RKHS based Dynamic State Estimator for non-Gaussian Radar Measurements

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***Abstract*—In the case of a non-linear system, the dynamic state of the targets (position, velocity, and acceleration) is estimated by an extended Kalman filter (EKF). The theory of EKF is established on the assumption that measurements follow Gaussian distribution. However, in practice, this assumption falls short and limits the application of EKF. In literature, to deal with the non-Gaussianity, the maximum correntropy criterion (MCC)-based EKF (EKF-MCC) has been studied well. The MCC, an information-theoretic criterion, claims to effectively deal with the system's non-Gaussianity. Nevertheless, like EKF, EKF-MCC also approximates the known system non-linearity with a Jacobian. The Jacobian provides the first-order approximation of the non-linearity and hinders the estimation accuracy achieved by EKF-MCC, particularly for complex target motion models. Therefore, in this work, firstly, we propose to use EKF-MCC for estimating the dynamic state of the target from non-Gaussian measurement. After that, utilizing MCC, we propose reproducing kernel Hilbert space (RKHS) based non-linear estimation of system non-linearity and using it with EKF-MCC. Amid non-linear estimation utilizing MCC, the proposed filter is named EKF-MCC-RKHS. The simulation performed to estimate the dynamic states of the complex constant acceleration (CA) target motion model validates the superiority of EKF-MCC-RKHS over recently introduced EKF-MCC and traditional EKF.**

***Index Terms*—EKF, EKF-MCC, EKF-MCC-RKHS, MCC, RKHS**

I. Introduction

With the rapid increase of radar sensors for surveillance and monitoring applications, adequate and accurate tracking demands have also increased. The Kalman filter (KF) plays a crucial role in tracking [1]–[3]. The KF filters the noisy radar measurements and estimates the target's dynamic state (position, velocity, and acceleration). In case of miss detections and sensor irregularities, the KF provides the predicted measurements to maintain the tracks. The KF is mainly used when the measurement function relating the measurements and the dynamic states is linear. However, a practical radar sensor provides the measurements in spherical coordinates, typically consisting of radial range, azimuth, and elevation. In this case, along with filtering, the dynamic state estimator has to convert the spherical coordinate to a Cartesian coordinate. The conversion from spherical to Cartesian coordinate is a nonlinear operation; therefore, the nonlinear extension of KF, the extended Kalman filter (EKF), is used [2], [3]. The utility of EKF in practical tracking applications is limited by the fact that the nonlinear measurement function is approximated by Jacobian, which effectively is a first-order approximation. The Jacobian approximation limits the accuracy achieved by EKF, particularly for estimating dynamic states for complex target motion models like constant acceleration (CA). Also, for most applications, the exact measurement function is unknown; for instance, not all radar sensors need to provide measurements in the same format; consequently, Jacobian for one sensor would not work for the other. Targeting the abovementioned issues in EKF, in [4], [5], reproducing kernel Hilbert space-based EKF (EKF-RKHS) has been proposed and validated for different target motion models.

In EKF-RKHS, the nonlinear measurement function is estimated at each time instant with the kernel recursive least squares (KRLS) method [6], which is essentially an RKHS-based nonlinear estimation technique. Afterward, the estimated measurement function is plugged into EKF, making the EKF-RKHS independent of knowing the exact non-linearity. Particularly in [5], the EKF-RKHS-based state estimator has been validated for different target motion models, and it has been shown that because of estimating the measurement function instead of approximating with Jacobian, EKF-RKHS achieves far better estimation accuracy as compared to EKF.

The theory of EKF and its advanced version EKF-RKHS is based on the assumption that the measurement follows the Gaussian distribution. The Gaussian assumption is made in the literature to ease the mathematical analysis of EKF and EKF-RKHS. However, the measurement doesn't need to be Gaussian distributed. Because the measurement noise is effectively the estimation noise that cannot be Gaussian distributed with a guarantee. Also, various surrounding effects around the radar sensor and random movement of the targets question the traditional Gaussian assumption for measurements. In light of the above discussion, the EKF and hence the EKF-RKHS performance will drastically degrade subject to the non-Gaussian distributed measurements. In [7], to make the EKF capable of dealing with the effects of non-Gaussianity, the maximum correntropy criterion (MCC)-based EKF (EKF-MCC) has been proposed. The EKF utilizes the minimum mean square error (MMSE) criterion for estimating unknown dynamic states and optimizes only the second-order error statistics, essentially an error covariance. However, unlike EKF, EKF-MCC uses MCC, an information-theoretic criterion considering the higher-order statistics for the estimation error.

In EKF-MCC, the consideration of higher-order error statistics gives an upper hand over EKF to minimize the effect of non-Gaussianity.

Nevertheless, like EKF, the implementation of the EKF-MCC requires the exact knowledge of the nonlinear measurement function. Even when the precise non-linearity is known, in EKF-MCC, the Jacobian is used, which provides the first-order approximation of the system non-linearity. Consequently, like EKF, the dependence of EKF-MCC on the exact knowledge of nonlinear measurement function and its approximation with Jacobian hinders the versatility and estimation accuracy achieved by EKF-MCC. Therefore, in the continuation of the research related to EKF-RKHS, in this work, we propose to estimate the nonlinear measurement function using reproducing kernel Hilbert's space (RKHS) based algorithm [8]. Subsequently, the estimated measurement function replaces the Jacobian in EKF-MCC; the filter is named EKF-MCC-RKHS. In EKF-MCC-RKHS, the non-linearity is estimated by the kernel recursive maximum correntropy (KRMC) algorithm, which, to deal with non-Gaussianity, utilizes MCC [9]. The summarized contribution of the proposed work are:

- The EKF-MCC is proved to be a suitable choice under measurements non-Gaussianity.
- The KRMC algorithm estimates the system non-linearity and replaces the Jacobian in EKF-MCC; hence achieves better estimation accuracy. The combined proposed filtering (EKF-MCC and RKHS-based KRMC) is termed EKF-MCC-RKHS.
- The EKF-MCC-RKHS because of using KRMC is no longer dependent on the exact knowledge of nonlinear measurement function. Moreover, MCC makes the proposed EKF-MCC-RKHS filter robust against measurement non-Gaussianity.

The organization of the paper is as follows. In Section II, the problem statement is described in detail. Next, Section III, covers the description of the proposed filtering scheme. Further, in section IV, simulations are performed over the practical target motion model and comparative conclusions are drawn between classical EKF, EKF-MCC, and EKF-MCC-RKHS. Lastly, conclusions are drawn in Section V.

Notations: Scalar variables (constants) are denoted by lower (upper) case letters. Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(.)^T$, $(.)^H$ and $(.)^*$ denote matrix/vector transpose, complex conjugate transpose, and scalar complex conjugation operation respectively. $\mathbb{E}[.]$ denotes statistical expectation and $\mathbb{R}$ denote the set of real numbers. $\mathbf{I}_n$ denotes the identity matrix of cardinality n and $\otimes$ denotes the Kronecker product.

II. Problem Statement

Let at the k^{th} time index of coherent pulse interval (CPI), the radar sensor reports the noisy measurement vector $\mathbf{y}_k \in \mathbb{R}^{n_m \times 1}$ of cardinality n_m from the target, moving in 3D space. The $\mathbf{y}_k$ either consist of the estimates of radial range ($\hat{r}_k$), azimuth ($\hat{\phi}_k$), and elevation ($\hat{\theta}_k$) or $\hat{r}_k$, radial velocity ($\hat{v}_k$), and $\hat{\phi}_k$. Observing $\mathbf{y}_k$, the primary objective is to estimate the state of the target $\mathbf{s}_k \in \mathbb{R}^{n_s \times 1}$ of cardinality n_s consisting of various target attributes. The evolving target's state w.r.t k can be modeled as

$$\mathbf{s}_k = \mathbf{f}(\mathbf{s}_{k-1}) + \mathbf{u}_k, \tag{1}$$

where, $\mathbf{s}_{k-1}$ is the target's state vector at $k-1^{st}$ instant, $\mathbf{f}(\cdot) \in \mathbb{R}^{n_s \times 1}$ governs how $\mathbf{s}_{k-1}$ evolves with k, and $\mathbf{u}_k \in \mathbb{R}^{n_s \times 1}$ modeling the error in the evolution of $\mathbf{s}_k$.

Consequently, $\mathbf{y}_k$ in terms of $\mathbf{s}_k$ is modeled as

$$\mathbf{y}_k = \mathbf{h}(\mathbf{s}_k) + \mathbf{w}_k, \tag{2}$$

where $\mathbf{h}(\cdot) \in \mathbb{R}^{n_m \times 1}$ is the non-linear measurement function relating $\mathbf{s}_k$ and $\mathbf{y}_k$, and $\mathbf{w}_k \in \mathbb{R}^{n_m \times 1}$ is the error modeling all types of estimation and surrounding noises.

The various attributes of the targets contained in $\mathbf{s}_k$ depend upon the considered target motion model. In this work, we have considered the CA target motion model. In CA, $\mathbf{s}_k$ consists of target positions, velocities and accelerations across x, y, and z coordinates. As a result, the total 9 parameters must be estimated, and CA covers the complete knowledge about the targets' kinematics. In CA motion model, the $\mathbf{s}_k$ is represented as

$$\mathbf{s}_k = [x_k, v_{xk}, a_{xk}, y_k, v_{y_k}, a_{y_k}, z_k, v_{zk}, a_{zk}]^T,$$

where, x_k, y_k, and z_k, v_{xk}, v_{y_k}, and v_{zk}, and a_{xk}, a_{y_k}, and a_{zk} are the target's positions, velocities, and accelerations, respectively moving in 3D space.

The $\mathbf{f}(s_{k-1})$ is given by

$$\mathbf{f}(\mathbf{s}_{k-1}) = [\mathbf{I}_3 \otimes \mathcal{F}]\mathbf{s}_{k-1},$$

where $\mathcal{F} = \begin{bmatrix} 1 & T_{\text{cpi}} & 0.5T_{\text{cpi}}^2 \\ 0 & 1 & T_{\text{cpi}} \\ 0 & 0 & 1 \end{bmatrix}$, and T_{cpi} is the CPI.

In this work, $\mathbf{u}_k$ is considered to be Gaussian distributed with zero mean vector and known covariance matrix $\mathbf{Q}_u = \mathbb{E}[\mathbf{u}_k {\mathbf{u}_k}^T]$, s.t. $\mathbf{u}_k \sim \mathcal{N}_{\mathbb{R}}(\mathbf{0}, \mathbf{Q}_u)$

$$\mathbf{Q}_u = [\mathbf{I}_3 \otimes \mathcal{T}]\sigma_a^2$$

where σ_a^2 is the acceleration variance, and

$$\mathcal{T} = \begin{bmatrix} \frac{T_{\text{cpi}}^4}{4} & \frac{T_{\text{cpi}}^3}{2} & \frac{T_{\text{cpi}}^2}{2} \\ \frac{T_{\text{cpi}}^3}{2} & T_{\text{cpi}}^2 & T_{\text{cpi}} \\ \frac{T_{\text{cpi}}^2}{2} & T_{\text{cpi}} & 1 \end{bmatrix}.$$

Further, in (2), it is worth noting that $\mathbf{s}_k$ is in Cartesian coordinate and $\mathbf{y}_k$ is in spherical coordinate, consequently, the $\mathbf{h}(\cdot)$ is nonlinear. Therefore, to estimate the hidden $\mathbf{s}_k$ from observable $\mathbf{y}_k$, a nonlinear extension of KF (EKF) could be used. However, EKF uses the MMSE criterion, which considers only the second-order error statistics and is suitable only when $\mathbf{w}_k$ is assumed to be Gaussian. In the line of the above statement, as $\mathbf{w}_k$ models the estimation and surrounding noises, the Gaussianity assumption is not certain. Therefore, in practice, the EKF may perform worse when used to estimate $\mathbf{s}_k$ from non-Gaussian $\mathbf{y}_k$.

In the literature, with the objective of making EKF free of Gaussianity assumption an ample amount of research has been done. Notably, in [7], MCC is suggested to use instead of MMSE, hence the filter named EKF-MCC. Further, in literature dealing with EKF-MCC, the non-Gaussianity of $\mathbf{w}_k$ is usually modeled as a Gaussian mixture [7], [9]. Therefore, in the presented work we have consider that $\mathbf{w}_k$ follows Gaussian mixture, which has the following form

$$\mathbf{w}_k \in p_1\mathcal{N}_{\mathbb{R}}(\boldsymbol{\mu}_w^1, \mathbf{R}_w^1) + p_2\mathcal{N}_{\mathbb{R}}(\boldsymbol{\mu}_w^2, \mathbf{R}_w^2), \quad (3)$$

where $\boldsymbol{\mu}_w^g$ and $\mathbf{R}_w^g$ for $g \in [1, 2]$ are the mean vector and covariance matrix of two separate Gaussian, respectively and p_1 and p_2 are the mixing proportions with $p_1 + p_2 = 1$.

From (3), the equivalent covariance matrix of $\mathbf{w}_k$ ($\mathbf{R}_w \in \mathbb{R}^{n_m \times n_m}$) is given by

$$\mathbf{R}_w = \mathbb{E}[(\mathbf{w}_k - \boldsymbol{\mu}_w)(\mathbf{w}_k - \boldsymbol{\mu}_w)^T]$$
$$= \sum_{g=1}^{2} p_g \mathbf{R}_w^g + \sum_{g=1}^{2} p_g(\boldsymbol{\mu}_w^g - \boldsymbol{\mu}_w)(\boldsymbol{\mu}_w^g - \boldsymbol{\mu}_w)^T,$$

where $\boldsymbol{\mu}_w = \sum_{g=1}^{2} p_g \mu_w^g$ is the mean of $\mathbf{w}_k$.

III. Proposed Filtering Scheme

In this section, firstly, MCC is briefly discussed. Later, the EKF-MCC is introduced, followed by a detailed description of the proposed EKF-MCC-RKHS.

A. Maximum correntropy Criterion

The MCC maximizes the correntropy between two random variables $X \in \mathbb{R}$ and $Y \in \mathbb{R}$. If $\mathcal{P}_{X,Y}$ is the joint distribution function of X and Y, the correntropy between X and Y is given by

$$\mathcal{C}(X, Y) = \mathbb{E}(\kappa(X, Y)) = \int \kappa(x, y)\mathcal{P}_{X,Y}, \quad (4)$$

where $\kappa(\cdot, \cdot) \in \mathbb{R}$ is the Gaussian Mercer Kernel (GMK) and is defined by

$$\kappa(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right),$$

where $\|\cdot\|$ is the l_2 norm, and σ is the kernel width.

In practice, as $\mathcal{P}_{X,Y}$ is unknown, the $\mathbb{E}$ of the random variable is approximated by the available limited samples. Therefore, if N samples of X and Y are available, (4) can be estimated as

$$\hat{\mathcal{C}}(X, Y) = \frac{1}{N}\sum_{i=1}^{N} K_\sigma(e_i), \quad (5)$$

where $e_i = x_i - y_i$ is the error, and $K_\sigma(e_i) = \kappa(e_i)$.

If Y is consider to be the estimate of X, then to yield the optimum value of Y closer to X, the MCC maximizes (5) as

$$Y = \hat{X} = \max_{y_i} \frac{1}{N}\sum_{i=1}^{N} K_\sigma(e_i) \quad (6)$$

In (6), the $K_\sigma(e_i)$ can be expanded in Taylor's series as $K_\sigma(e_i) = 1 - \frac{e_i^2}{2\sigma^2} + \frac{e_i^4}{2\sigma^4} - \cdots$. Hence, it is explicit that (6) contains the higher order terms of error, or more specifically, the MCC considers the higher order statistics of the error in optimization. However, MMSE considers only the error-squared term (e_i^2) in the optimization. Considering higher order statistics makes the MCC-based filter more robust against the non-Gaussianity. Also, the σ in (6) prevents the MCC-based filter from exploding for high error. Based on this fact, next, the EKF-MCC is introduced, which utilizes MCC to estimate $\mathbf{s}_k$ from non-Gaussian $\mathbf{y}_k$.

B. EKF-MCC

Let at k^{th} time instance, the estimate of $\mathbf{s}_{k-1}$ be $\hat{\mathbf{s}}_{k-1}$ and the corresponding error covariance matrix is $\mathbf{P}_{k-1} = \mathbb{E}[(\mathbf{s}_{k-1} - \hat{\mathbf{s}}_{k-1})(\mathbf{s}_{k-1} - \hat{\mathbf{s}}_{k-1})^T]$. Similar to EKF [2], the EKF-MCC propagates $\hat{\mathbf{s}}_{k-1}$ and $\mathbf{P}_{k-1}$ iteratively and yield the final estimate of $\mathbf{s}_k$ in two steps a) prediction and b) update. In prediction step, the EKF, predicts $\mathbf{s}_k$ and $\mathbf{P}_k$ with the help of following equations:

$$\mathbf{s}_k^p = \mathbf{F}_{k-1}\hat{\mathbf{s}}_{k-1},$$
$$\mathbf{P}_k^p = \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_u, \quad (7)$$

where $\mathbf{F}_{k-1} \in \mathbb{R}^{n_s \times n_s}$ is the motion model matrix and can be obtained from (1).

After prediction, in the update step, the EKF-MCC invokes the MCC criterion, and the predictions made in (7) are updated with the help of available measurement from radar sensor ($\mathbf{y}_k$). The following equations give the updated equations of EKF-MCC:

$$\mathbf{y}_k^p = \mathbf{H}_k\mathbf{s}_k^p, \quad (8)$$
$$\mathbf{K}_k^{MCC} = \bar{\mathbf{P}}_k^p\mathbf{H}_k^T(\mathbf{H}_k\bar{\mathbf{P}}_k^p\mathbf{H}_k^T + \bar{\mathbf{R}}_w)^{-1},$$
$$\hat{\mathbf{s}}_k = \mathbf{s}_k^p + \mathbf{K}_k^{MCC}(\mathbf{y}_k - \mathbf{y}_k^p),$$
$$\mathbf{P}_k = \bar{\mathbf{P}}_k^p - \mathbf{K}_k^{MCC}\mathbf{H}_k\bar{\mathbf{P}}_k^p.$$

where $\mathbf{H}_k = \frac{\partial \mathbf{h}(\mathbf{s}_k)}{\partial \mathbf{s}_k}\Big|_{\mathbf{s}_k = \mathbf{s}_k^p} \in \mathbb{R}^{n_m \times n_s}$ is the Jacobain matrix of $\mathbf{h}(\cdot)$ (evaluated at prediction $\mathbf{s}_k^p$), $\mathbf{y}_k^p$ is the predicted measurement, $\mathbf{K}_k^{MCC}$ and $\bar{\mathbf{P}}_k^p$ is the Kalman gain and predicted error covariance matrix of EKF-MCC, respectively, and $\hat{\mathbf{s}}_k$ and $\mathbf{P}_k$ are the final estimate of $\mathbf{s}_k$ and updated error covaraince matrix, respectively.

Further, in the representation of $\mathbf{K}_k^{MCC}$, the $\bar{\mathbf{P}}_k^p$ and $\bar{\mathbf{R}}_w$ are given by

$$\bar{\mathbf{P}}_k^p = \mathbf{C}_k^p\mathbf{E}_k^{s\,-1}\mathbf{C}_k^{p\,T},$$
$$\bar{\mathbf{R}}_w = \mathbf{C}_k^r\mathbf{E}_k^{y\,-1}\mathbf{C}_k^{r\,T},$$

where $\mathbf{C}_k^p$ and $\mathbf{C}_k^r$ are obtain from the Cholesky decomposition of $\mathbf{P}_k^p = \mathbf{C}_k^p\mathbf{C}_k^{p\,T}$ and $\mathbf{R}_w = \mathbf{C}_k^r\mathbf{C}_k^{r\,T}$, respectively, and

$$\mathbf{E}_k^s = diag[K_\sigma(e_1), K_\sigma(e_2), \cdots, K_\sigma(e_{n_s})],$$
$$\mathbf{E}_k^y = diag[K_\sigma(e_{n_s+1}), K_\sigma(e_{n_s+2}), \cdots, K_\sigma(e_{n_s+n_m})].$$

From (8), it can be inferred that in comparison to EKF's Kalman gain ($\mathbf{K}_k = \mathbf{P}_k^p\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^p\mathbf{H}_k^T + \mathbf{R}_m)^{-1}$), the $\mathbf{K}_k^{MCC}$

provides a mean to tackle the effect of non-Gaussianity with the help of σ. Also, for a high value of σ the $\mathbf{E}_k^s$ and $\mathbf{E}_k^y$ becomes identity matrix, correspondingly, $\bar{\mathbf{P}}_k^p = \mathbf{P}_k^p$ and $\mathbf{K}_k^{MCC} = \mathbf{K}_k$, eventually EKF-MCC converges to EKF. In the light of the above discussion, it is concluded that for a suitable value of σ, the EKF-MCC will have a different $\mathbf{K}_k^{MCC}$ to $\mathbf{K}_k$ of EKF, this makes EKF-MCC suitable to deal with non-Gaussianity.

Nevertheless, in (8), it is explicit that like EKF, EKF-MCC also linearizes $\mathbf{h}(\cdot)$ via Jacobian, which yields the first-order approximation of $\mathbf{h}(\cdot)$. Consequently, it restricts EKF-MCC from achieving high estimation accuracy and, in some cases, yielding inaccurate results when dealing with complex target motion models. Also, to obtain the Jacobian, and hence, to implement the further steps, EKF-MCC is bound to know the exact form of $\mathbf{h}(\cdot)$ (which depends on the radar sensor type and, in most cases, not known apriori). In the following subsection, we present an approach to deal with these short-comings in EKF-MCC. We describe the implementation of EKF-MCC-RKHS, which first estimates the $\mathbf{h}(\cdot)$ in RKHS using a well-known KRMC algorithm, which is effectively a non-linear adaptive algorithm in RKHS. Subsequently, the estimate of $\mathbf{h}(\cdot)$ replaces the Jacobian in EKF-MCC for performing further prediction and update.

C. EKF-MCC-RKHS

In this subsection, the KRMC algorithm used to implement EKF-MCC-RKHS is described. Firstly, referring to (2), the hidden state $\mathbf{s}_k$ is mapped to a high dimensional RKHS ($\mathcal{H}$) via an unknown implicit mapping function $\boldsymbol{\phi}(\cdot)$ [6], [10], [11]. Subsequently, $\mathbf{y}_k$, which originally is the nonlinear function of $\mathbf{s}_k$ is linearized in terms of $\phi(\mathbf{s}_k)$ in $\mathcal{H}$. Eventually, the estimate of $\mathbf{y}_k$ in $\mathcal{H}$ can be given by the well known Representer's theorem [12] as

$$\hat{\mathbf{y}}_k = \langle \boldsymbol{\Omega}, \boldsymbol{\phi}(\mathbf{s}_k) \rangle_{\mathcal{H}}, \tag{9}$$

where $\langle(\cdot,\cdot)\rangle_{\mathcal{H}}$is the inner product in $\mathcal{H}$ and $\boldsymbol{\Omega}$ is the unknown weight matrix in $\mathcal{H}$.

In the search of the optimized value of $\boldsymbol{\Omega}$ and for the available k pairs of $\mathbf{y}_k$ and $\mathbf{s}_k$, $\{(\mathbf{s}_0, \mathbf{y}_0), \ldots, (\mathbf{s}_{k-1}, \mathbf{y}_{k-1})\}$, the KRMC algorithm maximizes the following cost function

$$\mathcal{J}(\boldsymbol{\Omega}) = \max_{\boldsymbol{\Omega}} \quad \sum_{j=0}^{k-1} \beta^{k-1-j} K_{\sigma_1}(\mathbf{y}_j - \hat{\mathbf{y}}_j) + \frac{1}{2}\beta^{k-1}\lambda \left\|\boldsymbol{\Omega}\right\|^2, \tag{10}$$

where β is the forgetting factor ($0 < \beta \leq 1$), introduce in (10) to enhance the effect of latest estimates, $K_{\sigma_1}(\cdot)$ is the GMK with kernel width σ_1, and λ is the regularization factor.

Utilizing (9), yields

$$\mathcal{J}(\boldsymbol{\Omega}) = \max_{\boldsymbol{\Omega}} \quad \sum_{j=0}^{k-1} \beta^{k-1-j} K_{\sigma_1}(\mathbf{y}_j - \langle \boldsymbol{\Omega}, \boldsymbol{\phi}(\mathbf{s}_j)\rangle_{\mathcal{H}}) + \frac{1}{2}\beta^{j}\lambda \left\|\boldsymbol{\Omega}\right\|^2, \tag{11}$$

Evaluating the gradient of (11) and equating it to zero the solution of $\boldsymbol{\Omega}$ is given by

$$\boldsymbol{\Omega} = \boldsymbol{\Phi}_{k-1}(\boldsymbol{\Phi}_{k-1}^T \boldsymbol{\Phi}_{k-1} + \lambda\beta^{k-1}\sigma_1^2 \mathbf{B}_{k-1}^{-1})^{-1}\mathbf{y}_{k-1}, \tag{12}$$

where $\boldsymbol{\Phi}_{k-1} = [\boldsymbol{\phi}(\mathbf{s}_0), \ldots, \boldsymbol{\phi}(\mathbf{s}_{k-1})]^T$, and

$$\begin{aligned}\mathbf{B}_{k-1} = diag[&\beta^{k-1} K_{\sigma_1}(\mathbf{y}_0 - \langle \boldsymbol{\Omega}, \boldsymbol{\phi}(\mathbf{s}_0)\rangle_{\mathcal{H}}), \cdots, \\ &K_{\sigma_1}(\mathbf{y}_{k-1} - \langle \boldsymbol{\Omega}, \boldsymbol{\phi}(\mathbf{s}_{k-1})\rangle_{\mathcal{H}})].\end{aligned}$$

Substituting (12) in (9), yields

$$\hat{\mathbf{y}}_k = [\langle \boldsymbol{\phi}(\mathbf{s}_0), \boldsymbol{\phi}(\mathbf{s}_k)\rangle_{\mathcal{H}}, \ldots, \langle \boldsymbol{\phi}(\mathbf{s}_{k-1}), \boldsymbol{\phi}(\mathbf{s}_k)\rangle_{\mathcal{H}}]\mathbf{a}_{k-1}^T, \tag{13}$$

where $\mathbf{a}_{k-1} = (\boldsymbol{\Phi}_{k-1}^T \boldsymbol{\Phi}_{k-1} + \lambda\beta^{k-1}\sigma_1^2 \mathbf{B}_{k-1}^{-1})^{-1}$.

With an analogy of (13) with (2), it can be infer that the estimate of $\mathbf{h}(\cdot)$ at k^{th} time instant is given by

$$\hat{\mathbf{h}}_k = [\langle \boldsymbol{\phi}(\mathbf{s}_0), \boldsymbol{\phi}(\mathbf{s}_k)\rangle_{\mathcal{H}}, \ldots, \langle \boldsymbol{\phi}(\mathbf{s}_{k-1}), \boldsymbol{\phi}(\mathbf{s}_k)\rangle_{\mathcal{H}}]. \tag{14}$$

In (14), $\langle \boldsymbol{\phi}(\mathbf{s}_i), \boldsymbol{\phi}(\mathbf{s}_j)\rangle_{\mathcal{H}}$, represents the inner product of $\boldsymbol{\phi}(\mathbf{s}_i)$ and $\boldsymbol{\phi}(\mathbf{s}_j)$ in $\mathcal{H}$. Since $\boldsymbol{\phi}(\cdot)$ is inaccessible, (14) can be simplified by evoking a celebrated Mercer's theorem [8]. Mercer's theorem states that the inner product in $\mathcal{H}$ is efficiently calculated in Euclidean space by the use of $\kappa(\cdot,\cdot)$ (introduced in (4)).

Hence, utilizing Mercer's theorem, the simplified form of $\hat{\mathbf{h}}_k$ is given by

$$\hat{\mathbf{h}}_k = [\kappa(\mathbf{s}_0, \mathbf{s}_k), \ldots, \kappa(\mathbf{s}_{k-1}, \mathbf{s}_k)]. \tag{15}$$

The (15), suggests that unlike EKF-MCC, the knowledge of $\mathbf{h}(\cdot)$ is no longer needed. Instead, $\mathbf{h}(\cdot)$ can be iteratively estimated along with the prediction and update step of EKF-MCC. Moreover, at k^{th} instant, $\hat{\mathbf{h}}_k$ is based on the present state vector $\mathbf{s}_k$ and the past state vectors $\{\mathbf{s}_0, \ldots, \mathbf{s}_{k-1}\}$. This implies that at k^{th} time instant after obtaining $\hat{\mathbf{s}}_k$ from EKF-MCC, in (15), the $\hat{\mathbf{s}}_k$ replaces $\mathbf{s}_k$. Afterward, $\hat{\mathbf{h}}_k$ replaces $\mathbf{h}(\cdot)$ in EKF-MCC to do the further prediction and update steps. The process repeats iteratively for $k = 1, 2, \ldots, K$. Also, σ_1 in $\mathbf{a}_{k-1}$ provides an additional freedom to adjust the performance of EKF-MCC-RKHS against the non-Gaussianity.

The advantages of using EKF-MCC-RKHS over EKF-MCC and EKF are three-fold: (1) Unlike EKF-MCC and EKF, the implementation of EKF-MCC-RKHS is not restricted to knowing the exact form of $\mathbf{h}(\cdot)$. (2) Since, unlike EKF-MCC and EKF, the estimate of $\mathbf{h}(\cdot)$ is used, the EKF-MCC-RKHS will yield the estimate of $\mathbf{s}_k$ with higher accuracy. (3) Lastly, to estimate $\mathbf{h}(\cdot)$, since MCC is used in KRMC, like EKF-MCC, the EKF-MCC-RKHS is suitable to use with non-Gaussianity; however, provide better estimation accuracy than EKF-MCC.

The pseudo algorithm for EKF-MCC-RKHS is given in Algorithm 1, where $\mathbf{z}_k$, r_k, $\mathbf{Q}_k$, $\boldsymbol{\theta}_k$, $\mathbf{e}_k$, and $\mathbf{a}_k$ are as per [9].

IV. Simulation Results and Analysis

This section provides a thorough description of the simulation results performed to evaluate the performance of the proposed EKF-MCC-RKHS in comparison to EKF-MCC and EKF. The simulations are characterized in Scenario I and Scenario II. In Scenario I, the simulations are performed

Algorithm 1: Implementation of EKF-MCC-RKHS

Intialization:

$\mathbf{s}_0$, $\mathbf{y}(\mathbf{s}_0)$, $\hat{\mathbf{s}}_0$, $\mathbf{P}_0$, $\hat{\mathbf{h}}_0 = [\kappa(\hat{\mathbf{s}}_0, \hat{\mathbf{s}}_0)]$, $\mathbf{Q}_0 = (\lambda\beta\sigma_1^2 + \kappa(\hat{\mathbf{s}}_0, \hat{\mathbf{s}}_0))^{-1}$, $\mathbf{a}_0 = [1, \ldots, 1]^{1\times n_m}$

for $k = 1, 2, 3, \cdots, K$ **do**

EKF-MCC

Compute $\mathbf{s}_k^p$ and $\mathbf{P}_k^p$ using (7)

$\mathbf{y}_k^p = \mathbf{a}_{k-1}^T \hat{\mathbf{h}}_{k-1}^T$

$\hat{\mathbf{H}}_k = \left\{ \left(\frac{\partial \hat{\mathbf{h}}_{k-1}}{\partial \hat{\mathbf{s}}_{k-1}} \Big|_{\hat{\mathbf{s}}_{k-1} = \mathbf{s}_k^p} \right) \mathbf{a}_{k-1} \right\}^T$

Compute $\mathbf{K}_k^{MCC}$, $\hat{\mathbf{s}}_k$, and $\mathbf{P}_k$ using (8)

KRMC

$\hat{\mathbf{h}}_k = [\kappa(\hat{\mathbf{s}}_0, \hat{\mathbf{s}}_k), \ldots, \kappa(\hat{\mathbf{s}}_{k-1}, \hat{\mathbf{s}}_k)]$

$\mathbf{e}_k = \mathbf{y}_k - \mathbf{a}_{k-1}^T \hat{\mathbf{h}}_{k-1}^T$

$\mathbf{z}_k = \mathbf{Q}_{k-1} \hat{\mathbf{h}}_{k-1}^T$

$\theta_k = (\exp(-\frac{\|\mathbf{e}_k\|^2}{2\sigma_1^2}))^{-1}$

$r_k = \lambda\beta^k\sigma_1^2\theta_k + \kappa(\hat{\mathbf{s}}_k, \hat{\mathbf{s}}_k) - \mathbf{z}_k^T \hat{\mathbf{h}}_{k-1}^T$

$\mathbf{Q}_k = r_k^{-1} \begin{bmatrix} \mathbf{Q}_{k-1} r_k + \mathbf{z}_k \mathbf{z}_k^T & -\mathbf{z}_k \\ -\mathbf{z}_k^T & 1 \end{bmatrix}$

$\mathbf{a}_k = \begin{bmatrix} \mathbf{a}_{k-1} - r_k^{-1} \mathbf{z}_k \mathbf{e}_k^T \\ r_k^{-1} \mathbf{e}_k^T \end{bmatrix}$

end

to validate the better performance of EKF-MCC over EKF. Since EKF and EKF-RKHS use the same MMSE criterion, in Section I, the EKF-MCC is compared with EKF only. Henceforth, Scenario II evaluates the performance of EKF-MCC-RKHS compared to its counterparts EKF-MCC and EKF. For both scenarios, the $\mathbf{y}_k$ is assumed to be in spherical coordinate and follow Gaussian mixture with $\mathbf{w}_k \in p_1\mathcal{N}_{\mathbb{R}}(\mathbf{0}, \mathbf{R}_w^1) + p_2\mathcal{N}_{\mathbb{R}}(\mathbf{0}, \mathbf{R}_w^2)$, correspondingly, $\mathbf{R}_w = \sum_{g=1}^2 p_g \mathbf{R}_w^g$. The $p1 = 0.1$ and $p2 = 0.9$, the $\mathbf{R}_w^1 = \sigma_{w1}^2 \mathbf{I}_{n_m}$ and $\mathbf{R}_w^2 = \sigma_{w2}^2 \mathbf{I}_{n_m}$, where $\sigma_{w1}^2 = 0.15$ and $\sigma_{w2}^2 = 20$, respectively. Scenario I and Scenario II both are simulated for $K = 150$ CPI's i.e., $k = 1, 2, \cdots, 150$, $T_{cpi} = 0.01$ sec, and $\mathbf{Q}_u$ is according to (II), where $\sigma_a = 10^{-2}$. The $\hat{\mathbf{s}}_0$ and $\mathbf{P}_0$ for both scenarios are consider as $[1, 1, \cdots, 1]^{n_s \times 1}$ and $\mathbf{I}_{n_s}$, respectively. Further, since $\mathbf{y}_k$ is available in spherical coordinate ($\mathbf{y}_k = [\hat{r}_k, \hat{\theta}_k, \hat{\phi}_k]^T$), the associated non-linear $\mathbf{h}(\cdot)$ is given by

$$\mathbf{h}(\mathbf{s}_k) = \left[\sqrt{x_k^2 + y_k^2 + z_k^2}, \tan^{-1}\left(\frac{y_k}{x_k}\right), \tan^{-1}\left(\frac{\sqrt{x_k^2 + y_k^2}}{z_k}\right) \right]^T.$$

The target is assumed to move in 3D space in an indoor scene. Correspondingly, the maximum distance traveled by the target in x, y, and z directions is limited to 10 m. The performance of the filters for both Scenarios I and Scenario II are quantified with root mean square error (RMSE) along the positions, velocities, and accelerations in the x, y, and z directions. The RMSEs for the elements of $\hat{\mathbf{s}}_k$ are define as

$$\text{RMSE}(i) = \sqrt{\frac{1}{K} \sum_{k=0}^{K-1} (\mathbf{s}_k(i) - \hat{\mathbf{s}}_k(i))^2}; \quad i = 1, 2, \ldots, n_s$$

where i denotes the i^{th} element of $\mathbf{s}_k$ and $\hat{\mathbf{s}}_k$.

A. Scenario I

In Scenario I, the simulations are performed to evaluate the performance of EKF and EKF-MCC. For EKF-MCC, the $\sigma = 0.95$. The ground truth (GT) and estimated trajectories obtained from EKF and EKF-MCC are shown in Fig. 1. As depicted in Fig. 1, the trajectory estimated by EKF-MCC is close to GT. The improved performance of EKF-MCC over EKF amid MCC is shown in Fig. 2. In Fig. 2, the RMSE obtained by EKF-MCC and EKF in estimating all $n_s = 9$ parameters is shown. Referring to Fig. 1 and Fig. 2, it concluded that the EKF-MCC outperforms EKF when the system is affected by non-Gaussian noise.

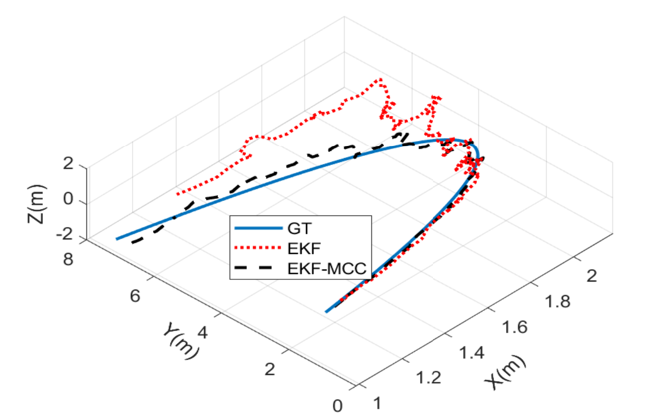

Fig. 1. Evolution of GT and estimated trajectories in 3D space with EKF and EKF-MCC for Scenario I.

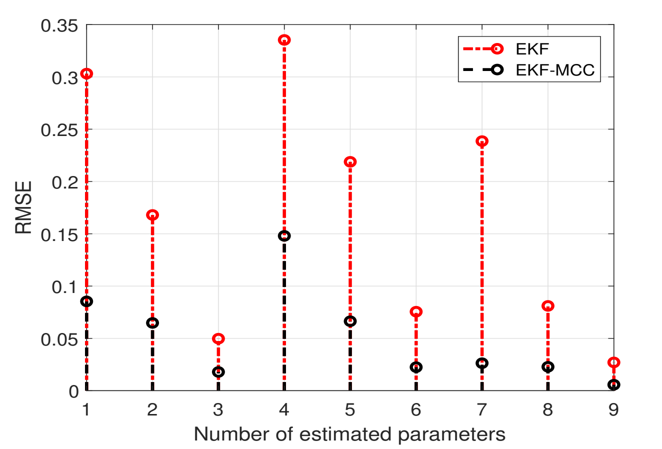

Fig. 2. RMSE in the estimation of all parameters (x, v_x, a_x, y, v_y, a_z, z, v_z, and a_z) with EKF and EKF-MCC for Scenario I.

B. Scenario II

The simulations to evaluate the comparative performance of EKF-MCC and EKF-MCC-RKHS are performed in Scenario II. It is already shown in Fig. 1 and Fig. 2 that EKF-MCC outperforms EKF. Nevertheless, for better clarity, in this subsection, the performance of EKF-MCC-RKHS is compared with EKF-MCC and EKF both. In the simulations, for EKF-MCC $\sigma = 0.95$ and for KRMC $\sigma_1 = 5$, also, β and λ are 1 and 0.004, respectively. Further, it is depicted in Fig. 3 that EKF-MCC-RKHS yield the estimated trajectory in close proximity to GT. Also, from Fig. 3, it can be inferred that EKF-MCC performs better than EKF with system non-Gaussianity; however, because of estimating $\mathbf{h}(\cdot)$ and using MCC, EKF-MCC-RKHS performs better than both EKF-MCC and EKF. Further, Fig. 4 validates the improved performance of EKF-MCC-RKHS over EKF-MCC and EKF in terms of RMSEs of all estimated parameters.

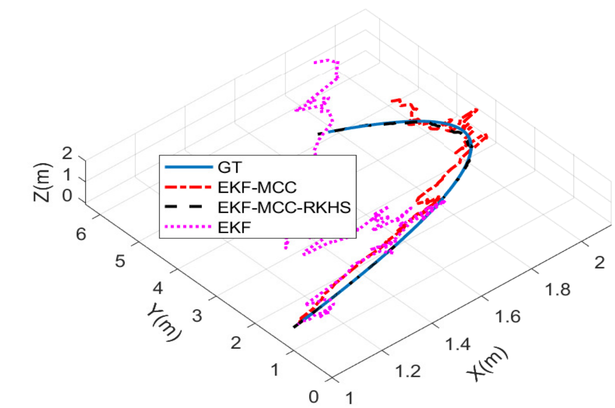

Fig. 3. Evolution of GT and estimated trajectories in 3D space with EKF, EKF-MCC and EKF-MCC-RKHS for Scenario II.

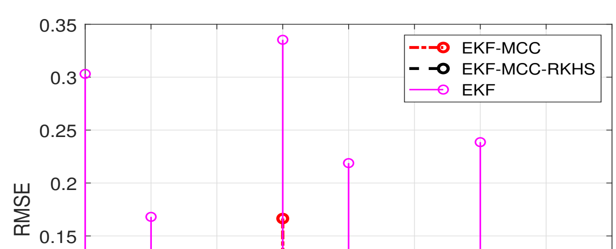

Fig. 4. RMSE in the estimation of all parameters (x, v_x, a_x, y, v_y, a_z, z, v_z, and a_z) with EKF, EKF-MCC, and EKF-MCC-RKHS for Scenario II.

V. Conclusions

In this paper, the problem of estimating the target's dynamic parameter in the presence of measurement non-linearity and non-Gaussianity is tackled. Firstly, to deal with the effects of non-Gaussianity, EKF-MCC is suggested as a better choice over EKF. For non-Gaussianity, unlike MMSE, the MCC utilizes higher-order error characteristics in estimation, providing estimates with good estimation accuracy. Further, to make EKF-MCC independent of knowing the exact non-linearity ($\mathbf{h}(\cdot)$), we proposed clubbing EKF-MCC filtering with RKHS-based KRMC. The KRMC uses the MCC and provides the estimates of $\mathbf{h}(\cdot)$ by linearizing the non-linear relation in RKHS. The combined filter EKF-MCC-RKHS, instead of approximating $\mathbf{h}(\cdot)$ via Jacobian, estimates $\mathbf{h}(\cdot)$ and hence provides better estimation accuracy over EKF-MCC. The simulation performed to estimate the target's dynamic parameters of the CA target motion model validates the superior performance of EKF-MCC-RKHS compared to its predecessors, EKF-MCC and EKF.

Future work could include the theoretical analysis of the convergence and estimation accuracy achieved by the proposed EKF-MCC-RKHS filter.

Acknowledgment

The authors' work is supported by the Luxembourg National Research Fund (FNR) through the SPRINGER Project No. 12734677.

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