A note on successive oligopolies and vertical mergers

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Abstract

In this paper we analyze how the technology used by downstream firms can influence input and output market prices. We show via an example that both these prices increase under a decreasing returns technology while the contrary holds when the technology is constant.

Keywords: successive oligopolies, vertical integration, technology, foreclosure

JEL classification: D43, L1, L22, L42

1 Introduction

We provide in this paper a framework to analyse vertical integration and horizontal mergers in a market context, based on early work of the authors ; see Gabszewicz and Zanaj (2006). The analysis of collusion in successive markets between downstream and upstream firms often relies on successive quantity oligopolies. In these markets, firms select *à la Cournot* the quantities of output of the good they produce, the output of the upstream firms serving as an input for the production of the output in the downstream market. Collusive agreements reduce the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them. Collusive outcomes are the Cournot equilibria corresponding to this reduced number of oligopolists in each market, which are then compared with those arising when downstream and upstream firms act independently. The link between upstream and downstream markets follows from the fact that the downstream firms' unit cost appears as the unit revenue for the upstream ones : the price paid for a unit of input for the firms in the former constitutes the unit receipt for the firms in the latter. In this paper, we consider the simplest situation in which, before any collusion takes place, the downstream market consists of two rival firms while the upstream one embodies three input suppliers. We

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apply to this context the methodology proposed above to study the effects of vertical integration on input and output prices.

We propose a model which makes explicit how the downstream and upstream markets are linked to each other *via* the *technology* used by the downstream firms to transform the input into the output. We consider two examples of technology-linked markets. The first corresponds to a decreasing returns technology while the other uses a constant returns technology. Our main finding is that the nature of the technology plays a crucial role on the effects of mergers concerning the input and output prices.

2 Successive oligopolies with decreasing returns

2.1 The context

Consider two *downstream* firms facing a linear demand $\pi(Q) = 1 - Q$ in the downstream market with Q denoting aggregate output. They share the same technology $f(z)$ to produce the output, namely

$$q = f(z) = z^{\frac{1}{2}}.$$

with z denoting the input used in the production of q. Consider also three upstream firms each producing the input z at the same linear total cost $C_j(s_j) = \beta s_j,\ j = 1, 2, 3$. We assume that this situation gives rise to two games. The players in the first game are the *two* downstream firms with output strategies q_i, while the players in the second are the *three* upstream firms with input strategies s_j.

The profits of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - q_{-i})q_i - pq_i^2,\ i = 1, 2$$

As a result of the strategic choice q_i, each firm i sends the input quantity signal $z_i(p) = q_i^2$ to the upstream market. When aggregating these signals, we get the demand function of input over which the upstream firms select their selling strategies s_j. The j_{th} upstream firm's profit Γ_j a the vector of strategies writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j})s_j - \beta s_j, \tag{1}$$

with $p(s_j, s_{-j})$ such that $\Sigma_{k=1}^3 s_k = \Sigma_{i=1}^2 z_i(p)$.

Given a price p in the input market, the best reply of downstream firm i in the downstream game obtains as

$$q_i = \frac{1 - q_{-i}.}{2p + 2} \tag{2}$$

Clearly, these best replies depend on the upstream market price p and we may compute the symmetric Nash equilibrium of the above game, contingent on the price p. Defining $q_i = q$ for $i = 1, 2$, re-expressing equation (2) and solving it in q, we get the symmetric solution

$$q_i^* = \frac{1}{(3+2p)} \tag{3}$$

so that we obtain

$$z_i^*(p) = z^*(p) = \frac{1}{(3+2p)^2};\ i = 1, 2. \tag{4}$$

The upstream firms then face a total demand $\sum_{i=1}^{2} z_i^*(p)$ of input equal to $2z^*(p)$. At a given vector of input strategies chosen by the upstream firms in the upstream game, the input price clearing the upstream market must satisfy

$$\frac{2}{(3+2p)^2} = \Sigma_{j=1}^{3} s_j,$$

so that we get

$$p(s_j, s_{-j}) = \sqrt{\frac{2}{4\Sigma_{j=1}^{3} s_j}} - \frac{3}{2}. \tag{5}$$

Substituting (5) into (1), the payoff function $\Gamma_j(s_j, s_{-j})$ of the upstream firm j in the upstream game rewrites as

$$\Gamma_j(s_j, s_{-j}) = (\sqrt{\frac{2}{4\Sigma_{j=1}^{3} s_j}} - \frac{3}{2})s_j - \beta s_j.$$

Notice that the profit function Γ_j is concave in $s_j, j = 1, 2, 3$, so that we can use the first order necessary and sufficient conditions to characterize the symmetric equilibrium. Accordingly, at the symmetric Nash equilibrium of the upstream game, we obtain

$$s^*(\beta) = \frac{25}{54\,(2\beta+3)^2}.$$

Finally, the equilibrium price $p^*(\beta)$ in the input market obtains as

$$p^*(\beta) = \frac{6}{5}\beta + \frac{3}{10}.$$

Consequently, substituting this equilibrium price into the equilibrium quantities $z^*$of input bought by each downstream firm, as given by (4), we get

$$z^*(\beta) = \frac{25}{36\,(2\beta+3)^2}$$

so that, from (3), we obtain

$$q_i^*(\beta) = q^*(\beta) = \frac{5}{6\,(2\beta+3)}.$$

Therefore, the resulting output price $\pi^*(\beta)$ in the downstream market obtains as

$$\pi^*(\beta) = \frac{6\beta+4}{6\beta+9}.$$

3 Vertical integration

Consider that one of the downstream firms vertically integrates with one of the upstream firms. After this merger, we move from an initial situation comprising globally five firms to a new one, with *two* firms in the downstream market and *a* duopoly in the upstream one. We assume complete foreclosure. Indeed, the integrated entity now internalizes output production by using the input provided at a marginal cost β by the upstream firm belonging to the new entity.

Let us first consider the game played among downstream firms operating in the downstream market after collusion takes place. The new sturcutre of the market is an asymmetric Cournot. The payoff of the integrated firm I is given by

$$\Pi_I(q_I,q_i) = (1 - q_I - q_i)q_I - \beta q_I^2,$$

with i denoting the downstream firm $i, i \neq I$, not belonging to the integrated entity which payoffs $\Pi_i(q_i, q_I)$ is given by

$$\Pi_i(q_i, q_I) = (1 - q_i - q_I)q_i - pq_i^2). \tag{6}$$

Since Π_I is concave, we may use the first order condition to get the best response function of the integrated entity in the downstream market game as

$$q_I = \frac{1-q_i}{2\beta+2}. \tag{7}$$

As for the downstream firm i, $i \neq I$, its best reply in the downstream market is conditional on the input price p realized in the upstream market, namely

$$q_i = \frac{1-q_I}{2+2p} \tag{8}$$

Solving the system of equations (7) and (8), we get the output quantities as

$$\begin{aligned} q_i &= \frac{2\beta+1}{4p+4\beta+4p\beta+3}, \\ q_I &= \frac{2p+1}{4p+4\beta+4p\beta+3}. \end{aligned}$$

Consequently, as expected, the downstream equilibrium is conditional on the input price obtained in the upstream market as a result of supply and demand in this market. There is only one downstream firm with input demand identical to the total demand in the upstream market, namely, $q_i(p) = (\frac{2\beta+1}{4p+4\beta+4p\beta+3})^2$. As for the supply, it comes from the strategies s_j, $j \neq I$, selected by the unintegrated upstream firms in this market. Consider one of the two upstream firms which do not belong to the entity. Its profit Γ_j at the vector of strategies (s_j, s_{-j}) writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j})s_j - \beta s_j,$$

with $p(s_j, s_{-j})$ such that $s_j + s_{-j} = q_i(p)$, namely

$$p(s_j, s_{-j}) = \frac{1}{4\beta+4}\left(\frac{\sqrt{s_j+s_{-j}}}{s_j+s_{-j}}(2\beta+1) - 4\beta - 3\right). \tag{9}$$

Therefore, at the symmetric equilibrium in the upstream market, each unintegrated firm supplies a quantity s_j^* of input which obtain as[1]

$$s^* = \frac{9}{32\,(2\beta+3)^2}.$$

Substituting the expression of s_j^* in the demand function (9) we get the input price p^* as

$$p^* = \frac{60\beta + 32\beta^2 + 21}{4\beta+4}.$$

Furthermore, substituting p^* in the expression of q_i and q_I, we obtain

$$\begin{aligned} q_i^* &= \frac{1}{8\,(2\beta+2)}, \\ q_I^* &= \frac{16\beta+23}{16\,(\beta+1)\,(2\beta+3)}. \end{aligned}$$

The price π^* of the output obtains as

$$\pi^* = \frac{1}{16}\frac{(2\beta+1)\,(16\beta+23)}{(\beta+1)\,(2\beta+3)}$$

.

We have now the equilibrium quantities and price for both cases with, and without, vertical integration. It can be shown from a direct comparison of prices that this merger determines both an increase in the output price and an increase in the input prices. Consider first the input market. The merger causes a decrease of both the number of demanders of input and suppliers of it. So,

[1]To determine the optimal input supply per firm see footnote 9 in Gabszewicz and Zanaj (2006).

at least theoretically, there would be *a priori* place for an increase or decrease of the input price. *Under a decreasing returns technology in the downstream market, the change in the input demand resulting from the merger countervails the change in the input supply always leading to an increase in the input price.* Concerning the output market, the merger reduces the production cost of the firm in the entity. But, on the other hand, the merger increases the production cost of the other firm, since it leads to a higher input price. It turns out that, in this framework, *the first effect is dominated by the second, so that the output price increases.*

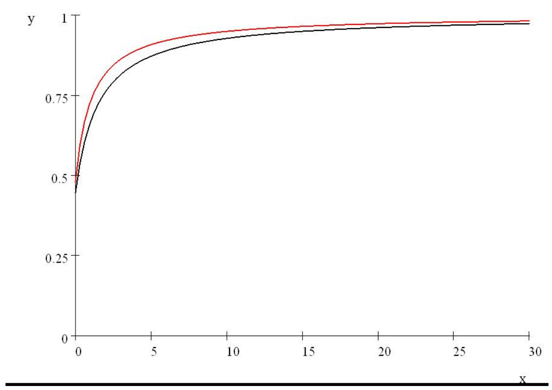

Figure 1: Output price with and without mergers

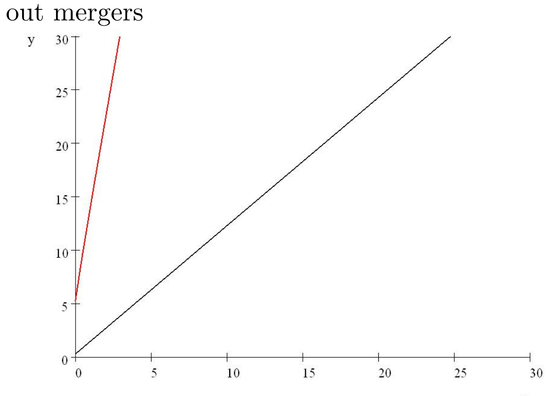

Figure 2: Input prices with and without mergers

4 Constant returns

We consider exactly the same case as above, with the exception that the technology $f(z)$ shared by the downstream firms is now given by

$$f(z) = \alpha z,\ \alpha > 0$$

as in Salinger (1988) and Gaudet and Van Long (1996)(with $\alpha = 1$ in the latter case).We also assume that $\alpha \geq \beta$: this assumption guarantees that the marginal cost of producing the input does not exceed its marginal product in the production of output[2]. The profits $\Pi_i(q_i, q_{-i})$ of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) now obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - q_{-i})q_i - pz_i.$$

As a result of the strategic choice q_i, each firm i sends an input quantity signal $z_i(p) = \frac{q_i}{\alpha}$ to the upstream market. Rewriting the profit function of each firm in terms of input z, we may compute the symmetric Nash equilibrium of the above game contingent on the input price p,namely

$$z^*(p) = \frac{\alpha - p}{3\alpha^2}, \tag{10}$$

so that

$$q^* = \frac{\alpha - p}{3\alpha}. \tag{11}$$

Given the input strategies chosen by the upstream firms in the second stage game, the input price clearing the upstream market must satisfy

$$\frac{2(\alpha - p)}{3\alpha^2} = \Sigma_{j=1}^3 s_j$$

so that, for this example, we get

$$p(s_j, s_{-j}) = \alpha - \alpha^2 \frac{3}{2} \Sigma_{j=1}^3 s_j \tag{12}$$

Substituting (12) into the payoff function $\Gamma_j(s_j, s_{-j})$ we have

$$\Gamma_j(s_j, s_{-j}) = \left(\alpha - \frac{3\alpha^2}{2} \Sigma_{j=1}^3 s_j\right) s_j - \beta s_j,$$

leading to the best response function

$$s_j(s_{-j}) = \frac{1}{3\alpha^2}(\alpha - \beta) - \Sigma_{-j=1}^2 s_{-j}.$$

Accordingly, at the symmetric equilibrium of the second stage game, we obtain

[2] We also assume that $\alpha \geq \beta$: this assumption guarantees that the marginal cost of producing the input does not exceed its marginal product in the production of output.

$$s^*(\beta) = \frac{(\alpha - \beta)}{6\alpha^2}.$$

Finally, the equilibrium price in the input market obtains as

$$p^*(\alpha, \beta) = \frac{1}{4}\alpha + \frac{3}{4}\beta. \tag{13}$$

Consequently, substituting this equilibrium price into the equilibrium quantities z_i^* of input bought by each downstream firm, as given by (10), we get

$$z^*(\alpha, \beta) = \frac{1}{4\alpha^2}\left(\alpha - \beta\right),$$

so that

$$q_i^*(\alpha, \beta) = \frac{1}{4\alpha}\left(\alpha - \beta\right).$$

Accordingly, the resulting output price $\pi^*(m, n)$ in the downstream market obtains as[3]

$$\pi^*(m, n) = \frac{1}{12\alpha}\left(6\alpha + 6\beta\right).$$

4.1 Modelling collusion with constant returns technology

Assume again that two firms, one upstream and one downstream merge. Let us first consider the game played between the two firms operating in the downstream market after collusion takes place. The payoff of the integrated firm I is given by

$$\Pi_I(q_I, q_i) = (1 - q_I - q_i)q_I - \beta\frac{q_I}{\alpha}$$

. As for the downstream firm i, not belonging to the integrated entity, it has as payoff

$$\Pi_i(q_i, q_I) = (1 - q_i - q_I)q_i - p(\frac{q_i}{\alpha}). \tag{14}$$

It is clear from the above payoffs that the main difference between the collusive and non collusive members in the downstream market comes from the fact that the former pays its input at marginal cost β while the latter buys it at the input price p. Since Π_I is concave, we may use the first order condition to get the best reply function of the integrated entity and the unintegrated firm. solving the resulting system, we get the equilibrium output quantities $q_I^*(\alpha, \beta, p)$ as

$$q_I^*(\alpha, \beta; p) = \frac{\alpha - \beta + (p - \beta)}{3\alpha}$$

and

$$q_i^*(\alpha, \beta; p) = \frac{\alpha - 2p + \beta}{3\alpha}. \tag{15}$$

[3] Notice that, in order to have $\pi^*(m, n) \geq p^*(m, n)$, - the requirement needed to guarantee the survival of firms in the downstream market -, no condition on α is required.

Consequently, as expected, the downstream equilibrium is conditional on the input price obtained in the upstream market as a result of supply and demand in this market. There is only one firm with total demand equal to $q_i^*(p;\beta) = \frac{\alpha-2p+\beta}{3\alpha}$. As for the supply, it comes from the strategies (s_j, s_{-j}), selected by the unintegrated upstream firms in this market. Consider the j_{th} upstream firm which does not belong to the entity. Its profit Γ_j at the vector of strategies (s_j, s_{-j}) writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j})s_j - \beta s_j,$$

with $p(s_j, s_{-j})$ such that $s_j + s_{-j} = z_i$, namely

$$p(s_j, s_{-j}) = \frac{(\alpha+\beta) - 3\alpha^2 (s_j + s_{-j})}{2} \tag{16}$$

. Accordingly, the payoff of the j-th upstream firm writes as

$$\Gamma_j(s_j, s_{-j}) = \left(\frac{(\alpha+\beta) - 3\alpha^2 (s_j + s_{-j})}{2} \right) s_j - \beta s_j.$$

Therefore , at the symmetric equilibrium in the upstream market, the unintegrated firm supplies a quantity $s_j^*$of input which obtains as

$$s_j^*(\alpha, \beta) = \frac{(\alpha-\beta)}{9\alpha^2}.$$

Substituting the expression of s_j^* in (16) we get the equilibrium input price $p^*(\alpha,\beta)$ as

$$p^*(\alpha,\beta) = \frac{1}{6}\alpha + \frac{5}{6}\beta. \tag{17}$$

Substituting (17) in (15) we get the output supply $q_i^*(\alpha,\beta)$ and $q_I^*(\alpha,\beta)$ of each unintegrated downstream firm, namely

$$q_i^*(\alpha,\beta) = \frac{2}{9\alpha}(\alpha-\beta);$$

$$q_I^*(\alpha,\beta) = \frac{7}{18\alpha}(\alpha-\beta).$$

Hence, the resulting output price $\pi^*$is given by

$$\pi^*(\alpha,\beta) = \frac{1}{18\alpha}(7\alpha+11\beta).$$

It can be easily checked (see graph 3 and 4) that, when downstream firms have a constant returns technology, and a vertical merger takes place, we get exactly the reverse result as in the case of decreasing returns: *in the case of a merger, both the output and input prices decrease.*

5 Concluding remarks

Starting from our previous work, we have analysed here the effects of vertical mergers in the framework of successive oligopolies on input and output prices, when these are determined by the market mechanism. Our analysis is based on a comparison between the market outcomes with, and without, merger. The interest of this approach, it seems to us, consists in showing how these effects are related to the technology used in the production of final goods. It is indeed the latter which determines the demand function of the downstream firms in the input market. When a merger forms, it creates a reduction in total input demand since the integrated entity produces output internally at marginal cost. Similarly, with complete foreclosure (as assumed), a merger reduces input supply since the entity does not participate in the upstream market anymore. The question is to know whether a demand decrease, combined with a supply decrease would lead to higher, or smaller prices in the two markets concerned by the merger. Surprisingly, the answer to this question is not unequivocal since, under decreasing returns to scale, both prices increase while the reverse is true in the case of constant returns !

Mergers are expected to influence market conditions because they internalize production costs reducing thereby the effects of double marginalization. Furthermore, they decrease competition in the input market by reducing the number of competitors. Thus a decrease in the output price is *a priori* expected from the reduction of double marginalisation, and this is indeed the case when downstream firms use a constant returns technology. By contrast, under decreasing returns, the reverse consequence is observed : the output price increases. Why is it so ? As usual, the reduction of double marginalisation increases the output supply of the entity. However, it increases the cost of production of the downstream which did not adhere to the entity, to such an extent that the reduction of its output level finally increases the output price: this decrease does not compensate the effect on price of the entity output increase. As stated above, mergers (under complete foreclosure) reduce the number of firms operating in the upstream market, and thus, it must be anticipated that their existence reduces competition and increases input price. However, under constant returns, it has the opposite effect. Why is it so ? The explanation lies in the behaviour of the unintegrated downstream firm. Due to the merger, this firm considerably reduces its production level in the output market and, accordingly, its demand in the input market becomes smaller at any input price. This leads to a reduction of the input price.

In this paper, we have only scratched a research territory which looks very promising. The theory of successive markets allows us to treat various forms of collusion based on the market mechanism, which provides a natural way to evaluate the profitability of such collusive arrangements. Another potential avenue for an alternative profitability evaluation would consist in relying on strategic market games. From this viewpoint, Gabszewicz *and al.* would constitute a natural departure point for researchers interested in the field.

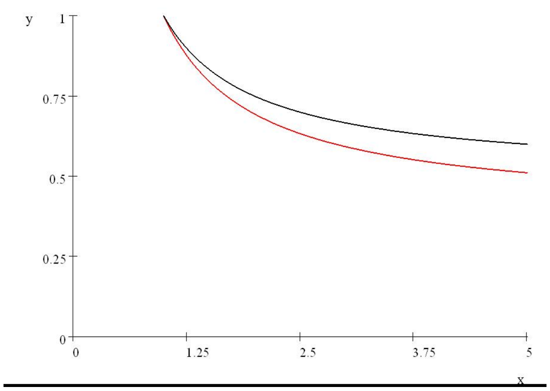

Figure 3: Output price with and without mergers

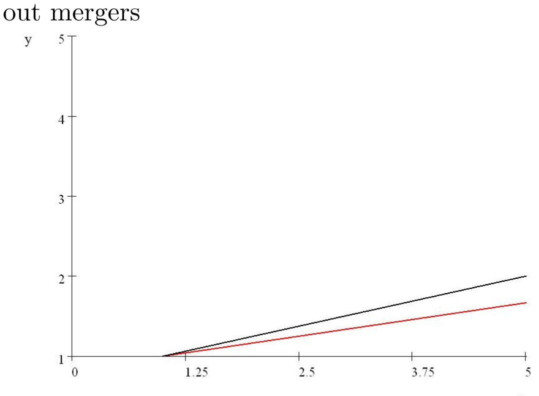

Figure 4: Input prices with and without mergers

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