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COMPETITION AND COOPERATION IN AVIATION: APPLICATIONS IN AIR CARGO AND AIRPORT COMMERCIAL PERFORMANCE

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Competition and Cooperation in Aviation: Applications in Air Cargo and Airport Commercial Performance

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Chapter 1

Introduction

Transportation plays a critical role in global commercial activities. It contributes to 5% of the European GDP (European Transport Sector, 2022) and 7.7% of the U.S. GDP (Bureau of Transportation Statistics, 2022) while enabling diverse economic activities. Among all modes of transportation, the aviation industry is a key contributor to global economic prosperity, enabling fast connection between continents and regional economies. Air transport carries about 1% of worldwide shipments by volume, but 35% by value, as such services are mainly used for high-value commodities (ATAG, 2020). The deregulation and liberalization of the aviation industry, dating back to the 1980s, have led to rapid transformations including operational and marketing changes. Deregulation has increased market competition in the aviation industry, which ultimately reduced air travel fares, increased air traffic volume, and developed the hub-and-spoke system (Peterson, 2018). After decades of deregulation, stakeholders in the aviation industry are facing a competitive market where they are required to develop in-depth insights and efficient strategies.

1.1. Research motivation

As key stakeholders in the aviation industry, airlines face uncertain demands with limited capacities. Consequently, revenue management techniques and concepts have been largely discussed to maximize airline operational profits for their survival in the competitive market. There is extensive literature on airline revenue management, facilitating the decision-making processes in a stochastic environment for both passenger and freight operations. Among different operational challenges, in-depth insights on the balance between short-term spot markets and long-term contracts for air freight operations, especially under uncertainty and competition, are still missing. COVID-19 drew the public's attention to this concern, when the short-term spot prices became remarkably unpredictable. For instance, the spot air freight rates from Frankfurt to North America increased from 1.76 USD per kilogram in 2019 to 4.89 USD per kilogram in 2022 (Statista, 2022b) and the financial performance of an airline or a logistics service provider would largely differ if it signed a long-term contract where a fixed price applies.

Airports were also challenged during COVID-19. The transportation demand, from the aspect of airports, is relatively volatile compared to other industries (Fuerst and Gross, 2018) and the pandemic greatly affected the airport transportation volume in a negative way. One strategy used to enhance airport financial performance is to expand non-aeronautical revenues. Non-aeronautical revenues enabled airports to survive in the crisis and remain competitive (InternationalAirportReview, 2022). Although considerable attention has been paid to study the determinants of non-aeronautical revenues, there is one topic to be further analyzed: the impacts of passenger dwell time on airport revenues. The main challenge with passenger dwell time is the inability to accurately trace passenger movements at airports. While survey data is often used in literature, it can be fairly biased as passengers who rush to the gates are unlikely to participate in the survey. Understanding the relationship between passenger dwell time and airport non-aeronautical revenues can potentially shed light on passenger behaviors at airports, contribute to managerial insights for airport operations, and enhance the competitiveness of airport financial performance.

Another critical stakeholder of the aviation industry is the policy maker, who needs to develop a comprehensive understanding of the impacts of the policies they set. As mentioned earlier, the deregulation of the aviation industry has been shown to positive impact to the industry. Albeit, with the recent wave of liberalization and a tendency to open skies arrangements, traffic movements in numerous places are still subject to bilateral air service agreements, which serve to protect national interests. Doganis (2019) estimates that currently there are about 1,500 bilateral air service agreements worldwide. While there is ample literature to guide policymakers in the context of passenger traffic and assess the impacts on welfare distribution, there is a lack of guidance on the impacts of air service agreements on the air cargo industry.

The above-mentioned challenges in the interaction between agents raise more and more attention, along with a significantly growing volume of transportation activities. In a market with fierce competition and uncertainty, agents need to gain insights onto the interaction with other agents and develop strategies to ensure profitability as well as market shares. The research objects and structures of this thesis are presented with more details in the following two subsections.

1.2. Research objectives

Different agents are involved in the delivery of aviation transportation services as illustrated in Figure 1.1. Typically, such activities include asset providers (APs) who provide transportation services such as air cargo companies and airlines, logistics service providers (LSPs) who aggregate demand from end customers, such as passengers and shippers, airports which provide aeronautical and non-aeronautical services, and policymakers who determine the boundary conditions for the activities of other agents. These agents interact with each other to exchange resources and share information, horizontally or vertically.

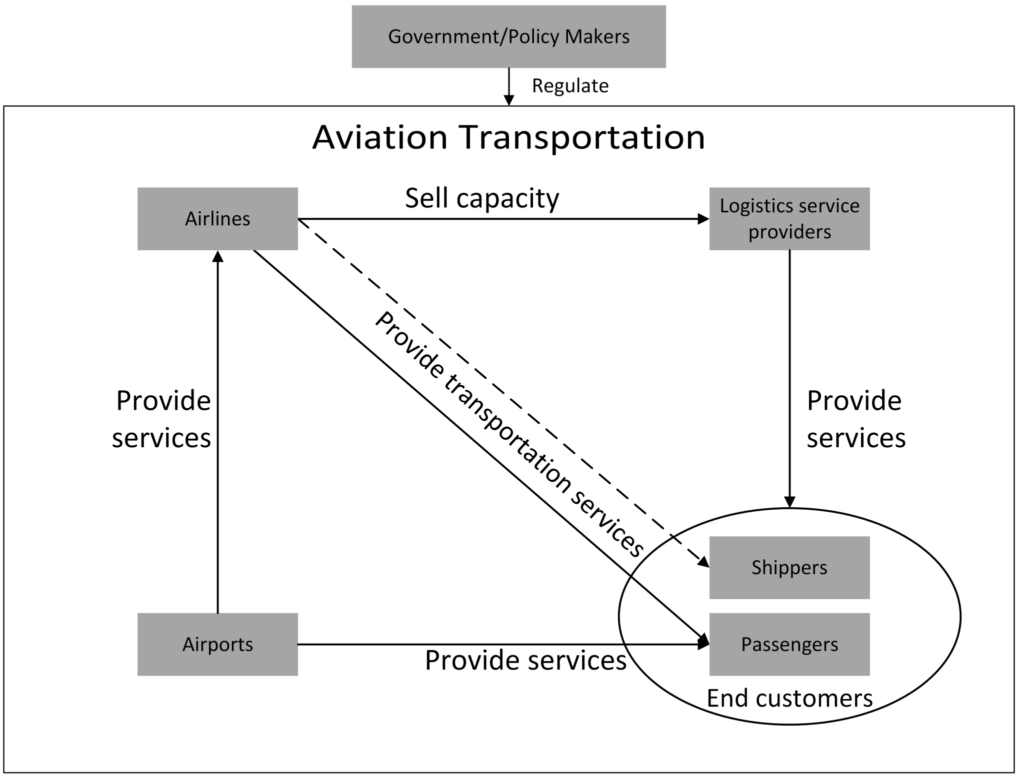


Figure 1.1 The interaction between agents in aviation industry

We first focus on asset providers, such as air cargo companies, who are often involved in fierce competition. They compete using price since transportation services are treated as commodities and end customers normally prefer the lowest price offer. In practice, digital technology enables transparent information and end customers can choose the service provider according to the price offers via online platforms. In theory, to attract customers, asset providers may undercut each other in price and consequently, they should price at their margins. However, this is not observed in real practices since asset providers are limited in their capacity, which is a long-term strategic decision and cannot be changed in the short-term. Such capacity constraints prevent asset providers from under-cutting each other to a certain extent, as the high-price asset provider may benefit from raising the price once the competitor, who offers a lower price, runs out of capacity. The price competition faced by asset providers is scantly discussed in the literature where most cargo revenue management papers assume a monopoly setting and competition among agents is not considered. It is hence of merit to explore how agents, such as asset providers, should price their capacity.

When facing competition, agents may look for cooperation, possibly in the form of airline alliances or vertical collaboration. Take the air cargo business for an example. An AP (such as an air cargo company) may, vertically, collaborate with its customer, an LSP, when both of them face demand uncertainty. Such cooperation can be achieved via long-term contracts, which specify quantities and prices ahead of spot markets. Termed *Hard Block*, the long-term contracts secure early sales for APs and simultaneously ensure capacities for LSPs. Consequently, APs may miss out on potential profits in the spot market when there is a surge in demand, and the LSPs may not be able to sell all purchased capacity to end customers in case of low demand. These practices raise a question: should the agents get involved in such cooperation at all? If yes, to what extent? In other words, how many units of capacity should they contract, and at what price? Some literature discusses the balance between long-term contracts and short-term spot markets (Amaruchkul et al., 2011b; Lin et al., 2017; Hellermann, 2006), but they have all abstracted away from competitive environments, which are prevalent in air cargo.

Airports, which play a critical role in aviation transportation, serve as gateways for air travel passengers. As shown in Figure 1.1, airports provide aeronautical and non-aeronautical services to airlines and end customers. As pointed out by Fuerst and Gross (2018), air travel demand is volatile, from the perspective of airports, compared to other industries, and it has been shown that airport operators are seeking to increase non-aeronautical revenues to improve the sustainability of financial status (Chen et al., 2020). Thereby, it is of interest to explore the determinants of non-aeronautical revenues. Considerable attention has been paid to investigate how passengers' demographics and travel purposes affect the revenues, but there is scant work discussing how passengers spend their dwell time at airports and how dwell time impacts non-aeronautical revenues. The main challenge is the low traceability of dwell time. Freathy and O'Connell (2012) conduct a survey to collect passenger dwell time and find a positive correlation between dwell time and the time of shopping. Appold and Kasarda (2006) estimate dwell time based on security waiting time and report no strong relationship between dwell time and commercial sales. Dwell time estimated from surveys can be biased since passengers who rush to their gates are less willing to be interviewed.

When considering the interplay between agents, the role of policymakers needs to be discussed as they critically affect aviation transportation. While there exist ample literature to guide policy makers in the context of passenger traffic (Gillen et al., 2002; Dresner and Tretheway, 1992; Ismaila et al., 2014; Oum et al., 2010), there is a lack of research when it comes to providing guidance in the context of air cargo operations. As highlighted by Kupfer et al. (2017), there are fundamental differences between passenger and cargo operations, such as the structure of the demand manifested through the LSPs. For instance, air service agreements, which define the scope of airborne traffic between different countries, are negotiated by policymakers to regulate the international aviation market. Many traditional bilateral agreements are signed to protect national interests since such agreements allow identical capacities for the asset providers from each side. However, limited literature has quantitatively explored the impacts of such agreements and it is not clear how the agreements affect the welfare distribution across agents, especially in air cargo operations.

This thesis is motivated by the outlined research gaps to improve the performance of the stakeholders in the aviation industry. It contributes to literature by modeling a competitive aviation market where various stakeholders interact with each other. With a comprehensive understanding of the competitive game, stakeholders can develop operational strategies to optimize profits.

1.3. Thesis structure

The dissertation contributes to the study of competition and cooperation strategies for aviation agents and the analysis of airport commercial performance, based on two main methods: game theory and empirical tools. Game-theoretical models help to analyze interactions for high-level strategic insights by abstracting away from operational details. Empirical tools help to conduct prescriptive analysis to build and test the significance of these relationships. This dissertation develops into three independent papers, among which one paper has been published in *Transportation Research Part B: Methodological* (Wu et al., 2022) and the other two papers are in final stages.

Chapter 2, entitled *Who benefits from air service agreements? The case of international air cargo operations*, studies how bilateral air service agreements, which regulate total cargo capacity in markets, affect total welfare as well as its distribution. We consider the interactions among four different types of agents: air cargo companies, end customers, LSPs, and policy makers. Policy makers sign bilateral air service agreements typically to protect national interests. They do so by specifying the services—e.g., capacities and frequencies—that each country is allowed to provide. Negotiating these terms often echoes the need of the APs from the counties of the respective policy makers. Limiting the attention to the case of two countries, hence, two policy makers and two APs (one from each country), the two APs are involved in a fierce price competition in spot markets given the nature of air cargo services: such transportation services are perceived as a commodity from the perspective of LSPs. On the one hand, the two APs can coordinate capacity via the agreement, negotiated by the policy makers. On the other hand, they compete with each other. This coopetition raises an interesting question: how should the APs coordinate the capacity while competing in prices? The challenge faced by the policy makers is to understand how welfare distributes among sectors under a bilateral service agreement and what interventions can be delivered to potentially improve the total welfare.

The situation is modeled as a two-stage game theoretical framework. In the first stage, the policy makers set the total capacity in markets via a reciprocal bilateral air service agreement, representing the two APs' benefits, and in the second stage, the two APs compete over prices in the spot market. We assume an uncertain and price-sensitive end customer demand. Solving backwards, the pricing strategies for the two APs in spot markets get characterized, capacity decisions are analyzed, and the welfare distribution gets explored. The main observation is that, comparing to a collaborative setting (monopoly), the coordinated duopoly results in lower capacity in market, lower profits to the APs, larger benefits to both end customers and LSPs, and overall lower total welfare. These results suggest that policy makers shall hold an open altitude towards a higher level of cooperation among the APs.

In Chapter 3, entitled *Contracting strategies for price competing firms under uncertainty*, an extension of the model developed in Chapter 2 is built to further explore the interactions between APs and LSPs. Specifically, Chapter 3 relaxes the assumption that all sales take place in spot markets. As such markets are characterized by demand uncertainty prior to entering the spot market, both APs and LSPs face uncertainty with respect to their profits. In practice, an AP and an LSP can also sign a contract ahead of the spot market when demand is uncertain. Such contracts entail certain capacity allocation from the AP to the LSP at a negotiated price. Thus, they secure early sales for APs and, at the same time, ensure capacity for the LSPs, possibly at a price lower than the one that would result in the spot market. However, if the AP commits too much capacity upfront, he may abstain from markets when spot demand is high and lose opportunities for enhanced profit while if the LSP purchases too much capacity he may end up with unused cargo space when realized demand is low. This leads to the following fundamental questions: how should the two agent types balance the contract and spot markets? Namely, how many capacity units should they contract, if at all, ahead of the spot market and at what price. Importantly, how should the carriers price-compete in the spot market?

To address the above-mentioned questions, a two-stage game-theoretical model is formulated, extended from the one in Chapter 2, involving two APs and two LSPs. In the first stage, given the APs' respective capacities, an AP can bargain with an LSP and contract a portion of the available capacity. A Nash bargaining framework is adopted to model the interaction between the AP and the LSP. In the second stage, the two APs bring their remaining capacities to the market while the residual spot market demand is realized.

Solving the model backwards, the equilibrium pricing strategies in the spot market is identified. In particular, when realized demand is sufficiently low, competition is severe and the two APs price at the margin; when demand is sufficiently high, the two APs price to clear the market; in intermediate demand realizations, equilibrium prices follow a mixed pricing strategy. Exploring the negotiation process, a uniformly distributed demand is assumed. We find that the quantity contracted decreases in the margin charged by the LSPs and generally decrease in the demand potential (i.e. the maximum possible demand realization). The corresponding contract price per unit of capacity contracted increases in this demand potential while being almost indifferent to the LSP's margin. The contracts are most valuable, i.e., yield the most benefit to the two agents, when demand potential is moderate, roughly ranging between half to twice the combined capacity of the competing carriers, while the LSP's margins are fairly low. Finally, it is observed that when the expected demand and the LSP margin are both sufficiently high, there is little benefit to enter a contract and a small amount of unit should be signed.

Chapter 4, entitled *Shopping or dinning? The effect of passenger dwell time on airport revenues*, shifts the attention from service providers to airports, which serve as gateways enabling movements of aircraft and passengers. A paper derived from this chapter is under final stage for submission. Along with the deregulation and liberalization of the aviation industry, airports are seeking new resources of revenue apart from providing traditional transportation values, to improve the sustainability of their financial performances. Developing non-aeronautical activities is a typical strategy for airports as they progressively act as shopping malls serving not only their passengers but also their employees and local residents. With free time at airports, passengers are likely to conduct retail activities and buy food and drinks. A key question is then raised: how does the time spent at airports translate into food/beverage consumption and retail sales and does this behavior change with airport size?

To address this question, the relationship between the duration time that passengers spend at airports, namely dwell time, and airport commercial revenues is discussed.

Instead of developing an analytical models as the first two chapters do, Chapter 4 applies empirical tools to derive insights based on real data. The data is collected from users who grant a mobile application the access to their footprints and demographics information, focusing on 89 major U.S airports from 2017 to 2019. Apart from the footprint data, the airport financial data as well as airport flight information are also collected. To analyze the panel data, a fixed effects regression model is built to quantify how dwell time affects different types of airport revenues, taking passenger demographics information, airport size, airport flight metrics and airport terminal design into consideration. Our results suggest that dwell time and passenger income level significantly affect airport operating revenues. We find that airport layout matters. Specifically, in an airport with efficient logistics design, passengers consume more food and beverage activities if they have longer dwell time, whereas in an airport with multiple concourses, passengers conduct more retail activities given more dwell time.

Chapter 2

Who benefits from air service agreements? The case of international air cargo operations[1]

2.1. Introduction

Air service agreements (ASAs) structure the scope of airborne traffic between different countries, including important decisions on flight capacities. Countries have negotiated bilateral ASAs to regulate air services between them since the mid-1940's (Doganis, 2019). Albeit the recent wave of liberalization and a tendency to open skies arrangements, traffic movements in numerous places are still subject to ASAs. For instance, Zhang et al. (2018) identify that China signed 136 ASAs with 117 countries between 1954 and 2016. Doganis (2019) estimates that currently there are about 1,500 ASAs worldwide.

Many traditional bilateral agreements exist to protect national interests. It is common to specify the service in terms of flight capacity that each state is allowed to provide. This value is typically equally shared between the airlines of either country, in reciprocity (Doganis, 2019; Forsyth, 2014). We refer to such capacities as *reciprocal capacities* to highlight the symmetry in the capacity decision between

[1] Co-authors: Anne Lange and Benny Mantin

the airlines of either country. ASAs generally distinguish between passenger services and cargo services and may spell out restrictions imposed on cargo only movements. For example, Malaysia has bilateral ASAs specifying a fixed number of all-cargo frequencies with Italy, Bangladesh and Egypt (Malaysian Aviation Commission, 2018). Moreover, the protocol signed by the U.S. and China[2] specifies the total weekly frequencies of flights for combination and all-cargo services that are available for each party.

Reciprocal capacities are often expressed in terms of flight frequencies for each asset provider (AP). Given the importance of the local industry, such capacity allocations generally echo the need of these players and as such can be perceived as a coordinated duopoly. In addition to political market actions, we have witnessed an increasing interest in collaboration between APs in recent years—more often in the passenger setting than in the cargo setting—to leverage revenue synergies and cost savings that may take diverse forms of cooperation, such as a strategic alliance, joint venture with or without full revenue sharing, and eventually mergers (Fageda et al., 2019). The core challenge faced by policy makers is to determine the capacity allocation to each of the APs and further determine whether to allow more intense collaboration between the two APs.

Air cargo is transported in dedicated freighter aircraft or in the belly-hold of passenger aircraft (Doganis, 2019). This paper focuses on capacity decisions of all-cargo APs as, according to Eurocontrol (2021), most of the air cargo is carried by dedicated all-cargo aircraft. This also allows us to abstract away from the uncertainty in belly capacity on passenger aircraft. Amaruchkul et al. (2011b) explain that belly capacity varies due to short term variations in boarding passenger numbers as well as their baggage.

While there exist ample literature to guide policy makers in the context of passenger traffic assessing impacts of different aviation policies on fares and traffic volume for passenger transport (e.g., Gillen et al., 2002; Dresner and Tretheway, 1992; Ismaila et al., 2014; Oum et al., 2010), there is, to the best of our knowledge, no such research providing guidance in the context of air cargo operations. Importantly, insights derived from passenger traffic do not directly translate to cargo traffic given the fundamental differences between these two domains. Kupfer et al. (2017) highlight some of the differences between the two markets, with the most notable one in the context of our research being the structure of the demand manifested through the role of the logistics service providers (LSPs). While in passenger markets, passengers can be perceived as atomistic agents, in cargo, the LSPs aggregate the end customer's demand and act on their behalf. The nature of the interaction between APs and their customers is thus altered: cargo asset providers actively trade with logistics

[2]The U.S. and China signed a protocol relating to civil air transport in 2007.

service providers. Consequently, our model reflects such interaction in the spot market equilibrium.

In our analysis we consider the interplay between four key stakeholders in the air cargo market: the cargo APs, the logistics service providers, the end customers, and the policy makers. The cargo APs compete with each other to sell their cargo capacities to LSPs, who aggregate the end customers' demand. The end customers buy the transportation service from the LSPs. The policy makers ultimately negotiate the capacity allocation and market structure in coordinated duopolies.

When negotiating ASAs, policy makers need to account for the following considerations. Air cargo transportation is a commodity service from the point of view of LSPs. Thus, there is a fierce price competition even between the two APs that operate within a specific ASA. If APs bring more capacity than needed to the market, prices are inclined to drop, whereas if they do not bring enough capacity to the market, they may end up with lost sales at a high price potential. Importantly, capacity is a long-term decision whereas price can be adjusted based on demand realizations. Specifically, determining aircraft fleet capacities involves aircraft acquisition, or leasing contracts, which are effective for a number of years. Demand in cargo markets can be subject to random fluctuations and accordingly can change from one day to the next. Thus, APs engage in long-term capacity choices while their prices may change based on realized demand.

In air cargo operations, APs sell capacity in different ways. They may sell capacity to LSPs via long-term contracts at the beginning of each season. As discussed by Hellermann (2006), a forward contract is a commonly adopted contract type, which reserves a certain amount of units at a negotiated price. Alternatively, APs may also opt for selling capacity several days ahead of the departure of the flights, which is normally referred as spot sales. In practice, APs combine both methods to address the risk they face and thus they need to trade-off capacity allocation between these two options citepLevin2012,Moussawi-Haidar2014,Amaruchkul2011. Our focus here is on the challenging spot market where APs face fierce competition. Our paper contributes to literature by modelling such competition in the market while studying the impacts of bilateral ASAs. We abstract from the details of the daily operations by assuming a spot market where two APs bring aggregated capacity. The generated understanding will allow studying long-term contracting in future work.

We seek to analyze the effect of reciprocal air cargo capacities specified in ASAs on the welfare distribution to the relevant actors in the air cargo market. To reach this goal, we develop a two stage game theoretical framework, where in the first stage the APs—through the policy makers' negotiation—determine the capacity allocations with uncertain demand, and in the second stage, once demand is realized, the APs engage in price competition to attract demand aggregated by the LSPs.

The cargo spot market with reciprocal capacities requires specific considerations in terms of modelling. We combine three key aspects for our analysis. First and foremost, the APs engage in price competition with capacity constraints (Betrand-Edgworth competition) where customers always prefer the lowest price. Second, the total end customer demand is uncertain. Third, reciprocal agreements entail identical capacities. Considering these features allows us to develop strategic insights for the actors in air cargo transportation.

Our research makes four contributions to the study of ASAs and air cargo strategy. First, we establish a novel duopoly spot market model for air cargo by considering the interaction between four types of agents—APs, LSPs, end customers and policy makers. Second, we characterize the spot market equilibrium in terms of pricing strategies, trading quantities and profit allocation. Notably, we illustrate that for relevant ranges of parameter settings, the APs are best advised to follow a mixed pricing strategy. Third, we investigate the optimal capacity decision for air cargo companies with demand uncertainty, but restricted by reciprocal capacity constraint via an ASA. We find that under uniformly distributed demand, APs are well advised to bring approximately 15% of the demand upper bound each to the spot market, with 30% of the demand upper bound in total, to reduce the competition level in the market. Finally, we analyze the welfare distribution across different agents and discuss managerial insights for policy makers.

The remainder is organized as follows. In Section 3.2, we review related literature. Section 3.3 illustrates our two-stage game-theoretical model and presents immediate results. Section 3.4 elaborates the welfare distribution in different cases and Section 2.5 demonstrates extended models, with three APs, with different demand distributions, with costly capacity, and with asymmetric capacities. Section 3.6 draws conclusions and suggests further research.

2.2. Literature review

International air cargo services, unlike many other industries, are regulated by governments via different air service agreements which may cap frequency, routes, and the level of freedom (Doganis, 2019). Extant literature holds findings for ASAs from a passenger transportation perspective. It has been argued that liberalized protocols between countries have positive impacts in comparison to ASAs, for instance, on airfares (Dresner and Tretheway, 1992), traffic volume (Adler et al., 2014), service level (Adler and Mantin, 2015) and economic welfare (Winston and Yan, 2015). However, as demonstrated by Gillen et al. (2002), some carriers may gain and others may lose encountering different levels of liberalization based on the simulation of the airline market between Canada and Japan. McHardy and Trotter (2006) also mention that passenger gains from deregulation may be exaggerated and ensuring actual gains requires proper policy maker intervention. Such considerations motivate policy makers' intention to protect their national carriers by employing traditional bilateral air service agreements. The agreement leads to a situation where air cargo companies from the two countries coordinate in capacity and compete in price. The cooperation between competitors is also referred as co-opetition, which is firstly introduced into public discussion by Nalebuff et al. (1996) from the perspective of business and game theory. Co-opetition is observed in various aviation practices, such as alliance and joint venture. There are both empirical research (Park and Russo, 1996; Goh and Uncles, 2003) and non-empirical research (Chen and Hao, 2013; Park et al., 2001) discussing insights of aviation co-opetition. However, ASAs have not been discussed from the aspect of co-opetition. The paper by McHardy and Trotter (2006) is one of the few that analytically investigate the distribution of gains and losses across different agents under various aviation policies. Such an approach provides comprehensive insights for policy makers on the implications of their policies. Our research allows to analytically assess welfare effects for the individual air cargo actors.

Our work relates to air cargo revenue management. While we focus on the mechanism to establish the market price across APs and find that a mixed pricing strategy is optimal in significant ranges of the solution space, existing research has studied cargo revenue strategies for individual carriers. Shah and Brueckner (2012) investigate price, frequencies and vehicle capacities of freight carriers considering brand loyalty of shippers in a freight-competitive context. They show how decisions are affected by parameters and explain the excess capacity in the freight industry by comparing the equilibrium to the social optimum. Amaruchkul et al. (2007) model the capacity booking process for a single-leg cargo flight as a Markov decision process to maximize expected profits. They propose heuristic methods to approach the optimal strategy. An extended model with multiple forwarders is studied by Amaruchkul et al. (2011b), focusing on the allocation of capacity before a booking horizon with stochastic usages.

Huang and Chang (2010) propose an approximation algorithm for two-dimensional air cargo capacity control and show that their method generates higher revenue than de-coupling control approaches in most cases. Besides single-leg operations, there is also literature focusing on cargo network revenue management. Barz and Gartner (2016) optimize revenue in an air cargo network by forming a dynamic programming and propose several approximations. Levina et al. (2011) consider a problem of controlling cargo accept/reject decisions with stochastic weight and volume modeled as a Markov decision process. They propose an approximation method and illustrate that the heuristic method is competitive. We explore pricing decisions in the air cargo spot market given competition and, further, study the effect of capacity decisions. Levin et al. (2012); Moussawi-Haidar (2014); Amaruchkul et al. (2011b) investigate the trade-off between long-term contract and short-term spot market and propose different dynamic control models to maximize the asset provider's revenue under monopoly setting. In contrast to their managerial approaches, our study gives insights to the welfare distribution that results from individual firm decision making, given bilateral ASAs.

Besides capacity, another critical decision for APs is pricing. Cargo tariffs are coordinated through IATA Tariff Conferences but final rates charged by APs in the market are very different from the tariffs (Doganis, 2019). In 2007, the anti-trust immunity of IATA Tariff Conferences has been withdrawn by the U.S Department of Justice (Order 2007-3-23)[3] since they were found to be anti-competitive. Along with the liberalization of aviation industry, air cargo tariffs are determined by APs according to the capacity and market conditions (IATA, 2021). The COVID-19 pandemic is an example where air cargo prices increased dramatically due to the shortage of capacity.

This paper takes the two-dimensional pricing structure of air cargo into consideration. Unlike passengers, air cargo can price along two dimensions—weight and volume—and capacity is constrained in both dimensions. In practice, APs offer prices based on chargeable weight, which is the maximum of the weight and volume equivalent. While our basic model simplifies the setting and assumes a single pricing instrument (Amaruchkul et al., 2011b; Lin et al., 2017), we also run a numerical study to show that the insights are not fundamentally changed with two-dimensional pricing structure.

Most of the cargo revenue management literature assumes a monopoly setting where competition among carriers is not taken into consideration (Amaruchkul et al., 2011b; Lin et al., 2017; Levin et al., 2012; Moussawi-Haidar, 2014). Competition is a major consideration in the cargo industry and must be modeled to reflect reality. Competition not only concerns the pricing mechanism in the spot market. Wang et al. (2017) study the entry problem of a mainline carrier into the upstream market considering the competition of promised delivery time between a mainline carrier and a regional carrier. Zhang et al. (2004) investigate the effects of a cargo alliance by two passenger airlines on competition in passenger markets and find that such an alliance will likely increase the partner's own outputs. Shiao and Hwang (2013) analyze the competitive strategies for air cargo carriers using a two-stage model. From two empirical cases, they find that combination airlines have competitive advantages compared to all-cargo airlines.

In a broader logistics context, capacity decisions with price competition have raised some attention. Zheng et al. (2017) propose the pricing strategy under of two competing ocean carriers with different risk preferences. Zhang et al.

[3]United States of America, Final Order issued by the Department of Transportation on the 30th of March, 2007.

(2010) investigate the revenue sharing equilibrium modeling the competition between airports and airlines and they find the airport has an incentive to charge strongly substitutable airlines for a share of concession revenue. Shang and Liu (2011) study a capacity game among firms which compete in promised delivery time and quality of service and characterize the equilibrium of capacity choices.

In practice, particularly in the international air cargo context under bilateral ASAs, there are only a limited number of airlines involved, potentially leading to fierce price competition with constrained capacity. Hence, it is relevant to investigate how price competition affects business actions. In the economic literature, there are numerous studies focusing on price competition using game-theoretical models. Osborne and Pitchik (1986) characterize the Nash equilibrium in a duopoly setting and find the capacity equilibrium coincides with the set of Cournot quantities with static demand. Levitan and Shubik (1978) and Vives (1986) characterize mixed pricing equilibria for oligopoly games. Allen and Hellwig (1993) illustrate a similar pricing equilibrium with a different demand allocation method where residual demand is proportionally split. These papers model the situation where the higher price offerers may end up selling nothing due to fierce price competition. This is of interest for the air cargo industry where customers often prefer the lowest price and lack loyalty. Different from their works, which focus on prices and profits, we also investigate the trading quantities under different outcomes, which then allow us to study welfare distribution. Also, we consider demand to be stochastic rather than static.

2.3. Model setup and preliminary analysis

Our model encapsulates the pricing decisions faced by competing APs when they interact in a spot market with LSPs, who aggregate demand from end customers. Ultimately, we seek to evaluate the impact of policy intervention, in the sense that policy makers sign bilateral agreements that outline capacity allowances to specific APs. These APs, which may be national carriers, are permitted to operate in markets as specified in the bilateral ASAs. As countries will often designate only one of its carriers to operate flights in each origin-destination pair under bilateral ASAs (Peter et al., 2015), we assume one AP per country. The results from the setting with two competing APs will later be compared with the setting where the two APs are allowed to collaborate closely and act as a single player, acting, effectively, as a monopoly.

We formulate the interaction between the four types of agents—the APs, the LSPs, the end customers and the policy maker—as a two-stage game theoretical framework. The first stage defines the capacity decisions. The policy makers, who negotiate ASAs, often represent, or act in the best interest of the asset providers. In other words, they ensure that the capacity allocation specified in ASAs best represents the APs' benefits. The two APs, AP_a and AP_b, make the capacity decision, k_a and k_b, at an identical unit cost c. Due to the reciprocity of the bilateral ASAs, our analysis enforces that the equilibrium capacity for each AP is identical. In other words, the two APs determine the optimal capacity while considering that the competitor is allowed for the same amount of capacity. We impose $k_a = k_b = k$, as a constraint, with k being the capacity brought by each AP to the market. This decision is a long term decision which is made well ahead of demand realization.

The second stage is a spot market where the two APs compete in price, by offering their spot prices p_a and p_b simultaneously, and sell capacity to LSPs. We assume that demand, d, is a random variable with pdf $f_d(d)$ and cdf $F_d(d)$. This demand, which is realized in the second stage, is aggregated by the LSPs and traded in the spot market. To reflect the end customers price sensitivity, we assume a down sloping linear demand-price function, $D(p) = d(1-p)$, in the spot market, with p normalized between a marginal price, $p = 0$ (for simplicity, it is normalized to zero), and a maximum price, $p = 1$. In our model, we do not consider the cost of lost sales since APs do not have an obligation to satisfy all demand and such cost should not fundamentally change the insights.

The LSPs, who act as intermediary agents between the end customers and the APs, apply a margin m to the end customers for each unit they trade. We assume there are sufficiently many LSPs active in the market such that no single LSP can unilaterally affect the price. The two APs bring the combined allocated capacities, $2k$, to the spot market and trade with the LSPs. Given the demand realization, the capacity may fall short of, or surpass, demand. The two APs are engaged in a price competition with limited capacities in the spot market. Hence, they are engaged in a Bertrand-Edgeworth type of competition. Accordingly, the two APs set their respective spot price p_a and p_b. Demand is satisfied first by the AP who offers the lower price and only then, customers buy from the AP with the higher price. Section 2.3.1 elaborates on the price competition between the two APs.

The modeling framework is illustrated in Figure 2.1 and a summary of notation is provided in the Appendix Table 3.4. To derive the sub-game perfect Nash equilibrium, we solve the model backwards. In Section 2.3.1 we solve the second stage of the game (the pricing and the spot market stage), whereas in Section 2.3.2 we proceed with the first stage (the capacity decisions).

2.3.1 Stage Two: Spot Market Equilibrium

In this stage, given the capacity decisions from the first stage, the two APs face price competition due to their customer's preference for a low price. For instance, if

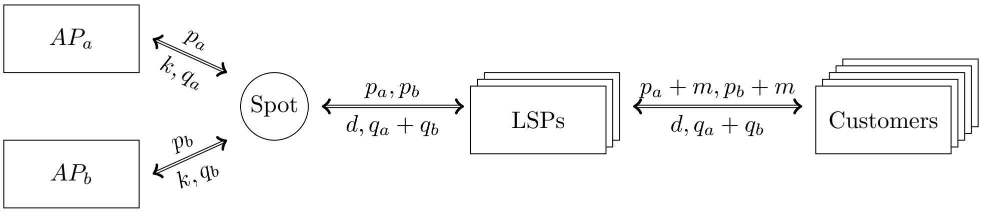


Figure 2.1 Benchmark model.

$p_a < p_b$, AP_a sells $q_a = \min(D(p_a), k)$, bounded by its capacity. AP_b can then sell the residual demand $q_b = \min(\max(0, D(p_b) - k), k)$. The expression of $\max(0, D(p_b) - k)$ guarantees the residual demand to be non-negative and the min function ensures the trading quantity is bounded from above by the capacity. We assume that LSPs who are willing to pay more get the priority to purchase. This assumption could also be explained by that the lower-price-offer company would choose customers in order to minimize its competitor's profits. Similar residual demand is adopted by Levitan and Shubik (1978), Osborne and Pitchik (1986), and Vives (1986).

The immediate profit of $\text{AP}_i, i \in \{a, b\}$, is given by

$$\pi_i(p_i, p_j, d, k) = \begin{cases} L(p_i) \equiv & p_i min(k, D(p_i)) & \text{if } p_i < p_j \\ E(p_i) \equiv & p_i min(k, \frac{D(p_i)}{2}) & \text{if } p_i = p_j \\ H(p_i) \equiv & p_i \min(k, max(0, D(p_i) - k)) & \text{if } p_i > p_j \end{cases} \tag{2.1}$$

where L (resp., H) denotes AP_i's revenue when this AP sets the lower (resp., higher) price, and E denotes the revenue when both set an equal price.

In Equation (2.1), the profit changes with different demand realizations and there may not exist a pure price equilibrium. For instance, when d is sufficiently small, the AP with the lower price satisfies all demand and the high-price AP realizes zero profit. In this case, the two APs undercut each other leading to a pure price equilibrium where both of them offer the marginal price. If d increases, the high-price AP can realize positive profits since the low-price AP cannot satisfy all demand at some point. When d is sufficiently large, the two APs fall into a pure price equilibrium where either increasing or decreasing the price unilaterally leads to a lower profit.

Based on the relationship between the profit functions from Equation (2.1), five qualitatively different outcomes may emerge as a function of the demand realizations. The outcomes are indexed by $i \in \{1, 2, 3, 4, 5\}$ reflecting increasing demand realizations (i.e., increasing d values). Outcome 1 corresponds to the case where $d \leq k$. In this case, d is sufficiently low such that the low price AP can satisfy all demand. This leads to a price undercutting sequence by the two APs, ultimately resulting with both setting a price of 0.

When $d > k$, the low price AP cannot satisfy the demand alone and accordingly, Outcomes 2 and 3 are both characterized by a cycle in the price response function. This occurs as each AP seeks to undercut the other AP's price, but at some point, further price undercutting is ruled out. Here, one of the APs has an incentive to deviate and increase the price to some higher value so as to leave the low price AP to sell out its capacity and remain as the only AP in the spot market allowing it to extract more surplus. However, by doing so the other AP may respond by setting a price marginally lower than the high price AP, thereby triggering the price undercutting mechanism which results with a cycle. In Outcome 2, $k < d \leq 2k$ the high price AP is unable to sell out its capacity, whereas in Outcome 3, in which $2k < d \leq 3k$, it may, for some sufficiently low prices, sell out its capacity. Outcome 2 is illustrated in Figure 2.2.

In Outcomes 4 and 5, the prevailing equilibrium prices are such that they lead to pure strategies. In Outcome 4, $3k < d \leq 4k$ and the profit maximizing price of the high price AP is bounded by its capacity. In Outcome 5, in which $4k < d$, the same condition holds, while the profit maximizing price when the two AP offer identical price are also bounded by their capacity.

These outcomes leads us to the characterization of the equilibrium prices and corresponding quantities for different realized demands. While the prices can be derived from Osborne and Pitchik (1986) or Vives (1986), we provide a complete characterization of the equilibrium outcomes. Our proof, while inspired by Osborne and Pitchik (1986), carries out the best-response price analyses under different outcomes, which are distinguished by demand thresholds, and shows that the distribution of the mixed pricing is atomless and continuous.

Proposition 2.1. *The equilibrium spot price is given by*

$$p_i^e = \begin{cases} 0 & \text{if } d \leq k \\ p_m & \text{if } k < d < 3k \\ \frac{d-2k}{d} & \text{if } 3k \leq d \end{cases} \tag{2.2}$$

where $p_m \in [\frac{(d-k)^2}{4kd}, \frac{d-k}{2d}]$ *following the cdf* $F_p(p_m) = \frac{4kdp_m-(d-k)^2}{4d^2p_m^2-4d^2p_m+8kdp_m}$, *with* $i \in \{a, b\}$. *The equilibrium sales are*

$$q_i = \begin{cases} \frac{d}{2} & \text{if } d \leq k \\ d(1-p_i)-k & \text{if } k < d < 3k \text{ and } p_i > p_{-i} \\ \frac{d(1-p_i)}{2} & \text{if } k < d < 3k \text{ and } p_i = p_{-i} \\ k & \text{if } k < d < 3k \text{ and } p_i < p_{-i} \text{ or } 3k \leq d \end{cases} \tag{2.3}$$

Some properties can be addressed for the mixed pricing equilibrium. When demand is intermediate, no pure pricing strategy exists since in the undercutting process, firms jump to a higher price for more profits. This leads to a cycle of pricing as illustrated in Figure 2.2. For instance, starting from N_1, where both APs post identical prices, one marginally drops the price (say to N_2) to capture the benefit of moving towards $L(p)$ leaving the competitor with the reduced profit at $H(p)$ (indicated by N_3), inducing the latter to further drop the price and move to, say, N_4. This sequence of price undercutting continues until one of them reaches the price $\underline{p}$ from which the other AP moves up to $\bar{p}$ rather than undercutting again. From here, the former AP moves closer to the latter by posting a price marginally below $\bar{p}$ thereby initiating the pricing cycle. Hence, we observe an Edgeworth Pricing Cycle, as identified by Maskin and Tirole (1988). Thus, firms end up with a mixed pricing strategy to avoid being predicted by the competitor and both APs have positive sales at any pairs of prices in the equilibrium range.

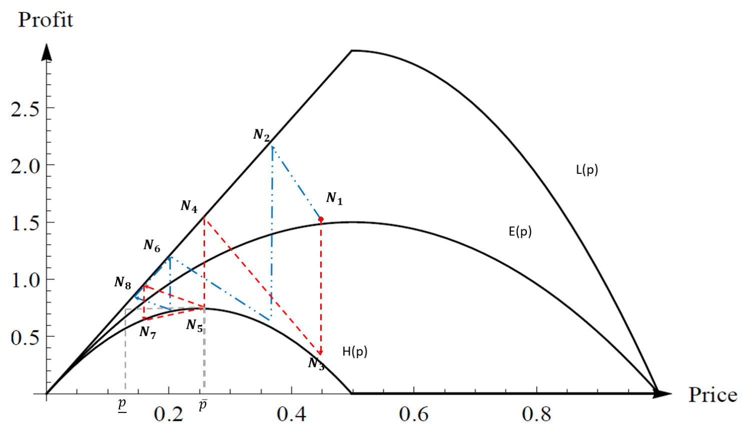


Figure 2.2 Illustration of an Edgeworth Pricing Cycle with intermediate demand

Figure 2.3 illustrates an example of equilibrium spot prices when $k = 10$. In this case, when $d \leq 10$, we have a pure pricing strategy with both setting a price of 0. When the demand is intermediate, the weighted average spot price is indicated by the solid line with the bounds $\underline{p}$ and $\bar{p}$ indicated by the dashed lines. Note that the weighted price is located towards the lower bound price as raising a lower price ensures sales while a higher price leads to higher unit profit at the risk of losing sales in low demand realizations. When $d > 30$, we have, again, a pure strategy in prices.

The equilibrium profit of each AP, $\pi_i^e(d, k)$, follows from Proposition 2.1:

Proposition 2.2. *For a given spot demand d, the equilibrium profit of each AP is*

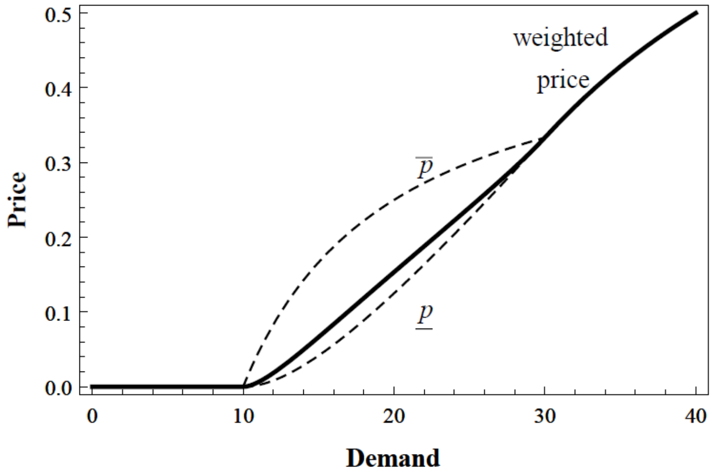


Figure 2.3 Equilibrium spot prices as a function of realized demand, given $k = 10$

given by:

$$\pi_i^e(d,k) = \begin{cases} 0 & \text{if } d \leq k \\ \frac{(d-k)^2}{4d} & \text{if } k < d < 3k \\ \frac{d-2k}{d}k & \text{if } 3k \leq d \end{cases} \quad (2.4)$$

which is non-decreasing and continuous in d.

This result allows an intuitive interpretation. Evidently, when d is sufficiently low, increasing d has no impact as the two APs price at the margin and make no profit. However, beyond this threshold, an increase in d translates directly to an increase in profit. A key factor of this profit generation is the available capacity. If k is too low, profit may be lost in case of a high realized d, but if k is too high, the range of zero profit expands.

More specifically, we illustrate the corresponding sales as well as profits as a function of the realized demand in Figure 2.4. An important observation from this figure is that the competing firms make no profit at all when demand is low due to the fierce competition they face, whereas their potential for profit is constrained by their limited capacity when demand is high. This implies that they face a challenging trade-off between fierce competition and capacity constraint when they determine their long-term capacity decisions. Hence, we next address the choice of optimal capacity.

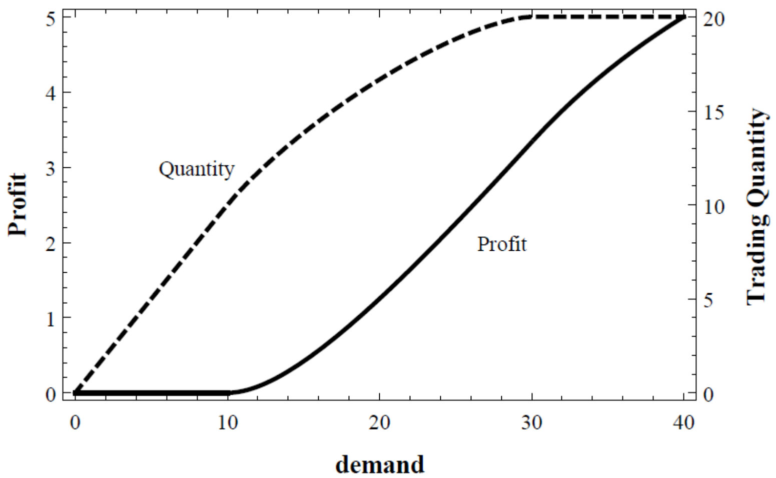


Figure 2.4 Total trading quantities and equilibrium profits as a function of realized demand d, given $k = 10$

2.3.2 Stage One: Capacity Decision

Under reciprocal bilateral ASAs, the two APs are constrained to bring identical capacity to the market, namely $k_a = k_b = k$, at a unit cost c. For simplicity of exposition, we normalize the cost to zero; however, in Section 2.5.3 we revisit this assumption to discuss how the unit cost impacts the optimal capacity and welfare distribution. Equation (2.4) provides the expected profit of each AP for a given pair of d and k. To find the profit maximizing capacity choice, we take the expectation over the realized demand. Specifically, we seek the value of k_D^* which maximizes each asset provider' expected profit $E_d[\pi_i^e(d,k)]$. Considering a general demand distribution F_d, we find the expected profit of each AP.

Proposition 2.3. *The expected profit of each asset provider is given by:*

$$\begin{aligned} E_d\left[\pi_i^e(d,k)\right] = &-\frac{2}{3}kF_d(3k) - \frac{1}{4}\int_k^{3k} F_d(d)dd + k + \frac{k^2}{4}\int_k^{3k}\frac{F_d(d)}{d^2}dd \\ &- 2k^2\frac{F_d(d)}{d}\bigg|_{3k}^{\infty} - 2k^2\int_{3k}^{\infty}\frac{F_d(d)}{d^2}dd \end{aligned} \tag{2.5}$$

Naturally, the expected profit of each AP depends on the capacity choice k. In order to gain further insights, we assume a uniformly distributed demand, i.e., $d \sim U(0,b)$. We have the following corollary.

Corollary 2.1. *Assume a uniformly distributed demand, $d \sim U(0,b)$. The expected*

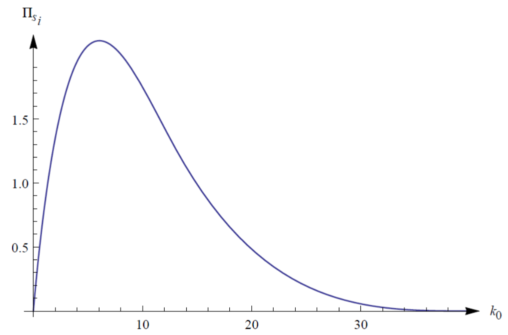


Figure 2.5 AP's expected profit in benchmark case, $d \sim U[0, 40]$

profit of each AP is

$$E_d\left[\pi_i^e(d,k)\right] = \frac{k^2}{4b}\ln 3 + \frac{k}{b}\left(b - 3k - 2k\ln\frac{b}{3k}\right) \tag{2.6}$$

which is approximately maximized at $k_D^* = \frac{-b}{4} \cdot W_{-1}\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right)^{-1} \approx 0.1512b$*, where* $W_{-1}(a)$ *is the lower branch of Lambert w function.*

We illustrate the behavior of the expected profits with a numerical example with uniformly distributed demand, $d \sim U[0, 40]$, in Figure 2.5. The optimal capacity is about 6.05 and substituting this capacity choice in Equation (2.6), yields an expected profit of about 2.11.

It is common to assume that more players in a market imply a higher level of competition possibly with a higher welfare. For instance, in a classic Cournot quantity competition, duopolies jointly bring more quantities than a monopoly due to competition which leads to a lower price. However, in our case, with uniform demand, the optimal capacity brought by each AP is about 15% of the demand upper bound, b. Jointly, they bring 30% of the demand upper bound to the market, which is less than the expected demand, $\frac{1}{2}b$. Such small amount results from the trade-off between the low amount of trading quantity with insufficient capacity and the high level of competition with overcapacity. A bilateral agreement enables the coordination of capacity between APs and less capacity is brought to the market to reduce the level of competition. Thereby, it is of interest to further investigate how welfare is distributed under such policy intervention.

2.4. Bilateral coordination of capacity

This section analyzes the impacts of the bilateral intervention by investigating the profit distribution among the three agents—APs, LSPs and customers—for the case of uniformly distributed demand. The welfare is then compared with the monopoly setting. We refer to this setting as the monopoly setting, having in mind settings where the two APs are allowed to collaborate tightly, such as a joint venture or a virtual merger.

2.4.1 Welfare distribution under coordinated duopoly

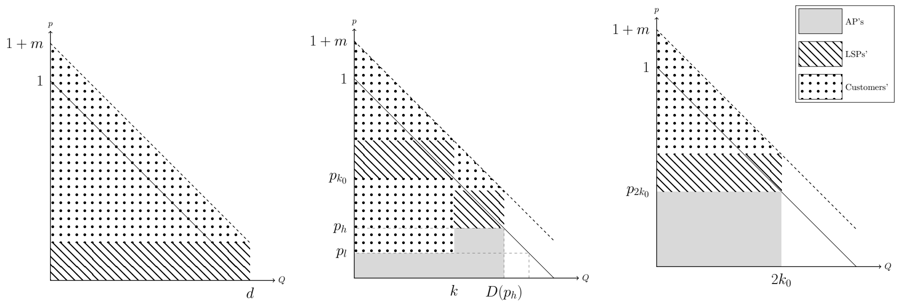
APs take profits as their welfare and the LSPs take total margins as the welfare. For end customers, the welfare comes from the surplus, which is the difference between prices and the willingness to pay. The distribution of welfare changes with demand realization as shown in Figure 2.6. The downward sloping solid line is the demand-price function in the spot market and the parallel downward sloping dashed line is the end customer demand function (which takes into account the margin charged by the LSPs).

In Figure 2.6a, the distinction between the end customers' and LSPs' surplus is as follows. The demand in the spot market is given by the solid sloped line while the real demand is shifted upwards by the addition of the margin m charged by the LSPs. Since the spot market price is zero, the entire area under the dashed demand line is split between the end customers and the LSPs, with the lower portion reflecting the margin m allocated to the LSPs.

In Figure 2.6c, a market price exists, $p_{\{2k\}}$, which determines the distribution of surplus. The rectangle below this price, is the APs' surplus, whereas the triangle above is split between the LSPs and the end customers in the same manner as in Figure 2.6a.

Consider Figure 2.6b, in which case we have two prices in the market – a higher price, p_h and a lower price p_l. Note that the lower price AP is limited by the capacity, k. Thus, this AP's surplus is the rectangle under p_l bounded to the right by k. The area above this rectangle is the surplus of the LSPs and the end customers with the former taking the standard part associated with the margin m. The high price AP gains the rectangle below the price p_h bounded from the left by k and from the right by $D(p_h)$. The share of the surplus above this rectangle is as discussed previously.

In order to formally analyze the welfare distribution in our duopoly setting, we firstly need to develop the expressions for the two APs' and LSPs' revenues as well as customers' surplus, denoted as π_a^D,π_b^D,π_l^D,π_c^D correspondingly. While Figure 2.6



(a) Overcapacity in Duopoly (b) Intermediate in Duopoly (c) Under-capacity in Duopoly

Figure 2.6 Welfare distribution among APs, LSPs and customers

	overcapacity	intermediate	undercapacity
Total traded quantity	d	$D(p_h)$	$2k$
AP's price	0	p_l, p_h	$p_{\{2k\}}$
Total revenue of APs	0	$kp_l + (D(p_h) - k)p_h$	$2kp_{\{2k\}}$
LSPs revenue	md	$mD(p_h)$	$m2k$
Customer surplus	$\frac{1}{2}d$	see Equation (2.7)	$k(1 - p_{\{2k\}})$

Note: $p_h(p_l)$ is the higher (lower) price charged by the two APs; $p_{\{2k\}}$ is the market clearance price of $2k$ units, namely $d\left(1 - p_{\{2k\}}\right) = 2k$.

Table 2.1 Duopoly welfare expressions given k and realized demand d

illustrates them graphically, Table 2.1 provides the formal expressions. The most complex analysis is in the intermediate range. Here we find that the APs' revenues depend on whether they raise the higher or the lower price in comparison to their competitor. Hence, the AP offering the lower price p_l sells all its capacity while the high price AP sells the residual demand $D(p_h) - k$, leading to the APs' welfare:

$$\pi_a^D(d,k) + \pi_b^D(d,k) = kp_l + (D(p_h) - k)p_h$$

The joint welfare of APs and customers is $\frac{1}{2}(p_h + 1)D(p_h) = \frac{1}{2}d - \frac{1}{2}dp_h^2$. Since prices are drawn from the equilibrium range when demand is intermediate, the weighted joint welfare by aggregating the prices is $\frac{1}{2}d - \frac{1}{2}d\int_{\underline{p}}^{\bar{p}} p_h^2 f_h(p_h)dp_h$ where $\underline{p}$ and $\bar{p}$ are the bounds of equilibrium prices and $f_h(p_h)$ is the probability of p_h being the higher price also derived in the proof of Lemma 2.1. Without deriving customer surplus directly, we obtain it by subtracting the welfare of APs from the joint welfare of APs and end customers:

$$\pi_c^D(d,k) = \frac{1}{2}d - \frac{1}{2}dp_h^2 - kp_l - (D(p_h) - k)p_h \tag{2.7}$$

Besides the welfare of APs and end customers, we still need to derive the welfare of LSPs which depends on the traded quantities. Here, we directly investigate the expected traded quantities by aggregating demand.

Lemma 2.1. *The total expected traded quantity on the spot market is given by:*

$$E_d[q_a + q_b] = 2k + kF_d(k) - 2kF_d(3k) - \int_0^k F_d(d)dd + \int_k^{3k} w(d,k) f_d(d)dd \tag{2.8}$$

where

$$\begin{aligned} w(d,k) &= \int_{\underline{p}}^{\bar{p}} d(1-p) f_h(p)dp \\ &= \frac{2(d-2k)(2d^3 - 7d^2k + 4dk^2 + 5k^3) + (d-3k)^2(d-k)^2 \ln - \frac{d-3k}{d-k}}{8(d-2k)^3} \end{aligned}$$

To derive more insights of the total traded quantity, we find a closed form expression for the expected traded quantity, which is stated in the following corollary.

Corollary 2.2. *Assume d follows a uniform distribution with $d \sim U(0,b)$ and k_D^* is the optimal coordinated capacity. The expected total traded quantity is*

$$E_d[q_a + q_b] = \frac{k_D^*}{8b}(16b + k_D^*(\pi^2 - 28)) \tag{2.9}$$

The expected total traded quantity is approximately $0.25b$, for $k_D^* \approx 0.1512b$, see Corollary 2.1.

Now, we can evaluate the welfare of the coordinated duopoly equilibrium.

Corollary 2.3. *If the asset providers bring k_D^* units each to the market, the total expected equilibrium welfare under policy intervention is approximated as:*

$$\begin{aligned} E_d[\Pi_D] &= E_d[\pi_a^D + \pi_b^D + \pi_l^D + \pi_c^D] \\ &\approx \frac{mk}{8b}(16b + k_D^*(\pi^2 - 28)) - \frac{3.8414}{b}{k_D^*}^2 + 2k_D^* - \frac{2{k_D^*}^2}{b} \ln \frac{b}{3k_D^*} \end{aligned} \tag{2.10}$$

Corollary 2.3 gives the total welfare under the coordinated duopoly setting. To gain more insights, we contrast it with the monopoly setting, where the two APs can (virtually) merge as a single operator. Although virtual mergers are often permitted, an anti-trust concern remains for this type of cooperation since reduced competition may lead to social welfare loss. Accordingly, in Section 2.4.2, we investigate the monopoly setting and compare the welfare distribution under both settings to study the effects on welfare distribution.

2.4.2 Welfare distribution under monopoly and comparison to the coordinated duopoly

To obtain the welfare when the two APs act as a monopoly, we first investigate the optimal capacity decision for a monopoly.

Corollary 2.4. *When demand is uniformly distributed, $d \sim U[0, b]$, the monopoly brings at least half of the demand upper bound in capacity to the market, $k^*_M \geq \frac{1}{2}b$. Consequently, the expected quantity traded under a monopoly setting, $E_d[q_a + q_b]$, equals one fourth of upper bound of demand, namely $\frac{1}{4}b$.*

Recall from Equation (2.9) that the expected traded quantity in the coordinated duopoly setting is approximately $0.25b$. Hence, we observe that the expected traded quantities in monopoly and coordinated duopoly settings are numerically identical for uniformly distributed demand in $[0, b]$. However, the total capacities in the two settings differ strongly. From Corollary 2.1 it follows that total capacity in the coordinated duopoly is approximately $0.3024b$, which is strictly lower than the capacity in the monopoly setting. Note that Corollary 2.4 assumes $c = 0$, and thus carrying idle capacity comes at no cost to the APs. Later in Section 2.5.3, we revisit this assumption and numerically show that this insight—that monopoly results with higher capacity than in the coordinated duopoly—qualitatively holds also for a strictly positive unit cost.

Figure 2.7 illustrates the APs' profit for both coordinated duopoly and monopoly. It depicts the expected profits of APs as a function of realized demand d given optimal capacity is chosen, following Proposition 2.2 and Corollary 2.1. The monopoly player sets the capacity to $\frac{1}{2}b$ and her profit is a linear function of d, see Equation (2.41) in the Appendix. This relationship is described by the blue dashed line in Figure 2.7. In contrast, the players in duopoly face price competition and this leads to the loss of total profits when demand is low, shown in the shadow area on the left. However, when demand is high the players miss a sales potential as they are constrained by their capacities. As it is optimal to bring less total capacity for the duopoly, this leads to a lower expected profit in this setting for high demand cases. This is shown in the shaded area on the right. Therefore, duopolies face a trade-off between fierce competition and constrained capacity.

As the expected traded capacities are essentially identical in both markets, the welfare discussion centers around the prices charged by the APs in either setting. The monopolist can set the price to maximize her profit without competition, whereas the APs in the coordinated duopoly set the price considering three impacts—their profit, the competition and the customers' response—leading to a small capacity brought to the market.

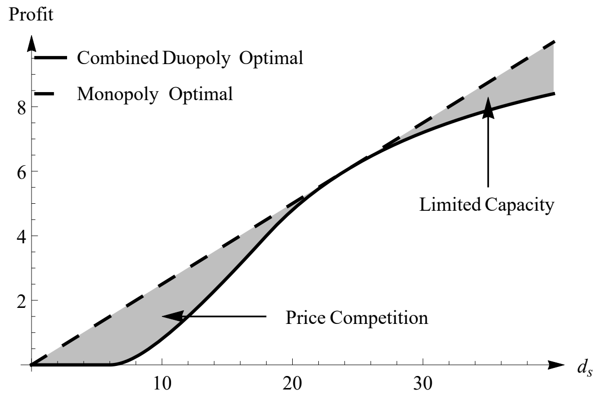


Figure 2.7 Comparison of asset providers' profits in monopoly and duopoly cases given optimal k

With the optimal capacities in both markets, we can investigate the welfare distribution. The graphic distribution of welfare under monopoly is in a similar shape of Figure 2.6c, with the trading quantity $D(p^*)$ and the trading price p^* replacing $2k_0$ and $p_{\{2k_0\}}$ correspondingly. Given a demand realization, the AP's welfare comes from sales which is $p^*D(p^*)$ with p^* to be the monopoly price. The LSPs gain $mD(p^*)$ since they apply a margin m to each unit traded. For customers, the welfare is the surplus which is the total willingness to pay minus the actual payment and can be derived as $\int_{p^*}^{1} D(p)dp$. Thus, the total expected welfare is given by

$$E_d[\Pi_M] = E_d\left[\frac{1}{4}d + \frac{m}{2}d + \frac{1}{8}d\right]. \tag{2.11}$$

Based on the above discussion, we have the following corollary (without further proof):

Corollary 2.5. *With a uniformly distributed demand $U[0, b]$, the total expected welfare under the monopoly setting is given by*

$$E_d[\Pi_M] = \frac{3}{16}b + \frac{m}{4}b.$$

Now, we compare the welfare of the monopoly and the coordinated duopoly with a uniform demand distribution.

Corollary 2.6. *Consider a uniformly distributed demand, $d \sim U[0, b]$. Compared with a monopoly setting, negotiated agreements result with lower profit to the APs, higher surplus to both LSPs and end customers, with a lower total social welfare.*

Agent	Welfare transition from monopoly to duopoly
AP	decrease
LSPs	increase
End customers	increase
Total	decrease

Table 2.2 Welfare Comparison

The welfare distribution under two different settings is illustrated in Table 2.2.

2.5. Extensions

We consider several extensions to our model: expansion of the market to more than two APs (Section 2.5.1), alternative demand distributions (Section 2.5.2), impact of capacity decisions on welfare in the presence of unit cost (Section 2.5.3), and asymmetric cargo capacity between the two APs (Section 2.5.4).

2.5.1 Spot market with three asset providers

In our model, we explored a coordinated duopoly. Some ASAs, however, permit more APs to serve. Will they bring more capacity to serve their customers? Below, we numerically extend our study to three APs.

As before, each AP brings k units to the market in a coordinated way and they price-compete for spot demand of d units. The demand-price function is identical to the basic model, namely $D(p) = d(1 - p)$. The price competition follows the same arrangement where customers always prefer the lower price. Let s denote the number of APs offering the same price, $s \in \{1, 2, 3\}$, and l denote the number of APs offering a lower price, $l \in \{0, 1, 2\}$. Accordingly, an AP sells $(D(p) - lk)^+ /s$ units. Vives (1986) discusses the rationing rules and Bertrand-Edgeworth equilibria in a similar setting where symmetric firms compete in price and the result is aligned with the duopoly market studied by Osborne and Pitchik (1986) as well as with our result of the basic model. In our numerical study, we only focus on the equilibrium pricing strategy and profits, which can be derived by following from their proposition of oligopoly settings. The following lemma—derived directly from Vives (1986)—characterizes the equilibrium properties for the three-AP spot market.

Lemma 2.2. *When there are three asset providers, the equilibrium spot prices they* *set are given by*

$$p_i^e = \begin{cases} 0 & \text{if } d \leq 2k \\ p_m & \text{if } 2k < d < 4k \\ \frac{d-3k}{d} & \text{if } 4k \leq d \end{cases} \tag{2.12}$$

where $p_m \in [\frac{(d-2k)^2}{4kd}, \frac{d-2k}{2d}]$ *following the cdf* $F_p(p_m) = \left(\frac{4kdp_m-(d-2k)^2}{4d^2p_m^2-4d^2p_m+12kdp_m}\right)^{\frac{1}{2}}$, *with* $i \in \{a, b, c\}$ *indicating the three asset providers, and the equilibrium profit is*

$$\pi_i^e = \begin{cases} 0 & \text{if } d \leq 2k \\ \frac{(d-2k)^2}{4d} & \text{if } 2k < d < 4k \\ \frac{d-3k}{d}k & \text{if } 4k \leq d \end{cases} \tag{2.13}$$

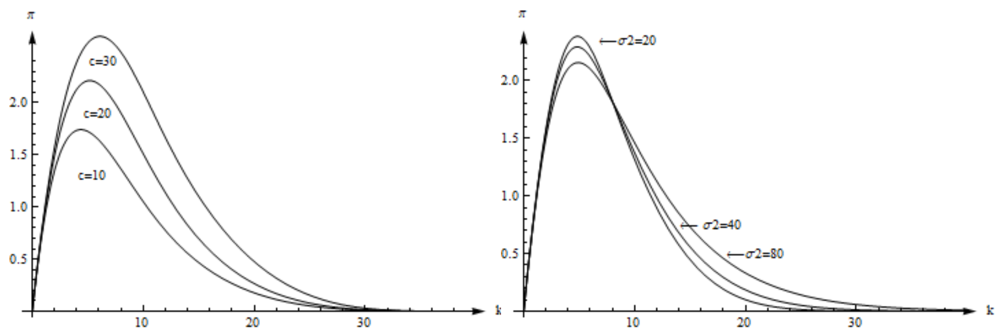
Comparing with the coordinated duopoly model, adding one AP, we find that the range of demand where all APs offer capacity at the marginal price is doubled (i.e., from $d \leq k$ to $d \leq 2k$), and that the profit of each AP decreases (comparing Equations (2.4) and (2.13)). Corollary 2.7 gives an upper bound on the optimal capacity decision for the three APs, denoted as k_T^*.

Corollary 2.7. *With three APs, assuming a uniformly distributed demand,* $d \sim U[0, b]$. *The optimal capacity is such that* $k_T^* \leq \frac{b}{4}$.

This upper bound implies that, if the APs offer the optimal capacity, they may face all three types of spot markets identified previously: overcapacity where they price at the marginal cost, intermediary capacity with mixed pricing strategy, and under-capacity where they price at a market-clearance price.

We further numerically investigate how the expected profit of each AP is affected by the choice of capacity. For instance, setting $b = 40$, the optimal capacity is about 3.9 units for each AP, which is approximately $0.10b$.[4] This is lower than $k_D^* = 0.15b$ (recall Corollary 2.1). Thus, the total capacity brought by three APs is slightly lower than when there are two APs. The result indicates that more APs do not fundamentally change the total capacity brought to the spot market. The number of APs, however, determines the distribution of profits and the the spot prices. Further analysis necessitates additional assumptions on how governments coordinate in the presence of multiple APs in one or both negotiating countries, or when more than two countries (i.e., leading to a multi-lateral ASA) negotiate. We leave this avenue for future work.

[4] As visually, the expected profit as a function of capacity assimilates that in Figure 2.5, we do not replicate the figure here.



(a) $d \sim Tri[0, 40, c]$ (b) $d \in \Gamma(\alpha, \beta)$ with identical mean

Figure 2.8 AP's expected profit as a function of k with different distributions

2.5.2 Capacity decisions with different demand distributions

To explore the robustness of our insights, we numerically study additional demand distributions.

Triangular distribution: Similar to the uniform distribution, this distribution, which is characterized by $d \sim Tri[a, b, m]$, is also bounded. To facilitate comparison with the uniform distribution studied thus far, we set the lower bound a to 0 and the upper bound b to 40, and vary the mode $m \in \{10, 20, 30\}$. Figure 2.8a demonstrates the expected profit of an AP as a function of the capacity. The optimal capacity k_D^* is 4.38, 5.18, and 6.09 when $c = 10$, 20, and 30, respectively. Naturally, k_D^* increases in the mode m, however, this increase in k_D^* is rather marginal.

Gamma distribution: This distribution does not have an upper bound and is characterized by a shape parameter, α, and a scale parameter, β. To compare with the results from the uniform distribution, we consider three mean-preserving gamma distributions. Specifically, since $E[d] = 20$ when $d \sim U[0, 40]$, we set $\alpha\beta = 20$ for three instances of the gamma distribution while letting the variance $\sigma^2 = \alpha\beta^2$ vary from 20 to 80. Figure 2.8b reveals that for all three gamma distributions with $\sigma^2 \in \{20, 40, 80\}$, the profit maximizing k is fairly consistent at around $k \approx 4.85$.

A two-dimensional demand structure: As discussed in the introduction part, air cargo is practically charged for chargeable weight, which is the maximum of the weight and volume equivalent. Here, we assume that both weight and volume equivalent follow an identical uniform distribution, namely $d_1 \sim U[0, b]$ and $d_2 \sim U[0, b]$. Thus, the chargeable weight $d_{max} = \max\{d_1, d_2\}$ follows the pdf of $f_{d_{max}}(d) = 2f_d(d)F_d(d) = \frac{2d}{b^2}$ for $d \in [0, b]$. Subsequently, we numerically analyze how capacity affects the expected profit of the APs and obtain a figure similar to Figure 2.5. Specifically, we find that the optimal capacity is about 6.98, which, again, is significantly smaller than the upper bound of either the weight or the volume distributions. It is higher than the case of the single-dimension price. This can be explained by the higher expected demand since the chargeable weight is the maximum of weight and volume. Thus, although the two-dimensional pricing structure quantitatively has an impact (it slightly increases the optimal capacity), qualitatively the insight persists (that APs bring only a small portion of demand).

From the results of different demand distributions discussed above, we observe that, indeed, the value of k_D^* is affected by the distribution of customer demand to a limited extent. Across different types of distributions, the optimal capacity ranges between 4.38 and 6.98, all of which are very close to the capacity in the uniformly distributed demand case. This confirms the insight that APs tend to bring a relatively small portion of the expected demand to the spot market, to avoid fierce competition, as enabled by ASAs.

2.5.3 Impact of capacity on welfare distribution

In this subsection, we conduct a numerical study to explore how capacity decisions affect welfare distribution between the three agents (i.e. the expected profits of APs and LSPs as well as the customer surplus). Specifically, our interest is in deviations from the APs' profit maximizing choice, ideally towards the welfare maximizing capacity.

Earlier, for simplicity of exposition, we have normalized the capacity cost to zero. This, however, trivially implies that welfare increases in capacity and, hence, policy makers should seek to set the highest possible k. Accordingly, we revisit our analysis to explicitly account for the unit cost, c. Assuming demand follows a uniform distribution over $U[0,b]$, with a unit cost c, an AP's expected profit, stated in Equation (2.6), becomes

$$E_d\left[\pi_i^e(d,k)\right] = \frac{k^2}{4b}\ln 3 + \frac{k}{b}\left(b - 3k - 2k\ln\frac{b}{3k}\right) - ck. \qquad (2.14)$$

This is approximately maximized at $k_D^{c*} = \frac{-(1-c)b}{4} \cdot W_{-1}\left(-\frac{1-c}{4e}\cdot 3^{\frac{9}{8}}\right)^{-1}$, where $W_{-1}(a)$ is the lower branch of Lambert w function. For instance, if $c = 0.2$, the optimal capacity from the APs' perspective is about $0.0939b$.

Figure 2.9 shows how the welfare is distributed between the APs, the customers and the LSPs with different k values, considering $d \sim U[0,40]$, a unit cost of capacity $c = 0.2$ and LSPs' margin $m = 0.4$. The APs' expected profits are maximized at about $k_D^{c*} = 3.76$, which can also be analytically derived from Equation (2.14). Such a small

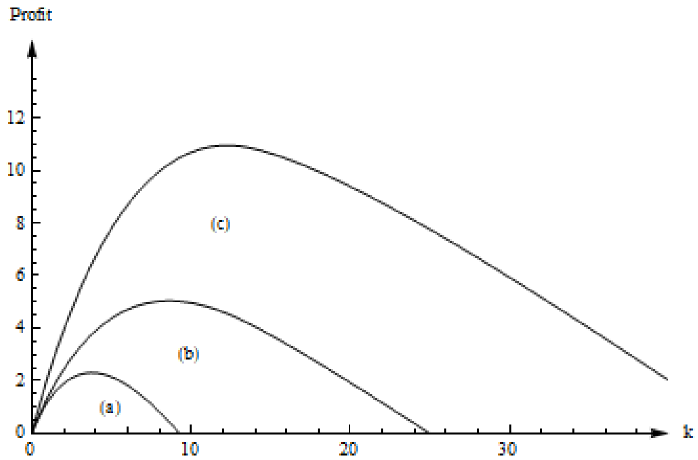
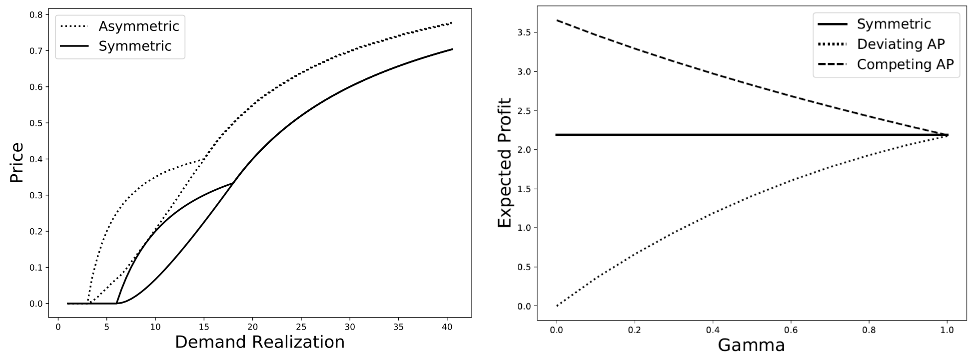


Figure 2.9 Stacked line graph of the expected welfare distribution across the asset providers (a), the end customers (b), and the LSPs (c) as a function of the capacity; $b = 40$, $m = 0.4$ and $c = 0.2$.

amount results from the trade-off between insufficient capacity and fierce competition, which is consistent with Corollary 2.1 assuming zero capacity cost. The total capacity brought to the market is about $0.2b$ and it is lower than the average demand, $0.5b$. This confirms our normalization that the unit cost of capacity quantitatively affect the capacity decision but not qualitatively.

Note, that increasing the capacity brought to the market from the APs' profit maximizing choice $2k_D^{c*}$, implies higher welfare, induced by higher LSPs' profit and greater consumer surplus. This, however, is at the expense of the APs. This results from increased competition, and subsequently more capacity traded at lower unit price. The LSPs benefit from the margin m, and customers from both larger quantity and lower price. Hence, the customers and LSPs prefer a market with more capacity. The APs are worse off due to the intense competition as well as the higher cost they incur.

The optimal welfare maximizing capacity is about 12.26, or $0.3b$. With such capacity, the APs make negative profits. In this sense, policy makers may wish to consider incentives, such as, subsidies, provided to the APs to induce them to increase the capacity they bring to the market. This is consistent with Corollary 2.6. That is, regardless whether capacity is costly or not, governments may wish to consider interventions to motivate APs to bring more capacity to the market. Another option is to encourage a virtual merge as per Section 3.4.



(a) Equilibrium prices when $\gamma = 0.5$ (b) Expected profits as a function of γ, $d \sim U[0, 40]$

Figure 2.10 APs' equilibrium spot prices and profits with asymmetric capacities, with $k_a = \gamma k_D^*$, $k_b = k_D^*$

2.5.4 Asymmetric spot market

Bilateral ASAs spell out the capacities that competing APs are permitted to operate in a market. Thus far, we assumed that both APs bring as much capacity as specified in the agreement, which is symmetric in nature. In practice, however, for a variety of reasons, APs may opt to deviate and bring less capacity to the market. Accordingly, this may result in a spot market where the competing APs have asymmetric capacities. The possibility of asymmetric capacities necessitates the analysis to explore if and how the spot market is affected.

Without loss of generality, we assume that AP_a is the asset provider who deviates from the amount of capacity specified in the agreement and brings, instead, k_a units to the spot market, where $k_a = \gamma k_D^*$ and $0 \leq \gamma \leq 1$, whereas AP_b brings $k_b = k_D^*$ units of capacity. To investigate the impact on the equilibrium spot market, we proceed with a numerical study. Given a pair of spot prices, p_a and p_b, and a realized spot demand d, the profit of each AP can be derived from Equation (2.1). These profit expressions allow us to numerically identify their best response functions, which ultimately lead to the equilibrium spot prices.

Figure 2.10a demonstrates the range of equilibrium spot prices for the asymmetric case with $\gamma = 0.5$ (dotted lines), as well as the symmetric case where both APs bring the agreed capacity (solid lines). Evidently, the equilibrium spot prices shift to the left and upwards due to the reduced capacity. Nevertheless, this price range is not qualitatively different from the symmetric solution. In both cases, the APs are involved in mixed pricing strategies when the demand is not too low or too high and they converge to a pure pricing strategy otherwise.

As the equilibrium prices are higher while the capacity is lower, it is not clear a priori whether the equilibrium profits are higher or lower. Hence, we derive the equilibrium profits following the same steps carried out for the symmetric case (see Proof of Proposition 2.2), and illustrate them in Figure 2.10b as a function of γ, assuming $d \sim U[0, 40]$. The figure reveals that the deviating AP, namely AP_a, is worse off as the price increase does not compensate for the drop in capacity. At the same time, the competing AP is better off as he experiences a price increase with no change in his capacity. This holds true for any $\gamma \in [0, 1)$. Thus, it is never optimal for an AP to bring less capacity than the amount permitted by the bilateral ASA.[5]

In conclusion, the spot pricing strategies, which are determined by the capacities brought to the spot market by the two APs as well as the realized demand, may differ from those derived under the symmetric case. However, as demonstrated above, these strategies are qualitatively consistent with those from the symmetric case. Importantly, we numerically showed that an AP does not have an incentive to deviate and bring less capacity, and accordingly, our analysis shall reflect their optimal decisions. In practice, APs may be subject to other objectives and constraints which are beyond the scope of our paper and, hence, are left for future exploration.

2.6. Conclusions

In this work, we study air cargo markets governed by bilateral ASAs, in the sense that capacity brought to the market by each of the asset providers is identical. Particularly, we investigate capacity and pricing strategies for air cargo carriers facing uncertain demand while engaging in price competition. We formulate the problem as a two-stage capacity and pricing game involving four types of agents—two asset providers, LSPs, end customers and the policy makers who coordinate the agreement.

We argue that cargo spot markets can be characterized by Bertrand-Edgeworth competition. We mathematically analyze the optimal pricing mechanism in the spot market as well as the optimal coordinated capacity choices. We characterize the price equilibrium in the spot market, given demand realizations and the corresponding traded quantities and profits. The asset providers shall adopt pure pricing strategies when realized demand is either sufficiently low or sufficiently high and shall change to a mixed pricing strategy when demand is within some intermediary level to avoid predictable price cuts by the competitor. Restricted by a bilateral air service agreement, which leads to reciprocal capacity for both treaty signatories, the competing asset providers bring less capacity to the market than a monopoly

[5] We can show that if the capacity spelled out by the ASA is merely indicatory, then both APs have an incentive to deviate and bring more capacity to the market.

asset provider would due to the trade-off between competition and sales: the more capacity they bring to the market, the more demand they can capture in particular when demand realization is high. However, this also exposes them to a larger range of demand realizations in which they face fierce price competition. Since the latter effect dominates, they end up bringing less capacity to the market.

Policy makers, who often favor restrictive bilateral ASAs to protect national asset providers, should be aware that encouraging a higher level of cooperation between asset providers may lead to a higher social welfare in this context. This observation results from that the competing asset providers demand the negotiators to limit the capacity allocation to restrict the level of competition in the market. Bearing that in mind, policy makers can encourage joint ventures or merged operations to take place to enhance total welfare.

To lend robustness to our model and insights, we consider several important extensions. First, we allowed for market expansion by assuming three, rather than two, asset providers. Second, we replaced the uniformly distributed demand with several alternative distributions, including one that accounts for the chargeable weight—a combination of weight and volume. Thirds, we explored the effect of different capacity decisions and their impact on welfare distribution while relaxing our zero cost assumption. Fourth, we allowed for asymmetric capacities in spot market to account for various practical considerations and studied how the spot market is affected.

Our model has some limitations. We assume that the LSPs and the end customers are all from the same two countries in an equal manner, while in practice the balance may be tilted with one country hosting more LSPs or customers than the other, or they may come from a third country altogether. Future research can account for some differentiation between LSPs. This paper focuses on bilateral ASAs and it is of interest to compare across different policies including open skies, to obtain more comprehensive understanding of how policies affect the joint welfare. Further, in some instances, asset providers may have more flexibility in their operations and schedules such that when demand realization is low, they can adjust their capacity planning by rescheduling flights to different destinations and as such, they will bring to the market less than the intended and coordinated capacity. Such operational issues can be modelled by flexible capacity at a cost. Our model focuses on the spot market whereas in practice APs and LSPs may engage in some long term capacity contracts prior to the spot market. Our competitive modelling framework lays out the foundations to study the comprehensive capacity negotiation strategies of APs and LSPs.

2.A. Proofs

Proof of Proposition 2.1-2.2. To prove the equilibrium pricing strategy in Proposition 2.1 and the equilibrium profit in Proposition 2.2, Lemmata 2.4 - 2.6 are developed.

We firstly analyze the immediate profit as a function of prices given different demand realization as shown in (2.1). It changes with different realized demand and we denote the thresholds of demand, which distinguish the profit outcomes, as d_i with $i \in \{1,2,3,4\}$, where d_i represents the transition between profit outcome i and $i+1$. We formally state it as the following lemma.

Lemma 2.3. *When the asset providers have symmetric capacities, their profit outcomes exhibit five distinct behaviors, with the transition thresholds between the 5 outcomes given by $d_i = ik$, $i \in \{1,2,3,4\}$.*

Proof. Let $p_{\{k\}} \equiv \max\left(0, 1-\frac{k}{d}\right)$ denotes the non-negative price clearing k units on the spot market with realized demand d and $p_{\{2k\}} \equiv \max\left(0, 1-\frac{2k}{d}\right)$ denotes the non-negative price clearing $2k$ units. The profit function (2.1) can be rewritten in a piece-wise form:

$$\pi_i(p_i, p_j, d, k) = \begin{cases} L(p_i) \equiv & \begin{cases} p_i k & \text{if } p_i \le p_{\{k\}} \\ p_i d(1-p_i) & \text{if } p_i > p_{\{k\}} \end{cases} \\ E(p_i) \equiv & \begin{cases} p_i k & \text{if } p_i \le p_{\{2k\}} \\ p_i \frac{d}{2}(1-p_i) & \text{if } p_i > p_{\{2k\}} \end{cases} \\ H(p_i) \equiv & \begin{cases} p_i k & \text{if } p_i \le p_{\{2k\}} \\ p_i(d(1-p_i)-k) & \text{if } p_{\{2k\}} < p_i \le p_{\{k\}} \\ 0 & \text{if } p_i > p_{\{k\}} \end{cases} \end{cases} \tag{2.15}$$

The outcome of this profit function changes with the relationship between k and the realized demand d and now we discuss different outcomes along the change of d.

Since $d(p) = d(1-p)$, $p_{\{k\}} = \max(0, 1-\frac{k}{d})$ and it is non-decreasing in d. Similarly, $p_{\{2k\}} = \max(0, 1-\frac{2k}{d})$ which is also non-decreasing in d. Hence, we have $\max(0, 1-\frac{k}{d}) \ge \max(0, 1-\frac{2k}{d}) \ge 0$, $p_{\{k\}} \ge p_{\{2k\}} \ge 0$. If $\max(0, 1-\frac{k}{d}) = 0$, then $\max(0, 1-\frac{2k}{d}) = 0$ as well.

When $d \le k$, $p_{\{k\}} = 0$ and $H(p) = 0$ for any p. When $p \in \left[0, p_{\{k\}}\right]$, $p_{\{k\}}$ becomes strictly positive and $H(p) > 0$. Hence, $d_1 = k$. When d continues increasing, $p_{\{2k\}}$ also becomes positive and this defines the second threshold, namely $d_2 = 2k$.

Note that $\frac{p_{\{k\}}}{2}$ is the unbounded profit maximization price of $H(p)$ when $p_{\{2k\}} < p \le p_{\{k\}}$. However, the maximized profit can be bounded by its capacity and this happens when $p_{\{2k\}} \geq \frac{p_{\{k\}}}{2}$. Then the maximum profit is obtained at $p_{\{2k\}}$. This defines the third threshold, $d_3 = 3k$, by solving $p_{\{2k\}} = \frac{p_{\{k\}}}{2}$. Similarly, d_4 defines whether the maximum profit of $E(p)$ is constrained by capacity. The unbounded profit maximization price of $E(p)$ is $\frac{1}{2}$ and the bounded profit maximization price is $p_{\{2k\}}$. Solving the equation $p_{\{2k\}} = \frac{1}{2}$ leads to $d_4 = 4k$. □

Let p_i^e, q_i^e and π_i^e denote the price, the quantity traded and the profit of Firm i correspondingly given the realized demand d, with $i \in \{a, b\}$, under equilibrium. We next characterize prices, sales and profits under equilibrium for each range defined in Lemma 2.3.

Lemma 2.4. *If $d < d_1$, then $p_i^e = 0$, $q_i^e = \frac{d}{2}$ and $\pi_i^e(d) = 0$, $i \in \{a, b\}$.*

Proof. When $d < d_1$, $d < k$ and the profit is given by:

$$\pi_i(p_i, p_j, d, k) = \begin{cases} L(p_i) \equiv & p_i d(1-p_i) & \text{if } p_i < p_j \text{ and } d < d_1 \\ E(p_i) \equiv & p_i \frac{d}{2}(1-p_i) & \text{if } p_i = p_j \text{ and } d < d_1 \\ H(p_i) \equiv & 0 & \text{if } p_i > p_j \text{ and } d < d_1 \end{cases} \tag{2.16}$$

implying that for each price set by the competition, the best price response is to cut down by ϵ. Namely, $p_i(p_j) = p_j - \epsilon$ for $p_j \geq \epsilon$, with $\epsilon > 0$. Hence, this process converges to the equilibrium price $p_i^e = 0$ with all d units being traded, each selling $q_i^e = \frac{d}{2}$ units, making zero profit, $\pi_i^e = 0$. □

Lemma 2.5. *If $d_1 < d < d_3$, there exists a mixed pricing strategy, with $p_i^e \in [\underline{p}, \bar{p}]$, $i \in \{a, b\}$, where $\underline{p} = \frac{(d-k)^2}{4kd}$, $\bar{p} = \frac{d-k}{2d}$, and cdf $F(p) = \frac{4kdp-(d-k)^2}{4d^2p^2-4d^2p+8kdp}$. The sales of two firms are given by:*

$$q_i^e(p_i, p_j) = \begin{cases} d(1-p_i) - k & \textit{if } p_i > p_j \\ \frac{d(1-p_i)}{2} & \textit{if } p_i = p_j \\ k & \textit{if } p_i < p_j \end{cases} \tag{2.17}$$

and the expected profits to given d are

$$\pi_i^e(d) = \frac{(d-k)^2}{4d} \tag{2.18}$$

Proof. We prove the two cases $d_1 < d < d_2$ and $d_2 \leq d < d_3$, separately.

When $d_1 < d < d_2$, the profit function is given by substituting $p_{\{2k\}} = 0$ and $p_{\{k\}} > 0$ into Equation (2.15). In this range, since $L(p) > E(p) > H(p)$, $p_i(p_j) = p_j - \epsilon$.

However, since $H(p^*) > L(p)$ for $p < \underline{p}$, we have the following response function:

$$p_i(p_j) = \begin{cases} p_j - \epsilon & \text{if } p_j \geq \underline{p} + \epsilon \\ \bar{p} & \text{if } p_j \leq \underline{p} \end{cases} \tag{2.19}$$

$\bar{p}$ maximizes $H(p)$ and $\underline{p}$ is the value of p which solves $L(p) = H(p)$, implying there is no pure strategy. Accordingly, we identify mixed pricing strategy and, since the firms have identical capacities, we seek for the equilibrium in symmetric strategies over the price range $[\underline{p}, \bar{p}]$ with cdf $F(p)$.

The proof of the mixed pricing strategy consists of four steps.

Step 1: there is no mass point in $F(p)$. We prove by contradiction. Suppose there is a mass point at some price $p \in [\underline{p}, \bar{p}]$, i.e., the probability of offering the price p is strictly positive, $\Pr(p_i = p) > 0$. Since there are countable mass points, there must exist a price $p - \epsilon$ where $\Pr(p_i = p - \epsilon) = 0$ with $\epsilon \to 0$. By relocating the mass point at p to $p - \epsilon$, implying $\Pr(p_i = p - \epsilon) = \Pr(p_i = p)$, AP_i's expected profit is changed by:

$$\begin{aligned}
&\Pr(p_i = p) \Pr(p_{-i} > p) \, (L(p - \epsilon) - L(p)) + \\
&\Pr(p_i = p) \Pr(p_{-i} = p) \, (L(p - \epsilon) - E(p)) + \\
&\Pr(p_i = p) \Pr(p - \epsilon < p_{-i} < p) \, (L(p - \epsilon) - H(p)) + \\
&\Pr(p_i = p) \Pr(p_{-i} = p - \epsilon) \, (E(p - \epsilon) - H(p)) + \\
&\Pr(p_i = p) \Pr(p_{-i} < p - \epsilon) \, (H(p - \epsilon) - H(p))
\end{aligned}$$

Differences only happen when AP_i originally charges price p. AP_{-i} may charge higher than p, between p and $p - \epsilon$ or lower than $p - \epsilon$. When $\epsilon \to 0$, all terms in the above equation approach to 0 except for the second term, which is positive. It implies that the deviation increases the payoff which violates the definition of an equilibrium and we reach a contradiction.

Step 2: the equilibrium profit equals $H(p^*)$, with $p^* = argmaxH(p)$ and $\bar{p} = p^*$. To prove this characteristic, we investigate the equilibrium profit of firm i at price $\bar{p}$. Since AP_{-i} adopts the equilibrium strategy, AP_i has identical profits at any price p within the equilibrium price range. Thus, the profit of AP_i at price $\bar{p}$ also equals the equilibrium profit. If AP_i offers $\bar{p}$, AP_{-i} always offers a lower price since there is no mass point. Thus, the profit of AP_i at $\bar{p}$ equals $H(\bar{p})$. Hence, the equilibrium profit is $H(p^*)$ after solving the optimization problem and the upper bound price equals $\frac{p_{\{k\}}}{2}$.

Step 3: the cdf is continuous. We prove by contradiction. Suppose there exists a range (p_1, p_2) with $p_2 \leq \bar{p}$, where $f(p) = 0$ for any p in this range. The profit at price p in the range is $L(p)(1 - F(p)) + H(p)F(p)$. Since $p_1 < p < \bar{p} = \frac{p_{\{k\}}}{2}$,

$L(p) > L(p_1)$ and $H(p) > H(p_1)$. Thus, the expected profit at p is greater than at p_1 since $F(p) = F(p_1)$ and hence, charging p with probability 1 generates more profit. We reach a contraction and the cdf is continuous in the equilibrium price range.

Step 4: the construction of the mixed pricing equilibrium. We construct the mixed pricing strategy ranging from $\underline{p}$ and $\bar{p}$, with distribution $F(p)$. Since the equilibrium profit is identical at any price p and $\underline{p} < \bar{p} < p_{\{k\}}$, $\underline{p}$, the lower bound price $\underline{p}$ can be identified by solving $L(\underline{p}) = H(\bar{p})$. Rewriting, we have $k\underline{p} = \frac{1}{2}p_{\{k\}}(d(\frac{1}{2}p_{\{k\}}) - k)$, for which the solution for $\underline{p}$ is $\frac{(d-k)^2}{4kd}$.

The profit of player i given the competitor adopting equilibrium mixed pricing strategy is :

$$\pi_i(p_i, F_j) = L(p_i)(1 - F_j(p_i)) + H(p_i)F_j(p_i) \tag{2.20}$$

where, AP_i offers the price p_i and the competitor adopts the mixed equilibrium strategy denoted as $F_j(p_i)$. It can be rewritten in the following way:

$$F_j(p_i) = \frac{L(p_i) - \pi_i(p_i, F_j)}{L(p_i) - H(p_i)} \tag{2.21}$$

Since the equilibrium profit, given both adopting equilibrium mixed pricing strategy, $\pi_i^e(F_i, F_j) = L(\underline{p}) = k\underline{p}$, the price distribution can be rewritten in the following way:

$$F_i = F_j = \frac{k(p - \frac{(d-k)^2}{4kd})}{2kp - pd + p^2 d} = \frac{4kdp_i - (d-k)^2}{-4dp_i(d - (1 - p_i) - 2k)} \tag{2.22}$$

It can be verified that $F(\underline{p}) = 0$ and $F(\bar{p}) = 1$.

If $d_1 < d < d_2$, $p_{\{2k\}} < \underline{p} < \bar{p} < p_{\{k\}}$. Consequently, $p_{\{2k\}} < p_i < p_{\{k\}}$ with $p_i \in [\underline{p}, \bar{p}]$, implying that AP_i sells k units if $p_i < p_{-i}$ and $d(1 - p_i) - k$ units otherwise. The expected profit, which is also the equilibrium profit by the definition of mixed pricing strategy, is given by:

$$\begin{aligned}
\pi_i^e &= \int_{\underline{p}}^{\bar{p}} p_i \left(\int_{\underline{p}}^{p_i} [d(1-p_i) - k] f(p_i) dp_{-i} + \int_{p_i}^{\bar{p}} k f(p_i) dp_{-i} \right) f(p_i) dp_i \\
&= \int_{\underline{p}}^{\bar{p}} p_i \left((d(1-p_i) - k) \int_{\underline{p}}^{p_i} f(p_i) dp_{-i} + k \int_{p_i}^{\bar{p}} f(p_i) dp_{-i} \right) f(p_i) dp_i \\
&= \int_{\underline{p}}^{\bar{p}} p_i \left((d(1-p_i) - k) F(p_i) + k (1 - F(p_i)) \right) f(p_i) dp_i \\
&= \int_{\underline{p}}^{\bar{p}} p_i f(p_i) \underbrace{\left((d(1-p_i) - 2k) F(p_i) + k \right)}_{(*)} dp_i
\end{aligned} \tag{2.23}$$

The form $(*)$ in the above expression can be simplified as $\frac{(d-k)^2}{4dp_i}$. Plugging back (*) and rewriting, we have

$$\pi_i^e = \frac{(d-k)^2}{4d} \int_{\underline{p}}^{\bar{p}} f(p_i)dp_i = \frac{(d-k)^2}{4d} \tag{2.24}$$

When $d_2 < d < d_3$, the profit and response functions are the same as in the case when $d_1 < d < d_2$. We can prove that in both cases the pricing range $[\underline{p}, \bar{p}]$ and $F(p)$ are identical, resulting in the same expected sales and expected profits. □

Now we proceed with the case where spot market demand is sufficiently high, denoted as under-capacity.

Lemma 2.6. *If $d > d_3$, $p_i^e = \frac{d-2k}{d}$, $q_i^e = k$, with a corresponding profit $\pi_i^e = \frac{d-2k}{d}k$, $i \in \{a, b\}$.*

Proof. We prove the two cases $d_3 < d < d_4$ and $d_4 \geq d$ separately. When $d_3 < d < d_4$, $d > 2k$. The best response function is given by

$$p_i(p_j) = \begin{cases} p_j - \epsilon & \text{if } p_i \geq p_{\{2k\}} + \epsilon \\ p_{\{2k\}} & \text{if } p_i \leq p_{\{2k\}} \end{cases} \tag{2.25}$$

Therefore, the resulting optimal prices are $p_i^e = p_{\{2k\}}$ where $p_{\{2k\}} = \frac{d-2k}{d}$. Since $p_a = p_b$, the two firms equally share the demand with $q_i^e = k$ and the corresponding profit is $\pi_i^e(d) = \frac{d-2k}{d}k$. Similar response function can be obtained for the case of $d > 2k$. □

Following Lemmata 2.4 - 2.6, the proof of Proposition 2.1 and Proposition 2.2 is completed. □

Proof for Proposition 2.3. i: The equilibrium profits for a given d follow from Proposition 2.1. Taking derivatives separately for each domain of d, π_i^e is then shown non-decreasing. Evaluating π_i^e at k and $3k$ shows the function is continuous.
ii: Integrating the equilibrium profit over the three domains of d, the expected profit function of each asset provider is given by:

$$
\begin{aligned}
E\left[\pi_i^e\right] =& \int_0^k 0 + \int_k^{3k} \frac{(d-k)^2}{4d} f_d(d) dd + \int_{3k}^{\infty} (1 - \frac{2k}{d}) k f_d(d) dd \\
=& \frac{1}{4} \int_k^{3k} d f_d(d) dd - \frac{1}{2} k \int_k^{3k} f_d(d) dd + \frac{k^2}{4} \int_k^{3k} \frac{f_d(d)}{d} dd \\
&+ k \int_{3k}^{\infty} \left[f_d(d) - \frac{2k}{d} f_d(d) \right] dd \\
=& \frac{3}{4} k F_d(3k) - \frac{1}{4} k F_d(k) - \frac{1}{4} \int_k^{3k} F_d(d) dd - \frac{1}{2} k F_d(3k) + \frac{1}{2} k F_d(k) \\
&+ \frac{k^2}{4} \frac{F(3k)}{3k} - \frac{k^2}{4} \frac{F(k)}{k} + \frac{k^2}{4} \int_k^{3k} \frac{F_d(d)}{d^2} dd \\
&+ k(1 - F_d(3k)) - \left[2k^2 \frac{F_d(d)}{d} \bigg|_{3k}^{\infty} + 2k^2 \int_{3k}^{\infty} \frac{F_d(d)}{d^2} dd \right] \\
=& -\frac{2}{3} k F_d(3k) - \frac{1}{4} \int_k^{3k} F_d(d) dd + k + \frac{k^2}{4} \int_k^{3k} \frac{F_d(d)}{d^2} dd \\
&- 2k^2 \frac{F_d(d)}{d} \bigg|_{3k}^{\infty} - 2k^2 \int_{3k}^{\infty} \frac{F_d(d)}{d^2} dd
\end{aligned}
$$

□

Proof of Corollary 2.1. Firstly, we can prove that $k^* \leq \frac{1}{3}b$ with k^* denoting the optimal capacity. Suppose $\frac{1}{3}b \leq k_1 < b \leq k_2$ and thus, $k_1 < b \leq 3k_1$ and $b \leq k_2$. From (2.4), the expected profit with $k = k_1$, $k = k_2$ are $\int_{k_1}^{b} \frac{(d-k_1)^2}{4d} f_d(d) dd$ and 0 respectively, implying $k^* < b$. Using Leibniz integral rule, the first order condition of the expected profit with $k = k_1$ is $\frac{1}{2b} \int_{k_1}^{b} \left(\frac{k_1}{d} - 1 \right) dd < 0$, which implies the expected profit decreases along k_1. Thus, $k^* \leq \frac{1}{3}b$, leading to $b \geq 3k^*$.

Given pdf $f_d(d) = 1/b$ and cdf $F_d(d) = d/b$, the expected profit function over demand d is written as:

$$
\begin{aligned}
E_d\left[\pi_i^e\right] =& -\frac{2}{3} k \frac{3k}{b} - \frac{1}{4} \int_k^{3k} \frac{d}{b} dd + k + \frac{k^2}{4} \int_k^{3k} \frac{d}{bd^2} dd - 2k^2 \frac{d}{bd} \bigg|_{3k}^{b} \\
&- 2k^2 \int_{3k}^{b} \frac{d}{bd^2} dd \\
=& -\frac{2k^2}{b} - \frac{1}{4b} \frac{d^2}{2} \bigg|_k^{3k} + k + \frac{k^2}{4b} \ln d \bigg|_k^{3k} - \frac{2k^2}{b} \ln d \bigg|_k^{3k} \\
=& \frac{k^2}{4b} \ln 3 + \frac{k}{b} \left(b - 3k - 2k \ln \frac{b}{3k} \right)
\end{aligned}
\tag{2.26}
$$

To find the profit-maximizing capacity k, the first order condition is given by:

$$\begin{aligned}\frac{\partial E_d\left[\pi_i^e\right]}{\partial k} &= -\frac{\ln 3}{2b}k + 1 - \frac{6}{b}k - \frac{\partial}{\partial k}\left(\frac{2k^2}{b}\ln\frac{b}{3k}\right) \\ &= \frac{1}{2b}\left(2b + (\ln 3 - 8)k - 8k\ln\frac{b}{3k}\right)\end{aligned} \tag{2.27}$$

The second order condition is given by:

$$\begin{aligned}\frac{\partial^2 E_d\left[\pi_i^e\right]}{\partial k} &= \frac{\ln 3 - 8}{2b} + \frac{4}{b}(\ln\frac{3k}{b} + k\frac{b}{3k}\frac{3}{b}) \\ &= \frac{1}{b}(\frac{1}{2}\ln 3 + 4\ln\frac{3k}{b})\end{aligned}$$

Noting that $\frac{\partial^2 E_d[\pi_i^e]}{\partial k} = 0$ when $k = 3^{-\frac{9}{8}}b$. Since $\frac{\partial^2 E_d[\pi_i^e]}{\partial k}$ increases when $k > 0$, $\frac{\partial^2 E_d[\pi_i^e]}{\partial k} < 0$ for $k < 3^{-\frac{9}{8}}b$ and $\frac{\partial^2 E_d[\pi_i^e]}{\partial k} > 0$ for $k > 3^{-\frac{9}{8}}b$. It implies that $E_d\left[\pi_i^e\right]$ is concave for $k < 3^{-\frac{9}{8}}b$ and convex for $k > 3^{-\frac{9}{8}}b$.

Since $0 < k < \frac{1}{3}b$, $\lim_{k\to 0}\frac{\partial E_d[\pi_i^e]}{\partial k} = 1$ using l'Hospital's rule and $\lim_{k\to\frac{1}{3}b}\frac{\partial E_d[\pi_i^e]}{\partial k} = 1+\frac{1}{6}(\ln 2-8) < 0$. Thus, the solution to $\frac{\partial^2 E_d[\pi_i^e]}{\partial k} = 0$ is unique and profit-maximizing. Let Equation (2.27) equal to zero

$$\begin{aligned}E_d'\left[\pi_i^e\right] = \frac{1}{2b}\left(2b + (\ln 3 - 8)k - 8k\ln\frac{b}{3k}\right) &= 0 \\ 2b + (\ln 3 - 8)k + 8k\ln\frac{3k}{b} &= 0 \\ 2 + (\ln 3 - 8)\frac{k}{b} + 8\frac{k}{b}\ln\frac{3k}{b} &= 0\end{aligned} \tag{2.28}$$

Notice that (2.28) is a transcendental equation which often does not have a closed form. However, it can be approximated by using the Lambert W-function. Let $x = \frac{k}{b}$ with $x > 0$ and rewrite it as following:

$$\begin{aligned}2 + (\ln 3 - 8)x + 8x\ln 3x &= 0 \\ \frac{2}{x} + (\ln 3 - 8) + 8\ln 3x &= 0 \\ (\ln 3 - 8) + \frac{2}{x} - 8\ln\frac{1}{3x} &= 0\end{aligned}$$

Let $y = \frac{1}{x}$, $A = \ln 3 - 8 - 8\ln\frac{1}{3}$, $B = 2$, $C = -8$. The above function could be further rewritten in the following way:

$$\begin{aligned} A + By + C\ln y &= 0 \\ By + C\ln y &= -A \\ \frac{B}{C}y + \ln y &= \frac{-A}{C} \end{aligned}$$

Raise a base e to the power of both sides and multiply both sides by $\frac{B}{C}$,

$$\begin{aligned} y \cdot e^{\frac{B}{C}y} &= e^{-\frac{A}{C}} \\ \frac{B}{C}ye^{\frac{B}{C}y} &= \frac{B}{C}e^{-\frac{A}{C}} \end{aligned}$$

Equation (2.A) is in the form of Lambert W function $xe^x = a$, where $x = \frac{B}{C}y$ and $a = \frac{B}{C}e^{-\frac{A}{C}}$. Therefore,

$$\frac{B}{C}y = W(\frac{B}{C}e^{-\frac{A}{C}})$$

Substitute back,

$$\frac{2b}{-8k} = W(\frac{2}{-8}e^{-\frac{\ln 3 - 8 - 8\ln\frac{1}{3}}{-8}})$$

Isolating k, we have

$$k = \frac{-b}{4} \cdot W\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right)^{-1} \tag{2.29}$$

Since $-\frac{1}{e} < -\frac{1}{4e} \cdot 3^{\frac{9}{8}} \approx -0.3165 < 0$, $W\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right)$ has two solutions $W_0\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right) \approx -0.5469$ and $W_{-1}\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right) = -1.6529$, leading to $k \approx 0.5471b$ and $k \approx 0.1512b$ correspondingly. However, since $k < \frac{1}{3}b$, we only take the second solution $W_{-1}\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right)$. Therefore, the expected profit is maximized when capacity $k = \frac{b}{-4} \cdot W_{-1}\left(-\frac{1}{4e} \cdot 3^{\frac{9}{8}}\right)^{-1} \approx 0.15b$ □

Proof of Lemma 2.1. From (3.33), the total quantity of the two *LSPs* equals d when $d \leq k$ and equals $2k$ when $3k \leq d$. When $k < d < 3k$, the probability of $p_h = p$ is $f_h(p) = 2F(p)f(p)$, where $F(p)$ (resp. $f(p)$) is the equilibrium density (resp. probability) function in (2.22). Therefore, when $k < d < 3k$, the expected sales, denoted as $w(d,k)$, is as following:

$$\begin{aligned} w(d,k) &= \int_{\underline{p}}^{\bar{p}} d(1-p)f_h(p)dp \\ &= \int_{\underline{p}}^{\bar{p}} d(1-p)2F(p)f(p)dp \\ &= \frac{2(d-2k)(2d^3-7d^2k+4dk^2+5k^3)+(d-3k)^2(d-k)^2\ln-\frac{d-3k}{d-k}}{8(d-2k)^3} \end{aligned} \tag{2.30}$$

Since the LSPs apply margin m to each units it sell, the expected profits $E_d(\pi_l)$ is the product of m and the expected quantity $E_d(q_l)$. Given Proposition 2.1, the expected total quantity could be written in the following way:

$$E_d[q_a+q_b] = \int_0^k df(d)dd + \int_k^{3k} w(d,k)f(d)dd + \int_{3k}^{\infty} 2kf(d)dd \tag{2.31}$$

Integrating by parts gives rise to Equation (2.8) □

Proof of Corollary 2.2. Given Lemma 2.1, the expected quantity assuming uniform demand distribution is given by:

$$\begin{aligned} E_d[q_a+q_b] &= 2k + k\frac{k}{b} - 2k\frac{3k}{b} - \int_0^k \frac{d}{b}dd + \int_k^{3k}\frac{1}{b}w(d,k)dd \\ &= 2k - \frac{11}{2b}k^2 + \int_k^{3k}\frac{1}{b}w(d,k)dd \end{aligned} \tag{2.32}$$

where $w(k,d)$ is given in (3.34). Focusing on the third term, we have

$$\begin{aligned} &\int_k^{3k}\frac{1}{b}w(d,k)dd \\ =&\int_k^{3k}\frac{1}{b}\frac{2(d-2k)(2d^3-7d^2k+4dk^2+5k^3)+(d-3k)^2(d-k)^2\ln-\frac{d-3k}{d-k}}{8(d-2k)^3}dd \\ =&\frac{1}{8b}\int_k^{3k}\frac{1}{(d-2k)^3} \\ &\left(2(d-2k)(2d^3-7d^2k+4dk^2+5k^3)+(d-3k)^2(d-k)^2\ln-\frac{d-3k}{d-k}\right)dd \end{aligned}$$

Substitute $d-2k$ by t and thus, $dd = dt$. Rewrite the equation:

$$\begin{aligned}&\int_k^{3k}\frac{1}{b}w(d,k)dd=\\&\frac{1}{8b}\int_{-k}^{k}\frac{1}{t^3}\left(2t(2t^3+5t^2k+k^3)+(t+k)^2(t-k)^2\ln\frac{k-t}{k+t}\right)dt\\=&\frac{1}{8b}\int_{-k}^{k}\left(4t+10k+\frac{2k^3}{t^2}+(t-\frac{2k^2}{t}+\frac{k^4}{t^3})\ln\frac{k-t}{k+t}\right)dt\end{aligned}$$

The above function could be rewritten as following:

$$\begin{aligned}\int_k^{3k}\frac{1}{b}w(d,k)dd=&\underbrace{\frac{1}{8b}\int_{-k}^{k}\left(4t+10k+(t-\frac{2k^2}{t})\ln\frac{k-t}{k+t}\right)dt}_{(**)}\\&+\underbrace{\frac{1}{8b}\int_{-k}^{k}\left(\frac{2k^3}{t^2}+\frac{k^4}{t^3}\ln\frac{k-t}{k+t}\right)dt}_{(***)}\end{aligned}\tag{2.33}$$

In (**), we evaluate the following two challenging components separately, $\int_{-k}^{k}t\ln\frac{k-t}{k+t}dt$ and $\int_{-k}^{k}\frac{1}{t}\ln\frac{k-t}{k+t}dt$. Since both of them are even functions with identical evaluations are identical at t and $-t$, we focus on the domain $t\in(0,k)$.

$$\begin{aligned}\int t\ln\frac{k-t}{k+t}dt=&\frac{1}{2}t^2\ln\frac{k-t}{k+t}-\frac{1}{2}\int\frac{2kt^2}{t^2-k^2}dt\\=&\frac{1}{2}t^2\ln\frac{k-t}{k+t}-\frac{1}{2}\int 2k\left(1+\frac{k^2}{(t-k)(t+k)}\right)dt\end{aligned}$$

Since by applying partial fraction decomposition, $\frac{1}{(t-k)(t+k)}=\frac{1}{2k(t-k)}-\frac{1}{2k(t+k)}$, we can rewrite:

$$\begin{aligned}\int t\ln\frac{k-t}{k+t}dt=&\frac{1}{2}t^2\ln\frac{k-t}{k+t}-\frac{1}{2}\int\left(2k+\frac{k^2}{t-k}-\frac{k^2}{t+k}\right)dt\\=&\frac{1}{2}t^2\ln\frac{k-t}{k+t}-kt-\frac{1}{2}k^2\ln\left|\frac{t-k}{t+k}\right|\\\int_0^k t\ln\frac{k-t}{k+t}dt=&-k^2\end{aligned}$$

Therefore, $\int_{-k}^{k}t\ln\frac{k-t}{k+t}dt=-2k^2$.

We next compute $\int_{-k}^{k}\frac{1}{t}\ln\frac{k-t}{k+t}dt$. Let $u=\frac{k-t}{k+t}$, and thus $t=k\cdot\frac{1-u}{1+u}$, $dt=-\frac{2k}{(u+1)^2}du$.

For $0 < t < k$, we have $0 < u < 1$.

$$\int_0^k \frac{1}{t} \ln \frac{k-t}{k+t} dt = \int_1^0 \frac{1+u}{k(1-u)} \ln u \cdot \frac{-2k}{(u+1)^2} du$$
$$= \int_1^0 \frac{-2}{(1-u)(1+u)} \ln u du$$

Substitute u by $1-x$. Thus, $du = -dx$ with x ranging from 0 to 1.

$$\int_0^k \frac{1}{t} \ln \frac{k-t}{k+t} dt = \int_0^1 \frac{2}{x(2-x)} \ln (1-x) dx$$
$$= \int_0^1 \left(\frac{1}{x} + \frac{1}{2-x} \right) \ln (1-x) dx$$
$$= \int_0^1 \frac{\ln (1-x)}{x} dx + \int_0^1 \frac{\ln (1-x)}{2-x} dx$$

The integration in the above function is challenging to be represented by elementary functions. However, we notice that $\int_0^1 \frac{\ln(1-x)}{x} dx$ is in the form of dilogarithm function, which is defined in the following way:

$$Li_2(z) = -\int_0^z \frac{\ln (1-x)}{x} dx$$

Special value for $z = 1$ is $Li_2(1) = \frac{1}{6}\pi^2$, implying $\int_0^1 \frac{\ln(1-x)}{x} dx = -\frac{1}{6}\pi^2$. Similarly,

$$\int_0^1 \frac{\ln (1-x)}{2-x} dx = -\ln (2-x) \ln (1-x)|_0^1 + \int_0^1 \frac{\ln (1-(x-1))}{x-1} dx$$

Applying L'Hopital's Rule, the first part in the above function is 0 and the second part could be rewritten as dilogarithm function as well:

$$\int_0^1 \frac{\ln (1-(x-1))}{x-1} dx = \int_{-1}^0 \frac{\ln (1-m)}{m} dm = -\int_0^{-1} \frac{\ln (1-m)}{m} dm$$

where $m = x+1$. Since $Li_2(-1) = -\frac{1}{12}\pi^2$, $\int_0^1 \frac{\ln(1-x)}{2-x} dx = \int_0^1 \frac{\ln(1-(x-1))}{x-1} dx = -\frac{1}{12}\pi^2$. Thus, $\int_0^k \frac{1}{t} \ln \frac{k-t}{k+t} dt = -\frac{\pi^2}{4}$ leads to $\int_{-k}^k \frac{1}{t} \ln \frac{k-t}{k+t} dt = -\frac{\pi^2}{2}$. Assembling all relevant terms, we have

$$
\begin{aligned}
(**) =& \frac{1}{8b}\int_{-k}^{k}(4t+10k)dt + \frac{1}{8b}\int_{-k}^{k}\left(t\ln\frac{k-t}{k+t} - \frac{2k^2}{t}\ln\frac{k-t}{k+t}\right)dt \\
=& \frac{20k^2}{8b} + \frac{1}{8b}(-2k^2 - 2k^2(-\frac{\pi^2}{2})) \\
=& \frac{1}{8b}\left(18k^2 + k^2\pi^2\right)
\end{aligned}
\tag{2.34}
$$

Now, we proceed with (***). Similarly, it is an even function and thus we can focus on the domain $(0,k)$.

$$
\begin{aligned}
(***) =& 2\cdot\frac{1}{8b}\int_0^k\left(\frac{2k^3}{t^2} + \frac{k^4}{t^3}\ln\frac{k-t}{k+t}\right)dt \\
=& \frac{k^3}{4b}\int_0^k\frac{1}{t^3}\left(2t + k\ln\frac{k-t}{k+t}\right)dt
\end{aligned}
$$

Integrating by parts:

$$
\begin{aligned}
(***) =& \frac{k^3}{4b}\left(-\frac{1}{2t^2}\left(k\ln\frac{k-t}{k+t} + 2t\right)\Bigg|_0^k - \int_0^k(-\frac{1}{2t^2})\left(\frac{2t^2}{t^2-k^2}\right)dt\right) \\
=& \frac{k^3}{4b}\left(-\frac{1}{2t^2}\left(k\ln\frac{k-t}{k+t} + 2t\right)\Bigg|_0^k + \int_0^k\frac{1}{t^2-k^2}dt\right)
\end{aligned}
$$

Using partial fraction decomposition, $\int\frac{1}{t^2-k^2}dt = \int\left(\frac{1}{2k(t-k)} - \frac{1}{2k(t+k)}\right)dt$. Thus,

$$
(***) = \frac{k^3}{4b}\left(-\frac{1}{2t^2}\left(k\ln\frac{k-t}{k+t} + 2t\right) + \frac{1}{2k}\ln\left|\frac{t-k}{t+k}\right|\right)\Bigg|_0^k
$$

Compute the limits separately.

$$
\begin{aligned}
\lim_{t\to k}(***) =& \frac{k^3}{4b}\left(-\frac{1}{2k}\ln\frac{0}{2k} + \frac{1}{2k}\ln\frac{0}{2k} - \frac{1}{k}\right) \\
=& -\frac{k^2}{4b}
\end{aligned}
$$

$$\begin{aligned}\lim_{t\to 0}(***) =& \frac{k^3}{4b}\left(-\frac{k}{2t^2}\ln\frac{k-t}{k+t}-\frac{1}{t}\right)\\ =& \frac{k^3}{4b}\left(-\frac{1}{t^2}\right)\left(\frac{k}{2}\ln\frac{k-t}{k+t}+t\right)\\ =& \frac{-k^3}{4b}\left(\frac{\frac{k}{2}\ln\frac{k-t}{k+t}+t}{t^2}\right)\end{aligned}$$

Since $\lim_{t\to 0}\frac{0}{0}$, we apply L'Hopital's Rule:

$$\lim_{t\to 0}(***) = \frac{-k^3}{4b}\cdot\frac{1-\frac{k^2}{k^2-t^2}}{2t}$$

We apply L'Hopital's Rule again:

$$\begin{aligned}\lim_{t\to 0}(***) =& \frac{k^3}{4b}\cdot\ln\frac{k+t}{k-t}\\ =& 0\end{aligned}$$

Therefore,

$$(***) = -\frac{k^2}{4b} \tag{2.35}$$

We substitute to (**) and (***) back in (2.33):

$$\begin{aligned}\int_k^{3k}\frac{1}{b}w(d,k)dd =& \underbrace{\frac{1}{8b}\int_{-k}^{k}\left(4t+10k+(t-\frac{2k^2}{t})\ln\frac{k-t}{k+t}\right)dt}_{(**)}\\ &+\underbrace{\frac{1}{8b}\int_{-k}^{k}\left(\frac{2k^3}{t^2}+\frac{k^4}{t^3}\ln\frac{k-t}{k+t}\right)dt}_{(***)}\\ =& \frac{1}{8b}\left(18k^2+k^2\pi^2\right)-\frac{k^2}{4b}\\ =& \frac{1}{8b}\left(16k^2+k^2\pi^2\right)\end{aligned} \tag{2.36}$$

Plug in the above results back to the expected quantity in (2.32):

$$\begin{aligned}E_d[q_a+q_b] =& 2k-\frac{11}{2b}k^2+\frac{1}{8b}\left(16k^2+k^2\pi^2\right)\\ =& \frac{k}{8b}(16b+k(\pi^2-28))\end{aligned} \tag{2.37}$$

□

Proof of Corollary 2.3. The LSPs' welfare under the coordinated duopoly can been derived by multiplying the margin m with the expected total traded quantity $E_d[q_a + q_b]$ which leads to the following equation:

$$E_d[\pi_l] = \frac{mk_D^*}{8b}(16b + k_D^*(\pi^2 - 28)) \tag{2.38}$$

We now investigate the joint expected welfare of asset providers and customers to avoid the complexity of identifying them separately especially in the intermediate equilibrium. It is expressed as:

$$\begin{aligned}
&E_d[\pi_a + \pi_b + \pi_c] \\
&= \int_0^k \frac{1}{2} d f_d(d) dd + \int_k^{3k} \left\{ \frac{1}{2}d - \frac{1}{2}d \int_{\underline{p}}^{\bar{p}} p_h^2 f_h(p_h) dp_h \right\} f_d(d) dd \\
&\quad + \int_{3k}^{\infty} \left\{ (1 + p_{\{2k\}})k \right\} f_d(d) dd \\
&= -\frac{23}{4b}k^2 + 2k - \frac{2k^2}{b} \ln \frac{b}{3k} + \int_k^{3k} \left\{ \frac{1}{2}d - \frac{1}{2}d \int_{\underline{p}}^{\bar{p}} p_h^2 f_h(p_h) dp_h \right\} f(d) dd
\end{aligned}$$

Due to the complex format f_h and the double integration, we again use Lambert function to approximate the result. The total welfare can be approximated as the following:

$$E_d[\pi_a + \pi_b + \pi_c] \approx -\frac{3.8414}{b}k^2 + 2k - \frac{2k^2}{b} \ln \frac{b}{3k} < -\frac{3.8413}{b}k^2 + 2k - \frac{2k^2}{b} \ln \frac{b}{3k} \tag{2.39}$$

and the following inequality holds:

$$-\frac{3.8415}{b}k^2 + 2k - \frac{2k^2}{b} \ln \frac{b}{3k} < E_d[\pi_a + \pi_b + \pi_c] < -\frac{3.8413}{b}k^2 + 2k - \frac{2k^2}{b} \ln \frac{b}{3k} \tag{2.40}$$

Hence, the total welfare under the coordinated duopoly can be approximated by adding (2.38) and (2.39). □

Proof of Corollary 2.4. The monopoly airline's profit is

$$\pi_{AP}^M = p \cdot \min(d(1 - p), k)$$

Thus, we have the monopoly airline's profit given d:

$$\pi_{AP}^M = \begin{cases} \frac{1}{4}d & \text{if } d \leq 2k \\ k(1 - k/d) & \text{otherwise} \end{cases} \tag{2.41}$$

d follows an arbitrary and unbounded but positive distribution. Aggregate the profit on the dimension of demand and the expected profit is given by:

$$E_d[\pi_{AP}^M] = -\frac{1}{2}kF_d(2k) + k - \frac{1}{4}\int_0^{2k} F_d(d)dd - k^2\int_{2k}^{\infty}\frac{f_d(d)}{d}dd$$

Similarly, assume demand follows uniform distribution $U[0,b]$ and we can prove that the optimal capacity $k^* \geq \frac{1}{2}b$. Suppose $k_1 < \frac{1}{2}b \leq k_2$ and the difference of the expected profits at k_1 and k_2 is given by:

$$\begin{aligned} E_d[\pi_{AP}^M(k_2)] - E_d[\pi_{AP}^M(k_1)] = &\int_0^b \frac{1}{4}df_d(d)dd - \int_0^{k_1}\frac{1}{4}df_d(d)dd \\ &- \int_{k_1}^b k_1(1-k_1/d)f_d(d)dd \end{aligned}$$

Since $\frac{1}{4}d > k_1(1-k_1/d)$, $E_d[\pi_{AP}^M(k_2)] > E_d[\pi_{AP}^M(k_1)]$. Hence, the optimal capacity $k_M^* \geq \frac{1}{2}b$. □

Proof of Corollary 2.6. From Corollary 2.1, the optimal capacity brought by the two coordinated duopolies is approximated as $0.1512b$ and thus the following inequality holds: $0.1511b < k_D^* < 0.1513b$. We use this inequality to compare the welfare.

We firstly compare the joint welfare of APs and customers. Taking the first order condition and the second order condition of (2.40), the upper bound of the joint welfare under the coordinated duopoly setting can be shown to increase with $0.1511b < k_D^* < 0.1513b$, leading to

$$\pi_{AP}^D + \pi_C^D < -\frac{3.8413}{b}k^2 + 2k - \frac{2k^2}{b}\ln\frac{b}{3k} \approx 0.1785b < 0.1786b \tag{2.42}$$

where the first inequality comes from (2.40).

Therefore, the joint welfare of APs and customers under the coordinated duopoly is less than the joint welfare of APs and customers under monopoly setting, which is $\frac{3}{16}b$.

It is also of interest to discover how the welfare of APs and the welfare of end customers change separately. From Corollary 2.1, the welfare of one AP can be reformed as $E_d\left[\pi_i(k,b)\right] = \frac{k^2}{4b}\ln 3 + \frac{k}{b}\left(b-3k\right) + \frac{2k^2}{b}\left(-\ln\frac{b}{3k}\right)$. We focus on the range of $\frac{1}{7}b < k < \frac{1}{6}b$ based on the inequality of optimal capacity and thereby we have $-\ln\frac{b}{3k} < -\ln 2$. Using this inequality, each AP's welfare is relaxed to $E_d\left[\pi_i(k,b)\right] < \frac{k^2}{4b}\ln 3 + \frac{k}{b}\left(b-3k\right) - \frac{2\ln 2}{b}k^2$ which can be shown to increase in the range of $\frac{1}{7}b < k < \frac{1}{6}b$. Thus, each AP's welfare can be further relaxed to $E_d\left[\pi_i(k,b)\right] < \frac{b}{144}(12 - 8\ln 2 + \ln 3) \approx$

$0.052b$. The total APs' welfare under the coordinated duopoly can then be proved less than that under monopoly: $\pi_a^D + \pi_b^D < \frac{b}{72}(12 - 8\ln 2 + \ln 3) < \frac{1}{8}b$.

The customers' welfare can be derived from the joint welfare of AP and customer knowing the APs' welfare.

$$\begin{aligned}\pi_c^D > & -\frac{3.8415}{b}k^2 + 2k - \frac{2k^2}{b}\ln\frac{b}{3k} \\ & - 2\left(\frac{k^2}{4b}\ln 3 + \frac{k}{b}(b-3k) + \frac{2k^2}{b}\left(-\ln\frac{b}{3k}\right)\right) \\ = & \left(-\frac{3.8415}{b} + \frac{6}{b} - \frac{2\ln 3}{4b}\right)k^2 + \frac{2k^2}{b}\ln\frac{b}{3k}\end{aligned}$$

The lower bound can be shown to obtain the minimum value when $k = 0.1511b$ by taking the first and second derivatives and hence $\pi_c^D > 0.7286b$ by replacing k as $0.1511b$. This implies that the welfare of customers under the coordinated duopoly is greater than the welfare of customers under monopoly.

Now we proceed with the welfare of LSPs. Under the duopoly the total LSPs welfare is $\frac{mk_D^*}{8b}(16b + k_D^*(\pi^2 - 28))$ which increases when k is in the range between $\frac{3}{20}b$ and $\frac{1}{6}b$. However, this range is to loose to compare the welfare and we narrow it down to a sub-range $\frac{151}{1000}b < k < \frac{1}{6}b$. The LSPs welfare can be then shown $\pi_{LSPs}^D > \frac{151}{8000000}mb\left(11772 + 151\pi^2\right) > 0.25mb$ where the LSPs' welfare under monopoly equals $\frac{mb}{4}$.

Since the APs' welfare is higher in monopoly setting and the others are higher in the coordinated duopoly setting, there may exist a threshold of m when the total welfare of the two settings is identical. The difference between two welfare is derived as following:

$$\begin{aligned}E_d[\pi^D - \pi^M] < & \frac{-3}{16}b - \frac{3.8413}{b}k^2 + 2k - \frac{2k^2}{b}\ln\frac{b}{3k} - \frac{1}{4}bm + \\ & \frac{km(16b + k(-28 + \pi^2))}{8b} \\ = & \left(\frac{(\pi^2 - 28)k^2}{8b} + 2k - \frac{b}{4}\right)m \\ & - \left(\frac{3.8413}{b} + \frac{2\ln\frac{b}{3k}}{b}\right)k^2 + 2k - \frac{3}{16}b\end{aligned}$$

where the first inequality comes from (2.39). Given that $0.1511b < k_D^* < 0.1513b$, the coefficient of m can be shown positive, namely $\frac{(\pi^2-28){k_D^*}^2}{8b} + 2k_D^* - \frac{b}{4} > 0$ and hence it increases with m. Let the upper bound equal to 0 and the threshold can be identified as following:

$$\tilde{m} = \frac{16\left(3.8413 + 2\ln\frac{b}{3k}\right)k^2 - 32kb + 3b^2}{2(\pi^2 - 28)k^2 + 32kb - 4b^2} \tag{2.43}$$

$\tilde{m}$ is approximately 15 and given that the price charged by asset providers is normalized between 0 and 1, m is 15 times of the maximum price charged by asset providers. This condition is barely satisfied in real market. Hence, we can conclude that the welfare of the coordinated duopoly setting is less than the welfare of monopoly setting. □

Proof of Lemma 2.2. The proof of this lemma directly follows the Proposition 1 in Vives (1986). Similar to the basic model, there exist three equilibrium situations in the spot market with three asset providers: under-capacity, intermediary capacity and overcapacity.

When the spot demand is sufficiently low, each firm has fierce competition and the price will be undercut to the marginal price, which is 0. Any price higher than the marginal price will lead to 0 demand allocated to the price offer. If the spot market is sufficiently high, all firms will offer the market clearance price, which is $1 - \frac{3k}{d}$. Any price lower than that will lead to a lower profit since the same quantity will be traded at a lower price. Any price higher than the market clearance will also bring less profits due to the demand elasticity. The threshold between the overcapacity and intermediary capacity case is obtained from the Proposition 1, namely $\frac{d}{2}$, which depends on the spot demand d and the number of asset providers.For $2k < d$, there exist a price p bringing positive amount of units to the highest price offer, leading to the jump of the price. The threshold between the intermediary capacity and the under-capacity is obtained from the Cournot quantity, namely $\frac{d}{n+1} = \frac{d}{4}$ with n indicating the number of symmetric firms in the competition.

For the intermediary equilibrium, where there is not pure pricing strategy but mixed pricing strategies, we identify the range of the prices. To obtain the upper bound of the price, which maximizes the profit given all competitors offering a lower price, we have $\bar{p} = argmax\{p \cdot (d \cdot (1-p) - 2k)\}$. Hence, $\bar{p} = \frac{d-2k}{d}$. Then, the equilibrium profit of each airline can be derived as $\bar{p} \cdot d \cdot (1 - \bar{p}) = \frac{(d-2k)^2}{4d}$ based on the definition of the equilibrium. The lowest price $\underline{p}$ is obtained as $\underline{p} = \frac{(d-2k)^2}{4dk}$ since it refers to the case where the asset provider is the lowest price offer and sells all its capacity. Noting that any price in this range gives identical profit, we can then derive the distribution by solving $(1-(F_p(p_m))^2)p_m k + (F_p(p_m))^2 p_m(d(1-p_m) - 2k)$, which is the expected profit considering when it is the highest price offer selling $(d(1-p) - 2k)$ and when it is not the highest price offer selling at its capacity. Hence, the CDF can be derived as $F_p(p_m) = \left(\frac{4kdp_m - (d-2k)^2}{4d^2p_m^2 - 4d^2p_m + 12kdp_m}\right)^{\frac{1}{2}}$. □

Proof of Corollary 2.7. The optimal capacity $k^* = argmax\ E_d[\pi(d,k)]$ where the profit given d and k is expressed in Lemma 2.2 and hence, the expectation can be elaborated as the following

$$E_d[\pi_i^e(d,k)] = 0 + \int_{2k}^{4k} \frac{(d-2k)^2}{4d} f_d(d)dd + \int_{4k}^{\infty} \frac{d-3k}{d} \cdot k \cdot f_d(d)dd$$

If $\frac{1}{2}b \leq k$, for $d \in [0,b]$, d is less than or equal to $2k$, implying that given any spot demand, the equilibrium profit is 0. This case is dominated by other cases and hence k should be greater than $\frac{1}{2}b$.

If $\frac{1}{4}b \leq k \leq \frac{1}{2}b$, given that $f_d(d) = \frac{1}{b}$, the expected profit of each asset provider can be simplified to

$$E_d[\pi_i^e(d,k)] = \int_{2k}^{b} \frac{(d-2k)^2}{4db} dd \tag{2.44}$$

By applying Leibniz integral rule, the first order condition of (2.44) with respect to k is obtained as $\int_{2k}^{b} \frac{8k-4d}{4db} dd$, which can be further written as the following

$$\frac{\partial E_d[\pi_i^e(d,k)]}{\partial k} = -1 + \frac{2k}{b} + \frac{2k}{b} \ln \frac{b}{2k}$$

For $\frac{1}{4}b \leq k \leq \frac{1}{2}b$, we have $\frac{2k}{b} \in [\frac{1}{2}, 1]$, within which range, the inequality of $\frac{\partial E_d[\pi_i^e(d,k)]}{\partial k} < 0$ always holds. Hence, the expected profit decreases in k, implying the minimum value of k in such range is the optimal capacity decision. Therefore, $k^* \leq \frac{1}{4}b$. □

2.B. Notation

Name	Meaning	Expression
k	capacity of an AP	
d	demand	
$f_d(d)$	pdf of end customer demand distribution	
$F_d(d)$	cdf of end customer demand distribution	
$f_p(p_m)$	pdf of equilibrium mixed pricing strategy	
p_i	spot market price per capacity unit set by AP_i	
q_i	quantities traded in spot market by AP_i	
m	unit margin charged by LSPs	
$D(p)$	demand-price function in spot market	$D(p) = d(1-p)$
$L(p_i)$	revenue of offering a lower price	(2.1))
$E(p_i)$	revenue of offering an equal price	(2.1)
$H(p_i)$	revenue of offering a higher price	(2.1)
p_i^e	equilibrium spot price of AP_i	(2.2)
$\pi_i(p_i, p_j)$	immediate profit of AP_i given the two spot prices	(2.1)
$\pi_i^e(.)$	equilibrium profit of AP_i	(2.4)
k_D^*	equilibrium capacity choice under coordinated duopoly	
k_D^{c*}	equilibrium capacity choice under coordinated duopoly with a unit cost c	
k_M^*	equilibrium capacity choice under monopoly	
k_T^*	equilibrium capacity choice with three APs	
p_l	the lower spot price	Table 2.1
p_h	the higher spot price	Table 2.1
$p_{\{\hat{k}\}}$	the price clearing $\hat{k}$ units	$1 - \frac{\hat{k}}{d}$
Π_M	total welfare under monopoly	
Π_D	total welfare under coordinated duopoly	

Table 2.3 Notation

Chapter 3

Contracting strategies for price competing firms under demand uncertainty[1]

3.1. Introduction

The key challenge faced by the APs is the share of their capacity to be traded through the long term contracts. Such contracts secure early sales, but may come at a lower price. In the air cargo market—a market which motivates our study—APs tend to trade more than half of their capacity via long term contracts. For instance, around 66% of air cargo volume from the Asia Pacific to Europe and Middle East was sold in the long term contract market in 2019 with a clear downtick associated with the COVID-19 pandemic (Xeneta, 2022).

In air cargo transportation, cargo airlines operate the assets – cargo aircraft – that will transport the cargo. LSPs aggregate the demand for cargo transportation from their end customers and approach APs to purchase some of their available capacity for cargo transportation. Typically, cargo capacity can be considered as a commodity as there is very little differentiation between the competing APs. Consequently, LSPs aim to purchase capacity at the lowest price possible, be it via long-term contracts or on the spot market.

However, very little is known on how APs and LSPs should best decide on the suitable

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share of long-term contract trading and spot market operations. Some fundamental trade-offs affect the situation. The AP decides on the provided capacity significantly ahead of the trading horizon and has an interest in selling all of its capacity and realize the highest price possible - yet, she is confronted with competition by other APs. Thus, the price needs to be low to attract LSPs, but high enough to have sufficient revenues. At the same time, the LSP generates revenues by transporting the cargo for its end customers. If he cannot secure capacity, he will miss a sales opportunity and realise sub-par revenues. However, if he purchases capacity at too high a price, its end customers may opt not to transport the cargo at all. Trading only in the spot market exposes both AP and LSP with a certain risk. If demand is low, the AP may not sell its entire capacity. If demand is high, the LSP may not be able to secure enough capacity to serve his customers.

Trading via a long-term contract affects the situation significantly. Primarily, it ensures secured sales for the AP and guarantees capacity availability for the LSP. At the same time, it also reduces both demand and capacity from the spot market – effectively altering the dynamics in the spot market. Reducing capacity increases the spot price, yet reducing demand decreases the spot price. Whether or not implementing long-term contracts is beneficial for both AP and LSP depends on the demand situation and the negotiated prices.

Extant literature specifically targeted at long-term contracts and spot market trades in air cargo is limited. Amaruchkul et al. (2011a), Lin et al. (2017) and Hellermann (2006) all position their work in this domain. However, none of them studies the effect of the competitive situation between APs themselves, and the negotiation between APs and LSPs.

Consequently, it is our objective to develop guidance for APs and LSPs regarding the decision on contract market and spot market trading. To do so, however, we will first need to develop a though understanding of the spot market characteristics. Our work addresses the following research questions:

1. How should APs price their cargo capacities in a competitive spot market to maximise profits?
2. What are the revenues that APs and LSPs can expect to realize in a competitive spot market under uncertain end customer demand?, and finally
3. How should APs and LSPs best balance their trades between contract and spot market?

We develop and analyze a two-stage game theoretical model with two APs and two LSPs. Our model includes competition between the two APs when it comes to pricing in the spot market, and reflects a negotiation between one AP and one LSP for a long-term contract.

Our findings include that the optimal pricing strategy in the spot market includes a mixed pricing range, specifically, if demand is not too high or too low, and we characterize the range, the pricing strategy as well as expected revenues. A numeric analysis of the contract negotiation stage lets us identify that the quantity contracted decreases in the margin charged by the LSPs and generally decreases in the market demand potential. The contract is most valuable, i.e., yields the most benefit to the two agents, when demand potential is moderate, roughly ranging between half to twice the combined capacity of the competing APs, while the LSP's margins are fairly low.

Our study makes the following three contributions. First, we identify the optimal pricing strategies for the asset providers formally and highlight the importance of a mixed pricing strategy. This contribution results from our novel model that describes the price competition in the spot market explicitly. Second, we explore how the optimal share of capacity traded in the spot market is affected by the uncertain end customer demand as well as the profit margin of the LSP. We can do so with the help of a Nash bargaining model for the contracting phase that allows us to exceed the findings of a leader-follower setup. Thirdly, monitor the impacts of a contract on both supply and demand of the spot market, which we judge as instrumental for understanding the balance in trading in contract and spot market.

The rest of the paper is organized as follows. In Section 3.2, we provide a literature review. In Section 3.3, we present the model, including a static and a dynamic spot market model that are required for modelling the long-term contract negotiations. In Section 3.4, we firstly analyze the optimal pricing strategies in both spot market models and compare them. We then explore the analytic and numeric results of the contract negotiation in Section 3.5. Section 3.6 summarizes.

3.2. Literature review

Freight contract and capacity allocation have been discussed in the literature of cargo freight revenue management, which is, however, quite scant when comparing to the large body of revenue management literature (Levin et al., 2012). Cargo exhibits several distinctive characteristics, which include, but not limited to, the aggregation of demand by LSPs, the competitive nature of the spot markets and capacity booking behavior. Some literature study the cargo capacity allocation problem for single-leg air cargo flights and discuss the optimal capacity control strategy for the booking process with stochastic demand and fixed capacity (Amaruchkul et al., 2007;

Amaruchkul and Lorchirachoonkul, 2011; Huang and Chang, 2010). Barz and Gartner (2016) and Levina et al. (2011) extend the discussion to a network context and study the stochastic dynamic capacity control problem by showing the structural properties. Further, Levin et al. (2012) investigate into the operational decisions of capacity management with both allotments and spot market demands. They formulate the spot market booking control problem as a dynamic program with approximations to its value function and solve the allotment selection problem using linear mixed-integer programs. Moussawi-Haidar (2014) conducts a work which also consider both allotment contracts and spot market, modeled as a discrete-time dynamic capacity control problem. He conducts numerical examples to solve industry-size problems with heuristic methods and derive an upper bound on the value function. Our focus is on the strategic, rather than the operational level of decisions. That is, instead of supporting the capacity control process, we seek to determine pricing decisions as well as the more strategic capacity allocation decisions between short-term and long-term markets. In practice, a long-term contract signed between an AP and an LSP contains multiple commercial terms, such as unit price, committed quantity, payment terms, service level, etc. Hellermann (2006) discussed contracts with various pricing structures and compared the fixed-commitment contract with the capacity-dependent contract. Similar to most literature, this paper simplifies the contract terms and focuses on prices and quantities.

Most of the literature on freight capacity and contract, as mentioned above, assume a monopoly setting. We contribute to this stream by accounting for the competition among agents. Other works that model competition include Shah and Brueckner (2012) and Shang and Liu (2011). Shah and Brueckner (2012) investigate price, frequency and vehicle capacity decisions of freight carriers considering the brand loyalty of shippers in a freight-competitive context. They show that the frequency decreases in costs and the number of carriers but increases in market size. They also explain the excess capacity in the freight industry by demonstrating that the equilibrium frequency is higher than the social optimum. Shang and Liu (2011) study a capacity game among firms which compete in promised delivery time and quality of service and characterize the equilibrium of capacity choices. We differ from these papers mainly by modeling the price competition between firms, where the end customers always prefer the lowest price until the capacity runs out.

Such price competition, which can also be referred as Bertrand-Edgeworth competition, has been numerously discussed in the economics literature that adopt game-theoretical approaches. Osborne and Pitchik (1986) characterize the Nash equilibrium in a duopoly setting and find the capacity equilibrium coincides with the set of Cournot quantities with static demand. Levitan and Shubik (1978) provide an explicit mixed strategy equilibrium solution for an oligopoly game. Allen and Hellwig (1993) illustrate a similar pricing equilibrium with a different demand allocation method where residual demand is proportionally split. The price competition, which are modeled and discussed in these papers, highly reflects the industrial practice that freight customers always prefer the lowest price offer. However, to the best of your knowledge, there is no study relating to freight operations has modelled the market as the Bertrand-Edgeworth competition. The most relevant paper is by Cohen et al. (2022), who develop a Bertrand competition game while considering capacity constraint, quality differentiation and customer heterogeneity and they characterize whether and when the price discrimination strategy is beneficial. As pointed out in their work, the Bertrand competition may lead to discontinuous "winner takes all" dynamics, which differs from other continuous demand model.

Our paper also captures the negotiations between an asset provider and an LSP. A few papers shed light on freight contract terms. Amaruchkul and Lorchirachoonkul (2011) studies capacity contracts between a carrier and a forwarder when certain parameters are private information. They study contracts with three terms including a lump sum payment, allotment and refund rate and obtain an upper bound on the information rent paid by the carrier. Gupta (2008) introduces flexible carrier-forwarder contracts which allow adjustments of contract parameters that do not make the forwarder financially worse off. He shows how the flexible contract improves the efficiency of capacity allocation for carriers. Hellermann (2006) proposes a contract with a reservation fee and execution fee between one forwarder and one carrier with one-dimensional capacity using a static game theoretic model. Lin et al. (2017) apply buy-back contract in air cargo industry between an asset provider and an intermediary considering both contract and spot markets although with two independent demands on these two markets. Our paper contributes by studying freight contracts in the presence of competition.

Most freight contract papers adopt a Stackelberg leader-follower game, as discussed above. However, a bargaining framework can be a better solution to modeling the negotiation process of contracts as argued by Lovejoy (2010). There are scant works of applying bargaining framework in supply chain management. Feng and Lu (2013) investigate the contracting behaviors of a two-tier supply chain in a static setting and illustrate how different game structures, Stackelberg game and bargaining game, critically affect firms' preferences over contract types and equilibrium contract terms. Mantin et al. (2014) analyze the role of third-party marketplaces for dual-format retailers in the bargaining process with its upstream supplier and show that how the dual-format improves its bargaining position by creating an outside option. We adopt a similar bargaining framework, which originates from Muthoo (1999), to model the negotiation process between an asset provider and an LSP with potentially different bargaining power.

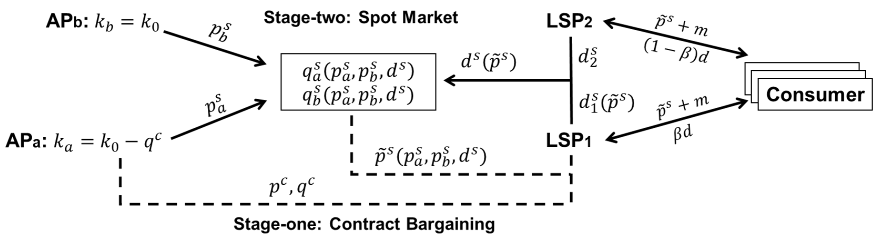


Figure 3.1 Modelling Framework

3.3. Model

We model the interactions between three types of agents: asset providers (henceforth, APs), who own and bring capacity to the market, end customers, who generate the demand for the asset providers' services, and the LSPs, who aggregate the demand from the end customers and trade directly with the asset providers.

Figure 3.1 illustrates the two-stage modeling framework. We assume two competing APs, denoted as AP_a and AP_b, with each endowed with the same capacity k_0. This reflects the symmetric choice of aircrafts or bilateral agreements [2]. The APs set their spot market prices: AP_a sets p_a^s and AP_b sets p_b^s to maximize individual profits. If they set different prices, demand is first served by the asset provider offering the lower price and only if this AP's capacity is exhausted, then demand is served by the other AP. We let q_a^s and q_b^s denote the quantities traded by AP_a and AP_b in the spot market, respectively.

We let d denote the demand from end customers, which is aggregated by the LSPs, who then interact with the APs through the spot market. For simplicity, we assume that there are only two competing LSPs. We assume that the realized demand, d, is allocated between the LSPs according to some proportion such a fraction β is allocated to LSP_1 and the remainder $1 - \beta$ is allocated to LSP_2. Different values of β reflect the size of LSPs and may affect the contract strategy ultimately. Equivalently, the two LSPs can be treated as one focal LSP, who bargains with one AP and possibly many others who share the remaining market. LSPs charge the end customers a fixed mark up, m, for their services. The value of m can be interpreted as a proxy for the degree of competition in this market. This mark-up is applied to the spot price for each transported unit of capacity.

An important feature of our model is demand uncertainty. At the beginning of the

[2] Countries negotiate air service agreements which structure the scope of airbone traffic and bilateral air service agreements normally specify the flight capacity that is equally shared between the airlines of either country (Wu et al., 2022)

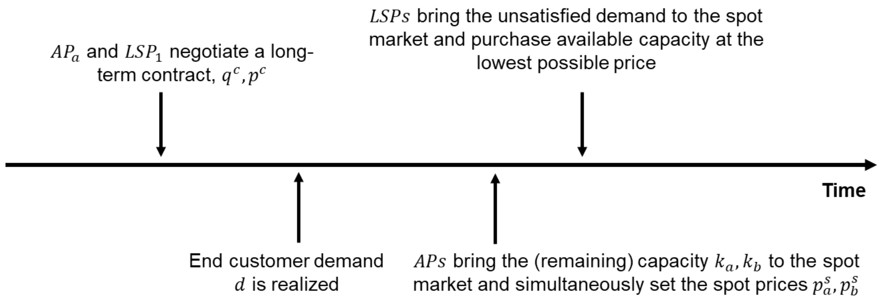


Figure 3.2 Sequence of events

horizon, demand is unknown, but assumed to follow some distribution. Specifically, we assume d follows a general probability density function $f_d(d)$ and corresponding cumulative density function $F_d(d)$. In the presence of demand uncertainty, asset provider and LSP have an incentive to sign a contract ahead of the spot market to secure capacity and sales. Our model assumes that AP_a and LSP_1 are the only parties to negotiate a long-term contract.

The sequence of events, which is illustrated in Figure 3.2, is as follows: In the first stage AP_a negotiates with LSP_1 the capacity to be allocated to this LSP and the corresponding price. They negotiate the amount of capacity units, q^c, $q^c \in [0, k_0)$, to be transferred from AP_a to LSP_1 and the unit price, p^c, to be paid to AP_a for this capacity. Note that $q^c = 0$ indicates failure of the negotiation process. In the second stage, demand d is realized and all demand not served from the long-term contracted capacity as well as all remaining capacity by both APs is brought to the spot market. The asset providers compete in price and LSPs purchase available capacity a the lowest possible price, following a demand-price function.

In the spot market, the asset providers simultaneously set their spot market prices, p_a^s and p_b^s, according to their strategies. Based on the two spot prices and the corresponding trading quantities, we define the weighted spot price as the following:

$$\tilde{p}^s = \frac{p_a^s \cdot q_a^s + p_b^s \cdot q_b^s}{q_a^s + q_a^s}, \tag{3.1}$$

where q_i^s with $i \in \{a, b\}$ refers to the capacity sold by either AP. The APs bring to the spot market their capacity, $k_a = k_0 - q_c \leq k_0 = k_b$, where q_c is the contracted quantity. Note that an AP is not guaranteed to sell its entire capacity, thus $q_i^s \leq k_i$.

The demand of the end customers follows a demand-price function in the form of $D(\tilde{p}^s) = d \cdot (1 - (\tilde{p}^s + m))$. Recall that the LSP charges the spot market price, $\tilde{p}^s$, plus the margin m to the end customers. The end customer demand-price function takes the form of $D^s(\tilde{p}^s) = d^s \cdot (1 - \tilde{p}^s)$ in the spot market, with $d^s = d_1^s + d_2^s$. Two details need to be considered when spelling the demand out explicitly. First, end customer demand is split amongst the LSPs with LSP_1 receiving a share β. Second, LSP_1 may have negotiated q^c units of capacity in the long-term market, and he uses it to serve some of his end customer demand. LSP_1 will only access the spot market to serve demand in excess of the q_c units he has. Thus, $d_1^s = \beta d - \min(\beta d, q_c)$ and $d_2^s = (1 - \beta)d$. The total spot market demand sums up to

$$d^s = d - \min(\beta d, q^c)$$

Note, the residual demand LSP_1 faces, d_1^s, is the excess units of demand willing to pay the spot market price $\tilde{p}^s$. There is a subtle point here as the LSP needs to carefully balance the realized demand d_1, the demand satisfied by the contract denoted as d_1^c, and the residual demand satisfied in spot market d_1^s.

In the spot market, the APs' profit functions are fundamental for their pricing decision. Since they compete over prices, given capacity constraints, they are engaged in a Bertrand-Edgeworth type of competition where the lower price offer captures all demand unless the capacity at this price runs out. For instance, AP_a sets the lower price, namely $p_a^s < p_b^s$, then bounded by its capacity, it sells $q_a^s = \min(k_a, D(p_a^s))$ whereas AP_b sells $q_b^s = \min(k_b, \max(0, D(p_b^s)) - k_a)$. Solving the demand allocation to the two competing APs, we follow Levitan and Shubik (1978) and Osborne and Pitchik (1986) and adopt their residual demand allocation rule.

In a nutshell, the profit of an AP in the spot market is the product of the price and the capacity sold at that price. Accordingly, given a spot market demand d^s, the profit function of AP$_i$, $i \in \{a, b\}$, is given by:

$$\pi_i(p_i^s, p_j^s) = \begin{cases} L_i(p_i^s) = & p_i^s \min\left(k_i, D(p_i^s)\right) & \text{if } p_i^s < p_j^s \\ E_i(p_i^s) = & p_i^s \min\left(k_i, \frac{k_i}{k_i+k_j} D(p_i^s)\right) & \text{if } p_i^s = p_j^s \\ H_i(p_i^s) = & p_i^s \min\left(k_i, \max\left(0, D(p_i^s) - k_j\right)\right) & \text{if } p_i^s > p_j^s \end{cases} \tag{3.2}$$

where L_i (resp., H_i) denotes AP_i's revenue when this AP sets the lower (resp., higher) price, and E_i denotes the revenue when both set the same price, with $i \in \{a, b\}$ and j indicating the competitor.

We explore a tractable, simplified market mechanism for the spot market first, before presenting a more elaborate, dynamic spot market model. We outline these two solution approaches in Section 3.3.1 and Section 3.3.2, respectively, and later in Section 3.3.3 we specify the details of the long-term contract between AP_a and LSP_1. In Section 3.4.3, we compare the differences between the spot market models.

3.3.1 Static spot market

In the static spot market, we simplify APs' pricing decisions by assuming that APs treat the spot demand d^s as an exogenous variable. Thus, in the static solution approach, the APs' pricing decisions do not take into account the interdependence of the spot prices and the spot demand. Consequently, the spot market is a standard Bertrand-Edgeworth competition and the equilibrium pricing strategy can be derived from (3.2), as further discussed in Section 3.4.1.

3.3.2 Dynamic spot market

The dynamic solution approach explicitly takes into account how end customer demand responses to the spot market price $\tilde{p}^s$. As $\tilde{p}^s$ increases, LSP_1 actually captures less end customer demand and q_c allows him to serve a higher share of his end customer demand. We use d_1^c to indicate the portion of demand satisfied by contracted capacity and the dynamic spot market model assumes $d_1^c = \min(\beta d, \frac{q^c}{1-\tilde{p}^s})$. Thus, demand in the spot market is a function of $\tilde{p}^s$:

$$d^s(\tilde{p}^s) = d - \min(\beta d, \frac{q^c}{1 - \tilde{p}^s}) \tag{3.3}$$

Rewriting (3.1), the weighted spot price $\tilde{p}^s$ extends to:

$$\tilde{p}^s(p_a^s, p_b^s, d^s) = \frac{p_a^s \cdot q_a^s(p_a^s, p_b^s, d^s) + p_b^s \cdot q_b^s(p_a^s, p_b^s, d^s)}{q_a^s(p_a^s, p_b^s, d^s) + q_b^s(p_a^s, p_b^s, d^s)} \tag{3.4}$$

The solution to the equation system of (3.3) and (3.4) captures the equilibrium of spot demand and weighted spot price for the dynamic spot market. In a dynamic spot market, APs know that different pairs of spot prices lead to different equilibrium spot demand and weighted spot price, ultimately different payoffs. Hence, APs determine the spot pricing strategy, with a realized end customer demand d, to optimize the spot profits, namely (3.2).

3.3.3 Long-term contract

Following Muthoo (1999), we model the negotiation for the long-term contract between AP_a and LSP_1 as a Nash bargaining framework. Such a framework assumes that upon entering negotiations, the two agents are endowed with bargaining powers. Specifically, the AP has a bargaining power τ and the LSP has the complementary power $1 - \tau$. The parameter τ determines the respective gains from trade, which is the difference between their combined profit if a contract is signed and their outside options, which is their combined profit if they walk away without agreeing to a contract. We denote AP_a's outside option by O_a and LSP_1's outside option by O_1. We denote the profits of AP_a and LSP_1 by Π_a and Π_1, respectively, if a contract is signed. Clearly, they only sign a contract if $\mathbb{E}[\Pi_i] > \mathbb{E}[O_i]$ with $i \in \{a, 1\}$.

The Nash Bargaining solution is the pair of q^{c*} and p^{c*} which solves:

$$\begin{aligned} &\max_{q^c, p^c} \left(\mathbb{E}_d[\Pi_a] - \mathbb{E}_d[O_a]\right)^{\tau} \cdot \left(\mathbb{E}_d[\Pi_1] - \mathbb{E}_d[O_1]\right)^{(1-\tau)}, \\ &s.t. \, \mathbb{E}_d[\Pi_a] \geq \mathbb{E}_d[O_a], \mathbb{E}_d[\Pi_1] \geq \mathbb{E}_d[O_1] \end{aligned} \tag{3.5}$$

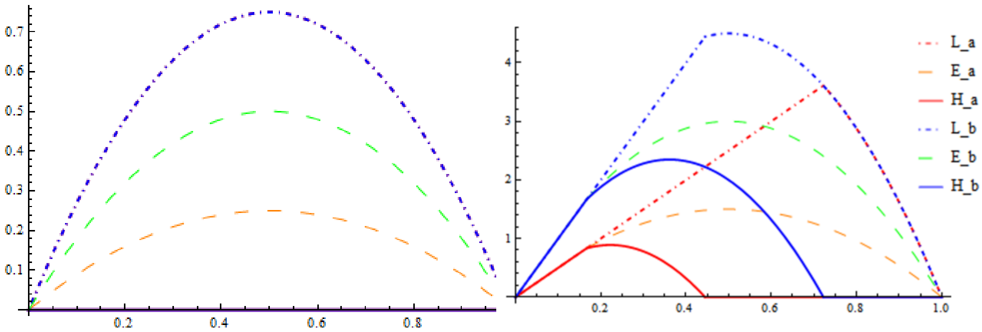
Note that the outside options of the agents reflect the agents' profits if all $2k_0$ units of capacity are traded via the spot market. This implies that solving the negotiation stage requires estimation of the expected revenues in the spot market conditional on q_c.

3.4. Spot market analysis

We first explore the tractable static spot market in Section 3.4.1 before presenting the dynamic spot market model in Section 3.4.2. Since we would like to understand the effective difference between both models, we contrast them in Section 3.4.3.

3.4.1 Static spot market

The AP's profit in the spot market, as given in (3.2) depends on the demand realization. Figure 3.3 illustrates two realizations by assuming $k_a = 5$ and $k_b = 10$. When d^s is sufficiently low, the AP with the lower price satisfies the entire demand and, hence, the high-price AP's profit is zero, $H(p) = 0$. This is illustrated in Figure 3.3a. When $d^s > k_a$, the smaller AP (AP_a in this case) may not be able to satisfy the demand alone, in which case $H_b(p)$ becomes positive for some low prices, shown in Figure 3.3b. $H_b(p)$'s linear part (for $p < 0.2$) indicates the price range where both APs are bounded by their capacities. In the middle price range ($0.2 < p < 0.45$), the low-price AP continues to exhibit a linear profit with respect to his own price since he is bounded by his capacity, while the high-price AP shifts to a quadratic profit function as he is no longer bounded by his capacity. In a higher price range ($0.45 < p < 0.7$), the small AP realizes zero profit if he offers the higher price and he sells his entire capacity if he offers the lower price. In contrast, the larger AP always makes a positive profit and he is not bounded by his capacity. A more detailed discussion is provided in the proof of Proposition 3.1.



(a) $d^s = 3$ (b) $d^s = 18$

Figure 3.3 Profit outcomes for two demand realizations, $k_a = 5$, $k_b = 10$

Formalizing the above illustration, we identify the optimal pricing strategy for different realizations of the spot demand d^s:

Proposition 3.1. *In the static spot market, given a spot demand d^s, the equilibrium static spot price is given by*

$$p_i^{ss}(d^s) = \begin{cases} 0 & \text{if } d^s \le k_a \\ p_i^m & \text{if } k_a < d^s < 2k_b + k_a \\ \frac{d^s - k_a - k_b}{d^s} & \text{if } 2k_b + k_a \le d^s \end{cases} \tag{3.6}$$

where p_i^m is a mixed pricing strategy defined over the following range:

$$[\underline{p}, \bar{p}] = \begin{cases} \left[\frac{1}{2}\left(1 - \sqrt{\frac{k_a}{d^s}\left(2 - \frac{k_a}{d^s}\right)}\right), \frac{d^s - k_a}{2d^s}\right] & \text{if } d^s < k_a + 2k_b - 2\sqrt{k_a k_b} \\ \left[\frac{(d^s - k_a)^2}{4k_b d^s}, \frac{d^s - k_a}{2d^s}\right] & \text{otherwise} \end{cases} \tag{3.7}$$

with the following cdf:

$$(F_a(p), F_b(p)) = \begin{cases} \left(\frac{k_b p - \pi_b^m}{k_b p - p(d^s(1-p) - k_a)}, \frac{k_a p - \pi_a^m}{k_a p - p(d^s(1-p) - k_b)}\right) & \text{if } \underline{p} \le p < \underline{p} \vee (p_{k_b} \wedge \bar{p}) \\ \left(\frac{p d^s(1-p) - \pi_b^m}{p k_a}, 1 - \frac{\pi_a^m}{p k_a}\right) & \text{if } \underline{p} \vee (p_{k_b} \wedge \bar{p}) \le p < \bar{p} \\ \left(\frac{p d^s(1-p) - \pi_b^m}{p k_a}, 1\right) & \text{if } p = \bar{p} \end{cases} \tag{3.8}$$

where π_i^m denotes the equilibrium profit by adopting mixed pricing strategies, $\pi_a^m = L_a(\underline{p}) = \underline{p} \cdot k_a$ and $\pi_b^m = H_b(\bar{p}) = \frac{1}{4} d^s \left(1 - \frac{k_a}{d^s}\right)^2$.

From (3.6), it is evident that with sufficiently low demand, there exists a pure pricing

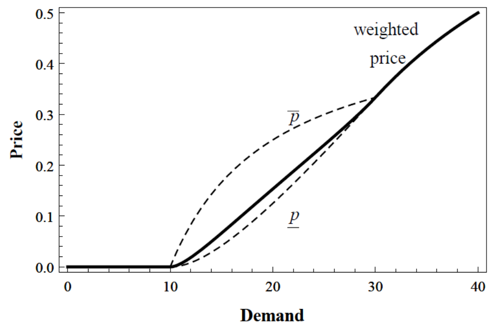


Figure 3.4 An illustration of the spot market weighted price and price bounds as a function of realized demand; $k_a = k_b = 10$

equilibrium where both APs set the marginal cost, assumed zero in our setting. When the spot demand exceeds the smaller AP's capacity, k_a, a price higher than the marginal cost may lead to positive profits as the competitor may not be able to capture the entire demand, in which case a mixed pricing strategy prevails. The two APs draw a price from the same range as specified in (3.7) according to their corresponding density functions as stated in (3.8). The range is identical for the two APs and depends on their capacities. The price upper bound only depends on k_a and d^s and decreases in k_a. The lower bound depends on both capacities and decreases in both k_a and k_b. This implies that given the size of the large AP, the spot price range is higher if the AP has low capacity since the larger AP can eventually behave as a monopolist at a higher price once the competitor runs out of capacity. As demand increases, the range of prices converges to the market clearance price, whereby all capacity is sold. These prices are demonstrated in Figure 3.4 for the case $k_a = k_b$.

In order to evaluate the spot market profit, we now identify the capacity traded in the spot market, as a function of the spot prices posted by the two APs.

Lemma 3.1. *Given a spot demand d^s, the total equilibrium quantity traded in the spot market is given by*

$$q^s(d^s) = \begin{cases} d^s & \text{if } d^s \leq k_a \\ d^s(1-p^s_a) & \text{if } k_a < d^s < 2k_b + k_a, p^s_a > p^s_b, p^s_a < p_{\{k_b\}} \\ \min(k_b, d^s(1-p^s_b)) & \text{if } k_a < d^s < 2k_b + k_a, p^s_a > p^s_b, p^s_a > p_{\{k_b\}} \\ d^s(1-p^s_b) & \text{if } k_a < d^s < 2k_b + k_a, p^s_a < p^s_b \\ k_a + k_b & \text{if } 2k_b + k_a \leq d^s \end{cases} \tag{3.9}$$

where $p^s_i(d^s)$, $i \in \{a, b\}$, follows the equilibrium static prices $p^{ss}_i(d^s)$ as stated in (3.6) *and $p_{\{k_b\}}$ indicates the price clearing when there are k_b units in a monopoly setting.*

When spot demand is sufficiently low, $d^s \leq k_a$, the equilibrium spot prices are 0 and the trading quantity then equals the spot demand d^s. When spot demand is intermediary, $k_a < d^s < 2k_b + k_a$, the equilibrium spot prices follow a mixed pricing strategy and Lemma 3.1 summarizes the three cases that emerge for different prices drawn from the price distribution (from (3.8)). In brief, the total quantity traded depends on the higher spot price while being constrained by the capacity of the larger AP, k_b. The low-price AP in this intermediate range is always constrained by his capacity, while the total quantity only depends on the higher spot market price. However, when the smaller AP_a sets a sufficiently high price, $p_a^s > p_{\{k_b\}}$, he does not serve any demand and the total quantity sold equals the amount traded by AP_b. Lastly, when the spot demand is sufficiently high, $d^s \geq 2k_b + k_a$, the two APs sell their entire capacities.

Knowing the equilibrium spot prices and quantities, we further extend the weighted spot price (3.1) to the expression shown in Lemma 3.2.

Lemma 3.2. *Assume $p_i^s < p_j^s$. The weighted spot price is as follows:*

$$\tilde{p}^s(p_i^s, p_j^s, d^s) = \begin{cases} p_i^s & \text{if } d^s < \frac{k_i}{1-p_j} \\ p_j^s + \frac{k_i(p_i - p_j)}{d^s(1-p_j)} & \text{if } \frac{k_i}{1-p_j} \leq d^s \leq \frac{k_i+k_j}{1-p_j} \\ \frac{p_i k_i + p_j k_j}{k_i + k_j} & \text{if } d^s > \frac{k_i+k_j}{1-p_j} \end{cases} \tag{3.10}$$

With a sufficiently small spot demand, the weighted spot price is identical to the lower price p_i^s since the low-price offer captures the entire demand. With an intermediary demand, the weighted spot price is a function of both APs' prices since they both capture some portion of the demand, as well as the low-price AP's capacity since it determines the number of demand units served at this price. With a sufficiently large demand, the two APs sell their entire capacities and the weighted spot price weighs these capacities.

We proceed with the equilibrium spot profits. Based on Proposition 3.1 and Lemma 3.1, the following lemma summarizes the equilibrium profits of APs given different realizations of spot market demand.

Lemma 3.3. *Given a spot market demand d^s, the equilibrium APs' spot profits $\pi_i^{ss}, i \in \{a, b\}$, are continuous and increasing in d^s, as given by the following equations:*

$$\pi_i^{ss} = \begin{cases} 0 & \text{if } d^s \leq k_a \\ \pi_i^m & \text{if } k_a < d^s < 2k_b + k_a \\ \frac{d^s - k_a - k_b}{d^s} \cdot k_i & \text{if } 2k_b + k_a \leq d^s \end{cases} \tag{3.11}$$

where π_i^m is the spot equilibrium profit with mixed pricing strategy as defined in Proposition 3.1.

We notice that, the size of the APs plays a role in the mixed pricing strategy the two firms adopt. Accordingly, we have the following characterization.

Lemma 3.4. *In the static spot market, when the APs employ mixed pricing strategies, with $k_a \leq k_b$, $F_b(d^s, p)$ first-order stochastically dominates $F_a(d^s, p)$ at any p within the equilibrium price bounds.*

The cdf of the mixed pricing strategy adopted by the large AP stochastically dominates the competitor's who brings less capacity to the spot market. It implies that the small AP is more likely to offer a lower spot price than the large AP given the realized spot demand. As the large AP may capture the entire demand if it were set to the lower price, the small AP offers a price from a stochastically dominated distribution, which implies a lower price in expectation, to increase the likelihood of capturing a share of the demand. The large AP can draw a higher price from the distribution and still be able to sell to the residual demand. As the small AP posts a lower price on average, we ask whether he is worse off compared to the large AP who sets, in expectation, a higher price.

Lemma 3.5. *Under the static spot market setting, $\frac{\pi_a^{ss}}{k_a} \leq \frac{\pi_b^{ss}}{k_b}$ holds for any realized d^s.*

It implies that the small AP is in an inferior position in the competition. The intuition is as follows. From Lemma 3.5, one can show that the equilibrium profit of the smaller AP is equal to or less than the larger AP given d^s according $\pi_a^{ss} \leq \pi_b^{ss}$. Accounting for their capacities, we can show that the profit per unit of capacity of the smaller AP is less than or equal to that of the larger AP, $\frac{\pi_a^{ss}}{k_a} \leq \frac{\pi_b^{ss}}{k_b}$.

Next, we explore the impact of d^s on the density function of the mixed pricing strategy. Intuitively, the expected spot market price should increase in the realized spot demand. Since the APs employ mixed pricing strategies, we prove this intuitive result by resorting to the stochastic dominance of the density functions given two different realizations of spot demand.

Lemma 3.6. *In the static spot market, when the APs employ mixed pricing strategies, with $k_a \leq k_b$, if $d_1^s < d_2^s$, $F_i(d_2^s, p)$ first-order stochastically dominates $F_i(d_1^s, p)$, $i \in \{a, b\}$*

Lemma 3.6 shows that the density function with a smaller d^s first-order stochastically dominates the density function with a larger d^s. It suggests that the expected equilibrium spot price increases in the realized spot market demand. The intuition is that a larger spot demand enables APs to price their capacities at higher levels. Thus, with the help of Lemma 3.3 we have a fully tractable model that allows us to assess the expected spot market profits, which are necessary for the estimation of

	$d^s \leq \frac{k_i}{1-p_j}$	$\frac{k_i}{1-p_j} < d^s < \frac{k_i+k_j}{1-p_j}$	$d^s \geq \frac{k_i+k_j}{1-p_j}$
$\frac{1}{2}d \leq \frac{q^c}{1-\tilde{p}^s}$	Case 1	Case 2	Case 3
$\frac{1}{2}d > \frac{q^c}{1-\tilde{p}^s}$	Case 4	Case 5	Case 6

Table 3.1 Six cases of d^s and $\tilde{p}^s$ in the dynamic spot market model

the agents' outside options in the long-term contracting stage. Recall, however, that the static spot market setting abstracts away from the interdependence of the spot prices and the spot demand. Thus, we next explore the dynamic spot market model to evaluate its fidelity.

3.4.2 Dynamic spot market

The dynamic spot market model extends the static model by fully modelling the elasticity of demand for LSP_1 spot market demand in excess of q_c. When APs set the spot prices, they account for the impacts of the spot prices on the spot demand and the spot demand is endogenously determined rather than a given value. For simplicity, we assume $\beta = \frac{1}{2}$, thus both LSPs equally share the market. Given a pair of spot prices, the solution to the equation system of Eq. (3.3) and Eq. (3.10) characterizes the spot demand and weighted spot price under the equilibrium dynamic spot market given a pair of spot prices.

We identify 6 cases of equilibrium involving d^s and $\tilde{p}^s$ as summarized in Table 3.1.

In Table 3.1, Cases 1-3 refer to the situation where only LSP_2 enters the spot market and LSP_1 satisfies all its demands through the contracted capacity. In Cases 4-6, both LSPs enter the spot market. The cases are also classified by the value of d^s, which affects the quantity traded in the spot market. We solve the 6 cases for d^s and $\tilde{p}^s$ and plug the value of d^s back to the six thresholds. Lemma 3.7 provides the thresholds with respect to the demand d and characterizes the equilibrium d^s and $\tilde{p}^s$ for the 6 cases.

Lemma 3.7. *Given a pair of spot prices, p_i^s and p_j^s, there are 6 equilibrium cases of the spot market demand d^s and the weighted spot price $\tilde{p}^s$, as characterized in Table 3.2.*

After characterizing the equilibrium of a dynamic spot market given a pair of spot prices, we now proceed with the equilibrium pricing strategy by deriving the best response function of the spot prices, namely $p_i^{s*}(p_j^s)$. As in the static model, we find that APs have pure strategies when spot demand is either sufficiently low or sufficiently high.

Table 3.2 Equilibrium of d^s and $\tilde{p}^s$ in dynamic spot market

Case	Threshold	$(d^s, \tilde{p}^s)$
1	$d < 2\cdot\min\left(\frac{k_i}{1-p_j^s}, \frac{q^c}{1-p_i^s}\right)$	$(\frac{1}{2}d, p_i)$
2	$\frac{2k_i}{1-p_j^s} < d < 2\cdot\min\left(\frac{k_i+k_j}{1-p_j^s}, \frac{q^c(1-p_j^s)+k_i(p_i^s-p_j^s)}{(1-p_j^s)^2}\right)$	$\left(\frac{1}{2}d, p_j + \frac{2k_i(p_i-p_j)}{d(1-p_j)}\right)$
3	$\frac{2(k_i+k_j)}{1-p_j^s} < d < \frac{2q^c(k_i+k_j)}{(k_i+k_j-p_i^s k_i-p_j^s k_j)}$	$(\frac{1}{2}d, \frac{p_i k_i+p_j k_j}{k_i+k_j})$
4	$\frac{2q^c}{1-p_i^s} < d < \frac{k_i}{1-p_j^s} + \frac{q^c}{1-p_i^s}$	$(d - \frac{q^c}{1-p_i}, p_i)$
5	$\max\left(\frac{2q^c(1-p_j^s)+2k_i(p_i^s-p_j^s)}{(1-p_j^s)^2},\ \min\left(\frac{2k_i-q^c}{1-p_j^s} - \frac{k_i(p_i^s-p_j^s)}{(1-p_j^s)^2}, \frac{k_i}{1-p_j^s} + \frac{q^c}{1-p_i^s}\right)\right) < d < \frac{q^c(k_i+k_j)}{(k_i+k_j-p_i^s k_i-p_j^s k_j)} + \frac{k_i+k_j}{1-p_j^s}$	$\left(\hat{d},\ \hat{p}\right)$ ‡
6	$d > \max\left(\frac{2q^c(k_i+k_j)}{(k_i+k_j-p_i^s k_i-p_j^s k_j)}, \frac{q^c(k_i+k_j)}{(k_i+k_j-p_i^s k_i-p_j^s k_j)} + \frac{k_i+k_j}{1-p_j^s}\right)$	$\left(d - \frac{q^c(k_i+k_j)}{k_i+k_j-p_i k_i-p_j k_j}, \frac{p_i k_i+p_j k_j}{k_i+k_j}\right)$

‡ $\hat{d} = \dfrac{-(d-q^c+k_i\frac{p_i-p_j}{1-p_j}-dp_j)-\sqrt{(d-q^c+k_i\frac{p_i-p_j}{1-p_j}-dp_j)^2-4dk_i(p_i-p_j)}}{2(p_j-1)}$

‡ $\hat{p} = \dfrac{-(d-q^c+k_i\frac{p_i-p_j}{1-p_j}+dp_j)-\sqrt{(d-q^c+k_i\frac{p_i-p_j}{1-p_j}-dp_j)^2-4dk_i(p_i-p_j)}}{-2d}$

Lemma 3.8. *In the dynamic spot market, when $d \leq \min(2(k_0-q^c), k_0)$, the equilibrium price is 0; when $d \geq 2k_b + \frac{2(k_a+k_b)k_b^2}{2(k_a+k_b)k_b-k_a q^c}$, the equilibrium price is $1-\frac{2k_b}{d}$.*

When demand falls in the intermediate range, we observe an Edgeworth Price Cycle similar to the static case. We are interested in characterizing the mixed pricing strategy. Lemma 3.9 states the solution for an AP's profit in the case of equal spot prices.

Lemma 3.9. *In the dynamic spot market, if $p_b^s = p_a^s$, the profit of AP_b is given by:*

$$\pi_b^s(p_b^s = p_a^s) = \begin{cases} p_a^s \cdot k_0 & \text{if } 0 \leq p_a^s \leq 1 - \frac{2(2k_0-q^c)}{d} \\ p_a^s \cdot (d(1-p_a^s)-q^c)\cdot\frac{k_0}{2k_0-q^c} & \text{if } 1-\frac{2(2k_0-q^c)}{d} < p_a^s \leq 1-\frac{2q^c}{d} \\ p_a^s \cdot \frac{1}{2}d(1-p_a^s)\cdot\frac{k_0}{2k_0-q^c} & \text{if } p_a^s \geq 1-\frac{2q^c}{d} \end{cases} \tag{3.12}$$

We further discuss the pricing strategy in the dynamic market in Section 3.4.3 by comparing to the pricing strategy in the static market via a numerical study.

3.4.3 Comparison of the two solution methods: static VS. dynamic

The static spot pricing solution approach simplifies the dynamic spot pricing where the spot market demand is determined endogenously. In the static spot market, APs' equilibrium pricing strategy follows a classic Bertrand-Edgeworth game but in the dynamic spot market, APs' equilibrium pricing strategy differs. In this section we compare the two solution approaches to obtain further insights. To compare the two pricing solutions, we first derive the equilibrium spot demand d^s and weighted spot price $\tilde{p}^s$. Noticing that, in the static spot market, the spot demand d^s is endogenously determined by the game, same to the dynamic spot market, although APs are assumed to treat spot demand as an exogenous variable.

As partly discussed in Section 3.3.2, the equilibrium of spot demand d^s and weighted spot price $\tilde{p}^s$ is the solution to the system equation, (3.1) and (3.3), which characterizes how the weighted spot price interacts with the spot demand given a realized d and a contract. The spot demand as a function of the weighted spot price, as stated in (3.3), is identical to both static and dynamic market. As for the weighted spot price, as stated in (3.1), the expression extends to the following:

$$\tilde{p}^s(p_a^s, p_b^s, d^s) = \int_{\underline{p}}^{\bar{p}} \int_{\underline{p}}^{\bar{p}} \frac{p_a^s \cdot q_a^s(p_a^s, p_b^s) + p_b^s \cdot q_b^s(p_a^s, p_b^s)}{q_a^s(p_a^s, p_b^s) + q_b^s(p_a^s, p_b^s)} \cdot f_b(p_b^s) \cdot f_a(p_a^s) \cdot dp_b^s \cdot dp_a^s \quad (3.13)$$

$f_i(p_i^s)$, with $i \in \{a, b\}$, denotes the pricing strategy of AP_i in the form of probability and the quantity q_i^s, traded in the spot market, depends on the relationship between the two spot prices. The weighted spot price results from the double integration of the spot prices and quantities.

Figure 3.5 illustrates the interaction between the spot demand and the weighted spot price for static spot market. The weighted spot price $\tilde{p}^s$ as a function of d^s is indicated by the dotted line and the spot demand d^s as a function of $\tilde{p}^s$ is indicated by the solid line. The intersection of the dotted line with x-axis at k_a is the threshold of the transition from overcapacity equilibrium, where APs face sufficiently low market demand, to intermediary equilibrium in spot market. The intersection of the solid line with x-axis represents the cases where all end customers are willing to purchase since the spot price is zero and LSP_1 brings all unsatisfied demand $\beta d - q^c$ to spot market. The intersection of the two curves is the equilibrium of the weighted spot price and the spot demand given a realized d.

An increase in d potentially leads to an increase in d^s, which may raise the spot prices and ultimately decrease the spot demand due to the interdependence between the weighted spot price and the spot demand. Lemma 3.10 analyzes the relationship

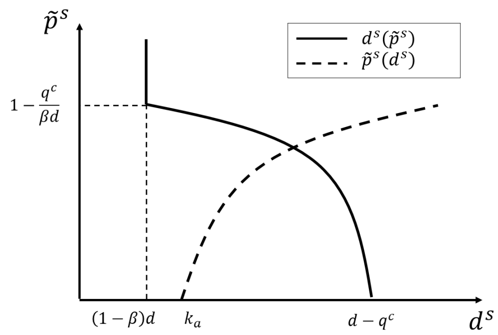


Figure 3.5 The interaction between spot market demand and weighted spot price for static spot market

between the realized end customer demand d and the equilibrium spot demand d^s for static spot market.

Lemma 3.10. *Assume $\beta = \frac{1}{2}$. The equilibrium d^s in the static spot market increases in d and it piece-wise linearly increases in d with the following relationship:*

$$d^s = \begin{cases} 0.5d & \text{if } d \leq 2q^c \\ d - q^c & \text{if } 2q^c < d \leq \max(k_0, 2q^c) \\ \frac{2k_0 - q^c}{2k_0} d & \text{if } d \geq \frac{3k_0 - q^c}{2k_0 - q^c} \cdot 2k_0 \end{cases}$$

Figure 3.6 demonstrates two instances of Lemma 3.10: when $q^c = 4$ and $q^c = 9$. The slope changes along with d as a joint effect of spot and contract. When d is sufficiently small, only LSP_2 brings demand to the spot market while LSP_1's demand is fulfilled by the contracted capacity, q^c. As d increases, there may exist a region where the spot market is overcapacitated, leading to marginal spot price, and both LSPs bring the entire demand minus q^c units to the spot market. For a sufficiently large d, the spot price becomes strictly positive and this results in less willingness to pay due to price sensitivity and thus, the slope is reduced.

Intuitively, more spot market demand brings asset providers to a better position where the equilibrium weighted spot price is higher. However, it is quite challenging to prove it in our setting due to the price competition in spot market. The two asset providers may adopt different mixed pricing strategies and thereby the weighted spot price in terms of quantities is derived from double integration, where d^s appears in the boundaries of the integration as well as the nominator and denominator of the core expression. Thereby, the solution of the equilibrium is not trivial.

Conjecture 3.1. *In the dynamic setting, the weighted spot market price $\tilde{p}^s$ increases in d^s.*

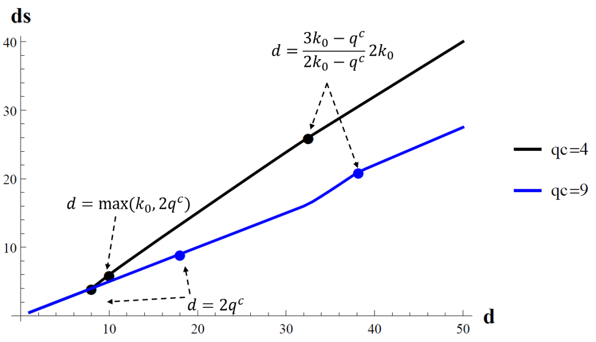
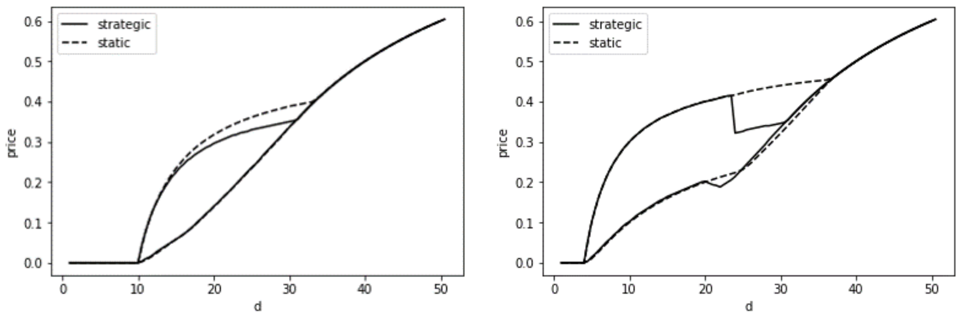


Figure 3.6 The relationship between the equilibrium d^s and d with static spot prices



(a) $q^c = 2$

(b) $[q^c = 8$

Figure 3.7 Comparison between static and dynamic spot prices

The next conjecture follows from Conjecture 3.1 and Lemma 3.10. It states that with a high end customer demand, the equilibrium weighted spot price is also high.

Conjecture 3.2. *In the dynamic setting, the weighted spot market price* $\tilde{p}^s$ *increases in* d*.*

We now compare the static spot pricing strategy with the dynamic pricing strategy. The prices are in a closed-form expressions for both strategies with either sufficiently low or sufficiently high demands. With intermediary demands, for the dynamic spot market, we numerically observe and identify the range of the mixed pricing strategy. For each realized d, we numerically compute the equilibrium spot demand d^s and the weighted spot price $\tilde{p}^s$. Figure 3.7 shows the equilibrium spot prices. The dashed line represents the static equilibrium prices and the solid one indicates the dynamic equilibrium prices.

We can observe that the thresholds of d, which distinguish the overcapacity equilibrium and the intermediary range, are numerically consistent under different pricing strategies. In the intermediary range, both solutions are mixed pricing strategies. The lower bounds of the price ranges are numerically matched whereas the upper bound of the price range of the dynamic solution is lower than the one of the static solution. This can be explained by the negative impacts of a high price on spot demand, especially when the contract quantity q^c is significant. Thereby, the APs who are farsighted by considering the inter-dependence between the price and demand in spot market may decrease the upper bound of the mixed pricing strategy. As we observed, the dynamic spot pricing strategy is quantitatively different but not qualitatively from the static spot pricing strategy.

3.5. Long-term contract analysis

We proceed with the long-term contracting stage. In Section 3.5.1, we analyze the Nash product and in Section 3.5.2 we conduct a numerical study to further illustrate the bargaining process.

3.5.1 Contract negotiation: Nash product

To identify the outcome of the bargaining process, we first express the agents' outside options. If bargaining fails, AP_a brings his entire capacity k_0 to the spot market whereas LSP_1 brings all the demand he faces, βd, to the spot market. Hence, $\mathbb{E}_d[O_a] = \mathbb{E}_d[\pi_a^{s*}|k_a = k_0]$ and $\mathbb{E}_d[O_1] = \mathbb{E}_d[\pi_1^{s*}|k_a = k_0]$. In this case, the spot market prices set by the APs do not affect the total spot market demand due to the lack of contract. We derive the APs' expected spot market profits in Proposition 3.2.

Proposition 3.2. *Assume that the spot market follows a static setting and d^s follows a distribution of f_{d^s}, the expected spot profit of AP_a is given by*

$$\mathbb{E}_{d^s}[\pi_a^{s*}] = \int_{k_a}^{k_a+2k_b-2\sqrt{k_a k_b}} \frac{1}{2}k_a \left(1 - \sqrt{\frac{k_a}{d^s}\left(2 - \frac{k_a}{d^s}\right)}\right) f_{d^s}(d^s)dd^s + \int_{k_a+2k_b-2\sqrt{k_a k_b}}^{k_a+2k_b} \frac{(d^s-k_a)^2}{4k_b d^s} k_a f_{d^s}(d^s)dd^s + \int_{k_a+2k_b}^{\infty} \frac{d^s-k_a-k_b}{d^s} k_a f_{d^s}(d^s)dd^s \tag{3.14}$$

and the expected spot profit of AP_b is given by

$$\mathbb{E}_{d^s}[\pi_b^{s*}] = \int_{k_a}^{k_a+2k_b} \frac{1}{4}d^s \left(1 - \frac{k_a}{d^s}\right)^2 f_{d^s}(d^s)dd^s + \int_{k_a+2k_b}^{\infty} \frac{d^s-k_a-k_b}{d^s} k_b f_{d^s}(d^s)dd^s$$

In the absence of a contract, f_{d^s} is identical to f_d. Otherwise, f_{d^s} depends on the realized end customer demand d as well as the weighted spot price.

Proposition 3.3. *AP_a's outside option is given by:*

$$\begin{aligned}\mathop{\mathbb{E}}_d[O_a] =& -\frac{2}{3}k_0 F_d(3k_0) - \frac{1}{4}\int_{k_0}^{3k_0} F_d(d)dd + k_0 + \frac{k_0^2}{4}\int_{k_0}^{3k_0}\frac{F_d(d)}{d^2}dd \\ & - 2k_0^2 \frac{F_d(d)}{d}\bigg|_{3k_0}^{\infty} - 2k_0^2\int_{3k_0}^{\infty}\frac{F_d(d)}{d^2}dd\end{aligned} \tag{3.15}$$

and LSP_1's outside option is given by

$$\begin{aligned}\mathop{\mathbb{E}}_d[O_1] =& \beta m\, \mathbb{E}[q_a^s + q_b^s] \\ =& \beta m\left[2k_0 + k_0 F_d(k_0) - 2k_0 F_d(3k_0) - \int_0^{k_0} F_d(d)dd + \int_{k_0}^{3k_0} w(d,k_0) f_d(d)dd\right]\end{aligned} \tag{3.16}$$

where

$$\begin{aligned}w(d,k_0) =& \int_{\underline{p}}^{\bar{p}} d(1-p) f_h(p)dp \\ =& \frac{2(d-2k_0)(2d^3 - 7d^2k_0 + 4dk_0^2 + 5k_0^3) + (d-3k_0)^2(d-k_0)^2 \ln -\frac{d-3k_0}{d-k_0}}{8(d-2k_0)^3}\end{aligned}$$

If AP_a and LSP_1 sign a contract, AP_a's profit consists of the profit from the contracted units, $\pi_a^c = q^c p^c$, plus the profit from units traded in the spot market, π_a^{s*}, $\Pi_a = \pi_a^{s*} + \pi_a^c$. Taking expectation over the end market demand we have

$$\mathop{\mathbb{E}}_d[\Pi_a] = \mathop{\mathbb{E}}_d[\pi_a^{s*}] + \pi_a^c \tag{3.17}$$

where $\mathbb{E}_d[\pi_a^{s*}]$ is given in (3.14).

LSP_1's profit stems from the total number of units sold to end customers. If end customer demand can be satisfied by the contracted quantity q^c, then LSP_1 does not trade in the spot market. Otherwise, he purchases q_1^s additional units in spot market to a total amount $q_1 = q_1^s + q_1^c$. The profit of LSP_1 is given by $\Pi_1 = \pi_1^{s*} + \pi_1^{c*} - q^c p^c$, where $\pi_1^{s*} = mq_1^s$ and $\pi_1^{c*} = mq_1^c$. Demand satisfied from the contracted units is $q_1^c = \min(q^c, d_1^c(1-\tilde{p}^s))$ and the quantity traded in the spot market by LSP_1 is $q_1^s = d_1^s(1-\tilde{p}^s)$. Taking expectation over the end market demand, LSP_1's profit is stated as

$$\mathop{\mathbb{E}}_{d}[\Pi_1] = \mathop{\mathbb{E}}_{d}[\pi_1^{s*}] + \mathop{\mathbb{E}}_{d}[\pi_1^{c*}] - q^c p^c \tag{3.18}$$

where $\mathbb{E}_d[\pi_1^{s*}] = m\,\mathbb{E}_d[q_1^s]$ and $\mathbb{E}[\pi_1^{c*}] = m\,\mathbb{E}_d[q_1^c]$.

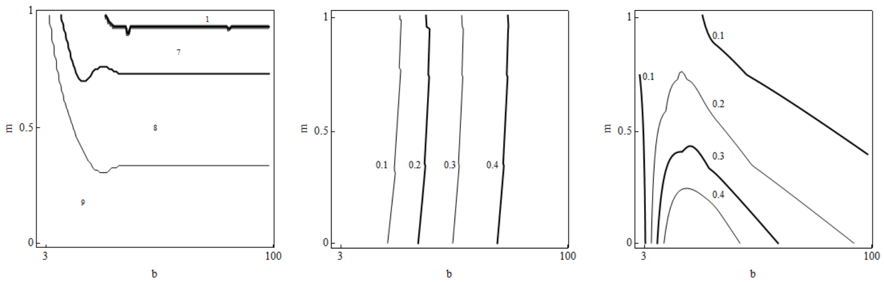
3.5.2 Numerical case study with uniformly distributed demand

To further understand the bargaining process, we conduct a numerical case study. We assume end customer demand to be uniformly distributed with $d \sim U[0, b]$ where b is the demand upper bound. We choose an initial capacity $k_0 = 10$ and equal bargaining powers between LSP_1 and AP_a, i.e., $\tau = 0.5$. We let the contract quantity q^c take integer values between 0 and 9, where $q^c = 0$ refers to the case without a contract and q^c maximally takes 9 instead of 10 to avoid a monopoly spot market. By solving the Nash bargaining product specified in (3.5), we find the optimal p^{c*} and q^{c*}.

The outside options of AP_a and LSP_1 are derived from (3.15) and (3.16), respectively. Since the outside option describes the case without a contract, the spot demand is identical to end customer demand and, hence, we replace $F_{d^s}(d^s)$ by $F_d(d^s)$. The expected profits of AP_a and LSP_1 after signing a contract are derived from (3.17) and (3.18), respectively. To compute these profits, we approximate the equilibrium of d^s and p^s from (3.3) and (3.13) given a realized d. We proceed with computing p^{c*} and q^{c*} which maximize the Nash product, which is defined in (3.5). Given q^c, the unique value of $p^{c*}(q^c)$ is numerically obtained.

Figure 3.8 illustrates the equilibrium contract quantity, equilibrium contract price and Nash bargaining profit for different values of b and m. Figure 3.8a shows the regions where the numbers indicate the q^{c*} for each area. The figure highlights that the optimal contract quantity decreases in the demand upper bound and the LSP's margin. It implies that the two agents should cooperate when they are confronted with the difficult situation of low demand and low margins for the LSP. There exists a region where only a small amount of units, 1 unit in the numerical example, shall be signed if the LSPs' margin and the demand potential are high. Another observation is that a large amount of units should be contracted in most cases.

The optimal contract price is shown in Figure 3.8b. It decreases in LSP margin and increases in the upper bound of demand, although it is more sensitive to the latter parameter. A higher contract price indicates a larger profit transfer from the LSP to the AP, implying the LSP benefits more in the contract. It is quite clear to see from the figure that the contract price depends significantly on b. As b increases, the AP will benefit from a favorable spot market where he will ultimately be able to raise monopolist prices. Another observation in this numerical analyses is that the contract prices are always lower than the expected spot price. This observation is aligned with



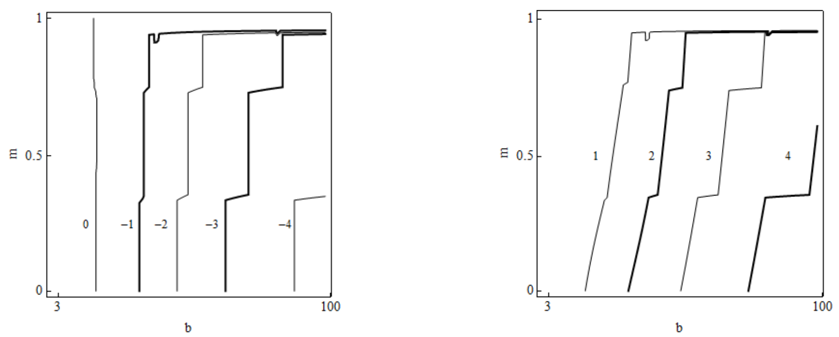
(a) Equilibrium contract quantity (b) Equilibrium contract price (c) Equilibrium contract profits

Figure 3.8 Equilibrium contract quantity, price and profits as a function of end customer demand d and LSP's margin m, given $k_0 = 10$ and $\tau = 0.5$

practice that long-term contracts are negotiated to trade units at a fixed and lower price comparing to the spot market. In our model, it brings an additional insight: The LSP will ultimately sell the transported units to the end customer at the spot price plus margin. But given that spot price is consistently above contract price, the difference between the two also results in a payoff for the LSP.

Figure 3.8c shows the extra profits achieved by the contract—the extra profits gained through a contract comparing to the case without a contract. It decreases in LSP's margin while firstly increases and then decreases in the upper bound of demand. This indicates that a contract is more beneficial when LSPs have a small margin and when demand upper bound is intermediary. The contract is most valuable, i.e., yields the most benefit to the two agents, when demand potential is moderate, roughly ranging between half to twice the combined capacity of the competing APs, while the LSP's margins are fairly low. It reflects the critical role of contracts as a hedging tool which brings more values with fierce competition either from the aspect of LSPs, namely low margin, or from the aspect of asset providers, namely intermediary demand which may lead to mixed pricing strategy.

To further understand who benefits from a contract, we plot the breakdown of extra profits for AP_a and LSP_1. Figure 3.9 shows whether the benefit of a contract comes from the AP or the LSP. Specifically, we present the extra profits before the internal transfer payment, $q^{c*} * p^{c*}$, between the two agents. We observe that when b is less than 19 units, which is about the total capacity, the benefits of a contract comes from both the AP and the LSP, who have positive values before the transfer payment. Otherwise, the benefits of the contract come from the LSP. It implies that when demand is less than the total capacity, both the AP and the LSP have an incentive to negotiate a contract to reduce the competition level in spot market. When demand is sufficiently high, the LSP is interested in a contract. This can be explained by



(a) Expected extra profits achieved of AP_a before transfer payment, $E_d[\Delta\Pi_a|q^{c*}] - q^{c*} * p^{c*}$

(b) Expected extra profits achieved from LSP_1 before transfer payment, $E_d[\Delta\Pi_1|q^{c*}] + q^{c*} * p^{c*}$

Figure 3.9 Expected extra profits obtained by AP_a and LSP_1 without p^c transferring profits, given $k_0 = 10$ and $\tau = 0.5$

the source of the LSP's profits: the trading quantity and the difference between the contract price and the spot price. When demand is high, a contract secures capacity for the LSP to increase the trading units above what she woudl secure in the spot market only. In addition, by removing capacity from the spot market, the spot price may rise which contributes to increasing the difference between the spot price and contract price.

In conclusion, Table 3.3 summarizes the optimal contracting strategy with different market parameters. First, with low m, the two agents should contract for a large amount of units in advance regardless of b. When b is small, both agents are willing to sign a contract at a low price. The AP can obtain extra profits from early sales as well from the spot market due to higher spot prices. The LSP benefits from the difference in spot price and contract price as the contracting inflates the spot price. The low contract price does not cost the LSP too much so that she still benefits from a higher spot price. When b is large, the agents will sign a contract at a high price. The AP foresees a good spot market with a large b and, hence, is only willing to sell capacity in advance at a high price. The LSP benefits from a contract by a high spot price and the secured capacity and therefore is willing to sign a contract.

Second, the contract is less favored with a high margin m. When b is small, the asset provider does benefit from a contract due to a higher spot price. The LSP may benefit from the difference between the high spot price and low contract price. As b increases, only a small amount of unit is signed. The AP faces a good spot market and, hence, is less willing to sign a contract in advance unless the contract price is sufficiently high.

	Small b	Large b
Small m	large q^{c*}, small p^{c*}	large q^{c*}, large p^{c*}
Large m	large q^{c*}, small p^{c*}	small q^{c*}, large p^{c*}

Table 3.3 Summary of optimal contracting strategy

3.6. Conclusions

We explore the interaction of asset providers (APs) and logistics service providers (LSPs) who have the opportunity to trade transport capacity at two occasions: in a long-term contracting market (sometimes referred to as allotment) and in a spot market. We aim to give guidance on the circumstances when it is beneficial to contract for transport capacity on the long-term horizon, and identify equilibrium prices and quantities to do so. APs decide on the capacity to bring to the market significantly before the selling horizon. LSPs need to purchase transport capacity for their uncertain end customer demand. The long-term market brings the benefit for the AP to secure sales, and for the LSP to secure capacity. Implicitly, the two of them jointly remove a fraction of demand and supply from the market. Once demand is realized, rather briefly before the transport takes place, both can again trade in a spot market. In the capacity-constrained spot market, multiple APs compete in price. In this paper, we study the strategies for APs and LSPs to best exploit the two sales options.

We formulate a stylized two-stage game theoretical model and analyze it. The first stage describes the option of trading capacities in a long-term contract and the second stage models how remaining capacity and demand are traded in the spot market. One, we model the price competition in the spot market explicitly. Two, we assume the realistic negotiation perspective when it comes to contract trade (with a Nash bargaining model). Three, we consider the interplay of the two trading situations – contract and spot market – to study optimal decisions of asset provider and LSPs.

Our model lets us establish the following findings. We characterize the equilibrium spot prices for both static and dynamic spot markets. Most notably, we fully present that APs should adopt a mixed pricing strategy for for intermediary demand realizations, we identify the range of such demand realizations and given the price distribution. Further, we use the findings on the spot market to explore optimal contract negotiation outcomes. We establish that the contract is most valuable when demand potential is moderate, roughly ranging between half to twice the combined capacity of the competing APs, while the LSP's margins are fairly low. More specifically, the optimal contract quantity decreases with demand potential and LSP's margin whereas the optimal contract price increases with demand and not sensitive to LSP's margin. We further identify that the AP benefits from a contract in case of low demand potential, partially through the implicit effect of reducing capacity from the spot market. It is only when the demand potential and the LSP margin are both high that the two agents would contract on a small amount of units. In this case, the AP may be able to gain from very high spot market prices and faces low risk of overcapacity. At the same time, the LSP does not depend significantly on the additional payoff from the difference in contract and market price. At last, we also establish numerically that the contract price is inferior to the expected spot price.

Our study introduces and analyzes freight markets from realistic yet thus far underrepresented assumptions as to competition among the agents. Naturally, it carries limitations whose future exploration will allow to gain additional insights. We have thoughtfully borrowed the end-customers' price demand function from other contexts, and it is worthwhile exploring whether other price-demand functions may represent the end-customers' behavior better. The analytical tractability forced us to derive concrete results for the case of a uniform demand distribution and exploring additional demand distributions shall firm our insights. We model the spot market as a pure price competition as the available capacity is a commodity. An opposing viewpoint more common in marketing literature and passenger transportation is that of partial customer loyalty. As for the contract bargaining, we numerically studied the case of identical negotiation power for asset provider and LSP whereas alternatives may be discussed in follow-up work. Our research may be extended beyond the duopoly setting of asset providers. In addition, it is relevant to address how *soft block* contracts may extend the range of beneficial arrangements by providing the LSP with the opportunity to return excess capacity from the contract trade to the spot market at a price premium.

This paper provides three major contributions. First, as a methodological contribution, we model price-competition between asset providers and discuss contracting and pricing strategies under uncertain demand. As pointed out in Section 3.2, existing literature on contracting and allotment management is mainly discussed in a monopoly setting. Hereby, this paper provides insights of the trade-off between contracts and spot market for asset providers facing competition. Second, as a analytical contribution, we identify the equilibrium pricing strategies in spot market with asymmetric capacities. That is, our model of spot market price competition clearly shows that asset providers are best advised to apply a mixed pricing strategy for a well defined range of the solution space. Given an exogenous spot demand, the distribution of larger asset provider's mixed pricing stochastically dominates the one of smaller asset provider. These finding are new to the domain and have not been identified by the extant literature. Third, as a managerial contribution, we highlight how APs and LSPs should make use of contract and spot selling opportunities to maximize their profits. Through a numerical study, we show that no contract shall be signed under certain market structure and the contract price can even be negative in some range of parameters.

3.A. Proof

Proof. Proof of Proposition 3.1 and Lemmata 3.1-3.3. To prove Lemma 3.1 and Proposition 3.1, we develop Lemmata 3.11-3.16 to characterize the equilibrium of the price competition. We start with analyzing the immediate profit as a function of prices given different demand realization.

Lemma 3.11. *Given $k_a \leq k_b$, the profits can be characterized by 7 possible outcomes with the following thresholds $d_{s_1} = k_a$, $d_{s_2} = k_b$, $d_{s_3} = k_a + k_b$, $d_{s_4} = 2k_a + k_b$, $d_{s_5} = k_a + 2k_b$ and $d_{s_6} = 2k_a + 2k_b$, separately.*

Proof. Proof of Lemma 3.11 The profit function Eq. (3.2) can be rewritten in the following way:

$$\pi_i^s(p_i^s, p_j^s) = \begin{cases} L_i(p_i^s) \equiv & \begin{cases} p_i^s k_i & \text{if } p_i^s \leq p_{\{k_i\}} \\ p_i^s d^s(1-p_i^s) & \text{if } p_i^s > p_{\{k_i\}} \end{cases} \\ E_i(p_i^s) \equiv & \begin{cases} p_i^s k_i & \text{if } p_i^s \leq p_{\{k_i+k_j\}} \\ p_i^s \frac{k_i}{k_i+k_j}(1-p_i^s) & \text{if } p_i^s > p_{\{k_i+k_j\}} \end{cases} \\ H_i(p_i^s) \equiv & \begin{cases} p_i^s k_i & \text{if } p_i^s \leq p_{\{k_i+k_j\}} \\ p_i^s(d^s(1-p_i^s)-k_j) & \text{if } p_{\{k_i+k_j\}} < p_i^s \leq p_{\{k_j\}} \\ 0 & \text{if } p_i^s > p_{\{k_j\}} \end{cases} \end{cases} \tag{3.19}$$

where $i \in \{a, b\}$. From the conditions in Eq. (3.19), $p_{\{k_a\}}$, $p_{\{k_b\}}$ and $p_{\{k_a+k_b\}}$ are critical thresholds of the profit functions. However, noticing that $p_{\{k_i\}} = 1 - \frac{k_i}{d^s}$ and k_i is given, d^s indeed determines the shape of the profit functions and $p_{\{k_a+k_b\}} < p_{\{k_b\}} < p_{\{k_a\}}$.

When d^s is sufficiently small, $p_{\{k_a\}} = \max\left(0, 1 - \frac{k_a}{d^s}\right) = 0$ and $H_b(p) = 0 \; \forall p$. As d^s increases, $p_{\{k_a\}}$ firstly becomes positive, implying the existence of a price p making H_b positive. Therefore, the first transition happens with a threshold $d_{s_1} = k_a$, where $p_{\{k_a\}} = 1 - \frac{k_a}{d^s} = 0$. In this case, both $L_a(p)$ and $H_b(p)$ are piece-wise functions, with the first domain bounded by AP_a's capacity.

When d^s further increases, at some point, $p_{\{k_b\}}$ becomes positive. Similarly, $H_a(p)$ and $L_b(p)$ turn to be piece-wise functions. The threshold is then identified to be $d_2 = k_b$.

d^s continuous to increase and $p_{\{k_a+k_b\}}$ becomes positive. A linear domain appears in $L_i(p)$, $E_i(p)$ and $H_i(p)$. Thus, the threshold is $d_{s_3} = k_a + k_b$. Meanwhile, we observe that the first domain of L_i and H_i are partly overlapping until p_i^s is greater than or equal to $p_{k_a+k_b}$. Afterwards, L_i continues linearly and H_i continues quadratically. The optimal H_a is obtained in second domain and the optimal price in the unconstrained second domain is $p_{opt} = \frac{1}{2}p_{\{k_b\}}$. However, along with the increase of d^s, the outcome is transited to the case where the optimal $H_a(p)$ is achieved at the intersection point of the two domains, and the optimal price now becomes $p_{\{k_a+k_b\}}$. Therefore, $p_{\{k_a+k_b\}} \geq p_{opt}$ guarantees the optimal point of $H_a(p)$ is at the intersection. Thus, $d_4 = 2k_a + k_b$ where $p_{\{k_a+k_b\}} = p_{opt}$.

When $d^s = d_4$, $H_b(p)$ is still optimized in the second domain. Thus, the next threshold is when $H_b(p)$ gets optimized at the intersection point of the two domains. The intersection locates at $p_{\{k_a+k_b\}}$ and thus $p_{\{k_a+k_b\}} \geq \frac{1}{2}p_{\{k_a\}}$ guarantees the optimal $H_b(p)$ is at the intersection. Thus, $d_{s_5} = k_a + 2k_b$.

The transition between the last two outcomes occurs when $E_i(p)$ optimizes at the intersection point as well. Similarly, $d_{s_6} = 2k_a + 2k_b$.

□

Lemma 3.12. *If $d^s < d_{s_1}$, $p_i^s = 0$, $q_{s_i} = \frac{k_i}{k_i+k_j}d^s$, $i \in \{a, b\}$, with a corresponding profit $\pi_i^s = 0$.*

Proof. Proof of Lemma 3.12 When $d^s < d_{s_1}$, the available capacity, $k_a + k_b$, is sufficiently large such that both APs intensively compete over the demand. Specifically, the profit is given by:

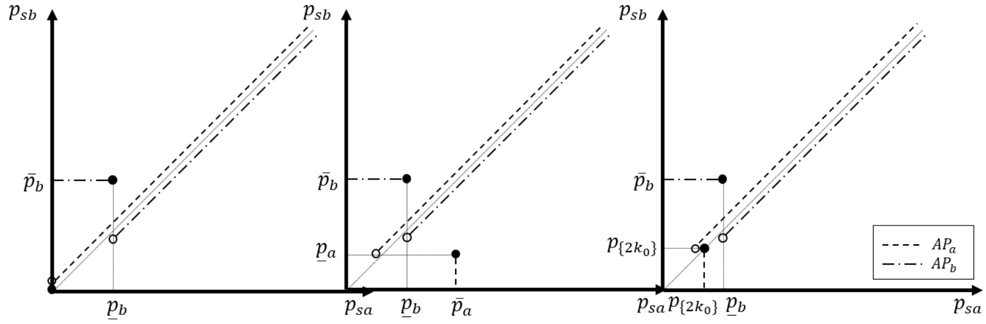
$$\pi_i^s(p_i^s, p_j^s) = \begin{cases} L_i(p_i^s) \equiv & p_i^s D(p_i^s) & \text{if } p_i^s < p_j^s \text{ and } d^s < d_{s_1} \\ E_i(p_i^s) \equiv & p_i^s \frac{k_i}{k_i+k_j} D(p_i^s) & \text{if } p_i^s = p_j^s \text{ and } d^s < d_{s_1} \\ H_i(p_i^s) \equiv & 0 & \text{if } p_i^s > p_j^s \text{ and } d^s < d_{s_1} \end{cases} \tag{3.20}$$

implying that for each price set by the competition, the best response is to under price. Namely, $p_i^s(p_j^s) = p_j^s - \epsilon$ for $p_j^s > 0$, with $\epsilon \to 0$. This process converges to $p_i^s = p_j^s = 0$, with d^s units being traded by APs and each of them selling $\frac{k_i}{k_i+k_j}d^s$.

□

Let $\bar{p_i}$ and $\underline{p_i}$, $i \in \{a, b\}$, denote the supremum and infimum, respectively, of the mixed pricing range of carrier i.

Lemma 3.13. *If $d_{s_1} < d^s < d_{s_5}$, there is no pure pricing strategy; however, there exists a mixed pricing strategy, with the following properties:*



(a) $d_{s_1} < d^s < d_{s_2}$ (b) $d_{s_2} < d^s < d_{s_4}$ (c) $d_{s_4} < d^s < d_{s_5}$

Figure 3.10 Response functions of two airlines when $d_{s_1} < d^s < d_{s_5}$

(a) *The two carriers share the same supremum and infemum with* $\bar{p} \equiv \bar{p}_b$ *and* $\underline{p} \equiv \underline{p_b}$

(b) *The cdf is continuous and strictly increasing except that the larger airline has a mass point at* $\bar{p}$.

(c) $G_a(F_a^*, F_b^*) = L_a(\underline{p})$ *and* $G_b(F_a^*, F_b^*) = H_b(\bar{p})$

Proof. Proof of Lemma 3.13

We first prove that no pure strategy in prices exists. By looking at the response functions of both airlines in Figure 3.10, no point of intersection is found, implying no pure strategy.

Next, we proceed with the mixed pricing strategy equilibrium. Denote $F_i(p)$ to be the cdf of the mixed pricing equilibrium with $i \in \{a, b\}$. Thus,

$$\pi_i(p) = H_i(p)\left(F_j(p) - \alpha_j(p)\right) + E_i(p)\alpha_j(p) + L_i(p)\left(1 - F_j(p)\right)$$

In the equilibrium, F_i makes the competitor's pricing strategy indifferent in terms of profit and we use G_i to represent the equilibrium profit, implying $G_i = G_i(F_i^*, F_j^*) = \pi_i(p)$ for any p in the support. By rearranging the equation,

$$F_j(p) = \frac{L_i(p) - G_i}{L_i(p) - H_i(p)} \tag{3.21}$$

The characteristics of the mixed strategy equilibrium are discussed as following.

(a) Let $\underline{p}_i$ and $\bar{p}_i$ to be the infimum and supremum of the cdf support of mixed strategy equilibrium, with $i \in \{a, b\}$. Suppose $\underline{p_i} < \underline{p_j}$.

The total expected profit of AP_i is given by $\mathbb{E}[\pi_i] = \int_{\underline{p_i}}^{\bar{p_i}} \pi_i(p) f_i(p) dp$, where $\pi_i(p_i)$ is the expected value of $\pi_i(p_i, p_{s_j})$ in Eq. (3.19) w.r.t p_{s_j}, given that the competitor adopts equilibrium strategy. For any price $p \in (\underline{p}_i, \underline{p}_j)$, AP_i is always the lowest price offer, implying $\pi_i(p) = L_i(p)$.

However, $L_i(p)$ does not generate equal number. If $f_i(p)$ is continuous and positive in any subset of $\left(\underline{p_i}, \underline{p_j}\right)$ in the range, it contradicts to the fact that equilibrium profit at any price is identical. If $f_i(p)$ is discrete, there must exist an optimal price maximizing $L_i(p)$ when $p \in \left(\underline{p_i}, \underline{p_j}\right)$ due to the concavity. Since $\underline{p_i}$ must be in the support of AP_i, $\underline{p_i}$ is the maximizing price, otherwise the profit will be improved by shifting the probability of offering $\underline{p_i}$ to the optimal price. Therefore, $L_i(p)$ is strictly decreasing in the range. At $\underline{p_i}$, the profit of AP_i equals to $L_i(\underline{p_i})$; at $\underline{p_j}$, the profit of AP_j equals to $L_i(\underline{p_j})(1 - F_j(\underline{p_j})) + E_i(\underline{p_j})\alpha_j(\underline{p_j})$. Knowing that $L_i(\underline{p_i}) > L_i(\underline{p_j}) > E_i(\underline{p_j})$, it is impossible to have identical equilibrium profit at $\underline{p_i}$ and $\underline{p_j}$, contradicting to the definition. Thus, $\underline{p_i} = \underline{p_j} = \underline{p}$.

Before proving another side, it is easy to know that the equilibrium profit of either airline is non-zero. Otherwise, they will deviate to a price at which $H(p) > 0$ and a positive payoff is guaranteed. Similar to the lower bound, we assume $\bar{p_i} < \bar{p_j}$. For any price $p \in (\bar{p_i}, \bar{p_j})$, the profit of AP_j equals $\pi_j(p) = H_j(p)$ since it is always the higher price offer. Thus, if $f_j(p)$ is continuous in the range, the profits at various prices are not identical, contradicting to the definition of equilibrium. If $f_j(p)$ is discrete in the range, there must exist an optimal price maximizing $H_j(p)$ due to the concavity and it must be $\bar{p_j}$. Thus, $H_j(p)$ is strictly increasing, implying $\bar{p_j} < \frac{1}{2}$. Considering the profit of AP_i at $\bar{p_i}$, $\pi_i(\bar{p_i}) = H_i(\bar{p_i})(F_j(\bar{p_i}) - \alpha_j(\bar{p_i})) + E_i(\bar{p_i})\alpha_j(\bar{p_i}) + L_i(\bar{p_i})(1 - F_j(\bar{p_i}))$. Since $H_i(p)$, $E_i(p)$, $L_i(p)$ are increasing if $p < \frac{1}{2}$, AP_i will improve its profits by offering a price higher than $\bar{p_i}$, contradicting to the assumption. Thus, $\bar{p_i} = \bar{p_j} = \bar{p}$.

(b) We firstly prove that there is no such price where both airlines have a mass point. Suppose at price p, with $p \in (\underline{p}, \bar{p})$, both airlines have a mass point, implying $\Pr(p_i^s = p) > 0$ for $i \in \{a, b\}$. Given the fact that there are countable mass points, there must exist a price $p - \epsilon$ where firm A does not have a mass point, namely $\Pr(p_{s_a} = p - \epsilon) = 0$ with $\epsilon \to 0$. By relocating the mass point at

p to $p-\epsilon$, AP_a's expected profit is changed by:

$$\begin{aligned}
&\Pr(p_a = p)\Pr(p_b > p)\,(L_a(p-\epsilon) - L_a(p)) + \\
&\Pr(p_a = p)\Pr(p_b = p)\,(L_a(p-\epsilon) - E_a(p)) + \\
&\Pr(p_a = p)\Pr(p-\epsilon < p_b < p)\,(L_a(p-\epsilon) - H_a(p)) + \\
&\Pr(p_a = p)\Pr(p_b = p-\epsilon)\,(E_a(p-\epsilon) - H_a(p)) + \\
&\Pr(p_a = p)\Pr(p_b < p-\epsilon)\,(H_a(p-\epsilon) - H_a(p))
\end{aligned}$$

When $\epsilon \to 0$, all items in the above equation approach to 0 except the second item, which is positive. It implies that the deviation increases the payoff which contradicts to the definition of equilibrium.

Then, we denote $F_a^*(p)$ and $F_b^*(p)$ to be the cumulative distribution function in the mixed price equilibrium. Similar to the symmetric case, the profit of an airline is given by

$$G_i(p, F_j^*) = L_i(p)(1 - F_j(p)) + E_i(p)\alpha_j(p) + H_i(p)(F_j(p) - \alpha_j(p)) \quad (3.22)$$

where $\alpha_i(p)$ represents the mass point at price p with $i \in \{a, b\}$. For any $p \in (\underline{p}, \bar{p})$ and $f_i(p) > 0$, the equilibrium profit of AP_i, denoted as $G_i(p, F_j^*)$, should be identical and optimal.

Now we investigate the mass point at $\bar{p}$ as well as the equilibrium profits, which is either neither firms has a mass point or only one firm has a mass point. The first case is not an equilibrium. Suppose neither airline has a mass point at price $\bar{p}$ and the equilibrium profit of AP_i equals $G_i(F_i^*, F_j^*) = H_i(\bar{p})$ at the upper bound price. Thus, $G_i(F_i^*, F_j^*) = H_i^*(\bar{p})$ as the equilibrium profit must be maximized. However, $p_i^{H*} \neq p_j^{H*}$ with p_i^{H*} (p_j^{H*}) maximizing $H_i(p)$ ($H_j(p)$), which contradicts to the identical price range of equilibrium.

Therefore, price $\bar{p}$ must be offered with a mass probability by one AP. AP_b can be proved to have a mass point at price $\bar{p}$. Suppose AP_a has a mass point at $\bar{p}$ and thus $F_a(\bar{p}-\epsilon)$ does not approach to 1 with $\epsilon \to 0$, implying $G_b(\bar{p}, F_a^*) > H_b(\bar{p})$ and $G_a(\bar{p}, F_b^*) = H_a(\bar{p})$ from Eq. (3.22). It can be proved that there is no mass point at $\underline{p}$ since the profit will be improved by deviating to $\underline{p} - \epsilon$ with $\epsilon \to 0$ and thus the equilibrium profit at $\underline{p}$ denoted by $G_i(\underline{p}, F_j^*)$ equals $L_i(\underline{p})$. Then, $L_a(\underline{p}) = H_a(\bar{p})$ and $L_b(\underline{p}) > H_b(\bar{p})$. The former equality leads to $k_a\underline{p} = \bar{p}(d^s(1-\bar{p}) - k_b)$. Knowing that $L_b(\underline{p}) = k_b\underline{p}$ and $H_b(\bar{p}) = \bar{p}(d^s(1-\bar{p}) - k_a)$, $H_b(\bar{p}) - L_b(\underline{p}) = (k_b - k_a)(\bar{p} - \underline{p}) > 0$ given $k_a < k_b$ contradicting to the assumption. Thus, AP_b is the agent to have mass point at $\bar{p}$ and AP_a does not offer $\bar{p}$. In this case, AP_b's equilibrium profit is H_b^* and AP_a's equilibrium profit is $L_a(\underline{p})$. The price bounds are $\underline{p}$ and $\bar{p}$ with $L_b(\underline{p}) = H_b(\bar{p}) = H_b^*$.

Now we proceed with the proof of non-existence of mass point in F_i^*. We investigate in a price range $[p_1, p_2]$ with $\underline{p} \leq p_1$ and $\bar{p} \geq p_2$, where any price in the domain is charged with positive pdf. Assume AP_i has a mass point at price p with $p \in [p_1, p_2]$. The profit at p of AP_j is stated as Eq. (3.22). The difference between the profits at p and $p - \epsilon$ is stated as bellow:

$$\begin{aligned}\pi_j(p-\epsilon) - \pi_j(p) = {} & H_j(p-\epsilon)(F_i(p-\epsilon) - \alpha_i(p-\epsilon)) + E_j(p-\epsilon)\alpha_i(p-\epsilon) \\ & + L_j(p-\epsilon)(1 - F_i(p-\epsilon)) - H_j(p)(F_i(p) - \alpha_i(p)) - E_j(p)\alpha_i(p) \\ & - L_j(p)(1 - F_i(p))\end{aligned}$$

Since any price within the range is charged with positive pdf, $F_i(p)$ is strictly increasing. Also, $F_i(p)$ is continuous since ϵ is sufficiently small so that there is no mass point between $p-\epsilon$ and p. Thus, $F_i(p) = F_i(p-\epsilon) + \int_{p-\epsilon}^{p} f_i(p)dp + \alpha_i(p)$ and the above equation can be rewritten as

$$\begin{aligned}\pi_j(p-\epsilon) - \pi_j(p) = {} & H_j\,(p-\epsilon)\left(F_i(p) - \alpha_i(p) - \int_{p-\epsilon}^{p} f_i(p)dp\right) + E_j(p-\epsilon)\alpha_i(p-\epsilon) \\ & + L_j(p-\epsilon)\left(1 - F_i(p) + \alpha_i(p) + \int_{p-\epsilon}^{p} f_i(p)dp\right) - H_j(p)(F_i(p) - \alpha_i(p)) \\ & - E_j(p)\alpha_i(p) - L_j(p)(1 - F_i(p)) \\ & = (H_j(p-\epsilon) - H_j(p))\,(F_i(p) - \alpha_i(p)) \\ & + (L_j(p-\epsilon) - H_j(p-\epsilon)) \int_{p-\epsilon}^{p} f_i(p)dp \\ & + (L_j(p-\epsilon) - L_j(p))(1 - F_i(p)) \\ & + (L_j(p-\epsilon) - E_j(p))\alpha_i(p)\end{aligned} \tag{3.23}$$

When ϵ approaches to 0, all items approach to 0 except for the last item which is non-zero, contradicting to the fact that profit at any price p within the domain should be identical. This implies that AP_i does not have a mass point when $p \in [p_1, p_2]$.

From Eq. (3.22), the cdf of AP_i with $p \in [p_1, p_2]$ is given by

$$F_i(p) = \frac{L_j(p) - G_j(F_i^*, F_j^*)}{L_j(p) - H_j(p)} \tag{3.24}$$

where $G_j(F_i^*, F_j^*)$ is the equilibrium profit of AP_i. $F_i(p)$ can be proved strictly increasing for $p \in (\underline{p}, \bar{p})$.

Taking the first order condition of $F_i(p)$ with respect to p, we have

$$f_i(p) = \frac{L_j^{'}(p) \cdot \left(G_j(F_i^*, F_j^*) - H_j(p)\right) + H_j^{'}(p)\left(L_j(p) - G_j(F_i^*, F_j^*)\right)}{\left(L_j(p) - H_j(p)\right)^2}$$

where the denominator is always positive. We know that $L_j^{'}(p) > 0$ and $H_b^{'}(p) \geq 0$ for $p \leq \bar{p}$. Since $G_j(F_i^*, F_j^*) \geq H_j(p)$ and $G_j(F_i^*, F_j^*) \leq L_j(p)$ with $i \in \{a, b\}$ for any $p \in (\underline{p}, \bar{p})$, the nominator of $f_a(p)$ is positive, implying $f_a(p)$ is positive and $F_a(p)$ is strictly increasing.

For firm b, $H_a^{'}(p) = 0$ for price $p > p_{\{k_b\}}$ implying the nominator is positive. For price $p \leq p_{\{k_b\}}$, $H_a^{'}(p)$ can be both negative and non-negative. If $H_a^{'}(p) < 0$ and $f_b(p) \leq 0$, $F_b(p)$ will decrease. However, since cdf should be non-decreasing, firm b cannot offer this price, implying $f_b(p) = 0$. Suppose there exist a price range (p_1, p_2) and $f_b(p) \equiv 0$ for any price p with $p_1 \geq \underline{p}$ and $p_2 \leq p_{\{k_b\}}$. Since $F_b(p_1) = F_b(p_2)$, the profit of firm a at p_1 and p_2 are $H_a(p_1)F_b(p_1) + L_a(p_1)(1 - F_b(p_1))$ and $H_a(p_2)F_b(p_1) + L_a(p_2)(1 - F_b(p_1))$ correspondingly with $F_b(p_1) = F_b(p_2)$. The profit function of firm a at $p \in (p_1, p_2)$ is $H_a(p)F_b(p_1) + L_a(p)(1 - F_b(p_1))$. Since $H_a(p)$ is concave, $2H_a(p) > H_a(p_1) + H_a(p_2)$. Noticing $L_a(p)$ is also concave, the profit at p is higher than the equilibrium profit, implying a deviation from equilibrium. Therefore, there does not exist a price range where any price is charged with $f_b(p) = 0$. Thus, $F_b(p)$ is also strictly increasing.

Then, it can be proved that there is no gap (p_3, p_4) where $f_i(p) \equiv 0$. If not, $F_i(p_3) = F_i(p_4)$, contradicting to the fact that $F_i(p)$ is strictly increasing with $p \in (\underline{p}, \bar{p})$.

Noticing that by taking the derivatives of $F_a(p)$ at price $\bar{p}$, we get $f_a(\bar{p}) = 0$, AP_a charges price $\bar{p}$ with $f_a(\bar{p}) = 0$. Therefore, assigning a mass point to F_b at price $\bar{p}$ does not affect the expected equilibrium profit. The mass probability at price $\bar{p}$ is $\alpha_b(\bar{p}) = 1 - F_b(\bar{p})$.

□

Lemma 3.14. *When $d_{s_1} < d^s < d_{s_5}$,*

$$(\underline{p}, \bar{p}) = \begin{cases} \left(\frac{1}{2}\left(1 - \sqrt{\frac{k_a}{d^s}\left(2 - \frac{k_a}{d^s}\right)}\right), \frac{d^s - k_a}{2d^s}\right) & \text{if } d^s < k_a + 2k_b - 2\sqrt{k_a k_b} \\ \left(\frac{(d^s - k_a)^2}{4k_b d^s}, \frac{d^s - k_a}{2d^s}\right) & \text{otherwise} \end{cases} \quad (3.25)$$

Proof. Proof of Lemma 3.14 As proved in Lemma 3.13, the price $\bar{p}$ depends on the price that maximizes $H_b(p)$. Referring to Eq. (3.19), $\bar{p} = \frac{d^s - k_a}{2d^s}$ since $d_{s_1} < d^s < d_{s_5}$. Based on the definition of equilibrium, the profit of AP_b at price $\underline{p}$ equals to the equilibrium profit, implying $L_b(\underline{p}) = H_b(\bar{p})$ due to the continuous cumulative function of AP_a. To compute the price $\underline{p}$, we discuss whether it falls in the first or the second domain of $L_b(p)$. Equivalently, if $L_b(p_{k_b}) < L_b(\underline{p}) = H_b(\bar{p})$, $\underline{p}$ falls in the linear part and otherwise in the quadratic part.

$$\begin{aligned} L_b(p_{k_b}) &< H_b(\bar{p}) \\ k_b p_{k_b} &< \frac{1}{2}(1 - \frac{k_a}{d^s})(d(1 - \frac{1}{2}(1 - \frac{k_a}{d^s})) - k_a) \end{aligned}$$

After rearranging,

$$d^{s2} - (2k_a + 4k_b)d^s + k_a^2 + 4k_b^2 > 0$$

Solve the quadratic equation and we have $d^s < k_a + 2k_b - 2\sqrt{k_a k_b}$ or $d^s > k_a + 2k_b + 2\sqrt{k_a k_b}$. Since $d^s < d_{s_5}$, the inequality holds only when the first condition is met. In that case, by solving $\underline{p}k_b = H_b^*$ and $\underline{p}d(1 - \underline{p}) = H_b^*$, where $H_b^* = \frac{1}{4}d(1 - \frac{k_a}{d^s})^2$, the price $\underline{p}$ is obtained. When $k_b < 4k_a$, the transition of $\underline{p}$ happens if $d_{s_2} < d^s < d_{s_3}$. Otherwise, the transition happens if $d_{s_3} < d^s < d_{s_4}$. □

Lemma 3.15. *When $d_{s_1} < d^s < d_{s_5}$, the cdf is given by:*

$$(F_a(p), F_b(p)) = \begin{cases} \left(\frac{k_b p - \pi_b}{k_b p - p(d^s(1-p) - k_a)}, \frac{k_a p - \pi_a}{k_a p - p(d^s(1-p) - k_b)}\right) & \textit{if } \underline{p} \le p < \underline{p} \vee (p_{k_b} \wedge \bar{p}) \\ \left(\frac{pd^s(1-p) - \pi_b}{pk_a}, 1 - \frac{\pi_a}{pk_a}\right) & \textit{if } \underline{p} \vee (p_{k_b} \wedge \bar{p}) \le p < \bar{p} \\ \left(\frac{pd^s(1-p) - \pi_b}{pk_a}, 1\right) & \textit{if } p = \bar{p} \end{cases} \tag{3.26}$$

with π_i to be the equilibrium profit of AP_i and $p_{k_i} = 1 - \frac{k_i}{d^s}$, $i \in \{a, b\}$.

Proof. Proof of Lemma 3.15 Since the cdf is derived from Eq. (3.24), the expression depends on the relationship between p and p_{k_b} due to the piece-wise function $L_b(p)$ and $H_a(p)$ when $p \in (\underline{p}, \bar{p})$, as shown in the lemma. □

Lemma 3.16. *If $d^s \ge d_{s_5}$, $p_i^s = \frac{d^s - k_a - k_b}{d^s}$, $q_{s_i} = k_i$, $i \in \{a, b\}$, with a corresponding profit $\pi_i^s = \frac{d^s - k_a - k_b}{d^s} k_i$.*

Proof. Proof of Lemma 3.16 We prove the two cases $d_{s_5} \ge d^s < d_{s_6}$ and $d_{s_6} \ge d^s$ separately. When $d_{s_5} \ge d^s < d_{s_6}$, $d^s > k_a + 2k_b$. The best response function is given by

$$p_i^s(p_j^s) = \begin{cases} p_j^s - \epsilon & \text{if } p_j^s \ge p_{\{k_a+k_b\}} + \epsilon \\ p_{\{k_a+k_b\}} & \text{if } p_j^s \le p_{\{k_a+k_b\}} \end{cases} \tag{3.27}$$

Therefore, the resulting optimal prices are $p_i^s = p_{\{k_a+k_b\}}$ where $p_{\{k_a+k_b\}} = \frac{d^s - k_a - k_b}{d^s}$.

Since $p_{s_a} = p_{s_b}$, the two firms proportionally share the demand with $q_{s_i} = k_i$ and the corresponding profit is $\pi_i^s(d^s) = \frac{d^s - k_a - k_b}{d^s} k_i$. □

Proposition 3.1 gets proved from Lemmata 3.11-3.16. □

Proof. Proof of Lemma 3.1 If the realized demand $d^s \leq d_{s_1}$, airlines are under fierce competition and both adopt the marginal price to satisfy the demand. If $d^s \geq d_{s_5}$, they sell at their capacity. If $d_{s_1} < d^s < d_{s_5}$, quantities depend on the two spot prices and either AP_a or AP_b offers the higher price. We know that $p_{\{k_a+k_b\}} < p < p_{k_a}$ from the properties of the equilibrium where p is a price in the mixed pricing range. From Eq. (3.19), if AP_a offers a higher price and $p_{s_a} < p_{\{k_b\}}$, AP_a sells $d^s(1 - p_{s_a}) - k_b$ and AP_b sells k_b, which implies the total quantity traded is $d^s(1 - p_{s_a})$. If $p_{s_a} > p_{\{k_b\}}$, AP_a sells nothing since there is no residual demand. AP_b sells k_b if $p_{s_b} \leq p_{\{k_b\}}$ and sells $d^s(1 - p_{s_b})$ if $p_{s_b} > p_{\{k_b\}}$. If AP_b offers a higher price, AP_b sells $d^s(1 - p_{s_b}) - k_a$ and AP_a sells k_a, implying that the total quantity is $d^s(1 - p_{s_b})$. □

Proof. Proof of Lemma 3.2 The proof is straightforward. With a sufficiently low spot demand, only the low price offer, AP_i sells and thereby the weighted spot price is identical to the lower price p_i^s. The threshold of d^s is derived from the situation where the remaining unsatisfied demand of AP_j is less than 0, namely when $d^s(1-p_j^s)-k_i \leq 0$. The second case is the situation where the lower price offer sells at the capacity and the higher price offer sells but less than its capacity. This indicates the threshold with the third case: $d^s(1 - p_j^s) - k_i \leq k_j$. The third case refers to the situation where both APs sell at their capacity. With simple algebra, the expression is derived. □

Proof. Proof of Lemma 3.3 The proof can be followed directly from Proposition 3.1. □

Proof. Proof of Proposition 3.2. From Lemma 3.12 and Lemma 3.16, the profits of the two airlines are 0 when $d^s \leq d_{s_1}$ and $\frac{d^s - k_a - k_b}{d^s} k_i$ with $i \in \{a, b\}$ when $d^s \geq d_{s_5}$. From Lemma 3.13, the profit of AP_a is $L_a(\underline{p})$ and the profit of AP_b is $H_b(\bar{p})$ when $d_{s_a} < d^s < d_{s_5}$. In this case, given Eq. (3.25), the profit of AP_a is

$$\pi_a = \begin{cases} \frac{1}{2}\left(1 - \sqrt{\frac{k_a}{d^s}\left(2 - \frac{k_a}{d^s}\right)}\right) k_a & \text{if } d^s < k_a + 2k_b - 2\sqrt{k_a k_b} \\ \frac{(d^s - k_a)^2}{4k_b d^s} k_a & \text{otherwise} \end{cases}$$

and the profit of AP_b is $\frac{1}{4} d^s \left(1 - \frac{k_a}{d^s}\right)^2$. □

Proof. Proof of Lemma 3.4 To show that $F_b(d^s, p)$ first-order stochastically dominates $F_a(d^s, p)$, it is equivalent to show $F_a(\cdot) \geq F_b(\cdot)$. The expressions of $F_a(\cdot)$ and $F_b(\cdot)$

depend on the value of $p_{\{k_b\}}$ as indicated in Eq. (3.8). Hence, there are three cases to be discussed: when $p_{\{k_b\}} < \underline{p}$, $p_{\{k_b\}} > \bar{p}$ and when $\underline{p} \leq p_{\{k_b\}} \leq \bar{p}$.

In the first case, $p_{\{k_b\}} < \underline{p}$ and from Eq. (3.8) the cdfs can be simplified as the following:

$$\begin{cases} F_a(d^s, p) = -\frac{d^s p}{k_a} - \frac{d^s}{4k_a p}(1 - \frac{k_a}{d^s})^2 + \frac{d^s}{k_a} \\ F_b(d^s, p) = 1 - \frac{1}{2p}(1 - \sqrt{\frac{k_a}{d^s}(2 - \frac{k_a}{d^s}})) \end{cases}$$

Denote $y = \frac{k_a}{d^s}$ and hence $0 \leq y \leq 1$ since the equilibrium pricing strategy is mixed strategy with intermediary demand, namely $d^s > k_a$, according to Proposition 3.1. Thereby, the difference between the cdfs are as following:

$$\begin{aligned} F_a(\cdot) - F_b(\cdot) &= -\frac{p}{y} - \frac{1}{4py}(1-y)^2 + \frac{1}{y} - 1 + \frac{1}{2p}(1 - \sqrt{y(2-y)}) \\ &= -\frac{1}{y}p - \frac{1}{p}(\frac{1}{4y} - 1 + \frac{y}{4} + \frac{\sqrt{y(2-y)}}{2}) + \frac{1}{y} - 1 \end{aligned}$$

Taking the first order condition with respective to p,

$$\frac{\partial(F_a(\cdot) - F_b(\cdot))}{\partial p} = (\frac{1}{4y} - 1 + \frac{y}{4} + \frac{\sqrt{y(2-y)}}{2})\frac{1}{p^2} - \frac{1}{y} \tag{3.28}$$

Taking the second order condition with respective to p,

$$\frac{\partial^2(F_a(\cdot) - F_b(\cdot))}{\partial p^2} = (\frac{1}{4y} - 1 + \frac{y}{4} + \frac{\sqrt{y(2-y)}}{2})\frac{-2}{p^3} \tag{3.29}$$

which is negative for $y \in [0, 1]$ and $p > 0$, indicating the first order condition decreases in this range. Then we identify the price $p_0 = \sqrt{\frac{1}{4} - y + \frac{1}{4}y^2 + \frac{1}{2}y\sqrt{y(2-y)}}$, with which the first order condition equals 0. Thereby, the first order condition is positive for $p \in [0, p_0]$ and negative for $p \in [p_0, \infty]$.

□

Proof. Proof of Lemma 3.6 With different realizations of spot demand d^s, the mixed pricing strategies differ in cdf and the price ranges. Denote two realizations of spot demand as d_1^s and d_2^s and assume $d_1^s < d_2^s$ without loss of generality. Correspondingly, the cdfs of the mixed pricing equilibrium are $F_i(d_1^s, p)$, $F_i(d_2^s, p)$ and the range of the prices are $[\underline{p}_1, \bar{p}_1]$ and $[\underline{p}_2, \bar{p}_2]$ with $i \in \{a, b\}$.

To show the stochastic dominance, the following inequality needs to be shown:

$$F_i(d_1^s, p) \geq F_i(d_2^s, p), \forall p \tag{3.30}$$

It can be shown that $\underline{p}$ and $\bar{p}$ increases in d^s. Hence, $\underline{p}_1 < \underline{p}_2$ and $\bar{p}_1 < \bar{p}_2$. If $\bar{p}_1 < \underline{p}_2$, there is no intersection of the two mixed pricing ranges. In this case, $F_i(d_1^s, p) \geq 0 = F_i(d_2^s, p)$ for $p \in [0, \bar{p}_1]$ and $F_i(d_1^s, p) = 1 \geq F_i(d_2^s, p)$ for $p \geq \bar{p}_1$, leading to Eq. (3.30) proved. If $\bar{p}_1 \geq \underline{p}_2$, there are more sub-cases to be discussed.

The first sub-case is when $p \in [0, \underline{p_2^s}]$ and $F_i(d_1^s, p) \geq 0 = F_i(d_2^s, p)$. The second sub-case is when $p \geq \bar{p}_1$ and $F_i(d_1^s, p) = 1 \geq F_i(d_2^s, p)$. Now we focus on the third sub-case when $p \in [\underline{p_2^s}, \bar{p}_1]$.

The expression of cdfs depends on the value of $p_{\{k_b\}}$, referring to the monopoly market clearance price for k_b units, as indicated in Eq. (3.8), with $p_{\{k_b\}} = 1 - \frac{k_b}{d^s}$. For convenience, we denote the piece-wise cdfs from Eq. (3.8) as the following functions taking d^s as the variable:

$$\begin{cases} (v_a(d^s), v_b(d^s)) \equiv & \left(\frac{k_b p - \pi_b^{s*}}{k_b p - p(d^s(1-p) - k_a)}, \frac{k_a p - \pi_a^{s*}}{k_a p - p(d^s(1-p) - k_b)} \right) & \text{if } \underline{p} \leq p < \underline{p} \vee (p_{k_b} \wedge \bar{p}) \\ (u_a(d^s), u_b(d^s)) \equiv & \left(\frac{p d^s(1-p) - \pi_b^{s*}}{p k_a}, 1 - \frac{\pi_a^{s*}}{p k_a} \right) & \text{if } \underline{p} \vee (p_{k_b} \wedge \bar{p}) \leq p < \bar{p} \\ (x_a(d^s), x_b(d^s)) \equiv & \left(\frac{p d^s(1-p) - \pi_b^{s*}}{p k_a}, 1 \right) & \text{if } p = \bar{p} \end{cases} \tag{3.31}$$

Considering that the cdfs are piece-wise functions, $F_i(d_1^s)$ and $F_i(d_2^s)$ may have similar expression given a p value, namely they fall in the same piece of domains divided by $p_{\{k_b\}}$. It is also possible that $F_i(d_1^s)$ and $F_i(d_2^s)$ fall in different pieces of domains given a p value.

Let $p_{\{k_{b1}\}}$ (resp., $p_{\{k_{b2}\}}$) denote the monopoly market clearance price given d_1^s (resp., d_2^s). Figure 3.11 shows two cases where the condition to show the stochastic dominance differs based on the value of p. For instance, if $p_{\{k_{b1}\}} < \underline{p_2}$, for a value $p \in [\underline{p_2}, p_{\{k_{b2}\}}]$, we need to show that $u_i(d_1^s) \geq v_i(d_2^s)$ as indicated in Eq. (3.31) and for a value $p \in [p_{\{k_{b2}\}}, \bar{p}_1]$, we need to show that $u_i(d_1^s) \geq u_i(d_2^s)$. If $\underline{p_2} < p_{\{k_{b1}\}} < \bar{p}_1$, there might exist three situations as indicated in Figure 3.11 where different inequalities need to be shown.

We have three cases: (i0 if $p \in [\underline{p_2}, \underline{p_2} \vee (p_{\{k_{b1}\}} \wedge \bar{p}_1)]$, we need to show $v_i(d_1^s) \geq v_i(d_2^s)$; ($ii$) if $p \in [\underline{p_2} \vee (p_{\{k_{b1}\}} \wedge \bar{p}_1), \underline{p_2} \vee (p_{\{k_{b2}\}} \wedge \bar{p}_1)]$, we need to show $u_i(d_1^s) \geq v_i(d_2^s)$; and ($iii$) if $p \in [\underline{p_2} \vee (p_{\{k_{b2}\}} \wedge \bar{p}_1), \bar{p}_1]$, we need to show $u_i(d_1^s) \geq u_i(d_2^s)$. Since F_a and F_b are different, we prove them separately. We first prove for F_a.

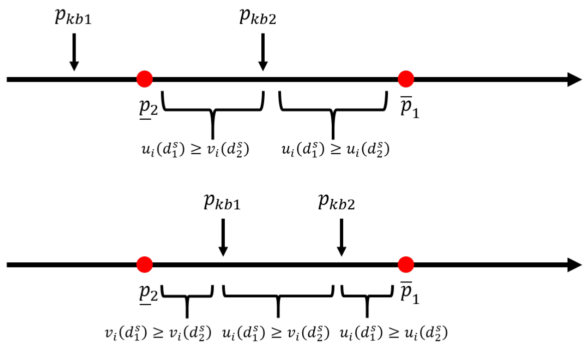


Figure 3.11 Demonstration of cases with different $p_{\{k_b\}}$

Case (i): we show that $v_a(d_1^s) \geq v_a(d_2^s)$ for $p \in \mathbb{P} \equiv [\underline{p_2}, \underline{p_2} \vee (p_{\{k_{b1}\}} \wedge \bar{p}_1)]$. In this case, it is equivalent to show that $v_a(d^s)$ is non-decreasing in d^s for $p \in [\underline{p_2}, \underline{p_2} \vee (p_{\{k_b\}} \wedge \bar{p}_1)]$ with $p_{\{k_b\}} = 1 - \frac{k_b}{d^s}$. From Eq. (3.31), we know that $v_a(d^s) = \frac{k_b p - \pi_b^{s*}}{k_b p - p(d^s(1-p)-k_a)}$ and it can be simplified to $v_a(d^s) = \frac{k_b p - \frac{1}{4}d^s + \frac{1}{2}k^a - \frac{k_a^2}{4d^s}}{k_b p - p(d^s(1-p)-k_a)}$ by plugging $\pi_b^{s*} = \frac{1}{4}d^s\left(1 - \frac{k_a}{d^s}\right)^2$. Taking the first order condition of $v_a(d^s)$ with respect to d^s, we have:

$$
\begin{aligned}
\frac{\partial v_a(d^s)}{\partial d^s} =& \frac{1}{\left(k_b p - p(d^s(1-p) - k_a)\right)^2} \cdot \\
& \left((-\frac{1}{4} + \frac{k_a^2}{4d_s}) \cdot (k_b p - p(d^s(1-p) - k_a)) - (k_b p - \frac{1}{4}d^s + \frac{1}{2}k^a - \frac{k_a^2}{4d^s})(-p + p^2) \right) \\
=& \frac{p}{\left(k_b p - p(d^s(1-p) - k_a)\right)^2 \cdot 4d^s} \cdot \\
& \left((-d^{s2} + k_a^2)(k_a + k_b - d^s(1-p)) - (4d^{s2}k_b p - d^{s3} + 2d^{s2}k_a - k_a^2 d)(p-1) \right) \\
=& \frac{p}{\left(k_b p - p(d^s(1-p) - k_a)\right)^2 \cdot 4d^s} \cdot \\
& \left(k_a d^{s2} - k_b d^{s2} + k_a^3 + k_a^2 k_b - 2k_a^2 d^s + 2k_a^2 d^s p + 4k_b d^{s2} p - 4k_b d^{s2} p^2 - 2k_a d^{s2} p \right)
\end{aligned}
\tag{3.32}
$$

Since $\frac{p}{\left(k_b p - p(d^s(1-p)-k_a)\right)^2 \cdot 4d^s} \geq 0$ for any p and d^s, we focus on the sign of the second term, denoted as ξ for convenience. Taking derivative of ξ with respect to p, $\partial\xi/\partial p = 2k_a^2 d^s + 4k_b d^{s2} - 8k_b d^{s2} p - 2k_a d^{s2}$, and solving $\partial\xi/\partial p = 0$ for p, we have $p = \frac{1}{2} - \frac{k_a}{4k_b}(1 - \frac{k_a}{d})$ denoted as $\tilde{p}$. As $\tilde{p}$ is greater than $\bar{p} = \frac{d^s - k_a}{2d^s}$, $\partial\xi/\partial p < 0$ for any d^s and $p < \bar{p}$. This indicates that $\xi|_{p\in\mathbb{P}} < \xi|_{p=\underline{p}}$. From Eq. (3.25), given the condition $p \in [\underline{p_2}, \underline{p_2} \vee (p_{\{k_b\}} \wedge \bar{p}_1)]$, we have $\underline{p} = \frac{(d^s-k_a)^2}{4k_b d^s}$. Hence, $\xi|_{p\in\mathbb{P}} < \xi|_{p=\underline{p}} = -\frac{({d^s}^2-k_a^2)(-d+k_a+2k_b)^2}{4k_b} < 0$ holds. This implies that $\frac{\partial v_a(d^s)}{\partial d^s} < 0$, i.e., that $v_a(d^s)$ decreases in d^s.

Case (ii): we show that $u_a(d_1^s) \geq u_a(d_2^s)$ for $p \in [\underline{p_2} \vee (p_{\{k_{b2}\}} \wedge \bar{p}_1), \bar{p}_1]$, which is equivalent to show that $u_a(d^s)$ is non-decreasing in d^s for $p \in [\underline{p_2} \vee (p_{\{k_b\}} \wedge \bar{p}_1), \bar{p}_1]$ with $p_{\{k_b\}} = 1 - \frac{k_b}{d^s}$. Recall from Eq. (3.31) that $u_a(d^s) = \frac{pd^s(1-p)-\pi_b^{s*}}{pk_a}$ and by substituting $\pi_b^{s*} = \frac{1}{4}d^s\left(1-\frac{k_a}{d^s}\right)^2$, it can be rewritten as $u_a(d^s) = \frac{1}{pk_a}\cdot(p(1-p)d^s - \frac{d^s}{4} + \frac{k_a}{2} - \frac{ka^2}{4d^s})$. Taking the first order condition of $u_a(d^s)$ with respect to d^s, $\frac{\partial u_a(d^s)}{\partial d^s} = \frac{1}{4p}(\frac{4p(1-p)}{k_a} - \frac{1}{k_a} + \frac{k_a}{{d^s}^2})$. Noticing that $\frac{1}{4p} > 0$, we need to figure out the sign of $\frac{4p(1-p)}{k_a} - \frac{1}{k_a} + \frac{k_a}{{d^s}^2}$, which can be rewritten as $\frac{1}{k_a}(4p(1-p)-1+\frac{k_a^2}{{d^s}^2})$. This expression is maximized at $p = \frac{1}{2}$ which is greater than the upper bound on the range, i.e., $\bar{p} = \frac{1}{2}(1-\frac{k_a}{d^s}) < \frac{1}{2}$. Thus, replacing p with this upper bound, we have that: $\frac{1}{k_a}\left(4p(1-p)-1+\frac{k_a^2}{{d^s}^2}\right) \leq \frac{1}{k_a}\left(4\cdot\frac{1}{2}(1-\frac{k_a}{d^s})(1-\frac{1}{2}(1-\frac{k_a}{d^s}))-1+\frac{k_a^2}{{d^s}^2}\right) = 0$. Hence, $\frac{\partial u_a(d^s)}{\partial d^s} < 0$, implying that $u_a(d^s)$ decreases in d^s.

Case (iii): we show that $u_a(d_1^s) \geq v_a(d_2^s)$ for $p \in [\underline{p_2} \vee (p_{\{k_{b1}\}} \wedge \bar{p}_1), \underline{p_2} \vee (p_{\{k_{b2}\}} \wedge \bar{p}_1)]$. In this case, p can be interpreted as a price between $p_{\{k_{b1}\}}$ and $p_{\{k_{b2}\}}$ bounded within $[\underline{p_2}, \bar{p}_1]$. Hence, there exists an intermediary demand, $d_1^s \leq \tilde{d^s} \leq d_2^s$, satisfying $p = 1 - \frac{k_b}{\tilde{d^s}}$. With $\tilde{d^s}$, the price p can be interpreted as the monopoly market clearance price for k_b units, $p_{\{\tilde{k_b}\}}$. Based on the previous two cases, the two inequalities, $u_a(d_1^s) \geq u_a(\tilde{d^s})$ and $v_a(\tilde{d^s}) \geq v_a(d_2^s)$, hold for $d_1^s \leq \tilde{d^s} \leq d_2^s$. Replacing p with $1 - \frac{k_b}{\tilde{d^s}}$ in $u_a(\tilde{d^s})$ and $v_a(\tilde{d^s})$, which have been simplified in the previous cases, we can show that $u_a(\tilde{d^s}) = v_a(\tilde{d^s})$. Therefore, $u_a(d_1^s) \geq u_a(\tilde{d^s}) = v_a(\tilde{d^s}) \geq v_a(d_2^s)$.

Using similar approach, we can solve for F_b, which has relatively simpler expressions than F_a, and can be shown to have similar characteristics for the case $\underline{p_2} < p_{\{k_{b1}\}} < \bar{p}_1$.

Therefore, $F_i(d_2^s)$ first-order stochastically dominates $F_i(d_1^s)$ with $d_1^s < d_2^s$ and $i \in \{a, b\}$. □

Proof. Proof of Lemma 3.7 We solve the six cases of equilibrium spot market. The approach is identical to all cases, which is to solve the equation system of Eq. (3.10) and Eq. (3.3). Take Case 1 as an example. In Case 1, $\tilde{p}^s = p_i^s$ according to Eq. (3.10) and $d^s = \frac{1}{2}d$ according to Eq. (3.3). Replace the value of d^s and $\tilde{p}^s$ back to the conditions of the case, we can obtain the thresholds of d for Case 1: $d < 2\min(\frac{k_i}{1-p_j^s}, \frac{q^c}{1-p_i^s})$. The same approach applies to other cases, although Case 5 is a bit tedious. □

Proof. Proof of Lemma 3.8 We first show the case when d is sufficiently low. Suppose both APs offer 0. It is clear that by offering a negative price, APs will be worse off. If AP_i decides to deviate from 0 by increasing to a strictly positive value p_i^s, we need to identify the condition on d making AP_i not better off to obtain the threshold of the overcapacity equilibrium. As a higher price offer, AP_i will be allocated $d^s(1-p_i^s)-k_j$ units, where d^s depends on the weighted spot price $\tilde{p}^s$.

From Eq. (3.3), we can discuss the value of d^s by cases. If $\frac{1}{2}d \geq d - q^c$, which is equivalent to $d \leq 2q^c$, the equilibrium $d^s = \frac{1}{2}d$. Hence, the units allocated to AP_i becomes $\frac{1}{2}d(1-p_i^s) - k_j$ and the inequality of $\frac{1}{2}d(1-p_i^s) - k_j \leq 0$ should holds for all p_i^s. Solve the inequality, we have $d \leq 2k_j$. Combine this constraint with the condition of this case, we have $d \leq \min(2k_j, 2q^c)$.

If $\frac{1}{2}d \leq d - q^c$, which is equivalent to $d \geq 2q^c$, the equilibrium d^s is not trivial. A positive unit allocated to AP_i indicates a positive weighted spot price $\tilde{p}^s$ since p_i^s is strictly positive. Hence, there should not exist a feasible $\tilde{p}^s$ for any p_i^s. From Eq. (3.10) and Eq. (3.3), the inequality of $\frac{k_j}{1-p_i} \geq d - q^c$, holding for any positive p_i^s, guarantees an empty set of strictly positive $\tilde{p}^s$, implying that AP_i is not better off by deviation. Solving the inequality, we have $d \leq q^c + k_j$. Combine this constraint with the condition of this case, we have $2q^c \leq d \leq q^c + k_j$.

Based on the results of the two cases and that $k_a \leq k_b$, we can summarize the condition of over-capacity as the following: $d \leq \min(2k_a, k_0) = \min(2(k_0 - q^c), k_0)$.

Now we proceed with the case where d is sufficiently high and provide a sketch of proof. Assume both APs set the price to be $1 - \frac{k_a+k_b}{d^s}$, which is the market clearance price for $k_a + k_b$ units. It can be easily shown that this price equals $1 - \frac{2k_b}{d}$ since the equilibrium $d^s = d - \frac{q^c}{1-\tilde{p}^s}$. First, neither of the AP is willing to sell at a lower price since the quantity traded is bounded by the capacity. Then we discuss the option for APs to increase the price. Assume AP_j increases the price by $\epsilon > 0$. The condition of d making the profit of AP_j not better off for any positive $\epsilon \to 0$ is the condition of the under-capacity equilibrium. □

Proof. Proof of Lemma 3.9 The result can be directly derived from the demand allocation function as the equilibrium of $\tilde{p}^s$ and d^s can be easily obtained due to identical spot prices. □

Proof. Proof of Proposition 3.3 In the outside option, the capacities brought by the two asset providers are identical, namely $k_a = k_b = k_0$ and the spot market equals the end customer demand, $d^s = d$.

Integrating the equilibrium profit over the three domains of d_s, the expected profit function of each airline is given by:

$$
\begin{aligned}
E\left[O_a\right] = &\int_0^{k_0} 0 + \int_{k_0}^{3k_0} \frac{(d-k_0)^2}{4d} f_d(d)dd + \int_{3k_0}^{\infty} (1-\frac{2k_0}{d}) k_0 f_d(d)dd \\
= &\frac{1}{4}\int_{k_0}^{3k_0} d f_d(d)dd - \frac{1}{2}k_0 \int_{k_0}^{3k_0} f_d(d)dd + \frac{{k_0}^2}{4}\int_{k_0}^{3k_0} \frac{f_d(d)}{d}dd \\
&+ k_0 \int_{3k_0}^{\infty} \left[f_d(d) - \frac{2k_0}{d} f_d(d) \right] dd \\
= &\frac{3}{4}k_0 F_d(3k_0) - \frac{1}{4}k_0 F_d(k_0) - \frac{1}{4}\int_{k_0}^{3k_0} F_d(d)dd - \frac{1}{2}k_0 F_d(3k_0) + \frac{1}{2}k_0 F_d(k_0) \\
&+ \frac{{k_0}^2}{4}\frac{F(3k_0)}{3k_0} - \frac{{k_0}^2}{4}\frac{F(k_0)}{k_0} + \frac{{k_0}^2}{4}\int_{k_0}^{3k_0} \frac{F_d(d)}{d^2}dd \\
&+ k_0(1-F_d(3k_0)) - \left[2{k_0}^2 \frac{F_d(d)}{d}\bigg|_{3k_0}^{\infty} + 2{k_0}^2 \int_{3k_0}^{\infty} \frac{F_d(d)}{d^2}dd \right] \\
= &-\frac{2}{3}k_0 F_d(3k_0) - \frac{1}{4}\int_{k_0}^{3k_0} F_d(d)dd + k_0 + \frac{{k_0}^2}{4}\int_{k_0}^{3k_0} \frac{F_d(d)}{d^2}dd - 2{k_0}^2 \frac{F_d(d)}{d}\bigg|_{3k_0}^{\infty} \\
&- 2{k_0}^2 \int_{3k_0}^{\infty} \frac{F_d(d)}{d^2}dd
\end{aligned}
$$

□

Now we derive the outside option for LSP_1. We can reduce the total quantity purchased by LSPs, identified as Eq. (3.9), to the following form:

$$
q_i^s = \begin{cases} \frac{d}{2} & \text{if } d \le k_0 \\ d(1-p_i^s) - k_0 & \text{if } k_0 < d < 3k_0 \text{ and } p_i^s > p_j^s \\ \frac{d(1-p_i^s)}{2} & \text{if } k_0 < d < 3k_0 \text{ and } p_i^s = p_j^s \\ k_0 & \text{if } k_0 < d < 3k_0 \text{ and } p_i^s < p_j^s \text{ or } 3k_0 \le d \end{cases} \tag{3.33}
$$

From this equation, the total quantity of two $LSPs$ equals d when $d \le k_0$ and equals $2k_0$ when $3k_0 \le d$. When $k_0 < d_s < 3k_0$, the probability of $p_{s_h} = p$ is $f_h(p) = 2F(p)f(p)$, where $F(p)$ (resp. $f(p)$) is the equilibrium density (resp. probability) function in Eq. (3.8). Therefore, when $k_0 < d < 3k_0$, the expected sales, denoted as

$w(d, k_0)$, is as following:

$$\begin{aligned} w(d, k_0) &= \int_{\underline{p}}^{\bar{p}} d(1-p) f_h(p) dp \\ &= \int_{\underline{p}}^{\bar{p}} d(1-p) 2F(p) f(p) dp \\ &= \frac{2(d-2k_0)(2d^3 - 7d^2 k_0 + 4d{k_0}^2 + 5{k_0}^3) + (d-3k_0)^2 (d-k_0)^2 \ln -\frac{d-3k_0}{d-k_0}}{8(d-2k_0)^3} \end{aligned} \tag{3.34}$$

Since the LSPs apply margin m to each units it sell, the joint expected profits of LSPs is the product of m and the expected quantity $\mathbb{E}[q_a^s + q_b^s]$. Given Proposition 3.1, the expected total quantity could be written in the following way:

$$\mathbb{E}[q_a + q_b] = \int_0^k d f_d(d) dd + \int_k^{3k} w(d, k) f_d(d) dd + \int_{3k}^{\infty} 2k f_d(d) dd \tag{3.35}$$

Integrating by parts gives rise to the outside option of LSP_1.

Proof. Table of Notation

Name	Meaning	Expression
k_0	capacity of an AP	
k_i	capacity of AP_i brought to spot market with $i \in \{a, b\}$	
p_i^s	spot market price per capacity unit set by AP_i with $i \in \{a, b\}$	
p^c	contract price per capacity unit agreed by AP_i and LSP_1	
$\tilde{p}^s$	weighted spot price according to traded quantities	
p_i^{ss}	equilibrium static spot price of AP_i	Eq. (3.6)
p^m	mixed pricing strategy in static spot market	Eq. (3.7)
d	end customer demand	
$f_d(d)$	pdf of end customer demand distribution	
$F_d(d)$	cdf of end customer demand distribution	
$F_i(p_m)$	cdf of equilibrium mixed pricing strategy in static spot market for AP_i with $i \in \{a, b\}$	
d^s	spot market demand	
d_i^s	spot market demand brought by LSP_i with $i \in \{1, 2\}$	
d_i^c	spot market demand satisfied by contract for LSP_i with $i \in \{1, 2\}$	
q_i^s	quantities traded in spot market by AP_i with $i \in \{1, 2\}$ and by LSP_i with $i \in \{a, b\}$	
m	unit margin charged by LSPs	
β	market share of LSP_1	
τ	bargaining power of LSP_1	
$D(p)$	demand-price function in spot market	$D(p) = d(1-p)$
$L_i(.)$	revenue of offering a lower price	Eq. (3.2)
$E(.)$	revenue of offering an equal price	Eq. (3.2)
$H(.)$	revenue of offering a higher price	Eq. (3.2)
$\pi_i(.)$	immediate profit of AP_i given the two spot prices	Eq. (3.2)
π_i^m	equilibrium profit by adopting mixed pricing strategies for AP_i given the two spot prices	
$\pi_i^{ss}(.)$	equilibrium static spot profit of AP_i	Eq. (3.11)
$p_{\{\hat{k}\}}$	the price clearing $\hat{k}$ units	$1 - \frac{\hat{k}}{d}$
O_i	the outside option for agent i, with $i \in \{a, 1\}$	

Table 3.4 Notation

Chapter 4

Shopping or Dining? On Passenger Dwell Time and Airport Revenues[1]

4.1. Introduction.

Airports act as gateways of cities serving passengers who may use air transportation services for business or tourism purposes In general, airports face a high transportation demand. According to Statista (2022a), there were 4.7 billion passengers traveled with airlines in 2020, namely more than 10 million passengers per day. Such traffic demand provides airports with two main sources of revenues, aeronautical and non-aeronautical revenues, with the latter accounting for 40% of all revenues before pre-pandemic times (Lucas, 2022). The two revenues are generated from two categories of activities, which are related to moving aircraft and retail concessions respectively.

High demand faced by airports brings great opportunities for both categories of revenues and non-aeronautical revenues are growing in significance. On the one hand, maintaining aeronautical revenue becomes increasingly challenging due to airlines' cost reductions. On the other hand, the demand is rather volatile compared to other industries (Fuerst and Gross, 2018) and thereby diversifying the portfolio of airport revenues enables airports to maintain a good financial performance. One hedging strategy is to expand non-aeronautical revenues. Indeed, it has been shown that airport operators are seeking to increase non-aeronautical revenues

[1]Co-authors: Chiara Morlotti and Benny Mantin

to improve the sustainability of their financial performance (Chen et al., 2020). Comparing to traditional aeronautical revenues, non-aeronautical revenues tend to generate higher margins (Lucas, 2022) and they, consequently, play a key role in maintaining competitiveness for airport business. It has been shown that non-aeronautical revenues are crucial in the recovery of business for airports after COVID-19 (InternationalAirportReview, 2022).

Given the significance of non-aeronautical revenues, considerable attention has been paid to explore airport financial performance and understanding of the determinants of airport revenues. One key determinant of airport non-aeronautical revenues is passengers' consumption behaviors and their socio-demographic characteristics. For instance, it is of interest to know whether business travelers spend more money at airports compared to economic travelers and if yes, how they differ in their behaviors (Torres et al., 2005; Fuerst et al., 2011; Graham, 2018). Apart from that, age groups and gender groups are also discussed in literature. Such insights help managers to develop efficient marketing strategies in daily decision marking process. Another key determinant is the layout of airport terminals which has also been discussed in literature regarding the impacts on airport revenues. More specifically, the size of commercial areas in terminals has been discussed to impact airport financial performance and it is argued that a higher share of food and beverage outlets negatively correlates with the commercial revenue per passenger (Fuerst and Gross, 2018). Some studies expand the focus to a broader scope of airport design. For instance, the short stay parking places and check-in facilities have been argued to improve retail income as these factors positively contribute to retail revenues (Volkova, 2009).

Dwell time is a critical factor for non-aeronautical revenues as sufficient dwell time enables passengers to browse and shop at airports. It is refers to the time interval between a passenger's arrival at the airport and the departure of the flight (Freathy and O'Connell, 2012). With free time at airports, passengers are likely to conduct retail activities and buy food and drinks. A key question is then raised: how does the time spent at airports translate into airport revenues. Although it may sound intuitive to assume that airport dwell time positively affects airport commercial activities, there lacks support towards this assumption. Some literature (Castillo-Manzano, 2010; Liu et al., 2014; Torres et al., 2005) show a positive relationship between the dwell time and airport non-aeronautical revenues, whereas Appold and Kasarda (2006) and Lu (2014) do not observe a significant impact of dwell time on commercial sales and buying tendency. The main challenge to develop an insightful understanding of the relationship between dwell time and airport revenues is the lack of traceability of passenger dwell time. The literature mainly relies on survey data containing self-reported waiting time at airport which can be subjective and likely biased. For instance, passengers who hurry to gates and, hence, do not spend any money at the airport, are less likely to fill in a survey.

This paper expands on these studies by analyzing a multi-source database which include dwell time data of airport visitors at 89 U.S. airports. Apart from dwell time, our analysis includes passenger income level and flight metrics such as flight haul distances and connecting ratios. We perform a fixed effects panel regression to explore how passenger dwell time at airports as well as passenger's income level impacts airport commercial revenues. More specifically, we find that dwell time has a positive impact on airport revenues and the impact varies with airport terminal design. Dwell time significantly and positively contribute to food and beverage revenues at airports with a linear or a finger pier terminal design, whereas dwell time affects retail revenues at airports with a concourse terminal design.

The rest of the paper is organized as follows. In Section 4.2, we provide a literature review and in Section 4.3 we present the data and the methodology that we include in this study. In Section 4.4, we run various models to analyze the impacts of dwell time. In Section 4.5, we further analyze additional data for a subgroup of airports. Section 4.6 summarizes.

4.2. Literature review

There exist an extensive literature on the determinants of airport revenues. Chen et al. (2020) review and summarize the determinants of airport retail revenues and categorize 26 attributes into five categories including airport characteristics and passenger related issues. Using panel data analysis, Lei and Papatheodorou (2010) measure the impact of LCC on airport non-aeronautical revenues and find that LCCs contribute less to airport commercial income comparing to other carriers. These two papers provide a comprehensive overview of critical determinants.

Socio-demographics of passengers are one of the focusing determinants of airport revenues. Fuerst et al. (2011) study the determinants and constraints of European airport commercial activities and find that revenues per passenger mainly depend on national income level in the region of the airport, the size of the airport and the percentage of leisure passengers. Commercial revenues per passenger also increases in the number of passengers, the ratio of commercial to total revenues and the number of flights. Fuerst and Gross (2018) analyze a panel of 75 airports in 30 countries and find that the greater the share of international passengers the higher is the level of commercial income. Castillo-Manzano (2010) interview $20,383$ passengers at 7 Spanish airports to understand passenger behaviors, focusing on the likelihood of commercial activities. The author observes that business passengers are not likely to make last-minute purchases and elderly passengers spend less on food and beverage activities. They also find that passengers from outside the Eurozone tend to spend more on food and beverage and that LCC passengers spend less on food and beverage. One survey-based research is conducted by Lu (2014) at two international airports in Taiwan to investigate passengers' shopping intentions. They observe that male passengers have less pre-planned shopping intentions and that elder passengers have stronger pre-planned shopping intentions. Liu et al. (2014) analyze an internet-based revealed preference survey which collects data from both travelers and non-travelers and they point out that the socio-demographic characteristics as well as travel related information have impact on passengers' activity patterns at airports. For instance, frequent travelers are less likely to shop at airports and travelers with higher incomes are more likely dine and shop. Based on a study of 75 U.S. airports, according to Appold and Kasarda (2006), the number of passengers has the most significant impact on airport financial performance and the average haul distance positively correlates to both food and beverage sales and non-food sales. In brief, socio-demographics of passengers significantly affect airport revenues.

Another relevant stream of literature focuses on the impacts of airport layout. Fuerst and Gross (2018) study the layout of airports and show that the size of airport and of commercial area significantly affect airport financial performance. Specifically, they argue that a higher share of food and beverage outlets is negatively associated with lower commercial revenue per passenger due to the low profit margin incurred by restaurant business. Volkova (2009) analyzes the determinants of airport retail revenues based on data from 13 European airports and observe that passengers outside of EU increase retail revenue per square meter at hub airports but not at regional airports. He also finds that the number of short stay parking places, check-in facility and the number of employees improve retail income. Chen et al. (2020) discuss research gaps including investigating the impacts of airport terminal design on airport retail performance. Van Oel and Van den Berkhof (2013) explore passengers' preferences towards airport design by employing discrete choice experiments in Schiphol airport and they find, for instance, that curvilinear roof and a curved layout are valued by passengers while signage has no impacts on passengers' preference. Adey (2008) points out that airport design, such as corridors and walls, constrain and guide passengers' movement, which likely affects passengers' commercial behaviors. Churchill et al. (2008) propose a method to evaluate the ease of airport terminal wayfinding to monitor passenger movement at airports. De Neufville et al. (2002) investigate the performance of airports with different terminal layout from the perspective of logistics efficiency and find that linear mid-field concourse airports with intelligent management is the best overall configuration for large airports. However, it is still not clear that whether different airport layouts lead to various consumption behaviors of passenger, which ultimately affect airport revenues. Further, if airport layout affects revenue, how should stakeholders leverage the insights to improve airport revenues.

One of the most significant determinants of airport revenues is dwell time but there is not agreement on the role of well time to the best of our knowledge. Freathy and O'Connell (2012) conduct a survey at an international airport to explore how passengers from different segments spend their time and money at the airside part of the airport. They identify a positive correlation between the time spent in commercial areas and the time spent in shopping, the effect of which depends on factors including gender, the composition of the group and the purpose of travel. Castillo-Manzano (2010) find that the dwell time prior to embarking positively correlates to passengers' decisions of buying food and beverages and purchasing goods. Liu et al. (2014) point out that with more available time, travellers' probabilities for all activities, including dinning and shopping, increase according to their internet-based survey. There are also papers that do not support the significance of dwell time. Appold and Kasarda (2006) estimate dwell time based on security point waiting time and survey reports and find no strong relationship between dwell time and commercial sales. Lu (2014) collects passengers' dwell time information in his survey from more than 400 passengers but the reported time on shopping is not significantly correlated with passengers' buying tendency, including both pre-planned and impulsive buying. As discussed earlier, the challenge of understanding the impacts of dwell time is the mainly due to the lack tractability. Most literature of dwell time is based on self-reported surveys, which can be imprecise and biased. In this work, we explore the impacts of dwell time on airport non-aeronautical revenues based on passengers' footprint data and study whether the impacts of dwell time vary with airport terminal designs.

4.3. Data and methods

4.3.1 Data sources

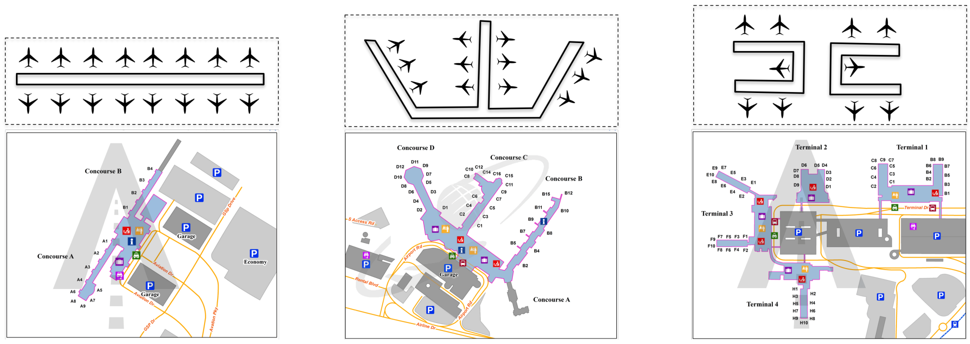
In this paper, we employ a unique data set from Placer.ai. Placer.ai is a firm that tracks mobile users' footprint via a dedicated application (to which the users consent). Its primary scope is foot traffic at retail outlets but it also provides data on airport dwell time[2]. In this study, we focus on visitors to U.S airports. The data provided by Placer.ai filters out short stay visitors (like meet and greet), employees of the facilities as well as taxi drivers (including UBER and the like). Placer.ai employs its data cleaning mechanism and releases the data only if it exceeds a certain confidence threshold (with at least 3000 data points). We use the tracked duration of stay of

[2] Placer.ai has created the dwell time data by extracting for many of the airports just for this study.

users as the dwell time of passengers. We focus on the top 100 airports in the U.S. based on annual enplanement ranking from 2019 as published by the Department of Transportation (DOT). Of the top 100 airports, 89 airports are included in the Placer.ai database as the number of visits at these airports exceeds the threshold set by Placer.ai. While Placer.ai data are available on a monthly basis, we aggregate the data on annual basis to align with the annual financial performance data of airports. Placer.ai data is available from 2017 onward and to avoid the impact of COVID on airports, we limit our analysis to the period between 2017 and 2019. Placer.ai also provides information regarding the hourly distribution of stays as well as socio-economic characteristics, such as income and age profile. Such data, unfortunately, are available from Placer.ai only for a subgroup of 55 airports.

The second data source is the FAA Form 5100-127 which summarizes annual operating and financial statistics for U.S. commercial airports. Of relevance to our study are the airports' operating revenues, which are broken down into aeronautical and non-aeronautical revenues. The latter source of revenues normally comes from commercial activities carried out at terminals and other service facilities. These consist of food and beverage revenues, retail stores and duty-free revenues, non-terminal facility lease revenues, services revenues, rental cars revenues and parking and transportation revenues. We shall note that Form 5100-127 provides the revenues achieved by airports which include concession fees and revenues earned from restaurants and coffee shops as well as revenues from retail operations in the terminals but these do not include sales made by third parties who operate their businesses at airports. For instance, food and beverage revenues are the concession fees and revenues to the airport from restaurant, snack bars and coffee shops located in the terminals. Retail stores and duty-free revenue contains the revenue achieved from retail and duty-free operations. As for service revenue, it is generated from services such as telecommunications, internet access and spas. These airports revenues serve as a proxy for retailers' actual revenues as the contract between an airport and a retailer normally consists of rent and minimum annual guarantees. Apart from that, airports are transitioning to revenue share contracts, such as Dallas/Fort Worth International Airport (TheMoodieDavittReport, 2020). Among the sub-categories of non-aeronautical revenues, the food and beverage revenues, retail stores and duty-free revenues and services revenues are the three main categories of key interest in the context of our study.

We also collect airport flight metrics from OAG—a travel data provider. Specifically, we collect three measures for each airport in our dataset: the average haul distance of flights departing from a specific airport, the percentage of connecting passengers and the percentage of seats offered by low cost carriers (LCC). The three metrics help us profile the flights and passengers of an airport, serving as control variables with impacts on revenues.



(a) Linear design: *Geer-Greenville Spartanburg International Airport (GSP)*

(b) Finger pier design: *Louis Armstrong International Airport (MSY)*

(c) Concourse design: *Fort Lauderdale/Hollywood International Airport (FLL)*

Figure 4.1 Airport terminal layouts

As mentioned in Section 4.2, the impact of airport terminal design on airport commercial revenues is worth of investigation. We collect terminal layout information from AirportGuide.com, which provides detailed layout design for a large number of airports. Instead of focusing on interior designs, we look at the overall structure of airport terminals and explore whether passengers behave differently. We follow the method proposed by Chen et al. (2020). Since no airport adopts transporter design and the three airports with satellites design can be also considered as concourse (TPA, MCO) or finger pier (SEA), we classify the airports in our data set into three major categories: linear, finger pier, and (midfield) concourse as illustrated in Figure 4.1.

4.3.2 Descriptive analysis

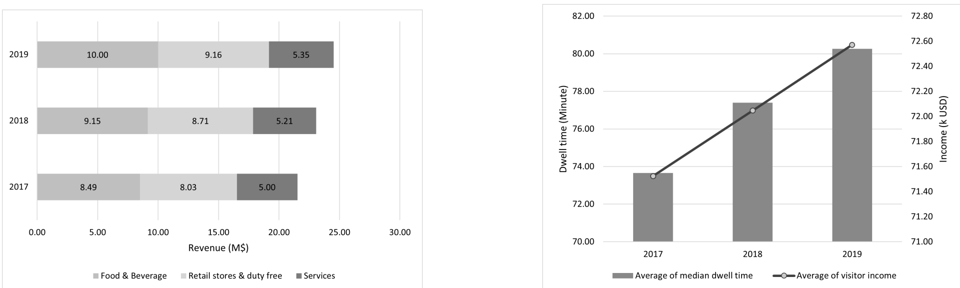
Operating revenues consist of aeronautical and non-aeronautical revenues, with the latter being responsible for around 45% of the total. In this work, we focus on non-aeronautical revenues as such revenues are directly related to passenger activities inside airport terminal. Table 4.1 summarizes the statistics for 2019, the most recent year in our panel.

As illustrated in Figure 4.2a, all three types of revenues increase over the three years.

To reduce the effect of outliers, in our study, we collect the median value of dwell time instead of the mean value as in rare cases passengers can stay in transition area for more than a day. As shown in Table 4.1, the average value of median dwell time is around 80 minutes. It is widely assumed that dwell time positively contributes to airport commercial activities but there is scarce evidence of such assumption mainly due to the lack of available data. From Figure 4.2b, dwell time is observed to increase

Variables	Definitions	Source	N	mean	sd
Total operating revenue (million $)	Total operating revenues per the airport's Statement of Revenues, Expenses, and Net Assets	FAA	89	244.57	322.31
Total non-aeronautical revenue (million $)	Total nonaeronautical revenues from airport operations	FAA	89	111.24	125.62
Food/beverage revenue (million $)	Concessions fees/revenues to the airport from restaurant, snack bars, coffee shops, and beverage stores located in the terminals	FAA	89	10.00	13.62
Retail stores/dutyfree revenue (million $)	Revenue from retail and duty free operations in the terminals	FAA	89	9.16	17.17
Services revenue (million $)	Revenues for services such as telecommunications, internet access, advertising, barbershops, shoeshine stands, spas	FAA	89	5.35	10.32
Dwell time (min)	The median duration of stay calculated from users' foot traffic	Placer.ai	89	80.22	9.40
Visitor income (thousand $)	The median income level of users	Placer.ai	89	72.48	7.72
Enplanement (million)	The number of scheduled enplanements	DOT	89	9	11
Flight count	The number of departing flights	DOT	89	72689	78381
Connecting seats	The ratio between the number of connecting seats and total seats	OAG	89	10.67%	15.57%
Haul distance	The average haul distance of flights	OAG	89	1473.88	752.85
LCC seats	The percentage of LCC departing seats	OAG	89	37.29%	24.94%
Terminal shape	L for linear shape; F for finger pier shape; C for concourse shape	AirportGuide	-	-	-

Table 4.1 Statistical summary of Year 2019



(a) The breakdown of mean non-aeronautical revenues from 2017 to 2019

(b) The average of median dwell time and the average of median visitor income from 2017 to 2019

Figure 4.2 Airport and passenger statistics overview

over time, which echoes the trend for revenues, and the average value of median household income increases from 71,520 USD to 72,570 USD in three years. To explore whether and how dwell time contributes to revenues, we include additional variables such as passenger socio-demographics and airport layout features in the analysis.

To control the impact of airport size, we collect two size measures for an airport: enplanement number and flight count. The first measure is consistent with Appold and Kasarda (2006) who point out that airport traffic accounts for the largest impact on airport revenues as the more passengers visit the airport, the more revenues are generated. Considering that airport size cannot be adjusted in a short run, we normalize the measure and focus on revenue per passenger. Accordingly, we explore how dwell time along with additional factors impact revenue per passenger. We define a measure, revenues per passenger, or RPP, which is the ratio of an airport's revenue and the number of passengers visited airports. Note that RPP can refer to different types of revenues. The second measure of airport size is the flight count. This measure serves as a control variable to account for the impact of airport size on different kinds of RPP.

As for flight information of airports, we collect data on the percentage of connecting passengers, average haul distance, and the percentage of LCC seats, from OAG database. As shown in Table 4.1, the average connecting passenger percentage across all 89 airports in 2019 is 11% and the average haul distance is 1474 kilometers. Low cost carriers accounts for about 37% of the offered seats.

As for the layout of airport terminals, 23 of them have a linear shape, 38 have a finger pier shape, and 28 of them have concourse shape. Figure 4.1 shows the three categories of airport terminal design with examples.

4.3.3 Model selection

Our objective is to explore whether more time spent by passengers at airports contributes to more commercial activities and if so, to which type of activity it contributes and by how much. Our empirical specification regresses the airport revenues on the focal independent variable—dwell time—together with other control variables. We exploit the fact that airport retail layouts, such as the assortment of shops and the size of commercial area, likely determine airport revenues. Accordingly, we include fixed effects for each airport in the model.

As discussed in Section 4.2, dwell time is expected to positively contribute to airport non-aeronautical revenues. To investigate whether the impact of revenues result from factors other than dwell time, we include additional variables to profile passengers more comprehensively. Visitor income level may affect revenues as rich people may contribute more to retail sales per passenger (Lu, 2014). Although we have taken the impact of airport size into consideration by focusing on revenue per passenger, we include the flight count of an airport into the model to investigate whether airport size affects RPP. A positive impact on RPP implies the benefit of economies scale.

Connecting passengers, may also affect RPP and hence we include the percentage of connecting seats of each airport into the model. Similarly, the average haul distance and the percentage of LCC seats are included as control variables. We use natural logs for variables on both sides of the specification to account for elasticity. Specifically, we apply a panel with fixed effects model and estimate the following equation:

$$\begin{aligned}\ln RPP_{it} = &\beta_0 + \beta_1 \ln DwellTime_{it} + \beta_2 \ln VisitorIncome_{it} \\ &+ \beta_3 \ln FlightCount_{it} + \beta_4 Connecting_{it} + \beta_5 \ln Haul_{it} \\ &+ \beta_6 LCC_{it} + \omega_i + \epsilon_{it}\end{aligned} \tag{4.1}$$

where *RPP* refers to the four types of revenues on per passenger basis, which are total non-aeronautical revenue, food and beverage revenue, retail revenue and service revenue. *DwellTime* is passenger dwell time, *VisitorIncome* estimates the household income level of passengers, *FlightCount* is the flight count of airports, *Connecting* is the percentage of connecting passenger of airports, *Haul* is the average haul distance and *LCC* refers to the percentage of LCC seats. Ultimately, ω_i is an airport fixed effect, and ϵ_{it} is an error term. We test the impact of airport layout by grouping the airports according to their terminal shapes as the layout design of an airport does not change during the panel.

4.4. Analysis

Panel regression analyses are discussed in this section. Section 4.4.1 shows the estimation of (4.1) for all 89 airports. To explore the role of airport layout design, Section 4.4.2 analyzes three sub-groups of airports with various terminal layout designs. Section 4.4.3 shows the results of employing SUR model and Section 4.4.4 runs alternative regressions for robustness checks.

4.4.1 Empirical results

We first test whether dwell time has a positive impact on airport commercial revenues. The panel data estimation results are shown in Table 4.2. It shows that 1% increase of dwell time leads to 0.4% increase in food and beverage RPP while it does not have a significant impact on total non-aeronautical RPP as well as other subcategory revenues. The observation slightly differs from the assumption that passengers will conduct more commercial activities if they have more available time. It indicates that, spending more time in airport does not necessarily encourage retail shopping behaviors but passengers likely purchase more food as time passes by.

Although the coefficients of visitor household income are positive, the impacts are not significant. It implies that wealthier passengers do not necessarily spend more money at the airport than other groups of passengers. As a size measure, the flight count negatively affects the total non-aeronautical RPP, with no significant impacts on the three subcategories. We can also observe that haul distance significantly and positively affect food and beverage RPP. It is likely because that passengers are inclined to dine at the airport or purchase food and beverage as a preparation for longer haul flights. As long-haul flight passengers are also likely international passengers, this argument is aligned with the observation that international passengers positively contribute to airport revenues (Volkova, 2009; Fuerst and Gross, 2018). The

	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.1199	0.4403*	0.4349	-0.7249
	(0.1074)	(0.2266)	(0.2627)	(0.9104)
Visitor income (log)	0.5154	1.0112	1.1031	0.6831
	(0.3145)	(0.7667)	(2.3326)	(3.7860)
Flight count (log)	-0.0615**	0.0469	-0.2220	0.2221
	(0.0267)	(0.0882)	(0.2154)	(0.3020)
Connect Pax	-0.4623	0.9380	-1.4760	5.2299
	(0.5156)	(0.8813)	(1.6733)	(11.3676)
Haul distance (log)	0.6787*	1.0747***	0.8219	1.2080*
	(0.3782)	(0.4078)	(0.4996)	(0.6347)
LCC passenger	-0.6091**	-1.5091	2.4344	-2.2185
	(0.2419)	(1.0149)	(3.7449)	(1.5351)
Constant	9.6869***	-0.4343	2.3329	2.0706
	(2.7834)	(4.4041)	(8.8211)	(16.2468)
Observations	265	264	259	254
R-squared	0.0986	0.1434	0.0156	0.0152
Number of IDs	89	89	87	85

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.2 Fixed effects estimation results

haul distance also weakly contributes to services RPP. One possible explanation is that passengers desire to purchase extra services such as spa and massage if they are flying for a longer distance. The percentage of LCC passengers negatively contributes to total non-aeronautical RPP. This is consistent with Castillo-Manzano (2010) who argues that LCC passengers have a lower likelihood to purchase food and beverage, while Graham (2018) observes that LCC passengers consume more food and beverage.

To test whether dwell time is endogenous with revenues, we employ a 2-stage least squares (2SLS) technique and introduce an instrumental variable—average delay of departure flights. On the one hand, delays of flights likely increases the dwell time of passengers. On the other hand, delays of flights should not correlate with airport non-aeronautical revenues. We compare the results of 2SLS and the panel regression with fixed effects model by running a Hausman test. As shown in Table 4.3, the null hypothesis of Hausman test for retail RPP is weakly rejected at 90% significance, while it is accepted for all other revenues. The results suggest that the two regressions are not systematically different and that dwell time is not significantly endogenous with respect to revenues per passenger.

	(1)	(2)	(3)	(4)
VARIABLES	Non-aeronautical RPP (log)	Food/beverage RPP (log)	Retail stores/dutyfree RPP (log)	Services RPP (log)
Chi2	6.25	0.10	10.99	1.85
Prob>Chi2	0.3963	1.0000	0.0887	0.9329

Test of H0: Difference in coefficients not systematic

Table 4.3 Hausman test results between the panel regression with fixed effects and the 2SLS model with instrumental variable

4.4.2 The role of airport layout

To take the layout design of terminals into consideration, we group airports according to their shapes of terminals. There are 23 linear shape airports, 38 finger pier shape airports and 28 concourse shape airports. We run panel regression with fixed effects for each of the three groups of airports separately and summarize the results in Table 4.4. It shows that dwell time significantly contributes to food and beverage consumption for linear and finger pier airports and contributes to retail sales for concourse airports.

	Variables	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
L	Dwell time (log)	0.2008	1.8575**	0.0771	0.4385
		(0.2158)	(0.8456)	(0.3934)	(1.3249)
F	Dwell time (log)	0.1575	0.9336***	0.9195	-1.5328
		(0.2905)	(0.3299)	(0.8966)	(1.8775)
C	Dwell time (log)	0.0348	0.0645	0.7192*	-2.1710
		(0.1039)	(0.3027)	(0.4217)	(1.9863)

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.4 Estimation results under three airport layouts

More specifically, 1% increase in dwell time contributes to 1.9% increase in food and beverage for linear airports and 0.9% for finger pier airports. Linear design airports have shorter walking distance with clear orientation (Chen et al., 2020) and they generally provide the most efficient configuration to minimize passengers' walking distance (De Neufville et al., 2002). It is likely that with more dwell time, passengers explore food and beverage options since linear design of airports is efficient from the aspect of logistics (De Neufville et al., 2002) and such design enables passengers to come back to their boarding gates efficiently. In finger pier designed airports, which normally consists of multiple linear hallways, passengers likely walk for a longer distance comparing to the linear design. Nevertheless, passengers still have clear understanding of the terminal layout since finger pier designed airports concentrate flows in a single space. Hence, passengers opt for food and beverage consumption if they have longer dwell time. By contrast to linear and finger pier airports, airports with concourse designs split flows into different concourses which function as independent but smaller scale terminals. It is shown that dwell time of passengers in such airports does not have significant impacts on food and beverage RPP and weak impact on retail sales. On the one hand, the complexity of terminal design may not encourage passengers to explore food and beverage consumption as the perception of the risk of missing flights may be higher than that in a simple designed airport. On another hand, footfall is generally believed to improve retails due to penetration rates (Chen et al., 2020). Passengers have longer walking distance in concourse designed airports and they get exposed to more retail opportunities. It is observed that 1% increase in dwell time leads to 0.7% increase in retail sales. As the three groups of airports vary in the impacts of dwell time, layout of airports is a critical factor to consider when analyzing airport commercial revenues.

4.4.3 Seemingly unrelated regressions

We first run a seemingly unrelated regressions (SUR) model contains multiple linear regressions simultaneously. Such model is often used to deal with contemporaneous correlation across equations and in our study, airport revenues may be complementary or substitutable to each other since passengers can normally contribute to one specific category of revenues at each moment. Table 4.5 shows the results with fixed effects of airports. Dwell time significantly affects the food and beverage revenues as well as retail sales per passenger. Among control variables, LCC seats significantly reduce revenues per passenger while haul distance of flights positively contributes to revenues.

4.4.4 Robustness checks

Table 4.6 shows the results of a pooled ordinary least squares regression, without fixed effects of airports. Dwell time significantly contributes to both food and beverage RPP and retail RPP. LCC seats and connecting passengers significantly reduce the total non-aeronautical revenues per passengers while increasing food and beverage consumption.

Considering that not all airports sign revenue sharing contracts with retailers but instead, adjust the concession fees the year after according to the real revenues made by retailers, we run a regression model with lagged independent variables. In Table 4.7, we can observe that dwell time only weakly contributes to retail sales per passenger and their percentage of connecting seats weakly contributes to food and beverage revenue per passenger. The LCC seats share negatively affect the total non-aeronautical revenue per passenger. Apart from minor differences, the lagged model is consistent with the general model, although the panel data only contains two years

Variables	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.0476	0.4199**	0.4060**	-0.7369
	(0.0613)	(0.2078)	(0.1724)	(0.9270)
Visitor income (log)	0.3508	1.2409	-0.7444	0.9730
	(0.2651)	(0.8984)	(0.7453)	(4.0079)
Flight count (log)	-0.0562***	0.0447	-0.0144	0.2126
	(0.0153)	(0.0518)	(0.0430)	(0.2310)
Connect Pax	-0.7409*	0.9253	-0.7762	5.2897
	(0.3965)	(1.3438)	(1.1147)	(5.9947)
Haul distance (log)	0.7291***	1.0587***	0.9448***	1.1734
	(0.0969)	(0.3285)	(0.2725)	(1.4653)
LCC passenger	-0.7727***	-1.4312***	-1.0633**	-2.0979
	(0.1615)	(0.5474)	(0.4541)	(2.4422)
Constant	10.5426***	-0.5340	8.9404**	1.2102
	(1.2359)	(4.1888)	(3.4748)	(18.6868)
Observations	253	253	253	253
R-squared	0.9879	0.9489	0.9865	0.7990

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.5 Estimation results: SUR model with fixed effects

Variables	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.1361	0.6345**	1.4849***	-0.4947
	(0.1336)	(0.2498)	(0.4454)	(0.7348)
Visitor income (log)	0.0242	0.8293**	0.7467**	-0.3652
	(0.2024)	(0.3291)	(0.3423)	(0.7236)
Flight count (log)	-0.0362	0.0012	-0.0391	0.1460
	(0.0472)	(0.0879)	(0.0885)	(0.1785)
Connect Pax	-1.0562***	0.8964**	0.0549	-1.5570*
	(0.2086)	(0.3754)	(0.2972)	(0.8088)
Haul distance (log)	-0.0334	-0.1009	0.6110***	0.2348
	(0.0653)	(0.1456)	(0.1575)	(0.2676)
LCC passenger	-0.3815***	0.3215**	-0.0893	-0.9087**
	(0.0772)	(0.1511)	(0.1769)	(0.4199)
Constant	16.5473***	7.7688***	-0.3709	13.6446***
	(0.9644)	(1.5348)	(2.8476)	(4.0426)
Observations	265	264	259	254
R-squared	0.3913	0.1669	0.1692	0.0522
Number of id				

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.6 Estimation results: OLS

of data using lagged form of independent variables, which is a quite short panel.

Variables	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time lag (log)	0.0578	0.0191	1.0020*	-0.3763
	(0.1275)	(0.4440)	(0.5242)	(0.8948)
Visitor income lag (log)	0.1787	1.1776	5.8156	-4.1565
	(0.3590)	(1.2202)	(7.6608)	(5.9102)
Flight count lag (log)	0.0376	0.1167	-0.2529	0.2124
	(0.0284)	(0.1169)	(0.2327)	(0.2157)
Connecting seats lag	-1.2938*	1.5760*	0.2552	25.6475
	(0.7286)	(0.8622)	(2.0843)	(23.9263)
Haul distance lag (log)	-0.0241	0.4618	-0.5083	0.3451
	(0.0775)	(0.3575)	(0.8482)	(0.5752)
LCC seats lag	-0.7578**	-1.2600	3.4917	-3.5046
	(0.3248)	(0.8160)	(2.8108)	(3.6616)
Constant	15.5506***	4.2264	-10.9380	25.7019
	(1.7115)	(5.8839)	(28.6634)	(25.9150)
Observations	176	176	172	169
R-squared	0.1408	0.1188	0.0181	0.0526
Number of id	89	89	87	85

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.7 Impacts of lagged variables: panel regression with fixed effects

4.5. The value of additional passenger characteristics

Thus far, passenger characteristics included only two features, dwell time and income levels. For a subset of airports, 55 in total, Placer.ai provided additional features such as the day of visits and the time of visits. While the list of features is long, given the limitations of our panel, we focus on four measures: *VisitsMonThu* indicates the percentage of visits on weekdays with Friday excluded. *VisitsLunch* shows the percentage of visits at noon and *PriorHome* tells the percentage of visits with prior location to be home. The last variable, *Elderly*, indicates the percentage of elderly passengers who are above 65.

Variables	Definitions	Source	N	mean	sd
Monday-Thursday visits	The percentage of visits between Monday and Thursday	Placer.ai	55	56.82%	1.21%
Lunch Visits	The percentage of visits between 11am and 2pm	Placer.ai	55	16.99%	2.27%
Visits from home	The percentage of visits coming from home	Placer.ai	55	36.93%	7.89%
Age¿=65	The percentage of visitors with age ¿= 65	Placer.ai	55	13.07%	3.40%

Table 4.8 Summary of additional passenger data

With these additional features, we conduct further analyses to explore how passenger visits contribute to revenues as shown in (4.2):

$$\begin{aligned}\ln RPP_{it} =& \beta_0 + \beta_1 \ln DwellTime_{it} + \beta_2 \ln VisitorIncome_{it} \\ &+ \beta_3 \ln FlightCount_{it} + \beta_4 Connecting_{it} + \beta_5 \ln Haul_{it} \\ &+ \beta_6 LCC_{it} + \beta_7 VisitsMonThu_{it} + \beta_8 VisitsLunch_{it} \\ &+ \beta_9 PriorHome_{it} + \beta_{10} Elderly_{it} + \omega_i + \epsilon_{it}\end{aligned} \tag{4.2}$$

Variables	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.0100	0.2878	0.6779*	0.1545
	(0.1168)	(0.3727)	(0.3821)	(1.0852)
Visitor income (log)	-0.2436	1.7216	-1.5046	-1.6840
	(0.4846)	(1.3774)	(1.5514)	(11.4019)
Flight count (log)	-0.0196	-0.0054	-0.1076	-0.1001
	(0.0396)	(0.0725)	(0.0768)	(0.3250)
Connect Pax	-0.8607*	0.2766	-1.3864	9.3805
	(0.4350)	(0.9893)	(1.3045)	(16.7149)
Haul distance (log)	1.0141***	1.4088***	1.0743***	1.3008*
	(0.3385)	(0.3990)	(0.3897)	(0.6695)
LCC passenger	-0.5928*	-1.7320**	-0.4705	-1.7223
	(0.3108)	(0.6579)	(0.6806)	(2.6298)
Visits Mon-Thu	2.5134	6.9160**	3.4417	7.1596
	(1.6849)	(3.2380)	(4.7706)	(12.0902)
Visits lunch	0.1762	2.2747*	-0.4555	0.7674
	(0.7298)	(1.1816)	(1.6969)	(4.8082)
Visits from home	-0.0328	-0.1256	1.3923	4.2725
	(0.4410)	(0.9821)	(1.1882)	(10.4435)
Age≥65	1.3145	0.7426	-3.0601	2.8626
	(0.7856)	(1.6178)	(2.0580)	(11.8074)
Constant	8.7503***	-9.1173	8.6504	3.6314
	(3.2203)	(5.7133)	(6.4289)	(37.7051)
Observations	165	165	164	162
R-squared	0.3848	0.2354	0.1691	0.0145
Number of id	55	55	55	54

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.9 Estimation results with additional visit data: panel regression with fixed effects

The results are provided by including additional variables of visits, the weak significance of dwell time in explaining food and beverage revenues per passenger in Table 4.2 is not observed in Table 4.9, while dwell time has weak impacts on retail sales. Among the new features, the percentage of visits from Monday to Thursday positively contributes to food and beverage revenues per passenger. The positive and weakly significant coefficient of the percentage lunch visits indicates that passengers go for lunch at airports. We can also observe that elderly passengers do not necessarily consume less than other age groups possibly because elderly passengers have high purchasing power (Perng et al., 2010) with low purchasing intention.

4.6. Conclusions

It is widely assumed that the longer passengers stay, the more revenues airports can make but this assumption lacks statistical evidence. This paper explores whether and how passenger dwell time affects airport non-aeronautical revenues. We analyze data from users who grant a mobile app the access to their footprints and demographics information, focusing on 89 U.S airports from 2017 to 2019. Apart from the footprint data, we also collect airport financial data as well as airport flight information. To analyze the panel data, we apply a fixed effects regression model to quantify how dwell time affects different types of airport revenues, taking passenger demographics information, airport layout and airport flight metrics into consideration.

Our regressions on 89 airports show that dwell time is positively but weakly contributes to food and beverage RPP while it has no significiant impacts on retail or service RPP. By grouping airports according to their terminal design layouts, we find that dwell time significantly and positively increases food and beverage RPP for airports with logistics efficiency, namely, linear design and finger pier design, whereas dwell time positively contributes to retail RPP for airports with multiple concourses. Overall speaking, our findings validate the assumption that dwell time increases airport non-aeronautical revenue per passenger but the impacts vary with airport terminal design.

Our findings are not only important for understanding passenger behaviors but also relevant for improving airport commercial operations. For instance, by observing that dwell time positively contributes to airport revenues, managers may come up with ways to increase passenger dwell time to enhance airport commercial performance. However, longer dwell time may lead to logistical inefficiency. Passengers who value logistical efficiency may opt for alternative transportation and ultimately the aeronautical revenues may get negatively affected. In this case, managers should be aware of the trade-off between non-aeronautical benefits and aeronautical profits. Additionally, the impacts of dwell time vary with airport designs and managers may take advantages of passengers' preference toward spending time at terminal. For example, in a linear design terminal, passengers are willing to go to restaurants given more dwell time since they are confident to reach back to the gate for boarding. Hence, for linear design airports, it is less critical to locate restaurants next to gates while retail shops can be located near gates to increase the penetration rate. Whereas for concourse design airports, it is critical to locate restaurants near gates to potentially reduce the perception of risks of missing flights due relatively complex terminal design. Apart from that, the multi-source dataset itself, on which our research is based, including passenger dwell time, contributes to the literature as many existing research focus on limited size of passengers and airports. One limitation of the present paper is the short period of panel data. A longer panel data would enable the exploration of long-term impacts of dwell time on airport revenues. For instance, it is of interest to investigate whether passenger dwell time affects aeronautical revenues in the long run. Apart from that, the lack of real sales data, including restaurant revenues and duty-free revenues, requires a more in-depth study to ascertain the effects of dwell time. Additionally, we outline some avenues for future research, such as: 1) how does COVID-19 have influenced dwell time and thus revenues? 2) what happens in different contexts (e.g., non-US) or in case of non private airports? 3) what is the trade-off of offering a faster service to passengers and revenues? The present research then provides a foundation for these more fine-grained studies if the required data becomes available.

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4.A. Tables

VARIABLES	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.2008	1.8575**	0.0771	0.4385
	(0.2158)	(0.8456)	(0.3934)	(1.3249)
Visitor income (log)	0.9966	-0.2798	0.4565	4.8719
	(0.5928)	(2.9061)	(1.9388)	(3.3964)
Flight count (log)	-0.0463	0.2861	0.0943	-0.2577*
	(0.0450)	(0.3513)	(0.1167)	(0.1390)
Connect Pax	0.2677	-0.9351	1.7461	-3.0191
	(0.7461)	(1.9948)	(2.3095)	(4.7430)
Haul distance (log)	-0.0938	-1.6206	-0.6398	1.2178
	(0.3431)	(2.2019)	(0.5852)	(1.8637)
LCC passenger	-1.0442**	-2.7889	-1.4515***	-2.8209*
	(0.3841)	(2.3212)	(0.4830)	(1.5682)
Constant	12.9596***	16.3559	14.9405	-14.3678
	(2.7506)	(16.0249)	(8.7303)	(15.8753)
Observations	68	68	68	65
R-squared	0.3309	0.2207	0.2102	0.1948
Number of id	23	23	23	22

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.10 Estimation results for linear shape airports(panel regression with fixed effects)

VARIABLES	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.1575	0.9336***	0.9195	-1.5328
	(0.2905)	(0.3299)	(0.8966)	(1.8775)
Visitor income (log)	0.5419	-0.5142	0.3347	6.2941
	(0.4844)	(1.2565)	(4.3232)	(4.1823)
Flight count (log)	-0.0387	-0.0094	-0.4699	0.8983
	(0.0350)	(0.0677)	(0.5171)	(0.6730)
Connect Pax	-0.7093	-1.6240	-6.2268	-23.0547*
	(1.1670)	(1.0326)	(5.3904)	(12.3413)
Haul distance (log)	-0.1244	-0.6392	0.1867	-2.7669
	(0.3321)	(0.8445)	(2.1002)	(2.7534)
LCC passenger	0.0858	-0.1811	11.1135	2.2209
	(0.6477)	(0.7258)	(13.0202)	(3.2872)
Constant	14.6661***	16.6102*	7.4808	3.7549
	(2.2852)	(8.8730)	(17.7987)	(22.1963)
Observations	113	112	107	105
R-squared	0.0148	0.1559	0.0555	0.2392
Number of id	38	38	36	35

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.11 Estimation results for finger pier shape airports(panel regression with fixed effects)

VARIABLES	(1) Non-aeronautical RPP (log)	(2) Food/beverage RPP (log)	(3) Retail stores/dutyfree RPP (log)	(4) Services RPP (log)
Dwell time (log)	0.0348	0.0645	0.7192*	-2.1710
	(0.1039)	(0.3027)	(0.4217)	(1.9863)
Visitor income (log)	0.2759	1.6886	2.8055	-15.7786
	(0.5386)	(1.5561)	(1.8506)	(15.5938)
Flight count (log)	-0.0153	0.0776	-0.0978	0.3761
	(0.0494)	(0.0657)	(0.0841)	(0.5569)
Connect Pax	-1.0773*	1.4630	-0.0074	17.7344
	(0.5297)	(1.1519)	(1.4889)	(17.8420)
Haul distance (log)	1.0429***	1.6979***	1.3200***	1.4233***
	(0.3188)	(0.1268)	(0.2967)	(0.4758)
LCC passenger	-0.9405*	-1.4860***	-0.3985	-3.3999
	(0.5060)	(0.4921)	(1.0317)	(4.6846)
Constant	7.7166**	-7.5841	-10.4750	72.6060
	(2.9678)	(6.9527)	(8.2411)	(67.4418)
Observations	84	84	84	84
R-squared	0.5202	0.5137	0.3075	0.0714
Number of id	28	28	28	28

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.12 Estimation results for concourse shape airports(panel regression with fixed effects)

Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) Avg departure delay	1.0000					
(2) Dwell_time	0.1350	1.0000				
(3) Non-aeuronautical RPP	0.0650	-0.3330	1.0000			
(4) Food & beverage RPP	-0.0740	0.2940	-0.0110	1.0000		
(5) Retail RPP	0.0840	0.4090	-0.0780	0.2670	1.0000	
(6) Service RPP	0.0150	0.0540	0.2150	0.1530	0.3780	1.0000

Table 4.13 Correlation

Chapter 5

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