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ORIGINAL RESEARCH

CFMT: a collaborative filtering approach based on the nonnegative matrix factorization technique and trust relationships

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Abstract
As a method of information filtering, the Recommender System (RS) has gained considerable popularity because of its efficiency and provision of the most superior numbers of useful items. A recommender system is a proposed solution to the information overload problem in social media and algorithms. Collaborative Filtering (CF) is a practical approach to the recommendation; however, it is characterized by cold start and data sparsity, the most severe barriers against providing accurate recommendations. Rating matrices are finely represented by Nonnegative Matrix Factorization (NMF) models, fundamental models in CF-based RSs. However, most NMF methods do not provide reasonable accuracy due to the dispersion of the rating matrix. As a result of the sparsity of data and problems concerning the cold start, information on the trust network among users is further utilized to elevate RS performance. Therefore, this study suggests a novel trust-based matrix factorization technique referred to as CFMT, which uses the social network data in the recommendation process by modeling user's roles as trustees and trusters, given the trust network's structural information. The proposed method seeks to lower the sparsity of the data and the cold start problem by integrating information sources including ratings and trust statements into the recommendation model, an attempt by which significant superiority over state-of-the-art approaches is demonstrated an empirical examination of real-world datasets.

Keywords Recommender system · Nonnegative matrix factorization · Gradient descent · Trust relationship · Collaborative filtering

1 Introduction

The information generated by the World Wide Web increases exponentially, highlighting the significant role of RSs as attractive tools for handling the information overload problem (Xu 2018). Having developed in parallel with the web, recommender systems now incorporate social information despite their initial demographic, content, and collaborative filtering bases (Bobadiila et al. 2013). A recommender system (RS) is a proposed solution to the information overload problem in social media, and algorithms such as collaborative filtering (CF) can generate personalized recommendations for users according to their behaviors (Anand and Bharadwaj 2011). There are three types of recommender systems, including collaborative filtering, content-based, and hybrid systems. In the content-based method, recommendations are made based on the user profile and item description similarity. In collaborative filtering, recommendations are provided to users according to other user's opinions. (Kalaï et al. 2018) Complex recommendations can also be made there since product content is disregarded in the recommendation process. It has turned CF into a popular filtering method, playing a vital role in many applications. A hybrid RS integrates collaborative filtering and content-based recommender systems. CF is addressed here as a popular recommender model commonly used in various applications, along with the drawbacks that it naturally suffers, the major one being data sparsity. In a model based on matrix factorization (MF), the users and items are mapped to the same latent feature space, the desired user/item features are trained on the existing ratings, and predictions are then made

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for unknown ratings based mainly on the inner products of related pairs of user-items and feature-vectors. The latent feature space dimension can be set to a low value with no accuracy reduction, as the rating matrix is too sparse and, therefore, usually of a low rank (Najafabadi et al. 2019).

CF uses user feedbacks to a limited set of items to make appropriate recommendations on the remaining items. CF methods are divided into two general categories: memory-based (Koren 2008) and model-based (Hofmann 2004). Model-based methods generate models to make recommendations and describe user behavior to predict item ratings. Despite their typically high recommendation, model-based methods have high computational complexity, turning into a severer issue for more users and items. On the other hand, memory-based methods, applied to the rating matrix, employ the user ratings on items for measurement of inter-user or inter-item similarity to select neighbors and recommend their items to the current user (Bobadilla et al. 2011). They are, therefore, faster than model-based methods but do not produce as accurate predictions. Memory-based methods exhibit greater scalability than model-based methods. (Luo et al. 2016). Both these methods have some disadvantages in item recommendation, where sparsity is highlighted, i.e., The other RS method, i.e., CF, is also associated with some drawbacks, such as the cold start problem, which occurs as a new user or new item enters the system (Lee and Brusilovsky 2017). More strictly, cold-start users/items are those on whom/which few ratings are available. Thus, a CF-based recommender's key function is to estimate missing rating values using those that are available (Guo et al. 2014a, b). We use a CF-based model for better predictions in recommender systems given the abundance of information, cold start, and data sparsity problems.

Benefitting from their excellent accuracy and scalability, matrix factorization methods are regarded as prominent ones among model-based CF approaches, where the principal idea is to draw both users and items into an analogous latent feature space. In an MF-based model, the users and items are mapped to the same latent feature space. The desired user/item features are trained on the existing ratings. Predictions are then made for unknown ratings based mainly on the inner products of related pairs of user-items and feature-vectors. The latent feature space dimension can be set to a low value with no decrease inaccuracy, as the rating matrix is too sparse and, therefore, usually of a low rank (Koren and Bell 2011). MF models are the basis of success in fulfilling some latent factor models (Eirinaki et al. 2018).

On the other hand, CF methods achieve high prediction accuracy and scalability using the MF technique. They predict user tastes in recommender systems based on collaborative filtering by presenting a novel approach, factorizing the rating matrix into two nonnegative matrices with elements ranging in [0, 1] and an understandable probabilistic meaning (Hernando et al. 2016). The personalized items are recommended to the users in a recommender system based on collaborating filtering according to their interests, rating behaviors, references, history, and many further aspects of information already provided by them. Neither the user's features nor those of the items are used by the recommender system, which instead employs user ratings or a user-item rating matrix, with elements representing to what extent the user likes the item (Panda et al. 2020). Therefore, researchers offer numerous MF techniques for CF (Chen et al. 2009; Luo et al. 2014; Xu 2018; Xu et al. 2012), all of which concentrate on fitting the rating matrix the user-item with low-rank approximations and applying it for more predictions. The matrix factorization mechanism is used widely in computer applications incorporated herewith collaborative filtering (Wang et al. 2019). It can be used to identify the critical features for reducing dimensionality and latent factors (Nilashi et al. 2018).

A common assumption in all RS methods mentioned above is user's independence and homogeneous distribution. Between-user social activities are not considered there, which does not meet the fact that friends typically ask for recommendations. To make more accurate analyses, therefore, some researchers have tried to address trust-based RSs. In other words, recommendation performance could be better using trust statements from social networks to incorporate information on social trust, involving explicit specification of neighbors trusted by users in providing recommendations. Lingam et al. (2017) and Ma et al. (2018) presented algorithms on recommender systems in online social networks and trust information connection and aggregation. It has been shown that the use of trust information can cause a significant improvement in recommendation performance. Besides the great improvement in ordinary user's recommendation performance, the gain is substantial for cold-start users (Wei et al. 2017). In this paper, Luo et al. (2014) inspired a CF method based on the NMF technique and trust relationships, referred to as CFMT or Collaborative Filtering Approach Based on the Nonnegative Matrix Factorization Technique and Trust Relationships, is proposed to overcome the cold start and sparsity issues. The proposed CFMT method uses trust information among users on social networks to decrease the adverse effects of the cold start and data sparsity problems on recommendation performance. The CFMT has two main objectives: to enhance accuracy for all users and to predict ratings for cold-start users (Chen et al., 2013a, b).

Among the methods available for the solution of the above two problems, ours is the only one that uses matrix factorization for the decomposition of the rating matrix and the trust matrix. The rating matrix is first filled out with the SVD method, and trust information is then added to it as for hidden factors. Unlike in the previous methods, a combination of trust and rating information is used in the proposed method as factorization. Moreover, SVD is used here to estimate the rating matrix so that a specific start point can be used for its completion. Gradient descent is also used in the proposed method to optimize the proposed model's objective function, which has high accuracy and speed.

Thus, the CFMT method seeks to provide high prediction accuracy for different datasets using trust statements and the recommendation process's NMF method. Furthermore, we develop an efficient algorithm for the offered formulation, which is evaluated by applying three different rating datasets from actual users obtained online. We attempt to indicate that the proposed method outperforms the current trust-based recommendations. The contributions of this study are as follows.

1. This study's novelty is to enhance the performance of CF recommendations in sparse data through the application of the SVD (De Lathauwer et al. 2000) and trust relationships to implicit data.
2. Trust information derived from social networks significantly improves recommendation performance when appropriately incorporated in the MF model. Thus, the proposed method uses the social network data in the recommendation process by modeling user's roles as trustees and trusters, given the trusted network's structural data.
3. We initialize the missing value in the rating matrix with SVD to start with a proper value rather than zero and increase the algorithm's convergence.
4. We introduce a novel trust-based method that incorporates trust in the nonnegative MF with the lowest time complexity to overcome the sparsity and cold start issues. The steps of CFMT are detailed in the relevant section.
5. We perform broad experiments on actual datasets to assess the proposed technique and contrast it with the previous CF methods. We attempt to demonstrate that the CFMT method can indeed cope with data sparsity effectively and is more accurate than the last trust-based methods in the recommendation process. Additionally, we seek to indicate the improved performance of CFMT compared to that of the previous extended versions of the CF technique, even for sparse data and few user-rated items, through different experimental results.

This paper is organized as follows. Some previous works are reviewed in Sect. 2. We introduce our method based on the NMF model and trust-based methods in Sect. 3. We explain the proposed algorithm in Sect. 4. In Sect. 5, we perform several experiments to evaluate the proposed method compared to other recent approaches. The paper is concluded in Sect. 6.

2 Related works and background

For the solution of the cold start problem in a recommender system, the long process of cluster-based matrix factorization is used (Hsieh et al. 2017). The first model of this type was proposed by Sarwar et al. (2000) using Singular Value Decomposition (SVD). However, it requires the unknown ratings to be filled artificially for carrying out an SVD process. After that, several MF-based methods have been proposed for solving CF problems. Hofmann (2004) makes a probabilistic semantic analysis to create a CF model. Moreover, Kurucz et al. (2007) proposed an MF model based on expectation maximization. Since then, recommendations based on MF models have attracted more consideration, and several sophisticated MF-based methods have been proposed and have provided real-world applications. Examples of this kind include a biased SVD model (Paterek 2007) and a probabilistic MF model (Mnih and Salakhutdinov 2008). Many extended versions of the SVD model were developed after its introduction to address sparse data. These include the context-aware recommendation algorithm with two-level SVD (Cui et al. 2018) and the co-factorization SVD model, introduced to enhance the single data source and resolution of the over-fitting problem in matrix factorization (Luo et al. 2019). Semantic data, which can help raise the accuracy of recommendation, were missing in the above methods, all focused on coordinating the user-item rating matrix with low-rank approximations and its application for more predictions. Furthermore, the idea of factorizing the matrix for CF has been applied to some related issues, e.g., video re-indexing, social recommendation, and mobile-user tracking (Narayanam and Narahari 2011). Recently, trust has come to be known as a means by which a social network is utilized to boost the quality of recommendation effectively (e.g., at the process of matrix factorization). Empirical studies have discovered the association between user similarity and trust. A trust-based RS's main function is to combine trust information and user ratings to enhance CF system's performance (Lai et al. 2013). There are two major trust-based methods in terms of user trust data, including implicit and explicit methods. Explicit trust involves trust values explicitly specified by users, whereas implicit trust concerns those that the similarity between users can predict. Ayub et al. (2018) used explicit trust, implicit trust, and user preference similarity for generating the target user's unified rating profile for more powerful, more accurate recommendations. The performance of CF-based recommender systems is also increased by the proposed unified approach in the presence of a limited set of ratings. A considerable number of successful explicit trust-based approaches can be seen in the literature. Trust-based approaches have been addressed to mitigate the sparsity problem and virtual and unreal ratings in the rating matrix. Reduction of sparsity problems and cold start is expected to be realized through the application of trust-based RSs. Furthermore, trust-based RSs are proven to outperform systems where the traditional CF approach is used (Bobadilla et al. 2013). Their project endeavors to utilize user-to-user and user-to-item relation as a possibility dispersal model through a fresh approach grounded on Rejection Sampling to determine its next stage (biased arbitrary walk). (Alexandridis et al. 2013, 2015) They deeply inquired about a novel type of social relationship-the membership and the combinable result with friendship. The CF recommender merges the social relations through a framework based on the graph, on the sparse and compact datasets attained from Last. FM. (Yuan et al. 2012). They will set forth an issue of their novel trust metric and its applications to a cluster to integrate these clusters into recommendation algorithms (Yuan et al. 2012). Their method is to resolve into factor the induced subgraph's adjacency matrix of the FoaF network of every user. This unorthodoxy aims to place the user's neighbors into lenient clusters (Alexandridis et al. 2017). They offer a united frame providing three specific recommendations in a solitary system: recommending items, recommending groups, and recommending friends. Meant of every sort of recommendation, they deeply examine the involvement of fusing the other two supplementary info resources to increase the algorithm's performance (Chen et al., 2013a, b). He et al. (2018) presented a trust inference approach capable of predicting the target user's implicit trust on every voting user based on a sparse explicit trust matrix. They then proposed an improved CF algorithm referred to as *iTrace*, benefitting from the predicted implicit and explicit trust for making recommendations within the CF framework. With latent vectors of rating and trust information, we can predict the ratings given by users. Ma et al. (2008) proposed *SocialRec* based on probabilistic MF using a latent feature space to connect social ratings with user trust. Ma et al. (2009) proposed the *RSTE* method, with an efficient probabilistic MF framework for RSs, employing the trusted users in the trusted network along with user ratings to make recommendations. Jamali and Ester (2010) proposed the *SocialMF* method, one of the most famous trust-aware algorithms. This work integrates trust propagation into the probabilistic MF model and improves prediction accuracy to a large extent. Guo et al. (2015) introduced the *TrustSVD* method, which considers both the implicit and the explicit influence of trust and ratings based on the *SVD++* algorithm. After that, Yang et al. (2017) proposed the *TrustMF* method considering the two aspects of trustor and trustee. A novel CF method referred to as *TCFACO* was proposed in Parvin et al. (2019) to predict missing ratings using trust statements as rich side information and Ant Colony Optimization (ACO) for low complexity. The proposed *TrustANLF* method integrates trust statements into the recommendation model as an additional source of information besides rating values to address data sparsity problems and cold start. Furthermore, the trust-based nonnegative MF model is solved using alternating direction optimization to reduce computational and memory costs and improve convergence speed (Parvin et al. 2018).

In the proposed method, matrix factorization has been used to decomposition the rating and trust matrices. The rating matrix is first filled out with the SVD method, and trust information is then added to the rating matrix as hidden factors. Unlike in the previous methods, a combination of trust and rating information is used in the proposed method as factorization. Moreover, SVD is used here to estimate the rating matrix so that a specific start point can be used for its completion. Gradient descent is also used in the proposed method to optimize the proposed model's objective function, which has high accuracy and speed. The compared and other relevant methods are mentioned in Sect. 5.2.

3 Problem definition

This section details how our method, referred to as CFMT, incorporates social trust information in the Regularized Single-element-based NMF method, called RSNMF (Luo et al. 2014), to enhance its prediction accuracy. CFMT is a fast method with proper scalability compared to well-known and state-of-the-art methods. Our method is effective when faced with cold-start users. The CFMT method incorporates trust information in the social network to help factorize the matrix. The proposed method integrates multiple information sources such as user ratings and trust statements into the recommendation process to reduce the data cold start and sparsity issues. Additional descriptions of the proposed method are provided in the following sections. The contributions of the proposed method can be summarized mainly by the following two aspects. (1) We initialize the missing values in the rating matrix with the SVD method to start with the appropriate value rather than zero. (2) We propose a novel trust-based method that incorporates trust in the nonnegative MF to reduce the cold start and sparsity issues. The detailed steps of CFMT are described in the next section.

3.1 Matrix factorization

A technique is presented in this paper that factorizes the rating matrix into two matrices, one is associated with users, and the other is to items. Each row of the former matrix represents the user vector associated with each user. Each column in the latter represents the item vector associated with each item (Hernando et al. 2016(. The main aim of a recommender system is to estimate an unknown rating in the rating matrix. For some items named I and a set of users named U, a rating matrix of user-items named R is a $|U| \times |I|$ matrix, where for each element, r_{ui} shows the rating given by user u to item i. In the MF-based model, the rating matrix is decomposed into two latent factors, Q and P, where Q is $|I| \times f$, P is $|U| \times f$, and $f \leq \min(|U|, |I|)$ includes the

Table 1 Notations

Symbol	Description
R	User-item rating matrix in the RS
R_K	Known entries
U	User sets
I	Item sets
P	User feature matrix
Q	Item feature matrix
λ_P	Regularizing coefficients for P
λ_Q	Regularizing coefficients for Q
β	Importance of the social regularization
r_{ui}	the user u's rating on item i
K	the set of the (u, i) pairs for which r_{ui} is known in rating matrix R
U_k	the set of users who have rated the items in rating matrix R
I_k	the set of items that have been rated in rating matrix R
U_i	the set of users who have rated on item i
I_u	the set of items rated by user u
$\|\|_F$	Frobenius Norm
F	Feature dimension

dimensions of the latent space. The MF process is executed through minimization of the following objective function:

$$\arg\min \varepsilon(P,Q) = \left\|R - PQ^T\right\|_F^2, \quad s.t.\ P,\ Q \geq 0 \tag{1}$$

where ‖.‖ indicates the Frobenius norm. The function in Eq. (1) measures the discrepancy between R and PQ^T and enables Q and P to be nonnegative by integrating the non-negativity constraints.

3.2 Relationship between the user-trust matrix and the user-item matrix

The matrix factorization technique is failed to obtain acceptable results unless using Meta information such as demographic information, clustering, and association rules. In this work, the trust information is used as an extension to the rating matrix. In the trust matrix, which is a binary one, a user either trust others or not, and he is either trusted by others or not. When two users trust each other (whether or not mutually), i.e., they are friends in technical terms, they also have particular tastes in common, and similar items are recommended to them in a social network based on the trust, which can be effective in the solution of the cold start problem in the recommender system. Based on the same theory, the trust matrix is formulated after decomposition into two other matrices, as detailed in Eqs. 3 and 4. The rating matrix is formulated in the same way. Finally, a specific objective function takes shape by integrating the two matrices, as shown in Eqs. 4–12, and the problem is optimized using gradient descent.

In this work, the recommendation's purpose is to predict unknown ratings given by a user (e.g., u) to an item (e.g., i) that is not previously known using two matrices, including a user trust and a user-item matrix. We assume an RS comprising m user and n items, respectively, and a user-item matrix $R=[r_1, r_2,\ldots, r_n]$, where each component $r_{\{ui\}} \varepsilon R$ indicates the rating that user u gives to item$i$, and the notations are explained in Table1.

Therefore, the MF process is formulated as follows.

$$\begin{aligned}\arg\min \varepsilon(P,Q) &= \frac{1}{2}\left\|R - PQ^T\right\|_F^2 \\ &= \frac{1}{2}\sum_{(u,i)\in R_k}\left(r_{ui} - \sum_{k=1}^{f} p_{uk}q_{ik}\right)^2 \quad s.t.\ P,\ Q \geq 0\end{aligned} \tag{2}$$

We also assume that a trusted network is displayed by a graph G = (V; E), where V indicates m users and E shows the trust relationships between the users. Thus, the adjacency matrix $T = [t_{a,b}]_{m\times m}$ can be used to define the trust relationships between the users, where $t_{a,b}$ represents the trust between users a and b, i.e., $t_{a,b}= 1$ means that user a trusts user b, whereas $t_{a,b} = 0$ represents the distrust relationship. Moreover, matrix T is very sparse. In brief, the trust matrix and the rating matrix are combined to predict unknown ratings (Guo et al. 2015).

4 Proposed algorithm

The proposed MF method is based on the combining the rating and trust information to overcome CF's problems. One of the primary challenge for rating matrix is its sparsity. For solving that, we initialize unknown ratings with the SVD method. SVD is a famous MF method that is used for the generation of low-rank approximations. The SVD of $A \in R_{m\times n}$ with rank r is defined as $A = U\ S\ V^T$, where $V \in R_{n\times n}$ and $U \in R_{m\times m}$ are orthogonal, and $S \in R_{m\times n}$ is a diagonal matrix with r non-zero components, which are the singular values of$A$. The columns of U and V represent the eigenvectors of AA^T andA^TA, respectively. Therefore, the efficient dimensions of these three matrices, U, V, and$S$, are $m \times r$, $n \times r$, and $r \times r$, respectively. The initial diagonal elements $(\sigma 1,\ \sigma 2,\ \ldots,\ \sigma r)$ of S have the property$\sigma 1 \geq \sigma 2 \geq \ldots \geq \sigma r > 0$.

A key attribute of *SVD*, precious in RSs, is its capability of providing the optimal approximation to the original matrix A using multiplication of three smaller matrices. In this study, this method is applied to obtain the estimated values, and R is thus initialized with SVD to ensure that the ratings are better than zero.

Our CFMT method is built on top of a well-known method known as RSNMF, proposed by Luo et al. (2014). In a CF problem, Tikhonov regularization is practical in enhancing both convergence rate and prediction accuracy. Because of R's sparse nature, a joint selection of the regularizing terms of feature matrices is their Frobenius norm Luo et al. (2016). Thus, after the addition of the regularizing terms, the objective function in Eq. (1) is changed into:

$$\begin{aligned} \arg\min \varepsilon(P,Q) &= \frac{1}{2}\left\|R - PQ^T\right\|_F^2 + \frac{1}{2}\lambda_P \left\|p_u\right\|_F^2 + \frac{1}{2}\lambda_Q \left\|q_i\right\|_F^2 \quad s.t.\ P,\ Q \geq 0 \\ &= \frac{1}{2}\sum_{(u,i)\in R_k}\left(r_{ui} - \sum_{k=1}^{f} p_{uk}q_{ik}\right)^2 + \frac{1}{2}\lambda_P \sum_{k=1}^{f} p_{uk}^2 + \frac{1}{2}\lambda_Q \sum_{k=1}^{f} q_{ik}^2 \end{aligned} \tag{3}$$

To compute the similarity among users, we use the absolute value of the cosine similarity among them. The cosine similarity between user's u and f is calculated as follows:where u and f represent two users with p-dimensional vectors ($u = \{a_1, a_2, \ldots, a_p\}$ and $f = \{b_1, b_2, \ldots, b_p\}$). It can be seen from the equation that the value of similarity is between 0 and 1. Moreover, the similarity value of two completely similar users will be equal to 1, and this value will be equal to 0 for completely dissimilar users (Ar and Bostanci 2016). A natural, easy way for a combination of

where λ_Q and λ_P are constant values indicating the regularizing coefficients for Q and P, respectively, and $\|\|_F$ means the Frobenius norm Eq. (3) involves the objective function RSNMF. For social trust to be considered in the proposed recommendation model, a social regularization term is imposed to minimize the similarities between the tastes of each user u_i and those of his friends (Ma et al. 2011). More specifically, if the friends of user u_i are listed in $F^+_{(i)}$, it is assumed that the interests of u_i must be similar enough to those of his friends in $F^+_{(i)}$. It formulated as follows:

$$T_1(P) = \frac{\beta}{2}\sum_u \sum_{f\in F^+(u)} \left\|P_u - P_f\right\|_F^2 \quad , \ \beta > 0 \tag{4}$$

where β controls the influence of the trust regularization, one of the advantages of this approach is the indirect propagation of user tastes. For instance, if user P_u trusts userP_f, and user P_g trusts userP_u, the distance between feature vectors $\left(P_u, P_f\right)$, and $\left(P_u, P_g\right)$ is minimized, which results in minimization of the similar tastes of user P_u and both his outlink friends $F^+_{(u)}$ and his inlink friends$F^-_{(i)}$. This fact is formulated as follows:

$$T_2(P) = \frac{\beta}{2}\sum_u \sum_{f\in F^+(u)} \left\|P_u - P_f\right\|_F^2 + \frac{\beta}{2}\sum_u \sum_{g\in F^-(u)} \left\|P_u - P_g\right\|_F^2 \tag{5}$$

Furthermore, if user P_u s and user P_f's are very close, the user P_f must be closer to the P_u friend's tastes and P_f must not be very close if the users are dissimilar. Therefore, Eq. (5) is rewritten as follows.

$$\begin{aligned} T_3(P) = &\frac{\beta}{2}\sum_u \sum_{f\in F^+(u)} W(u,f)\left\|P_u - P_f\right\|_F^2 \\ &+ \frac{\beta}{2}\sum_u \sum_{g\in F^-(u)} W(u,g)\left\|P_u - P_g\right\|_F^2 \end{aligned} \tag{6}$$

the rating matrix (Eq. (3)) with the trust matrix of a user to amplify the performance of the RS and relieve the sparsity and cold start problems is to merge the rating value and trust effect linearly, as follows:

$$\begin{aligned} \mathop{\arg\min}_{s.t.\ P,\ Q\geq 0} \varepsilon(P,Q) = &\frac{1}{2}\sum_{(u,i)\in R_k}\left(r_{ui} - \sum_{k=1}^{f} p_{uk}q_{ik}\right)^2 \\ &+ \frac{1}{2}\lambda_P \sum_{k=1}^{f} p_{uk}^2 + \frac{1}{2}\lambda_Q \sum_{k=1}^{f} q_{ik}^2 \\ &+ \frac{\beta}{2}\sum_{u\in U_i}\sum_{f\in F^+(u)} W(u,f).\sum_{k=1}^{f}(P_{uk} - P_{fk})^2 \\ &+ \frac{\beta}{2}\sum_{u\in U_i}\sum_{g\in F^-(u)} W(u,g).\sum_{k=1}^{f}(P_{uk} - P_{gk})^2. \end{aligned} \tag{7}$$

Hence, we use gradient descent to find a solution minimizing $\varepsilon(P,Q)$, to update parameters p_{uk} q_{ik} and, and to reduce $\varepsilon(P,Q)$ in Eq. (8), Luo et al. (2014). Therefore, the update rule for the optimization problem without the non-negative constraint is formulated as follows:

$$\begin{aligned} p_{uk} &= p_{uk} - \eta_{uk}\frac{\partial\varepsilon}{\partial p_{uk}} \\ &= p_{uk} - \eta_{uk}.2\Bigg(-\sum_{i\in R_k} q_{ik}\left(r_{ui} - \sum_{k=1}^{f} p_{uk}q_{ik}\right) \\ &\quad + \lambda_p p_{uk} + \beta\sum_{f\in F^+(u)}\left(W(u,f)(p_{uk} - p_{fk})\right) \\ &\quad + \beta\sum_{g\in F^-(u)}\left(W(u,g)(p_{uk} - p_{gk})\right)\Bigg) \end{aligned} \tag{8}$$

Moreover, to find an optimum solution, we use the gradient descent method to search for a q_{ik} n answer.

$$q_{ki} = q_{ki} - \eta_{ki} \frac{\partial \varepsilon}{\partial q_{ki}} \quad \Rightarrow \quad q_{uk} = q_{uk} - \eta_{ki}.2\left(- \sum_{u \in U_i} p_{uk}\left(r_{ui} - \sum_{k=1}^{f} p_{uk} q_{ki} \right) + \lambda_Q q_{ki} \right) \tag{9}$$

Since $\eta_{u,k}$ and $\eta_{k,i}$ in Eq. (10) and Eq. (11) are positive learning rates with some notational abuse; we were inspired from RSNMF to turn $\eta_{u,k}$and $\eta_{k,i}$ as the original $2\eta_{u,k}$, and $2\eta_{k,i}$ respectively, to have a concise form [40]. Thus, Eq. (10) and Eq. (11) are reformulated to:

$$p_{uk} = p_{uk} + \eta_{uk} \sum_{i \in I_u} q_{ki} r_{ui} - \eta_{uk} \sum_{i \in I_u} (q_{ki}\, \hat{r}_{ui} + \lambda_P p_{uk}) - \eta_{uk}\beta \sum_{f \in F^+(u)} W(u,f).(p_{uk} - p_{fk}) - \eta_{uk}\beta \sum_{g \in F^-(u)} W(u,g).(p_{uk} - p_{gk}) \tag{10}$$

$$q_{ki} = q_{ki} + \eta_{ki} \sum_{u \in U_i} p_{uk}(r_{ui} - \hat{r}_{ui}) - \lambda_Q q_{ki} \Rightarrow q_{ki} \leftarrow q_{ki} + \eta_{ki} \sum_{u \in U_i} p_{uk} r_{ui} - \eta_{ki} \sum_{u \in U_i} (p_{uk}\, \hat{r}_{ui} + \lambda_Q q_{ki}) \tag{11}$$

We can also use the principle of NMF to change $\eta_{u,k}$, and $\eta_{k,i}$ to manipulate the negative components to retain non-negativity, Luo et al. (2014). The harmful ingredients in Eqs. (12, 13) are:

$$-\eta_{uk}\left(\sum_{i \in I_u} (q_{ki}\, \hat{r}_{ui} + \lambda_P p_{uk}) + \beta \sum_{f \in F^+(u)} W(u,f).(p_{uk} - p_{fk}) + \beta \sum_{g \in F^-(u)} W(u,g).(p_{uk} - p_{gk}) \right)$$

for p_{uk},and

$-\eta_{ki} \sum_{u \in U_i} \left(p_{uk}\, \hat{r}_{ui} + \lambda_Q q_{ki} \right)$ for q_{ki}.

Then, following the principle of NMF, we set:

$$\eta_{uk} = \frac{p_{uk}}{\sum_{i \in I_u} \left(q_{ki}\, \hat{r}_{ui} + \lambda_P p_{uk} \right) + \beta \sum_{f \in F^+(u)} W(u,f).\left(p_{uk} - p_{fk} \right) + \beta \sum_{g \in F^-(u)} W(u,g).\left(p_{uk} - p_{gk} \right)}$$

and

$\eta_{ki} = \frac{q_{ki}}{\sum_{u \in U_i} \left(p_{uk}\, \hat{r}_{ui} + \lambda_Q q_{ki} \right)}$.

Therefore, p and q are rewritten as follows:

$$p_{uk} = p_{uk} \frac{\left(\sum_{i \in I_u} q_{ki} r_{ui} \right)}{|I_u| \lambda_P p_{uk} + \sum_{i \in I_u} (q_{ki}\, \hat{r}_{ui}) + \beta \sum_{f \in F^+(u)} W(u,f).(p_{uk} - p_{fk}) + \beta \sum_{g \in F^-(u)} W(u,g).(p_{uk} - p_{gk})} \tag{12}$$

$$q_{ki} = q_{ki} + q_{ki} \frac{\sum_{u \in U_i} \left(p_{uk} r_{ui} - \left(p_{uk} \hat{r}_{ui} + \lambda_Q q_{ki} \right) \right)}{\sum_{u \in U_i} \left(p_{uk} \hat{r}_{ui} + \lambda_Q q_{ki} \right)} = q_{ki} \frac{\sum_{u \in U_i} (p_{uk} r_{ui})}{\sum_{u \in U_i} \left(p_{uk} \hat{r}_{ui} + \lambda_Q q_{ki} \right)} = q_{ki} \frac{\sum_{u \in U_i} (p_{uk} r_{ui})}{|U_i| \lambda_Q q_{ki} + \sum_{u \in U_i} (p_{uk} \hat{r}_{ui})} \tag{13}$$

4.1 Algorithm design

We design the pseudo-code of the CFMT method in Algorithm 1. This algorithm has several inputs, including trust matrix

T, rating matrix R, and parameters λ and β. It should be noted that the parameters involved in the optimization process must be initialized with random positive numbers for an efficient update. For implementing the NMF algorithm, the regularizing coefficients λ_P and λ_Q are set to appropriate values, and the feature dimension is set to 10. Moreover, the complexity of each line in Algorithm 1 is calculated. First, we randomly initialize the parameters with small values and apply the MF method (SVD) to the original rating matrix R. Then, we continue to train the model until the objective function is converged to an appropriate value. Finally, we return the learned matrices P and Q as outputs.

Table 2 Computational complexity

Algorithm	Complexity
TrustANLF	$O\ (t \times f^2 \times \lfloor\Omega\rfloor)$
ANLF	$O\ (t \times f \times \lfloor\Omega\rfloor)$
Social MF	$O\ (\lfloor U\rfloor \times t^2 \times f + \lfloor\Omega\rfloor \times \lfloor U\rfloor \times f)$
TrustMF	$O\ ((\lfloor\Omega\rfloor + \lfloor T_k\rfloor) \times t \times f)$
Social Rec	$O\ (t \times f \times \lfloor\Omega\rfloor \times + \lfloor T\rfloor) \times \lfloor U\rfloor)$
RSTE Rec	$O\ (\lfloor U\rfloor \times \lfloor\Omega\rfloor \times t^2 \times f)$
Trust SVD	$O\ ((\lfloor\Omega\rfloor + \lfloor T\rfloor) \times f \times \max(\lvert f^+\rvert, \lvert f^-\rvert, f))$

Algorithm 1. *CFMT[1]*

Inputs: R, U, I, T, f

Outputs: *predicted user-item rating matrix.*

Begin	***Complexity***
Initialize randomly $\lambda_P, \lambda_Q, \beta$	$O(1)$
Initialize $s = 0$, n = max imum number of iterations	$O(1)$
Initialize f	$O(1)$
Step 1: Initializing with an approximate value	–
Apply the SVD method to the original rating matrix R	$O(U \times I^2)$
Step 2: Finding the nonnegative features P, Q	–
While $s \leq n$ **&&** no convergence **do**	$s \times$
for $k = 1$ **to** f	$f \times$
for each user u in U	$U \times$
Update $p_{u,k}$ According to (12)	$O((\lvert I_u\rvert + \lvert F_u^+\rvert + \lvert F_u^+\rvert))$
end for	–
for each item i in I	$I \times$
Update $q_{i,k}$ According to (13)	$O(\lvert U_i\rvert)$
end for	–
end for	–
$s = s + 1$;	–
End while	–
End	–

4.2 Complexity analysis

The primary computational cost of training the CFMT method is imposed by evaluating the objective function ε. Here, the intricacy of CFMT is summarized by the CFMT analysis algorithm intricacy is outlined by which the following results are obtained. Let f and s be the dimensions of the feature space and the number of iterations, respectively. We compute the computational time of each step in the CFMT algorithm. Let U_i and I_u represent the set of users having rated item I and the set of items rated by the user, respectively. Thus, we find that the complexity costs consist of two main parts:

1. the initialization step of the parameters, with a computational complexity of

$$C1 = O(3 \times O(1)) + O\left(U \times I^2\right);$$

2. the training process, with a computational complexity of

$$C2 = s \times f\left[U \times O\left(\left(\left|I_u\right| + \left|F_u^+\right| + \left|F_u^+\right|\right)\right) + I \times O\left(\left|U_{i\cdot}\right|\right)\right]$$

Because $\left|R_k\right| >> \max\{|u|, |I|\}$, and the values of $\left|I_u\right|$ and $\left|U_i\right|$ are minimal even with an increase in data size, the final overall computational cost is summarized as follows:

Table 3 Statistics of the three data sets

Features	*Epinions*	*FilmTrust*	*Ciao*
#user	40,163	1508	30,444
#items	139,738	2071	72,665
#ratings	664,824	35,497	1,625,480
#trusters	33,960	609	6,792
#trustees	49,288	732	7,297
#trusts	487,183	1853	111,781

$Cost \cong s \times f[U + I]$.

Given this formula, the complexity appears to be of degree 3; however, since the value of f is constant as an initialized parameter, the complexity is computed as the product of s and max $\{|u|, |i|\}$. Hence, the complexity is quadratic. Computational complexity is compared in Table 2 between well-known and state-of-the-art methods and the proposed CFMT method. It is clear from the table that complexity is lower in the proposed method than in the others.

4.3 Impact of feature dimension

In this section, we analyze the effect of the number of latent dimensions of user and item latent vectors on the performance of CFMT. Generally, it is known from the literature that the performance of the recommendation improves as the number of latent dimensions increases Sarwar et al. (2000). Below shows the performance concerning the number of latent dimensions of our proposed model. Interestingly enough, while experiments on the Epinion dataset show the trend of performance improvement as the number of latent dimensions increases, we could not discover any particular directions from the experiments on Filmtrust. Although precisely interpreting each latent dimension's meaning is infeasible, we assume that it represents the profile of user's interest and item's features. For datasets with relatively small users and items, such as Filmtrust, a large number of latent dimensionality would surpass the inherent number of users and item's profiles. However, for datasets like Epinion, which is composed of many users and items, the performance of recommendation improves as the number of latent dimensionality increases. Nevertheless, if the number of dimensions is too large, the complexity will significantly increase. Therefore, we need to find a fair number of latent dimensions to balance the trade-off between the performance and the complexity; thus, a value of $f = 10$ is an appropriate setting.

5 Experiments

Several experiments were conducted, using which the recommendation qualities of the approaches utilized in this work were compared to those of well-known CF and trust-based recommendation methods to demonstrate the proposed algorithm's effectiveness. In the experiments, five-fold cross-validation was used for training and testing. All the methods were implemented in the Java programming language on a PC with 6-GB RAM and a Core i7 processor. Moreover, all the methods were implemented on top of LibRec[1] 2.0.0, where most of these well-known methods have been implemented.

We selected Epinions (Massa and Avesani 2007), FilmTrus (Guo et al. 2013) And Ciao (Guo et al. 2014) as the datasets used for performing the experiments on the data. These datasets were employed in our experiments to determine the stability of the CFMT method. They contain both item ratings and social trust relationships. The general specifications of these datasets are shown in Table 3.

Also, we initialize the value of algorithm parameters as *Factors = 10, β = 0.5, α = 0.5 and max.iter = 100*. Such values are reported by the most of the related works like *Ayub* Ayub et al. (2018), TrustANLF Parvin et al. (2018), TrustMF Yang et al. (2017), TrustSVD Guo et al. (2015), and ANLF Luo et al. (2016).

5.1 Comparison with social-based methods

In evaluation processes, Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are two standard metrics for assessment of the performance of prediction (Guo et al. 2015). Therefore, the RMSE and MAE of the proposed method and the other methods were computed to evaluate all the method's performance. RMSE would always be greater than MAE, and both RMSE and MAE could range within [0 ∞]. The comparison results are reported using the MAE and RMSE valuation measures, defined as follows $r_{u,i}$: the actual rate, $\hat{r}_{u,i}$ the estimated rate, and Z is the set of ratings that user u has given to item i.

$$RMSE = \sqrt{\frac{\sum_{(u,i)} (\hat{r}_{u,i} - r_{u,i})^2}{Z}} \tag{14}$$

$$MAE = \frac{\sum_{(u,i)} \left|\hat{r}_{u,i} - r_{u,i}\right|}{Z} \tag{15}$$

For evaluation of the performance of the proposed method(CFMT), it was compared to several well-known and

[1] https://guoguibing.github.io/librec/.

Table 4 Comparison with social-based methods in terms of all the above factors

Methods	Error Metrics					
	MAE			RMSE		
	Datasets					
	FilmTrust	Epinions	Ciao	FilmTrust	Epinions	Ciao
RSTE (Ma et al. 2009)	0.680	0.873	0.560	0.851	1.10	0.773
TrustMF (Yang et al. 2017)	0.721	0.877	0.505	0.919	1.184	0.710
SocialMF (Jamali et al. 2017)	0.698	0.862	0.637	0.852	1.104	0.905
TrustSVD (Guo et al. 2015)	0.607	0.834	0.723	0.787	1.094	0.955
Social Rec (Ma et al. 2008)	0.712	0.862	0.571	0.916	1.104	0.803
ANLF (Luo et al. 2016)	0.711	0.901	0.659	0.893	1.234	0.841
RSNMF (Luo et al. 2014)	0.813	1.172	0.712	1.022	1.324	0.923
TrustANLF (Parvin et al. 2018)	0.584	0.785	0.519	0.777	1.063	0.720
Ayub (Ayub et al. 2018)	0.668	0.944	0.794	0.868	1.307	1.110
GA (Ar et al. 2016)	0.672	0.953	0.796	0.882	1.302	1.089
ITrace (He et al. 2018)	0.665	0.976	0.803	0.878	1.352	1.098
CFMT (proposed method)	0.584	0.775	0.504	0.789	1.031	0.716

Table 5 Comparison with social-based methods in terms of the cold start problem

Methods	Error Metrics					
	MAE			RMSE		
	Datasets					
	FilmTrust	Epinions	Ciao	FilmTrust	Epinions	Ciao
RSTE	0.618	0.930	0.878	0.775	1.269	1.150
TrustMF	0.619	0.934	1.073	0.882	1.373	1.311
SocialMF	0.589	0.919	1.014	0.818	1.312	1.266
TrustSVD	0.650	0.861	0.725	0.845	1.117	0.939
Social Rec	0.757	0.919	1.014	0.939	1.312	1.266
ANLF	0.787	0.956	0.796	0.986	1.157	0.981
RSNMF	0.751	1.053	0.895	0.911	1.301	1.23
TrustANLF	0.607	0.842	0.716	0.784	1.090	0.928
Ayub	0.633	0.502	0.805	0.824	0.878	1.211
GA	0.652	0.695	0.838	0.868	0.952	1.185
ITrace	0.668	0.725	0.878	0.881	1.123	1.213
CFMT	0.632	0.823	0.836	0.854	1.062	1.141

state-of-the-art methods, including RSTE Ma et al. (2009), TrustMF Yang et al. (2017), SocialMF Jamali and Ester (2010), TrustSVD Guo et al. (2015), SocialRec Ma et al. (2008), RSNMF Luo et al. (2014), ANLF Luo et al. (2016), *Ayub* Ayub et al. (2018), ITrace He et al. (2018), TrustANLF Parvin et al. (2018) and metaheuristic-based and trust-based methods such as GA Ar and Bostanci (2016). The results of these methods in terms of MAE and RMSE are shown in Table 4. The results demonstrate that the CFMT method is consistently superior to the other's best approach in most cases. CFMT and TrustANLF have better performance than the trust-based models. According to the results, it is imperative to use trust relationships in the recommendation process when generating recommendations. This experiment indicates the importance of trust inference of relationships in the CFMT method, which provides good results. It can also be observed that (1) models combining RSNMF and trust relationships exhibit enhanced performance. For example, the average improvement of CFMT on the Epinions dataset is 8.6% in MAE. CFMT obtains the lowest error due to the combination of the RSNMF method and the trust relationships.

Table 6 Comparison with rating-based methods

Methods	Error Metrics					
	MAE			RMSE		
	Data Set					
	FilmTrust	Epinions	Ciao	FilmTrust	Epinions	Ciao
UAvg	0.729	0.925	0.456	0.943	1.19	0.678
IAvg	0.696	0.823	0.255	0.925	1.09	0.480
PMF	0.753	1.35	0.24	1.02	1.81	0.42
SVD++	0.699	0.912	0.745	0.891	1.205	1.00
RSNMF	0.661	0.973	0.841	0.884	1.101	0.659
LLORMA	0.848	1.62	0.803	1.041	2.03	1.014
ANLF	0.711	1.15	0.659	0.893	1.31	0.841
TrustANLF	0.584	0.785	0.519	0.777	1.063	0.720
CFMT	0.582	0.775	0.506	0.789	1.031	0.708

5.2 Performance in the face of cold-start users

In this section, the performance of the CFMT method in the presence of cold-start users was investigated. It is a vital challenge for the success of RSs to address new users or users with few rating counts, as this kind of user emerges highly frequently in real-world applications, which necessitates handling cold-start users as an important challenge in existing systems. The efficiency of the previously mentioned methods is shown in Table 5, where it can be seen that the use of the trust statements significantly raises the performance quality of the recommendation. This outcome is noteworthy. It discloses that it is possible to relieve the shortage of user ratings for new and cold-start users by incorporating the trust relationships among users. In this set of experiments, the trust-based method's performance is less than that of the CFMT method in most cases. For instance, the average improvement in CFMT in terms of MAE in the Epinions and FilmTrust data sets is 9.4% and 2.9%, respectively.

It is noteworthy that most users express their interests in numerous real applications only on a limited number of items. Such users with up to 5 expressed ratings are referred to as cold-start users, who count for more than half of the total number of users in both FilmTrust and Epinions. This gives significant importance to the performance of any recommendation method for cold-start users. The experiments show that CFMT, TrustSVD, and Ayub have better performance than the other methods because we consider both inlink and outlink effects of user ratings and user trusts to solve the cold start data sparsity problems. A comparison demonstrates that the approach proposed in this study has higher performance in terms of predictive accuracy than the trust-based recommendation methods.

5.3 Comparison with other models

As another significant issue, the well-known rating-based approaches' performance states were investigated and compared with the CFMT method. Specifically, the efficiency of the CFMT method was compared to that of the baseline and rating-based algorithms, including UAvg, IAvg, PMF (Mnih and Salakhutdinov 2008), SVD++(Koren 2008), RSNMF Luo et al. (2014), LLORMA Lee et al. (2013), ANLF Luo et al. (2016), TrustANLF Parvin et al. (2018). The results of the conducted experiments with the above five algorithms on the data sets are presented in Table 6. The proposed algorithm was first run with the trust relationships, and it was then compared with those in which only rating information is utilized. A comparison with rating-based models demonstrated that the approach proposed in this study exhibits superior performance in terms of the accuracy of predictions.

5.4 Parameter-sensitive tests

The effects of the parameters of the proposed method were evaluated. These parameters include λ_P, λ_Q, and β. Several experiments were performed to clarify how changes in these parameters could change the accuracy of recommendations. The results obtained over the parameters λ_P, λ_Q, and β are reported in Fig. 1. In these experiments, the feature dimension f was set to 10. For the first experiment, the values of λ_Q and β were set to a fixed value of 1, and the CFMT was then examined as the value of λ_P increased from 0.001 to 3. The results reported in Fig. 1 show that as the value of λ_P decreases, the proposed method cannot converge well. In other words, the value of λ_P is linked with the performance of our model. As for the appropriate value of λ_P, which obtains the desired result in different datasets, the

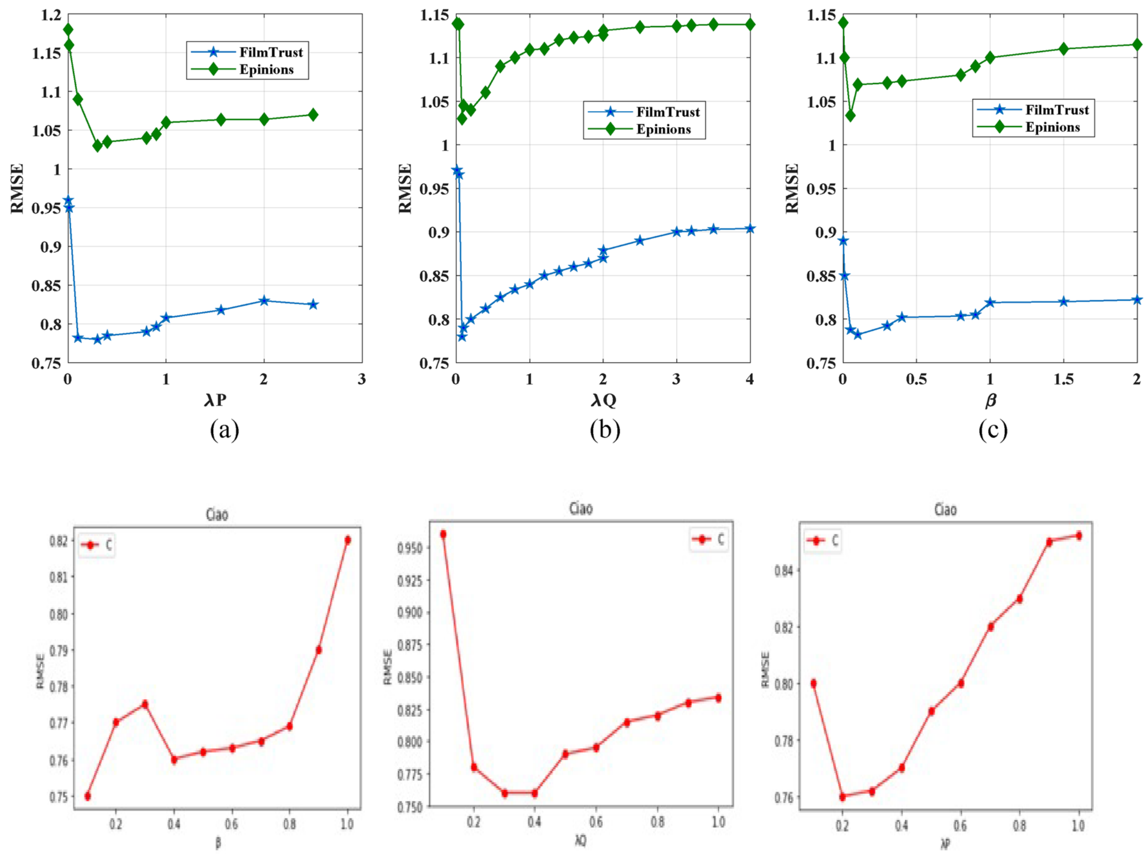

Fig. 1 **a** Effect of parameter λ_P, **b** effect of parameter λ_Q, and **c** effect of parameter β

value $\lambda_P = 0.3$ is appropriate. Moreover, the value $\lambda_Q = 0.08$ is an appropriate setting in other datasets.

The β parameter quantifies the amount of social network information used by the proposed algorithm to complete the observed rating matrix. It can be seen that when β has a significantly small value, the availability of information on trust relationships among the users is forgotten by the algorithm, and only the perceived user rating is utilized for factorization. In contrast, if an immense value is assigned to β, the trust information will govern the learning process, resulting in weaker performance. Hence, to avoid harm to recommendation performance, it is required to obtain a reasonable value for social regularization, which is realized by analyzing how the combination of these parameters influences recommendation performance. The most appropriate value of β, which gives an excellent performance, likely varies from one dataset to another. It is, therefore, reasonable to assign a value of 0.05 to this parameter.

5.5 Scalability analysis

The scalability of CFMT in terms of training time for application cases to various dataset percentages was examined, particularly for the range from 0.1 to 1% with 0.1 intervals. The results in Fig. 2 indicate a linear increase in training time as the training data's quantity is increased. Thus, this approach is prone to use for datasets on a large scale, and it is considered a capable method for large-scale CF problems. Consequently, the method can be used in big data applications with small numbers of tuning proceedings. CFMT can undoubtedly improve the capability of RSs of meeting industrial requirements.

5.6 Impact of feature dimension

This section investigates the effect of latent space dimensions on the FilmTrust, Epinions, and Ciao datasets. We also study the function of the proposed method for various dimensions of the latent space. Figure 3a shows the effect of multiple measurements on the FilmTrust dataset, and Fig. 3b shows the effect of the dimensional change on the Epinions dataset. Moreover, Fig. 3c shows the effect of the dimensional change on the Ciao dataset. It is known that the larger the dimensions of the features, the higher the efficiency since there is a large amount of data information available, but it should be noted that an

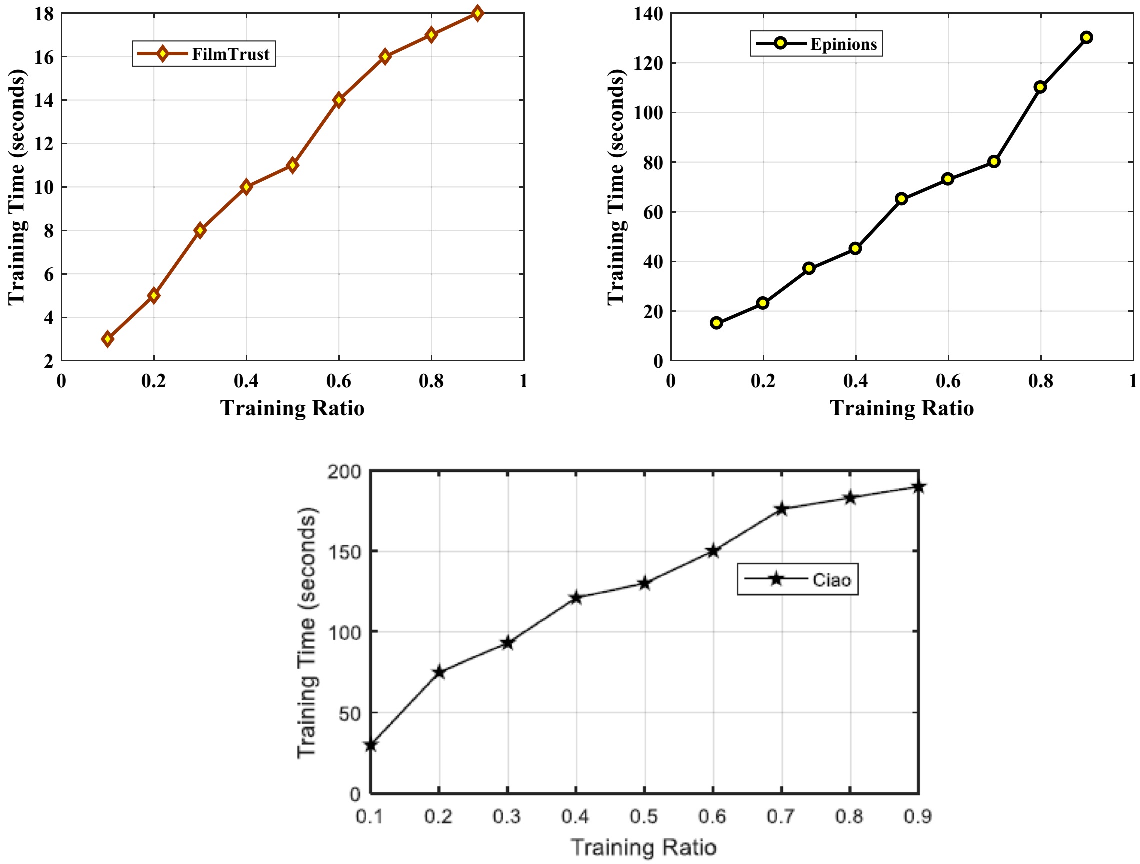

Fig. 2 Scalability of *CFMT* across all the datasets $[f = 10]$

increase in the dimensions of the latent space increases the time complexity of the algorithm. Therefore, we need to obtain the right amount to have proper performance. An algorithm usually appears to be better in reaching an optimal result in a small space, and it also has less time complexity. As shown in Fig. 4, the proposed method provides good results overall datasets for k = 10 (solution). In other words, the value k = 10 provides an optimal result on three datasets.

5.7 Comparison of run time

In this section, the new trust-based method's run time is compared to those of other methods. Table 7 shows the different algorithms' run times in 100 iterations on the FilmTrust, Epinions, and Ciao datasets. As demonstrated by the results, the proposed method's performance on the FilmTrust dataset is better than in all the other methods except SocialRec. It has the lowest run time in 100 iterations. Furthermore, the proposed method exhibits a more insufficient run time for the Epinions and Ciao dataset. It is capable of providing better results than other methods for large and widespread datasets.

5.8 Statistical significance test

Statistical significance t-test was used to evaluate the proposed method more accurately than other methods. This statistical test is utilized to show the mean difference between the two samples. We used the paired t-test to show whether our proposed method is significantly different from other methods. We based the null-hypothesis that there is an insignificant difference in the proposed method than other methods. If the p-value obtained is less than the significance level (hereon 0.05), the null hypothesis is rejected and indicates a statistically significant difference in the proposed method. As an alternative, we base hypothesis 1 on that the proposed method enjoys a significant improvement, so if the p-value is less than the significant level, it means confirming hypothesis 1. We used the fivefold cross-validation method to conduct this test. In this method, we divided each data set into five sections. In each replication, four sections were

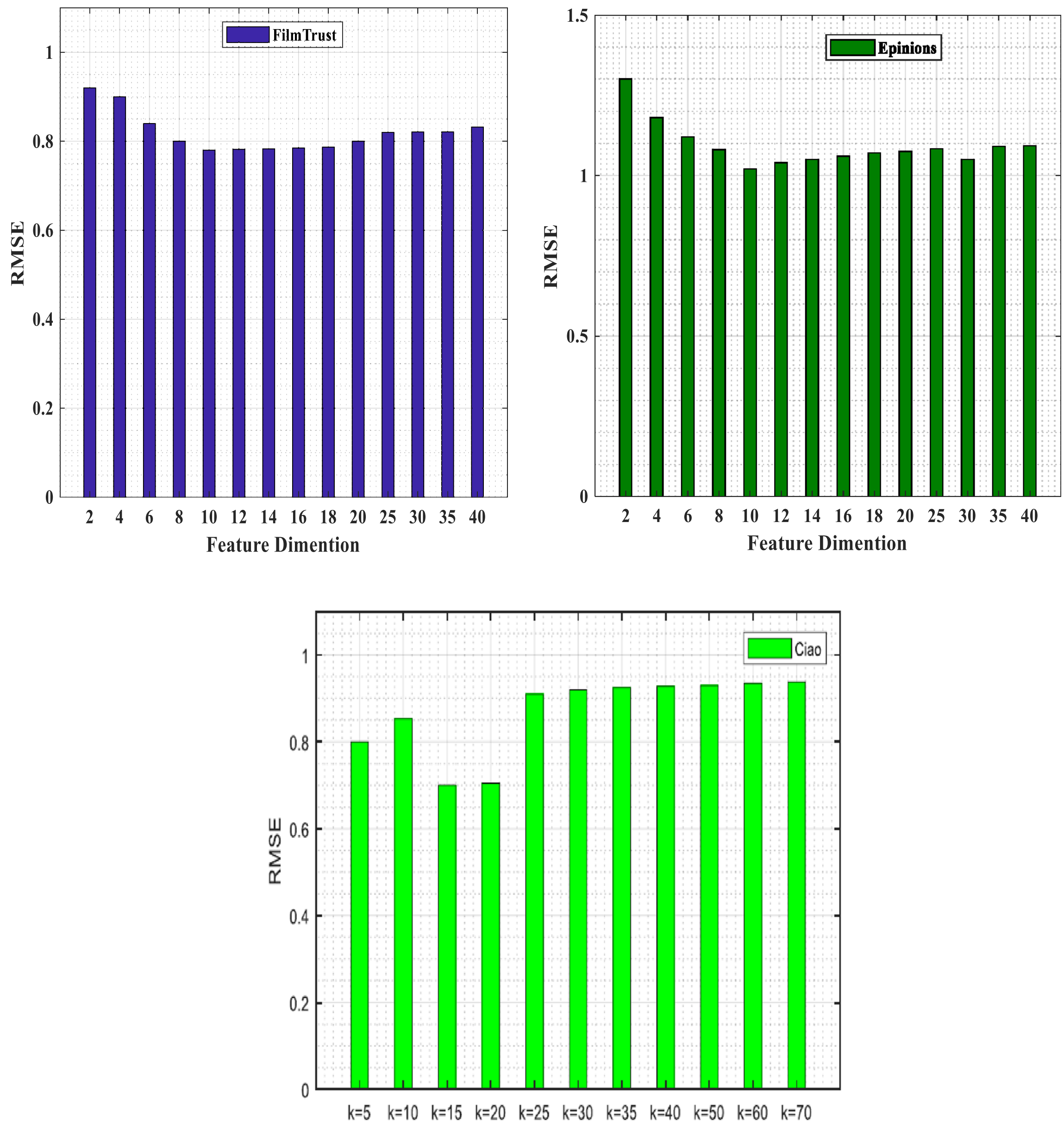

Fig. 3 Impact of latent dimensionality on the three datasets

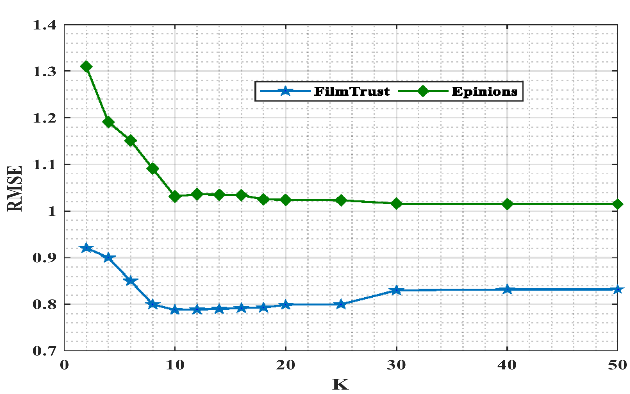

Fig. 4 Impact of feature dimension (f) on CFMT

Table 7 Run time comparison on the two datasets

Methods	Datasets		
	FilmTrust	Epinions	Ciao
RSTE	16.23	94.45	88.6
TrustMF	19.14	110.11	114
SocialMF	15.45	61.10	93.33
TrustSVD	24.22	119.88	93.55
SocialRec	14.89	48.75	84
CFMT	15.21	55.63	95

Table 8 Statistical significance t-test result for MAEs achieved from fivefold cross-validation

Methods		FilmTrust	Epinions	Ciao
RSTE	Mean	0.6725	0.8705	0.56
	Variance	0.0006917	0.0004917	6.67E-05
	p-value	9.72E-04	8.54E-04	2.31E-04
TrustMF	Mean	0.716	0.8705	0.505
	Variance	0.0005667	0.0002523	1.27E-05
	p-value	2.41E-04	1.65E-04	0.5367166
SocialMF	Mean	0.688	0.76875	0.6345
	Variance	0.0008667	0.0001563	0.000023
	p-value	1.62E-05	1.29E-03	2.92E-06
TrustSVD	Mean	0.6095	0.83425	0.726
	Variance	0.0004917	0.0002949	2.87E-05
	p-value	4.46E-02	1.53E-03	1.06E-05
Social Rec	Mean	0.71275	0.8695	0.57225
	Variance	0.0001449	0.0002917	4.92E-06
	p-value	1.32E-03	1.29E-03	2.57E-05
ANLF	Mean	0.7035	0.9025	0.65725
	Variance	0.0001583	0.0003103	2.29E-05
	p-value	1.43E-03	2.49E-04	1.95E-05
RSNMF	Mean	0.8155	1.1665	0.713
	Variance	0.0008917	0.0009877	3.87E-05
	p-value	2.06E-03	6.24E-05	2.03E-05
TrusANLF	Mean	0.5865	0.784	0.516
	Variance	0.0008917	0.000284	2.07E-05
	p-value	3.65E-01	6.23E-02	4.21E-02
Ayub	Mean	0.6655	0.94675	0.7915
	Variance	0.0004917	7.83E-05	3.57E-05
	p-value	1.39E-02	1.17E-05	1.62E-06
GA	Mean	0.6745	0.96025	0.7845
	Variance	0.0010917	2.76E-05	7.10E-05
	p-value	1.93E-02	5.78E-05	1.45E-05
ITrace	Mean	0.65	0.9805	0.80775
	Variance	0.0005667	2.17E-05	2.09E-05
	p-value	1.22E-03	3.79E-05	7.08E-07
CFMT	Mean	0.5595	0.76875	0.50375
	Variance	0.0010917	0.0001563	8.25E-06
	p-value	–	–	–

randomly selected for testing and the remaining one for testing. For each method, five values of MAE were used in the statistical test. The results of this test are shown in the table. The proposed method has a significant difference except for the cases that are highlighted in the table. Looking at the Table 8, we find out that in the Filmtrust and Epinions datasets, the TrustSVD method is insignificantly different from the proposed method. In the Ciao dataset, the least difference is related to the TrustMF method.

6 Conclusion

In this study, we attempted to use the utility trust statements in the recommendation process. An MF-based method was proposed that integrates rating information and trust relationships between users to overcome the sparsity and cold start issues. The importance of trust as a useful tool to resolve CF problems in traditional RSs was experimentally examined. The study results indicated the improved accuracy of the recommendations due to the incorporation of inlink and outlink trust relationships. The CFMT method provided a higher speed and lowered computation cost than the well-known methods and sufficient effectiveness in confrontation with cold start and data sparsity. We used social trust relationships as an additional source of information to make more accurate predictions at a lower computational cost and yet with higher accuracy. Overall, the results demonstrated the significant superiority of CFMT to the state-of-the-art and well-known methods in prediction recommendation accuracy. Future research can continue this study line by using deep learning methods to automatically extract features involving rating and trust information.

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