IET Generation, Transmission & Distribution

Research Article

Energy and reserve co-optimisation – reserve availability, lost opportunity and uplift compensation cost

ISSN 1751-8687
Received on 8th May 2018
Revised 28th September 2018
Accepted on 16th October 2018
doi: 10.1049/iet-gtd.2018.5480
www.ietdl.org

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Abstract: Ancillary services used for active power balancing are called balancing services or operating reserves and their provision is vital for maintaining power system frequency at the nominal value. In a deregulated environment, where generation is unbundled from transmission and distribution operations, independently owned generating companies may elect to provide operating reserves. However, it is not easy to calculate the exact cost of reserve provision and, therefore, bid for it accurately. Although the cost efficiency of reserve provision can be improved by co-optimising energy and reserve markets, generating companies can still encounter monetary losses caused by the provision of reserve. Currently, these losses are compensated based on ex-post calculations. Hence, current energy and reserve prices do not adequately factor in the ex-post compensation caused by reserve provision. This study proposes an energy and reserve co-optimisation with an explicit consideration of two compensation mechanisms, i.e. lost opportunity and uplift payments. The problem is structured as a bilevel model. The upper level is a mixed-integer unit commitment problem and the lower level is a continuous economic dispatch problem. The case study shows that the proposed model increases market efficiency.

Nomenclature

Sets and indices

N_b set of buses, indexed by b
N_d set of demands, indexed by d
N_i set of conventional generators, indexed by i
N_j set of generator's piecewise start-up cost curve segments, indexed by j
N_k set of generator's operating cost curve segments, indexed by k
N_l set of lines, indexed by l
N_t set of time periods, indexed by t
N_w set of wind farms, indexed by w

Non-negative continuous variables in the upper-level problem

c_i^{TG} total generation cost for generator i during period t ($)
c_i^{RA} total reserve cost for generator i during period t ($)
c_i^{LO} lost opportunity cost for generator i during period t ($)
c_i^{UL} uplift cost for generator i during period t ($)
v_i^{TG} total generation revenue for generator i during period t ($)
v_i^{RA} total reserve revenue for generator i during period t ($)

Free continuous variables in the upper-level problem

p_i^{ER} profit of generator i during period t in energy and reserve co-optimisation algorithm ($)

Binary variables in the upper-level problem

$x_{t,i}^{\mathrm{ON}}$ generator on/off status (1 if generator i is on during period t, 0 otherwise)
$x_{t,i}^{\mathrm{SU}}$ generator start-up status (1 if generator i is started up during period t, 0 otherwise)
$x_{t,i}^{\mathrm{SD}}$ generator shut-down status (1 if generator i is shut down during period t, 0 otherwise)
$y_{t,i,j}^{\mathrm{SU}}$ generator start-up cost identification matrix (1 if generator i is started up during period t after being off for $T_{i,j}^{\mathrm{SU}}$ to $T_{i,j+1}^{\mathrm{SU}}$ periods, 0 otherwise)

Non-negative continuous variables in the lower-level problem

$g_{t,i}^{\mathrm{TG}}$ total generation of generator i during period t (MW)
$g_{t,i,k}^{\mathrm{IN}}$ incremental output on cost segment k of generator i during period t (MW)
$r_{t,i}^{\mathrm{UP}}$ reserve up provision of generator i during period t (MW)
$r_{t,i}^{\mathrm{DN}}$ reserve down provision of generator i during period t (MW)
$g_{t,i}^{\mathrm{WG}}$ output of wind farm w during period t (MW)

Free continuous variables in the lower-level problem

$f_{t,l}^{\mathrm{TL}}$ power flow through transmission line l during period t (MW)
$\Theta_{t,b}^{\mathrm{TB}}$ voltage angle at bus b during period t (rad)

Dual non-negative continuous variables in the lower-level problem

$\gamma_{t,i,k}^{\mathrm{IN}}$ generator dispatch; incremental generation shadow price on segment k of generator i during period t
$\gamma_{t,i}^{\mathrm{MX}}$ generator dispatch; maximum power generation shadow price of generator i during period t
$\gamma_{t,i}^{\mathrm{MN}}$ generator dispatch; minimum power generation shadow price of generator i during period t
$\alpha_{t,i}^{\mathrm{UP}}$ generator ramping; up ramp shadow price of generator i during period t
$\alpha_{t,i}^{\mathrm{DN}}$ generator ramping; down ramp shadow price of generator i during period t
$\omega_{t,w}^{\mathrm{WG}}$ wind power generation; shadow price of wind farm w during period t
$\phi_{t,l}^{\mathrm{MX}}$ power flow; maximum power flow shadow price of transmission line l during period t
$\phi_{t,l}^{\mathrm{MN}}$ power flow; minimum power flow shadow price of transmission line l during period t

$\theta_{t,b}^{\mathrm{MX}}$ voltage angle; maximum voltage angle shadow price of transmission bus b during period t
$\theta_{t,b}^{\mathrm{MN}}$ voltage angle; minimum voltage angle shadow price of transmission bus b during period t

Dual free continuous variables in the lower-level problem

$\beta_{t,b}^{\mathrm{PB}}$ power balance shadow price at bus b during period t
$\gamma_{t,i}^{\mathrm{TG}}$ generator dispatch; total power generation shadow price of generator i during period t
ρ_{t}^{UP} reserve provision; up reserve system-wide shadow price during period t
ρ_{t}^{DN} reserve provision; down reserve system-wide shadow price during period t
$\phi_{t,l}^{\mathrm{TF}}$ power flow; total power flow shadow price of transmission line l during period t
$\theta_{t,b}^{\mathrm{RF}}$ voltage angle; reference bus voltage angle shadow price of transmission bus b during period t

Parameters in the upper-level problem

$C_{i,j}^{\mathrm{SU}}$ start-up cost of generator i at start-up segment j (\$)
C_{i}^{NL} no-load cost of generator i (\$)
$C_{i,k}^{\mathrm{IN}}$ incremental cost of generator i at segment k (\$/MW)
C_{i}^{RU} reserve up provision cost of generator i (\$/MW)
C_{i}^{RD} reserve down provision cost of generator i (\$/MW)
P_{i}^{EO} total daily profit of generator i obtained through energy-only optimisation (\$)
$X_{i}^{\mathrm{ON}_0}$ initial on/off status of generator i
$T_{i}^{\mathrm{UP}_0}$ initial required minimum up time of generator i (h)
$T_{i}^{\mathrm{DN}_0}$ initial required minimum down time of generator i (h)
T_{i}^{UP} minimum up time of generator i (h)
T_{i}^{DN} minimum down time of generator i (h)
$T_{i,j}^{\mathrm{SU}}$ start-up cost blocks j of generator i (h)

Parameters in the lower-level problem

$M_{i/w/d,b}^{\mathrm{TG/WG/PD}}$ thermal generator/wind farm/demand transmission network incidence matrix. Value 1 denotes that unit $i/w/d$ is connected to bus b, and 0 otherwise
$M_{l,b}^{\mathrm{TL}}$ transmission line network map. Value 1 at bus b denotes beginning of the line, −1 denotes end of the line, and 0 denotes no incidence
$D_{t,d}$ demand forecast during period t for demand d (MW)
$W_{t,w}$ wind generation forecast during period t for wind farm w (MW)
R_{t}^{UP} system-wide up reserve requirements during period t (MW)
R_{t}^{DN} system-wide down reserve requirements during period t (MW)
$G_{i,k}^{\mathrm{IN}}$ maximum output of generator i at segment k (MW)
G_{i}^{MX} maximum output of generator i (MW)
G_{i}^{MN} minimum stable output of generator i (MW)
A_{i}^{UP} ramp up rate of generator i (MW/h)
A_{i}^{DN} ramp down rate of generator i (MW/h)
B_{l}^{TL} susceptance of line l (S)
F_{l}^{MX} maximum capacity of line l (MW)

1 Introduction

Electricity markets can be designed to clear different market products, e.g. energy and ancillary services, separately, as in European countries, or jointly, as in the USA. Typically, the day-ahead market participants first submit price-quantity energy and reserve bids and offers. Next, the market operator determines the cleared quantities and prices based on the supply–demand equilibrium for each product. Additional information on energy and reserve co-optimisation market models can be found in [1, 2], on the current status of electricity markets in the USA in [3], and on long-term co-optimisation of energy and reserve in [4, 5].

This paper adopts a taxonomy typical to the US markets [6, 7]. Ancillary services are required by system operators to ensure system reliability. In terms of the active power supply–demand balance, the ancillary services imply three types of frequency control capabilities: primary, secondary and tertiary. Secondary control is often called regulation, while tertiary control is divided into ten-minute spinning, ten-minute non-spinning and thirty-minute operating reserve (e.g. in ISO New England). For sake of clarity, this paper considers the spinning reserve, or simply reserve, that unifies both the secondary and tertiary regulation capabilities.

We emphasise that the presented model is only suitable for US markets and not for European markets, where generators do not submit clearly defined fixed, start-up and variable costs. Instead, all cost components are within an ordered block. Therefore, uplift costs cannot be defined and remunerated as in the US markets and generators should account for such costs when they submits orders for selling electricity. Details on the current market designs in Europe are available in [8].

Market algorithms with complete cost allocations are very important because they provide fair energy and reserves prices, which in return entail sufficient signals for new generation/grid investments and prevent scarcity periods. High share of renewable generation, due to increased uncertainty, could cause periods with insufficient generation or, more frequently, insufficient reserve capacities. Some power systems tackle this issue by providing capacity payments or creating capacity markets in order to create sufficient revenues for flexible generation. According to [9, 10], system operators should reach for out-of-market measures only after all possibilities for market upgrades are depleted. Uplift cost payments are a commonly used out-of-market measure for compensating the costs that occur when generators are committed, but their revenues are not allocated within the market pricing. In order to calculate them accurately, uplift costs should be assigned according to some principles of cost causation. Although [11] indicates that ISO-NE's markets could generally procure sufficient resources to meet reliability requirements, sometimes additional generators are committed, causing uplift payments. Reasons for this could be local reliability commitments (local reserve requirements, voltage support, constraints in distribution network) or system-wide capacity and congestion management needs. Provision of ancillary services can incur additional costs, such as additional start-ups, increased wear and tear due to increased cycling, keeping a unit at part load, reduced efficiency of plants etc. Operation of a unit at part load because it cannot be dispatched to its full capacity results in lost opportunity cost, which is another type of out-of-market measure. In general, if there were no ancillary services provision, units with marginal production costs lower than locational marginal price (LMP) would sell their energy in the power pool. From perspective of a market participant, lost opportunity cost represents a portion of the energy market profit a generator loses when backed down to use some of its capacity to provide reserves. From the perspective of a system operator, lost opportunity cost represents an additional cost associated with supplying energy from more expensive units because it needs to meet the reserve requirements [12, 13].

An extensive overview of out-of-market corrections used in electricity markets is presented in [14]. The authors conclude that out-of-market corrections may have a significant impact on generating units' revenue and that using simplified deterministic market model does not accurately represent the outcomes of the stochastic unit commitment models. A study on different energy and reserve market designs, with analysis of the lost opportunity cost allocation, is available in [15]. The findings indicate that co-optimising energy, reserve and lost opportunity cost leads to non-differentiable optimisation problems of a bilevel structure. In practice, this complexity can be overcome by means of sequential approximations, when a relaxed or simplified problem is solved until an acceptable solution is achieved [16]. Recently, such methods have gained some interest. For example, convex hull pricing [16–18] is seen as an emerging alternative to obtain market clearing prices for energy and reserve; however, this approach has not yet been adopted by any US ISO and requires further investigation to overcome certain challenges [17]. We refer interested readers to [17] for further discussions on convex pricing and its comparison to the current practices.

Benefits of pricing schemes using algorithms with complete cost allocation are twofold: it derives timely and correct signals for new generation/grid investments and does not over or under-compensate specific costs. Reference [19] indicates that US RTO and ISO total uplift annual payments range from \$50 million (ISO-NE in 2009) to more than \$800 million (PJM in 2013). The correct pricing and explicit calculation of uplift costs, as well as lost opportunity costs, is thus of high value for power system and can significantly decrease total operating costs. LMPs are sufficient to compensate for variable costs of all committed generators, but they might be insufficient for costs such as fixed and start-up costs emerging from non-convexity of the binary scheduling algorithm (part of the upper-level problem in our model). This kind of problem is important for flexible peaking units which operate in short runs during peak periods. Our model considers both unit commitment and economic dispatch through a single-level equivalent, thus incorporating uplift cost calculation whose only goal is to remunerate generators if LMPs are insufficient to cover their daily operating costs. Lost opportunity cost has other goal, i.e. to remunerate generators for lost revenue because their output was backed down due to reserve requirements and constraints. More detailed explanation of lost opportunity costs can be found in report [12].

This paper proposes a new model that co-optimises energy and reserve services and explicitly accounts for the uplift and lost opportunity payments while optimising the social welfare, instead of resorting to likely suboptimal post-clearing out-of-market corrections. The benefits of the proposed formulation are threefold: (i) prices reflect all generators costs and therefore provide accurate short and long-term signals, (ii) implicit calculation of lost opportunity and uplift costs avoids over and under-compensation of generators, and (iii) remuneration of lost opportunity and uplift costs is verifiable.

The proposed model is a bilevel programme where the upper-level problem represents a unit commitment problem, while the lower-level problem performs economic dispatch based on the commitment decisions. The bilevel programme is then reformulated into a single-level equivalent mathematical programme with equilibrium constraints (MPEC) using the duality-based approach and linearised using the Karush–Kuhn–Tucker's optimality conditions and the 'big M' method. The lower-level problem is convex with respect to lower-level decision variables only. This convexity makes it possible to invoke the duality transformation and strong duality theorem to convert the original bilevel programme into a single-level equivalent. In the single-level equivalent, there is no separation between the upper- and the lower-level variables in mathematical sense, i.e. all variables are associated with the resulting single-level problem. Since the single-level problem contains binary variables, uplift payments are used to offset the effect of these non-convexities.

The rest of the paper is organised as follows. Section 2 describes the proposed bilevel model and its transformation into a single-level equivalent. Section 3 demonstrates its application on a modified ISO New England test bed. Section 4 concludes the paper and provides insights for future research.

2 Model

The proposed bilevel model consists of the upper- and lower-level problems. The upper-level problem minimises the total generation (TG), reserve availability (RA), lost opportunity (LO), and uplift (UL) costs subject to multi-period, mixed-integer constraints on binary commitment decisions on thermal generators. The lost opportunity and uplift costs in the upper-level objective function are minimised explicitly using the dual variables of the lower-level problem. The lower-level problem solves a multi-period, network-constrained market clearing parametrised by the upper-level binary commitment decisions subject to technical characteristics of thermal generators, transmission network constraints, renewable generation, load forecasts and reserve requirements. The proposed bilevel model is then converted into a single-level equivalent using the duality-based approach. The equivalent is then linearised using the KKT optimality conditions.

2.1 Upper-level problem

The objective function of the upper-level problem is

$$\min_{\Xi_{\mathrm{UPL}}} c^{\mathrm{UPL}} = \sum_{i=1}^{N_i} \left(c_i^{\mathrm{TG}} + c_i^{\mathrm{RA}} + c_i^{\mathrm{LO}} + c_i^{\mathrm{UL}} \right), \tag{1}$$

where $\Xi_{\mathrm{UPL}} = \left\{ c_i^{\mathrm{LO}}, c_i^{\mathrm{UL}}, x_{t,i}^{\mathrm{ON}}, x_{t,i}^{\mathrm{SU}}, x_{t,i}^{\mathrm{SD}}, y_{t,i,j}^{\mathrm{SU}} \right\}$ and decision variables c_i^{TG}, c_i^{RA}, c_i^{LO} and c_i^{UL} are defined as

$$c_i^{\mathrm{TG}} = \sum_{t=1}^{N_t} \left[\sum_{j=1}^{N_j} \left(C_{i,j}^{\mathrm{SU}} \cdot y_{t,i,j}^{\mathrm{SU}} \right) + C_i^{\mathrm{NL}} \cdot x_{t,i}^{\mathrm{ON}} + \sum_{k=1}^{N_k} \left(C_{i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} \right) \right], \tag{2}$$

$$c_i^{\mathrm{RA}} = \sum_{t=1}^{N_t} \left(C_i^{\mathrm{RU}} \cdot r_{t,i}^{\mathrm{UP}} + C_i^{\mathrm{RD}} \cdot r_{t,i}^{\mathrm{DN}} \right), \tag{3}$$

$$v_i^{\mathrm{TG}} = \sum_{t=1}^{N_t} \left[g_{t,i}^{\mathrm{TG}} \cdot \sum_{b=1}^{N_b} \left(M_{i,b}^{\mathrm{TG}} \cdot \beta_{t,b}^{\mathrm{TG}} \right) \right], \tag{4}$$

$$v_i^{\mathrm{RA}} = \sum_{t=1}^{N_t} \left(\rho_t^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{UP}} + \rho_i^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{DN}} \right), \tag{5}$$

$$p_i^{\mathrm{ER}} = v_i^{\mathrm{TG}} + v_i^{\mathrm{RA}} - c_i^{\mathrm{TG}} - c_i^{\mathrm{RA}}, \tag{6}$$

$$c_i^{\mathrm{LO}} \geq P_i^{\mathrm{EO}} - p_i^{\mathrm{ER}}, \tag{7}$$

$$c_i^{\mathrm{UL}} \geq c_i^{\mathrm{TG}} + c_i^{\mathrm{RA}} - v_i^{\mathrm{TG}} - v_i^{\mathrm{RA}}. \tag{8}$$

Note that since objective function (1) considers both variables c_i^{LO} and c_i^{UL}, an overcompensation to the generators might occur. This issue is elaborated in Section 3. Case Study, where all the simulations consider only one of these two variables, except Sim 7, which takes both terms for demonstration purposes. Equation (2) computes the total generation cost for each generator and includes the start-up, no-load and incremental fuel costs. The cost of upward and downward reserve provided by each generator is computed in (3) with respect to the energy-only market clearing outcome. Equation (4) evaluates the energy revenue $\left(v_i^{\mathrm{TG}}\right)$, where the amount of generated electricity $\left(g_{t,i}^{\mathrm{TG}}\right)$ is multiplied by the locational marginal price $\left(\beta_{i,b}^{\mathrm{TG}}\right)$. Equation (5) calculates revenues acquired from the up and down reserve provision of each generator $\left(v_i^{\mathrm{RA}}\right)$, i.e. operating reserve prices $\left(\rho_t^{\mathrm{UP}}, \rho_t^{\mathrm{DN}}\right)$ are multiplied by the amount of provided reserves $\left(r_{t,i}^{\mathrm{UP}}, r_{t,i}^{\mathrm{DN}}\right)$. Equation (6) uses the costs and revenues from (2)–(5) in order to calculate the total daily profit $\left(p_i^{\mathrm{ER}}\right)$ for each generator from the energy and reserve provision. Equation (7) computes the lost opportunity cost for each generator. Positive variable c_i^{LO} attains non-zero values only if provision of reserve services incurs monetary loses to generators relative to the energy-only case. P_i^{EO} is the total daily profit of generator i calculated in the energy-only case, which is pre-computed before solving the proposed bilevel optimisation and used as a reference point for computing out-of-market corrections. The calculation flowchart is visualised in Fig. 1, which shows four main processes within the proposed methodology: initialisation of the models, Energy-Only Optimisation (EOO) model run (to obtain P_i^{EO} values), Energy and Reserve Co-optimisation (ERO) run (requires P_i^{EO} values from EOO) and dispatch orders to the generating units. Equation (8) computes the generator uplift costs as the difference between total generators costs and revenues and attains non-zero

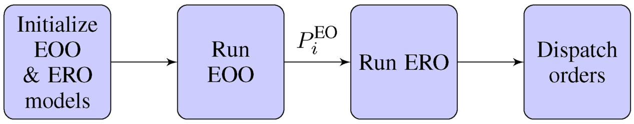

Fig. 1 *Calculation flowchart*

values when compensating for the revenue insufficiency. The left-hand sides of (7) and (8) compute the values of c_i^{LO} and c_i^{UL}, which are defined as non-negative decision variables (see Nomenclature). When the right-hand sides of (7) and (8) attain negative values (i.e. no out-of-market compensation needed), the model will set c_i^{LO} and c_i^{UL} to zero to preserve feasibility. When the right-hand sides of (7) and (8) attain positive values, the inequalities turn into strict equalities since both c_i^{LO} and c_i^{UL} are minimised in objective function (1). Note that all terms in (7) and (8) refer to a 24-h period and that $\beta_{tb}^{\mathrm{PB}}, \rho_t^{\mathrm{UP}}$ and ρ_t^{DN} in (7) and (8) are dual variables of the lower-level problem that parametrise the upper-level problem.

Objective function (1) is constrained as follows:

$$x_{t,i}^{\mathrm{SU}} - x_{t,i}^{\mathrm{SD}} = x_{t,i}^{\mathrm{ON}} - x_{t-1,i}^{\mathrm{ON}}, \quad \forall t \in (1, N_t] \tag{9}$$

$$x_{t,i}^{\mathrm{SU}} + x_{t,i}^{\mathrm{SD}} \le 1, \quad \forall t \in [1, N_t] \tag{10}$$

$$x_{t,i}^{\mathrm{ON}} = X_i^{\mathrm{ON}_0}, \quad \forall t \in \left[1, T_i^{\mathrm{UP}_0} + T_i^{\mathrm{DN}_0}\right] \tag{11}$$

$$\sum_{\tau = t - T_i^{\mathrm{UP}} + 1}^{t} x_{\tau,i}^{\mathrm{SU}} \le x_{t,i}^{\mathrm{ON}}, \quad \forall t \in \left[T_i^{\mathrm{UP}}, N_t\right] \tag{12}$$

$$\sum_{\tau = t - T_i^{\mathrm{DN}} + 1}^{t} x_{\tau,i}^{\mathrm{SD}} \le 1 - x_{t,i}^{\mathrm{ON}}, \quad \forall t \in \left[T_i^{\mathrm{DN}}, N_t\right] \tag{13}$$

$$y_{t,i,j}^{\mathrm{SU}} \le \sum_{\tau = T_{i,j}^{\mathrm{SU}}}^{T_{i,j+1}^{\mathrm{SU}} - 1} x_{t-\tau,i}^{\mathrm{SD}}, \quad \forall t \in \left[T_{i,j+1}^{\mathrm{SU}}, N_t\right], j \in \left[1, N_j\right) \tag{14}$$

$$\sum_{j=1}^{N_j} y_{t,i,j}^{\mathrm{SU}} = x_{t,i}^{\mathrm{SU}}, \quad \forall t \in [1, N_t], j \in \left[1, N_j\right] \tag{15}$$

Equation (9) models binary logic on the binary commitment, start-up and shut-down decisions of generators. Constraint (10) prevents simultaneous start-ups and shut-downs of generators. Constraint (11) enforces the consequent commitment decisions from the previous optimisation horizon during the first $T_i^{\mathrm{UP}_0} + T^{\mathrm{DN}_0}$ periods, if necessary. Constraints (12) and (13) account for the minimum up and down time constraints. The number of periods since the last shut down of a generator is computed in (14), while (15) selects the segment of the start-up cost curve that corresponds to the value set in (14). Detailed information on modelling piecewise start-up cost is available in [20].

2.2 Lower-level problem

The objective function of the lower-level problem is

$$\min_{\Xi_{\mathrm{PLL}}} c^{\mathrm{PLL}} = \sum_{t=1}^{N_t} \sum_{i=1}^{N_i} c_{t,i}^{\mathrm{TG}}, \tag{16}$$

where $\Xi_{\mathrm{PLL}} = \left\{g_{t,i}^{\mathrm{TG}}, g_{t,i,k}^{\mathrm{IN}}, r_{t,i}^{\mathrm{UP}}, r_{t,i}^{\mathrm{DN}}, g_{t,i}^{\mathrm{WG}}, f_{t,l}^{\mathrm{TL}}, \Theta_{t,b}^{\mathrm{TB}}\right\}$ are the primal lower-level decision variables. Note that, in line with [15], the primal lower-level objective function contains only term $c_{t,i}^{\mathrm{TG}}$. However, term $c_{t,i}^{\mathrm{RA}}$ could be added to this objective function, which would result in a different right-hand side of dual constraints (34) and (35), where zeros would be replaced with C_i^{RU} and C_i^{RD}.

This objective function is subject to the following constraints (dual variables of the lower-level constraints are provided after a colon):

$$\begin{aligned}&\sum_{i=1}^{N_i} M_{i,b}^{\mathrm{TG}} \cdot g_{t,i}^{\mathrm{TG}} + \sum_{w=1}^{N_w} M_{w,b}^{\mathrm{WG}} \cdot g_{t,w}^{\mathrm{WG}} + \sum_{l=1}^{N_l} M_{l,b}^{\mathrm{TL}} \cdot f_{t,l}^{\mathrm{TL}} \\ &= \sum_{d=1}^{N_d} M_{d,b}^{\mathrm{PD}} \cdot D_{t,d} \colon (\beta_{t,b}^{\mathrm{PB}}), \quad \forall t \in [1, N_t], b \in [1, N_b]\end{aligned} \tag{17}$$

$$g_{t,i}^{\mathrm{TG}} = \sum_{k=1}^{N_k} g_{t,i,k}^{\mathrm{IN}} \colon (\gamma_{t,i}^{\mathrm{TG}}), \quad \forall t \in [1, N_t], i \in [1, N_i] \tag{18}$$

$$\begin{aligned}&g_{t,i,k}^{\mathrm{IN}} \le G_{i,k}^{\mathrm{IN}} \cdot x_{t,i}^{\mathrm{ON}} \colon \left(\gamma_{t,i,k}^{\mathrm{IN}}\right), \\ &\forall t \in [1, N_t], i \in [1, N_i], k \in [1, N_k]\end{aligned} \tag{19}$$

$$\begin{aligned}&g_{t,i}^{\mathrm{TG}} + r_{t,i}^{\mathrm{UP}} \le G_i^{\mathrm{MX}} \cdot x_{t,i}^{\mathrm{ON}}, \quad \colon (\gamma_{t,i}^{\mathrm{MX}}), \\ &\forall t \in [1, N_t], i \in [1, N_i]\end{aligned} \tag{20}$$

$$g_{t,i}^{\mathrm{TG}} - r_{t,i}^{\mathrm{DN}} \ge G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{ON}} \colon \left(\gamma_{t,i}^{\mathrm{MN}}\right), \forall t \in [1, N_t], i \in [1, N_i] \tag{21}$$

$$\begin{aligned}&g_{t,i}^{\mathrm{TG}} + r_{t,i}^{\mathrm{UP}} - g_{t-1,i}^{\mathrm{TG}} + r_{t-1,i}^{\mathrm{DN}} \le A_i^{\mathrm{UP}} \cdot x_{t-1,i}^{\mathrm{ON}} \\ &+ G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SU}} \colon (\alpha_{t,i}^{\mathrm{UP}}), \quad \forall t \in (1, N_t], i \in [1, N_i]\end{aligned} \tag{22}$$

$$\begin{aligned}&-g_{t,i}^{\mathrm{TG}} + r_{t,i}^{\mathrm{DN}} + g_{t-1,i}^{\mathrm{TG}} + r_{t-1,i}^{\mathrm{UP}} \le A_i^{\mathrm{DN}} \cdot x_{t,i}^{\mathrm{ON}} \\ &+ G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SD}} \colon \left(\alpha_{t,i}^{\mathrm{DN}}\right), \quad \forall t \in (1, N_t], i \in [1, N_i]\end{aligned} \tag{23}$$

$$\sum_{i=1}^{N_i} r_{t,i}^{\mathrm{UP}} = R_t^{\mathrm{UP}} \colon \left(\rho_t^{\mathrm{UP}}\right), \quad \forall t \in [1, N_t] \tag{24}$$

$$\sum_{i=1}^{N_i} r_{t,i}^{\mathrm{DN}} = R_t^{\mathrm{DN}} \colon \left(\rho_t^{\mathrm{DN}}\right), \quad \forall t \in [1, N_t] \tag{25}$$

$$g_{t,w}^{\mathrm{WG}} \le W_{t,w} \colon \left(\omega_{t,w}^{\mathrm{WG}}\right), \quad \forall t \in [1, N_t], w \in [1, N_w] \tag{26}$$

$$f_{t,l}^{\mathrm{TL}} = B_l^{\mathrm{TL}} \cdot \sum_{b=1}^{N_b} M_{l,b}^{\mathrm{TL}} \cdot \Theta_{t,b}^{\mathrm{TB}} \colon \left(\phi_{t,l}^{\mathrm{TF}}\right), \quad \forall t \in [1, N_t], l \in [1, N_l] \tag{27}$$

$$-F_l^{\mathrm{MX}} \le f_{t,l}^{\mathrm{TL}} \le F_l^{\mathrm{MX}} \colon \left(\phi_{t,l}^{\mathrm{MN}}, \phi_{t,l}^{\mathrm{MX}}\right), \quad \forall t \in [1, N_t], l \in [1, N_l] \tag{28}$$

$$\Theta_{t,b}^{\mathrm{TB}} = 0 \colon \left(\theta_{t,b}^{\mathrm{RF}}\right), \quad \forall t \in [1, N_t], b \equiv b^{\mathrm{REF}} \tag{29}$$

$$-\pi \le \Theta_{t,b}^{\mathrm{TB}} \le \pi \colon \left(\theta^{\mathrm{MN}}, \theta_{t,b}^{\mathrm{MX}}\right), \forall t \in [1, N_t], b \in [1, N_b] \setminus b^{\mathrm{REF}}. \tag{30}$$

Nodal power balance is enforced in (17), where the first two terms on the left-hand side are the power generation injections from thermal and wind power generators, the third left-hand term is the net injection of the incidental transmission lines, and the right-hand side term is the demand. Note that parameters $M^{\mathrm{TG}}, M^{\mathrm{WG}}, M^{\mathrm{TL}}, M^{\mathrm{PD}}$ map thermal generators, wind farms, transmission lines and demand to network buses. Constraint (18) computes the total power output of thermal generators based on the power output segments modelled in (19). Maximum and minimum limits on the total power output and available reserve of thermal generators are enforced in (20) and (21). Ramp rates of thermal generators are imposed in (22) and (23). Equations (24) and (25) enforce the total upward and downward reserve requirement to be provided by thermal generators. Output of wind power generators is limited to their forecast values in (26). Transmission line flows are computed in (27) using a lossless dc power flow approximation, while the power flow limits are enforced in (28). Equation (29) sets voltage angle at the reference bus to zero. The voltage angles at other buses are bounded in (30).

2.3 Dual lower-level problem

Dual of the lower-level problem (16)–(30) is

$$\begin{aligned}\max_{\Xi_{\mathrm{DLL}}} c^{\mathrm{DLL}} = \sum_{t=1}^{N_t}\Bigg[&\sum_{b=1}^{N_b}\beta_{t,b}^{\mathrm{PB}}\cdot\sum_{d=1}^{N_d}M_{d,b}^{\mathrm{PD}}\cdot D_{t,d}\\ &+\sum_{i=1}^{N_i}x_{t,i}^{\mathrm{ON}}\cdot\left(G_i^{\mathrm{MN}}\cdot\gamma_{t,i}^{\mathrm{MN}}-\sum_{k=1}^{N_k}G_{i,k}^{\mathrm{IN}}\cdot\gamma_{t,i,k}^{\mathrm{IN}}-G_i^{\mathrm{MX}}\cdot\gamma_{t,i}^{\mathrm{MX}}\right)\\ &+R_t^{\mathrm{UP}}\cdot\rho_t^{\mathrm{UP}}+R_t^{\mathrm{DN}}\cdot\rho_t^{\mathrm{DN}}-\sum_{w=1}^{N_w}W_{t,w}\cdot\omega_{t,w}^{\mathrm{WG}}\\ &-\sum_{l=1}^{N_l}F_l^{\mathrm{MX}}\cdot(\phi_{t,l}^{\mathrm{MX}}+\phi_{t,l}^{\mathrm{MN}})-\sum_{b=1}^{N_b}\pi\cdot\left(\theta_{t,b}^{\mathrm{MX}}+\theta_{t,b}^{\mathrm{MN}}\right)\Bigg]\\ &-\sum_{t=1}^{N_t}\sum_{i=1}^{N_i}\Big[\alpha_{t,i}^{\mathrm{UP}}\cdot\left(A_i^{\mathrm{UP}}\cdot x_{t-1,i}^{\mathrm{ON}}+G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SU}}\right)\\ &+\alpha_{t,i}^{\mathrm{DN}}\cdot\left(A_i^{\mathrm{DN}}\cdot x_{t,i}^{\mathrm{ON}}+G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SD}}\right)\\ &+\sum_{j=1}^{N_j}\left(C_{i,j}^{\mathrm{SU}}\cdot y_{t,i,j}^{\mathrm{SU}}\right)+C_i^{\mathrm{NL}}\cdot x_{t,i}^{\mathrm{ON}}\Big]\end{aligned}\tag{31}$$

subject to:

$$\begin{aligned}&\sum_{b=1}^{N_b}M_{i,b}^{\mathrm{TG}}\cdot\beta_{t,b}^{\mathrm{PB}}+\gamma_{t,i}^{\mathrm{TG}}-\gamma_{t,i}^{\mathrm{MX}}+\gamma_{t,i}^{\mathrm{MN}}-\alpha_{t,i}^{\mathrm{UP}}+\alpha_{t+1,i}^{\mathrm{UP}}+\alpha_{t,i}^{\mathrm{DN}}\\ &\quad-\alpha_{t+1,i}^{\mathrm{DN}}\le 0:\left(g_{t,i}^{\mathrm{TG}}\right),\quad\forall t\in[1,N_t),i\in[1,N_i]\end{aligned}\tag{32}$$

$$-\gamma_{t,i}^{\mathrm{TG}}-\gamma_{t,i,k}^{\mathrm{IN}}\le C_{i,k}^{\mathrm{IN}}:\left(g_{t,i,k}^{\mathrm{IN}}\right),\forall t\in[1,N_t],i\in[1,N_i],k\in[1,N_k]\tag{33}$$

$$-\gamma_{t,i}^{\mathrm{MX}}-\alpha_{t,i}^{\mathrm{UP}}-\alpha_{t+1,i}^{\mathrm{DN}}+\rho_t^{\mathrm{UP}}\le 0:\left(r_{t,i}^{\mathrm{UP}}\right),\ \forall t\in[1,N_t),i\in[1,N_i]\tag{34}$$

$$-\gamma_{t,i}^{\mathrm{MN}}-\alpha_{t+1,i}^{\mathrm{UP}}-\alpha_i^{\mathrm{DN}}+\rho_t^{\mathrm{DN}}\le 0:\left(r_{t,i}^{\mathrm{DN}}\right),\ \forall t\in[1,N_t),i\in[1,N_i]\tag{35}$$

$$\sum_{b=1}^{N_b}M_{w,b}^{\mathrm{WG}}\cdot\beta_{t,b}^{\mathrm{PB}}-\omega_{t,w}^{\mathrm{WG}}\le 0:\left(g_{t,w}^{\mathrm{WG}}\right),\ \forall t\in[1,N_t],w\in[1,N_w]\tag{36}$$

$$\begin{aligned}&\sum_{b=1}^{N_b}M_{l,b}^{\mathrm{TL}}\cdot\beta_{t,b}^{\mathrm{PB}}+\phi_{t,l}^{\mathrm{TF}}-\phi_{t,l}^{\mathrm{MX}}+\phi_{t,l}^{\mathrm{MN}}\\ &=0:\left(f_{t,l}^{\mathrm{TL}}\right),\quad\forall t\in[1,N_t],l\in[1,N_l]\end{aligned}\tag{37}$$

$$-\sum_{l=1}^{N_l}B_l^{\mathrm{TL}}\cdot M_{l,b}^{\mathrm{TL}}\cdot\phi_{t,l}^{\mathrm{TF}}+\theta_{t,b}^{\mathrm{RF}}=0:\left(\Theta_{t,b}^{\mathrm{TB}}\right),\forall t\in[1,N_t],b=b^{\mathrm{REF}}\tag{38}$$

$$\begin{aligned}&-\sum_{l=1}^{N_l}B_l^{\mathrm{TL}}\cdot M_{l,b}^{\mathrm{TL}}\cdot\phi_{t,l}^{\mathrm{TF}}-\theta_{t,b}^{\mathrm{MX}}+\theta_{t,b}^{\mathrm{MN}}=0:\left(\Theta_{t,b}^{\mathrm{TB}}\right),\\ &\forall t\in[1,N_t],b\in[1,N_b],b\ne b^{\mathrm{REF}},\end{aligned}\tag{39}$$

where $\Xi_{\mathrm{DLL}}=\{\gamma_{t,i,k}^{\mathrm{IN}},\gamma_{t,i}^{\mathrm{MX}},\gamma_{t,i}^{\mathrm{MN}},\alpha_{t,i}^{\mathrm{UP}},\alpha_{t,i}^{\mathrm{DN}},\omega_{t,w}^{\mathrm{WG}},\phi_{t,l}^{\mathrm{MX}},\phi_{t,l}^{\mathrm{MN}},\theta_{t,b\ne b\_rf}^{\mathrm{MX}},\theta_{t,b\ne b\_rf}^{\mathrm{MN}},\beta_{t,b}^{\mathrm{PB}},\gamma_{t,i}^{\mathrm{TG}},\rho_t^{\mathrm{UP}},\rho_t^{\mathrm{DN}},\phi_{t,l}^{\mathrm{TF}},\theta_{t,b=b\_rf}^{\mathrm{RF}}\}$ are dual lower-level variables. In constraints (32)–(39), the primal variables associated with each dual constraint are referenced after a colon.

2.4 Conversion to MPEC

Since the lower-level problem is linear and convex with respect to the lower-level variables, the strong duality condition can be formulated:

$$c^{\mathrm{PLL}}=c^{\mathrm{DLL}}\tag{40}$$

Finally, the proposed MPEC is given as follows:

$$\begin{aligned}&\mathrm{Eq.}\,(1)\\ &\mathrm{subjectto:}(7)-(8),(9)-(15),\\ &(17)-(30),(32)-(39),(40)\end{aligned}\tag{41}$$

2.5 Linearisation

MPEC (41) is nonlinear due to (i) the multiplication of continuous primal lower-level and continuous dual lower-level variables in (7) and (8), and (ii) the multiplication of binary upper-level and continuous dual lower-level variables in (40).

Three nonlinear terms of type (i) are $g_{t,i}^{\mathrm{TG}}\cdot\sum_{t=1}^{N_t}M_{i,b}^{\mathrm{TG}}\cdot\beta_{t,b}^{\mathrm{PB}}$, $r_{t,i}^{\mathrm{UP}}\cdot\rho_t^{\mathrm{UP}}$ and $r_{t,i}^{\mathrm{DN}}\cdot\rho_t^{\mathrm{DN}}$. After using some of the KKT conditions (for more information please see Appendix), (7) and (8) are replaced with the following expressions:

$$\begin{aligned}c_i^{\mathrm{LO}}\ge\sum_{t=1}^{N_t}\Bigg[&P_{t,i}^{\mathrm{EO}}-\sum_{k=1}^{N_k}\left(C_{i,k}^{\mathrm{IN}}\cdot g_{t,i,k}^{\mathrm{IN}}+G_{i,k}^{\mathrm{IN}}\cdot\gamma_{t,i,k}^{\mathrm{IN}}\cdot x_{t,i}^{\mathrm{ON}}\right)\\ &-G_i^{\mathrm{MX}}\cdot\gamma_{t,i}^{\mathrm{MX}}\cdot x_{t,i}^{\mathrm{ON}}+G_i^{\mathrm{MN}}\cdot\gamma_{t,i}^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{ON}}\\ &-\alpha_{t,i}^{\mathrm{UP}}\cdot(A_i^{\mathrm{UP}}\cdot x_{t-1,i}^{\mathrm{ON}}-G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SU}})\\ &-\alpha_{t,i}^{\mathrm{DN}}\cdot(A_i^{\mathrm{DN}}\cdot x_{t,i}^{\mathrm{ON}}-G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SD}})\\ &+\sum_{j=1}^{N_j}\left(C_{i,j}^{\mathrm{SU}}\cdot y_{t,i,j}^{\mathrm{SU}}\right)+C_i^{\mathrm{NL}}\cdot x_{t,i}^{\mathrm{ON}}+\sum_{k=1}^{N_k}\left(C_{i,k}^{\mathrm{IN}}\cdot g_{t,i,k}^{\mathrm{IN}}\right)\\ &+C_i^{\mathrm{RU}}\cdot r_{t,i}^{\mathrm{UP}}+C_i^{\mathrm{RD}}\cdot r_{t,i}^{\mathrm{DN}}\Bigg],\end{aligned}\tag{42}$$

$$\begin{aligned}c_i^{\mathrm{UL}}\ge\sum_{t=1}^{N_t}\Big[&c_i^{\mathrm{TG}}+c_i^{\mathrm{RA}}\\ &-\sum_{k=1}^{N_k}\left(C_{i,k}^{\mathrm{IN}}\cdot g_{t,i,k}^{\mathrm{IN}}+G_{i,k}^{\mathrm{IN}}\cdot\gamma_{t,i,k}^{\mathrm{IN}}\cdot x_{t,i}^{\mathrm{ON}}\right)\\ &-G_i^{\mathrm{MX}}\cdot\gamma_{t,i}^{\mathrm{MX}}\cdot x_{t,i}^{\mathrm{ON}}+G_i^{\mathrm{MN}}\cdot\gamma_{t,i}^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{ON}}\\ &-\alpha_{t,i}^{\mathrm{UP}}\cdot\left(A_i^{\mathrm{UP}}\cdot x_{t-1,i}^{\mathrm{ON}}-G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SU}}\right)\\ &-\alpha_{t,i}^{\mathrm{DN}}\cdot\left(A_i^{\mathrm{DN}}\cdot x_{t,i}^{\mathrm{ON}}-G_i^{\mathrm{MN}}\cdot x_{t,i}^{\mathrm{SD}}\right)\Big].\end{aligned}\tag{43}$$

Note that (42) and (43) contain multiplications of binary upper-level and continuous dual lower-level variables as in (40). Equations (40), (42) and (43) can be easily linearised using the 'big M' method [21].

3 Case study

3.1 Test system

The proposed model is tested on a 8-zone ISO-NE testbed that consists of 76 conventional generators (5 nuclear, 11 coal, 16 oil, and 44 gas) with the total generation capacity of 23 GW and the peak demand of 18 GW. Hourly wind generation profiles are from [22]. We refer interested readers to [23] for further information on the testbed. We assume that reserve requirements are deterministic and use the (3 + 5)% rule, i.e. at all periods the reserve requirement is equal to the sum of the 3% of the demand forecast and 5% of the wind generation forecast. Upward and downward reserve requirements are considered symmetric. The marginal cost of reserve provision for conventional generators, C_i^{RU} and C_i^{RD}, is computed from [12].

In the remainder of the paper, we use the following abbreviations: EOO – energy-only optimisation; ERO – energy and reserves optimisation; LOC –lost opportunity cost; RAC – reserve allocation cost; TGC – total generation cost; ULC – uplift cost. Discussion and results in Subsections B and C include only TGC and RAC terms in objective function. In Subsection D, this simulation is called Sim 2.

Table 1 Daily revenue and cost results (in 10^6 \$) for model validation and ULC and LOC interpretation

Generator	EOO Pr.	ERO Pr.	RAC	LOC	TGC + RAC	ULC
nuclear 1	2.0415	2.0056	0	0.0359	0.3226	0
nuclear 2	0.9350	0.8146	0	0.1204	0.3561	0
coal 1	0.2007	0.1955	0.0010	0.0053	0.0799	0
coal 2	0.1057	0.0842	0.0033	0.0216	0.1384	0
oil 1	1.1278	1.3079	0.0079	0	1.3159	0
oil 2	0.2507	0.2606	0.0047	0	0.3417	0
gas 1	0.0829	0.0650	0.0046	0.0179	0.0918	0
gas 2	−0.0189	−0.0199	0.0018	0.0011	0.0499	0.0199

Table 2 LOC (in 10^6 \$) for different objective functions

Simulation Generator	Sim 1 TGC	Sim 2 TGC + RAC	Sim 3 TGC + LOC	Sim 4 TGC + RAC + LOC	Sim 5 TGC + ULC	Sim 6 TGC + RAC + ULC	Sim 7 T + R + L + U	Sim 8 T + R + L&U
nuclear 1	0.0069	0.0359	0	0	0	0	0.0606	0
coal 1	0.0025	0.0053	0	0	0	0	0.0094	0
oil 1	0	0	0	0	0	0	0	0
oil 2	0	0	0	0	0	0	0.0008	0
gas 1	0.0187	0.0179	0.0181	0.0263	0	0.0330	0.0044	0
gas 2	0	0.0011	0	0	0	0	0	0
gas 3	0	0	0	0	0.0011	0	0	0
gas 4	0.0293	0.0330	0	0.0361	0	0.0452	0.0201	0.0096
daily total	0.7409	0.7951	0.1620	0.4942	0.0704	0.5586	0.6394	0.2016

3.2 Lost opportunity cost

This section compares LOC of eight generating units (two units of each type). These units are listed in Table 1 in the ascending order of their flexibility and marginal cost, i.e. from relatively inflexible nuclear units to rather flexible gas-fired units.

Since the LOC payments in Table 1 aim to compensate for losses caused by reserve provision, these values are computed as the difference between the profits of EOO and ERO. The cost of reserve provision is computed in column RAC, which is zero for a unit that does not provide reserve services. Table 1 indicated that reserve providers are flexible generating units.

To better understand the results in Table 1, let us analyse unit Gas 1. This unit provides reserve and therefore the RAC payment is non-zero. Its ERO profit is lower than the EOO profit, which indicates that the offer for reserve provision of this unit is not sufficient to cover the costs for the standby of reserve capacity and the LOC it incurs. Take notice that reserve provision revenue is also included in LOC calculus in (7). The price of reserves (ρ_t^{UP} and ρ_t^{DN}) is determined in the same manner as the price of energy $\left(\beta_{t,b}^{\mathrm{PB}}\right)$. LMPs are determined as the dual variables of the power balance (17), while operating reserve prices are determined as dual variables of the reserve requirement constraints (24) and (25). All of these constraints create final prices based on the last, marginal generator. In case of unit Oil 1, there is no LOC payment, i.e. its profit increases when reserve constraints are enforced. This is because it has low cost of reserve allocation (C_i^{RU} and C_i^{RD}). Therefore, the cleared operating reserve price is sufficient to cover the reserve allocation costs. Additionally, since reserve requirements tend to decrease the output of some less expensive units, unit Oil 1 can increase its generation and consequently its profit form selling energy.

Relatively inflexible units, e.g. Nuclear 1, Nuclear 2, are also entitled to LOC payments, even though they do not provide any reserve. These LOC payments occur because during certain time periods the cleared LMPs are below the marginal cost of inflexible units, or even become negative. These effects are observed due to the discrete nature of commitment decisions and relatively high minimum power output limits of some generating units.

3.3 Uplift cost

This subsection analyses the ULC payments in Table 1, which are intended to ensure the full cost recovery to the generating units. ULC is computed as a difference of total operating cost of a given unit and its revenue, as given in (8). The ULC payment is zero for a given unit, if its total revenue is greater or equal than its operating cost.

Let us analyse unit Gas 2. It operates for 9 h and performs one start-up procedure during that time, i.e. it needs to be compensated for the start-up cost. The cleared market prices are such that the revenue cannot fully cover unit's operating cost and Gas 2 is at a loss during that day. Hence, the ULC payment should recover the operating cost of this unit.

On the other hand, unit Gas 1 receives no ULC payment, i.e. its revenue can cover all costs. It performs one start-up procedure but operates for 16 h. Revenues during those operating hours are sufficient to cover fixed, start-up, variable and reserve costs.

Relatively inflexible units (all except gas turbines) operate at constant power output throughout the day because of the low production costs. Since they never act as marginal generators, they never suffer from negative daily revenue streams. Peak units (Gas 2), on the other hand, operate in a load-following regime few hours a day and often become marginal generators. Therefore, the cleared prices are sometimes insufficient to cover their total daily TGC + RAC.

3.4 Benefits of the proposed model

Results of different simulations used for determining the benefits of the proposed algorithm are displayed in Tables 2–4. The column headlines indicate what kind of objective function was used, i.e. what terms are taken into account in (1). All the simulations are subject to the same ERO constraints. For example, objective function of Sim 1 is composed just of total generation cost – TGC, while Sim 7 takes into account TGC, RAC, ULC and LOC. Sim 8 differs from Sim 7 as it includes LOC through ULC, i.e. c_i^{LO} is included in (8) as additional revenue:

$$c_i^{\mathrm{UL}} \geq c_i^{\mathrm{TG}} + c_i^{\mathrm{RA}} - v_i^{\mathrm{TG}} - v_i^{\mathrm{RA}} - c_i^{\mathrm{LO}}. \tag{44}$$

LOC and ULC from the viewpoint of the system operator are costs, but from the viewpoint of generating units they are additional revenue. Therefore, if ULC exists, for a specific unit, it

Table 3 ULC (in 10^6 \$) for different objective functions

Simulation Generator	Sim 1 TGC	Sim 2 TGC + RAC	Sim 3 TGC + LOC	Sim 4 TGC + RAC + LOC	Sim 5 TGC + ULC	Sim 6 TGC + RAC + ULC	Sim 7 T + R + L + U	Sim 8 T + R + L&U
nuclear 1	0	0	0	0	0	0	0	0
coal 1	0	0	0	0	0	0	0	0
oil 1	0	0	0	0	0	0	0	0
oil 2	0	0	0	0	0	0	0	0
gas 1	0	0	0	0	0	0	0	0
gas 2	0	0.0199	0	0	0.0147	0	0.0154	0.0175
gas 3	0.0006	0.0005	0	0	0.0042	0	0.0017	0
gas 4	0	0	0	0	0	0	0	0.0096
daily total	0.0641	0.0595	0.1308	0.1394	0.0361	0.1435	0.0461	0.2559

Table 4 Objective function values considering out-of-market (OOM) mechanisms (in 10^6 \$) for different objective functions

Simulation	Sim 1	Sim 2	Sim 3	Sim 4	Sim 5	Sim 6	Sim 7	Sim 8
OF + OOM	TGC	TGC + RAC	TGC + LOC	TGC + RAC + LOC	TGC + ULC	TGC + RAC + ULC	T + R + L + U	T + R + L&U
OF	8.2517	8.3692	8.5453	8.9978	8.2720	8.6348	9.0520	8.6248
OF + LOC	8.9926	9.1643	/	/	8.3424	9.1934	/	/
OF + ULC	8.6158	8.8272	8.6761	9.1372	/	/	/	/
OF + LOC + ULC	9.0567	9.2238	/	/	/	/	/	/

should be decreased for the value of LOC. If ULC does not exist, ULC should be the same as LOC, which is the idea behind Sim 8.

Tables 2 and 3 display LOC $\left(c_i^{\mathrm{LO}}\right)$ and ULC $\left(c_i^{\mathrm{UL}}\right)$ values for eight randomly chosen units. Most of the chosen units are the same as in Table 1 (Nuclear 1 and Coal 1 are replaced with Gas 3 and 4 to better present changes in LOC and ULC). Last row in both tables displays total daily LOC and ULC summed over all 76 units.

All the base-load units (coal and nuclear) incur LOC in the first two simulations, but if we include either LOC or ULC in the objective function, their LOC falls to zero. ULCs for all base-load units are zero even in Sim 1 and Sim 2. Peak units behave differently than base units. For example, Oil 1 is a peak unit with low cost for reserve capacity allocation and earns more when reserve is co-optimised with energy. Thus, its LOC is zero in all the simulations. Gas 3 because it operates only few hours a day, its total daily revenue is not sufficient to cover its costs, which entails ULC, as can be seen in Table 3. Generally, observing the last row in Tables 2 and 3, one can see that incorporating ULC or LOC in the objective function decreases their values. Since constraints for LOC and ULC use the same formulations for revenue, taking into account one of them (either LOC or ULC) in the objective function will in most cases lower the other as well. LOC of unit Gas 1 is zero in Sim 5, where ULC is minimised as a part of the objective function. On the other hand, in Sim 3, where LOC is considered in the objective function, LOC is greater than zero. Some differences between units of the same technology exist because they do not have the same technical features (min/max power, ramp limits, start-up costs, marginal generation costs, …). Even if all the generator characteristics were the same, they could still have different values due to system requirements and discrete nature of some constraints. For example, if two generators with exactly the same features are offline and the system needs 1 MW of power at peak, the algorithm will start only one of those two units. This unit will have non-zero ULC, while the other one will have zero ULC. The decision which of the two units will be started will be randomly decided by the solver, as both decisions result in the same objective function value.

Table 4 compares total system costs obtained using different objective functions and ex-post calculated ULC and LOC added to the resulting objective function in case the objective function did not consider them. To clarify, the bottom four rows display objective function value and objective function value with added out-of-market LOC and/or ULC. The presented results clearly indicate the cost-effectiveness of including LOC and ULC in co-optimisation of energy and reserves. Objective function value of Sim 2 is over \$0.2556 mil. lower than of Sim 8. However, Sim 2 requires additional out-of-market payments, resulting in overall generator payments \$9.2238 mil., which is higher than total system cost when both LOC and ULC are considered in the objective function, i.e. in Sim 8 (\$8.6248 mil.). The difference is \$0.5990 mil., which presents a 6.4941% decrease in total system costs. This brings us to the conclusion that the following three issues are resolved when using the proposed method:

i. transparency issue related to the non-market-oriented prices and gathering of funds to pay these costs,
ii. incorrect allocation of the costs (with out-of-market remuneration some of the units would be under- and some over-compensated),
iii. high payments for the final consumers (with out-of-market remuneration final consumer payments are 6.4941% higher).

When only ULC or LOC payments are added to the objective function, the savings are lower. If only ULC is added to the objective function (Sim 4), the overall cost is \$8.9978 mil., which is \$0.1665 mil. (1.8505%) more efficient than OF + LOC cost of Sim 2 (\$9.1643 mil.). The same applies for adding ULC in the objective function, where the overall cost is reduced from \$8.8272. to \$8.6348 mil. (2.2282% savings). These numbers clearly indicate the benefits of the proposed model, especially when using the combined ULC and LOC implicit allocation.

Computational time for the eight different simulations performed on the 8-zone ISO-NE system ranges from 55 to 66 seconds, which indicates that the proposed procedure is applicable in today's electricity markets.

4 Conclusion

Participating in electricity markets that co-optimise energy and reserve products may hamper the ability of some generating units to fully recover their operating cost. This paper proposes an approach to ensure full cost recovery of generating units using LOC and ULC payments. These payments are co-optimised with energy and reserve provision in a market clearing procedure. As a result, the proposed approach leads to a reduction of the system-wide operating cost that in some instances exceeds 6%.

Q1

5 References

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6 Appendix

6.1 Linearisation – multiplication of variables

This appendix presents the technique used for linearisation of the multiplication of continuous PLL and continuous DLL variables. This type of nonlinear product can be equivalently expressed in terms of other dual variables in order to linearise it using the KKT optimality conditions, namely primary feasibility conditions and complementary slackness conditions. First, we use a derivative of the Lagrangian function with respect to the total generation:

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial g_{t,i}^{\mathrm{TG}}} = & -M_{i,b}^{\mathrm{TG}} \cdot \beta_{t,b}^{\mathrm{PB}} - \gamma_{t,i}^{\mathrm{TG}} - \gamma_{t,i}^{\mathrm{MX}} + \gamma_{t,i}^{\mathrm{MN}} \\ & -\alpha_{t,i}^{\mathrm{UP}} + \alpha_{t+1,i}^{\mathrm{UP}} + \alpha_{t,i}^{\mathrm{DN}} - \alpha_{t+1,i}^{\mathrm{DN}} = 0.\end{aligned} \tag{45}$$

Equation (45) is multiplied with $g_{t,i}^{\mathrm{TG}}$ and reformulated into (46). Then, both sides of (46) are summed over the observed horizon to obtain (47):

$$\begin{aligned}M_{i,b}^{\mathrm{TG}} \cdot \beta_{t,b}^{\mathrm{PB}} \cdot g_{t,i}^{\mathrm{TG}} = & \, g_{t,i}^{\mathrm{TG}} \cdot (- \gamma_{t,i}^{\mathrm{TG}} - \gamma_{t,i}^{\mathrm{MX}} \\ & + \gamma_{t,i}^{\mathrm{MN}} - \alpha_{t,i}^{\mathrm{UP}} + \alpha_{t+1,i}^{\mathrm{UP}} + \alpha_{t,i}^{\mathrm{DN}} - \alpha_{t+1,i}^{\mathrm{DN}}),\end{aligned} \tag{46}$$

$$\begin{aligned}\sum_{t=1}^{N_t} \left(M_{i,b}^{\mathrm{TG}} \cdot \beta_{t,b}^{\mathrm{PB}} \cdot g_{t,i}^{\mathrm{TG}} \right) = & \sum_{t=1}^{N_t} \Big[g_{t,i}^{\mathrm{TG}} \cdot \big(-\gamma_{t,i}^{\mathrm{TG}} \\ & -\gamma_{t,i}^{\mathrm{MX}} + \gamma_{t,i}^{\mathrm{MN}} - \alpha_{t,i}^{\mathrm{UP}} + \alpha_{t+1,i}^{\mathrm{UP}} + \alpha_{t,i}^{\mathrm{DN}} - \alpha_{t+1,i}^{\mathrm{DN}} \big) \Big]\end{aligned} \tag{47}$$

Equation (47) still contains nonlinear products. In order to linearise the first term on the right-hand side, constraint (18) has been used. This results in (48):

$$\gamma_{t,i}^{\mathrm{TG}} \cdot g_{t,i}^{\mathrm{TG}} = \sum_{k=1}^{N_k} \gamma_{t,i}^{\mathrm{TG}} \cdot g_{t,i,k}^{\mathrm{IN}} . \tag{48}$$

In order to further transform (48), derivative of the Lagrangian function with respect to the incremental generation is used, resulting in (49), further rearranged in (50):

$$\frac{\partial \mathscr{L}}{\partial g_{t,i,k}^{\mathrm{IN}}} = -C_{i,k}^{\mathrm{IN}} + \gamma_{t,i}^{\mathrm{TG}} - \gamma_{t,i,k}^{\mathrm{IN}} = 0, \tag{49}$$

$$\gamma_{t,i}^{\mathrm{TG}} = C_{i,k}^{\mathrm{IN}} + \gamma_{t,i,k}^{\mathrm{IN}} . \tag{50}$$

$\gamma_{t,i}^{\mathrm{TG}}$, as defined in (50), is inserted in (48) resulting in:

$$\gamma_{t,i}^{\mathrm{TG}} \cdot g_{t,i}^{\mathrm{TG}} = \sum_{k=1}^{N_k} \left(C_{i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} + \gamma_{t,i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} \right). \tag{51}$$

Linearisation of the second term on the right-hand side of (51) can be achieved using complementary slackness condition (52), associated with the constraint (19). It is further rearranged to (53):

$$\gamma_{t,i,k}^{\mathrm{IN}} \cdot \left(G_{i,k}^{\mathrm{IN}} \cdot x_{t,i}^{\mathrm{ON}} - g_{t,i,k}^{\mathrm{IN}} \right) = 0, \tag{52}$$

$$\gamma_{t,i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} = G_{i,k}^{\mathrm{IN}} \cdot \gamma_{t,i,k}^{\mathrm{IN}} \cdot x_{t,i}^{\mathrm{ON}} . \tag{53}$$

Multiplication $\gamma_{t,i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}}$, as defined in (53), is inserted in (51), resulting in:

$$\gamma_{t,i}^{\mathrm{TG}} \cdot g_{t,i}^{\mathrm{TG}} = \sum_{k=1}^{N_k} \left(C_{i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} + G_{i,k}^{\mathrm{IN}} \cdot \gamma_{t,i,k}^{\mathrm{IN}} \cdot x_{t,i}^{\mathrm{ON}} \right). \tag{54}$$

In order to linearise the second and the third term on the right-hand side of (47), complementary slackness conditions associated to (20) and (21) are used. The obtained (55) and (56) are further rearranged to (57) and (58):

$$\gamma_{t,i}^{\mathrm{MX}} \cdot \left(G_i^{\mathrm{MX}} \cdot x_{t,i}^{\mathrm{ON}} - g_{t,i}^{\mathrm{TG}} - r_{t,i}^{\mathrm{UP}} \right) = 0, \tag{55}$$

$$\gamma_{t,i}^{\mathrm{MN}} \cdot \left(-G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{ON}} + g_{t,i}^{\mathrm{TG}} - r_{t,i}^{\mathrm{DN}} \right) = 0, \tag{56}$$

$$\gamma_{t,i}^{\mathrm{MX}} \cdot g_{t,i}^{\mathrm{TG}} = G_i^{\mathrm{MX}} \cdot \gamma_{t,i}^{\mathrm{MX}} \cdot x_{t,i}^{\mathrm{ON}} - \gamma_{t,i}^{\mathrm{MX}} \cdot r_{t,i}^{\mathrm{UP}}, \tag{57}$$

$$\gamma_{t,i}^{\mathrm{MN}} \cdot g_{t,i}^{\mathrm{TG}} = G_i^{\mathrm{MN}} \cdot \gamma_{t,i}^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{ON}} + \gamma_{t,i}^{\mathrm{MN}} \cdot r_{t,i}^{\mathrm{DN}} . \tag{58}$$

Owing to the second terms in both (57) and (58) still contain a multiplication of continuous variables, further substitutions are required. Derivative of the Lagrangian function with respect to the reserve up and down provision is used to obtain (59) and (60), which, after rearranging, result in (61) and (62):

$$\frac{\partial \mathscr{L}}{\partial r_{t,i}^{\mathrm{UP}}} = -\gamma_{t,i}^{\mathrm{MX}} - \alpha_{t,i}^{\mathrm{UP}} - \alpha_{t+1,i}^{\mathrm{DN}} - \rho_t^{\mathrm{UP}} = 0, \tag{59}$$

$$\frac{\partial \mathscr{L}}{\partial r_{t,i}^{\mathrm{DN}}} = -\gamma_{t,i}^{\mathrm{MN}} - \alpha_{t+1,i}^{\mathrm{UP}} - \alpha_{t,i}^{\mathrm{DN}} - \rho_t^{\mathrm{DN}} = 0, \tag{60}$$

$$\gamma_{t,i}^{\mathrm{MX}} = -\alpha_{t,i}^{\mathrm{UP}} - \alpha_{t+1,i}^{\mathrm{DN}} - \rho_t^{\mathrm{UP}}, \tag{61}$$

$$\gamma_{t,i}^{\mathrm{MN}} = -\alpha_{t+1,i}^{\mathrm{UP}} - \alpha_{t,i}^{\mathrm{DN}} - \rho_t^{\mathrm{DN}} . \tag{62}$$

Dual variables $\gamma_{t,i}^{\mathrm{MX}}$ and $\gamma_{t,i}^{\mathrm{MN}}$, as defined in (61) and (62), are inserted in (57) and (58), which leads to (63) and (64):

$$\gamma_{t,i}^{\mathrm{MX}} \cdot g_{t,i}^{\mathrm{TG}} = G_i^{\mathrm{MX}} \cdot \gamma_{t,i}^{\mathrm{MX}} \cdot x_{t,i}^{\mathrm{ON}} + \alpha_{t,i}^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{UP}} + \alpha_{t+1,i}^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{UP}} + \rho_t^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{UP}}, \tag{63}$$

$$\gamma_{t,i}^{\mathrm{MN}} \cdot g_{t,i}^{\mathrm{TG}} = G_i^{\mathrm{MN}} \cdot \gamma_{t,i}^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{ON}} - \alpha_{t+1,i}^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{DN}} - \alpha_{t,i}^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{DN}} - \rho_t^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{DN}}. \tag{64}$$

Complementary slackness conditions associated with constraints (22) and (23) are

$$\alpha_{t,i}^{\mathrm{UP}} \cdot \left(A_i^{\mathrm{UP}} \cdot x_{t-1,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SU}} - g_{t,i}^{\mathrm{TG}} - r_{t,i}^{\mathrm{UP}} + g_{t-1,i}^{\mathrm{TG}} - r_{t-1,i}^{\mathrm{DN}}\right) = 0, \tag{65}$$

$$\alpha_{t,i}^{\mathrm{DN}} \cdot \left(A_i^{\mathrm{DN}} \cdot x_{t,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SD}} + g_{t,i}^{\mathrm{TG}} - r_{t,i}^{\mathrm{DN}} - g_{t-1,i}^{\mathrm{TG}} - r_{t-1,i}^{\mathrm{UP}}\right) = 0. \tag{66}$$

These are used to linearise the last three terms in (63) and (64), as well as the last four terms in (47). Equations (65) and (66) can be rearranged to (67) and (69). Please note that these refer to $\forall t \in [2, N_t]$, while the initial conditions in form of (68) and (70) also must be met.

$$\alpha_{t,i}^{\mathrm{UP}} \cdot \left(g_{t,i}^{\mathrm{TG}} - g_{t-1,i}^{\mathrm{TG}}\right) = \alpha_{t,i}^{\mathrm{UP}} \cdot (A_i^{\mathrm{UP}} \cdot x_{t-1,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SU}} - r_{t,i}^{\mathrm{UP}} - r_{t-1,i}^{\mathrm{DN}}), \forall t \in [2, N_t] \tag{67}$$

$$\alpha_{1,i}^{\mathrm{UP}} \cdot \left(g_{1,i}^{\mathrm{TG}} - G_i^{\mathrm{TG0}}\right) = \alpha_{1,i}^{\mathrm{UP}} \cdot (A_i^{\mathrm{UP}} \cdot X_i^{\mathrm{ON0}} + G_i^{\mathrm{MN}} \cdot x_{1,i}^{\mathrm{SU}} - r_{1,i}^{\mathrm{UP}} - R_i^{\mathrm{DN0}}), \tag{68}$$

$$\alpha_{t,i}^{\mathrm{DN}} \cdot (- g_{t,i}^{\mathrm{TG}} + g_{t-1,i}^{\mathrm{TG}}) = \alpha_{t,i}^{\mathrm{DN}} \cdot (A_i^{\mathrm{DN}} \cdot x_{t,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SD}} - r_{t,i}^{\mathrm{DN}} - r_{t-1,i}^{\mathrm{UP}}), \forall t \in [2, N_t] \tag{69}$$

$$\alpha_{1,i}^{\mathrm{DN}} \cdot \left(-g_{1,i}^{\mathrm{TG}} + G_i^{\mathrm{TG0}}\right) = \alpha_{1,i}^{\mathrm{DN}} \cdot (A_i^{\mathrm{DN}} \cdot x_{1,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{1,i}^{\mathrm{SD}} - r_{1,i}^{\mathrm{DN}} - R_i^{\mathrm{UP0}}). \tag{70}$$

Both sides of (67) and (69) are then summed over the observed time period (including $t = 1$ through (68) and (70)) and rearranged to (71) and (72):

$$-\alpha_{1,i}^{\mathrm{UP}} \cdot G_i^{\mathrm{TG0}} + \sum_{t=1}^{N_t - 1} g_{t,i}^{\mathrm{TG}} \cdot \left(\alpha_{t,i}^{\mathrm{UP}} - \alpha_{t+1,i}^{\mathrm{UP}}\right) + g_{Nt,i}^{\mathrm{TG}} \cdot \alpha_{Nt,i}^{\mathrm{UP}} = \sum_{t=1}^{N_t} \alpha_{t,i}^{\mathrm{UP}} \cdot \left(A_i^{\mathrm{UP}} \cdot x_{t-1,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SU}} - r_{t,i}^{\mathrm{UP}} - r_{t-1,i}^{\mathrm{DN}}\right), \tag{71}$$

$$\alpha_{1,i}^{\mathrm{DN}} \cdot G_i^{\mathrm{TG0}} + \sum_{t=1}^{N_t} g_{t,i}^{\mathrm{TG}} \cdot \left(\alpha_{t+1,i}^{\mathrm{DN}} - \alpha_{t,i}^{\mathrm{DN}}\right) - \alpha_{Nt,i}^{\mathrm{DN}} \cdot g_{Nt,i}^{\mathrm{TG}} = \sum_{t=1}^{N_t} \alpha_{t,i}^{\mathrm{DN}} \cdot \left(A_i^{\mathrm{DN}} \cdot x_{t,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SD}} - r_{t,i}^{\mathrm{DN}} - r_{t-1,i}^{\mathrm{UP}}\right). \tag{72}$$

Multiplications $\gamma_{t,i}^{\mathrm{TG}} \cdot g_{t,i}^{\mathrm{TG}}$, as defined in (54), $\gamma_{t,i}^{\mathrm{MX}} \cdot g_{t,i}^{\mathrm{TG}}$, as defined in (63), and $\gamma_{t,i}^{\mathrm{MN}} \cdot g_{t,i}^{\mathrm{TG}}$, as defined in (64), along with the sums from the left-hand sides of (71) and (72), are inserted in (47), resulting in (after rearranging the terms):

$$\begin{aligned} &\sum_{t=1}^{N_t} (M_{i,b}^{\mathrm{TG}} \cdot \beta_{t,b}^{\mathrm{PB}} \cdot g_{t,i}^{\mathrm{TG}}) \\ &= \sum_{t=1}^{N_t} \Bigg[- \sum_{k=1}^{N_k} \left(C_{i,k}^{\mathrm{IN}} \cdot g_{t,i,k}^{\mathrm{IN}} + G_{i,k}^{\mathrm{IN}} \cdot \gamma_{t,i,k}^{\mathrm{IN}} \cdot x_{t,i}^{\mathrm{ON}}\right) \\ &\quad - G_i^{\mathrm{MX}} \cdot \gamma_{t,i}^{\mathrm{MX}} \cdot x_{t,i}^{\mathrm{ON}} - \rho_t^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{UP}} \\ &\quad + G_i^{\mathrm{MN}} \cdot \gamma_{t,i}^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{ON}} - \rho_t^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{DN}} \\ &\quad - \alpha_{t,i}^{\mathrm{UP}} \cdot (A_i^{\mathrm{UP}} \cdot x_{t-1,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SU}}) \\ &\quad - \alpha_{t,i}^{\mathrm{DN}} \cdot (A_i^{\mathrm{DN}} \cdot x_{t,i}^{\mathrm{ON}} + G_i^{\mathrm{MN}} \cdot x_{t,i}^{\mathrm{SD}}) \Bigg]. \end{aligned} \tag{73}$$

Equation (73) still contains two nonlinear terms, $\rho_t^{\mathrm{UP}} \cdot r_{t,i}^{\mathrm{UP}}$ and $\rho_t^{\mathrm{DN}} \cdot r_{t,i}^{\mathrm{DN}}$. Nevertheless, by substituting (4), (5), and (73) into the lost opportunity cost (7) and uplift compensation cost (8) constraints, nonlinear terms will subtract each other and the only remaining nonlinearity will be multiplication of a binary variable and a dual continuous variable, which can be easily linearised using the ‘big M’ method, finally resulting in (42) and (43).

IET-GTD20185480

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