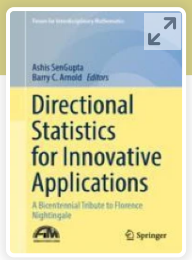

Directional Statistics for Innovative Applications pp 161–186

Generalized Skew-Symmetric Circular and Toroidal Distributions

Andriette Bekker, Najmeh Nakhaei Rad, Mohammad Arashi & Christophe Ley

Chapter | First Online: 16 June 2022

Part of the Forum for Interdisciplinary Mathematics book series (FFIM)

Abstract

Existing circular and toroidal distributions are mostly symmetric; however, many datasets possess asymmetric patterns. Due to the increasing need for asymmetric distributions in recent times, driven by complex modern datasets, in this chapter a new approach is introduced to generate skewed distributions from symmetric distributions, for modeling both circular and toroidal skewed data. This new family of asymmetric distributions, called *generalized skew-symmetric* distributions, includes some well-known distributions as special cases, such as the circular models of Umbach

and Jammalamadaka (Stat Probab Lett 79:659–663, 2009) [24] and Abe and Pewsey (Stat Pap 52:683–707) [1] and the toroidal model of Ameijeiras-Alonso and Ley (Biostatistics, 2020. https://doi.org/10.1093/biostatistics/kxaa039) [2]. General properties of the new models are studied, and we see that the proposed distributions are able to provide wider ranges of skewness as their competitors. To illustrate the practical implementation and usefulness of our new general skewing approach, we compare our models to competitors from the literature on several real datasets.

References

1. Abe, T., Pewsey, A.: Sine-skewed circular distributions. Stat. Pap. **52**(3), 683–707 (2011)

2. Ameijeiras-Alonso, J., Ley, C.: Sine-skewed toroidal distributions and their application in protein bioinformatics. Biostatistics (2020). https://doi.org/10.1093/biostatistics/kxaa039

3. Ameijeiras-Alonso, J., Ley, C., Pewsey, A., Verdebout, T.: On optimal tests for circular reflective symmetry about an unknown central direction. Stat. Pap. (2019). https://doi.org/10.1007/s00362-019-01150-7

4. Ardia, D., Mullen, K., Peterson, B., Ulrich, J., Boudt, K.: DEoptim: global optimization by differential evolution.

R package version 2.2-5 (2020)

5. Azzalini, A.: A class of distributions which includes the normal ones. Scand. J. Stat. **12**, 171–178 (1985)

6. Azzalini, A., Capitanio, A.: Distributions generated by perturbation of symmetry with emphasis on a multivariate skew-t distribution. J. Roy. Stat. Soc.: Ser. B **65**, 367–389 (2003)

7. Azzalini, A., Capitanio, A.: The Skew-Normal and Related Families. IMS Monographs, Cambridge University Press, Cambridge (2014)

8. Balakrishnan, N.: Discussion on "Skew multivariate models related to hidden truncation and/or selective reporting" by B. C. Arnold and R. J. Beaver. Test **11**, 37–39 (2002)

9. Buttarazzi, D., Pandolfo, G., Porzio, G.C.: A boxplot for circular data. Biometrics **74**(4), 1492–1501 (2018)

10. Huang, W.J., Chen, Y.H.: Generalized skew-Cauchy distribution. Stat. Probab. Lett. **77**(11), 1137–1147 (2007)

11. Jander, R.: Die optische Richtungsorientierung der Roten Waldameise (*Formica ruea* L.). Zeitschrift für vergleichende Physiologie, **40**(2), 162–238 (1957)

12. Jupp, P.E., Regoli, G., Azzalini, A.: A general setting for symmetric distributions and their relationship to general distributions. J. Multivariate Anal. **148**, 107–119 (2016)

13. Ley, C., Babić, S., Craens, D.: Flexible models for complex data with applications. Annu. Rev. Stat. Appl. (2020)

14. Ley, C., Verdebout, T.: Simple optimal tests for circular reflective symmetry about a specified median direction. Stat. Sinica **24**, 1319–1339 (2014)

15. Ley, C., Verdebout, T.: Skew-rotationally-symmetric distributions and related efficient inferential procedures. J. Multivariate Anal. **159**, 67–81 (2017)

16. Ley, C., Verdebout, T.: Modern Directional Statistics. Chapman and Hall/CRC Press, Boca Raton, Florida (2017)

17. Liu, D., Peddada, S.D., Li, L., Weinberg, C.R.: Phase analysis of circadian-related genes in two tissues. BMC Bioinf. **7**(1), 77–87 (2006)

18. Mardia, K.V.: Statistics of Directional Data. Academic Press, New York (1972)

19. Meintanis, S., Verdebout, T.: Le Cam maximin tests for symmetry of circular data based on the

characteristic function. Stat. Sinica **29**, 1301–1320 (2019)

20. Pewsey, A.: Testing circular symmetry. Can. J. Stat. **30**(4), 591–600 (2002)

21. Price, K., Storn, R.M., Lampinen, J.A.: Differential Evolution: A Practical Approach to Global Optimization. Springer Science and Business Media, Berlin (2006)

22. SenGupta, A., Ong, S.H.: A unified approach for construction of probability models for bivariate linear and directional data. Commun. Stat.-Theory Methods **43**, 2563–2569 (2014)

23. Storn, R., Price, K.: Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. J. Global Optim. **11**(4), 341–359 (1997)

24. Umbach, D., Jammalamadaka, S.R.: Building asymmetry into circular distributions. Stat. Probab. Lett. **79**(5), 659–663 (2009)

Acknowledgements

Christophe Ley's research is supported by the FWO Krediet aan Navorsers grant with reference number 1510391N. This work was further based upon research supported in part by the National Research Foundation (NRF) of South Africa, SARChI Research Chair UID:

71199; Ref.: IFR170227223754 grant No. 109214; Ref.: SRUG190308422768 Grant No. 120839, and DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS), South Africa. The opinions expressed and conclusions arrived at are those of the authors and are not necessarily to be attributed to the CoE-MaSS and the NRF. We would like to thank the reviewers for their thoughtful comments and efforts toward improving our manuscript.

Author information

Authors and Affiliations

Department of Statistics, University of Pretoria, Pretoria, 0002, South Africa

Andriette Bekker, Najmeh Nakhaei Rad & Mohammad Arashi

Department of Mathematics and Statistics, Mashhad Branch, Islamic Azad University, Mashhad, Iran

Najmeh Nakhaei Rad

DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS), Johannesburg, South Africa

Najmeh Nakhaei Rad

Department of Statistics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran

Mohammad Arashi

Department of Applied Mathematics, Computer Science and Statistics, Ghent University, 9000, Ghent, Belgium

Christophe Ley

Department of Mathematics, University of Luxembourg, Esch-sur-Alzette, Luxembourg

Christophe Ley

Corresponding author

Correspondence to Christophe Ley .

Editor information

Editors and Affiliations

Applied Statistics Unit, Indian Statistical Institute, Kolkata, West Bengal, India

Prof. Ashis SenGupta

Department of Statistics, University of California, Riverside, CA, USA

Prof. Barry C. Arnold

Rights and permissions

Copyright information

About this chapter

Cite this chapter

Bekker, A., Nakhaei Rad, N., Arashi, M., Ley, C. (2022). Generalized Skew-Symmetric Circular and Toroidal Distributions. In: SenGupta, A., Arnold, B.C. (eds) Directional Statistics for Innovative Applications. Forum for Interdisciplinary Mathematics. Springer, Singapore. https://doi.org/10.1007/978-981-19-1044-9_9

DOI	Published	Publisher Name
https://doi.org/10.1007/978-981-19-1044-9_9	16 June 2022	Springer, Singapore

Print ISBN	Online ISBN	eBook Packages
978-981-19-1043-2	978-981-19-1044-9	Mathematics and Statistics Mathematics and Statistics (R0)