2
$c_{lease}(v)$			-					-					$4.2v^1$					$4.2v^2$		



(a) Profit MSP1 with package

(b) Profit MSP1 without package

(c) Profit MSP2 with package

(d) Profit MSP2 without package

Figure 6.11: Example 2: Comparison profit variation with and without the mobility package

In the figure a clear transformation of the network equilibrium is observed due to the introduction of the mobility package. In the absence of a package, car-sharing 1 fails to sustain profitability and exits the market, while car-sharing 2 manages to maintain a viable fleet and generate positive profits. However, with the introduction of the mobility package, car-sharing 1 becomes more appealing due to its collaboration with the bus service. Conversely, car-sharing 2 experiences a significant decline in profitability.

On the user's side, it is experienced a reduction of the TTC (from 20000 to 19820), when introducing the mobility package. Furthermore, Table 6.9 presents a comparison of flow variations at equilibrium between the two cases, with and without the MaaS package. As previously mentioned, the attractiveness of car-sharing 2 diminishes, and there is a slight decrease in PC usage. Notably, all options involving the presence of the bus service demonstrate an increase in flow, with a rising preference for car-sharing 1.

It is interesting to observe how through this analysis, local authorities have the opportunity to apply into different strategies that can effectively increase PT usage in comparison to PC. The results also demonstrate that with appropriate incentives, individuals may be more inclined to subscribe to MaaS. In the long run, this could have a positive impact on reducing car ownership and increasing loyalty towards sustainable modes of transportation.

Table 6.9: Example 2: Results comparison

Flows without Package							
		PC	**Bus**	**Bus with Subscription**	**Bus + MSP1**	**MSP1**	**MSP2**
OD1	**User Class1**	179	121				
OD2	**User Class2**	55	190	155			
OD2	**User Class3**	157					43
Flows with Package							
		PC	**Bus**	**Bus with Package**	**Bus + MSP1**	**MSP1**	**MSP2**
OD1	**User Class1**	181		117	2		
OD2	**User Class2**	9	70	195	127		
OD2	**User Class3**	188					12

6.5 Conclusion

In this chapter, an EPEC has been introduced to investigate equilibrium strategies in multi-modal transport networks involving multiple MSPs and users. To address this problem, an algorithm based on DM has been proposed. This algorithm sequentially solves individual MPECs until equilibrium conditions are met. This equilibrium signifies that no participant in the process has a motivation to change their choices, as it would result in reduced profit for MSPs or increased travel costs for users. The algorithm iteratively applies the solution approaches outlined in Sections 5.3 for the upper-level and 4.4.2 for the lower-level. Finally, the proposed general solution algorithm has been applied to two illustrative examples as detailed in Section 6.4.

In the first numerical example, the competitive dynamics between two car-sharing services within a multi-modal transport system were analysed. The results revealed the intricate, nonlinear, and non-convex nature of the profit function. Multiple distinct EPEC solutions were identified, illustrating the non-uniqueness of equilibrium points. Moreover, the analysis of the profit surfaces and equilibrium points emphasized the impact of the prioritization of car-sharing services within the diagonalization process.

After analysing the equilibrium solutions computed above, and examining the profit surfaces, a notable observation emerges: there exist two distinct regions of equilibria that appear to constitute continuous sets of solutions aligned along two primary lines. A least squares fit to the multiple equilibrium points computed reveals that these lines have equations:

$$v^2 = -0.9517v^1 + 159.1468 \tag{6.3}$$

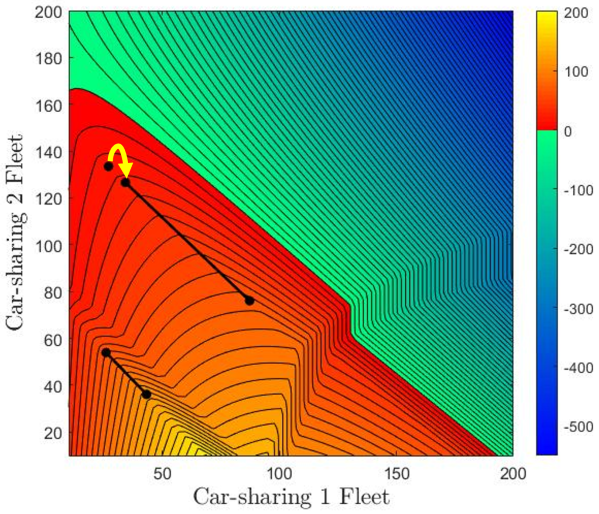
for the upper line, and:

$$v^2 = -1.0309v^1 + 80.7030 \tag{6.4}$$

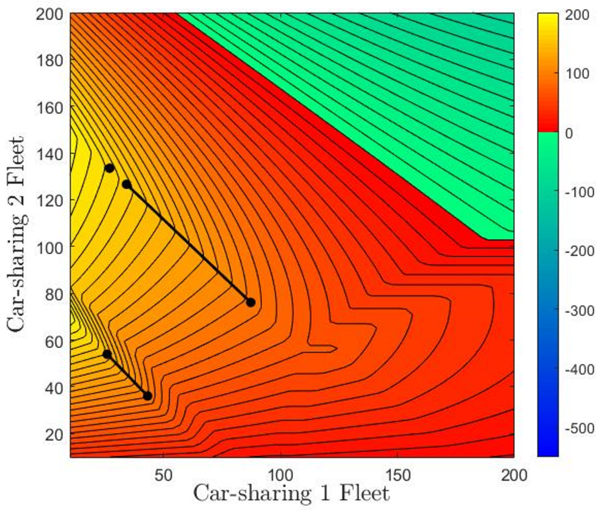
for the lower line; where v^1 is the feet size of car-sharing 1, and v^2 is the fleet size of car-sharing 2.

From a visual inspection of the contour plots displayed in Figure 6.3 it may appear that equilibrium solutions occur all along these lines from $v^1 \in [0, 69]$ for the lower line and approximately $v^1 \in [0, 104]$ for the upper line. However, numerical investigations suggest this is not the case. Each linear set of solutions has boundary points, shown by the line-end dots in Figure 6.12. There appear to be a continuum of equilibrium points within each marked segment. However, points on the extrapolation of these line segments, but outside the marked end points, do not meet the equilibrium criteria and when used as initial points for the algorithm, the nearest boundary point is returned as the equilibrium solution, as shown by the yellow arrow in Figure 6.12a. This deserves further investigation, to understand the occurrence of continuous or separated equilibria in different scenarios.

Generally, this example provided insights into the complexity of competitive transportation networks and the decision-making processes of MSPs. Furthermore, the impact of different user classes on the model was explored, demonstrating how changes in demand between these classes influenced profit surfaces due to shifts in path flows based on distinct cost perceptions.



(a) Car-sharing 1



(b) Car-sharing 2

Figure 6.12: Example 1: Profit variation with equilibrium segments

Lastly, the proposed methodology was applied to a more complex network, introducing collaboration through a mobility package and assessing competition. The results have important implications for understanding market dynamics and devising strategies within the transportation market.

This Chapter provides insight into the complexity and potential applicability of the proposed methodology. The model aims to capture the intricate dynamics that can result from various market strategies. However, it's crucial to recognize that user choices play a fundamental role, often determining the survival of specific mobility services. To truly comprehend the intricate dynamics that unfold when competitors enter the market or when suppliers collaborate through packages, it's imperative to consider all actors involved. Nevertheless, modeling systems like MaaS is exceptionally complex, and this research can be viewed as an initial step toward a clearer understanding of these contexts for potential application in large-scale networks.

The following Chapter presents the overarching conclusions of this thesis along with potential future research developments.

Chapter 7

Conclusion

7.1 Summary

In recent years, the rising prevalence of PC usage and its adverse impact on CO2 emissions have prompted the emergence of alternative mobility solutions within urban transportation networks. These solutions are proposed as alternatives to PC usage and as first/last mile options that complement PT systems. However, as highlighted in Chapter 1, these solutions, despite their rapid proliferation across various countries, have not yet led to a significant reduction in single-car usage. Additionally, the presence of these different services has intensified competition among MSPs, sometimes resulting in a struggle for survival within the transportation system.

In response to this evolving landscape, the concept of MaaS has emerged as a potential solution. MaaS aims to offer a wide range of mobility services into a single platform, allowing users to access multiple options through bundled packages at discounted prices. As discussed in Chapter 2, pilot projects implementing the MaaS concept have been initiated worldwide. However, their success has been confined to the pilot phase and has not translated into large-scale adoption.

Several factors contribute to this limited success. For instance, it can be challenging to facilitate collaboration within a single platform of multiple MSPs from both the private and public sectors, each of which has a different business model and market strategies. Furthermore, changing user habits to align with a novel and somewhat unclear transportation concept presents a substantial hurdle.

Nevertheless, not only within the framework of MaaS, but in the broader context of transportation, it is evident that the system has grown increasingly complex. This complexity arises from the interactions between the diverse MSPs, marked by both collaborative and competitive strategies, as well as the complex multi-modal choices made by users.

In order to address the main research question, 'What are the conditions that allow meeting the objectives of multiple actors in a multi-modal transport system?' a review of the existing transportation literature was conducted. This review aimed to gain insights into the contextual development, structural complexities, key participants, and modelling of these intricate transportation systems. First, in Chapter 2, an extensive literature review on MaaS revealed the challenges associated with integrated systems involving multiple actors. While numerous pilot projects have been developed it seems fundamental to investigate the impact of this system on all participating actors. Existing simulation and mathematical models have made attempts to address this challenge, but they have limitations in comprehensively representing the full range of features within this multi-modal transport system. Specifically, they often fall short in including the multiple available transportation modes, the varying needs and preferences of users, collaboration strategies offered through mobility packages, and the intricate interaction between users and different MSPs. These aspects constitute the primary characteristics addressed in this thesis. From this considerations, a key sub-question arises: 'How can a network model be defined to effectively incorporate the diverse attributes of users and services within a multi-modal transport system?'. In order to be able to address this question, and considering that a mathematical approach is taken in this thesis, in Chapter 3 a detailed literature review of classical transportation models have been conducted. It's important to note that there are currently no models that comprehensively capture all the elements of such a complex system, and the significance of this chapter lies in the notion that understanding how different aspects have been modeled in classical transportation literature is fundamental for modeling a multi-actor, multi-modal transportation system. This understanding helps leverage concepts from the literature in order to extend, and potentially combine them. Specifically, the following aspects have been considered: the modeling of multi-modal networks, representing user choices and formulating traffic assignment, defining supplier market strategies to study the dynamics between suppliers and users.

In Chapter 4, the first sub-research question is thoroughly addressed, and the first contribution of this thesis is presented as a combination of various components. A supernetwork representation is introduced, which directly incorporates various multi-modal constraints, including mobility packages and the daily trip chains of diverse user. This network is designed to include the different transportation options available at each trip connection, offering users multi-modal routes to accommodate their travel needs. To account for the heterogeneity among travellers, a classification system is employed, dividing users into distinct classes based on their socio-economic characteristics and daily trip chains. As a result, users within the same class are subject to costs with identical weights. Within this network, each MSP owns specific layers. Each layer consists of a set of links, representing both the source of revenue generated by their service and the associated costs.

Revenue is influenced by the number of users utilising their mobility service, while costs are contingent on the service's characteristics and the fleet size chosen by the MSP.

To address the second research sub-question:'Given a set of strategies from diverse actors, what is the emerging behavior of the whole multi-modal transport system?', Chapter 4 introduces two distinct formulations for user and MSPs representations. For users' representation, Wardrop's first equilibrium principle has been taken into account, considering that users are assigned to the transportation network under the condition that at equilibrium all the paths with positive flow have minimum cost. It is also taken into consideration that link costs are perceived in a non-separable manner, indicating that the cost of a particular link is influenced by the costs of parallel links with similar infrastructural characteristics. This complexity necessitates the formulation of the UE as a VI problem. A solution algorithm based on an adaptation of the EM has been proposed, showing promising results in terms of computational time and, in the tested scenarios, ensures the uniqueness of the solution. As for MSPs providing mobility services within this multi-modal network, they are assumed to be profit maximizers. This means their sustainability in the market is based on their services generating a positive profit. To represent a broad range of services, an additional contribution of this thesis is the definition of a general profit maximization formulation able to describe different MSP and their market behavior.

To investigate the interactions between strategies employed by MSPs and the modal choices made by users, a bi-level structure has been established by connecting the proposed mathematical formulations. In Chapter 5, a Stackelberg congestion game is adopted. At the upper-level, a single MSP applies different strategies to increase the profit, while being subject to lower-level traffic assignment, represented as a VI. This problem results in an MPEC. To identify the upper-level fleet size that maximizes total profit, various potential solution points are assessed using a SQP algorithm. For each upper-level value, the corresponding lower-level equilibrium solution is evaluated through an adaptation of the EM method. This iterative process continues until the MSP no longer has an incentive to change the fleet size, as such changes would lead to reduced profit. This algorithm is applied to different examples to highlight the model's characteristics and to illustrate how the strategies of other MSPs can impact the overall outcomes of the entire system.

In Chapter 6, the analysis is extended to consider the strategies of multiple MSPs adjusting their fleet sizes independently and with a self-interested approach to maximize their individual profits. This problem is framed as an EPEC, which is considered as a sum of several MPECs. This chapter constitutes the main contribution of this thesis. EPEC have been rarely used in transportation context, and most of the time they are applied to uni-modal context.

Within this thesis, an EPEC is defined where each MSP aims to optimize their service capacity to maximize profit while accounting for the strategies of other MSPs and user multi-modal travel choices. One of the complexity of this extended version arises from the fact that from a change of one of the upper-level strategies derives a change of the lower-level equilibrium and also the strategies of other upper-level MSPs. To address this, a DM is employed. This method tackles the EPEC by solving one MPEC at a time while keeping the decision variables of other MSPs fixed. This algorithm is applied to two examples to analyse the dynamics between two competing MSPs and to explore scenarios in which collaboration occurs through a mobility package with shared pricing. The results show the complexity of the problem. Particularly, it has been experienced non-uniqueness of the solution, and emphasized the impact of the prioritization of car-sharing services within the diagonalization process. It has been observed that these results change when the upper-level profit has a smoother variation, that happened when varying the demand or changing the pricing strategies. Furthermore, in a more complex scenario, it becomes evident that the equilibrium of the system can change when collaborative strategies are introduced.

The proposed methodology has demonstrated promising results while also highlighting the complexity of the overarching problem. There exist several limitations within the current model, and there are multiple opportunities for future developments to expand upon this study. In the following section, an examination of these limitations and unexplored features will be conducted, along with the presentation of potential directions for further research.

7.2 Future Research

The proposed research has shown interesting results in its application on small-scale scenarios. However, several are the possible research directions that can be explored in order to better understand the behaviours of the model and of the different actors of the transportation network.

One promising research direction, although only partially explored thus far, could serve as an excellent starting point for investigating the large-scale applications of the methodology introduced in Chapter 4. Specifically, Li et al. (2022) proposed a Differentiable Bi-level Programming approach to solve large-scale Stackelberg congestion games. This problem shares the same bi-level structure as the one discussed in this thesis. At the upper level, a leader seeks to maximize their own profit, while at the lower level, the equilibrium problem is formulated as a path-based VI. The proposed solution approach incorporates concepts from Machine Learning applications. More precisely, the authors utilise the imitative logit dynamics (ILD) to reformulate the lower-level routing problem. This approach models the players' behavior in such a way that they adapt their strategies by emulating previous successful experiences. In this context, the authors interpret the ILD as the result of all travelers simultaneously minimizing their expected costs using a mirror descent (MD) method. They demonstrate that ILD can be represented as a differentiable program (DiP). This representation allows them to compute the gradient of the leader's objective using automatic differentiation (AD), a technique used for numerical function evaluation. The structure of the proposed DiP is reminiscent of a deep neural network (DNN). The hidden layers in this DNN map the choices of followers from one day (or stage) to the next, based on their perceived utility, while the leader's decision can be incorporated into these layers as weights or trainable parameters. Following this approach, given a starting point, the DNN can forecast the choices made by followers and the leader's payoff for each day.

Exploring the potential applicability of this methodology to the MPEC formulation presented in this thesis could be interesting, as it might offer a means to expand the proposed analysis to large-scale networks.

An additional limitation of the proposed methodology lies in the requirement for the full enumeration of all possible paths. Currently, these paths are manually enumerated, with an attempt to consider only the reasonable ones, i.e. omitting paths where users leave home with their PC and come back with PT. This manual enumeration is necessary to account for multi-modal constraints and package subscriptions. However, when considering the application of this methodology to large-scale networks, it becomes evident that this approach is no longer feasible. Therefore, a potential research direction could involve the development of an algorithm capable of generating paths that simultaneously captures all network constraints.

Furthermore, while this thesis has explored certain aspects of MSPs' market strategies, a deeper investigation into the underlying mechanisms of their pricing strategies could be highly valuable. It would be worthwhile to individuate pricing and compensation mechanisms that could help mitigate the PoA while increasing service profitability. As highlighted in Chapter 6, it's evident that the pricing schemes adopted by MSPs have a strong influence on the PoA and the occurrence of multiple optima. Therefore, it seems interesting to develop optimal pricing schemes, which are currently exogenously imposed in the existing model, with the aim of reducing or minimizing the PoA. Moreover, within the context of shared mobility packages, it's also worth considering how the revenue-sharing could potentially impact the profit of the different MSPs. Additionally, an analysis of how revenue-sharing strategies for these packages could be determined based on the relative influence of different MSPs in the transportation market could provide valuable insights.

Based on the findings from Chapter 6, it's evident that under certain conditions, a unique solution to the problem may be observed, while in other cases, multiple solutions may emerge, when varying the initial conditions or which MSP's profit maximization is evaluated first. In the first example presented in Section 6.4, the small-size network allowed for a comprehensive assessment of profit variations for both suppliers. However, this approach becomes considerably more intricate when evaluating multiple upper-level variables simultaneously. Therefore, it would be interesting to explore if it is possible to determine, without an exhaustive evaluation, whether a specific problem is characterized by a single equilibrium or multiple equilibria.

Furthermore, a critical aspect to delve into is the behaviour of the solution points discussed in the conclusion section (6.5). The results reveal that equilibrium points are situated in two main and distinct areas. A visual examination of the contour plots in Figure 6.3 suggests that these points align with two specific lines (Equations 6.3 and 6.4). However, a numerical analysis has demonstrated that only the points within the segments shown in Figure 6.12 are indeed solution points. The reason behind this result remains unclear, and it would be beneficial to investigate further. One hypothesis is that this outcome might be related to the choice of the solution algorithm employed, specifically the DM.

Finally, concerning the DM, it is worth exploring whether this algorithm is suitable for larger networks and whether, in these cases, it can effectively converge. Additionally, this thesis has only focused on examples evaluating maximum two upper-level suppliers. It would be valuable to understand if the DM remains an appropriate choice when increasing the number of suppliers competing at the upper-level.

In this section, a brief outline of several potential research directions has been presented. Nevertheless, it is evident that the research undertaken has opened up a broad spectrum of possibilities for future investigations.

Appendix A

List of Publications

A.1 Conferences

- An equilibrium model for Mobility-as-a-Service
  C. Bandiera, R.D. Connors, F. Viti
  BIVEC Transport Research Days 2021

- Assessing Profit Maximization and Fleet Management in a Multimodal Network Design Problem
  C. Bandiera, R.D. Connors, F. Viti
  Proceedings of the 25th International Conference of Hong Kong Society for Transportation Studies, HKSTS 2021: Sustainable Mobility

- Evaluating Mobility Service Providers' Strategies in an Activity-Based Supernetwork
  C. Bandiera, R.D. Connors, F. Viti
  10th symposium of the European Association for Research in Transportation (hEART2022)

- Competition and Cooperation between Suppliers in Multimodal Network Design Problems
  C. Bandiera, R.D. Connors, F. Viti
  Eleventh Triennial Symposium on Transportation Analysis conference (TRISTAN XI)

- Mobility Service Providers' Equilibrium Strategies in Multimodal Networks
  C. Bandiera, R.D. Connors, F. Viti
  11th symposium of the European Association for Research in Transportation (BIVEC Transport Research Days 2023 / hEART2023)

Bibliography

Aashtiani, H. Z., & Magnanti, T. L. (1981). Equilibria on a congested transportation network. *Siam Journal on Algebraic and Discrete Methods*, *2*, 213–226 (cit. on pp. 49, 56, 57).

Adler, N., Brudner, A., & Proost, S. (2021). A review of transport market modeling using game-theoretic principles. *European Journal of Operational Research*, *291*(3), 808–829 (cit. on p. 63).

Ameli, M., Lebacque, J.-P., & Leclercq, L. (2020). Simulation-based dynamic traffic assignment: Meta-heuristic solution methods with parallel computing. *Computer-Aided Civil and Infrastructure Engineering*, *35*(10), 1047–1062 (cit. on p. 49).

Arentze, T. T., & Molin, E. J. E. Travelers' preferences in multimodal networks: Design and results of a comprehensive series of choice experiments. In: 2013 (cit. on p. 71).

Arentze, T. T., & Timmermans, H. J. P. (2004). Multistate supernetwork approach to modelling multi-activity, multimodal trip chains. *International Journal of Geographical Information Science*, *18*, 631 –651 (cit. on pp. 46, 47).

Axhausen, K. W., Zimmermann, A., Schönfelder, S., Rindsfüser, G., & Haupt, T. (2002). Observing the rhythms of daily life: A six-week travel diary. *Transportation*, *29*, 95–124 (cit. on p. 75).

Barreto, L., Amaral, A., & Baltazar, S. (2018). Mobility as a service (maas) in rural regions: An overview. *2018 International Conference on Intelligent Systems (IS)*, 856–860 (cit. on p. 40).

Bazaraa, M. S., Sherali, H. D., & Shetty, C. M. (2013). *Nonlinear programming: Theory and algorithms*. John wiley & sons. (Cit. on p. 80).

Becker, H., Balać, M., Ciari, F., & Axhausen, K. W. (2018). Assessing the welfare impacts of shared mobility and mobility as a service (maas). *Transportation Research Part A: Policy and Practice* (cit. on p. 34).

Beckmann, M., McGuire, C. B., & Winsten, C. B. (1956). *Studies in the economics of transportation*. Yale University Press. (Cit. on pp. 49, 50, 52).

Bekhor, S., & Toledo, T. (2005). Investigating path-based solution algorithms to the stochastic user equilibrium problem. *Transportation Research Part B: Methodological*, *39*(3), 279–295 (cit. on p. 49).

Bertrand, J. (1883). Théorie mathématique de la richesse sociale. *Journal des savants*, *67*(1883), 499–508 (cit. on p. 57).

Bingfeng, S., Ming, Z., Xiaobao, Y., & Ziyou, G. (2017). Bi-level programming model for exclusive bus lanes configuration in multimodal traffic network. *Transportation research procedia*, *25*, 652–663 (cit. on p. 62).

Bowman, J. L., & Ben-Akiva, M. E. (2001). Activity-based disaggregate travel demand model system with activity schedules. *Transportation research part a: policy and practice*, *35*(1), 1–28 (cit. on p. 12).
Boyce, D. E., Mahmassani, H. S., & Nagurney, A. A retrospective on beckmann, mcguire and winsten's "studies in the economics of transportation". In: 2005 (cit. on pp. 50, 55).
Boyles, S. D., Lownes, N. E., & Unnikrishnan, A. (2020). Transportation network analysis. *Volume I* (cit. on p. 48).
Butler, L., Yigitcanlar, T., & Paz, A. (2021). Barriers and risks of mobility-as-a-service (maas) adoption in cities: A systematic review of the literature. *Cities*, *109*, 103036 (cit. on p. 19).
Calderón, F., & Miller, E. J. (2020). A literature review of mobility services: Definitions, modelling state-of-the-art, and key considerations for a conceptual modelling framework. *Transport Reviews*, *40*(3), 312–332 (cit. on p. 35).
Carlier, K., Inro, T., Fiorenzo-Catalano, S., & Lindveld, C. D. R. A supernetwork approach towards multimodal travel modeling. In: 2003 (cit. on pp. 45, 46).
Cartenì, A. C., Bifulco, G. N. B., & Papola, A. P. (2010). An activity-based approach for complex travel behaviour modelling (cit. on p. 12).
Cascetta, E. (2009, January). *Transportation system analysis: Models and applications*. (Cit. on pp. 45, 47, 74).
Charnes, A, & Cooper, W. (1958). Extremal principles for simulating traffic flow in a network. *Proceedings of the National Academy of Sciences*, *44*(2), 201–204 (cit. on p. 57).
Chiu, Y.-C., Bottom, J., Mahut, M., Paz, A., Balakrishna, R., Waller, T., & Hicks, J. (2011). Dynamic traffic assignment: A primer. *Transportation research circular* (cit. on p. 82).
Christina Hvid, S. R., Nicolai B. Sørensen. (2018). The story of 'minrejseplan' - a major step towards maas. http://asp.vejtid.dk/Artikler/2018/ITSWC/9073.pdf. (Cit. on p. 40)
Christodoulou, G. (2008). Price of anarchy. In M.-Y. Kao (Ed.), *Encyclopedia of algorithms* (pp. 665–667). Springer US. (Cit. on p. 124).
Cisterna, C., Bigi, F., Tinessa, F., & Viti, F. Analysis of maas membership attributes: An agent-based approach. In: *62*. 2022, 483 – 490 (cit. on p. 34).
Cisterna, C., Bigi, F., & Viti, F. (2023, March). The impact of total cost of ownership on maas system appeal using an agent-based approach. (Cit. on p. 34).
Cisterna, C., Giorgione, G., & Viti, F. (2021). Explorative analysis of potential maas customers: An agent-based scenario [The 12th International Conference on Ambient Systems, Networks and Technologies (ANT) / The 4th International Conference on Emerging Data and Industry 4.0 (EDI40) / Affiliated Workshops]. *Procedia Computer Science*, *184*, 629–634 (cit. on p. 34).
Clienti. (2018). Concept, identity and some. https://clienti.dk/en/cases/nt-minrejseplan/. (Cit. on p. 40)
Cohen, A., & Shaheen, S. (2018). *Planning for shared mobility*. (Cit. on p. 26).
Colson, B., Marcotte, P., & Savard, G. (2007). An overview of bilevel optimization. *Annals of operations research*, *153*, 235–256 (cit. on p. 59).
Commission, E. (2023). Sustainable urban mobility. https://transport.ec.europa.eu/transport-themes/urban-transport/sustainable-urban-mobility_en. (Cit. on p. 14)

Connors, R., & Watling, D. (2014). Assessing the demand vulnerability of equilibrium traffic networks via network aggregation. *Networks and Spatial Economics*, *15*, 365–395. https://doi.org/10.1007/s11067-014-9251-9 (cit. on p. 69)

Cottle, R. W. (1966). Nonlinear programs with positively bounded jacobians. *SIAM Journal on Applied Mathematics*, *14*(1), 147–158 (cit. on p. 56).

Cottle, R. W., & Su, C.-L. Equilibrium problems with equilibrium constraints: Stationarities, algorithms, and applications. In: 2005 (cit. on pp. 63, 64, 116).

Cournot, A. A. (1838). *Recherches sur les principes mathématiques de la théorie des richesses*. L. Hachette. (Cit. on p. 57).

Cruz, C., & Sarmento, J. (2020a). “mobility as a service” platforms: A critical path towards increasing the sustainability of transportation systems. *Sustainability*, *12*, 6368 (cit. on p. 30).

Cruz, C., & Sarmento, J. (2020b). “mobility as a service” platforms: A critical path towards increasing the sustainability of transportation systems. *Sustainability*, *12*, 63–68 (cit. on p. 28).

Dafermos, S. (1980). Traffic equilibrium and variational inequalities. *Transportation Science*, *14*, 42–54 (cit. on pp. 49, 52, 76).

Dafermos, S. C. (1971). An extended traffic assignment model with applications to two-way traffic. *Transportation Science*, *5*(4), 366–389 (cit. on pp. 47, 52).

Dafermos, S. C. (1972). The traffic assignment problem for multiclass-user transportation networks. *Transportation science*, *6*(1), 73–87 (cit. on p. 47).

Dafermos, S. C., & Sparrow, F. T. (1969). The traffic assignment problem for a general network. *Journal of Research of the National Bureau of Standards B*, *73*(2), 91–118 (cit. on p. 49).

de Luca, S., & Mascia, M. (2021). Adaptive travel mode choice in the era of mobility as a service (maas): Literature review and the hypermode mode choice paradigm. *Models and Technologies for Smart, Sustainable and Safe Transportation Systems* (cit. on p. 31).

Djavadian, S., & Chow, J. Y. J. (2017a). Agent-based day-to-day adjustment process to evaluate dynamic flexible transport service policies. *Transportmetrica B: Transport Dynamics*, *5*, 281–306 (cit. on p. 36).

Djavadian, S., & Chow, J. Y. J. (2017b). An agent-based day-to-day adjustment process for modeling ‘mobility as a service’ with a two-sided flexible transport market. *Transportation Research Part B-methodological*, *104*, 36–57 (cit. on p. 36).

Eckhardt, J., Aapaoja, A., Nykänen, L., & Sochor, J. Mobility as a service business and operator models. In: In *Paper presented at the 12th its european congress, strasbourg, france*. 2017 (cit. on pp. 28, 29).

El-Agroudy, M., Abou-Senna, H., & Radwan, E. (2021). Mobility-as-a-service: Simulation of multi-modal operations in low-density cities. *Transportation Research Record: Journal of the Transportation Research Board*, *2676* (cit. on p. 34).

Eren, E., & Uz, V. E. (2020). A review on bike-sharing: The factors affecting bike-sharing demand. *Sustainable cities and society*, *54*, 101882 (cit. on p. 16).

Esztergár-Kiss, D., Kerényi, T., Mátrai, T., & Aba, A. (2020). Exploring the maas market with systematic analysis. *European Transport Research Review*, *12*, 1–16 (cit. on p. 28).

Esztergár-Kiss, D., & Aba, T. M. A. (n.d.). Hands-on experience of a maas demonstration in budapest. *23rd EURO Working Group on Transportation Meeting, EWGT 2020, 16-18September 2020, Paphos, Cyprus* (cit. on p. 41).

Eurostat. (2023). Number of cars per inhabitant increased in 2021. https://ec.europa.eu/eurostat/web/products-eurostat-news/w/ddn-20230530-1#:~:text=In%202021%2C%20the%20average%20number,central%20and%20eastern%20EU%20members.. (Cit. on pp. 12, 13)

Fan, W., Khan, M. B., Ma, J., & Jiang, X. (2014). Bilevel programming model for locating park-and-ride facilities. *Journal of Urban Planning and Development*, *140*, 04014007 (cit. on p. 60).

Farahani, R. Z., Miandoabchi, E., Szeto, W. Y., & Rashidi-Bajgan, H. (2013). A review of urban transportation network design problems. *Eur. J. Oper. Res.*, *229*, 281–302 (cit. on p. 44).

Fellendorf, M., & Vortisch, P. (2010). Microscopic traffic flow simulator vissim. *Fundamentals of traffic simulation*, 63–93 (cit. on p. 34).

Fenton, P. (2020). Sustainable mobility and transport in stockholm: Moving from eccentric to business as usual. https://civitas.eu. (Cit. on p. 40)

Ferrero, F., Perboli, G., Rosano, M., & Vesco, A. (2018). Car-sharing services: An annotated review. *Sustainable Cities and Society*, *37*, 501–518 (cit. on p. 15).

Fiorenzo-Calatano, M. S. Choice set generation in multi-modal transportation networks. In: 2007 (cit. on pp. 45, 46).

Fotouhi, H., & Miller-Hooks, E. (2021). Optimal time-differentiated pricing for a competitive mixed traditional and crowdsourced event parking market. *Transportation Research Part C: Emerging Technologies*, *132*, 103409 (cit. on p. 65).

Frank, M., Wolfe, P., et al. (1956). An algorithm for quadratic programming. *Naval research logistics quarterly*, *3*(1-2), 95–110 (cit. on p. 80).

Friedel, A. (2020). Free floating car sharing map. https://maphub.net/Augustin/carshare. (Cit. on pp. 15, 16)

Friedel, A. (2021). Free floating micro mobility map europe. https://maphub.net/Augustin/micro. (Cit. on p. 18)

Fu, X., & Lam, W. H. K. (2014). A network equilibrium approach for modelling activity-travel pattern scheduling problems in multi-modal transit networks with uncertainty. *Transportation*, *41*, 37–55 (cit. on p. 47).

Fu, X., Lam, W. H. K., Chen, B. Y., & Liu, Z. (2020). Maximizing space-time accessibility in multi-modal transit networks: An activity-based approach. *Transportmetrica A: Transport Science*, *18*, 192 –220 (cit. on p. 60).

Fudenberg, D., & Tirole, J. (1991). *Game theory*. MIT press. (Cit. on p. 57).

García, R., & Marin, A. (2002). Parking capacity and pricing in park'n ride trips: A continuous equilibrium network design problem. *Annals OR*, *116*, 153–178 (cit. on p. 62).

Garus, A., Alonso, B., Aloso Raposo, M., Ciuffo, B., et al. (2022). Impact of new mobility solutions on travel behaviour and its incorporation into travel demand models. *Journal of Advanced Transportation*, *2022* (cit. on p. 18).

Gayo. (2020). One app for all your transport. https://gaiyo.com/. (Cit. on p. 42)

Goedopweg. (2020). Travel app: Mobility as a service. https://goedopweg.nl/projecten/mobility-as-a-service. (Cit. on p. 42)

Gouvernement.lu. (2020). Le portail de covoiturage copilote intègre l'application klaxit. https://gouvernement.lu/fr/actualites/toutes_actualites/communiques/2020/01-janvier/28-covoiturage-copilote.html. (Cit. on p. 16)

Gu, Y., Cai, X., Han, D., & Wang, D. Z. (2019). A tri-level optimization model for a private road competition problem with traffic equilibrium constraints. *European Journal of Operational Research*, *273*(1), 190–197 (cit. on p. 64).

Guo, Z., Deride, J., & Fan, Y. (2016). Infrastructure planning for fast charging stations in a competitive market. *Transportation Research Part C-emerging Technologies*, *68*, 215–227 (cit. on p. 63).

Han, D., & Lo, H. K. (2004). Solving non-additive traffic assignment problems: A descent method for co-coercive variational inequalities. *European Journal of Operational Research*, *159*(3), 529–544 (cit. on p. 48).

Harker, P. T. (1984). A variational inequality approach for the determination of oligopolistic market equilibrium. *Mathematical Programming*, *30*, 105–111 (cit. on p. 117).

Harker, P. T., & Pang, J.-S. (1990). Finite-dimensional variational inequality and nonlinear complementarity problems: A survey of theory, algorithms and applications. *Mathematical programming*, *48*(1-3), 161–220 (cit. on p. 56).

Hartman, P., & Stampacchia, G. (1966). On some non-linear elliptic differential-functional equations (cit. on p. 52).

Hasan, M. K., & Dashti, H. M. (2007). A multiclass simultaneous transportation equilibrium model. *Networks and spatial economics*, *7*(3), 197–211 (cit. on p. 48).

Haurie, A., & Marcotte, P. (1985). On the relationship between nash—cournot and wardrop equilibria. *Networks*, *15*(3), 295–308 (cit. on p. 57).

Heikkilä, S. Mobility as a service - a proposal for action for the public administration, case helsinki. In: 2014 (cit. on p. 26).

Hensher, D., Ho, C., & Reck, D. (2021a). Mobility as a service and private car use: Evidence from the sydney maas trial. *Transportation Research Part A: Policy and Practice*, *145*, 17–33 (cit. on pp. 30, 41).

Hensher, D. A. (2020). What does intelligent mobility add to sustainability? https://imoveaustralia.com/wp-content/uploads/2020/05/David-Hensher-iMOVE-MaaS-webinar-slide-deck-1-May-2020-v2.pdf. (Cit. on p. 41)

Hensher, D. A., Mulley, C., Ho, C., Wong, Y., Smith, G., & Nelson, J. D. (2020). *Understanding mobility as a service (maas): Past, present and future* (1st ed.). Elsevier. (Cit. on pp. 31, 41).

Hensher, D. A., Mulley, C., & Nelson, J. D. (2021b). Mobility as a service (maas) – going somewhere or nowhere? *Transport Policy*, *111*, 153–156 (cit. on p. 19).

Ho, C. Q., Hensher, D. A., Mulley, C., & Wong, Y. Z. (2018). Potential uptake and willingness-to-pay for mobility as a service (maas): A stated choice study. *Transportation Research Part A: Policy and Practice*, *117*, 302–318 (cit. on p. 29).

Hoerler, R., Stünzi, A., Patt, A., & Del Duce, A. (2020). What are the factors and needs promoting mobility-as-a-service? findings from the swiss household energy demand survey (sheds). *European Transport Research Review*, *12*, 1–16 (cit. on p. 19).

Horni, A., Nagel, K., & Axhausen, K. W. Introducing matsim. In: In *The multi-agent transport simulation matsim*. Ubiquity Press, 2016, pp. 3–7 (cit. on p. 34).

iMove Australia. (2020). Mobility as a service: Progress and new insights from an australian trial webinar. https://imoveaustralia.com/news-articles/personal-public-mobility/sydney-maas-trial-is-midway-through/f. (Cit. on p. 41)

Jelbi. (2020). Jelbi. one for all. https://www.jelbi.de/en/home/. (Cit. on p. 29)
Jiang, Z., Lei, C., & Ouyang, Y. (2020). Optimal investment and management of shared bikes in a competitive market. *Transportation Research Part B: Methodological*, *135*, 143–155 (cit. on p. 63).
Jittrapirom, P., Caiati, V., Feneri, A.-M., Ebrahimigharehbaghi, S., González, M. J. A., & Narayan, J. Mobility as a service: A critical review of definitions, assessments of schemes, and key challenges. In: 2017 (cit. on p. 26).
Jittrapirom, P., Marchau, V., van der Heijden, R., & Meurs, H. (2020). Future implementation of mobility as a service (maas): Results of an international delphi study. *Travel Behaviour and Society*, *21*, 281–294 (cit. on p. 28).
Kamargianni, M. (2020). Outcomes, challenges, lessons learned. http://www.maas4eu.eu/wp-content/uploads/2020/11/09_MaaS4EU-Final-Event-Presentation-Outcomes-Challenges-and-Lessons-Learned.pdf. (Cit. on p. 41)
Kamargianni, M., Li, W., Matyas, M., & Schäfer, A. W. (2016). A critical review of new mobility services for urban transport. *Transportation research procedia*, *14*, 3294–3303 (cit. on p. 30).
Kamargianni, M., & Matyas, M. The business ecosystem of mobility-as-a-service. In: 2017 (cit. on pp. 19, 26–28).
Kamargianni, M., Yfantis, L., Muscat, J., Azevedo, C. L., & Ben-Akiva, M. Incorporating the mobility as a service concept into transport modelling and simulation frameworks. In: Transportation Research Board (TRB) 98th Annual Meeting 2019. Washington D.C., USA, 2019 (cit. on p. 37).
Karamardian, S. (1969). The nonlinear complementarity problem with applications, part 1. *Journal of Optimization Theory and Applications*, *4*, 87–98 (cit. on p. 57).
Karlsson, I., Sochor, J., & Strömberg, H. (2016). Developing the 'service' in mobility as a service: Experiences from a field trial of an innovative travel brokerage. *Transportation research procedia*, *14*, 3265–3273 (cit. on p. 40).
Karlsson, I., Mukhtar-Landgren, D., Smith, G., Koglin, T., Kronsell, A., Lund, E., Sarasini, S., & Sochor, J. (2020). Development and implementation of mobility-as-a-service – a qualitative study of barriers and enabling factors [Developments in Mobility as a Service (MaaS) and Intelligent Mobility]. *Transportation Research Part A: Policy and Practice*, *131*, 283–295 (cit. on p. 27).
Khobotov, E. (1987). Modification of the extra-gradient method for solving variational inequalities and certain optimization problems. *USSR Computational Mathematics and Mathematical Physics*, *27*(5), 120–127 (cit. on p. 81).
Koh, A., & Shepherd, S. (2010). Tolling, collusion and equilibrium problems with equilibrium constraints. *European Transport Trasporti Europei*, 3–22 (cit. on p. 64).
Koh, A., Shepherd, S., & Watling, D. (2013). Competition between two cities using cordon tolls: An exploration of response surfaces and possibilities for collusion. *Transportmetrica A: Transport Science*, *9*(10), 896–924 (cit. on p. 64).
Kooti, F., Grbovic, M., Aiello, L. M., Djuric, N., Radosavljevic, V., & Lerman, K. Analyzing uber's ride-sharing economy. In: In *Proceedings of the 26th international conference on world wide web companion*. 2017, 574–582 (cit. on p. 15).
Korpelevich, G. M. The extragradient method for finding saddle points and other problems. In: 1976 (cit. on pp. 80, 81).

Koutsoupias, E., & Papadimitriou, C. Worst-case equilibria. In: In *Stacs*. *99*. Springer. 1999, 404–413 (cit. on p. 124).

Krylatov, A., Raevskaya, A., & Ageev, P. (2023). Optimization strategies for the bilevel network design problem with affine cost functions. *Annals of Mathematics and Artificial Intelligence*, *91*(2-3), 329–347 (cit. on p. 60).

König, D., Eckhardt, J., Aapaoja, A., Sochor, J., & Karlsson, M. (2016). Deliverable 3: Business and operator models for mobility as a service (maas). maasifie project funded by cedr. (cit. on pp. 26, 28, 35).

Labbé, M., & Marcotte, P. (2021). Bilevel network design. *Network Design with Applications to Transportation and Logistics*, 255–281 (cit. on p. 60).

Leyffer, S., & Munson, T. (2010). Solving multi-leader–common-follower games. *Optimisation Methods & Software*, *25*(4), 601–623 (cit. on p. 117).

Li, J., Yu, J., Wang, Q., Liu, B., Wang, Z., & Nie, Y. M. (2022). Differentiable bilevel programming for stackelberg congestion games. *arXiv preprint arXiv:2209.07618* (cit. on p. 141).

Li, Q., & Liao, F. (2020). Incorporating vehicle self-relocations and traveler activity chains in a bi-level model of optimal deployment of shared autonomous vehicles. *Transportation Research Part B: Methodological*, *140*, 151–175 (cit. on p. 60).

Li, S. (2020). Pilot 2 manchester. http://www.maas4eu.eu/wp-content/uploads/2020/11/05_MaaS4EU-Final-Event-Manchester-Pilot.pdf. (Cit. on p. 41)

Liao, F., Arentze, T. A., & Timmermans, H. J. P. (2010). Supernetwork approach for multimodal and multiactivity travel planning. *Transportation Research Record*, *2175*, 38 –46 (cit. on p. 47).

Liu, P., Liao, F., Huang, H.-J., & Timmermans, H. (2015a). Dynamic activity-travel assignment in multi-state supernetworks [21st International Symposium on Transportation and Traffic Theory Kobe, Japan, 5-7 August, 2015]. *Transportation Research Procedia*, *7*, 24–43 (cit. on p. 47).

Liu, P., Liao, F., Tian, Q., Huang, H., & Timmermans, H. (2020). Day-to-day needs-based activity-travel dynamics and equilibria in multi-state supernetworks. *Transportation Research Part B: Methodological* (cit. on p. 47).

Liu, Y., Zhu, N., & Ma, S. (2015b). Simultaneous optimization of transit network and public bicycle station network. *Journal of Central South University*, *22*, 1574–1584 (cit. on p. 45).

Liu, Z., & Nagurney, A. (2007). Financial networks with intermediation and transportation network equilibria: A supernetwork equivalence and reinterpretation of the equilibrium conditions with computations. *Computational Management Science*, *4*, 243–281 (cit. on p. 46).

Lo, H. K., Yip, C. W., & Wan, Q. Modeling competitive multi-modal transit services. In: 2002 (cit. on p. 60).

Luo, Z.-Q. T., Pang, J. S., & Ralph, D. Mathematical programs with equilibrium constraints. In: 1996 (cit. on pp. 58, 61, 63, 99–101).

MaasAlliance. (2018). Mobility as a service: Positioning scotland for an emerging global market. https://maas-alliance.eu/wp-content/uploads/sites/7/2018/02/MaaS-Positioning-Scotland-for-an-Emerging-Global-Market_PUBLIC.pdf. (Cit. on p. 40)

MaaSAlliance. (2019a). Maas of the month: Enabling smart. mobility in stockholm. https://maas-alliance.eu/wp-content/uploads/sites/7/2019/10/MaaS-of-the-Month-Fluidtime-final.pdf. (Cit. on p. 40)

MaaSAlliance. (2019b). Maas of the month: Supporting maas with pilots. https://maas-alliance.eu/wp-content/uploads/sites/7/2019/08/MaaS-of-the-Month-Supporting-MaaS-with-pilots.pdf. (Cit. on p. 42)

Maerivoet, S., & De Moor, B. (2000). Transportation planning and traffic flow models". *Applied Mathematics*, *60*(3), 916–938 (cit. on p. 84).

Marcotte, P. (1991). Application of khobotov's algorithm to variational inequalities and network equilibrium problems. *INFOR: Information Systems and Operational Research*, *29*(4), 258–270 (cit. on p. 81).

MATLAB. (2023). Constrained nonlinear optimization algorithms. https://www.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html. (Cit. on p. 101)

Matyas, M., & Kamargianni, M. A stated preference experiments for mobility-as-a-service plans. In: In *2017 5th ieee international conference on models and technologies for intelligent transportation systems (mt-its)*. IEEE. 2017, 738–743 (cit. on p. 29).

Meurs, H., Sharmeen, F., Marchau, V., & van der Heijden, R. (2020). Organizing integrated services in mobility-as-a-service systems: Principles of alliance formation applied to a maas-pilot in the netherlands [Developments in Mobility as a Service (MaaS) and Intelligent Mobility]. *Transportation Research Part A: Policy and Practice*, *131*, 178–195 (cit. on p. 28).

Meurs, H., & Timmermans, H. J. P. Mobility as a service as a multi-sided market: Challenges for modeling. In: 2017 (cit. on p. 35).

Michelot, C. (1986). A finite algorithm for finding the projection of a point onto the canonical simplex of r n. *Journal of Optimization Theory and Applications*, *50*, 195–200 (cit. on p. 82).

Mitropoulos, L., Kortsari, A., Mizaras, V., & Ayfantopoulou, G. (2023). Mobility as a service (maas) planning and implementation: Challenges and lessons learned. *Future Transportation*, *3*, 498–518 (cit. on p. 42).

Montella, B., Gallo, M., & D'Acierno, L. (2000). Multimodal network design problems. *WIT Transactions on the Built Environment*, *3*, 405–414 (cit. on pp. 44, 45).

Moovit. (2022). City of tampa launches mobility as a service (maas) app. https://moovit.com/press-releases/moovit-tampa-partnership/. (Cit. on p. 42)

Mounce, R., & Carey, M. (2015). On the convergence of the method of successive averages for calculating equilibrium in traffic networks. *Transportation science*, *49*(3), 535–542 (cit. on p. 52).

Musolino, G., Rindone, C., Vitale, A., & Vitetta, A. (2023). Pilot survey of passengers' preferences in mobility as a service (maas) scenarios: A case study. *Transportation Research Procedia*, *69*, 328–335 (cit. on p. 29).

MyCorridor. (2020). Final event presentations. http://www.mycorridor.eu/pilots. (Cit. on p. 41)

Nagurney, A. (1998). *Network economics: A variational inequality approach* (Vol. 10). Springer Science & Business Media. (Cit. on pp. 52, 53, 55, 80).

Nagurney, A. (2000). A multiclass, multicriteria traffic network equilibrium model. *Mathematical and Computer Modelling*, *32*, 393–411 (cit. on pp. 54, 55, 76, 79, 80).

Nagurney, A., Cruz, J., Dong, J., & Zhang, D. (2005). Supply chain networks, electronic commerce, and supply side and demand side risk. *European journal of operational research*, *164*(1), 120–142 (cit. on p. 46).

Nagurney, A., & Toyasaki, F. (2003). Supply chain supernetworks and environmental criteria. *Transportation Research Part D: Transport and Environment*, *8*(3), 185–213 (cit. on p. 46).

Nair, R., & Miller-Hooks, E. (2014). Equilibrium network design of shared-vehicle systems. *Eur. J. Oper. Res.*, *235*, 47–61 (cit. on p. 62).

Nash, J. F. (1951). Non-cooperative games. *Classics in Game Theory* (cit. on pp. 57, 63).

Nasuha, N., & Rohani, M. (2018). Overview of application of traffic simulation model. *MATEC Web of Conferences*, *150*, 03006 (cit. on p. 34).

Navigogo. (2018). Navigogo: Scotland's first maas platform. https://www.the-espgroup.com/project/navigogo/. (Cit. on p. 40)

Nguyen, T. K., Hoang, N. H., & Vu, H. L. (2022). A unified activity-based framework for one-way car-sharing services in multi-modal transportation networks. *Transportation Research Part E: Logistics and Transportation Review* (cit. on p. 62).

Noor, M. A. (1987). On the nonlinear complementarity problem. *Journal of mathematical analysis and applications*, *123*(2), 455–460 (cit. on p. 56).

O'Brien, O., Rabello, R., Chou, S., Benicchio, T., & DeMaio, P. (2023). The meddin bike - sharing the world map. https://bikesharingworldmap.com/index.php#/all/2.6/0/51.5/. (Cit. on p. 17)

Pagoni, I., Gatto, M., Tsouros, I., Tsirimpa, A., Polydoropoulou, A., Galli, G., & Stefanelli, T. (2022). Mobility-as-a-service: Insights to policymakers and prospective maas operators. *Transportation Letters*, *14*(4), 356–364 (cit. on p. 27).

Panicucci, B., Pappalardo, M., & Passacantando, M. (2007). A path-based double projection method for solving the asymmetric traffic network equilibrium problem. *Optimization Letters*, *1*, 171–185 (cit. on p. 82).

Pantelidis, T. P., Chow, J. Y., & Rasulkhani, S. (2020). A many-to-many assignment game and stable outcome algorithm to evaluate collaborative mobility-as-a-service platforms. *Transportation Research Part B: Methodological*, *140*, 79–100 (cit. on p. 37).

Papadimitriou, C. Algorithms, games, and the internet. In: In *Proceedings of the thirty-third annual acm symposium on theory of computing*. 2001, 749–753 (cit. on p. 124).

Parliament, E. (2023). Reducing carbon emissions: Eu targets and policies. https://www.europarl.europa.eu/news/en/headlines/society/20180305STO99003/reducing-carbon-emissions-eu-targets-and-policies#:~:text=EU%20climate%20change%20goals%20and,neutrality%20by%202050%20legally%20binding.. (Cit. on p. 13)

Perrone, L. F., & Fujimoto, R. M. A framework for new generation transportation simulation. In: 2006. https://api.semanticscholar.org/CorpusID:14554125 (cit. on p. 33)

Pham, H. D., Shimizu, T., & Nguyen, T. V. (2021). A Literature Review on Interactions Between Stakeholders Through Accessibility Indicators Under Mobility as a Service Context. *International Journal of Intelligent Transportation Systems Research volume*, *19*(2), 468–476 (cit. on p. 37).

Polydoropoulou, A., Pagoni, I., Tsirimpa, A., Roumboutsos, A., Kamargianni, M., & Tsouros, I. (2020). Prototype business models for mobility-as-a-service [Developments in Mobility as a Service (MaaS) and Intelligent Mobility].

Transportation Research Part A: Policy and Practice, *131*, 149–162 (cit. on pp. 28, 35).
Pozo, D., Sauma, E., & Contreras, J. (2017). Basic theoretical foundations and insights on bilevel models and their applications to power systems. *Annals of Operations Research*, *254*, 303–334 (cit. on pp. 57, 58).
Primeaux, P. D., & Stieber, J. (1994). Profit maximization: The ethical mandate of business. *Journal of Business Ethics*, *13*, 287–294 (cit. on p. 72).
Punzo, G., Panarello, D., & Castellano, R. (2022). Sustainable urban mobility: Evidence from three developed european countries. *Quality & Quantity*, *56*(5), 3135–3157 (cit. on p. 12).
Åsa Ström Hildestrand. (2018). Minrejseplan and plustur – two ways to maximise mobility in rural, northern denmark. https://www.mambaproject.eu/wp-content/uploads/2018/07/WP2_good-practice-cases_-MinRejseplan-and-Plustur-FINAL.pdf. (Cit. on p. 40)
Rashidi, E., Parsafard, M., Medal, H., & Li, X. (2016). Optimal traffic calming: A mixed-integer bi-level programming model for locating sidewalks and crosswalks in a multimodal transportation network to maximize pedestrians' safety and network usability. *Transportation Research Part E: Logistics and Transportation Review*, *91*, 33–50 (cit. on p. 60).
Reck, D., & Axhausen, K. Multimodal transportation plans: Empirical evidence on uptake, usage and behavioral implications from the augsburg maas trial. In: 2020 (cit. on p. 41).
Saw, K., Katti, B. K., & Joshi, G. J. Literature review of traffic assignment: Static and dynamic. In: 2015 (cit. on p. 48).
Scheffer, A., Connors, R., & Viti, F. (2021). Trip chaining impact on within-day mode choice dynamics: Evidences from a multi-day travel survey. *Transportation Research Procedia*, *52*, 684–691 (cit. on p. 47).
Schneider, F., Ton, D., Zomer, L.-B., Daamen, W., Duives, D., Hoogendoorn-Lanser, S., & Hoogendoorn, S. (2021). Trip chain complexity: A comparison among latent classes of daily mobility patterns. *Transportation*, *48*, 953–975 (cit. on p. 12).
Shaheen, S. A., & Chan, N. D. Mobility and the sharing economy: Potential to overcome first- and last-mile public transit connections. In: 2016 (cit. on p. 18).
Shaheen, S. A., Sperling, D., & Wagner, C. (1999). A short history of carsharing in the 90's (cit. on p. 14).
Sheffi, Y. (1985). *Urban transportation networks* (Vol. 6). Prentice-Hall, Englewood Cliffs, NJ. (Cit. on pp. 45, 50, 51).
Sheffi, Y., & Daganzo, C. F. (1978). Hypernetworks and supply-demand equilibrium obtained with disaggregate demand models. *Transportation Research Record*, *673*, 113–121 (cit. on pp. 45, 46).
Siklósi, V. (2020). Pilot 1 budapest. http://www.maas4eu.eu/wp-content/uploads/2020/11/03_MaaS4EU-Final-Event-Pilot-Intro.pdf. (Cit. on p. 41)
SimMobility. (2023). Mobility futures collaborative. https://mfc.mit.edu/simmobility. (Cit. on p. 37)
Simon-Kucher. (2018). Mobility as a service: Germany's first mobility flat rate. https://www.simon-kucher.com/en/blog/mobility-service-germanys-first-mobility-flat-rate. (Cit. on p. 41)

Singh, S. S. (2012). *Projection based algorithms for variational inequalities* (Doctoral dissertation). The University of North Carolina at Chapel Hill. (Cit. on p. 80).

Sinha, A., Malo, P., & Deb, K. (2017). A review on bilevel optimization: From classical to evolutionary approaches and applications. *IEEE Transactions on Evolutionary Computation*, *22*, 276–295 (cit. on pp. 58–60).

Smile. (n.d.). Smile-einfach mobil. https://smile-einfachmobil.at/index_en.html. (Cit. on p. 40)

Smith, G., & Hensher, D. A. (2020). Towards a framework for mobility-as-a-service policies. *Transport Policy*, *89*, 54–65 (cit. on p. 27).

Smith, M. J. (1979). The existence, uniqueness and stability of traffic equilibria. *Transportation Research Part B-methodological*, *13*, 295–304 (cit. on pp. 49, 52).

Sochor, J., Strömberg, H., & Karlsson, I. (2014). Travelers' motives for adoprting a new,innovative travel service: Insight from the ubigo field operational test in gothenburg, sweden. *21st World Congress on Intelligent Transportation Systems, Detroit* (cit. on p. 40).

Sochor, J., Strömberg, H., & Karlsson, I. (2015a). Implementing mobility as a service: Challenges in integrating user, commercial, and societal perspectives. *Transportation Research Record*, *2536(1)*, 1–9 (cit. on p. 40).

Sochor, J., Arby, H., Karlsson, M., & Sarasini, S. (2017). A topological approach to mobility as a service: A proposed tool for understanding requirements and effects, and for aiding the integration of societal goals. *Research in transportation business and management*, *27*, 3–14 (cit. on p. 30).

Sochor, J., Karlsson, I. M., & Strömberg, H. (2016). Trying out mobility as a service: Experiences from a field trial and implications for understanding demand. *Transportation Research Record*, *2542*(1), 57–64 (cit. on p. 29).

Sochor, J., Strömberg, H., & Karlsson, I. M. The added value of a new, innovative travel service: Insights from the ubigo field operational test in gothenburg, sweden. In: In *Internet of things. iot infrastructures: First international summit, iot360 2014, rome, italy, october 27-28, 2014, revised selected papers, part ii 1*. Springer. 2015, 169–175 (cit. on p. 32).

Sprumont, F., Scheffer, A. H. M., Caruso, G., Cornélis, E., & Viti, F. (2022). Quantifying the relation between activity pattern complexity and car use using a partial least square structural equation model. *Sustainability* (cit. on p. 75).

Stackelberg, H. V. (1952). *The theory of market economy*. Oxford University Press, Oxford. (Cit. on pp. 57, 63).

Steffensen, S., & Bittner, M. (2014). Relaxation approach for equilibrium problems with equilibrium constraints. *Computers Operations Research*, *41*, 333–345 (cit. on p. 63).

Storme, T., Vos, J. D., Paepe, L. D., & Witlox, F. (2020). Limitations to the car-substitution effect of maas. findings from a belgian pilot study. *Transportation Research Part A-policy and Practice*, *131*, 196–205 (cit. on pp. 30, 40).

Susilo, Y. O., & Axhausen, K. W. (2014). Repetitions in individual daily activity–travel–location patterns: A study using the herfindahl–hirschman index. *Transportation*, *41*, 995–1011 (cit. on p. 75).

Szeto, W. Y., & Jiang, Y. (2014). Transit assignment: Approach-based formulation, extragradient method, and paradox. *Transportation Research Part B-methodological*, *62*, 51–76 (cit. on p. 80).

Tirachini, A., Hensher, D. A., & Rose, J. M. (2014). Multimodal pricing and optimal design of urban public transport: The interplay between traffic congestion and bus crowding. *Transportation Research Part B-methodological*, *61*, 33–54 (cit. on p. 45).
TomTom. (2023). How did daily travel affect co2 emissions? https://www.tomtom.com/traffic-index/. (Cit. on pp. 12, 13)
Trivector. (2020). Evaluation of ubigo stockholm. civitas eccentric measure 3.5. https://civitas.eu. (Cit. on p. 40)
Tsouros, I., Tsirimpa, A., Pagoni, I., & Polydoropoulou, A. (2021). Maas users: Who they are and how much they are willing-to-pay. *Transportation Research Part A: Policy and Practice*, *148*, 470–480 (cit. on pp. 29, 31).
Uber. (2023). Earn money by driving or get a ride now. https://www.uber.com. (Cit. on p. 15)
van Nes, R. Design of multimodal transport networks: A hierarchical approach. In: 2002 (cit. on p. 45).
van den Berg, V. A., Meurs, H., & Verhoef, E. T. (2022). Business models for mobility as an service (maas). *Transportation Research Part B: Methodological*, *157*, 203–229 (cit. on p. 38).
Vij, A., & Dühr, S. (2022). The commercial viability of mobility-as-a-service (maas): What's in it for existing transport operators, and why should governments intervene? *Transport Reviews*, *42*(5), 695–716 (cit. on p. 27).
Von Neumann, J., & Morgenstern, O. (1947). Theory of games and economic behavior, 2nd rev (cit. on p. 57).
W Axhausen, K., Horni, A., & Nagel, K. (2016). *The multi-agent transport simulation matsim*. Ubiquity Press. (Cit. on p. 34).
Wan, Q. K., & Lo, H. K. (2009). Congested multimodal transit network design. *Public Transport*, *1*, 233–251 (cit. on p. 60).
Wang, G., Li, Y., Xu, M., & Gao, Z. (2021). Operating a public–private mixed road network via determining tradable credits and road tolls: An equilibrium problem with equilibrium constraints approach. *International Journal of Sustainable Transportation*, *15*(2), 87–96 (cit. on p. 64).
Wardrop, J. G. (1952). Road paper. some theoretical aspects of road traffic research. *Proceedings of the institution of civil engineers*, *1*(3), 325–362 (cit. on pp. 48, 49, 69, 79).
Watling, D. (2006). User equilibrium traffic network assignment with stochastic travel times and late arrival penalty. *European journal of operational research*, *175*(3), 1539–1556 (cit. on p. 79).
Watling, D. P., Shepherd, S. P., & Koh, A. (2015). Cordon toll competition in a network of two cities: Formulation and sensitivity to traveller route and demand responses. *Transportation Research Part B-methodological*, *76*, 93–116 (cit. on pp. 64, 116).
Whim. (2023). Whim. how to get there. https://whimapp.com/. (Cit. on pp. 26, 29, 32)
Wong, Y., & Hensher, D. (2021). Delivering mobility as a service (maas) through a broker/aggregator business model. *Transportation*, *48* (cit. on pp. 32, 35).
Wong, Y. Z., Hensher, D. A., & Mulley, C. (2020). Mobility as a service (maas): Charting a future context [Developments in Mobility as a Service (MaaS) and Intelligent Mobility]. *Transportation Research Part A: Policy and Practice*, *131*, 5–19 (cit. on pp. 14, 27, 28).

Wong, Y. Z., Mulley, C., & Prize, A. W. Stated preference design for mobility as a service (maas) broker/aggregator contracts. In: 2018 (cit. on p. 28).

Wright, S., Nocedal, J., et al. (1999). Numerical optimization. *Springer Science*, *35*(67-68), 7 (cit. on p. 101).

Xi, H., Aussel, D., Liu, W., Waller, S., & Rey, D. (2022). Single-leader multi-follower games for the regulation of two-sided mobility-as-a-service markets. *European Journal of Operational Research* (cit. on p. 36).

Yang, H., Xiao, F., & Huang, H.-J. (2009). Private road competition and equilibrium with traffic equilibrium constraints. *Journal of Advanced Transportation*, *43*, 21 –45 (cit. on p. 64).

Yang, S., Wu, J., Sun, H., Qu, Y., & Wang, D. Z. W. (2022). Integrated optimization of pricing and relocation in the competitive carsharing market: A multi-leader-follower game model. *Transportation Research Part C: Emerging Technologies* (cit. on p. 63).

Ye, J., Jiang, Y., Chen, J., Liu, Z., & Guo, R. (2021). Joint optimisation of transfer location and capacity for a capacitated multimodal transport network with elastic demand: A bi-level programming model and paradoxes. *Transportation Research Part E: Logistics and Transportation Review*, *156*, 102540 (cit. on p. 61).

Yong, L. Nonlinear complementarity problem and solution methods. In: In *Artificial intelligence and computational intelligence: International conference, aici 2010, sanya, china, october 23-24, 2010, proceedings, part i 2*. Springer. 2010, 461–469 (cit. on p. 56).

Yumuv. (2020a). Empirical use and impact analysis of maas. https://gis.ethz.ch/en/projects/finished-projects.html. (Cit. on p. 42)

Yumuv. (2020b). Urban modes of transport in a bundle. https://yumuv.ch/en. (Cit. on p. 42)

Zankl, C., Kranzinger, S., Research), I. Z. S., (CERTH/HIT), K. T., & (RSM), G. T. (2020). Mobility as a service in a multimodal european cross-border corridor (mycorridor). deliverable d6.2: Pilot results consolidation. http://www.mycorridor.eu/project-library/. (Cit. on p. 41)

Zhang, L., Yang, H., Wu, D., & Wang, D. (2014). Solving a discrete multimodal transportation network design problem. *Transportation Research Part C: Emerging Technologies*, *49*, 73–86 (cit. on p. 62).

Zhou, J., Lam, W. H., & Heydecker, B. G. (2005). The generalized nash equilibrium model for oligopolistic transit market with elastic demand. *Transportation Research Part B: Methodological*, *39*(6), 519–544 (cit. on p. 63).

Zhou, Y., Li, L., & Zhang, Y. (2023). Location of transit-oriented development stations based on multimodal network equilibrium: Bi-level programming and paradoxes. *Transportation Research Part A: Policy and Practice*, *174*, 103729 (cit. on p. 61).

Zorn, E. (2020). Pilot 3 luxembourg. http://www.maas4eu.eu/wp-content/uploads/2020/11/07_MaaS4EU-Final-Event-Luxembourg-Pilot.pdf. (Cit. on p. 41)