RANGE-ISL MINIMIZATION AND SPECTRAL SHAPING IN MIMO RADAR SYSTEMS VIA WAVEFORM DESIGN

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ABSTRACT

In this paper, we look at a waveform design problem for colocated Multiple-Input Multiple-Output (MIMO) radar systems. Under continuous phase constraint, we aim to minimize the range-Integrated Sidelobe Level (ISL) with a compatible spectral response. In this regard, we define the range-ISL function in the time domain first, and then express it in the frequency domain using the Parseval relation. Following that, we incorporate weights on the range-ISL in the frequency domain to apply spectral compatibility. As a result, we have a multi-variable, non-convex, NP-hard optimization problem. We proposed an iterative algorithm based on the Coordinate Descent (CD) method to obtain a local optimum solution. We show the performance of the proposed method and compare it to the counterparts in the numerical results.

Index Terms— Waveform Design, MIMO Radar, ISL minimization, Spectral Shaping

1. INTRODUCTION

Spectrum congestion has become an urgent issue with lots of radio services competing for the limited usable spectrum, such as wireless communications and active Radio Frequency (RF) sensing. In this situation of spectrum congestion, radars must deal with simultaneous broadcasts from other RF systems; given the requirement for significant bandwidth in both systems, spectrum sharing with communications is a very plausible scenario [1–4]. Although complex allocation procedures are in place to control spectral usage, strict allocations result in wasteful spectrum utilization when subscriptions are low. In this regard, cognitive and smart spectrum utilization provides an adaptable and reusable option for improved system performance in the development of smart sensing systems [5].

Cognitive radar systems are intelligent sensors that interact with their surroundings. To improve performance, these radars optimally adapt their transmit, receive, and other parameters to the environment [6]. The management of resources in cognitive Multiple-Input Multiple-Output (MIMO) radar is becoming critical for the next generation of active sensing and communications [7]. Spatial, spectral and temporal (range) are the three main resources in MIMO radar systems [8]. Transmit beampattern shaping is used in the spatial domain to control the spatial distribution of transmit power [9–13]. In the spectral domain, spectral shaping plays an important role in spectral sharing for MIMO radar and other RF systems coexistence [1–4]. Low auto and cross-correlation sidelobes levels are required in the time (range) domain to avoid masking weak targets within the range sidelobes of a strong target and to obtain orthogonality respectively [14–16]. Waveform design and windowing [17] are the most commonly used approaches for resource management [17].

In this paper, we consider the range-Integrated Sidelobe Level (ISL) minimization problem in time (range) domain with spectral compatibility via waveform design. We consider solving the problem under continuous phase constraint which is basically a constant modulus waveform. In general, there are two approaches to tackle this problem. First, solving an optimization problem where, the range-ISL minimization and spectral compatibility are the objective function and constraint, respectively [18]. Another approach is solving a bi-objective optimization problem of range-ISL and spectral shaping with weighted sum method [19]. To address the problem, we consider one objective function, that we use the Parseval theorem to express the range-ISL function in frequency domain. The objective function is then weighted to control the waveform's spectral response. Then we proposed an iterative algorithm based on the Coordinate Descent (CD) method to solve the problem, which decreases the objective function monotonically in each iteration. The numerical results demonstrate the effectiveness of the proposed method.

We organize this paper as follows. In section 2, we introduce the system model and problem statement. Section 3 presents the proposed CD based framework whose performance is numerically assessed in section 4

Notations: The following notations is adopted. Bold lower and uppercase letters for vectors matrices, respectively. **Diag** $\{\mathbf{a}\}$, $(.)^T$, $(.)^*$, $(.)^r$, $|.|$, $\lfloor . \rceil$ and $\odot$ denote the diagonal matrix of vector $\mathbf{a}$, transpose, sequence reversal, conjugate, absolute value, round and Hadamard product respectively. The letter j represents the imaginary unit (i.e., $j = \sqrt{-1}$).

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2. SYSTEM MODEL AND PROBLEM FORMULATION

Let $\mathbf{X} \in \mathbb{C}^{M \times N}$ be the transmitted sequence in MIMO radar system with M transmitters and the sequence length of N. Let $\mathbf{x}_m$ be the transmitted sequence of m^{th} transmitter,

$$\mathbf{x}_m = [x_{m,1}, x_{m,2}, \ldots, x_{m,N}]^T \in \mathbb{C}^N. \tag{1}$$

$x_{m,n}$ denotes the n^{th} sample of the m^{th} transmitter. The aperiodic cross-correlation of $\mathbf{x}_m$ and $\mathbf{x}_{m'}$ is defined as [16],

$$r_{m,m'}(k) \triangleq \sum_{n=1}^{N-k} x_{m,n} x^*_{m',n+k}, \tag{2}$$

where $m, m' \in \{1, \ldots, M\}$ are the transmit antennas indices and $l \in \{-N+1, \ldots, N-1\}$ is the lag of cross-correlation. If $m = m'$, (2) represents the aperiodic auto-correlation of signal $\mathbf{x}_m$. The zero lag of auto-correlation represents the peak of the matched filter output and contains the energy of sequence, while the other lags ($l \neq 0$) are referred to the sidelobes. The range-ISL can therefore be expressed by [15],

$$\text{ISL} = \sum_{m,m'=1}^{M} \sum_{l=-N+1}^{N-1} |r_{m,m'}(k)|^2 - MN^2. \tag{3}$$

Please note that, the MN^2 term in (3) is the peak of the matched filters output, where is a constant value for constant modulus waveforms ($\sum_{m=1}^{M} |r_{m,m}(0)|^2 = MN^2$). Therefore the range-ISL as objective function can be equivalently written as $f(\mathbf{X}) \triangleq \sum_{m,m'=1}^{M} \sum_{l=-N+1}^{N-1} |r_{m,m'}(l)|^2$.

Based on Parseval theorem the range-ISL minimization problem can be written in the frequency domain as, $f(\mathbf{X}) = \frac{1}{2N-1} \sum_{m,m'=1}^{M} \sum_{k=-N+1}^{N-1} |R_{m,m'}(k)|^2$, where, $R_{m,m'}(k)$ indicates the Fourier transform of $r_{m,m'}(l)$. Let us assume that $\mathbf{F} \triangleq [\mathbf{f}_0, \ldots, \mathbf{f}_{2N-2}] \in \mathbb{C}^{(2N-1)\times(2N-1)}$ is the Discrete Fourier Transform (DFT) matrix, where, $\mathbf{f}_k \triangleq [1, e^{-j\frac{2\pi k}{2N-1}}, \ldots, e^{-j\frac{2\pi k(2N-2)}{2N-1}}]^T \in \mathbb{C}^{2N-1}$, $k = \{0, \ldots, 2N-2\}$. Hence, the $2N-1$ points Fourier transform of the cross correlation between the m^{th} and the m'^{th} transmitters can be written as $\mathbf{v}_{m,m'} \triangleq \mathbf{F}\bar{\mathbf{x}}_m \odot \mathbf{F}\bar{\mathbf{x}}^{r^*}_{m'}$ ($R_{m,m'}(k) \triangleq v_{m,m'}(k)$). Here $\bar{\mathbf{x}}_m, \bar{\mathbf{x}}_{m'} \in \mathbb{C}^{(2N-1)}$ are the $N-1$ zero pad versions of $\mathbf{x}_m$ and $\mathbf{x}_{m'}$ respectively, i.e. $\bar{\mathbf{x}}_m \triangleq [\mathbf{x}_m; \underbrace{0; \ldots; 0}_{N-1}]$. In this regard, we consider the following optimization problem,

$$\begin{cases} \min\limits_{\mathbf{X}} \quad f(\mathbf{X}) = \sum\limits_{m,m'=1}^{M} \left\| \mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_m \odot \mathbf{F}\bar{\mathbf{x}}^{r^*}_{m'} \right\|_2^2 \\ s.t. \quad x_{m,n} \in \mathcal{X}_\infty, \end{cases} \tag{4}$$

where, $\mathbf{w} = [w_0, \ldots, w_{2N-2}]^T$, $0 \leq w_k \leq 1$, $k \in \{0, \ldots, 2N-2\}$, and $\mathcal{X}_\infty$ indicates the continuous phase sequence. More precisely, $\mathcal{X}_\infty = \{e^{j\phi} | \phi \in \Omega_\infty\}$, where $\Omega_\infty = [0, 2\pi)$. Please note that, choosing $w_k = 1$, leads to range-ISL minimization. Besides, by choosing an appropriate value for $\mathbf{w}$ the spectral response can be shaped. In this regards, let $\mathcal{U} = \cup_{k=1}^{K} (u_{k,1}, u_{k,2})$ be the K number of normalized frequency stop-bands ($K \leq 2N-1$), where $0 \leq u_{k,1} < u_{k,2} \leq 1$ and $\cap_{k=1}^{K} (u_{k,1}, u_{k,2}) = \varnothing$. Thus, the undesired discrete frequency bins are given by $\mathcal{V} = \cup_{k=1}^{K} (\lfloor (2N-1)u_{k,1} \rceil, \lfloor (2N-1)u_{k,2} \rceil)$. Therefore the weights $\mathbf{w}$ can be obtained by,

$$w_k = \begin{cases} 1 & k \in \mathcal{V} \\ 0 & k \notin \mathcal{V} \end{cases}, \quad k \in \{1, \ldots, 2N-1\}. \tag{5}$$

Problem (4) is a multi-variable, non-convex and NP-hard optimization problem. In the following we proposed a CD-based method to obtain a local optimum solution.

3. PROPOSED METHOD

In CD based methods we need to consider one entry of $\mathbf{X}$ as being the only variable while keeping the others fixed; for this identified variable, we optimize the objective function. This methodology is efficient when the objective function can be written in a simplified form for that identified variable [20]. To this end, First we express the problem with respect to t^{th} transmitter, then we express it with the d^{th} sample. Let $\mathbf{x}_t$ be the only variable block, while other blocks are held fixed and stored in the matrix $\mathbf{X}_{-t} \triangleq [\mathbf{x}_1^T; \ldots; \mathbf{x}_{t-1}^T; \mathbf{x}_{t+1}^T; \ldots; \mathbf{x}_M^T] \in \mathbb{C}^{(M-1)\times N}$. In this case, the function $f(\mathbf{X})$ can be decomposed to a term independent of the optimization variable $\mathbf{x}_t$, and two other terms, one indicating the auto-correlation of $\mathbf{x}_t$, and the other is its cross-correlation with the other sequences of the set $\mathbf{X}_{-t}$. Precisely,

$$f(\mathbf{X}) = f_{au}(\mathbf{x}_t) + f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t}) + f_m(\mathbf{X}_{-t}). \tag{6}$$

Since, $f_m(\mathbf{X}_{-t})$ does not depend on $\mathbf{x}_t$, therefore it can be ignored in the objective function. Thus, it can be shown that,

$$\begin{aligned} f_{au}(\mathbf{x}_t) &= \left\| \mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_t \odot \mathbf{F}\bar{\mathbf{x}}_t^{r^*} \right\|_2^2, \\ f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t}) &= \sum_{\substack{m=1 \\ m \neq t}}^{M} \left\| \mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_t \odot \mathbf{F}\bar{\mathbf{x}}_m^{r^*} \right\|_2^2, \end{aligned} \tag{7}$$

Let us assume that $x_{t,d}$ is the only variable at $(i)^{th}$ iteration of the optimization procedure. In this regards $f(\mathbf{X})$ can be written with respect to $x_{t,d}$ as (see Appendix),

$$f(x_{t,d}, \mathbf{X}^{(i)}_{-(t,d)}) = a_2 x_{t,d}^2 + a_1 x_{t,d} + a_0 + a_{-1} x^*_{t,d} + a_{-2} {x^*_{t,d}}^2, \tag{8}$$

where the coefficients are given in the Appendix. Here, $\mathbf{X}^{(i)}_{-(t,d)} \triangleq \mathbf{X}^{(i)}|_{x_{t,d}=0}$ refers to the fixed entries. By substi-

Algorithm 1 : Spectral Compatible Range-ISL minimization.

Input: $\mathbf{X}^{(0)}$ and $\mathbf{w}$.
Initialization: $i := 0$.
Optimization:
while $\frac{1}{\sqrt{MN}}\left\|\mathbf{X}^{(i)} - \mathbf{X}^{(i-1)}\right\|_F < \zeta$ **do**
  $i := i + 1$;
  for $t = 1, \ldots, M$ **do**
    for $d = 1, \ldots, N$ **do**
      Update $x_{t,d} = e^{j\phi^\star}$, using (11);
      $\mathbf{X}^{(i)} = \mathbf{X}^{(i)}_{-(t,d)}|_{x_{t,d}=x^{(i)}_{t,d}}$;
    end for
  end for
end while
Output: $\mathbf{X}^\star = \mathbf{X}^{(i)}$.

tuting $x_{t,d} = e^{j\phi}$, (8) can be written as,

$$f(\phi^{(i)}) = \sum_{n=-2}^{2} a_n e^{jn\phi}, \tag{9}$$

which depends only on parameter ϕ [1]. In this case, the optimal $\phi^\star$ can be calculated by finding real roots of the first order derivative of the objective function and evaluating the objective value in these points and boundaries. In this regards the derivative of $f(\phi)$ with respect to ϕ can be written as,

$$f'(\phi) = \frac{df(\phi)}{d\phi} = \sum_{n=-2}^{2} jna_n e^{jn\phi}, \tag{10}$$

Readily it can be shown that $f'(\phi) = je^{j2\phi}(-2a_{-2}e^{-j4\phi} - a_{-1}e^{-j3\phi} + 0 \times e^{-j2\phi} + a_1 e^{-j\phi} + 2a_2)$. Using the slack variable $z \triangleq e^{-j\phi}$, the critical points can be achieved by obtaining the real roots of fourth degree polynomial of $\sum_{n=0}^{4} q_n z^n = 0$, where $q_0 \triangleq 2a_2$, $q_1 \triangleq a_1$, $q_2 \triangleq 0$, $q_3 \triangleq -a_{-1}$ and $q_3 \triangleq -2a_{-2}$. Let $z_{1,2,3,4}$ be the roots of the aforementioned polynomial function. Therefore the stationary points of $f(\phi)$ with respect to ϕ is $\phi_{1,2,3,4} \triangleq -j\ln(z_{1,2,3,4})$ and subsequently the optimum phase is,

$$\phi^\star = \arg\min_{\phi} \left\{ f(\phi) \,|\, \phi \in \phi_{1,2,3,4} \right\}. \tag{11}$$

Hence, the variable $x_{t,d}$ will be updated by $x_{t,d} = e^{j\phi^\star}$. This procedure will continue for other entries until the stationary point is obtained. We consider $\frac{1}{\sqrt{MN}}\left\|\mathbf{X}^{(i)} - \mathbf{X}^{(i-1)}\right\|_F < \zeta$, ($\zeta > 0$) as stopping criterion of optimization. The proposed method is summarized in **Algorithm 1**.

4. NUMERICAL RESULTS

In this section, we consider evaluating the performance of the proposed method and compare it with the state-of-the-art counterparts. In this regard, we consider a set of random phase codded sequences ($\mathbf{X}_0 \in \mathbb{C}^{M\times N}$) as an initial waveform. Here, every code entry is given by, $x^{(0)}_{m,n} = e^{j\phi^{(0)}_{m,n}}$, where $\phi^{(0)}_{m,n}$ is a random real variable uniformly distributed in $[0, 2\pi)$. Besides, we assume that the stopping condition for **Algorithm 1** is set at $\zeta = 10^{-6}$.

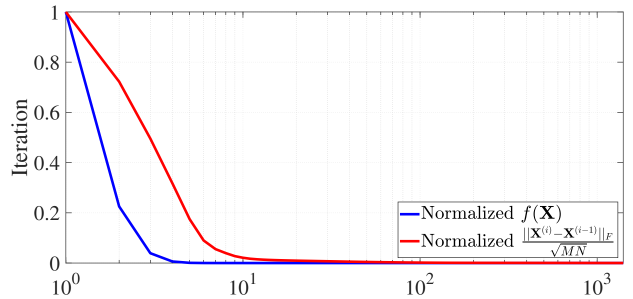


Fig. 1: Convergence behavior of the objective function and argument ($M = 4$, $N = 128$, and $\mathcal{U} = [0.3, 0.35] \cup [0.7, 0.8]$)

4.1. Convergence

Figure 1 shows the convergence behavior of the proposed algorithm in two aspects of objective function and argument. As can be seen, the objective function decreases monotonically which was expected due to using CD method. Besides, converging $\frac{1}{\sqrt{MN}}\left\|\mathbf{X}^{(i)} - \mathbf{X}^{(i-1)}\right\|_F$ to zero indicates that the argument converges to the stationary point.

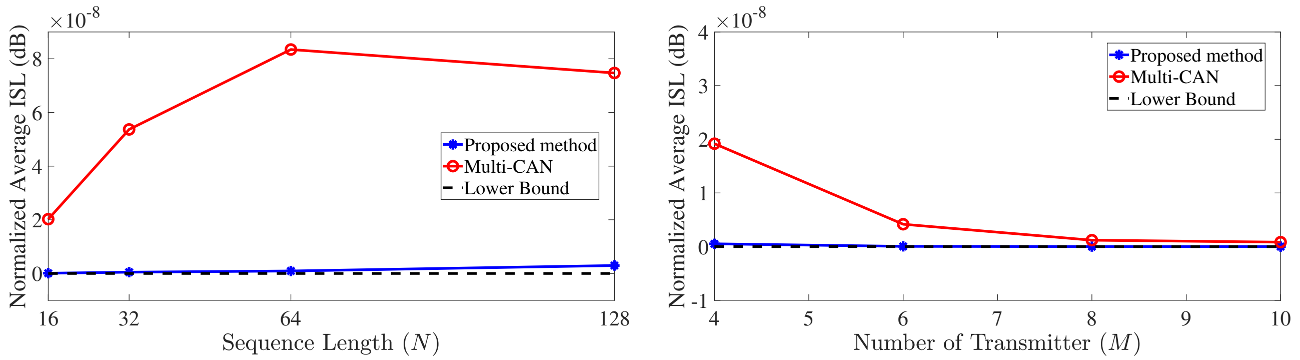
4.2. Range-ISL Minimization

The range-ISL minimization can be obtained by setting $w_k = 1, k \in \{1, \ldots, 2N-1\}$. Figure 2a and Figure 2b compare the average range-ISL of the proposed method with Multi-Cyclic Algorithm-New (CAN) [21] and the lower bound which is given by, $N^2M(M-1)$ [22], with different sequence length and number of transmitters, respectively. As can be seen in both cases the proposed method offers lower range-ISL compared to Multi-CAN.

4.3. Spectral Shaping

Figure 3 compares the performance of the proposed method with [23] in terms of spectral shaping. By choosing $\theta = 1$, [23] offers the optimum spectral response. Observe that, not only the proposed method offers deeper null rather than [23], but also the ISL of the proposed method and [23] are 84080.1307 and 87924.6796, respectively. This indicates that the proposed method offers lower ISL rather than [23].

[1] For convenience we use ϕ instead of $\phi_{t,d}$ in the rest of the paper.



(a)

(b)

Fig. 2: Comparison of average range-ISL of the proposed method with Multi-CAN and the lower bound, (a) different sequence length ($M = 4$) and (b) different number of transmitters ($N = 128$) (number of trials = 10 and $L = 32$).

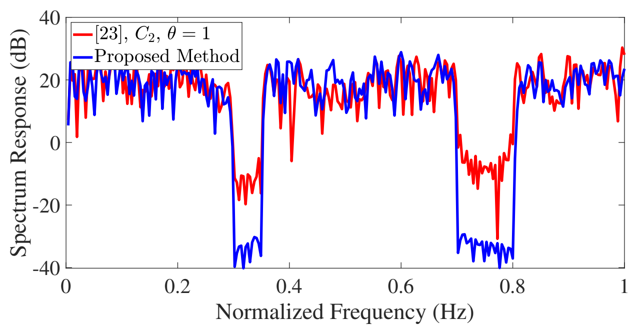


Fig. 3: The spectrum response of the proposed method with [23] ($M = 4$, $N = 128$ and $\mathcal{U} = [0.3, 0.35] \cup [0.7, 0.8]$)

5. CONCLUSION

In this paper, we address the range-ISL minimization and spectral shaping problem in MIMO radar systems. In this regard, we express the range-ISL function in the spectral domain, and then incorporating the weights on the spectral domain gives us the ability to control the spectral response. In order to design the waveform, in the first step, we express the optimization problem with respect to one variable. Then we propose an iterative algorithm based on CD which in each iteration finds the critical points with respect to one variable and obtain the optimum solution through them. The simulation results shows the monotonicity decreasing of the objective function and better performance of the proposed method rather than the state of the art, in terms of range-ISL minimization and spectral shaping.

Appendix

Calculating the coefficients of $f_{au}(\mathbf{x}_t)$: The k^{th} entry of $\mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_t$ and $\mathbf{F}\bar{\mathbf{x}}_t^{r^*}$ min (7) can be written as,

$$(\mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_t)_k = \alpha_1 \bar{x}_{t,d} + \gamma_1, \quad (\mathbf{F}\bar{\mathbf{x}}_t^{r^*})_k = \beta_2 \bar{x}^*_{t,d} + \gamma_2.$$

where, $\alpha_1 \triangleq w_k e^{-j\frac{2\pi(k-1)(d-1)}{2N-1}}$, $\beta_2 \triangleq e^{-j\frac{2\pi(k-1)(N-d)}{2N-1}}$, $\gamma_1 \triangleq \sum_{\substack{n=1\\ n\neq 2N-d}}^{2N-1} w_k \bar{x}^*_{t,N-n+1} e^{-j\frac{2\pi(k-1)(n-1)}{2N-1}}$ and $\gamma_2 \triangleq \sum_{\substack{n=1\\ n\neq d}}^{2N-1} \bar{x}_{t,n} e^{-j\frac{2\pi(k-1)(n-1)}{2N-1}}$ and $\bar{x}_{t,d}$ denotes the d^{th} entry of $\bar{\mathbf{x}}_t$. Therefore,

$$(\mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_t^{r^*} \odot \mathbf{F}\bar{\mathbf{x}}_t)_k = \alpha_{tdk}\bar{x}_{t,d} + \beta_{tdk}\bar{x}^*_{t,d} + \gamma_{tdk}, .$$

where, $\alpha_{tdk} \triangleq \alpha_1\gamma_2$, $\beta_{tdk} = \beta_2\gamma_1$, $\gamma_{tdk} = \alpha_1\beta_2 + \gamma_1\gamma_2$. Substituting the aforementioned equation in $f_{au}(\mathbf{x}_t)$ we have,

$$f_{au}(\mathbf{x}_t) = a_{au,0}x_{t,d}^2 + a_{au,1}x_{t,d} + a_{au,2} + a_{au,3}x^*_{t,d} + a_{au,4}{x^*_{t,d}}^2.$$

where, $a_{au,0} \triangleq \frac{1}{2N-1}\sum_{k=1}^{2N-1}\alpha_{tdk}\beta_{tdk}$, $a_{au,4} \triangleq a^*_{au,0}$, $a_{au,1} \triangleq \frac{1}{2N-1}\sum_{k=1}^{2N-1}(\alpha_{tdk}\gamma^*_{tdk} + \beta^*_{tdk}\gamma_{tdk})$, $a_{au,3} \triangleq a^*_{au,1}$ and $a_{au,2} \triangleq \frac{1}{2N-1}\sum_{k=1}^{2N-1}(|\alpha_{tdk}|^2 + |\beta_{tdk}|^2 + |\gamma_{tdk}|^2)$.

Calculating the coefficients of $f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t})$: By some mathematical manipulation, it can be shown that,

$$f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t}) = \sum_{\substack{m=1\\ m\neq t}}^{M}\|\mathbf{V}_m\bar{\mathbf{x}}_t\|_2^2,$$

where, $\mathbf{V}_m \triangleq \textbf{Diag}(\mathbf{w} \odot \mathbf{F}\bar{\mathbf{x}}_m^{r^*}) \in \mathbb{C}^{(2N-1)\times(2N-1)}$. By some mathematical manipulation it can be shown that the k^{th} entry of $\mathbf{V}_m\bar{\mathbf{x}}_t$ can be obtained by, $(\mathbf{V}_m\bar{\mathbf{x}}_t)_k = \alpha_{mdk}x_{t,d} + \gamma_{mdk}$, where, $\alpha_{mdk} \triangleq v_{mdk}$ and $\gamma_{mdk} \triangleq \sum_{\substack{n=1\\ n\neq d}}^{2N-1} v_{mnk}x_{t,n}$ and v_{mnk} denotes the n^{th} and k^{th} entry of matrix $\mathbf{V}_m$. Substituting the aforementioned equation in $f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t})$,

$$f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t}) = a_{cr,0}x_{t,d} + a_{cr,1} + a_{cr,2}x^*_{t,d}.$$

where, $a_{cr,0} \triangleq \sum_{\substack{n=1\\ n\neq d}}^{2N-1}\bar{x}^*_{t,n}v_{mnd}$, $a_{cr,2} \triangleq a^*_{cr,0}$ and $a_{cr,1} \triangleq v_{mdd} + \sum_{\substack{n=1\\ n\neq d}}^{2N-1}\sum_{\substack{n'=1\\ n'\neq d}}^{2N-1}\bar{x}^*_{t,n}v_{mnn'}\bar{x}_{t,n'}$. Adding, $f_{au}(\mathbf{x}_t)$ and $f_{cr}(\mathbf{x}_t, \mathbf{X}_{-t})$ we have,

$$f_{cr,au}(\mathbf{x}_t, \mathbf{X}_{-t}) = a_2x_{t,d}^2 + a_1x_{t,d} + a_0 + a_{-1}x^*_{t,d} + a_{-2}{x^*_{t,d}}^2,$$

where, $a_2 \triangleq a_{au,0}$, $a_1 \triangleq a_{au,1} + a_{cr,0}$, $a_0 \triangleq a_{au,2} + a_{cr,1}$, $a_{-1} \triangleq a_1^*$ and $a_{-2} \triangleq a_2^*$.

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