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On uncertainty when it affects successive markets[*]

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Abstract

In this paper, we examine how uncertainty can affect successive markets, when uncertainty can jointly influence both the upstream and downstream markets' conditions. The main result of the paper is that the equilibrium input and output quantities under stochastic dependence can be higher or lower than the corresponding quantities in the case of certainty equivalence depending on how much dependent are the events.

JEL classification: L1, L120

1 Introduction

In this paper, we examine how uncertainty can affect successive markets, when uncertainty can jointly influence both the upstream and downstream markets' conditions. Generally, shocks affecting the economic environment influence simultaneously most markets in the same chain of value, and not each of them separately. Yet, to the best of our knowledge, most existing studies in microeconomics, consider that uncertainty affects just one market in isolation (Sandmo 1971, Leland 1972, Sheshinski and Dreze 1976, Gabszewicz and Poddar 1997, Grimm and Zoettl 2006, among others), excluding thereby that the spillovers resulting from uncertainty might be transfered from market to market in the same chain[1]. These spillovers follow from the technological linkage between firms producing the input and those using this input in their own production process.

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[1]These frequent and, quite often, unpredictable shocks can affect not only a sector but even the economy as a whole. Of course, the study of this more general case would require a general equilibrium approach.

An example of the chain effects which can occur when uncertainty affects more than one market is provided by a consumption product the demand of which would be high, whatever the price, when the weather is good and low in the opposite case. Moreover, assume that simultaneously, the resource needed to produce that good would be scarce if weather is bad, and abundant in the opposite case. Then good weather simultaneously leads to high output demand function and abundant resources so that restricting the analysis to the downstream market only, does not take into account the effect of weather on the upstream market.

We analyse uncertainty in successive markets using a microeconomic set up. To this end, we first introduce a simple model to describe how a downstream firm and an upstream firm interact in a deterministic environment. Then, we introduce uncertainty on output demand function in the downstream market and, as a consequence, on the demand function for input in the upstream market. Furthermore, we assume that the cost function for producing the input is also uncertain. We consider two cases: stochastic independence and stochastic dependence.

We find that if shocks affecting different markets are dependent, the equilibrium quantity in both downstream and upstream markets under stochastic dependence, is higher or smaller than the equilibrium quantity under certainty equivalence (or stochastic independence) according to the degree of stochastic dependence between the two random variables, namely output demand function and input cost function.

The effect of uncertainty on output demand has been extensively questioned in the existing literature. It is shown in a number of papers (e.g. Sandmo, 1971, Leland, 1972) that firm's attitude to bear the inherent risk of production has important effects on the firm's willingness to produce, i.e. on its choice of optimal level of production. Uncertainty places a crucial on the profitability of entry of new firms in a market and on how much capacity to built to deter entry by the incumbent firms (Gabszewicz and Poddar, 1997). Sheshinski and Dreze (1976) analyse uncertainty on output demand in a single commodity competitive market and zero price elasticity demand. These authors find that free entry and competition may lead to excess capacity on the average and that the price is lower than the minimum average cost.

2   The model

Consider a monopoly firm who faces an uncertain output demand function $p(q)$, twice differentiable with $p'(q) < 0$ and $p''(q) < 0$, which is

$$p(q) = \begin{cases} p^+(q) & \text{with probability} \quad \pi \\ p^-(q) & \text{with probability } 1 - \pi \end{cases}$$

with π denoting the probability of getting a high demand function. We assume that $p^+(q) = p(q) + \gamma$, while $p^-(q) = p(q) - \gamma$, $\gamma > 0, p(q) - \gamma \geq 0$, $\forall q$. Thus, the output demand function $p(q)$ is a random variable with two values. The

downstream monopoly uses the input z to produce the output using a linear technology $f(z) = \alpha z$, $\alpha > 0$. This input z is produced by a monopoly upstream firm in the upstream market with cost function $c(z)$, with $c'(z) > 0$, $c''(z) < 0$, given by

$$c(z) = \begin{cases} c^+(z) \text{ with probability } \rho \\ c^-(z) \text{ with probability } 1 - \rho. \end{cases}$$

where, for $\forall z, c^+(z) > c^-(z)$. Thus, the input cost function $c(z)$ is a random variable with two values. It follows that the set of possible states of nature reduces to four elements, namely:

a- both the output demand function and the cost function of producing input are high;

b- the output demand function is high and the cost function of producing input is low;

c- the output demand function is low and the cost function of producing input is high;

d- both the output demand function and the cost function of producing input are low.

3   Certainty equivalent

In the certainty equivalent scenario, output demand is deterministic and equal to its actuarial value, namely $E(p(q)) = \pi p^+(q) + (1-\pi)p^-(q) = p(q) - \gamma(1 - 2\pi)$. Denote by r the input price. Thus, the profit of the downstream monopolist in this case is equal to

$$\Pi(r, q) = (p(q) - \gamma(1 - 2\pi)) q - \frac{r}{\alpha} q, \tag{1}$$

The first order condition of the downstream monopolist writes as:

$$\frac{dp(q)}{dq} q + p(q) - \gamma(1 - 2\pi) - \frac{r}{\alpha} = 0 \tag{2}$$

Taking into account technology, (2) can be reexpressed as the input demand $r(z)$, namely

$$r(z) = \left(\frac{dp(\alpha z)}{d(\alpha z)} \alpha z + p(\alpha z) - \gamma(1 - 2\pi) \right) \alpha. \tag{3}$$

The profit $\Gamma(z)$ of the upstream firm writes as

$$\Gamma(z) = r(z)z - c(z).$$

Consequently, we can obtain the input supply function $s(z)$, from the first order condition:

$$r(z) + r'(z)z - c'(z) = 0.$$

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Applying the input market clearing condition, it follows that $r(z^*) = s(z^*)$, where z^* is the equilibrium input quantity. Accordingly, the profit of the upstream monopolist writes as

$$\Gamma(z) = \left(\frac{dp(\alpha z)}{d(\alpha z)} \alpha z + p(\alpha z) - \gamma (1 - 2\pi) \right) \alpha z - \left(\rho c^+(z) + (1 - \rho) c^-(z) \right)$$

We make the simplifying assumption that $c^{+'}(z) = c^{-'}(z) = c'(z)$, it follows that the first order condition writes as

$$\left(\frac{d^2 p(\alpha z)}{d(\alpha z)^2} \alpha^2 z + 2 \frac{dp(\alpha z)}{d(\alpha z)} \alpha \right) \alpha z +$$

$$+ \left(\frac{dp(\alpha z)}{d(\alpha z)} \alpha z + (p(\alpha z) - \gamma (1 - 2\pi)) \right) \alpha - c'(z) = 0,$$

while the second order condition obtains as

$$\left(\frac{d^3 p(\alpha z)}{d(\alpha z)^3} \alpha^3 z + 3 \frac{d^2 p(\alpha z)}{d(\alpha z)^2} \alpha^2 \right) \alpha z + \left(\frac{d^2 p(\alpha z)}{d(\alpha z)^2} \alpha^2 z + 2 \frac{dp(\alpha z)}{d(\alpha z)} \alpha \right) \alpha - c''(z).$$

Since $c"(z) < 0$, the profit of the upstream monopolist is concave if the demand for input faced by this firm is concave. Furthermore, concavity of input demand obtains from the following condition on output demand :

$$\left(\frac{d^3 p(q)}{dq^3} q + 2 \frac{d^2 p(q)}{dq^2} \right) < 0 \tag{4}$$

Consequently, when (4) holds, the solution given by the FOC exists and is interior. For instance, the condition (4) is satisfied in the following example. Consider a monopoly firm who faces a demand for its output which can be $p(q) = 1 + \gamma - q^3$ with probability π and $1 - \gamma - q^3$ with probability $1 - \pi$. This monopoly uses the technology $f(z) = z$. Maximising the profit $\Pi(q)$ in the certainty equivalent case, $\Pi(q) = \left((\pi(1 - q^3 + \gamma) + (1 - \pi)(1 - q^3 - \gamma))q - rq \right)$, we obtain $q(r) = z(r) = \sqrt[3]{\frac{1}{4} (\gamma (2\pi - 1) + (1 - r))}$. The upstream monopolist faces the above input demand and a linear stochastic production cost $c(z)$, equal to $\beta^+ z$ with probability ρ and $\beta^- z$ with probability $1 - \rho$. Taking into account the equality of demand and supply, i.e. $r(z) = s(z)$, in the upstream market, we get $r(z) = \gamma (2\pi - 1) - 4z^3 + 1$. Accordingly, the profit function of the upstream monopolist writes as $\Gamma(z) = \left(2\pi\gamma - \gamma - 4z^3 + 1 \right) z - \beta z$, with $\beta = \rho\beta^+ + (1 - \rho)\beta^-$. It is checked that condition (4) holds. Taking the FOC leads to $z^* = \sqrt[3]{\frac{1}{16} (2\pi\gamma - \gamma - \beta + 1)}$. Substituting z in r, we obtain $r^* = \frac{\beta - 3\gamma + 6\pi\gamma + 3}{4}$. Finally, substituting r^* into $q(r)$, we obtain $q^* = \sqrt[3]{\frac{1}{16} (2\pi\gamma - \gamma - \beta + 1)}$ and $p^* = \frac{\beta + \gamma - 2\pi\gamma + 15}{16}$.

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4 Uncertainty

Assume now that both the downstream and the upstream firms know the distribution probability on output demand. Furthermore assume that the downstream firm behaves as a price taker in the input market. In other words, we assume that the upstream firm is aware of the effect of uncertainty on output demand, while the downstream firm ignores the effect of this uncertainty on the input price[2]. Finally, the upstream firm is assumed to know the probability distribution of its own production cost function.

We consider now the consequences of this uncertain emvironment on the market solution in two different scenarios: stochastic independent, and stochastic dependent, events.

4.1 Stochastic independence

Under stochastic independence, the states of nature a, b, c and d occur with probability $\pi\rho, \pi(1-\rho), (1-\pi)\rho, (1-\pi)(1-\rho)$, respectively. The expected profits of the downstream monopolist now obtains as

$$
\begin{aligned}
E\Pi(q,r) &= \pi\rho\left(p^+(q)q - q\frac{r}{\alpha}\right) + \pi(1-\rho)(p^+(q)q - q\frac{r}{\alpha}) + \\
&\quad + (1-\pi)\rho\left(p^-(q)q - q\frac{r}{\alpha}\right) + (1-\pi)(1-\rho)\left(p^-(q)q - q\frac{r}{\alpha}\right),
\end{aligned}
$$

which can be rewritten as

$$
E\Pi(q,r) = \left[\pi q p^+(q) - (1-\rho)\frac{q}{\alpha}r\right] + \left[(1-\pi)\,qp_l(q) - \rho\frac{q}{\alpha}r\right]. \tag{5}
$$

Comparing the objective functions (1) and (5), we see that the output quantities maximizing profits of the downstream monopolist coincide in both cases.

Since the objective function of the downstream monopolist under stochastic independence is the same as in the case of certainty equivalent, the input demand is also the same in both cases. Therefore, the objective function for the upstream monopoly under stochastic independence is also the same as in the certainty equivalent scenario. Hence, *the market solution under independent stochastic uncertainty coincides with the market solution which appears in the case of certainty equivalence.*

4.2 Stochastic dependence

Consider now the case of stochastic dependence. We keep the assumption that the downstream firm does not know the effect of uncertainty on the input price and, accordingly, takes the input price as given whatever the state. Furthermore, for sake of simplicity, we assume that Pr(high output demand ı low input

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[2]This assumption is in the spirit of the traditional framework of successive markets where it is assumed that the downstream firm(s) take the input price as given (Salinger 1988, Gaudet and Von Long 1996...).

marginal cost $)=\Pr($low output demand $\mid$ high input marginal cost $)=\delta$. Hence the states of nature $a,\ b,\ c$ and d now occur each with probabiliies

a : $\Pr\left(\text{high output demand}\cap\text{low input marginal cost}\right)=\delta(1-\rho)$;

b : $\Pr\left(\text{low output demand}\cap\text{low input marginal cost}\right)=(1-\delta)\left(1-\rho\right)$;

c : $\Pr\left(\text{low output demand}\cap\text{high input marginal cost}\right)=\delta\rho$;

d : $\Pr\left(\text{high output demand}\cap\text{high input marginal cost}\right)=(1-\delta)\rho$.

Then, the expected profit function of the downstream firm writes as

$$E\Pi(q,r)=\left(p^{+}(q)\left(\delta\left(1-2\rho\right)+\rho\right)+p^{-}(q)\left(1-\left(\delta\left(1-2\rho\right)+\rho\right)\right)\right)q-\frac{q}{\alpha}r.$$

Define θ as the probability to have a high output demand function under stochastic dependence, namely $\theta=\delta\left(1-2\rho\right)+\rho$, with $0\leq\delta\left(1-2\rho\right)+\rho\leq1$, or $\frac{\rho}{1-2\rho}\leq\delta$. Then the expected profit obtains as

$$E\Pi(q,r)=\left(p^{+}(q)\theta+p^{-}(q)\left(1-\theta\right)\right)q-\frac{q}{\alpha}r. \tag{6}$$

Since the objective function (**??**) is the same as (5), except that the probability of high demand is now given by θ and the probability of low demand is $1-\theta$, the input demand is similar to the input demand under stochastic independence with these corresponding probabilities. Hence, following the same procedure as in the case of certainty equivalent case, we obtain the market solution from the first order condition of (6) :

$$\left(\frac{d^{2}p(\alpha z)}{d(\alpha z)^{2}}\alpha^{2}z+2\frac{dp(\alpha z)}{d(\alpha z)}\alpha\right)\alpha z+$$

$$+\left(\frac{dp(\alpha z)}{d(\alpha z)}\alpha z+\left(p(\alpha z)-\gamma\left(1-2\theta\right)\right)\right)\alpha-c'(z)=0.$$

Thus, we obtain the following:

Proposition 1 *The output and input quantities produced at equilibrium under stochastic dependence are respectively higher equal or lower than the quantities produced under certainty equivalence (or stochastic independence), if and only if the probability π of a high output demand function, under certainty equivalence, is higher, equal or lower, than the corresponding probability θ under stochastic dependence.*

Notice that $\frac{\partial\theta}{\partial\rho}\lessgtr0$ iff $\delta\gtrless\frac{1}{2}$. Thus, the probability of having a high output demand function (resp. output quantity) increases with the probability of high input cost function (resp. input quantity), when the probability of high output demand conditional on low input marginal cost does not exceed $\frac{1}{2}$.

As for the prices, both the output and input prices under stochastic dependence can be either higher equal or lower than the price under certainty equivalence (or stochastic independence). Of course, if θ is higher than π, then the demand functions both in the downstream and upstream markets shift upwards. However, the intersection with the quantity chosen by the monopolist in its own market does not allow to conclude about the sign of the change in the corresponding price.

5 Conclusion

In this paper, we examine how uncertainty can affect successive markets, when uncertainty can jointly influence both the upstream and downstream markets' conditions. Our main interest is to analyse the effect of uncertainty on the production choice of the upstream and the downstream firm, when uncertainty in each market originates from stochastic independent and/or dependent events. The contribution of this paper to the existing literature is twofold. We provide a microeconomic founded framework to deal with uncertainty in different markets which constitute a value chain, and secondly, we show how the combined uncertainty affects the production choice. A natural extension of the present paper would be to analyse successive oligopolies and thus consequence of uncertainty on the profitability of vertical integration and horizontal mergers.

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