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Deregulation and Efficiency in Slot-Constrained Airports

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Deregulation and Efficiency in Slot-Constrained Airports*

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Abstract

We investigate the presence of inefficiency in slot allocation when a coordinator allocates slots on destination markets served by monopoly and duopoly airlines, and the number of available peak-time slots is constrained by airport capacity. When an airport maintains regulated per-passenger fees, we observe the emergence of allocative inefficiency. Conversely, in scenarios where an airport has the autonomy to set fees, we find that, in line with empirical evidence, fee deregulation resolves these allocative inefficiencies by increasing per-passenger fees. However, the improvement in allocation efficiency may be counterbalanced by the rise in fees, potentially impacting overall welfare.

Keywords: Slot allocation, Endogenous fee, Airport capacity.
JEL codes: R41,H21,H23.

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1 Introduction

In the past decades, growth in air traffic has outstripped the development of runways and other passenger-handling infrastructures. As a result, many airports are experiencing critical shortages of infrastructure capacity, particularly during their demand peak times. As a consequence, air transport authorities must be increasingly concerned with the allocation of airport slots (or permits) that are attributed to airlines for the access to airport infrastructure necessary for the departure or landing of their fleets within specific time frames. A proper evaluation of the industry's conduct about slot allocation seems very much welcome.

In addition to that, the recent trends in airport regulation are moving towards less governmental involvement. First, many traditionally public-owned airports have undergone privatization. Starting with the privatization of airports in the UK in the late 1980s, more and more airports have been (either fully or partially) privatized worldwide (e.g. Oum *et al.* 2004, Winston, C., Ginés 2009). According to IATA (2017), the share of fully privately owned airports in Europe increased from 9% to 16% between 2010 and 2016 while the share of mixed ownership models increased from 13% to 25% over the same period.[1] As the ownership of airports changes from public to private, the objective of airports is expected to shift from social benefits to profit maximization.[2]

Second, there are calls for the dismantlement of regulation and less stringent price monitoring. As pointed out in ACI (2017), "The role of a regulator and its oversight function is to monitor and ensure there is no significant abuse of market power. Strict forms of price regulation result in allocative inefficiencies which affect economic incentives adversely." For instance, some airport authorities determine a ceiling on the increase in passenger revenues obtained from basic airport services.[3]

To this matter, the most relevant regulatory point is the management of per-passenger fees. Indeed, the airports' income in the past years was mainly based on passenger charges (i.e. passenger service charges, security, and transfer charges), rather than fees that apply directly to aircraft operators (ICAO, 2013).[4] In Europe, for example, airport passenger charges paid on the average airfare to fly from European airports more than doubled between 2006 and 2016 (ICAO, 2013, IATA, 2017). Given the growing importance of the per-passenger revenue, the analysis of the impact of the deregulation on this revenue on the overall airport slot strategy

[1] Worldwide, among the 100 busiest airports for passenger throughput, 46% have private sector participation. And 41% of global airport traffic is handled by airports that are managed and/or financed by private stakeholders (IATA, 2017).

[2] In addition, many public airports are self-financed and operated under binding budget constraints, which entices them to implement profit-oriented activities.

[3] This then defines the maximum annual revenue per passenger for each year in the regulatory period (e.g., Airports Regulation Document 2017-2021, 2017).

[4] The passenger-based revenues represent 63% of total aeronautical income according to ICAO (2013). In this regard, see Zhang (2012) and Czerny *et al.* (2017) for discussions about airport improvement fees, which are used to charge passengers for airport infrastructure development and/or debt repayment, and are becoming a more important revenue source for airports.

seems highly policy relevant.

The purpose of the present paper is to revisit the slot allocation problem in a setting where airport authorities liberalize the level of charges to passengers. The key assumption is that an airport can generate per-passenger revenue, which reflects real world practice.[5] Traditionally airports levied a single uniform Departing Passenger Charge (DPC), payable by the airline, which was perceived to meet the cost of providing terminal services.[6] Airport liberalization thus brings a question about the size and the effect of the markups that unregulated airports charge to passengers above their costs.

Given the problem at hand, the purpose is to represent the functioning of airports listed as "Level 2" or "Level 3" in the Worldwide Airport Slot Guidelines (WASG, 2022). These are airports whose capacity infrastructure is not generally adequate to meet the demands of airport users at peak times. In level 2 or 3 airports, slot allocation is managed by slot "facilitators" or "coordinators", respectively.[7] The coordinators/facilitators are organizations responsible for slot allocation in line with the national slot regulations. In their operating process, these organizations must follow the instructions described in the Worldwide Airport Slot Guidelines and "optimize the benefits of consumers, taking into account the interests of airports and airlines" (WASG, 2022, p. 8). However, these organizations are predominantly operated and funded by airports whose revenues increase with the number of passengers. In the EU, the organization structure of slot coordinator implies a strong influence of (big) airports: for example, the Spanish coordinator is part of AENA, a public industrial entity that is the owner and manager of almost all Spanish airports (Ranieri *et al.*, 2013).[8] Our modeling of slot allocation mirrors the fact that, twice a year, the International Air Transport Association (IATA) organizes scheduling conferences where slot coordinators and airlines collaborate to enhance the initial slot assignments for the upcoming season (Lenoir, 2016).

Our framework is based on Picard *et al.* (2019), where a capacity-constrained airport serves destinations with duopoly airlines and establishes its slot allocation. Consistent with empirical evidence,[9] their result shows the existence of slot allocation inefficiency. The present paper extends this analysis to a situation where monopoly and duopoly airlines serve airport destinations, a slot coordinator performs the slot allocation, and airports have the freedom to

[5] Airports are allowed to levy a uniform per-passenger fee for flight activities. Regarding passenger service charges, ICAO (2012) recommends that "these (passenger service) charges should be levied through the aircraft operators where practicable. The need for consultations between airport entities and users at the local level with a view to alleviating collection problems should be emphasized."

[6] The charge covers all the terminal infrastructure, provision of check-in desks, baggage system and security screening. The DPC can be split into separate charges for passengers (mainly basic infrastructure and security screening), a fee per bag, rental of the check-in desk, self-service check-in kiosks, etc. All these charges are levied on the airline.

[7] The roles of the facilitator at a Level 2 airport and the coordinator in a Level 3 airport are administratively similar, but are governed by different principles for managing scheduling processes.

[8] Slot coordinators operating within the EU and the UK include ACL, ACS, AENA, Assoclearance, COHOR, FHKD, SACN and SCA (European Commission, 2011).

[9] See, among others, Zografos *et al.* (2013), Katsaros and Psaraki (2012) and Airports Council International Europe (2009).

set their passenger fees.

More specifically, we assume that slot allocation is handled by a slot coordinator/facilitator whose aim is to "optimize the benefits of consumers, taking into account the interests airports and airlines" (WASG, 2022, p.8). We consider slot allocation among destination markets served by both monopolies and duopolies.[10] Then, we analyze the case in which the airport is "unregulated", in the sense that it is free to set per passenger fees without any regulatory constraint.[11] Our findings confirm the existence of allocative inefficiency in a more general setting than Picard *et al.* (2019). In addition, we show that allocative inefficiency resulting from unused peak slots would vanish at an unregulated private airport. The distortion of price sets downward pressure on the distortion of allocative efficiency, and no allocative inefficiency appears. Interestingly, in an unregulated environment, the airport fee would never be set to a too low level by a private airport. These results are consistent with empirical regularities. For instance, Bel and Fageda (2010) find that the airports controlled by private companies that are not subject to regulation fix higher prices than regulated airports.

Next, the paper investigates the effect of liberalizing per-passenger fees in terms of social welfare. As said above, liberalization solves allocation inefficiency and in turn has a positive effect on welfare. By contrast, the increase in per-passenger fees decreases passengers' surplus for given slot allocation. It follows that, if the airport profit maximizing per passenger fees are very high, the negative effect on welfare offsets the positive effect of slot reallocation. The latter scenario maintains the usual reserves against the liberalization of fees even if the use of resources is more efficient.

Our model abstracts away from the practice of airport-slot grandfathering. While grandfathering is common at many airports, it is not endorsed by organizations such as IATA for Level 2 and 3 airports where slot utilization is constrained. For instance, the IATA's recent Worldwide Airport Slot Guidelines explicitly state: "the concepts of historic precedence and series of slots do not apply at Level 2 airports" (WASG 2022, p.30). It however leaves room for grandfathering by suggesting later on to prioritize "approved services that plan to operate unchanged from the previous equivalent season" (ibidem). Thus, in the aviation industry, slot coordinators apply grandfathering by giving a priority only to airlines on the slots that they occupy at least 80% of the time during the previous season.

In practice though, slot coordinators possess a definite degree of flexibility in their decision-making. To comply with the capacity declared by airports, they are able to displace or reject slot requests in accordance with the guidelines defined by the WASG. As a case in point, approximately 10% of the slots end up being either relinquished or canceled by the conclusion of the season (Odoni, 2020). Similarly, 32% of slots were free to be reshuffled by the slot coordinator in CDG airport in 2018 (Pouget *et al.*, 2023). Hence, the priority of historical

[10] By contrast Picard *et al.* (2019) assumed the airport serving either only monopoly or only duopoly airlines.

[11] Throughout the paper, we will use the term *regulated* to the case where the passenger-based airport revenue is determined exogenously by policymakers throughout, and *unregulated* to the case where this revenue is determined by the airport.

slots is applicable only in the short term and does not equate to a 'right' conferred by a formal grandfathering system. In the long run, all slots get reallocated in some or other ways according to economic interests and technical possibilities. The implications of slot reallocation procedures are therefore important to understand. This paper investigates a long-run situation where slot coordination seeks to augment the number of passengers and therefore impacts airport revenues. This creates incentives for slot discrimination in competitive destinations while airport deregulation may mitigate the associated inefficiencies.

Related literature. The present study, to the best of our knowledge, is the first that combines the analysis of slots and pricing policies, and thus, it is related to both literatures.

In the literature on slot allocation, as well as Picard *et al.* (2019), Barbot (2004) models different slot periods as vertically differentiated products with high or low quality, by letting airlines determine their number of flights. Verhoef (2010) and Brueckner (2009) evaluate the effect of the adoption of a slot allocation in comparison with the alternative policy of congestion pricing. Both contributions show that slot trading or auctioning and the first best congestion pricing give the same level of passenger volume and welfare. Unlike the present contribution, they do not let the airport allocate slots without charges. Verhoef (2010) and Brueckner (2009) are generalized by Basso and Zhang (2010), who introduce airport profits into the analysis. In this case, the adoption of slot allocation or congestion pricing brings about different results.

The literature on airport pricing policies is rich. To cite some relevant contributions, Ivaldi *et al.* (2015) and Martín and Socorro (2009) assume that airports negotiate prices with the airlines and charge them for the use of the aeronautical facilities at the airport, and they charge the passengers through the prices of non-aeronautical facilities. Lin and Zhang (2017) assume private airports levy per-flight charges on hub carriers, which could be either movement-related or weight-related, and per-passenger charges to maximize profits. Czerny (2013) assumes in the area of aeronautical services, the airport is a monopoly provider and charges a price per passenger to airlines. These papers, however, do not discuss the interplay between optimal per-passenger fee choice and slot allocation.

The remainder of this paper is organized as follows. The baseline model is presented in Section 2, while the results are outlined in Section 4. A welfare appraisal is developed in Section 6, and Section 7 develops a numerical example. Section 8 sets forth the conclusion.

2 The Model

We study an airport that offers a continuum of destinations with mass M served by a single airline (monopolies) and a continuum of destinations with mass N served by two airlines. Each destination has a market size $z \in [\underline{z}, \overline{z}]$. Market sizes of monopoly destinations are distributed with c.d.f. F and p.d.f. $f > 0$; those of duopoly market destinations with c.d.f. G and p.d.f. $g > 0$. Accordingly, the mass of destinations with size $\mathrm{d}z$ is given by $Mf(z)\mathrm{d}z$ and $Ng(z)\mathrm{d}z$ in the monopoly and duopoly destinations.

Every destination market is vertically differentiated with respect to peak and offpeak travel. A peak period represents the time window that consists of the most desirable travel times in a day, whilst an offpeak period contains all the rest time intervals. Examples of peak periods are 7:00-9:00 a.m. for a morning peak and 5:00-7:00 p.m. for an afternoon peak.

Under vertical differentiation (Gabszewicz and Thisse, 1979), all potential passengers have preferences for peak load hours. The slot qualities for peak and offpeak slots are given by s_1 and s_0, respectively, where $s_1 > s_0 > 0$. Passengers differ by their peak-travel taste $v \in [0, 1]$, v being uniformly distributed. They are endowed with the utility function, $U_i(v, p_i) = vs_i - p_i$ where p_i is the ticket price with $i = 1$ if they fly at peak or $i = 0$ off peak. The peak and offpeak times capacities are denoted as K and L. We consider that the peak period can be running at full capacity while, as in many airports, offpeak period capacity is unconstrained and can accommodate the movements in all monopoly destinations and the double movements in duopoly destinations.[12] This means that $K < M + 2N < L$.

Our analysis studies the behavior of a "slot coordinator" that manages the slot allocations and may be called "coordinator" for the sake of brevity. The airport activity is rewarded by passenger fees $\phi \geq 0$. Passenger fees can be collected directly from passengers or may be collected as charges on aircraft movements that are proportional to aircraft sizes, i.e. the number of passengers. We consider regulated and unregulated airports. For this purpose, we consider that regulation carries over passenger fees so that unregulated airports are authorized to set their passenger fees. For simplicity, the airport and the airlines operate under constant returns to scale. Given the linear demand system (see below), we can normalize the marginal cost to zero without loss of generality. Hence, airline fares and passenger fee can be interpreted as markups.

The timing is as follows. In the first stage, the unregulated airport sets its preferred per-passenger fees. If the airport is regulated, the fee is exogenously given by the regulation authority. In the second stage, the slot coordinator allocates peak and offpeak slots to airlines of the airport. In the third stage, if a destination is served by a single airline, the monopoly operator sets its seat supplies based on the slot allocation. If two airlines serve a destination, they non-cooperatively choose their seat supplies based on the slot allocation. The equilibrium concept is the Cournot-Nash equilibrium. Finally, passengers choose to travel, purchase flight tickets and pursue their travel.

We begin by determining the passenger and airline choices, then discuss the slot coordinator choice and finally the airport choice with and without regulation on the passenger fee.

[12] Evidence of unconstrained slots during offpeak times can be found, among others, in Barnhart *et al.*, (2012) for Newark airport (EWR), Swaroop *et al.* (2012) for several American airports, and Dray (2020) in a study of worldwide airports.

3 Passenger and airline choices

In this section, we study the passengers' seat demand and airline' seat supply. Because destinations are independent for both travellers and airlines, travel demand and airline decisions can be studied separately for each market. We therefore study destinations served by monopolies and duopolies separately.

3.1 Monopoly destination markets

We identify each monopoly airline by its market size z and time period $i \in \{0,1\}$. Given the above preference, at a price $p_i(z)$, the mass of passengers choosing to travel is equal to $q_i(z) = z\left[1 - p_i(z)/s_i\right]$. The inverse demand function is therefore $p_i(q,z) = (1 - q/z)\, s_i$ and, the monopoly airline profit $\pi_i(q,z) = \left[p_i(q,z) - \phi\right] q$. It can readily be shown that the profit maximizing number of seats is equal to $q_i(z) = zq_i$ where

$$q_i = \frac{s_i - \phi}{2s_i}.$$

Because $q_1 > q_0$, the number of passengers is larger at peak time. The resulting travel prices are equal to $p_i(z) = p_i = (s_i + \phi)/2$, which are also larger at peak time. The optimal monopoly profit $\pi_i(z) = z\,(s_i - \phi)^2 / (4s_i)$ is positive for any slot under the assumption $\phi \leq s_0$.

3.2 Duopoly destination markets

Duopoly airlines engage in a seat capacity competition that results in a Cournot-Nash equilibrium (see Picard *et al.*, 2019). Let us consider two airlines a and b flying to a same destination with market size z. For conciseness, we dispense with reference to market size z when it does not lead to confusion.

Same time slot Consider that the two airlines a and b supply q^a and q^b seats in each flight departing in the same slot $i \in \{0,1\}$. Passengers choose to travel only if $s_i v - p \geq 0$. The total travel demand is thus given by $q = z\,(1 - p/s_i)$. This gives the inverse demand function $p\left(q^a, q^b; z\right) = s_i\left[1 - \left(q^a + q^b\right)/z\right]$. Airline a's profits is given by $\pi^a = \left[p\left(q^a, q^b; z\right) - \phi\right] q^a(z)$ while airline b's profit has a symmetric expression. In a Cournot-Nash equilibrium, each airline chooses the number of seats that maximizes its profit, taking as given by the competitor's seat capacity. After establishing and solving the first order conditions, one gets the equilibrium seat capacities $q^a(z) = q^b(z) = zq_{ii}$ where

$$q_{ii} \equiv \frac{s_i - \phi}{3s_i}, \tag{1}$$

and where double subscript $ii \in \{00, 11\}$, indicates whether the two airlines are flying during offpeak (00) or peak (11) slots, respectively (we use two subscripts for duopoly destinations). Quite naturally, seat supplies are proportional to the market size z and fall with passenger fee. Seat supply and profits are positive under the above assumption of low enough passenger fee, $\phi \leq s_0$. The equilibrium prices are equal to $p^a(z) = p^b(z) = p_{ii}$ where $p_{ii} \equiv (s_i + 2\phi)/3$. Since the passenger fee is a cost for airlines, prices are increasing functions of ϕ.

Different time slots Consider now that airlines a and b supply q^a and q^b seats in flights departing respectively in the offpeak and peak slot. In the equilibrium, there exist two ticket prices p^a and p^b such that passengers decide to fly on and off the peak time. Let us say without loss of generality that flight a uses the offpeak slot. The passenger indifferent between flying and staying, is given by a taste parameter $v^a = p^a/s_0$. The passenger indifferent between flying on and off peak has taste parameter given by $v^b = \left(p^b - p^a\right)/(s_1 - s_0)$. Therefore, demands for off and on peak flights are equal to $q^b = z(1 - v^b)$ and $q^a = z(v^b - v^a)$. Plugging the previous values in those expressions gives the inverse demand functions $p^a\left(q^a, q^b\right) = s_0\left[1 - \left(q^a + q^b\right)/z\right]$ and $p^b\left(q^a, q^b\right) = s_1\left[1 - \left(q^a s_0/s_1 + q^b\right)/z\right]$, which gives the profits $\pi^a = \left[p^a\left(q^a, q^b\right) - \phi\right] q^a$ and $\pi^b = \left[p^b\left(q^a, q^b\right) - \phi\right] q^b$. In a Cournot-Nash equilibrium, each airline chooses the aircraft seat capacity that maximizes its profit, taking as a given the competitor's seat capacity. Establishing and solving first order conditions yields the equilibrium seat capacities $q^a(z) = z\ q_{01}$ and $q^b(z) = z\ q_{10}$ where

$$q_{01} \equiv \frac{s_0 s_1 - \phi\left(2s_1 - s_0\right)}{(4s_1 - s_0)s_0} \quad \text{and} \quad q_{10} \equiv \frac{2s_1 - s_0 - \phi}{4s_1 - s_0}, \tag{2}$$

where double the subscripts 01 and 10 denote the respective airlines on offpeak and peak times. Again, seat capacities linearly increase in market size z. As it can be shown that $q_{01} < q_{10}$, offpeak flights supply fewer seats than peak ones. Offpeak flights have also lower price-cost margins and are therefore less profitable. To ensure positive seat supplies and profits, we assume the condition $\phi < \overline{\phi}$ where

$$\overline{\phi} \equiv \frac{s_0 s_1}{2s_1 - s_0}.$$

The seat supplies of the above slot configurations rank as follows: $q_{10} > q_{11} > q_{00} > q_{01}$. Peak flights carry larger numbers of passengers than offpeak ones; this difference is more acute when airlines are allocated to different travel periods. Furthermore, one can check that more passengers fly when both airline are not off the peak: $q_{01} + q_{10} > 2q_{00}$. Finally, more passenger fly when duopoly airlines as are set apart for low enough fees. More formally, iff

$$q_{01} + q_{10} > 2q_{11} \Longleftrightarrow \phi < \hat{\phi} \equiv \frac{s_0 s_1}{2\left(3s_1 - s_0\right)} < \overline{\phi}.$$

One can check that, under this condition, duopoly airlines serve more passengers than monopolies for given z ($2q_{11} > q_1$ and $q_{01} + q_{10} > q_1$).

4 Slot coordinator choice

According to the Worldwide Airport Slot Guidelines, the prime objective of airport slot coordination is to organize available airport capacity "in order to optimize benefit to consumers, taking into account the interest of airports and airlines" (WASG 2022, p.8). Given its limited information on each passenger surplus, the slot coordinator's board can only take into account the flow of passengers in the airport. We therefore consider that it allocates slots with the objective of finding the larger passenger traffic subject to the peak capacity constraint of the airport.

The slot coordinator problem is to assign monopoly and duopoly aircraft movements to peak or offpeak slots. We associate each monopoly destination with market size z with the index $m_0(z) = 1$ to monopoly airlines using in an offpeak slot and $m_1(z) = 1$ and to ones using a peak time slot ($m_0(z) + m_1(z) = 1$). We similarly associate a duopoly destination with market size z with the index $n_{11}(z)$ if the two airlines take the peak slot, $n_{01}(z)$ if one airline takes the peak slot and the other does not, and $n_{00}(z)$ if the two airlines take the offpeak slot ($n_{00}(z) + n_{01}(z) + n_{11}(z) = 1$). For the sake of conciseness, we dispense the reader with references to specific market size z and support $[\underline{z}, \bar{z}]$ whenever it creates no ambiguity. Accordingly, the slot coordinator chooses the sets of slot allocation functions $n = (n_{00}, n_{01}, n_{11})$ and $m = (m_0, m_1)$ that maximizes the number of passengers

$$P(m, n, \phi) = \int z\left(q_0 m_0 + q_1 m_1\right) M \mathrm{d}F + \int z\left[2q_{00}n_{00} + (q_{01} + q_{10})n_{01} + 2q_{11}n_{11}\right] N \mathrm{d}G, \quad (3)$$

subject to

$$\int m_1 M \mathrm{d}F + \int \left(n_{01} + 2n_{11}\right) N \mathrm{d}G \leq K, \quad (4)$$

$$m_0 + m_1 = 1, \quad (5)$$

$$n_{00} + n_{01} + n_{11} = 1. \quad (6)$$

Replacing m_0 and n_0 from (5) and (6), we can write the Lagrangian function as

$$\begin{aligned} \mathcal{L} = \int z \{ & \left[q_0\left(1 - m_1\right) + q_1 m_1\right] Mf - \mu m_1 Mf \\ & + \left[2q_{00}\left(1 - n_{01} - n_{11}\right) + (q_{01} + q_{10})n_{01} + 2q_{11}n_{11}\right] Ng \\ & - \mu\left(n_{01} + 2n_{11}\right) Ng \} \,\mathrm{d}z + \mu K, \end{aligned} \quad (7)$$

where $\mu \geq 0$ is the Khun-Tucker multiplier associated to the capacity constraint. This solution approach extends Picard *et al.* (2019) to a mix of monopoly and duopoly destinations.

Pointwise differentiating the Lagrangian function (7) with respect to $m_1(\cdot)$, one readily finds the marginal benefit to set monopoly destinations z on the peak slot

$$\mathcal{L}_1(z) \equiv \frac{\partial \mathcal{L}}{\partial m_1} = \left[z\left(q_1 - q_0\right) - \mu\right] Mf. \tag{8}$$

This marginal benefit in terms of passenger number $\mathcal{L}_1(z)$ increases in z and has a root at $\mathcal{L}_1(z) = 0$ at

$$z_1 \equiv \frac{\mu}{q_1 - q_0} = \frac{2\mu\left(s_0 s_1\right)}{\phi(s_1 - s_0)}. \tag{9}$$

As a result, monopoly airlines are put on-peak if $z \geq z_1$ and offpeak otherwise.

Further pointwise differentiation with respect to $n_{01}(\cdot)$ and $n_{11}(\cdot)$ gives the marginal benefits for different slots and for same peak slots in the destination with market size z. That is,

$$\begin{aligned}
\mathcal{L}_{01}(z) &\equiv \frac{\partial \mathcal{L}}{\partial n_{01}} = \left[z\left(q_{01} + q_{10} - 2q_{00}\right) - \mu\right] Ng, \\
\mathcal{L}_{11}(z) &\equiv \frac{\partial \mathcal{L}}{\partial n_{11}} = \left[z\left(2q_{11} - 2q_{00}\right) - 2\mu\right] Ng.
\end{aligned}$$

As a result, the coordinator has incentives to put the two airlines on the same peak slot if $\mathcal{L}_{11}(z) \geq \max\{\mathcal{L}_{01}(z), 0\}$, on two different slots if $\mathcal{L}_{01}(z) \geq \max\{\mathcal{L}_{11}(z), 0\}$ and on the same offpeak slot if $0 \geq \max\{\mathcal{L}_{01}(z), \mathcal{L}_{11}(z)\}$. The marginal benefits $\mathcal{L}_{01}(z)$ and $\mathcal{L}_{11}(z)$ are increasing in z with intercepts at $-\mu$ and -2μ. They have positive roots at :

$$\begin{aligned}
z_{01} &\equiv \frac{\mu}{q_{01} + q_{10} - 2q_{00}} = \frac{3\mu s_0\left(4s_1 - s_0\right)}{\left(s_1 - s_0\right)\left(s_0 + 2\phi\right)}, \\
z_{11} &\equiv \frac{\mu}{q_{11} - q_{00}} = \mu\frac{3s_0 s_1}{\phi\left(s_1 - s_0\right)}.
\end{aligned} \tag{10}$$

One can note that the configuration of slot allocation depends on the level of per-passenger fees. More specifically, if $\phi \in [0, \widehat{\phi})$ ("small passenger fees"), the slope of $\mathcal{L}_{01}(z)$ is larger than of that $\mathcal{L}_{11}(z)$ and the root z_{01} is lower than z_{11}. In this case, $\mathcal{L}_{01}(z) > \mathcal{L}_{11}(z)$ for all $z > 0$. The opposite applies for $\phi \in \left[\widehat{\phi}, \overline{\phi}\right)$ ("large passenger fees").

It is convenient to study the slot coordinator choice according to the level of per-passenger fees. Figure 1 shows differences in terms of marginal benefits according to the two scenarios. It illustrates the marginal benefits to passengers from destination markets served by monopolies and duopolies, according to whether $\phi \in \left[0, \widehat{\phi}\right)$ and $\phi \in \left[\widehat{\phi}, \overline{\phi}\right]$. Here, z represents the size of the destination market. To ease the exposition, the marginal benefits of monopolies are displayed in red. The slot coordinator sets peak slots to destination markets z that ensure the highest marginal benefit. In particular, he sets the monopoly flights on the peak for every destination market with size with $\mathcal{L}_1(z) \geq 0$, i.e. with $z \geq z_1$, and off peak for every destination market with size $\mathcal{L}_1(z) < 0$ ($z \in (0, z_1)$).

Likewise, the slot coordinator sets the two duopoly flights on the peak if $\mathcal{L}_{11}(z) \geq \max\{\mathcal{L}_{10}(z), 0\}$, and assigns only one duopoly flight on the peak if $\mathcal{L}_{01}(z) \geq \max\{\mathcal{L}_{11}(z), 0\}$. As illustrated in Figure 1 it becomes apparent that, with small fees $\left(\phi \in \left[0, \widehat{\phi}\right)\right)$, duopolies adopting peak-offpeak slot configurations consistently accommodate a larger number of passengers compared to peak monopolies. Peak monopolies, in turn, carry more passengers than duopolies with peak-peak slot configurations. In contrast, when facing high fees $\left(\phi \in \left[\widehat{\phi}, \overline{\phi}\right]\right)$, the optimal configuration for maximizing passenger numbers depends on the market size. Larger duopoly markets, for instance, are assigned two peak slots, leading to the highest passenger count. Further discussion of this topic will take place below, with a detailed focus on small and high fees separately.

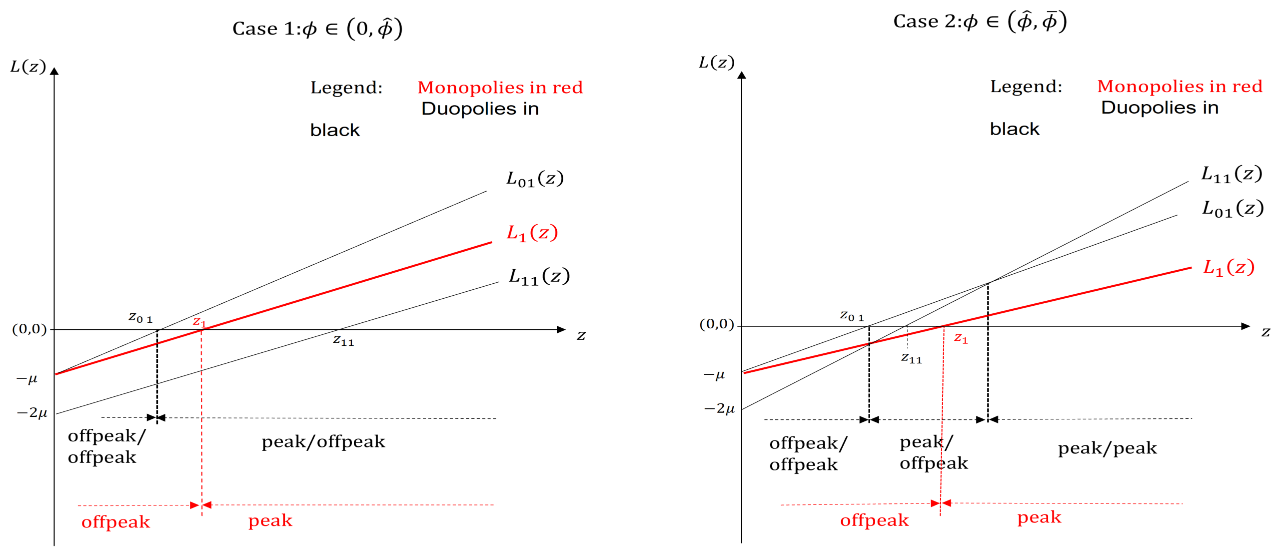

Figure 1: Passengers' marginal benefits by destination

Small passenger fees For $\phi \in \left(0, \widehat{\phi}\right)$, the slot coordinator's marginal benefit $\mathcal{L}_{01}(z)$ is always larger than $\mathcal{L}_{11}(z)$. This means that the slot coordinator never chooses to put both duopoly airlines on the peak slot. Rather, duopoly airlines are put both on-peak and offpeak ($n_{11}(z) = 0$, $n_{01}(z) = 1$) if $z > z_{01}$ and offpeak and offpeak ($n_{11}(z) = n_{01}(z) = 0$) otherwise.

Let us consider the case where some slots are unused because the capacity constraint is not met. That is,

$$M[1 - F(z_1)] + N[1 - G(z_{01})] < K.$$

Since the multiplier is then equal to $\mu = 0$, the thresholds z_1 and z_{01} are also equal to zero and the values of c.d.f. $G(z_1)$ and $F(z_{01})$ are null. This implies that $M + N < K$. In words, each destination can be allocated at *at least* a peak slot. Under this condition, the slot coordinator has incentives to 'organize' the discrimination in duopoly airline destinations and restrict one flight on peak time. This results in stronger passenger discrimination in the destination markets served by two airlines. When the coordinator allocates one airline to offpeak slot, it is because the latter sets a so low offpeak fare that the total passenger demand increases in this destination. The existence of unused peak slots results in *slot allocative inefficiency*. There exist empty peak slots in which a second duopoly flight could be allocated. This condition does not depend on the market size distributions F and G across monopoly and duopoly destination structures.

Finally, if $M + N \geq K$, the airport capacity is too small to allocate a peak slot to every destination. The above constraint is binding and $\mu > 0$. There is not allocative inefficiency. Furthermore, the slot coordinator trades off allocating peak slots to monopoly and duopoly airlines as the airport capacity falls. Indeed, in this case, μ rises so that both z_1 and z_{01} increase. The coordinator reduces both the number of monopoly and duopoly destinations on the peak slot.

For destinations served by monopolies, the number of passengers is always higher if one peak slot is conceded to the monopoly airline.

Figure 2 shows the total number of passengers in duopoly airline destinations according to the airport peak slot capacity when per-passenger fees lie in the range $\phi \in \left[0, \widehat{\phi}\right)$: the bold line "10" is the allocation to peak-offpeak duopolies, while the bold line "11" represents the allocation of peak-peak duopolies. The shaded area "Mix" represents the possible combinations between allocating peak-peak and peak-offpeak duopolies. The allocation of peak-offpeak duopolies "10" yields a total number of passengers that is higher than the one of peak-peak duopolies "11" and also any combination of the two allocations in any destinations. Accordingly, the slot coordinator always prefers to allocate just one peak slot rather than two to any destination served by duopolies.

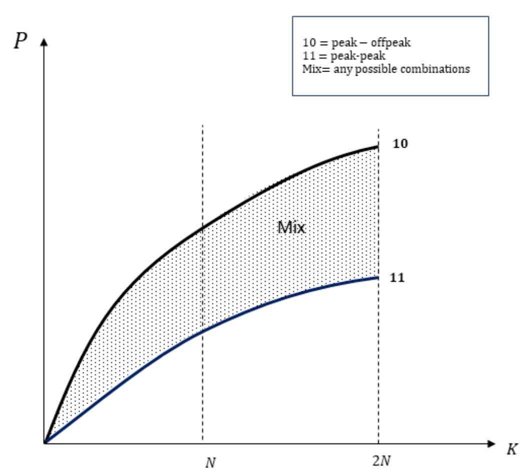

Figure 2: Total number of airport passengers when $\phi \in [0, \widehat{\phi})$.

Large passenger fee When passenger fees are set to $\phi \in \left[\widehat{\phi}, \overline{\phi}\right)$, the slot coordinator adopts a richer slot allocation for duopoly airlines. The marginal profit $\mathcal{L}_{01}(z)$ and $\mathcal{L}_{11}(z)$ intersects at

$$z^* \equiv \frac{\mu}{2q_{11} - q_{01} - q_{10}} = \mu \frac{3s_0 s_1 (4s_1 - s_0)}{(s_1 - s_0)(6s_1 - 2s_0)\left(\phi - \hat{\phi}\right)}, \tag{11}$$

where it can be shown that $z^* \geq z_{01}$ for any $\phi \in [\hat{\phi}, \bar{\phi}]$. This means that, duopoly airlines are set off the peak for $z < z_{01}$ because both $\mathcal{L}_{01}(z)$ and $\mathcal{L}_{11}(z)$ are negative; then, airlines are put on different slots for $z \in [z_{01}, z^*]$ because $\mathcal{L}_{01}(z)$ is positive and larger than $\mathcal{L}_{11}(z)$; finally, both airlines are put on peak slots for $z > z^*$ as $\mathcal{L}_{11}(z)$ is positive and larger than $\mathcal{L}_{01}(z)$. As all slot are used by the coordinator, there is *no slot allocation efficiency* for large enough passenger fee, $\phi > \hat{\phi}$.

Figure 3 shows the total number of passengers in duopoly airline destinations according to the airport peak slot capacity when $\phi \in [\hat{\phi}, \bar{\phi}]$. In this case, there exist mixes of peak-peak and peak-offpeak slot configurations that entice more passengers to fly. The slot coordinator therefore maximizes the total number of passengers by appointing a mix of peak-peak and peak-offpeak slots to destinations served by duopoly airlines.

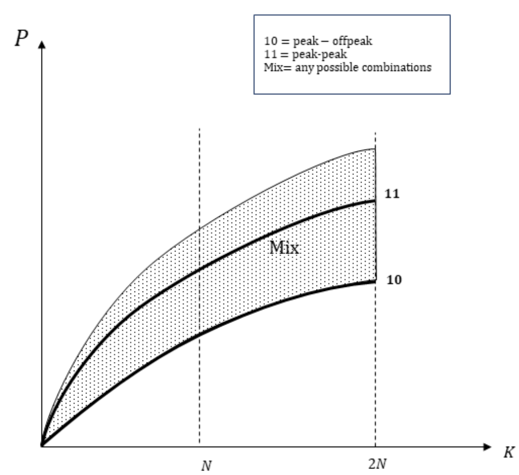

Figure 3: Total number of airport passengers when $\phi \in (\widehat{\phi}, \overline{\phi})$.

We summarize the above analysis in the following proposition:

Proposition 1 *For any passenger fees $\phi < \hat{\phi}$, the slot allocation is inefficient if $M + N < K$.*

This proposition makes relevant the role of the regulator's policy about passenger fees in the allocation of airport slots. By setting a too small fee, it entices coordinators to inefficiently use their peak slots and strengthen the discrimination between passengers. The question is whether this property holds when the airport is authorized to choose its own fee.

5 Airport choice

For a few decades, airport management has been subject to waves of deregulation and privatization. Airports in Australia and New Zealand are run under private ownership and many European airports are operated under public and private ownership. Many regulatory constraints on their activity revenues have been relaxed. The purpose of this section is to analyze the impact of airport management on slot allocation. We therefore consider airports that are not regulated and authorized to choose their passenger fees. We show that airports want to set passenger fees that refrain the slot coordinator from adopting an inefficient slot allocation.

The airport makes profits out of the revenues from the passenger fees. Accordingly, it airport chooses the level of per-passenger fees ϕ that maximizes its profit

$$\Pi = \phi P\left(m^*, n^*, \phi\right),$$

where the number of passenger is given by (3) and the slot allocation functions m^* and n^* are the slot coordinator's optimal allocations as derived in the previous section.

Note first that the slot coordinator and airport have congruent objective functions in terms of slot allocation. As a result, the slot allocation that is optimal for the slot coordinator is also optimal for the airport. Thus, using the fact that the slot allocation function maximizes the total number of passengers ($\partial P/\partial n = \partial P/\partial m = 0$ at $m = m^*$ and $n = n^*$), the optimal fee is given by the following first order condition:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\phi} = \frac{\partial \Pi}{\partial \phi} = P + \phi \frac{\partial P}{\partial \phi} = 0. \tag{12}$$

Note also that the seat numbers q_i and q_{ij}, $i, j \in \{0, 1\}$, are linear functions of ϕ. Hence, at the coordinator's optimal allocations m^*, n^*, the function $P\left(m^*, n^*, \phi\right)$ is also a linear function of ϕ. For the sake of clarity, we redefine the seat numbers q_i and q_{ij} as $q_i = q_i^o + q_i'\phi$ and $q_{ij} = q_{ij}^o + q_{ij}'\phi$ where q_i^o and q_{ij}^o are positive intercepts while q_i' and q_{ij}' negative slopes (scalars). Then, it can be seen that (12) is quadratic function of ϕ and yields a unique, positive optimal fee given by

$$\phi^* = -\frac{1}{2}\frac{\left[\begin{array}{c}\int z\left(q_0^o m_0^* + q_1^o m_1^*\right) M\mathrm{d}F \\ +\int z\left[2q_{00}^o n_{00}^* + (q_{01}^o + q_{10}^o) n_{01}^* + 2q_{11}^o n_{11}^*\right] N\mathrm{d}G\end{array}\right]}{\left[\begin{array}{c}\int z\left(q_0' m_0^* + q_1' m_1^*\right) M\mathrm{d}F \\ +\int z\left[2q_{00}' n_{00}^* + (q_{01}' + q_{10}') n_{01}^* + 2q_{11}' n_{11}^*\right] N\mathrm{d}G\end{array}\right]} > 0, \tag{13}$$

which is a function of the chosen slot allocation (m^*, n^*). Can the coordinator choose an inefficient slot allocation where duopoly airlines cannot fly on peak time to the same destination? We have shown that this situation occurs if the passenger fee is smaller than $\widehat{\phi}$.

Small passenger fees If the passenger fee is smaller than $\widehat{\phi}$, the slot coordinator never allocates two flights on peak time to the same destination ($n_{11}^* = 0$). For the sake of exposition, consider the airport that operates only duopoly destinations ($N > M = m_{ij}^* = 0$). The optimal fee (13) simplifies to

$$\phi^* = -\frac{1}{2}\frac{\int z\left[2q_{00}^o n_{00}^* + (q_{01}^o + q_{10}^o) n_{01}^*\right] N\mathrm{d}G}{\int z\left[2q_{00}' n_{00}^* + (q_{01}' + q_{10}') n_{01}^*\right] N\mathrm{d}G} > \frac{2q_{00}^o}{-2\left(q_{01}' + q_{10}'\right)} = \frac{7}{12} s_0 \frac{4s_1 - s_0}{s_1},$$

where we used the fact that $q_{01}^o + q_{10}^o > 2q_{00}^o$ and $-(q_{01}' + q_{10}') > -2q_{00}' > 0$ to bound the numerator of the LHS ratio by below and its denominator by above. The last equality results from substituting and simplifying the values of the intercept q_{00}^o and slopes q_{01}' and q_{10}'. The RHS can be shown to be larger than $\widehat{\phi}$. So, the airport never sets a fee small enough that the coordinator does not use all peak slots, in particular, if $N > K$. In the next proposition, we show that the same argument can be repeated for the airport with any mix of monopoly and duopoly airline destinations. Hence, there exists no equilibrium in the range of fees $[0, \widehat{\phi})$.

Proposition 2 *In an unregulated airport with a mix of monopoly and duopoly destinations, the equilibrium passenger fee ϕ^* is never lower than $\widehat{\phi}$. Hence, slot allocation is always efficient.*

Proof. See Appendix. ■

Proposition 2 implies that equilibrium passenger fees are larger than $\widehat{\phi}$. The striking consequence of Proposition 2 is that allocative inefficiency is eliminated once the airport obtains the right to set its passenger fee. The intuition is as follows: When a regulated airport is imposed a low fee by the regulation authority, the slot coordinator finds it beneficial to keep some peak slots unused to entice the duopoly airlines to attract a larger number of passengers in the off-peak period. This is natural: Since the fee does not change between low and high valuation passengers, the coordinator chooses to expand the passenger numbers, even if this strategy leaves slots unused. When the airport faces no fee regulation, the airport would manipulate fees in such a way that the given schedule is not optimal anymore. In turn, by excluding the peak-offpeak allocative schedules for each destination, allocation inefficiency is also precluded.

Large passenger fees When $\phi > \widehat{\phi}$, the airport choice is described by the values of the thresholds z_1, z_{01} and z^*, and the passenger fee ϕ that satisfy the definitions (9), (10) and (11) and the capacity constraint

$$M\left[1-F\left(z_1\right)\right]+N\left[G(z^*)-G(z_{01})\right]+2N\left[1-G(z^*)\right]\leq K.$$

Then, for any type of airport, the optimal fee (13) is given by

$$\phi^* = -\frac{1}{2}\frac{\left[\begin{array}{c} q_0^o \int^{z_1} zM\mathrm{d}F + q_1^o \int_{z_1} zM\mathrm{d}F \\ +2q_{00}^o \int^{z_{01}} zN\mathrm{d}G + (q_{01}^o + q_{10}^o)\int_{z_{01}}^{z^*} zN\mathrm{d}G + 2q_{11}^o \int_{z^*} zN\mathrm{d}G \end{array}\right]}{\left[\begin{array}{c} q_0' \int^{z_1} zM\mathrm{d}F + q_1' \int_{z_1} zM\mathrm{d}F \\ +2q_{00}' \int^{z_{01}} zN\mathrm{d}G + (q_{01}' + q_{10}')\int_{z_{01}}^{z^*} zN\mathrm{d}G + 2q_{11}' \int_{z^*} zN\mathrm{d}G \end{array}\right]}. \tag{14}$$

In the Appendix, we show that this optimal fee exceeds the critical value $\widehat{\phi}$ that supports an equilibrium (i.e. $\phi^* > \hat{\phi}$). However, given the analytical complexity of the present analysis, we break down this study for the special cases of monopoly and duopoly airlines. We finally consider the more general case with both monopolies and duopolies in a numerical simulation in Section 7.

The following proposition shows the equilibrium fees when all airlines are monopoly. For convenience, denote as

$$L^m(x) = \int^{F^{-1}(x)} z\mathrm{d}F(z)/\int z\mathrm{d}F(z),$$

the share of passenger demand below the percentile x of its maximal demand.

Proposition 3 *Suppose an airport that is unregulated and serves all monopoly airlines. Then, the optimal passenger fee is given by:*

$$\phi_M^* = \frac{1}{2}\left[\frac{1}{s_1} + \left(\frac{1}{s_0} - \frac{1}{s_1}\right)L^m(1-K/M)\right]^{-1},$$

where $\phi_M^* > \widehat{\phi}$.

Proof. In Appendix. ■

In Proposition 3, the passenger fee increases with an expansion of airport capacity (larger K) because the airport can put more flights on the peak period where each airline attracts more passengers. Conversely, the fee decreases with an expansion of the number of destinations (larger M) because the airport includes more offpeak flights with smaller passenger loads and therefore lower airport revenues. When slot capacity equates the number of destinations, the airport is not capacity constrained and sets its passenger fee to $s_1/2$, which is half of the passenger's gross surplus from flying on peak time. When slot capacity is close to zero, all aircraft fly offpeak, the per passenger fee is equal to $s_0/2$, which half that same surplus offpeak.

Those values reflect the surplus sharing (double marginalization) in the business chain of airport and airline monopolies.[13]

We now turn on the analysis in which the airport serves only duopoly destination markets. The optimal fee is obtained in a similar way but is cumbersome to display (see Appendix). Nevertheless, to get analytical tractability, we can assume that destination markets are uniformly distributed: $G : [0,1] \to 1$, $G(z) = z$. The following proposition holds.

Proposition 4 *Suppose an airport is unregulated and serves only duopoly airlines. Then, the optimal per-passenger fee is given by:*

$$\phi_D^* = \frac{1}{4} \frac{2\left(\frac{4}{s_0} - \frac{1}{s_1}\right)^2 - \left(\frac{1}{s_0} - \frac{1}{s_1}\right)\left(\frac{2}{s_0} - \frac{1}{s_1}\right)\left(2 - \frac{K}{N}\right)^2}{\frac{1}{s_1}\left(\frac{4}{s_0} - \frac{1}{s_1}\right)^2 + \frac{1}{s_0}\left(\frac{1}{s_0} - \frac{1}{s_1}\right)\left(\frac{3}{s_0} - \frac{1}{s_1}\right)\left(2 - \frac{K}{N}\right)^2},$$

where $\phi_D^* > \widehat{\phi}$. It is smaller than $\overline{\phi}$ if $s_1 \leq 3s_0/2$.

Proof. In Appendix. ■

A quick glance to Proposition 4 shows that ϕ_D^* decreases with $2 - K/N$. Hence, it decreases with the number of duopoly destinations and increases in the airport's capacity. Larger slot capacity allows the airport to put more airlines on peak slots and increases the passengers' demand, and hence the airport revenues. As the number of destinations increases for the same capacity, the number of passengers flying at off peak time increases, implying a lower demand and a lower price. Furthermore, it can readily be checked that a proportional increase in s_0 and s_1 (by the same multiplier m) increases ϕ_D^* in a proportional way (by m). It can be shown that an increase of s_1/s_0 also increases the fee ϕ_D^* (see Appendix).

6 Welfare

In this section, we evaluate the welfare impact of liberalizing per-passenger fees. Since airport and airline operating costs are normalized to zero, airport profits come from total per-passenger fees, whereas airline profits are ticket income less total per-passenger fees paid to the airport. In turn, passenger surplus is represented by the total gross utility generated from flying minus all ticket payments. Since monetary transfers between airlines and airport cancel out, and so do transfers between passengers and airlines, then social welfare equals the sum of passengers' gross utility in all $M + N$ destination markets.

[13] Finally, this fee is admissible only if monopoly airlines survive in the off-peak market, i.e., if $\phi^* < s_0$. That occurs if

$$\frac{s_1 - 2s_0}{2s_1 - 2s_0} < L^m(1 - K/M).$$

This condition imposes a lower boundary on M. If off-peak flights are rare, they are obliged to pay the high fee that corresponds to on-peak fligths and therefore cannot survive. The condition nevertheless holds if $s_1 < s_0/2$.

We first analyze the airport with only monopolies and then with only duopolies in all destination markets, respectively. In the Section 7, we will investigate the effects of liberalization of an airport serving both monopolies and duopolies using a numerical simulation.

6.1 Monopoly destinations

When the slot coordinator assigns slot i to a monopoly airline with market size z, its flight generates a gross utility equal to $z \int_{1-q_i}^{1} v s_i dv$. It can readily be shown that the social welfare generated in the destination market z is equal to $w_i z$ where

$$w_i = \frac{(s_i - \phi)(3s_i + \phi)}{8s_i}, \tag{15}$$

which falls with higher passenger fee since $\mathrm{d}w_i/\mathrm{d}\phi = -\left(\phi + s_i\right)/\left(4s_i\right) < 0$.

For capacity unconstrained airports ($M \leq K$), social welfare is equal to $W = M \int_0^\infty z w_1 \mathrm{d}G(z)$, which also falls in ϕ. For capacity constrained airports ($M > K$), social welfare writes as

$$W = M \int_{z_1}^{\infty} z w_1 \mathrm{d}G(z) + \int_0^{z_1} z w_0 \mathrm{d}G\left(z\right), \tag{16}$$

where both integrands are functions of w_1 and w_0, which each fall in ϕ. By contrast, because z_1 satisfies the capacity constraint $M\left(1 - F\left(z_1\right)\right) = K$, it is independent of ϕ. As a result, social welfare always decreases with higher ϕ whatever the capacity level.

Proposition 5 *Social welfare decreases with higher passenger fees ϕ in an airport serving a set of monopoly destination markets.*

The implication of Proposition 5 is that, for given slot allocation, social welfare increases with an decrease of per passenger fees ϕ.

6.2 Duopolies

When the slot coordinator assigns two airlines to a destination market z to the same type of time slot $i \in \{0, 1\}$, each flight generates a gross utility $\frac{1}{2} z \int_{1-2q_{ii}}^{1} v s_i dv$. Conversely, when the coordinator assigns each duopoly airline to different a time slot, the offpeak flight generates a gross utility equal to $z \int_{1-q_{01}-q_{10}}^{1-q_{10}} v s_0 dv$ while the peak flight yields $z \int_{1-q_{10}}^{1} v s_1 dv$. Welfare value of a flight with a destination market size z is equal to $z w_{ij}$, where

$$w_{ii} = \frac{(s_i - \phi)(2s_i + \phi)}{9s_i}, \tag{17}$$

for two flights in the same time period $i \in \{0,1\}$, and

$$w_{01} = \frac{[s_1(s_0 - 2\phi) + s_0\phi]\,[3s_1s_0 + \phi\,(2s_1 + s_0)]}{2s_0\,(4s_1 - s_0)^2}, \tag{18}$$

$$w_{10} = \frac{s_1(2s_1 - s_0 - \phi)(6s_1 - s_0 + \phi)}{2\,(4s_1 - s_0)^2}, \tag{19}$$

for two flights in different time periods. We have $dw_{ii}/d\phi < 0$ while $dw_{01}/d\phi > 0$, $dw_{10}/d\phi < 0$ and $d(w_{01} + w_{10})\,/d\phi < 0$.

For small enough fees $\phi < \widehat{\phi}$, social welfare is rewritten as

$$W_{01}\,(\phi) = \int_{z_{01}}^{\infty} z(w_{01} + w_{10})N\mathrm{d}G(z) + \int_{0}^{z_{01}} z(2w_{00})N\mathrm{d}G(z), \tag{20}$$

where z_{01} is the equilibrium value that depends on μ given by the capacity constraint $K \geq \int_{z_{01}}^{\infty} N\mathrm{d}G(z)$.

For larger fee ϕ increasing above $\widehat{\phi}$, z^* decreases from infinite value and social welfare becomes

$$W_{11}\,(\phi) = \int_{z^*}^{\infty} z(2w_{11})N\mathrm{d}G(z) + \int_{z_{01}}^{z^*} z(w_{01} + w_{10})N\mathrm{d}G(z) + \int_{0}^{z_{01}} z(2w_{00})N\mathrm{d}G(z), \tag{21}$$

where z_{01} and z^* are equilibrium values and depend on μ given by $K \geq \int_{z^*}^{\infty} 2N\mathrm{d}G(z) + \int_{z_{01}}^{z^*} zN\mathrm{d}G(z)$.

We first put light on the fact that the slot coordinator inefficiently uses peak slots when capacity accommodates one peak slot in each destination $(M + N < K)$ and the passenger fee ϕ is exogenous $\phi < \widehat{\phi}$. What is thus the welfare benefit resulting by imposing the coordinator to allocate duopoly airlines with destination market sizes $z \in [z^*, \infty)$ in the same peak slot? Denoting the initial threshold as z'_{01} and the final ones as z_{01} and z^*, the welfare difference is given by

$$W_{11} - W_{01} = \int_{z^*}^{\infty} z(2w_{11} - w_{01} - w_{10})N\mathrm{d}G(z) - \int_{z'_{01}}^{z_{01}} z(w_{01} + w_{10} - 2w_{00})N\mathrm{d}G(z),$$

where all terms in parentheses are positive. The first integral expresses the benefit for the passengers flighting on the new peak-peak destinations. The second integral expresses the welfare loss for the passengers obliged to move onto offpeak time. In the case of unused peak slots and allocative inefficiency ($\mu = 0$), the welfare benefit is definitively positive because z'_{01} and z_{01} are both equal to zero. Otherwise, welfare improvement depends on the above balance.

Second, we study the impact of deregulation of the fee ϕ while letting the coordinator choose the slot allocation for the given fee. First, consider the case where $\phi < \widehat{\phi}$. The welfare change due to a marginal increase in ϕ is equal to

$$\frac{\mathrm{d}W_{01}}{\mathrm{d}\phi} = \int_{z_{01}}^{\infty} z\frac{\mathrm{d}(w_{01}+w_{10})}{\mathrm{d}\phi}N\mathrm{d}G(z) + \int_{0}^{z_{01}} 2z\frac{\mathrm{d}w_{00}}{\mathrm{d}\phi}N\mathrm{d}G(z) \tag{22}$$

$$-\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}z_{01}\left(w_{01}+w_{10}-2w_{00}\right)Ng(z_{01}), \tag{23}$$

where the two first terms have negative integrand and the bracket in the third term is positive. We have

$$\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi} = \frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{\mu}{q_{01}+q_{10}-2q_{00}}\right) = \frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{3\mu s_0\left(4s_1-s_0\right)}{\left(2\phi+s_0\right)\left(s_1-s_0\right)}\right).$$

On the one hand, if slot capacity is slack, then we have $\mu = 0$ so that $dz_{01}/d\phi = 0$ and $dW_{01}\left(\phi\right)/d\phi < 0$. Although a higher fee mitigates the slot allocation inefficiency, social welfare falls with higher passenger fees. On the other hand, if the slot capacity binds such that $K = \int_{z_{01}}^{\infty} N\mathrm{d}G(z)$, it is clear that z_{01} is independent of ϕ. So, $dz_{01}/d\phi = 0$ and therefore $dW_{01}/d\phi < 0$. To sum up, $dW_{01}\left(\phi\right)/d\phi < 0$ for any $\phi < \widehat{\phi}$. Hence the regulator has no incentives to set small fees. Secondly, in the case where $\phi > \widehat{\phi}$, it can be shown that welfare also decreases with higher fee (see Appendix).

Proposition 6 *In an airport serving a set of duopoly destination markets, welfare decreases with higher ϕ if $\phi \neq \widehat{\phi}$. For uniform G, it has a positive jump at $\phi = \widehat{\phi}$.*

Proof. See Appendix. ■

The second part of the proposition is proven by assuming uniform distribution, but the jump is also confirmed in the numerical simulation devoted to study the general case with monopolies and duopolies (see Section 7). Intuitively, the welfare improvement stems from the resolution of allocative inefficiency. This is proven by the fact that no jump occurs in the welfare analysis when the airport serves only monopolies: indeed, in that situation no allocative inefficiency occurs.

The proposition implies that an increase in passenger fee may improve welfare if it entices the slot coordinator to reallocate duopoly flights to peak slots when the fee reaches the value of $\widehat{\phi}$. This result may be explained by noticing the increase in competition when the slot coordinator sets peak/peak or offpeak/offpeak duopolies rather than peak/offpeak ones. Indeed, with only one peak time per destination offered, passengers gain in terms of relatively lower prices, irrespective of the benefit in terms of departing hours.

Once the reallocation is set though, social welfare decreases again as per-passenger fees increase. It follows that deregulation has a positive effect on social welfare only if the optimal airport choice is not too far from the threshold $\widehat{\phi}$. Otherwise, the negative effect of higher fees would more than offset the positive effect of reallocation.

7 An example

When we consider destinations served by both monopolies and duopolies, analytical tractability precludes us to find some findings, namely, (i) we are not able to explicitly obtain a closed form solution of the optimal per passenger fee in the case where $\phi > \widehat{\phi}$, and (ii) we cannot derive the analytical welfare comparison. In this section we cover these aspects by developing a numerical example, to show that results are consisted with the findings obtained analytically in the rest of the paper.

In the example, we mimic the situation of an airport like Los Angeles (LAX) with $M = 56$ monopoly destinations and $N = 28$ duopoly destinations in April 2023 (other configurations are disregarded here). For this airport, the passenger distribution in monopoly destinations can be approximated by the Pareto distribution $F(z) = (z/\overline{z})^{\alpha_M}$, where $\overline{z} = 10^6$ (passenger per year) and $\alpha_M = 0.25$, while the distribution of duopoly destinations is summarized by the Pareto distribution $G(z) = (z/\overline{z})^{\alpha_N}$, where $\overline{z} = 4.9 * 10^6$ (passenger per year) and $\alpha_N = 0.18$. For the example, we collect all flights into a single pair of peak and offpeak periods and assume a number of flights given by $M + 2N = 114$. For the sake of the discussion, we set the peak capacity to $K = 110$ so that all flights cannot be allocated to the peak period.

In 2023, average fares at LAX vary about USD 250. We assume that three quarter of the fare pays for airport and airline marginal costs. Hence, assuming zero airport passenger fee and considering a monopoly destination, the airline price, or equivalently markup, is equal to $p_1 = s_1/2 = 0.25 * 250$, gives $s_1 = 125$. We finally assume that the consumer's benefit to fly on peak is about USD 35 so that $s_0 = 90$. It follows that $\widehat{\phi} = 19.73$ and $\overline{\phi} = 70.31$. The total number of airport passengers at $K = 110$ is equal to $53 * 10^6$ passengers. This is slightly below the 2023 flow of sixty-six million passengers because of our omission of many airport movements.

Figure 4 shows the use of slot capacity in this setting. As shown in Proposition 1, allocative inefficiency emerges for $\phi \in \left(0, \widehat{\phi}\right)$, since some peak slots are left unused.

We turn now to verify the existence of an equilibrium with liberalized fees and $\phi \in \left[\widehat{\phi}, \overline{\phi}\right)$. Figure 5 shows the optimal per-passenger fee levied by the airport. It shows that ϕ^* lies in the range $\left[\widehat{\phi}, \overline{\phi}\right)$, consistent with the slot configuration chosen by the coordinator. In other words, the numerical simulation ensures that, unlike the case for $\phi \in \left(0, \widehat{\phi}\right)$, an equilibrium with unregulated fees may exist when the airport serves both monopoly and duopoly airlines.

Finally, Figure 6 considers the variation of welfare, airport and airline profits with the increase in per-passenger fees. The figure shows a decrease in welfare following an increase in ϕ, with the exception of the positive jump at $\widehat{\phi}$, which we formally presented in Proposition 6. The upward jump at $\widehat{\phi}$ implies a drastic welfare improvement in slot allocation, which is valued at USD 110 million. However, as Figure 2 shows, this improvement is smaller than the welfare loss of raising the fee from zero to $\widehat{\phi}$, which amounts to USD 515 million. Hence, in this example, airport liberalization cannot improve welfare compared to a regulated airport with

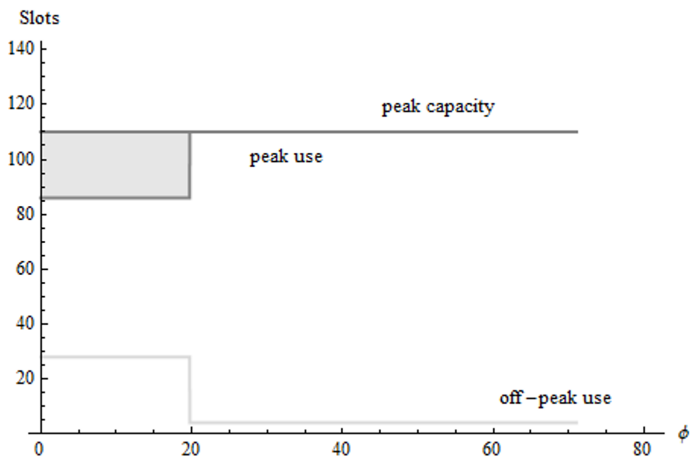

Figure 4: Use of slot capacity

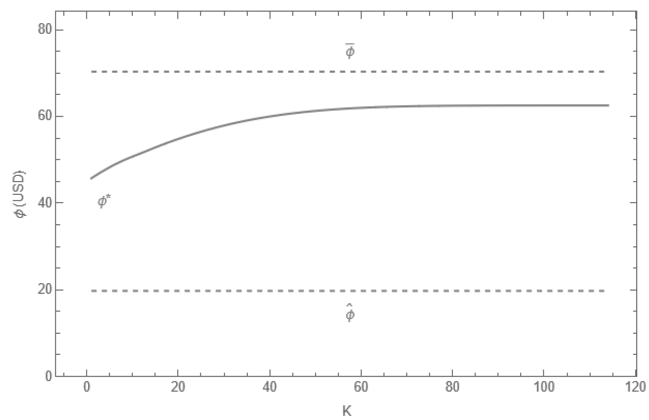

Figure 5: Airport profit maximizing passenger fee as function of capacity

zero passenger fee. The figure finally shows that the fall in welfare is due to a decrease in the passengers' welfare, as well as the decrease in airline profits.

8 Conclusions

In this paper we have carried forward the problem of allocative inefficiency. Starting from the findings of Picard *et al.* (2019) for airports with regulated per-passenger fees serving duopoly airlines, we have extended the analysis by first considering both monopolies and duopolies in the regulated case, showing the emergence of allocative inefficiency and then by evaluating the welfare effects of liberalization.

Whenever unregulated per-passenger fees have a positive effect on an efficient use of airport infrastructure, it improves welfare because of the allocation efficiency gain, but the increase in per-passenger fees also worsens the welfare level. Thus from a policy perspective, liberalization may have a negative effect on welfare, provided that the number of peak slots available is

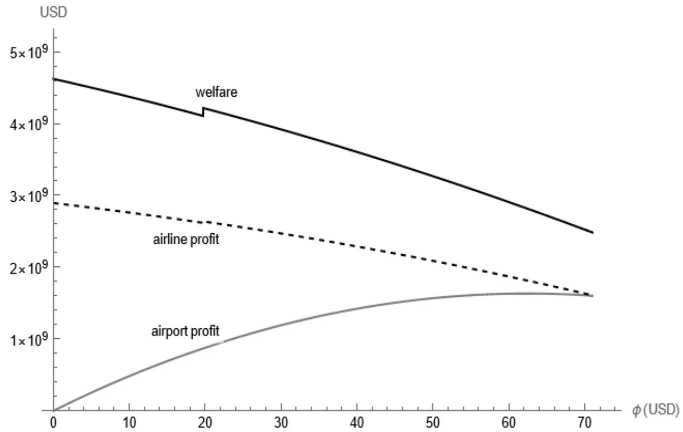

Figure 6: Welfare, airport and airlines profit

relatively limited, and the passengers' benefit from flying at peak times is sufficiently higher than the offpeak alternative. We hope, with our findings, to provide guidance to airport regulators.

A possible limitation of the present analysis is related to the fact that delays are not modelled. In our analysis, the adverse impact of congestion or delay on passenger satisfaction might be represented through a diminishing function linked to the count of unused slots. In essence, unutilized slots could generate a favorable external effect. Consequently, the extent of allocative inefficiency associated with unused slots in the existing model could be overestimated. It's important to note that this constraint is mitigated when our analysis is concentrated on level-2 airports, where concerns regarding delays are less pronounced.

References

[1] ACI, 2017. *Policy Brief: Airport ownership, economic regulation and financial performance.* https://store.aci.aero/product/aci-policy-brief-airport-ownership-economic-regulation-and-financial-performance/, Airports Council International.

[2] Airports Council International Europe, 2009. Report. https://www.aci-europe.org/, Airports Council International Europe.

[3] Airports Regulation Document 2017-2021, 2017. Directorate General of Civil Aviation of the Ministry of Public Works. Approved by the Spanish Ministry of Transport. https://www.aena.es/sites/Satellite?blobcol=urldata&blobkey=id&blobtable=MungoBlobs&blobwhere =1576855984887&ssbinary=true.

[4] Barbot, C. 2004. Economic effects of re-allocating airports slots: a vertical differentiation approach. *Journal of Air Transport Management* **10:** 333-343.

[5] Barnhart, C., Fearing, D., Odoni, A., and Vaze, V. 2012. Demand and capacity management in air transportation. *EURO Journal on Transportation and Logistics* **1**: 135-155.

[6] Basso, L.J., and Zhang. A. 2010. Pricing vs. slot policies when airport profits matter. *Transportation Research Part B* **44**: 381-391.

[7] Bel, G., Fageda, X., 2010. Privatization, regulation and airport pricing: an empirical analysis for europe. *Journal of Regulatory Economics* **37**: 142-161.

[8] Brueckner, J.K., 2009. Price vs. quantity-based approaches to airport congestion management. *Journal of Public Economics* **93**: 681-690.

[9] Czerny, A. I., 2013. Public versus private airport behavior when concession revenues exist. *Economics of Transportation* **2**: 38-46.

[10] Czerny, A. I., Cowan, S., Zhang, A., 2017. How to mix per-flight and per-passenger based airport charges: The oligopoly case. *Transportation Research Part B: Methodological* **104**: 483-500.

[11] Dray, L., 2020. An empirical analysis of airport capacity expansion. *Air Transport Management* **87**, 101850.

[12] European Commission, 2011. *Impact Assessment of Revisions to Regulation 95/93.* EC, Brussels. https://ec.europa.eu/transport/sites/transport/files/modes/air/studies/doc/airports/2011-03-impact-assessment-revisions-regulation-95-93.pdf.

[13] Gabszewicz, J. and Thisse, J.-F., 1979. Price competition, quality and income disparities. *Journal of Economic Theory* **20**: 340-359.

[14] IATA, 2017. *IATA Economics Briefi:Economic Benefits from Effective Regulation of European Airports.*

https://www.iata.org/contentassets/9c80e4e8c52243149dcd6d50c76b6ea0/economic-benefits-of-lower-airport-charges-2017.pdf, international Air Transport Association.

[15] ICAO, 2012. *ICAO's policies on charges for airports and air navigation services* **9082**. https://www.icao.int/publications/Documents/9082_9ed_en.pdf/, Airports Council International.

[16] ICAO, 2013. *ATConf/6-WP/88.*

https://www.icao.int/Meetings/atconf6/Documents/WorkingPapers/ATConf.6.WP.088.2.en.pdf, Airports Council International.

[17] Ivaldi, M., Sokullu, S., Toru, T., 2015. Airport prices in a two-sided market setting: Major us airports. *CEPR Discussion Paper* **DP10658**.

[18] Katsaros, A., Psaraki, V., 2012. Slot misuse phenomena in capacity-constrained airports with seasonal demand: the greek experience. *Transportation Planning and Technology* **35**: 790-806.

[19] Lenoir, N. 2016. Airport slots and aircraft size at EU airports. *Research for TRAN Committee, European Parlament.*

https://www.europarl.europa.eu/RegData/etudes/IDAN/2016/585873/IPOL_IDA(2016)585873_EN.pdf

[20] Lin, M. H., Zhang, Y., 2017. Hub-airport congestion pricing and capacity investment. *Transportation Research Part B: Methodological* **101**: 89–106.

[21] Martín, J. C., Socorro, M. P., 2009. A new era for airport regulators through capacity investments. *Transportation Research Part A: Policy and Practice* **43**: 618-625.

[22] Odoni, A. R. 2020. *A Review of Certain Aspects of the Slot Allocation Process at Level 3 Airports under Regulation 95/93.* Report No. ICAT-2020-09, MIT International Center for Air Transportation (ICAT) Department of Aeronautics and Astronautics, Cambridge, MA 02139 USA.

[23] Oum, T. H., Zhang, A., Zhang, Y., 2004. Alternative forms of economic regulation and their efficiency implications for airports. *Journal of Transport Economics and Policy* **38**: 217-246.

[24] Picard, P. M., Tampieri, A., Wan, X., 2019. Airport capacity and ineffciency in slot allocation. *International, Journal of Industrial Organization* **62**, 330-357.

[25] Pouget, L., Ribeiro, N.A., Odoni, A.R., Antunes, A.P. 2023. How do airlines react to slot displacements? Evidence from a major airport. *Journal of Air Transport Management* **106**, 102300.

[26] Ranieri, A., Alsina, N., Castelli, L., Bolic, T., Herranz, R. 2013. Airport slot allocation: performance of the current system and options for reform. *Third SESAR Innovation Day Conference Proceedings*.

[27] Swaroop, P., Zou, B., Ball, M. O., and Hansen, M. 2012. Do more US airports need slot controls? A welfare based approach to determine slot levels. *Transportation Research Part B: Methodological* **46**: 1239-1259.

[28] Verhoef, E.T., 2008. Congestion pricing, slot sales and slot trading in aviation. *Transportation Research Part B: Methodological* **44**: 320-329.

[29] *WASG Guidelines Second Edition* 2022.

https://www.iata.org/contentassets/4ede2aabfcc14a55919e468054d714fe/wasg-edition-2-english-version.pdf

[30] Winston, C., Ginés, d. R., 2009. *Aviation infrastructure performance: A study in comparative political economy.* Brookings Institution Press.

[31] Zhang, A., 2012. *Airport improvement fees, benefit spillovers, and land value capture mechanisms.* Lincoln Institute of Land Policy.

Appendix

Proof of Proposition 2

We first determine the optimal passenger fee. As seat numbers as linear functions of ϕ, we redefine them as $q_i = q_i^o + q_i'\phi$ and $q_{ij} = q_{ij}^o + q_{ij}'\phi$ where (q_i^o, q_{ij}^o) are positive intercept and (q_i', q_{ij}') negative slopes. Differentiating the Lagrangian function (7) w.r.t. ϕ yields

$$\begin{aligned}\mathcal{L}_\phi = \int z \, \{ \; & (q_0 n_0 + q_1 m_1)\, Mf + (\phi q_0' n_0 + \phi q_1' m_1)\, Mf \\ & + [2q_{00}n_{00} + (q_{01} + q_{10})n_{01} + 2q_{11}n_{11}]\, Ng \\ & + [2\phi q_{00}' n_{00} + \phi(q_{01}' + q_{10}')n_{01} + 2\phi q_{11}' n_{11}]\, Ng \; \} \mathrm{d}z + \mu K.\end{aligned}$$

Given that q_i' and q_{ij}' are negative, the marginal profit $\mathcal{L}_\phi$ is a linearly decreasing function of ϕ. Since q_i^o and q_{ij}^o are strictly positive, $\mathcal{L}_\phi$ is strictly positive at $\phi = 0$. Therefore, the airport always have incentives to raise the fee strictly above zero. Moreover, for any given slot allocation, the root of $\mathcal{L}_\phi$ yields a unique, optimal and positive fee given by

$$\phi^* = -\frac{1}{2} \frac{\left[\begin{array}{c} \int z\,(q_0^o m_0 + q_1^o m_1)\, M\mathrm{d}F \\ + \int z\,[2q_{00}^o n_{00} + (q_{01}^o + q_{10}^o)n_{01} + 2q_{11}^o n_{11}]\, N\mathrm{d}G \end{array} \right]}{\left[\begin{array}{c} \int z\,(q_0' m_0 + q_1' m_1)\, M\mathrm{d}F \\ + \int z\,[2q_{00}' n_{00} + (q_{01}' + q_{10}')n_{01} + 2q_{11}' n_{11}]\, N\mathrm{d}G \end{array} \right]} > 0. \tag{24}$$

Can the slot coordinator choose an inefficient slot allocation where no duopoly airline flies on peak time to the same destination? We have shown that this situation occurs if the passenger fee is smaller than $\widehat{\phi}$. However, the airport never chooses such a small fee, even if it fills its capacity. So, let us consider the case where $\phi < \widehat{\phi}$ so that $n_{11} = 0$. The optimal fee reduces to

$$\phi^* = -\frac{1}{2} \frac{\int z\,(q_0^o m_0 + q_1^o m_1)\, M\mathrm{d}F + \int z\,[2q_{00}^o n_{00} + (q_{01}^o + q_{10}^o)n_{01}]\, N\mathrm{d}G}{\int z\,(q_0' m_0 + q_1' m_1)\, M\mathrm{d}F + \int z\,[2q_{00}' n_{00} + (q_{01}' + q_{10}')n_{01}]\, N\mathrm{d}G}. \tag{25}$$

First, consider that the airport operates only monopoly destinations: $M > N = 0$. Then, its optimal fee reduces to

$$\phi^* = -\frac{1}{2} \frac{\int z\,(q_0^o m_0 + q_1^o m_1)\, M\mathrm{d}F}{\int z\,(q_0' m_0 + q_1' m_1)\, M\mathrm{d}F} > \frac{q_0^o}{-2q_0'} = \frac{1}{2} s_0, \tag{26}$$

where we use $q_1^o = q_0^o$ and $-q_0' > -q_1' > 0$ to put lower boundary for the numerator and a higher boundary for the denominator. It can be shown that the RHS of this inequality is larger than $\widehat{\phi}$.

Second, consider an airport that operates only duopoly destination ($N > M = 0$). It sets an optimal fee equal to

$$\phi^* = -\frac{1}{2}\frac{\int z\left[2q^o_{00}n_{00} + (q^o_{01} + q^o_{10})n_{01}\right] N\mathrm{d}G}{\int z\left[2q'_{00}n_{00} + (q'_{01} + q'_{10})n_{01}\right] N\mathrm{d}G} > \frac{2q^o_{00}}{-2\left(q'_{01} + q'_{10}\right)} = \frac{7}{12}s_0\frac{4s_1 - s_0}{s_1}, \tag{27}$$

where we used $q^o_{01} + q^o_{10} > 2q^o_{00}$ and $-(q'_{01} + q'_{10}) > -2q'_{00} > 0$ to bound the numerator of the LHS ratio by below and its denominator by above. Again, this can also be shown to be larger than $\hat{\phi}$.

Finally, consider an airport with monopolies and duopolies ($M > 0, N > 0$). We can put the same lower and upper bounds in the numerator and denominator of (24). Then,

$$\phi^* > \frac{1}{2}\frac{q^o_0\int zM\mathrm{d}F + 2q^o_{00}\int zN\mathrm{d}G}{-q'_0\int zM\mathrm{d}F - (q'_{01} + q'_{10})\int zN\mathrm{d}G} = \frac{1}{2}\frac{q^o_0 + 2q^o_{00}X}{-q'_0 - (q'_{01} + q'_{10})X}, \tag{28}$$

where $X \equiv \left(\int zN\mathrm{d}G\right) / \left(\int zM\mathrm{d}F\right)$ is a hyperbolic function of X with a negative root, a negative vertical asymptote and a positive horizontal asymptote. As X rises from zero to infinity, the RHS of (28) monotonically moves between two values that are equal the RHS of (26) and (27). Since the latter are both larger than $\widehat{\phi}$, the RHS of (28) also lies above $\widehat{\phi}$. To sum up, there exist no equilibrium with small fees $\phi \in [0, \widehat{\phi})$.

Proof of Proposition 3

For monopoly airline destinations only ($M > N = 0$), the above system reduces to the three conditions

$$\phi^* = -\frac{1}{2}\frac{q^o_0\int^{z_1} zM\mathrm{d}F + q^o_1\int_{z_1} zM\mathrm{d}F}{q'_0\int^{z_1} zM\mathrm{d}F + q'_1\int_{z_1} zM\mathrm{d}F} = \frac{1}{2}\frac{\int^{z_1} z\mathrm{d}F + \int_{z_1} z\mathrm{d}F}{\frac{1}{s_0}\int^{z_1} z\mathrm{d}F + \frac{1}{s_1}\int_{z_1} z\mathrm{d}F} \in [\frac{s_0}{2}, \frac{s_1}{2}],$$

$$M\left[1 - F(z_1)\right] \leq K,$$

and

$$z_1 = \frac{\mu}{q_1 - q_0} = \frac{2\mu\left(s_0 s_1\right)}{\phi(s_1 - s_0)}.$$

Hence, if $M \leq K$, $\mu = 0$, $z_1 = 0$, and $\phi^* = s_1/2$. Otherwise, if $M \geq K$, $z_1 = F^{-1}\left(1 - K/M\right)$ and

$$\phi^*_M = \frac{1}{2}\left[\frac{1}{s_1} + \left(\frac{1}{s_0} - \frac{1}{s_1}\right)L^m(1 - K/M)\right]^{-1} \in \left[\frac{s_0}{2}, \frac{s_1}{2}\right],$$

where $L^m(x) = \int^{F^{-1}(x)} zdF(z)/\int zdF(z)$ represents the share of passenger demand below the percentile x of its total demand. This is an increasing function.

Proof of Proposition 4

For duopolies $N > M = 0$, with $\phi \in [\hat{\phi}, \bar{\phi}]$, we have four conditions for four variables ϕ, z_{01}, z^* and μ.

$$\phi^* = -\frac{1}{2}\frac{2q^o_{00}\int^{z_{01}} zN\mathrm{d}G + (q^o_{01}+q^o_{10})\int_{z_{01}}^{z^*} zN\mathrm{d}G + 2q^o_{11}\int_{z^*} zN\mathrm{d}G}{2q'_{00}\int^{z_{01}} zN\mathrm{d}G + (q'_{01}+q'_{10})\int_{z_{01}}^{z^*} zN\mathrm{d}G + 2q'_{11}\int_{z^*} zN\mathrm{d}G},$$

$$N\left[G(z^*) - G(z_{01})\right] + 2N\left[1 - G(z^*)\right] \leq K,$$

$$z_{01} \equiv \frac{\mu}{q_{01}+q_{10}-2q_{00}} = \frac{3\mu s_0 (4s_1 - s_0)}{(2\phi + s_0)(s_1 - s_0)},$$

and

$$z^* \equiv \frac{\mu}{2q_{11} - q_{01} - q_{10}} = \mu\frac{3s_0 s_1 (4s_1 - s_0)}{(s_1 - s_0)(6\phi s_1 - s_0 s_1 - 2\phi s_0)}.$$

If the capacity constraint is not binding, $\mu = 0$ so that $z_{01} = z^* = 0$. All duopolies fly on peak. The constraint becomes: $2N \leq K$. The optimal fee is

$$\phi^* = -\frac{1}{2}\frac{2q^o_{11}\int zN\mathrm{d}G}{2q'_{11}\int zN\mathrm{d}G} = \frac{s_1}{2}.$$

The passenger fee is the same as in monopoly airlines. However, the duopolies board more passengers in so that the airport makes more profit. Their passenger supply is therefore more elastic to the passenger fee.

If the capacity constraint binds, we get $\mu > 0$. We define

$$a(\phi) \equiv \frac{z^*}{z_{01}} = \frac{q_{01}+q_{10}-2q_{00}}{2q_{11}-q_{01}-q_{10}} = \frac{s_1(2\phi + s_0)}{2\phi(3s_1 - s_0) - s_0 s_1},$$

which is function that decreases from $(8s_1 - 3s_0)/s_0$ to 1 when ϕ increases from $\hat{\phi}$ to $\bar{\phi}$. We can express the equilibrium with the following two non-linear equations with two unknowns z_{01} and ϕ^* :

$$2 - K/N = G(z_{01}) + G\left[z_{01}a(\phi)\right], \tag{29}$$

$$\phi^* = \frac{1}{2}\frac{2q^o_{11}Z^{\mathrm{d}} + (2q^o_{00} - q^o_{01} - q^o_{10})Z^{\mathrm{d}}(z_{01}) + (q^o_{01} + q^o_{10} - 2q^o_{11})Z^{\mathrm{d}}\left[z_{01}a(\phi^*)\right]}{-2q'_{11}Z^{\mathrm{d}} - (2q'_{00} - q'_{01} - q'_{10})Z^{\mathrm{d}}(z_{01}) - (q'_{01} + q'_{10} - 2q'_{11})Z^{\mathrm{d}}\left[z_{01}a(\phi^*)\right]}, \tag{30}$$

where $Z^{\mathrm{d}}(z) = \int^z \zeta dG(\zeta)$ and $Z^{\mathrm{d}} = \int zdG(z)$.

Remind the definitions of $L^{\mathrm{d}}(x) = \int^{G^{-1}(x)} zdG(z)/\int zdG(z)$ and $Z^{\mathrm{d}} = \int zdG(z)$. Inverting the capacity constraint, we get $z_{01}a(\phi) = G^{-1}\left[2 - K/N - G(z_{01})\right]$. So, $Z^{\mathrm{d}}\left[z_{01}a(\phi^*)\right] = \int^{z_{01}a(\phi^*)} zdG(z) = L^{\mathrm{d}}\left[2 - K/N - G(z_{01})\right] * Z^{\mathrm{d}}$. Also, $Z^{\mathrm{d}}(z_{01}) = L^{\mathrm{d}}\left[G(z_{01})\right] * Z^{\mathrm{d}}$. The optimal fee writes as the following function of z_{01}:

$$\begin{aligned}\phi^* &= \frac{1}{2}\frac{2q^o_{11}+\left(2q^o_{00}-q^o_{01}-q^o_{10}\right)L^{\mathrm{d}}\left[G\left(z_{01}\right)\right]+\left(q^o_{01}+q^o_{10}-2q^o_{11}\right)L^{\mathrm{d}}\left[2-K/N-G(z_{01})\right]}{-2q'_{11}-\left(2q'_{00}-q'_{01}-q'_{10}\right)L^{\mathrm{d}}\left[G\left(z_{01}\right)\right]-\left(q'_{01}+q'_{10}-2q'_{11}\right)L^{\mathrm{d}}\left[2-K/N-G(z_{01})\right]}\\ &= \frac{3}{4}s_0s_1\frac{2\left(4s_1-s_0\right)+\left(5s_1-s_0\right)\left\{L^{\mathrm{d}}\left[G\left(z_{01}\right)\right]-L^{\mathrm{d}}\left[2-K/N-G(z_{01})\right]\right\}}{s_0\left(4s_1-s_0\right)+s_1\left(s_1-s_0\right)L^{\mathrm{d}}\left[G\left(z_{01}\right)\right]+\left(s_1-s_0\right)\left(3s_1-s_0\right)L^{\mathrm{d}}\left[2-K/N-G(z_{01})\right]}.\end{aligned}$$

For uniform distribution $G(z)=z$ on $[0,1]\to[0,1]$, this gives $Z^{\mathrm{d}}(z)=z^2/2$ and $L^{\mathrm{d}}(x)=\int^x zdz/\int zdz=x^2$. The system becomes

$$\begin{aligned}\phi &= \frac{1}{2}\frac{2q^o_{11}\left(1+a(\phi)\right)^2-\left(q^o_{01}+q^o_{10}-2q^o_{00}\right)\left(2-K/N\right)^2-\left(2q^o_{11}-q^o_{01}-q^o_{10}\right)\left(a(\phi)\right)^2\left(2-K/N\right)^2}{-2q'_{11}\left(1+a(\phi)\right)^2-\left(2q'_{00}-q'_{01}-q'_{10}\right)\left(2-K/N\right)^2-\left(q'_{01}+q'_{10}-2q'_{11}\right)\left(a(\phi)\right)^2\left(2-K/N\right)^2},\\ z_{01} &= \frac{2-K/N}{1+a(\phi)}.\end{aligned}$$

After plugging the values of q^o_{ij},q'_{ij} and $a\left(\phi\right)$, we simplify the first equation to

$$\phi^*=\frac{2s_1s_0\left(4s_1-s_0\right)^2-s_1s_0\left(s_1-s_0\right)\left(2s_1-s_0\right)\left(2-K/N\right)^2}{4s_0\left(4s_1-s_0\right)^2+4s_1\left(s_1-s_0\right)\left(3s_1-s_0\right)\left(2-K/N\right)^2}.$$

Then one can show that $\phi^*>\widehat{\phi}$ if and only if

$$\begin{aligned}&3\left(2s_1-s_0\right)\left(2s_1s_0\left(4s_1-s_0\right)^2-s_1s_0\left(s_1-s_0\right)\left(2s_1-s_0\right)\left(2-K/N\right)^2\right)\\ &>\\ &s_0s_1\left(s_0\left(4s_1-s_0\right)^2+4s_1\left(s_1-s_0\right)\left(3s_1-s_0\right)\left(2-K/N\right)^2\right),\end{aligned}$$

which is true for all K/N when holds for $K/N=0$, i.e.,

$$\left(12s_1-7s_0\right)\left(4s_1-s_0\right)^2-4\left(s_1-s_0\right)\left(3s_0^2-16s_0s_1+24s_1^2\right)>0.$$

This simplifies to

$$48s_1^2\left(s_1-s_0\right)+40s_1^3+8s_1\left(s_1-s_0\right)+5s_0^3>0.$$

Hence, $\phi^*>\widehat{\phi}$.

We also give the condition for which the optimal solution ϕ^* is in the interval $[0,\overline{\phi}]$. Using the definition of $\overline{\phi}$ and simplifying, we get $\phi^*\leq\overline{\phi}$ iff

$$2\frac{\left(2s_1-3s_0\right)}{\left(s_1-s_0\right)}<\left(2-K/N\right)^2.$$

This is always true if the LHS is negative; that is, if $s_1<3s_0/2$.

We finally show that an increase of s_1/s_0 also increases the fee ϕ^*. Indeed, let $x=s_1/s_0>1$ and $y=\left(2-K/N\right)^2\in[0,2]$. Then, $\mathrm{d}\phi^*/\mathrm{d}x>0$ iff $y^2x^2\left(x-1\right)^2+y\left(4x-1\right)\left(2x-1\right)\left(14x^2+1\right)-$

$2(4x-1)^4 < 0$. The LHS is a convex quadratic function of y that accepts a single positive root and has negative values at $y = 0$ and $A = 2$. So, the condition holds for all $y \in [0, 2]$.

Proof of Proposition 6

We prove that the welfare function decreases in ϕ and that it has a positive jump at $\widehat{\phi}$.

Welfare function decreasing in ϕ

To prove the first part of the proposition, we need to prove that welfare decreases with higher ϕ above $\widehat{\phi}$. When ϕ lies above $\widehat{\phi}$, social welfare becomes

$$\begin{aligned}\frac{\mathrm{d}W_{11}}{\mathrm{d}\phi} &= \int_{z^*}^{\infty} 2z\frac{\mathrm{d}w_{11}}{\mathrm{d}\phi}N\mathrm{d}G(z) + \int_{z_{01}}^{z^*} z\frac{\mathrm{d}(w_{01}+w_{10})}{\mathrm{d}\phi}N\mathrm{d}G(z) + \int_{0}^{z_{01}} 2z\frac{\mathrm{d}w_{00}}{\mathrm{d}\phi}N\mathrm{d}G(z) \\ &- \frac{\mathrm{d}z^*}{\mathrm{d}\phi}z^*(2w_{11}-w_{01}-w_{10})Ng(z^*) - \frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}z_{01}(w_{01}+w_{10}-2w_{00})Ng(z_{01}),\end{aligned} \tag{31}$$

where the three first terms are negative because their integrands are negative, while the parentheses in the last two terms are positive. The values of z_{01} and z^* are equilibrium values and depends on μ given by

$$K \geq \int_{z^*}^{\infty} 2N\mathrm{d}G(z) + \int_{z_{01}}^{z^*} zN\mathrm{d}G(z).$$

When this capacity constraint is slack, $\mu = 0$ so that $dz_{01}/d\phi = dz^*/d\phi = 0$ and therefore $dW_{01}(\phi)/d\phi < 0$. When the constraint binds, we have $\mu > 0$ and

$$G(z^*) + G(z_{01}) = 2 - \frac{K}{N}.$$

This imposes

$$\frac{\mathrm{d}z^*}{\mathrm{d}\phi}g(z^*) + \frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}g(z_{01}) = 0. \tag{32}$$

Therefore z^* and z_{01} move in opposite directions.

Using the definitions of z_{01} and z^* we have

$$\begin{aligned}\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi} &= \frac{1}{q_{01}+q_{10}-2q_{00}}\frac{\mathrm{d}\mu}{\mathrm{d}\phi} + \mu\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{1}{q_{01}+q_{10}-2q_{00}}\right) \\ \frac{\mathrm{d}z^*}{\mathrm{d}\phi} &= \frac{1}{2q_{11}-q_{01}-q_{10}}\frac{\mathrm{d}\mu}{\mathrm{d}\phi} + \mu\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{1}{2q_{11}-q_{01}-q_{10}}\right)\end{aligned}$$

where the ratio in each first term is positive for $\phi > \widehat{\phi}$ and each last term can be checked to be negative. To be compatible with (32), it must be that $\frac{\mathrm{d}\mu}{\mathrm{d}\phi} > 0$: by contradiction, if $\frac{\mathrm{d}\mu}{\mathrm{d}\phi} \leq 0$, both $\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}$ and $\frac{\mathrm{d}z^*}{\mathrm{d}\phi}$ are negative, which is impossible with (32). Intuitively, capacity constraint represents a higher value of the private airport when ϕ is larger.

Also, let us write

$$\frac{\mathrm{d}\ln z_{01}}{\mathrm{d}\phi} = \frac{\mathrm{d}\ln\mu}{\mathrm{d}\phi} - \frac{\mathrm{d}\ln\left(q_{01}+q_{10}-2q_{00}\right)}{\mathrm{d}\phi}$$
$$\frac{\mathrm{d}\ln z^*}{\mathrm{d}\phi} = \frac{\mathrm{d}\ln\mu}{\mathrm{d}\phi} - \frac{\mathrm{d}\ln\left(2q_{11}-q_{01}-q_{10}\right)}{\mathrm{d}\phi}$$

So, we have

$$\frac{\mathrm{d}\ln z_{01}}{\mathrm{d}\phi} > \frac{\mathrm{d}\ln z^*}{\mathrm{d}\phi} \iff \frac{\mathrm{d}\ln\left(q_{01}+q_{10}-2q_{00}\right)}{\mathrm{d}\phi} < \frac{\mathrm{d}\ln\left(2q_{11}-q_{01}-q_{10}\right)}{\mathrm{d}\phi}$$

After replacement and simplifications, this is equivalent to

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\phi}\ln\left(\frac{s_0s_1-\phi\left(2s_1-s_0\right)}{(4s_1-s_0)s_0}+\frac{2s_1-s_0-\phi}{4s_1-s_0}-2\frac{s_0-\phi}{3s_0}\right)\\
&<\frac{\mathrm{d}}{\mathrm{d}\phi}\ln\left(2\frac{s_1-\phi}{3s_1}-\frac{s_0s_1-\phi\left(2s_1-s_0\right)}{(4s_1-s_0)s_0}-\frac{2s_1-s_0-\phi}{4s_1-s_0}\right)\\
&\iff 2s_0\frac{4s_1-s_0}{\left(2\phi+s_0\right)\left(s_0s_1+2\phi s_0-6\phi s_1\right)}<0\\
&\iff \frac{s_0s_1}{2\left(3s_1-s_0\right)}<\phi\\
&\iff \hat{\phi}<\phi
\end{aligned}$$

which holds true. Therefore, since z_{01} and z^* move in opposite direction, it must be that

$$\frac{\mathrm{d}\ln z_{01}}{\mathrm{d}\phi} > 0 > \frac{\mathrm{d}\ln z^*}{\mathrm{d}\phi} \iff \frac{\mathrm{d}z_{01}}{\mathrm{d}\phi} > 0 > \frac{\mathrm{d}z^*}{\mathrm{d}\phi}$$

Finally, a sufficient condition for $\frac{\mathrm{d}W_{11}}{\mathrm{d}\phi} < 0$ is that the second line in (31) is negative. That is, we successively get

$$\begin{aligned}
-\frac{\mathrm{d}z^*}{\mathrm{d}\phi}z^*(2w_{11}-w_{01}-w_{10})Ng(z^*)-\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}z_{01}(w_{01}+w_{10}-2w_{00})Ng(z_{01})<0 &\iff\\
\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}g(z_{01})z^*(2w_{11}-w_{01}-w_{10})Ng(z^*)+\frac{\mathrm{d}z_{01}}{\mathrm{d}\phi}z_{01}(w_{01}+w_{10}-2w_{00})Ng(z_{01})>0 &\iff\\
z^*(2w_{11}-w_{01}-w_{10})+z_{01}(w_{01}+w_{10}-2w_{00})>0,&
\end{aligned}$$

which is true.

Positive jump at $\widehat{\phi}$

To prove the existence of a positive jump at $\widehat{\phi}$, we need to determine the welfare levels of W_{11} and W_{10} about $\widehat{\phi}$.

$$\begin{aligned} W_{11}\left(\widehat{\phi}\right) - W_{01}\left(\widehat{\phi}\right) = & \int_{z^*\left(\widehat{\phi},\mu^+\right)}^{\infty} z(2w_{11})N\mathrm{d}G(z) + \int_{z_{01}\left(\widehat{\phi},\mu^+\right)}^{z^*\left(\widehat{\phi},\mu^+\right)} z(w_{01}+w_{10})N\mathrm{d}G(z) \\ & + \int_{0}^{z_{01}\left(\widehat{\phi},\mu^+\right)} z(2w_{00})N\mathrm{d}G(z) - \int_{z_{01}\left(\widehat{\phi},\mu^-\right)}^{\infty} z(w_{01}+w_{10})N\mathrm{d}G(z) \\ & - \int_{0}^{z_{01}\left(\widehat{\phi},\mu^-\right)} z(2w_{00})N\mathrm{d}G(z), \end{aligned}$$

where we denote the threshold by their components ϕ and μ, and where superscript $-$ and $+$ refer to ϕ to be on the left $\left(\phi < \widehat{\phi}\right)$ or right $\left(\phi > \widehat{\phi}\right)$ of $\widehat{\phi}$, respectively. Rearranging and simplifying, we get

$$W_{11}\left(\widehat{\phi}\right) - W_{01}\left(\widehat{\phi}\right) = (2w_{11}-w_{01}-w_{10})N\int_{z^*\left(\widehat{\phi},\mu^+\right)}^{\infty} z\mathrm{d}G(z) + (w_{01}+w_{10}-2w_{00})N\int_{z_{01}\left(\widehat{\phi},\mu^+\right)}^{z_{01}\left(\widehat{\phi},\mu^-\right)} z\mathrm{d}G(z) \tag{33}$$

Notice that the first part of equation (33) is always positive. By contrast, the second part is positive only if

$$\int_{z_{01}\left(\widehat{\phi},\mu^+\right)}^{z_{01}\left(\widehat{\phi},\mu^-\right)} z\mathrm{d}G(z) > 0.$$

To verify it, we have to derive the explicit values of μ and the consequent thresholds $z_{01}\left(\widehat{\phi},\mu^-\right)$ and $z_{01}\left(\widehat{\phi},\mu^+\right)$ when we assume uniform distribution. The values of μ^-, μ^+, $z_{01}\left(\widehat{\phi},\mu^-\right)$ and $z_{01}\left(\widehat{\phi},\mu^+\right)$ depend on the value of K compared to N, respectively. Thus we have to find the values of μ and z_{01} in four possible cases.

Begin with $\phi < \hat{\phi}$, starting from $N > K$. In this case, we need to verify whether we have either $\mu = 0$ and slack constraint $N\left[1 - G(z_{01})\right] < K$, or $\mu > 0$ and binding constraint $N\left[1 - G(z_{01})\right] = K$. Suppose $\mu = 0$, then $z_{01} = 0$ and $N\left[1 - G(0)\right] = N > K$, which contradicts the slack constraint $N\left[1 - G(z_{01})\right] = N < K$. It follows that $\mu > 0$ and given by the binding capacity $N\left[1 - G(z_{01})\right] = K$, we get

$$N\left[1 - G\left(\frac{3\mu s_0\left(4s_1 - s_0\right)}{\left(s_1 - s_0\right)\left(s_0 + 2\phi\right)}\right)\right] = K,$$

$$\Longleftrightarrow$$

$$\frac{\left(s_1 - s_0\right)\left(s_0 + 2\phi\right)}{3s_0\left(4s_1 - s_0\right)}G^{-1}\left(1 - K/N\right) = \mu,$$

which, with uniform distribution, it becomes:

$$\mu = \mu^{-}_{N>K} \equiv \frac{(s_1 - s_0)(s_0 + 2\phi)}{3s_0(4s_1 - s_0)}(1 - K/N).$$

By plugging $\mu^{-}_{N>K}$ into z_{01}, we get

$$z^{-}_{01,N>K} = 1 - K/N.$$

We now turn to $\phi < \hat{\phi}$ when $N < K$. In this case, we need to verify whether we have either $\mu = 0$ and slack constraint $N[1 - G(z_{01})] < K$, or $\mu > 0$ and binding constraint $N[1 - G(z_{01})] = K$. Suppose then that $N[1 - G(z_{01})] = K$ implies $\mu \leq 0$. For this, we state $[1 - G(z_{01})] = K/N > 1 \iff G(z_{01}) < 0$, which is impossible. So,

$$\mu = \mu^{-}_{N<K} = 0 \text{ and } z^{-}_{01,N<K} = 0.$$

Consider next $\phi > \hat{\phi}$, starting from $N > K$. Now we need to verify whether we have either $\mu = 0$ and slack constraint $2N[1 - G(z^*)] + N[G(z^*) - G(z_{01})] < K$, or $\mu > 0$ and binding constraint $2N[1 - G(z^*)] + N[G(z^*) - G(z_{01})] = K$. If we suppose $\mu = 0$, then $z_{01} = z^* = 0$, and the slack constraint is $2N[1 - G(0)] + N[G(0) - G(0)] = N < K$, which contradicts $N > K$. So, it should be that $\mu > 0$ given by $2N[1 - G(z^*)] + N[G(z^*) - G(z_{01})] = K \iff 2 - K/N = G(z^*) + G(z_{01})$. For uniform distribution,

$$\mu = \mu^{+}_{N>K} \equiv \frac{(s_1 - s_0)(2 - K/N)}{3s_0(4s_1 - s_0)\left(\frac{s_1}{(6s_1 - 2s_0)\left(\phi - \hat{\phi}\right)} + \frac{1}{(s_0 + 2\phi)}\right)},$$

which, for $\phi = \hat{\phi}$, becomes $\lim_{\phi \to \hat{\phi}} \mu^{+}_{N>K} \equiv 0$, from which we get $z^{+}_{01,N>K} = 0$.

We are left with the case $\phi < \hat{\phi}$ If and $N < K$. Now we need to verify again if we have either $\mu = 0$ and slack constraint $2N[1 - G(z^*)] + N[G(z^*) - G(z_{01})] < K$, or $\mu > 0$ and binding constraint $2N[1 - G(z^*)] + N[G(z^*) - G(z_{01})] = K$. Accordingly, we suppose that $\mu = 0$. Then, the slack constraint $2N[1 - G(0)] + N[G(0) - G(0)] = 2N < K$, which cannot be the case when $2N > K$ as considered above. So, $\mu > 0$ for $2N > K$ and, with uniform distribution, $\mu = \mu^{+}_{N<K} = \mu^{+}_{N>K} = 0$. To sum up, in the case of a uniform distribution, we get

Table 1

		μ	z_{01}
$\phi < \hat{\phi}$	$N > K$	$\frac{(s_1 - s_0)(s_0 + 2\phi)(1 - K/N)}{3s_0(4s_1 - s_0)}$	$1 - K/N$
	$N < K$	0	0
$\phi > \hat{\phi}$	$N > K$	0	0
	$N < K$	0	0

We are now in a position to compare W_{11} and W_{01} in the two cases $N > K$ and $N < K < 2N$. Consider first $N > K$. By plugging the results from Table 1 in (33), we get

$$\int_{z_{01}\left(\widehat{\phi},\mu^{+}_{N>K}\right)}^{z_{01}\left(\widehat{\phi},\mu^{-}_{N>K}\right)} z\mathrm{d}G(z) = \int_{0}^{1-K/N} z\mathrm{d}G(z) > 0.$$

Second, consider $N < K$. Then,

$$\int_{z_{01}\left(\widehat{\phi},\mu^{+}\right)}^{z_{01}\left(\widehat{\phi},\mu^{-}\right)} z\mathrm{d}G(z) = \int_{0}^{0} z\mathrm{d}G(z) = 0.$$

Therefore, in any possible scenario we have a positive jump in the level of welfare when $\phi = \widehat{\phi}$, by assuming uniform distribution.