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Maximum likelihood characterization of rotationally symmetric distributions on the sphere

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Abstract

A classical characterization result, which can be traced back to Gauss, states that the maximum likelihood estimator (MLE) of the location parameter equals the sample mean for any possible univariate samples of any possible sizes n if and only if the samples are drawn from a Gaussian population. A similar result, in the two-dimensional case, is given in von Mises (1918) for the Fisher–von Mises–Langevin (FVML) distribution, the equivalent of the Gaussian law on the unit circle. Half a century later, Bingham and Mardia (1975) extend the result to FVML distributions on the unit sphere $(\mathcal{S}^{k-1} := \{\mathbf{v} \in \mathbb{R}^k : \|\mathbf{v}\| = 1\})$, $k \geq 2$. In this paper, we present a general MLE characterization theorem for a large subclass of rotationally symmetric distributions on (\mathcal{S}^{k-1}) , $k \geq 2$, including the FVML distribution.

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