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Satellite Routing with Quantum Annealing: Collecting Space Debris and On-orbit Servicing

Priyank Dubey[a b], Andreas M. Hein [c]

[a] *Visiting Student, SpaSys-SnT, University of Luxembourg, Luxembourg* . priyank.dubey@ext.uni.lu
[b] Corresponding Author
[c] *Associate Professor, SpaSys-SnT, University of Luxembourg, Luxembourg* . andreas.hein@uni.lu

Abstract

Optimizing satellite routes for multiple space debris collection and multiple on-orbit servicing can be a very complex problem due to the large number of variables and constraints that need to be taken into account. Factors such as the location and movement of the debris and servicing targets in the orbit, the capabilities of the satellite, and the constraints on the satellite's fuel and power usage all need to be considered. Additionally, the problem may be further complicated by the need to consider multiple objectives, such as minimizing fuel usage while maximizing debris collection or servicing coverage. Classical approach to solve this problem includes heuristics and metaheuristics methods like Genetic Algorithm, Particle Swarm Optimization, Ant Colony Optimization and mixed-integer programming. In the current paper, we plan to implement Quantum annealing based algorithm for optimizing satellite routes. It is a quantum computing method that can be used to optimize satellite routes. The principle behind quantum annealing is to use quantum-mechanical effects to find the global minimum of a function. In the context of satellite routing, this function would represent the cost or energy required for a satellite to travel a certain route. The satellite's routes would be represented by variables in the function, and the quantum annealer would use quantum-mechanical effects to search for the lowest-energy route, which would correspond to the optimal path for the satellite to take. We plan to use Ising model to implement quantum annealing for satellite routing. It can used to represent the cost function as a set of binary variables interacting with each other through pairwise interactions. The interactions between the variables would represent the different constraints and objectives of the routing problem, such as fuel usage and debris collection. The goal would be to find the configuration of variables that minimizes the cost function, which corresponds to the optimal satellite route. A complete mathematical model will be generated, and numerical analysis will be performed based on the presented technique.

Keywords: Quantum computing for space, Space technology, Quantum routing algorithm , Satellite routing, Space debris, On-orbit servicing

1. Introduction

The exponential growth in the number of satellites and various other objects present in the Earth's orbit has resulted in a progressively congested space environment. The increasing proliferation of human-made artifacts, encompassing inactive satellites, spent rocket stages and hardware fragments, presents a substantial hazard not only to functioning satellites but also to the safety of future space missions [1]. Furthermore, the concept of on-orbit servicing, which involves the repair, refueling, and enhancement of satellites that are already situated in orbit, has attracted significant attention. Furthermore, the concept of on-orbit servicing, which involves the repair, refueling, and enhancement of satellites that are already situated in orbit, has attracted significant attention [2]. The utilization of dedicated satellites for the purpose of collecting space debris and providing on-orbit servicing has emerged as a feasible solution to address these challenges. These initiatives possess the capacity to increase the operational lifespan of satellites and reduce the probability of catastrophic collisions occurring in orbit.

The Defense Advanced Research Projects Agency's (DARPA) Orbital Express mission in 2007 which demonstrated autonomous docking and fluid transfer between two spacecraft, laying the groundwork for future on-orbit servicing missions [3]. NASA's Restore-L project aimed to refuel a government-owned satellite and demonstrate robotic manipulation in space [4]. Commercial ventures like Northrop Grumman and Astroscale are working on commercial OOS solutions, such as Northrop Grumman's MEV-1, which successfully docked with an aging communications satellite [5].

Nevertheless, the process of routing and scheduling these servicing satellites is not a straightforward task. The satellite is required to successfully traverse a complex network of orbital trajectories, each presenting distinct limitations pertaining to the principles of physics, energy management, timing synchronization, and safety considerations [6]. Moreover, the mission objectives may encompass various aspects, including the prioritization of high-risk debris and the optimization of fuel consumption [7]. There are several heuristics and meta-heuristics problem-solving methods have been proposed so far proposed in order to achieve the objective while considering the constraints mention above. In [8], a vehicle routing problem with time windows (VRPTW)-based hybrid genetic algorithm is proposed to tackle the single objective static multi-satellite collection scheduling problem (m-SatCSP). [9][10] presents an ant colony optimization (ACO) metaheuristic that features a special solution representation for the fuel tanker location and routing decision. [11] seek to maximize the weighted number of completed tasks subject to constraints related to the movements of the space robots, refueling depots, and tasks. Similarly, several other studies present different algorithm for the problems [12][13][14][15].

However, the task of satellite routing aims to identify the most cost-efficient and time-effective paths for both space debris collection and on-orbit servicing. Optimizing these routes can lead to substantial cost savings, potentially reducing overall mission expenses by up to 30% [16]. Current computational methods can handle complexities involving around 400 variables within an acceptable time frame [17]. As the demands for space-based services and debris mitigation grow, the classical algorithms will soon reach their computational limits. Looking ahead, there will be a need for approaches that can rapidly solve problems involving thousands of variables. The increase in the number of variables has a tendency to exponentially increase the computational time required to solve these problems on classical computers. One area in which quantum computers exhibit significant potential is their ability to address computational challenges by efficiently encoding and solving large-scale problems [18]. This is achieved through the utilization of quantum algorithms, such as the quantum approximate optimization algorithm (QAOA), which enables expedited problem-solving capabilities.

Quantum-based routing algorithms offer a promising avenue for meeting these future challenges, enabling quicker and more efficient optimization [19]. Currently, there are two prevailing paradigms in the field of quantum computing: annealing-based quantum computers and gate-model quantum computers [20][21]. The preceding models are distinguished by their utilization of quantum annealing, a technique that yields the lowest energy state of a specified quantum Hamiltonian, also referred to as an energy function. In the context of optimization problems, the quantum Hamiltonian is established based on the fitness function associated with the problem, such that the solution to the problem is represented by the lower energy state. Quantum-gate devices, in contrast, are distinguished by their utilization of quantum circuits, which consist of operations known as quantum gates. These quantum gates can be likened to the classical logic gates found in conventional circuits. Therefore, the application of quantum gates in a sequential manner is employed to manipulate the states of qubits until the final solution of the problem is achieved.

In this paper, we apply quantum annealing to the satellite routing problem with applications to space debris removal and in-orbit servicing.

This paper is structured as follows: In Sect. 2, we provide an basic introduction to Quantum Annealing along with Ising model, QUBO. We have discussed about the step by step problem formulation methodology in Sect. 3. In Sect. 4, we provide the results for both classical and quantum solution. In Sect. 5, we finally conclude the paper with the conclusion.

2. Quantum Annealing

The recent advancements in the manipulation of quantum states have given rise to the concept of quantum computation and simulation. Initially rooted in theoretical concepts, this notion has now expanded to promote an established sector with vast potential for technological applications [22]. The quantum annealing technique was formally introduced in the scientific community through the publication by [23], followed by its practical validation by [24]. The development and subsequent commercialization of a superconducting circuit quantum Ising glass annealing machine was achieved by D-wave systems [25]. Subsequently, a significant transformation has occurred as a result of a proliferation of remarkable research papers in both theoretical and experimental domains. The extensive and rigorous investigations conducted over the past two decades have ultimately resulted in the emergence of a novel era characterized by quantum information and technologies. The field of quantum annealing (QA) has experienced significant growth due to the development and commercialization of programmable QA machines, known as quantum annealers, which possess thousands of qubits. There are several literature which presents an overview of the QA based protocol and delves into recent theoretical and experimental advancements in QA that leverage the benefits of quantum tunneling for identifying the minimum of a classical energy function [22][26][27].

2.1. Ising Model

QA usually focuses toward finding the ground state of a generic Ising model, which may include random biases and/or random many-body interactions [28]. In statistical mechanics, the Ising model is a mathematical model used to describe how spins behave on a lattice. It's one of the simplest representations of magnetism's core physical principles. Spins are a type of discrete variable in this model; they can take on the values +1 (spin up) or -1 (spin down). Each of these spins interacts with its immediate neighbors, creating a lattice structure.

Consider an Ising Hamiltonian H_P the subscript P stands for the problem Hamiltonian. It is assumed that H_P is a classical many body Ising Hamiltonian described in terms of the z components of the Pauli operator $\left\{\sigma_j^z\right\}$. Consider a driver Hamiltonian H_D which is not commutative with H_P and has the trivial ground state. A simple choice for H_D is the transverse field: $H_D=-\sum_j \sigma_j^T$, so that H_D does not commute with H_P. The total Hamiltonian of QA is given as

$$H(t)=A(t)H_D+B(t)H_P, \quad (1)$$

where $A(t)$ and $B(t)$ are the scheduling function satisfying $A\left(t_i\right) \gg B\left(t_i\right)$ at the initial time t_i and $A\left(t_f\right) \ll B\left(t_f\right)$ at the final time t_f so that $H(t)$ interpolates between H_D at $t=t_i$ and H_P at $t=t_f$. The initial state at $t=t_i$ is set at the ground state of $H_D \approx H\left(t_i\right)/A\left(t_i\right)$. If the change in $H(t)$ with t is considerably small, the spin state evolves adiabatically (i.e., always stays in the ground state of the instantaneous Hamiltonian) and arrives at the ground state of H_P at $t=t_f$ which we seek. This constitutes the basic notion of the QA, also known as the adiabatic quantum computation. Consider the following Hamiltonian with ferromagnetic nearest neighbour interactions in one dimension:

$$H=-J\sum_j \sigma_j^z\sigma_{j+1}^z-\Gamma\sum_j \sigma_j^x. \quad (2)$$

where J denotes the strength of the interaction and Γ is the strength of the non-commuting transverse field. Here $H_D=-\sum_j \sigma_j^x$ and $H_P=-\sum_j \sigma_j^z\sigma_{j+1}^z$. The transverse field Γ is annealed to reach the ground state of H_P from the ground state of H_D.

The efficacy of quantum annealing (QA) is dependent upon the rate at which the Hamiltonian evolves over time. The adiabatic theorem of quantum mechanics establishes the criterion for adiabatic time evolution [29].

$$\frac{\max\left[\left|\left\langle 1(t)\left|\frac{dH(t)}{dt}\right|g(t)\right\rangle\right|\right]}{\min[\Delta(t)]^2} \ll 1, \quad (3)$$

where $|g(t)\rangle$ and $|1(t)\rangle$ are the instantaneous ground and first-excited states at time t, respectively, and $\Delta(t)$ denotes the instantaneous energy gap above $|g(t)\rangle$. The min and max functions are taken with respect to the variable t. Thus, roughly speaking, QA works better for larger $\Delta(t)$.

2.2. Quadratic Unconstrained Binary Optimization (QUBO)

Quadratic unconstrained binary optimization (QUBO), alternatively referred to as unconstrained binary quadratic programming (UBQP), is a combinatorial optimization problem that finds extensive application in various domains, including finance, economics, and machine learning. The QUBO problem is classified as NP-hard. In the field of theoretical computer science, various classical problems such as maximum cut, graph coloring, and the partition problem have been formulated in terms of embeddings into QUBO [30].

Consider a set of binary vectors of a fixed length $n>0$ is denoted by $\mathbb{B}^n$, where $\mathbb{B}=\{0,1\}$ is the set of binary values (or bits). We are given a real-valued upper triangular matrix $Q\in\mathbb{R}^{n\times n}$, whose entries Q_{ij} define a weight for each pair of indices $i,j\in\{1,\ldots,n\}$ within the binary vector. We can define a function $f_Q:\mathbb{B}^n\rightarrow\mathbb{R}$ that assigns a value to each binary vector through

$$f_Q(x)=x^\top Qx=\sum_{i=1}^n\sum_{j=i}^n Q_{ij}x_ix_j \quad (4)$$

Intuitively, the weight Q_{ij} is added if both x_i and x_j have value 1 . When $i=j$, the values Q_{ii} are added if $x_i=1$, as $x_ix_i=x_i$ for all $x_i\in\mathbb{B}$. The QUBO problem consists of finding a binary vector x^* that is minimal with respect to f_Q, namely

$$x^*=\underset{x\in\mathbb{B}^n}{\arg\min} f_Q(x)$$

In general, x^* is not unique, meaning there may be a set of minimizing vectors with equal value w.r.t. f_Q. The complexity of QUBO arises from the number of candidate binary vectors to be evaluated, as $|\mathbb{B}^n|=2^n$ grows exponentially in n.

The Ising model and QUBO are equivalent via a linear transformation of the variables. The binary quadratic model (BQM) class contains Ising and quadratic unconstrained binary optimization (QUBO) models used by samplers such as the D-Wave system. QUBO is very closely related and computationally equivalent to the Ising model, whose Hamiltonian function as defined earlier :

$$H(\sigma) = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j - \mu \sum_j h_j\sigma_j$$

with real-valued parameters h_j, J_{ij}, μ for all i, j. The spin variables σ_j are binary with values from $\{-1, +1\}$ instead of $\mathbb{B}$. Moreover, in the Ising model the variables are typically arranged in a lattice where only neighboring pairs of variables $\langle ij \rangle$ can have nonzero coefficients. Applying the identity $\sigma \mapsto 2x - 1$ yields an equivalent QUBO problem [31]

$$\begin{aligned}
f(x) &= \sum_{\langle ij \rangle} -J_{ij}\left(2x_i - 1\right)\left(2x_j - 1\right) + \\
&\sum_j \mu h_j \left(2x_j - 1\right) \\
&= \sum_{\langle ij \rangle} \left(-4J_{ij}x_ix_j + 2J_{ij}x_i + 2J_{ij}x_j - J_{ij}\right) + \\
&\sum_j \left(2\mu h_j x_j - \mu h_j\right) \\
&\text{using } x_j = x_jx_j \\
&\text{and using } \sum_{\langle k=ji \rangle} = \sum_{\langle ik=j \rangle} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{i} Q_{ij}x_ix_j + C
\end{aligned}$$

where

$$Q_{ij} = \begin{cases} -4J_{ij} & \text{if } i \neq j \\ \sum_{\langle ik=j \rangle} \left(2J_{ki} + 2J_{ik}\right) + 2\mu h_j & \text{if } i = j \end{cases}$$

$$C = -\sum_{\langle ij \rangle} J_{ij} - \sum_j \mu h_j$$

and using the fact that for a binary variable $x_j = x_jx_j$. As the constant C does not change the position of the optimum x^*, it can be neglected during optimization and is only important for recovering the original Hamiltonian function value.

3. Problem Formulation

The problem can be viewed in the context of space operations as one of planning optimal routes for on-orbit servicing of satellites or collecting space debris. This is similar to the Vehicle Routing Problem (VRP) in terrestrial logistics, but with additional complexities due to space and orbital mechanics' three-dimensional nature. Also, in space operations, the concept of distance goes beyond the simple spatial separation between two objects. Objects in space are not stationary but in orbit around celestial bodies, making it difficult to travel between them. To move between objects, one must consider the orbital paths of both objects, which requires changing velocity and fuel expenditure. Even if the target is at the same orbital altitude, matching velocities are necessary for docking or transferring cargo. Velocity matching is another step that requires precise control and more fuel. Orbital manoeuvres are resource-intensive, such as Hohmann transfers, which require careful planning and additional fuel [32]. Therefore, minimizing distance in space operations is often a more complex optimization problem, aiming to minimize the total expenditure of resources, particularly fuel, required for orbital manoeuvres and velocity matching. Time is also a critical resource, as missions may have limited window periods. The true objective is to minimize the complexity and resource expenditure of the entire operation from start to finish.

In the context of space operations, consider a framework is proposed wherein a central mothership, referred to as M, serves as the central hub for all activities related to orbital servicing and retrieval of space debris. The mothership is stationed at a predetermined orbital location and is responsible for coordinating servicing missions aimed at a group of ten satellites or fragments of space debris, referred to collectively as S. The objects within the domain of S are designated for either maintenance or collection, based on predictive requirements. In order to execute these specialized operations, M utilizes a specific group of dedicated servicing satellites, referred to as T, which are each equipped with the necessary capabilities for on-orbit servicing or space debris capture. The primary objective is to reduce the expenses related to satellite maintenance and debris retrieval. This expense may encompass various resources such as time, fuel, or other relevant factors. We model this problem as Vehicle Routing Problem (VRP) variant in QUBO with the constraints. Both Classical and Quantum solvers are used to solve this problem. We IBM DOcplex interface coupled with the CPLEX optimizer as a classical solver and quantum algorithms QAOA and VQE coupled with using conventional optimizers SPSA and COBYLA as quantum solution. We present the results of each scenario in terms of associated cost and iterations. We also visualize the solution on a graph using NetworkX.

Our main is to minimize cost associated with servicing the satellites or collecting debris. This cost could be time, fuel, or some other resource. Mathematically, this is represented as C_{ij} between the $ith - jth$ space debris or service receiver satellites, weighted by the decision variable X_{ij}. For our case, to minimize the time or fuel cost for servicing a subset of 10 satellites predicted to require on-orbit servicing or for collecting space debris, using 3 servicing satellites launched from a mothership. The decision variable x_{ij} can be defined as:

$$X_{ij} = \begin{cases} 1, & \text{if } ij \text{ route is considered for solution} \\ 0, & \text{otherwise} \end{cases}$$

Hence, we can write our objective function as: Minimize

$$\text{Objective } = \min \sum_{i \in S} \sum_{j \in S} C_{ij} \cdot X_{ij} \qquad (5)$$

where C_{ij} is the distance (or fuel cost, or time cost) between the $ith - jth$ space debris or service receiver satellites. The cost function is subjected to constraints:

$$\begin{array}{ll} \sum_{i \in S} x_{ij} = 1 & \forall j \in S \backslash \{0\} \\ \sum_{j \in S} x_{ij} = 1 & \forall i \in S \backslash \{0\} \end{array} \qquad (6)$$

$$\begin{array}{ll} \sum_{i \in S} x_{i0} = M & \forall j \in S \backslash \{0\} \\ \sum_{j \in S} x_{i0} = M & \forall i \in S \backslash \{0\} \end{array} \qquad (7)$$

$$\sum_{i \neq V} \sum_{j \in V} x_{ij} \geq r(V), \quad \forall V \subseteq S \backslash \{0\}, V \neq \emptyset \qquad (8)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in S$$

The above constraints ensure that there is only one route towards S_i and leaving S_j, all the T should leave M and come back to M and impose the routes must be connected respectively. Hence, we have our objective function in QUBO form 5 subjected to constraints 6, 7 and 8.

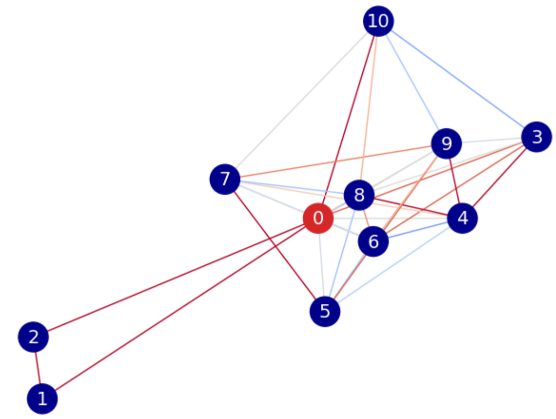

Figure 1: Visualization of the complexity of the problem as a graph using NetworkX in the cool-warm color scheme. There are 34 possible edges of the problem. The mothership or central hub is colored in red, and the other space debris/satellites for on-orbit servicing are in dark blue.

4. Results

The solution of the QUBO problem 5 is examined through the utilization of both classical and quantum solvers. The comprehensive examination of the complexities and potential solutions of the satellite routing problem is achieved by employing both classical and quantum computational methods. By employing this dual methodology, we are able to assess the comparative advantages and disadvantages of individual computational paradigms, thus providing a comprehensive approach to enhancing the efficiency of satellite routes for activities such as the collection of space debris and the execution of on-orbit services.

4.1. Classical Solution

In the context of satellite routing for tasks such as space debris collection or on-orbit servicing, the QUBO model is a robust mathematical formulation that captures the problem's complexities. The goal is to minimize a quadratic function with binary variables, which could represent decisions such as whether or not to route a satellite to a specific location. When using a classical computing approach to solve this QUBO model, the IBM DOcplex [33] interface coupled with the CPLEX optimizer [34] constitutes a powerful toolkit.

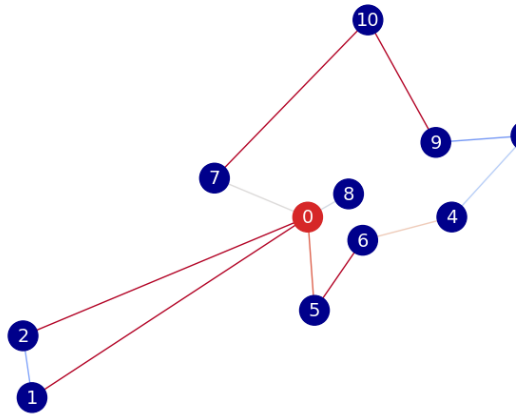

Figure 2: Visualization of the optimized solution from CPLEX's optimizers using NetworkX in the coolwarm color scheme. The mothership or central hub is colored in red, and the other space debris/satellites for on-orbit servicing are in dark blue.

Once the problem is formulated as a QUBO, the CPLEX optimization algorithm suite can be utilized. CPLEX possesses a variety of algorithms, such as simplex algorithms, interior-point methods, and branch-and-cut algorithms, which are highly effective for mixed-integer quadratic programming problems such as the one we're addressing. Due to its quadratic terms, the QUBO model is inherently nonlinear; however, CPLEX's optimizers are designed to efficiently handle such complexities. Under certain conditions, CPLEX's classical algorithms guarantee global optimally, which means that the solution you get is the best one possible given the constraints. This reliability is very important for operations with high stakes, like satellite routing, where making decisions that aren't the best could lead to higher costs or even mission failure.

Figure. 2 shows the optimized solution of QUBO problem 5 formed with the mentioned satellite routing scenario. The system facilitates an optimal connection between space debris or satellites requiring servicing (S). The provided solution is considered the best due to its ability to minimize resource consumption and associated costs. The color scheme employed in this study represents the range of costs associated with nodes, with nodes of maximum cost depicted in red and nodes of minimum cost transitioning to blue. For the rest of the results, we will consider this solution an optimal solution for comparison.

4.2. Quantum Solution

After obtaining the quadratic program formulation for our model, the Qiskit optimization library can be employed to convert it into its QUBO representation [35]. In the present scenario, there are inequality limitations that typically necessitate the inclusion of additional variables, sometimes referred to as slack variables. In the context of this particular scenario, the quantity of variables expands from 68 to 196 in order to account for the inclusion of inequality restrictions. Another limitation pertains to the number of qubits that can be simulated. The IBM-provided $qasm - simulator$, specifically in its runtime mode, is the most practical method for simulating programs using more than 20 qubits in the absence of a supercomputer. The $qasm - simulator$has a limitation where it can only simulate a maximum of 32 qubits. In the context of actual devices, a comparable issue arises where our ability to do experiments is limited to a 7-qubit device that we have access to.

As a result, we can reduce the number of variables by selecting the optimal solution for the slack variables. We select the optimal solution for some of the variables that reflect the routes from node i to node j and let the quantum computer determine the optimal solution for the rest: Using 7 variables, replace the others with the optimal solution. From the 7 variables left, 3 have an optimal solution equal to 1 and 4 equal to 0. We analyzed to assess the suitability of the existing model with perfect solutions by employing two conventional optimizers, namely Simultaneous perturbation stochastic approximation (SPSA) and Constrained Optimization BY Linear Approximation (COBYLA), for two quantum algorithms: Quantum Approximate Optimization Algorithm (QAOA) with 2 layer repeats and variational quantum eigensolver (VQE) with the two Local ansatz. In both cases, a total of 1024 shots were used.

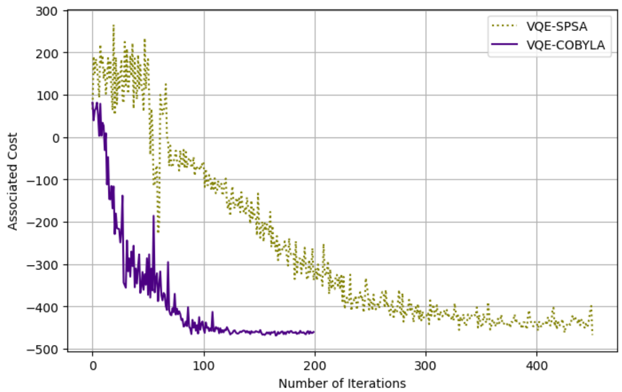

Figure 3: Simulation for associated cost vs iteration for solution by using VQE-SPSA and VQE-COBYLA.

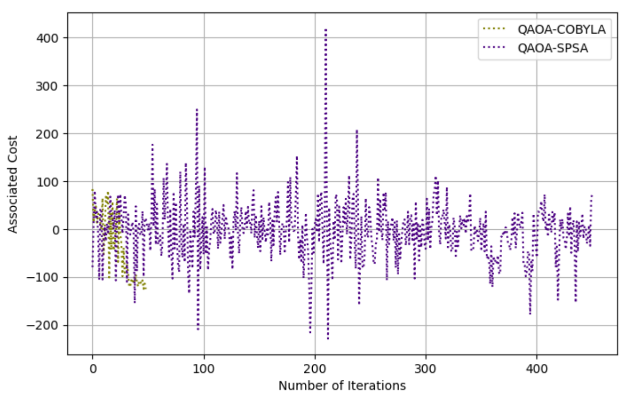

Figure 4: Simulation for associated cost vs iteration for solution by using QAOA-SPSA and QAOA-COBYLA.

The cost vs iteration simulation using the Variational Quantum Eigensolver (VQE) method, in conjunction with classical Simultaneous Perturbation Stochastic Approximation (SPSA) and Constrained Optimization by Linear Approximation (COBYLA) optimizers, is illustrated in Figure. 3. It is evident that both methods yielded approximately identical minimum costs. In contrast to VQE-COBYLA, VQE-SPSA requires a greater number of iterations to converge to the final minimum. The graphical depiction of these concepts can be seen in Figure. 5. Based on the analysis of Figure. 3, it can be deduced that the outcomes of both the graphical representation graphs and CPLEX's optimizers resulted in an optimal solution or the same path between the starting point S, as depicted in Figure.5.

In similar fashion, Figure. 4 illustrates the cost versus iteration simulation utilizing the Quantum Approximate Optimization Algorithm (QAOA), in conjunction with classical Simultaneous Perturbation Stochastic Approximation (SPSA) and Constrained Optimization by Linear Approximation (COBYLA) optimizers. It is evident that both methods do not yield the same minimum cost in the end. The QAOA-COBYLA algorithm achieved convergence to the final value after 100 iterations, yielding an associated cost of approximately -100. Despite the presence of considerable noise during the optimization process of the Quantum Approximate Optimization Algorithm with Simultaneous Perturbation Stochastic Approximation (QAOA-SPSA), it manages to converge towards the optimal solution. The QAOA-SPSA algorithm requires a greater number of iterations to converge to the final cost when compared to the VQE-COBYLA algorithm. The visual representations of these concepts can be observed in Figure. 5. Based on the analysis of Figure. 5, it can be deduced that the QAOA-SPSA approach yielded an optimal solution or a path that is identical to the one predicted by CPLEX's optimizers, as illustrated in Figure. 2 .

5. Conclusion

In this research, we propose a framework for orbital servicing and debris retrieval, involving a central mothership and dedicated satellites. The goal is to reduce expenses related to satellite maintenance and debris retrieval. The problem is modelled as a Vehicle Routing Problem and solved using both classical and quantum solvers. Results are presented in terms of associated costs and iterations, and visualized on a graph using NetworkX. Optimal outcomes are observed in certain quantum algorithms, while others do not exhibit the same level of performance. The representation of the initial problem requires 68 variables while the QUBO representation needs 196 variables. Due to limited qubits, we reduced the number of variables to 7 choosing the correct set of variables for the slack variables. Nevertheless, the quantum algorithm yields optimal solutions that are comparable to those obtained from classical algorithms. This suggests that the utilization of the quantum framework may present a feasible strategy for enhancing the efficiency of satellite servicing and space debris collection missions in the near term, dependent upon the availability of quantum computers with a greater number of qubits. This research thus lays the groundwork for future studies that could significantly improve the efficiency and cost-effectiveness of these crucial space operations.

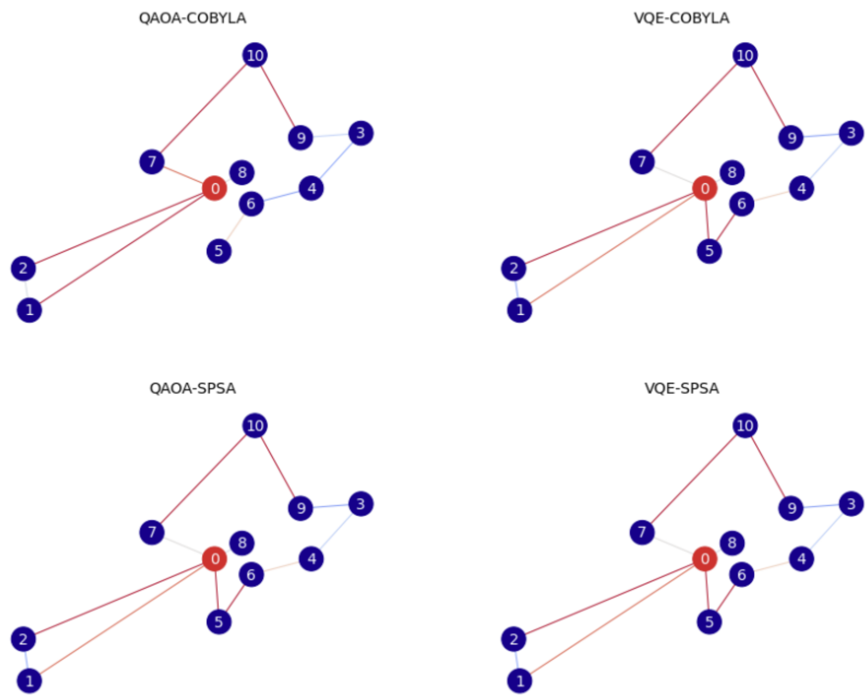

Figure 5: Visualization of solutions from VQE-SPSA, VQE-COBYLA, QAOA-SPSA and QAOA-COBYLA optimizers using NetworkX in the cool- warm color scheme. The mothership or central hub is colored in red, and the other space debris/satellites for on-orbit servicing are in dark blue

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