Successive oligopolies and decreasing returns

Skerdilajda Zanaj

Centre de Recherche en Économie Appliquée (CREA)

Université du Luxembourg

Avenue de la Faiencerie, 162A, L1511,

Luxembourg

July, 2009

Abstract

In this paper, we propose an example of successive oligopolies where the downstream firms share the same decreasing returns technology of the Cobb-Douglas type. We stress the differences between the conclusions obtained under this assumption and those resulting from the traditional example considered in the literature, namely, a constant returns technology. We find that when firms use a decreasing returns technology rather than a linear one: *(i)* the profit of a downstream firm can decrease, when the upstream market is more competitive; *(ii)* the input price does not tend to the corresponding marginal cost when the number of firms in both markets tends to infinite; *(iii)* double marginalization is lower.

Keywords: successive oligopolies, technology.

JEL classification: D43, L13, L22, L40

1 Introduction

The literature on successive oligopolies is traditionally based on examples. In particular, the firms producing the final output - downstream firms -, are assumed to have the same Cobb-Douglas linear technology $f(z) = z$, with z denoting the amount of the single input used in the production process. This simplifying assumption concerns, nevertheless, the crucial feature of industries composed of a chain of markets. Indeed, the link between these markets is the input demand of downstream firms to input suppliers, which depends on the output technology. Consequently, while the homogeneous or linear transformation of the input to the output used in the existing literature, is suitable for all industries in which the output production consists simply on the distribution of the good to the final consumers, such assumption rules out most other industries.

In this paper, we go on with the analysis of successive oligopolies in the same spirit, but now introduce the alternative assumption that downstream firms share the same *decreasing returns* technology. In order to allow for comparisons between the two cases, we assume that the downstream firms use the Cobb-Douglas production function $f(z) = \sqrt{z}$ in the production process so that the two technologies belong to the same class of production functions. Our concern is whether the main conclusions reached under the constant returns assumption still hold when this decreasing returns technology is substituted to the linear one.

As it will be established in this paper, comparing the market solution obtained with the above decreasing returns technology with the linear case, shows that several features are different from those observed under constant returns. First, contrary to the linear case, the profit of a downstream firm, under the decreasing returns technology, may well be decreasing with the number of upstream firms. Second, we find that, again in contrast with the linear case, increasing simultaneously the number of firms in both markets (upstream and downstream), does not let the input market price to converge to the competitive one, namely the marginal cost of producing the input.

Furthermore, comparing the market solutions corresponding to the example of constant returns and decreasing returns technology in the output market, we show that double marginalization is less severe under decreasing returns than under constant returns, reflecting the fact that the cost per unit is higher in the latter than in the former. It is well-known that double-marginalization problem can be alleviated or even be avoided through the use of more sophisticated contracts (Tirole, 1989), but in industries like the cable TV in the US, the business practice is to charge a price per subscriber, thus, in this indutry linear pricing is applied and double marginalisation is an issue. Therefore, evaluating its size gives insighits to the profitability of eventual vertical aggreements.

Finally, we compare the effects of mergers under both assumptions fixing the number of firms in each market. It turns out that, for this particular example, the effects of vertical integration on prices are going in opposite directions. While the input and output prices both increase in the former when vertical integration takes place, both decrease in the latter.

The above discrepancies between market behavior corresponding to alternative technological conditions reveal how fragile are the theoretical conclusions obtained when analyzing the interplay of firms' strategies in successive markets. To get robust conclusions, a general theoretical framework for analyzing successive oligopolistic markets is clearly required.

The paper is organized as follows. In the next section, we develop two games, one in the downstream and the other in the upstream market, to obtain the industry equilibria. In section 2, according to the technology used in the downstream market, we analyze the effects of number of firms in profits, the asymptotic properties of input and output prices, the size of double marginalization, and finally, the effects of technology on collusive agreements. Section 3 concludes.

2 Industry equilibria under constant returns

In this section, we recall the example of successive oligopolies considered in the literature like in Gaudet and Van Long (1996), Ordover et al (1990), or Salinger (1988). There are two markets, the downstream and upstream one, with identical firms in each of them. In these markets, firms select non cooperatively the quantities of the good they produce, the good produced by the upstream firms serving as the only input used in the production of the final output in the downstream market. The link between the two markets follows from the fact that the downstream firms' unit cost appears as the unit revenue for the upstream ones : the price paid for a unit of input for the firms in the former constitutes the unit receipt for the firms in the latter. In the downstream market, firms share the same technology $f(z)$ given by

$$f(z) = z$$

as in Salinger (1988) and Gaudet and Van Long (1996). The profits $\Pi_i(q_i, q_{-i})$ of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) now obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{k \neq i} q_k) q_i - p z_i,$$

with p denoting the input price. As a result of the strategic choice q_i, each firm i sends an input quantity signal $z_i(p) = q_i$ to the upstream market. Given the price p, the best reply of downstream firm i in the downstream game obtains as

$$z_i(z_{-i}; p) = \frac{1 - p - \Sigma_{k \neq i} z_k}{2}, i = 1, ..., n. \tag{1}$$

We may compute the symmetric Nash equilibrium of the above downstream game contingent on the price p. Defining $z_i = z$ for $i = 1...n$, re-expressing equation (1) and solving it in z, we get at the symmetric solution

$$z^*(p) = \frac{1 - p}{(n + 1)}; \tag{2}$$

so that

$$q^*(p) = \frac{1 - p}{(n + 1)}. \tag{3}$$

Now assume that there are m identical upstream firms who produce the input z at the same linear total cost βs_j, $j = 1, .., m$, $\beta \geq 0$. We assume that $\beta \leq$

1[1]. This gives rise to another game whose players are the m upstream firms with strategies $s_j, j = 1, ..m$. Given a n-tuple $(s_1, ..., s_j, ..s_m)$ of input strategies chosen by the upstream firms in the second stage game, the input price clearing the upstream market must satisfy

$$n\frac{(1-p)}{(n+1)} = \Sigma_{k=1}^{m} s_k$$

so that, for this example, we get

$$p(\Sigma_{k=1}^{m} s_k) = 1 - \frac{n+1}{n}\Sigma_{k=1}^{m} s_k. \tag{4}$$

Given a vector of strategies $(s_1, ...s_j, ..s_m)$, the j_{th} upstream firm's profit Γ_j writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j})s_j - \beta s_j. \tag{5}$$

Substituting (4) into the payoff function $\Gamma_j(s_j, s_{-j})$ we have

$$\Gamma_j(s_j, s_{-j}) = \left(1 - \frac{n+1}{n}\Sigma_{k=1}^{m} s_k\right) s_j - \beta s_j,$$

leading to the best response function

$$s_j(s_{-j}) = \frac{n(1-\beta)}{2(n+1)} - \frac{(1+n)\Sigma_{k\neq j} s_k}{2(n+1)},\ j = 1, ..., m.$$

Accordingly, at the symmetric equilibrium of the second stage game, we obtain

$$s^*(m,n) = \frac{n\,(1-\beta)}{(n+1)(m+1)}.$$

Finally, the equilibrium price in the input market obtains as

$$p^*(m,n) = \frac{1+m\beta}{m+1}. \tag{6}$$

Consequently, substituting this equilibrium price into the equilibrium quantities z_i^* of input bought by each downstream firm, as given by (2), we get

$$z^*(m,n) = \frac{m(1-\beta)}{(n+1)(m+1)},$$

so that

$$q_i^*(m,n) = \frac{m(1-\beta)}{(n+1)(m+1)}.$$

Accordingly, the resulting output price $\pi^*(m,n)$ in the downstream market obtains as

$$\pi^*(m,n) = \frac{(1+m+n)+mn\beta}{(n+1)(m+1)}. \tag{7}$$

The profit $\Pi_i(m,n)$ of a downstream firm at equilibrium in the downstream game writes as

$$\Pi_i(m,n) = \frac{m^2(\beta-1)^2}{(n+1)^2(m+1)^2}. \tag{8}$$

This market solution can be now used to determine some properties of profits and prices.

Number of firms and profits

It is easily seen that $\frac{\partial \Pi_i(m,n)}{\partial m} > 0$. In the setup of successive oligopolies, an increase of the number of input suppliers has a direct and an indirect effect on the profit of the downstream firms. The direct effect is on the input price. A higher

m, clearly decreases the input price, $\frac{\partial p^*(m,n)}{\partial m} < 0$, because competition in the upstream market becomes fiercer. The indirect effect in on the output price. In fact, the decrease of input price as a consequence of an increase of m, decreases the output price $\frac{\partial \pi^*(m,n)}{\partial p} \frac{\partial p}{\partial m} < 0$. In other words, given that downstream firms behave competitively in the input market, a lower input cost, translates into a lower output price in the downstream market.

Asymptotic properties of input and output prices

Another property that we can investigate is the effect of entry of new firms in each market. To do so, we use a replication procedure introduced by Debreu and Scarf (1963) in the framework of a pure exchange economy: we replicate a number of time, say $\tau, \tau = 1, 2, ...$, the basic economy described above. In the $\tau-$th replica, there are τn downstream and τm upstream firms and the downstream market demand is given by $\tau(1 - Q)$. The corresponding prices of the $\tau-$th replica are the expressions (6) and (7), where n and m become τn and τm. It can be easily shown[2] that the market solution resulting from free entry in each market obtains by taking the limit of these expressions when $\tau \to \infty$. We compute

$$\lim_{\tau \to \infty,} \pi^*(\tau m, \tau n) = \beta$$

and

$$\lim_{\tau \to \infty,} p^*(\tau m, \tau n) = \beta.$$

Thus, as expected, under constant returns, when the number of firms in each market both tend to infinity at the same speed, the equilibrium output price converges to its marginal cost, and similarly for the input price. Furthermore both prices converge to their competitive counterpart.

In the following sections, using the example of a Cobb-Douglas function, we investigate whether these properties still hold when the technology used by the downstream firms has decreasing returns.

3 Industry equilibria under decreasing returns

In this section, we keep most of the traditional assumptions used in the constant returns example proposed by the literature: linear demand in the downstream market, identical production function for the downstream firms and identical linear total cost for the upstream firms. Also we assume, as usual, that downstream firms are price takers in the input market. Thus, we are completely in line with the traditional example considered in the literature on successive oligopolies, but one : the production function of the downstream is no longer linear, but with decreasing returns. Consider again n *downstream* firms facing the linear demand $\pi(Q) = 1 - Q$ in the downstream market. All of them share the same technology $f(z)$ to produce the output, which is now

$$q = f(z) = \sqrt{z}.$$

The profits of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{k \neq i} q_k) q_i - p q_i^2.$$

Given a vector of strategies $(q_1, ...q_i, ..q_n)$, the resulting input demand $\Sigma_{k=1}^n z_k(p)$ in the upstream market obtains as

$$\Sigma_{k=1}^n z_k(p) = \Sigma_{k=1}^n q_k^2.$$

The m identical upstream firms produce the input z at the same linear total cost βs_j, $j = 1, .., m$, $\beta > 0$. Given a vector of strategies $(s_1, ...s_j, ..s_m)$, the j_{th} upstream firm's profit Γ_j writes as

$$\Gamma_j(s_j, s_{-j}) = p(s_j, s_{-j})s_j - \beta s_j. \tag{9}$$

Given an n-tuple of strategies $(q_1, ...q_i, ..q_n)$ and a $m-$tuple of strategies $(s_1, ...s_j, ..s_m)$, we may compute the symmetric Nash equilibrium of each of the above games under the condition that *the input price balances supply and demand in the input market.* The explicit values of the symmetric Nash equilibrium in each of the above games are derived in Appendix 1.

Denoting $(q^*(p), ...q^*(p), ..q^*(p))$ and $(s^*(p), ...s^*(p), ..s^*(p))$ the symmetric solution of each game, they must satisfy the equality

$$n\left(q^*(p)\right)^2 = ms^*(p). \tag{10}$$

4 Number of firms and profits

In this section, we study how the profit of downstream firms depend on the number of firms in the input market when output firms use a decreasing returns technology. As we saw above, under oligopoly with a constant returns technology, decreasing linear production cost must necessarily increase downstream firms' profits. Accordingly, since increasing the number of upstream firms leads to a decrease in the input price, the resulting profit of downstream oligopolists must necessarily increase under a constant returns technology.

Does this simple reasoning still applies when returns are decreasing? It turns out that this is not always the case.

Proposition 1 *In spite of fiercer competition, the profit of downstream firms may well decrease with the number of firms in the upstream market. For instance, when the number of firms in the downstream market does not exceed 3, profits of a downstream firm always decreases when the number of upstream firms increases.*

Proof. see appendix 3. ■

Similarly to the case of constant returns technology, an increase in the number of input suppliers has two effects, one on the input price and the other on the output price. The difference is that, in the case of decreasing returns, the *size* of these effects is different and can be such that the indirect effect on output price may well overcome the direct effect on the input price, leading finally to a decrease of the downstream firms' profits when m increases.

Seade (1985) has shown that, under Cournot oligopoly, it is not necessarily true that a decrease of production cost leads to an increase in profits, a proposition analogous to ours. Seade (1985) uses conditions on the elasticity on the market demand function to identify when decreasing cost can increase profits. In our case, this phenomenon is obtained in a chain of markets and, consequently, the condition bears on the technology of the downstream firms, as well on the relative number of firms in the markets.

5 Asymptotic properties of input and output prices

We have seen above that when downstream firms use a constant returns technology, asymptotically, both these prices tend to the corresponding marginal costs. This property fails to hold in our example of a decreasing returns technology! Contrary to intuition, we show in the next proposition that, under decreasing returns, the input price may well not converge to its marginal cost

Proposition 2 *There exists decreasing returns technologies for which the equilibrium input price does not converge to upstream firms' marginal cost, when the number of replications of the basic economy tends to infinity.*

Proof. see Appendix 3. ■

The intuition of this proposition can be described as follows. Under the decreasing returns technology $f(z) = \sqrt{z}$, the equilibrium quantity produced by each downstream firm tends to zero when the number of replicas tends to infinity. Accordingly, the marginal product of input tends itself to infinity, making impossible the equality of supply and demand in the upstream market. The volume of input demand can be matched with input supply only by dampening demand with a price which remains strictly higher than the marginal cost of producing the input, whatever the number of replicas[3]. In the linear case, marginal productivity remains constant whatever large the number of replicas, which prevents a similar phenomenon to arise.

Finally, comparing the market solution with constant and decreasing returns technology in the downstream market, we can compare the size of double marginalization and the effects of collusive agreements.

6 Double marginalization under constant vs decreasing returns

It is well known that the vertical integration of a downstream monopolist and an upstream monopolist is profitable, because the profit of the integrated entity will exceed the combined profits corresponding to market solution ex-ante the agreement. The reason for this is the presence of double marginalisation. This result is extended to successive oligopolies in Gaudet and Van Long (1996), where the technology of downstream firms is assumed to be constant returns. Here, we address the size of double marginalization according to the type of technology used by downstream firms to produce the output: decreasing or constant returns. Double marginalization is defined as the sum of the markup exercised by the upstream firms, $p^* - \beta$, and the markup applied by the downstream firms, $\pi^* - p^*$, which yields $\pi^* - \beta$. Therefore, to compare double marginalization according to the downstream technology, we compare output prices under the two technologies. From the direct comparison of output prices we obtain that:

Proposition 3 *Double marginalization is lower when downstream firms use* *the decreasing returns technology* $f(z) = \sqrt{z}$ *than under the constant returns technology* $f(z) = z$.

Proof. see Appendix 3. ■

This difference in the size of double marginalization, due to the technology, is important because it entails different consequences of vertical agreements on profitability of mergers, as we now see in the next section[4].

7 The effects of technology on collusive agreements

Collusive agreements between upstream and downstream firms eliminate double marginalization, which yields lower prices for the consumers of the final product. On the other hand, these vertical integration agreements can lead to foreclosure of rivals firms in the downstream market, which has the opposite effect on the price of the final product. Finally, the global effect depends on the size of double marginalization; which itself depends, as shown before, on the technology used by the downstream firms. In this section, we use the above example of decreasing returns technology in successive oligopolies, to analyze and compare the effects of vertical integration according to the technology used in the downstream market.

Collusive agreements reduce the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them (see Salant, Schwitzer and Reynolds (1983)). Collusive outcomes are the Cournot equilibria corresponding to this reduced number of oligopolists in each market.

Assume that k downstream firms i, $i = 1, ..., k$, say, and h upstream firms $j, j = 1, ..., h$, say, collude and maximize joint profits (notice that all firms, $h + k$, merge in one entity). We assume that $k < n$ and $h < m$ [5]. After this merger, we move from an initial situation comprising globally $n + m$ firms to a new one, with $n - k + 1$ firms in the downstream market and $m - h$ in the upstream one. Indeed, the integrated entity now internalizes output production by using the input provided by the h upstream firms belonging to the new entity. This general formulation covers as particular cases mergers including either only downstream firms, or only upstream ones, which correspond to the usual case of horizontal merging of firms.

The payoff of the integrated firm I is given by

$$\Pi_I(q_I, q_{-I}) = (1 - q_I - \sum_{k \neq I} q_k)q_I - \beta q_I^2.$$

where q_I denotes the quantity of output produced by the integrated entity. As for the downstream firms $i, i \neq I$, not belonging to the integrated entity, they have as payoffs[6]

$$\Pi_i(q_i, q_I, q_{-i}) = (1 - q_i - \sum_{k \neq i}^{n} q_k)q_i - pq_i^2). \tag{11}$$

Following the upstream and the downstream games explained above, we derive in Appendix 3, the equilibrium output and input quantities and prices, for the entity of $h + k$ firms and the non-integrated upstream and downstream firms. Comparing these variables with those obtained when downstream firms use a linear technology, it is possible to analyze how collusive agreements can be affected by technology.

To this end, using an illustration with two downstream and three upstream firms, we show that a collusive agreement between one downstream and one upstream firm can have diametrically opposite consequences depending on whether the technology is constant, or decreasing returns (see Gabszewicz and Zanaj 2007). In the first case, the collusive agreement leads to a *decrease* in both the input and output prices, while the reverse holds under decreasing returns. Moreover, the profitability of mergers also depends on technology. Indeed, with constant returns, it can be shown that when $n = m = 7$, only vertical integration of one downstream and one upstream firm can be profitable. These results are very different from those obtained when firms in the downstream firm use constant returns, as in Salinger (1988). For the same parametric values, Salinger (1988) shows that under vertical integration, the number of profitable mergers is much larger.

8 Conclusion

In this paper, we propose an example of successive oligopolies where the downstream firms share the same decreasing returns technology of the Cobb-Douglas type. We stress the differences between the conclusions obtained under this assumption and those resulting from the traditional example considered in the literature, namely, a constant returns technology. We find that when firms use a decreasing returns technology rather than a linear one: *(i)* the profit of a downstream firm can decrease, when the upstream market is more competitive; *(ii)* the input price does not tend to the corresponding marginal cost when the number of firms in both markets tends to infinite; *(iii)* double marginalization is lower and, finally, *(iv)* vertical integration may arise less frequently, and may lead to higher prices for final consumers. These discrepancies between market behavior corresponding to alternative technological conditions reveal how fragile are the theoretical conclusions obtained when analyzing the interplay of firms' strategies in successive markets only using a linear technology in the downstream market, as it is done so far in the literature. To get robust conclusions, a general theoretical framework for analyzing successive oligopolistic markets is clearly required.

References

[1] Debreu G. and Scarf H., "A Limit Theorem on the Core of an Economy", 1963, *International Economic Review*, Vol. 4, 235-246

[2] Gabszewicz J. J. and Zanaj S., 2007, "A note on vertical mergers and successive oligopolies", *Communication & strategies,* Volume 4th quarter 2007, Issue: 68.

[3] Gaudet G. and Van Long N., "Vertical Integration, Foreclosure and Profits in the Presence of Double Marginalisation", *Journal of Economics and Management Strategy*, 1996, Vol. 5(3), 409-432

[4] Hansen, T. and Gabszewicz, J., 1972. "Collusion of factor owners and distribution of social output," *Journal of Economic Theory*, Elsevier, vol. 4(1), pages 1-18, February.

[5] Hart O. and Tirole J., 1990, "Vertical integration and market foreclosure", *Brookings papers on economic activity: Microeconomics*, 205:286.

[6] Ordover, J. and Saloner, G. and Salop, S.: "Equilibrium vertical foreclosure", *American Economic Review*, 1990, Vol. 80, 127-142

[7] Salant S., Schwitzer Sh. and Reynolds R. , "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium, *The Quarterly Journal of Economics*, 1983, Vol. 98, 185-99

[8] Salinger M., "Vertical mergers and market foreclosure", *The Quartely Journal of Economics*, 1988, Vol. 103, 345-356

[9] Seade J, 1985, "Profitable cost increases and the shifting of taxation : equilibrium responses of markets in oligopoly", Warwick discussion papers, n 260.

[10] Tirole, J., 1989, The theory of Industrial Organisation. *Cambridge: MIT press*

Notes

[1] This assumption guarantees that the marginal cost of producing the input does not exceed its marginal product in the production of output.

[2] In the τ_{th}-replica, the prices at which demand is equal to supply both in the downstream and upstream markets, do not depend on the number τ, but depend only on m and n. Indeed, at the symmetric equilibrium in the upstream market, the input quantities supplied by the m upstream firms have to be multiplied by τ in the τ_{th}-replica; similarly for the quantities demanded by the n downstream firms in the downstream market. Consequently, the equality of supply and demand in the upstream market eliminates the $\tau-$ factor in each side of the equality. A similar reasoning applies for the symmetric price equilibrium in the downstream market. It follows that the study of the behaviour of the upstream and downstream markets when the number of replicas increases is equivalent to the study of the limit equilibrium prices and quantities, when the number of firms is τn and τm , instead of n and m, in each market, respectively. This replication procedure thus leads to increase, simultaneously and at the same speed, the number of firms in each market.

[3] It would be interesting to extend this result to the general class of decreasing returns Cobb-Douglas production functions $f(z) = z^{\alpha}$, $\alpha < 1$. Unfortunately, it turns out the solutions of the model leads to cumbersome computations when $\alpha \neq \frac{1}{2}$.

[4] Here, we are interested in successive Cournot oligopolies and consequently in the size of double marginalization. Clearly, if firms play Bertrand or prices are not linear, for instance, firms can use two-part tariffs, double marginalisation disappears. In this cases, the cause and the profitability of vertical agreements are different (Tirole, 1989, and Hart and Tirole, 1990).

[5] This assumption guarantees that there always exists at least one unintegrated firm on each side of the upstream market so that the integrated entity cannot exclude the unintegrated downstream firms to have access to the input. A similar assumption in another approach to collusion has been used by Gabszewicz and Hansen (1972).

[6] Notice that the set $\{k : k \neq i\}$ includes the index I.

Appendix 1: Decreasing returns technology

In this section of the Appendix we derive the equilibrium quantities and prices when downstream firms use decreasing returns technology. The profits of the i_{th} downstream firm at the vector of strategies (q_i, q_{-i}) obtains as

$$\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{k \neq i} q_k) q_i - p q_i^2.$$

Taking the first derivative and solving it in q, we get at the symmetric solution

$$q^*(p) = \frac{1}{(n + 2p + 1)};\ i = 1...n. \tag{12}$$

Similarly, re-expressing equation (**??**), and solving it for s, we obtain

$$s^*(p) = \frac{\beta - p(ms^*)}{\partial p(ms)/\partial s}. \tag{13}$$

The input price p^* must satisfy the system of equations $n\,(q^*(p))^2 = ms^*(p)$, (12) and (13). To derive the explicit equilibrium price, we can proceed as follows. First, we identify the total demand for input at the symmetric solution of the first game, using (12) namely $\frac{n}{(n+2p+1)}$. Then, using the input clearing market condition, the equality $\frac{n}{(n+2p+1)^2} = \Sigma_{k=1}^m s_k(p)$ has to be satisfied at any vector of strategies $(s_1, ..., s_j, ..s_m)$ in the input market. Accordingly, the equality

$$p(\Sigma_{k=1}^m s_k) = \sqrt{\frac{n}{4\Sigma_{k=1}^m s_k}} - \frac{n+1}{2}. \tag{14}$$

must hold for any vector of strategies in the input market. Substituting (14) into the profit function of an upstream firm, $\Gamma_j(s_j, s_{-j})$ we have

$$\Gamma_j(s_j, s_{-j}) = (\sqrt{\frac{n}{4\Sigma_{k=1}^m s_k}} - \frac{n+1}{2}) s_j - \beta s_j,$$

Notice that the profit function $\Gamma_j(s_j, s_{-j})$ is concave in $s_j, j = 1, ...m$, so that we can use the first order necessary and sufficient conditions to characterize an equilibrium. Accordingly, at the symmetric Nash equilibrium of the upstream game, we obtain

$$s^*(m,n) = \frac{n\left(2m-1\right)^2}{4m^3\left(2\beta+1+n\right)^2}.$$

Hence the profit $\Gamma_j(m,n)$ of an upstream firm at the symmetric equilibrium of the upstream game obtains as

$$\Gamma_j(m,n) = \frac{n(2m-1)}{8\left(n+2\beta+1\right)m^3}.$$

Finally, the equilibrium price $p^*(m,n)$ in the input market obtains as

$$p^*(m,n) = \frac{n+1+4m\beta}{2\left(2m-1\right)}.$$

Consequently, substituting this equilibrium price into the equilibrium quantities $q^*$of output selected by the downstream firms, as given by (12), we get

$$q^*(m,n) = \frac{2m-1}{2m\left(2\beta+n+1\right)}$$

so that, given the technology, the equilibrium input quantities used by downstream firms writes as

$$z^*(m,n) = \frac{\left(2m-1\right)^2}{4m^2\left(2\beta+n+1\right)^2}$$

Therefore, the resulting output price $\pi^*(m,n)$ in the downstream market obtains as

$$\pi^*(m,n) = 1 - \frac{n\,(2m-1)}{2m\,(2\beta+n+1)}.$$

The profit $\Pi_i(m,n)$ of a downstream firm at equilibrium in the corresponding game is thus equal to

$$\Pi_i(m,n) = \frac{1}{8}\,(4m\beta+4m+n-1)\,\frac{2m-1}{m^2\,(2\beta+n+1)^2}.$$

Notice that $\Pi_i > 0$, - a requirement needed to guarantee the survival of firms in the downstream market.

Appendix 2: Proofs of propositions

Proposition 1:

Proof. The derivative of the profit of a downstream firm is $\frac{3m+n-mn-1+2m\beta}{4(n+1)^2m^3}$. Hence, the sign depends only on the sign of the numerator, $3m+n-mn-1+2m\beta$. The derivative is positive iff $m < \frac{1-n}{2\beta+3-n}$, and negative iff $m > \frac{1-n}{2\beta+3-n}$. It is immediate that the last expression is always true for $\beta > \frac{n-3}{2}$. ■

Proposition 2:

Proof. We consider the situation where the number of replicas τ tends to infinity. So, we calculate the limit for $\tau \to +\infty$ of the expression for the input price:

$$\lim_{\tau\to\infty} p^*(\tau m, \tau n) = \frac{1}{4}\frac{n}{m} + \beta.$$

Clearly, the price p^* at the limit does not converge to β, unless m converges to infinite more quickly than n. ■

Proposition 3:

Proof. Consider $\frac{(1+m+n)+mn\beta}{(n+1)(m+1)} + \frac{n(2m-1)}{2m(2\beta+n+1)} < 1$.We prove that the inequality is false. It is easy to check that the first derivative with respect to α of each side of the left hand side of the inequality is negative. We know that $\beta \leq 1$. Therefore, assuming $\beta = 1$, we can consider the inequality $\frac{\beta(1+m+n)+mn\beta}{\beta(n+1)(m+1)} + \frac{n(2m-1)}{2m(2\beta+n+1)} < 1$ where we just make the left hand side bigger. The solution of such inequality is a subset of the original inequality.

Solving for β, we find that it is true only for $\beta < 0$. This is not an admissible set of β, therefore the inequality is false, and the price with decreasing returns technology is smaller than the price with constant returns. ■

Appendix 3: Vertical integration solution

Following the solution of the game in the benchmark model, at the symmetric equilibrium in the upstream market, each unintegrated firm supplies a quantity s_j^* of input which obtains as

$$s^*(k,h) = \frac{1}{4}\frac{(2m-2h-1)^2(n-k)}{(n-k+2\beta+2)^2(m-h)^3}.$$

while the input price writes as

$$p^*(k,h) = \frac{1}{4(\beta+1)}\Big((n-k+2\beta+2)\Big(2(2\beta+1)(m-h)^2-1\Big)-2\beta(n-k)\Big).$$

Then, substituting p^* in the expression of q_i and q_I, we obtain

$$
\begin{aligned}
q_i(k,h) &= \frac{1}{2\left(n-k+2\beta+2\right)\left(m-h\right)^2}, \\
q_I(k,h) &= \frac{2\left(n-k+2\beta+2\right)\left(m-h\right)^2-\left(n-k\right)}{4\left(n-k+2\beta+2\right)\left(\beta+1\right)\left(m-h\right)^2}.
\end{aligned}
$$

The price of the output then obtains as

$$
\begin{aligned}
\pi^*(k,h) &= (1-\frac{1}{2\left(n-k+2\beta+2\right)\left(m-h\right)^2}\left(n-k\right) \\
&\quad -\frac{2\left(n-k+2\beta+2\right)\left(m-h\right)^2-\left(n-k\right)}{4\left(n-k+2\beta+2\right)\left(\beta+1\right)\left(m-h\right)^2}).
\end{aligned}
$$