

Equation-based and data-driven modeling strategies for industrial coating processes

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Abstract

Computational Fluid Dynamics (CFD) and Machine Learning (ML) approaches are implemented and compared in an industrial Chemical Vapor Deposition process for the production of cutting tools. In this work, the aim is to analyze the pros and cons of each method and propose a blend of the two approaches that is suitable in industrial applications, where the process is too complicated to address with first-principles models and the data do not allow the implementation of data-hungry methods. Both approaches accurately predict the coating thickness (Mean Absolute Percentage Error (MAPE) of 6.0% and 4.4% for CFD and ML respectively for the test case reactor). CFD, despite its increased computational cost, both in terms of de-

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veloping and also calibrating for the application at hand, provides meaningful insight and illuminates the process. On the other hand, ML can provide predictions in a time-efficient manner, and is thus appropriate for inline and concurrent predictions. However, it is limited by the available data and has low extrapolation ability. Equation-based and data-driven methods are combined by exploiting a handful of CFD results for efficient interpolation in a reduced space defined by the principal components of the dataset, by implementing Gappy POD. This allows for the accurate reconstruction of the full state-space with limited data.

Keywords: Chemical Vapor Deposition, Coating, supervised learning, CFD modeling, industrial application, production data

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³ **1. Introduction**

⁴ Chemical Vapor Deposition (CVD) processes are popular in a wide range
⁵ of applications, including microelectronics (Creighton and Parmeter, 1993),
⁶ sensors (Ozaydin-Ince et al., 2011) and wear resistant coatings (Kathrein
⁷ et al., 2003). The coating process involves the nonlinear interplay of physical
⁸ mechanisms, such as diffusion and convection, with a plethora of homoge-
⁹ neous and heterogeneous chemical reactions. The competition between the
¹⁰ different mechanisms determines the process outcome and the product qual-
¹¹ ity. It is therefore a fine example of a process that is too complicated to
¹² study with first-principles models, such as Computational Fluid Dynamics
¹³ (CFD) and where the data is often not enough to implement sophisticated

14 data-driven strategies. Taking all of the above into consideration, the ob-
15 jective of this work is to investigate the potential benefit of simplified CFD
16 process models, accompanied by purely data-driven predictions using Ma-
17 chine Learning (ML) approaches. Both methods are driven by the size and
18 type of the available production data. In the CFD case, the data are used
19 for calibration and validation and in the ML case for regression.

20 Computational Fluid Dynamics is a valuable tool for studying deposi-
21 tion processes (Kleijn and Hoogendoorn, 1991; Kleijn et al., 2007; Cheimar-
22 ios et al., 2012; Cho and Mountziaris, 2013; Psarellis et al., 2018; Koronaki
23 et al., 2019; Papavasileiou et al., 2022), since it allows the investigation of
24 the flow field inside the reactor, as well as the main physical and chemical
25 pathways that lead to the deposition of thin film coatings. Nevertheless,
26 modeling industrial-scale deposition applications using CFD presents several
27 challenges: Firstly, dealing with the complexity of the process, which often
28 has several unknowns and secondly, the large scale of real applications.

29 Specifically, the actual chemical reactions that lead to deposition, includ-
30 ing their rates, are often unknown. Therefore, it is not possible to predict
31 the effect of the interplay between transport phenomena and chemical kinet-
32 ics on the deposition rate, necessitating the development of a kinetic model
33 (Topka et al., 2022). Even when a chemical reaction scheme is available,
34 some of its parameters may need to be fitted for the specific application.
35 This parameter fitting involves an increased computational cost, as it usu-
36 ally requires numerous simulations (Gakis et al., 2015; Koronaki et al., 2016;
37 Gkinis et al., 2017, 2019). Nevertheless, CFD has been applied to several
38 CVD applications, shedding light on previously “opaque” processes (Fotiadis

39 et al., 1990; Liu and Xiao, 2015; Aviziotis et al., 2017) while also allowing
40 to predict their outcomes (Endo et al., 2004). Although attempts have been
41 made towards increasing the efficiency of CFD models by implementing re-
42 duced order modeling methods (Gkinis et al., 2019; Spencer et al., 2021),
43 developing an efficient and accurate model in an industrial setting remains a
44 challenging and time-consuming task.

45 In the era of Industry 4.0, digitalization has become one of the main
46 drivers of innovation (Kagermann, 2015) and production data are becoming
47 more and more available. The industry is trying to exploit this data, seek-
48 ing improvement in several domains, including: maintenance management
49 (Saxena and Saad, 2007; Susto et al., 2015; Wu et al., 2019; Dalzochio et al.,
50 2020) quality management (Kim et al., 2012, 2018; Carvajal Soto et al., 2019;
51 Iqbal et al., 2019; Wang et al., 2022), production planning and control (Priore
52 et al., 2018; Tulsyan et al., 2018; Ma et al., 2019; Agarwal et al., 2020; Deng
53 et al., 2022), supply chain management (Du and Jiang, 2019), process out-
54 come predictions (Cai et al., 2020; Azadi et al., 2022; Dai et al., 2022; Malley
55 et al., 2022) and process optimization (He et al., 2021; Galvis et al., 2022).
56 Furthermore, digital twins (Boyes and Watson, 2022) are becoming increas-
57 ingly popular in the process industry (Hürkamp et al., 2020; Rasheed et al.,
58 2020; Perno et al., 2022), as well as in other, diverse applications (Urcun
59 et al., 2021; Kalaboukas et al., 2023). Although the application of sophisti-
60 cated methods such as Deep Neural Networks (DNNs) (Blakseth et al., 2022;
61 Deshpande et al., 2022), Physics Informed Neural Networks (PINNs) (Raissi
62 et al., 2019) and manifold learning (Koronaki et al., 2023) has been demon-
63 strated on controlled small scale problems, several challenges still remain

64 when incorporating ML in everyday industrial practice. Addressing these
65 challenges is one of the main objectives of this work.

66 The industrial application in this work is the coating of cutting tools with
67 $\alpha\text{-Al}_2\text{O}_3$ for increased wear resistance. Concerning CFD, the goal is to pro-
68 pose the best possible simplified model, based on the available data which are
69 necessary for verification and validation. This leads to a 2D, time-dependent
70 CFD model, presented in detail in previous work (Papavasileiou et al., 2022).
71 The proposed model implements representative boundary conditions and em-
72 ploys a simple reaction scheme for the $\alpha\text{-Al}_2\text{O}_3$ deposition with the goal of
73 reducing the computational cost.

74 Concerning ML, the first task is to pre-treat the available data, upon
75 which the choice of method depends on. Addressing mixed types of data
76 (categorical and numerical) is a common challenge in many applications, not
77 restricted to deposition processes. Several regression models are trained to
78 predict the $\alpha\text{-Al}_2\text{O}_3$ coating thickness using characteristics of the reactor
79 set-up and process conditions as inputs. In this work, the focus lies more on
80 tree-based methods (James et al., 2021b) which are the best-performing for
81 the given data-set.

82 The two approaches are initially compared in their ability to accurately
83 and efficiently predict the alumina coating thickness of the cutting tool in-
84 serts. Specifically, the advantages and disadvantages of each strategy are as-
85 sessed in terms of accuracy, interpretability, extrapolation ability and com-
86 putational cost. As a final step, the two approaches are merged through
87 the implementation of the Gappy Proper Orthogonal Decomposition (Gappy
88 POD) method (Everson and Sirovich, 1995; Willcox, 2006). The latter, is

89 popular for optimal sensor placement, and here it adapted to propose a suffi-
90 cient number of known data from which we can infer quantities that are not
91 measurable.

92 The manuscript is structured as follows: A concise overview of the process
93 and the available production data is given in Section 2. The implemented
94 methods (CFD, ML and Gappy POD) are presented in Sections 3 and 4. The
95 results of each method are analyzed and compared in Section 5, followed by
96 the conclusions in Section 6.

97 2. Process description

98 A two-step coating process takes place inside the studied industrial-scale,
99 commercial CVD reactor (Sucotec SCT600TH). First, a Ti(C,N) base layer
100 of about $9 \mu\text{m}$ is grown on the cemented carbide cutting inserts, such as the
101 ones shown in Fig. 1a. Subsequently, an alumina layer is deposited under
102 a $\text{AlCl}_3\text{--CO}_2\text{--HCl--H}_2\text{--H}_2\text{S}$ chemical system. The temperature and pressure
103 for the alumina coating step are $T=1005^\circ\text{C}$ and $p=80$ mbar, respectively
104 (Hochauer et al., 2012). The alumina coating deposition step of the process
105 takes approximately 3 hours.

106 The CVD reactor consists of 40-50 perforated disks, stacked one on top
107 of the other, whereon the inserts are placed. In Fig. 1b, a schematic of three
108 such disks is shown for clarity. The mixture of gas reactants, enters the
109 reactor via perforations on a rotating cylindrical tube, placed in the center
110 of the structure of the stacked disks. There are two antipodal perforations for
111 each disk level. There is a 60° angle difference between the axis connecting
112 the inlet holes for each disk level. The rotational motion of the inlet tube

113 (rotating with a rotational speed of 2 RPM) causes the process to have an
114 inherent periodic nature. The interior geometry of the reactor changes from
115 production run to production run, since the geometry of the inserts (and the
116 disks on which they are placed), changes based on production requirements.

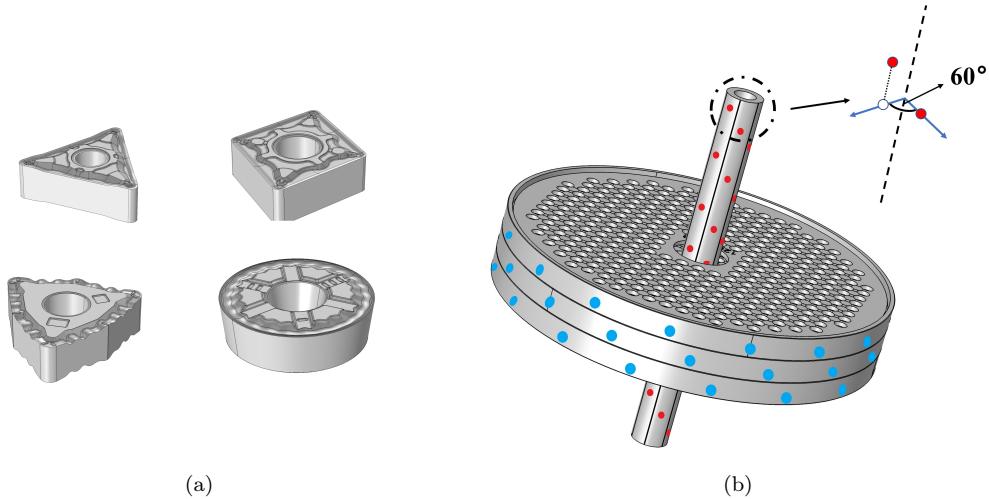


Figure 1: (a) Indicative geometries of the coated cutting tools. (b) A 3D representation of a 3-disk part of the reactor. The inlet perforations on the rotating inlet tube are shown in red. The outlet perforations for each disk are shown in blue.

117 The main goal of the process is to achieve uniform coating thickness,
118 since this uniformity also leads to uniform product longevity (Bar-Hen and
119 Etsion, 2017). Ideally, coating thickness uniformity would be achieved across
120 all production runs, reactors, and production sites. However, this is not
121 always the case. For this reason, a way of predicting the coating thickness of
122 the inserts given the reactor set-up is needed. Furthermore, coming up with
123 a systematic way of assessing the factors that influence the coating thickness
124 uniformity is also highly important.

125 *2.1. Available data*

126 For the $\text{Ti}(\text{C},\text{N})/\alpha\text{-Al}_2\text{O}_3$ multi-layer coating, the thickness measure-
127 ments are performed via the Calotest method. A small spherical cavity
128 is ground on the coated inserts using a rotating ball of known geometry,
129 providing a tapered cross-section of the film when viewed under an optical
130 microscope (Łepicka and Grądzka-Dahlke, 2019). This way, the thickness of
131 both the $\text{Ti}(\text{C},\text{N})$ and $\alpha\text{-Al}_2\text{O}_3$ coating layers can be calculated. Measure-
132 ments are usually taken for 3 positions on 5 disks of interest. Therefore, 15
133 thickness measurements are available for each production run. A 2D rep-
134 resentation of the reactor indicating the points where thickness is typically
135 measured is shown in Fig. 2. These measurements allow for not only for
136 the calibration and validation of the CFD model, but also for several ML
137 approaches.

138 Apart from coating thickness measurements, the dataset also contains
139 several features concerning the process and the reactor setup, which will
140 serve as inputs to the machine-learning model. The production “recipe” used
141 for the coating is the available feature providing information regarding the
142 process. Setup-wise, there is a plethora of available features for each disk of
143 the reactor, including:

- 144 1. The position of each disk inside the reactor.
- 145 2. The number of inserts placed on each disk.
- 146 3. The type of insert placed on each disk. Each type of insert has different
147 geometrical characteristics.
- 148 4. The type of disk used. The type of disk used is always relative to the
149 type of insert placed on top of it.

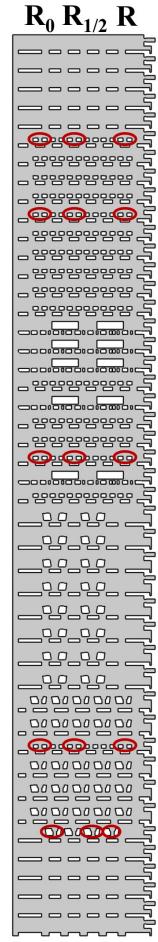


Figure 2: Positions with available α -Al₂O₃ thickness values from the production data for our test case. In general, across different production runs, the R position (the one closest to the reactor outlet) is the one with the highest amount of data. For this reason, the ML models are trained to make predictions for inserts placed in this position.

150 5. The surface area of the inserts placed on the disk.

151 These features allow for the creation of more features, such as the total
152 surface area and the standard deviation of the surface area of the inserts
153 that are coated inside the reactor. Another feature that can be created is the
154 difference between the nominal surface area of the production “recipe” and
155 the actual insert surface area inside the reactor. Furthermore, for each disk,
156 we can exploit the information available for its neighboring disks.

157 This way, we end up with several features, of which thirteen are used as
158 inputs after being pre-processed. These features are summarized in Table 1.
159 Considering the coating thickness measurements as outputs, we can train
160 several supervised learning models to make coating thickness predictions per
161 disk. In this context, during training, a labeled set of inputs is provided and
162 specifically here, the inputs are the aforementioned features and the labels
163 are the $\alpha\text{-Al}_2\text{O}_3$ coating thickness measurements.

164 **3. Computational ingredients**

165 *3.1. ML methods*

166 For the data-driven approach to the problem, the implementation of an
167 assortment of machine learning methods for the prediction of coating thick-
168 ness inside the reactor is investigated. All methods implemented fall into
169 supervised learning methods.

170 In supervised learning, each one of the input variables x_i is associated with
171 a response (or output) y_i (James et al., 2021a). The goal of the ML strategy
172 is to train a model able to relate the input variables x_i to the output y_i . This
173 way, future observations can be predicted and the relationship between the

Table 1: Summary of the features included in the training of the regression models.

Feature	Type	Pre-processing
Number of inserts on disk	Numerical (integer)	standardization
Surface area of inserts on disk	Numerical (float)	standardization
Disk position	Numerical (integer)	standardization
Total surface area of inserts inside the reactor	Numerical (float)	standardization
Surface area standard deviation	Numerical (float)	standardization
Nominal “recipe” surface area - actual surface area	Numerical (float)	standardization
Production “recipe”	Categorical	binary encoding
Insert geometry	Categorical	binary encoding
Disk geometry	Categorical	binary encoding
Insert geometry – disk above	Categorical	binary encoding
Insert geometry – disk below	Categorical	binary encoding
Disk geometry – disk above	Categorical	binary encoding
Disk geometry – disk below	Categorical	binary encoding

174 inputs and the output can be interpreted. Here, the goal is to predict the
 175 α -Al₂O₃ coating thickness (a continuous target variable) from several inputs,
 176 using a regression method. The specific methods include but are not limited
 177 to:

178 • Linear methods, such as linear, lasso or ridge regression.
 179 • Non-linear methods, such as polynomial regression.

180 • Tree-based methods, such as regression trees and their ensemble ver-
181 sions: random forests, gradient boosted regression trees and extreme
182 gradient boosted regression trees.

183 • Artificial neural networks.

184 During the early phases of this research, several techniques were utilized,
185 including linear, lasso, and ridge regression, as well as support vector ma-
186 chines and Gaussian process regression. Preliminary findings indicated that
187 tree-based methods outperformed the other techniques, and as a result, the
188 focus of this study is on tree-based methods.

189 The models' accuracy will be evaluated via two different metrics, namely
190 the mean absolute error (MAE) and the mean absolute percentage error
191 (MAPE). When the model is trained or tested on N observations and for
192 each observation i the prediction is \hat{y}_i while the actual value is y_i , MAE and
193 MAPE can be written as follows:

$$194 \quad \text{MAE} = \frac{1}{N} \sum_{i=1}^N |\hat{y}_i - y_i| \quad (1)$$

$$195 \quad \text{MAPE} = \frac{1}{N} \sum_{i=1}^N \frac{|\hat{y}_i - y_i|}{y_i} \quad (2)$$

196 Two different computational costs pertain to each ML model, the training
197 time (t_{train}) and the prediction time (t_{pred}) of the model. Both of these costs
198 are expressed in CPU time.

199 3.1.1. Tree-based methods

200 Tree-based methods work by partitioning the space of the inputs X into
201 a set of rectangles. Afterwards, a simple model (e.g. a constant) is fit in each

202 partition. The process starts by splitting the entire input space in two based
203 on a variable of the input space and its value. The optimal variable and split
204 point are chosen in order to achieve an accurate fit. Then, either or both
205 of the resulting regions are split again in two, once again using the optimal
206 input and split point. This procedure continues until a stopping criterion
207 has been met. The occurring binary splits allow for model interpretability
208 since the entire sample space can be described by a single tree. Tree-based
209 methods can be used for both regression and classification purposes (Hastie
210 et al., 2009a).

211 The prediction accuracy of a single tree is often not as high as that of other
212 methods. Furthermore, a small change in the data can lead to an entirely
213 different tree layout. These two issues and especially the predictive perfor-
214 mance of the trees can be rectified by combining multiple trees through the
215 implementation of ensemble methods such as bagging and boosting (James
216 et al., 2021b).

217 The concept behind ensemble methods is to build a prediction model
218 by combining a number of simpler base methods, in two steps: First, a
219 number of base learners must be created from the available data. The second
220 step involves the combination of these learners into one ensemble predictor.
221 The most common ensemble tree-based methods are random forests, bagged
222 trees and gradient boosted trees. These methods, however, have some key
223 differences between them.

224 Random forests and bagged trees, discussed here, operate similarly. They
225 both build B regression trees and each tree is trained using bootstrap-
226 sampled (i.e. sample a particular data-point and then reintroduce it to the

dataset), versions of the original dataset. Bagging regression methods provide a prediction by averaging the outputs of the B trees that they consist of. If $\hat{y}_{i,b}$ is the prediction of each grown tree, then the final prediction of the bagging method $\hat{y}_{i,bag}$ is given by:

$$\hat{y}_{i,bag} = \frac{1}{B} \sum_{b=1}^B \hat{y}_{i,b} \quad (3)$$

Random forests and bagged trees differ only in the amount of input features N_{input} that are considered when building each tree. In bagged trees, all available features are considered. On the contrary, in random forests, a random subset of p input features is considered. This serves the purpose of de-correlating the individual trees, since the trees are not always built by selecting the global optimal features, but by selecting the optimal feature from a randomly sampled subset of the input features (James et al., 2021b).

Gradient boosting and extreme gradient boosting are boosting methods. In the case of boosting methods, contrary to bagging methods, the B base trees are created sequentially. First, the first tree of the ensemble is created. Afterwards, each created tree is fitted to the difference between the value predicted by the previous tree and the real output. This way, each tree improves the shortcomings of the previous one. There is no averaging of the result of the B trees in this case (Hastie et al., 2009b).

Therefore, after building the b^{th} tree which outputs γ_{jb} and is trained on the residual of the output of the ensemble after the previous tree has been built, the output of the ensemble $f_b(x)$ can be written as:

$$f_b(x) = f_{b-1}(x) + \lambda \cdot \sum_{j=1}^J \gamma_{jb} I(x \in R_{jm}) \quad (4)$$

250 where I is the indicator function, and λ is the learning rate of the boosting
 251 procedure. λ serves the purpose of scaling the contribution of the output of
 252 each tree to the final prediction of the ensemble.

253 The result of the model is the output of the ensemble after the final tree
 254 has been built. Boosting methods are more prone to overfitting for large
 255 values of B than bagging methods. For this reason, B needs to be carefully
 256 selected through cross-validation.

257 *3.1.2. Challenges*

258 Applying data-driven methods to a real-world dataset presents several
 259 challenges. First and foremost, the dataset needs to be “cleaned”: Given that
 260 the production dataset is derived from different production sites, different
 261 reactors, and different people, it is bound to contain some errors. These
 262 errors must be identified and corrected before any type of analysis. Then,
 263 there is the question of the format of the data. Even when the data is neatly
 264 organized in an SQL database, it still needs to be extracted and formatted
 265 (using the pandas python library (McKinney, 2010), for example) so that
 266 it can be used to train models in a python framework. Afterwards comes
 267 the question of data type. In this particular application, there are both
 268 numeric and alphanumeric features (features that contain names instead of
 269 values). Since several of the implemented methods are not compatible with
 270 alphanumeric (categorical) features, those features need to be encoded in a
 271 way (i.e. binary encoding, one-hot encoding (Potdar et al., 2017)) that allows

272 them to be used in our models. Finally, once the data is ready, the task is
273 to find the best performing model and to determine the hyperparameters
274 that influence performance. Therefore, a hyperparameter optimization step
275 must also be included. By following this step-by-step approach, we can
276 establish a data pipeline specific to our data that allows us to overcome
277 all the aforementioned challenges. This however requires experience, input
278 from the process experts, along with a clear understanding of the data.

279 *3.2. CFD modeling: Implementation & challenges*

280 For this specific application, a digital “replica” of the process would have
281 to be a 3D, time-dependent full reactor (40-50 disks) model which would
282 include a complex reaction scheme. A complex reaction scheme, would lead
283 to more degrees of freedom and an increased number of kinetic parameters
284 that would need to be fitted. Apart from this, given the rotation of the
285 inlet tube (and therefore the fact that the problem is not axisymmetric) a
286 moving mesh would also need to be implemented. This would translate into
287 a computationally intractable task. If we consider that the reactor interior
288 geometry changes on a day-to-day basis, since the geometries of inserts and
289 the disks on which they are placed change based on production quotas, a
290 computationally expensive model is not a suitable method to study this in-
291 dustrial application. For this reason, aiming to drive the computational cost
292 down, the problem was approached as follows:

293 • The problem is modeled in 2D.

294 • The boundary conditions for both the inlet and the outlet are selected
295 in a way that is representative of their 3D characteristics.

296 • The model takes into account only 7-disk parts of the reactor in a divide
297 and conquer approach.

298 • A simpler reaction scheme that still leads to accurate results is used.

299 To efficiently tackle the challenges of the process, a 2D, time-dependent
300 model that accounts for the transport of mass, momentum, and species inside
301 the reactor is proposed. The COMSOL Multiphysics® software was used for
302 the CFD modeling. The interested reader can seek detailed information
303 in the recent work of Papavasileiou et al. (2022); here the key points are
304 summarized for completeness.

305 A reaction scheme consisting of a homogeneous reaction in the gas phase
306 and a heterogeneous reaction for the deposition of α -Al₂O₃ is part of the
307 model. The following assumptions are made: a) laminar and incompressible
308 flow, b) constant temperature of in the entire reactor domain, c) ideal gas
309 phase. The CFD model accounts for 7-disk “building blocks” of the reactor,
310 in order to keep the computational cost low. To account for the rotation of
311 the inlet tube, pulse velocity boundary conditions are applied at the inlets.
312 To represent the placement of the holes on the inlet tube in the 2D com-
313 putational geometry, a phase difference is included between the boundary
314 conditions of each disk. A similar approach is taken for the outlet perfora-
315 tions. Since they are not aligned, pressure boundary conditions are applied
316 at every other disk (1st open, 2nd closed and so forth). In order to model
317 the deposition of α -Al₂O₃ under the AlCl₃–CO₂–HCl–H₂–H₂S chemical sys-
318 tem, we implement a simple reaction scheme based on the work of Schierling
319 et al. (1999). Implementing this simpler scheme results in a lower computa-

320 tional cost. The simulations account for two full rotations (or periods) of the
321 feeding tube.

322 **4. Combining equation-based and data-driven approaches using**
323 **Gappy POD**

324 In this work, the Gappy POD method is used for the reconstruction of sev-
325 eral 7-disk reactor snapshots acquired using the aforementioned CFD model
326 using limited - or “gappy” data. Gappy POD was first introduced by Everson
327 and Sirovich (1995) and then implemented, among others, to a CFD airfoil
328 application by Willcox (2006) and for non-linear fracture mechanics modeling
329 (Kerfriden et al., 2013). Optimal sensor placement is another problem that
330 can be solved using the Gappy POD method, as indicated in the works of
331 Willcox (2006) and Jo et al. (2019). This is achieved by finding the optimal
332 way of filling the “gaps” in the data, or in other words, selecting the sensor
333 positions that give the most information possible.

334 A concise overview of the method, along with the procedure followed for
335 the acquisition of data and the metrics used for the evaluation of the method,
336 are presented in the following paragraphs.

337 *4.1. Overview*

338 In this section, the Gappy POD method is summarized for completeness.
339 Let’s consider a dataset \mathbf{X} of M vectors (represented as d -dimensional real
340 vectors x_1, \dots, x_M). A POD basis, $\Phi \in \mathbb{R}^{N \times M}$, of \mathbf{X} is computed, such that
341 \mathbf{X} can be approximated as a linear combination of p vectors:

342

$$\tilde{\mathbf{X}} = \sum_{j=1}^p c^j \Phi^j \quad (5)$$

343 or in matrix-vector format:

344

$$\tilde{\mathbf{X}} = \Phi \cdot c \quad (6)$$

345 The size of the truncated POD basis Φ is selected based on the error
 346 between the actual vector \mathbf{X} and the reconstructed approximation $\tilde{\mathbf{X}}$:

347

$$\text{reconstruction error} = \|\mathbf{X} - \tilde{\mathbf{X}}\| \quad (7)$$

348 Another factor that can be taken into account when selecting the size of
 349 the truncated basis is the total energy retained by the selected number of
 350 modes. For each basis vector j , the relative importance (E_j) is given by:

351

$$E_j = \frac{\lambda_j}{\sum_{i=1}^p \lambda_i} \quad (8)$$

352 and therefore, the total energy retained for the k retained modes is given by:

353

$$E_{\text{total}} = \sum_{j=1}^k E_j \quad (9)$$

354 Let us consider a vector X' that is spanned by the same basis Φ and that
 355 only m values of this vector are known, such that the partial vector X'_{partial}
 356 can be defined:

357

$$X'_{\text{partial}} = m \cdot X', m \in \mathbb{R}^{m \times N} \quad (10)$$

358 The goal is to find coefficients c' , such that an approximation \tilde{X}' of the
 359 vector X' can be defined as :

360

$$\tilde{X}' = X' \cdot c' \quad (11)$$

361 then:

362

$$X'_{\text{partial}} \approx m \cdot X' \cdot c' \quad (12)$$

363 Finding the values of c' that satisfy the above leads to an optimization
 364 problem, which results in the solution of the linear system:

365

$$M \cdot c' = (m \cdot \Phi)' \cdot X'_{\text{partial}} \quad (13)$$

366 with $M = (m \cdot \Phi)' \cdot (m \cdot \Phi)$ 367 *4.2. CFD data sampling*

368 Snapshots, i.e. vectors containing information regarding the system's
 369 state at a specific time, of 12 different 7-disk reactor parts will be used for
 370 the implementation of the Gappy POD method. For each reactor part, there
 371 31 available time-instances (each one with 1 second time difference from the
 372 previous). This way, the full dataset consists of 372 vectors.

373 At each time-instance, 4 quantities of interest are sampled along the lines
 374 connecting inlet-outlet at each disk level. The points of these lines are then
 375 interpolated at 250 specific query points using linear interpolation. In this
 376 manner, 250 evenly spaced points along each line are obtained. An example
 377 of the lines along which the quantities of interest are sampled is demonstrated
 378 in Fig. 3.

379 The quantities of interest at each point are:

380 1. The velocity magnitude (U).

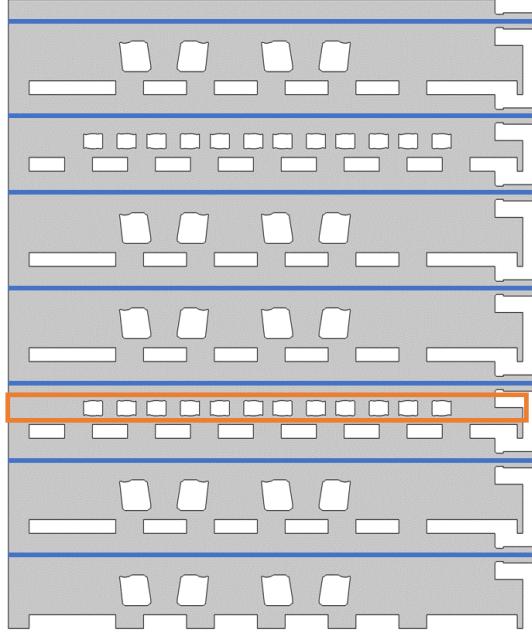


Figure 3: In blue: The seven lines along which the 4 quantities of interest ($U, p, C_{AlCl_3}, C_{H_2O}$) are sampled. In orange: The disk with available thickness measurements. The thickness measurements, as well as the α - Al_2O_3 deposition rates at the inserts of this disk, are also included for our implementation of Gappy POD.

- 381 2. The pressure (p).
- 382 3. The concentration of the precursor $AlCl_3$ (C_{AlCl_3}).
- 383 4. The concentration of water (C_{H_2O}).

384 Furthermore, the deposition rates as predicted by the CFD model along
 385 with the available thickness data for 3 positions ($R_0, R_{1/2}, R$) for each 7-disk
 386 reactor part, are included in each snapshot. An overview of the resulting
 387 dataset after sampling and organizing the vectors is presented in Fig. 4.

388 It is worth noting that a plethora of input parameters influences the

389 final product, the most important of which include the configuration of the
 390 reactor's interior geometry and the production "recipe". The latter includes
 391 all the steps and chemical species involved in the production of a single
 392 coating layer. In this work, to make the simulations tractable, the focus lies
 393 on a single "recipe" for a single product and various geometries, without loss
 394 of generality.

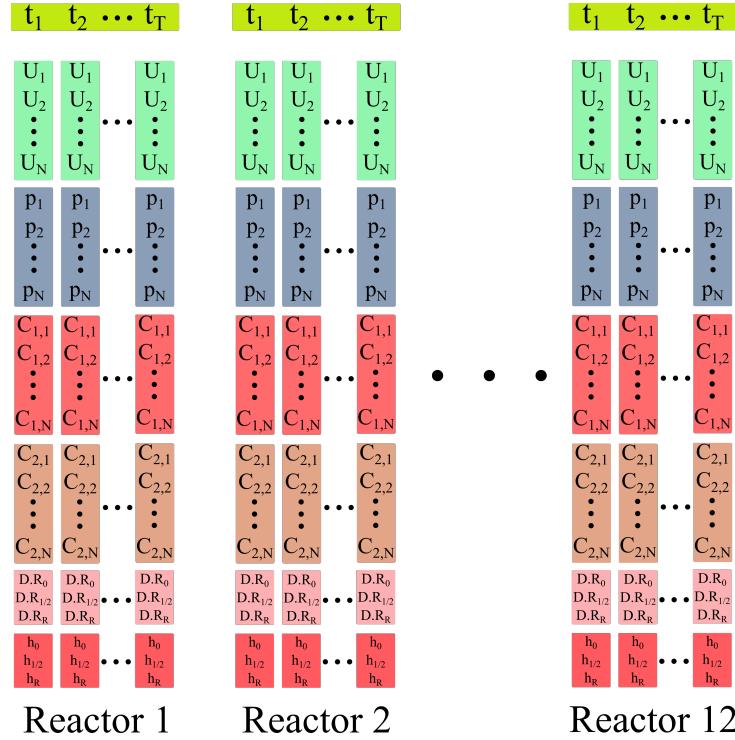


Figure 4: The final matrix considered for the Gappy POD method. A total of 31 time-instances for 12 different reactor geometries have been sampled. These contain all 4 quantities of interest (velocity magnitude, pressure, precursor concentration (C_1), water concentration (C_2) along with the calculated deposition rates (D.R) and the coating thickness measurements (h) taken from the production data. In our case, $T = 31$ (number of time-instances per reactor) and $N = 1750$ (total number of points: 7 lines containing 250 points each).

395 *4.3. Performance metrics*

396 The performance of the Gappy POD approach will be evaluated using the
397 Root Mean Squared Error (RMSE) between: a) the Gappy POD reconstruc-
398 tion and the POD reconstruction, b) the Gappy POD reconstruction and the
399 snapshots of the reactor given by the CFD model. The RMSE between two
400 values (\hat{y}_i and y_i) for N observations can be written as follows:

401
$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2} \quad (14)$$

402 *4.4. Mask selection*

403 The effectiveness of Gappy POD depends on the condition number of
404 matrix M , which is defined in Eq. (13). The matrix M is created from the
405 inner products of the "gappy" POD vectors, which are the elements of the
406 original POD vectors corresponding to the known elements of X' . Since
407 these vectors are no longer orthogonal, the matrix M is fully populated. For
408 orthogonality to be preserved, the known element positions and non-zero
409 elements of M must be appropriately arranged. Additionally, the diagonal
410 entries of M must not be too small, indicating that the POD basis element
411 at that point should not be small. The condition number of the matrix
412 M reflects these requirements, with a smaller condition number indicating
413 greater satisfaction of these conditions. This analysis is detailed in (Willcox,
414 2006), in the context of optimal sensor placement, and in (Alonso et al.,
415 2004a,b), which consider the angle between the measurement subspace and
416 the low dimensional space that spans the data.

417 To determine the known values of the vector X' in a more systematic

418 manner, a greedy algorithm similar to the one proposed by Willcox (2006) is
419 implemented. However, in our case, the mask elements are selected in a way
420 that reduces the reconstruction error. Considering m known values of each
421 snapshot X' , then the greedy algorithm implemented works as follows:

- 422 1. Initialize by randomly selecting m known values.
- 423 2. Starting with the first mask element, loop through all the possible
424 positions for the known values and calculate the reconstruction error
425 for each resulting mask.
- 426 3. Find the position of the element that minimizes the reconstruction error
427 and place the first element there.
- 428 4. Repeat steps 2-3 for all remaining mask elements.

429 This way, we can efficiently find positions for the mask elements that
430 yield an acceptable reconstruction error. It should be noted, however, that
431 this does not always lead to the globally optimal positions.

432 5. Results

433 5.1. CFD model

434 5.1.1. CFD model parameters

435 To elaborate on the model summary made in Section 3.2, further infor-
436 mation regarding the CFD model parameters is given in this section.

437 The prescribed inlet boundary conditions are inlet velocity conditions.
438 For each disk, the gas feed velocity is a time-dependent pulse function that
439 mirrors the inlet tube rotation, varying between 0 and V_{\max} . There is a phase
440 difference between the pulses of each disk. V_{\max} and the aforementioned

441 phase difference are determined based on the experimental conditions and
442 geometry, taking into account: a) the 2 RPM rotational speed of the inlet
443 tube, b) the total inlet gas flow rate, c) the number of disks per run, d)
444 the two antipodal perforations per disk, e) the diameter of the perforations
445 (0.002 m), and f) the 60° angle difference between the perforations of each
446 disk.

447 Outlet pressure boundary conditions are applied at every other disk level.
448 This way, we account for the real geometry where the outlet perforations are
449 not aligned. This results in a model where only the first, the third, the fifth,
450 and the seventh outlet from the top are considered open.

451 Seven different chemical species are considered, along with a simplified
452 reaction scheme for the deposition of α -Al₂O₃. The molar fractions at the
453 inlet are the following: CO₂ (0.0385), AlCl₃ (0.0169), HCl (0.0210), H₂O
454 (10⁻⁶), CO (10⁻⁶), H₂ (0.9203), and H₂S (0.0033).

455 The process conditions for the alumina coating step are $T=1005^{\circ}\text{C}$ and
456 $p=80$ mbar, as indicated in (Hochauer et al., 2012). Further information can
457 be found in the recent work of Papavasileiou et al. (2022).

458 *5.1.2. CFD model predictions*

459 The CFD model has been tested for 4 different 7-disk reactor geometries.
460 All four 7-disk geometries are building blocks of the test case reactor, whose
461 2D representation is shown in Fig. 2. It is possible to predict the α -Al₂O₃
462 coating thickness with a maximum relative error of 8% and within 5% mean
463 absolute percentage error for each 7-disk geometry, when compared to the
464 available production data. The maximum observed mean absolute percent-
465 age error for the α -Al₂O₃ coating thickness is 4.33%. Simulations for each

466 geometry consist of about 10^6 degrees of freedom. The solution time for each
467 geometry is approximately 3 core hours on an 11th Gen Intel(R) Core(TM)
468 i7-1185G7 processor. The results of the CFD simulations are summarized in
469 Fig. 5.

470 *5.2. Data-driven predictions*

471 We implement the following tree-based methods: a) Regression Trees,
472 b) Random Forests, c) Gradient Boosting Regression Trees (GBRT) and
473 eXtreme Gradient Boosting Regression Trees (XGBoost). All the methods
474 have comparable performance. Among them, the best performing is XGBoost
475 and the results below focus on its predictions.

476 The dataset contains a total of 6114 observations and is split into a train-
477 ing set and a test set, using a ratio of 75/25. Each one of these observations
478 contain thickness measurements at the R position for a particular disk (cf.
479 Fig. 2), corresponding to a number of inputs, detailed in Section 2.1. The nu-
480 matical features were standardized, and the categorical features were encoded
481 using binary encoding.

482 *5.2.1. Hyperparameter selection*

483 Optimal model performance, is influenced by the choice of hyperparam-
484 eters for each method. The most important hyperparameters of the imple-
485 mented tree-based ensemble methods are:

486 1. The maximum depth of the trees (d_{\max}), i.e. the number of bifurcations
487 of the main “branch” of the tree. Selecting too large a tree depth
488 can lead to overfitting, which in essence means that the model fails to
489 generalize accurately.

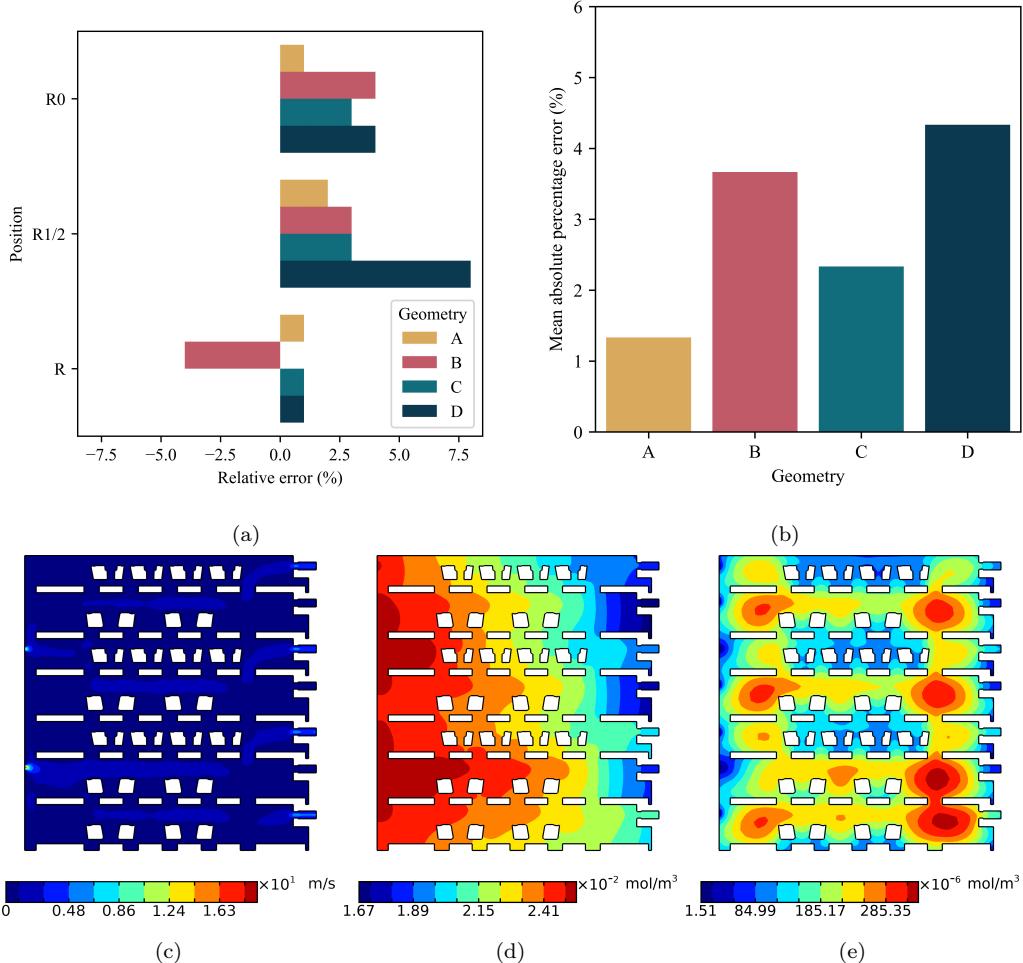


Figure 5: (a) Relative error for the CFD predictions for 3 different positions with available production data inside the reactor. Simulations are performed for four different 7-disk geometries in total. (b) Mean absolute percentage error (averaged over the 3 positions for which data are available) for the CFD simulations for the 4 different reactor geometries. (c) Velocity magnitude, (d) Precursor Concentration and (e) Water Concentration inside the reactor at a certain time during the deposition.

490 2. The number of trees (B). A large number of trees reduces the variance
 491 of bagging methods, however it can lead to overfitting in the case of

492 boosting methods.

493 3. For boosting methods specifically, another important hyperparameter
494 is the learning rate (λ). The choice of λ usually affects the optimal
495 B . For example, a very small λ usually requires a large B to achieve
496 satisfactory performance.

497 Searching for the optimal model hyperparameters in an exhaustive man-
498 ner is a computationally expensive task. The time required for all 5 tree-
499 based methods using an exhaustive grid search approach performing 10-fold
500 cross-validation was 43 core hours on an 11th Gen Intel(R) Core(TM) i7-
501 1185G.

502 To demonstrate here the effect of d_{\max} , results are shown for fixed values
503 of B and λ (cf. Fig. 6). For a constant number of trees ($B = 10000$), boosting
504 methods show better performance for low values of d_{\max} . On the contrary,
505 bagging methods indicate better performance for higher values of d_{\max} .

506 Overall, for all the hyperparameters tested, boosting methods appear to
507 outperform their bagging counterparts. Out of the two boosting methods, the
508 XGBoost method displays higher training and predicting speed. Specifically,
509 for the same training set and the same hyperparameters ($B = 10000$, $d_{\max} =$
510 5 and $\lambda = 0.01$), the average training time over 10 cross-validation splits
511 is 16.5s for the XGBoost model and 99.5s for the GBRT model. Moreover,
512 the average prediction time is 20ms for the XGBoost model and 333ms for
513 the GBRT model. Therefore, due to its lower computational cost, further
514 hyperparameter tuning will take place for the XGBoost algorithm, in order
515 to find the optimal hyperparameter combination.

516 After selecting the optimal value of maximum depth, we further inves-

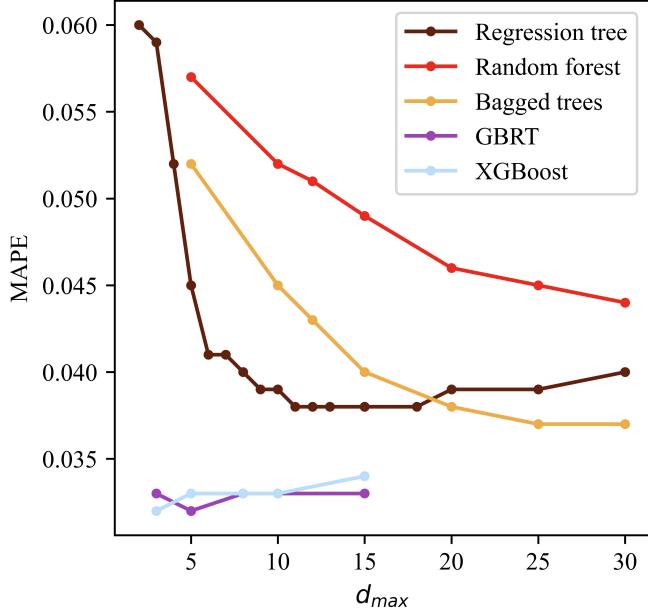


Figure 6: MAPE vs d_{max} for all methods after 10-fold cross-validation. $B = 10000$ for all ensemble methods. $\lambda = 0.01$ for the boosting methods. For the base method (regression tree) and bagging methods (Bagged Trees and Random Forests) increasing the maximum depth of the trees leads to a reduced MAPE. For the boosting methods (GBRT and XGBoost), the MAPE increases when increasing the maximum depth of the trees. Random forest regression performing worse than the simple regression tree can be attributed to the fact that it only considers a subset of available features when building each tree of the ensemble.

517 tigate the effect of the number of trees B on the accuracy of the XGBoost
 518 model. As indicated in Table 2, the accuracy of the model drastically im-
 519 proves when $B \geq 500$, nevertheless, the trade-off is in the form of increased
 520 computational cost.

521 Following hyperparameter optimization and tuning, the final values se-
 522 lected for the XGBoost model are the following: $d_{max} = 5$, $B = 10000$,

Table 2: XGBoost model results after cross-validation for various values of B , where $d_{\max} = 5$ and $\lambda = 0.01$. As expected, an increased number of base predictors improves the performance of the ensemble boosting method. However, it also increases the training time and prediction time of the model. All metrics are averaged over 10 cross-validation splits.

Number of trees (B)	MAPE	\bar{t}_{train} (s)	\bar{t}_{pred} (ms)
10000	3.1%	16.3	20
5000	3.3%	8.0	14
2000	3.4%	3.3	9
1000	3.6%	1.7	9
500	3.9%	0.9	8
200	12.6%	0.4	8
100	33.8%	0.2	10

523 $\lambda = 0.01$.

524 *5.2.2. Machine learning outcomes*

525 Two more accuracy metrics are introduced here, the mean square error
 526 (MSE) and the coefficient of determination (R^2). When the model is trained
 527 or tested on N observations and for each observation i the predicted value is
 528 \hat{y}_i while the actual value is y_i and the average of the actual values is \bar{y} , MSE
 529 and R^2 can be written as follows:

530
$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2 \quad (15)$$

531

$$R^2 = 1 - \frac{\sum_{i=1}^N (\hat{y}_i - y_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (16)$$

532 The prediction error of XGBoost regression model for the training set,
 533 reaches a MAPE of 0.9%, versus 3.1% for the test set. The prediction ac-
 534 curacy of the XGBoost model on the training set and on the test set can
 535 be summarized in Figs. 7a and 7b respectively. Due to the confidentiality
 536 of the production data, absolute α -Al₂O₃ thickness values cannot be pre-
 537 sented. Therefore, only relative error values and normalized thickness values
 538 are presented.

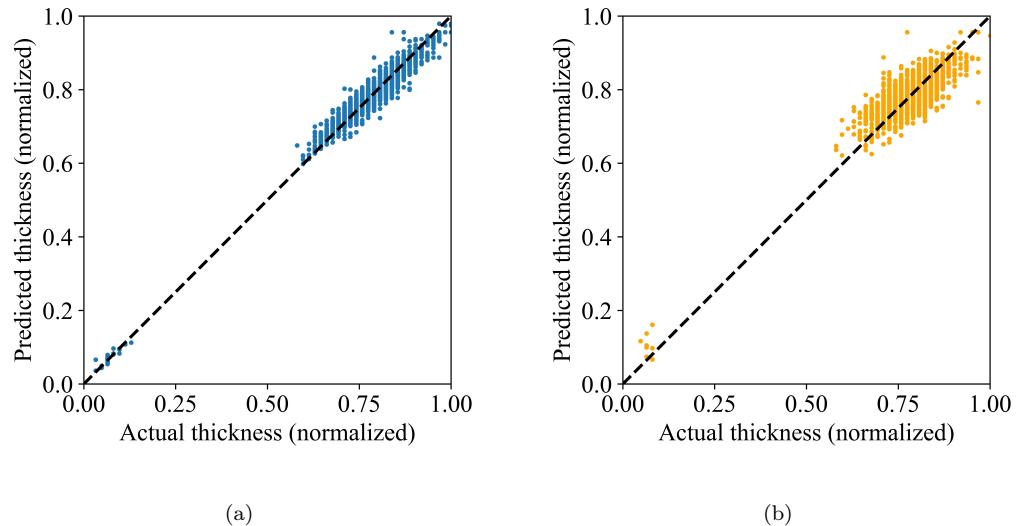


Figure 7: (a) Training set performance: MSE:0.005 | MAE:0.051 | MAPE:0.9% | R^2 :0.980.
 (b) Test set performance: MSE:0.059 | MAE:0.187 | MAPE:3.1% | R^2 :0.753.

539 *5.3. CFD vs ML*

540 *5.3.1. Predictive accuracy*

541 For the test-case reactor set-up presented in Fig. 2, the prediction results

542 for the position closest to the outlet for both methods are given in Table 3.

543 Disk position is counted from the bottom to the top of the reactor.

Table 3: XGBoost prediction accuracy vs CFD prediction accuracy for the coating thickness of inserts closest to the reactor outlet (R position). Errors relative to the available production data are presented. The high error in the prediction of the CFD model for the 6th reactor disk can be attributed to the fact that it is the bottom-most disk of the simulated 7-disk geometry, and therefore the effect of the inlets and outlets that are below it is not taken into account.

Disk position	CFD prediction	XGBoost prediction
39	3.2%	3.5%
35	1.0%	-3.1%
23	-4.0%	-7.0%
10	1.0%	-5.5%
6	20.6%	-2.8%
MAPE	6.0%	4.4%
Total prediction time (s)	43200	0.1

544 Despite the significant difference in the computational effort involved in
545 the CFD model in comparison to the ML regression model, both methods
546 have comparable accuracy on the test-case. CFD predictions for the test
547 reactor have a mean absolute percentage error of 6%, while XGBoost makes
548 predictions with a mean absolute percentage error of 4.4%. The high error
549 in the prediction of the CFD model for the 6th reactor disk (20.6%) can be

550 attributed to the fact that it is the bottom-most disk of the simulated 7-disk
551 geometry and therefore the effect of the inlets and outlets that are below it
552 is not taken into account. This can be solved by an extra 7-disk simulation,
553 where the disk of interest won't be in the bottom-most position. This would
554 of course further increase the computational cost of the CFD approach. The
555 maximum observed absolute relative error for the predictions of the XGBoost
556 model on the test-case reactor is 7%.

557 *5.3.2. Computational performance*

558 Although the predictive accuracy of the two approaches is similar, they
559 demonstrate a very noticeable contrast when it comes to their computational
560 performance. Specifically, in the case of CFD, making predictions for an en-
561 tire production run would require 4 or 5 7-disk simulations. This corresponds
562 to a computational cost of 12 to 15 core hours. On the other hand, using
563 the XGBoost model to make predictions for an entire production run comes
564 with a computational cost of less than 1 core second. This translates to a
565 reduction of more than 99.99% in required resources.

566 *5.4. Gappy POD*

567 Results of our Gappy POD implementation will be presented for two
568 different cases:

569 1. The case of the full dataset.
570 2. The case of a single reactor.

571 In each case, the dataset consists of time-instances of the state vector,
572 over a period of 30 secs. Therefore, the full dataset eventually consists of 372
573 snapshots, whereas in the single reactor dataset, it consists of 31 vectors.

574 In both cases, 87.5% of the available snapshots are used to derive the
575 POD basis of the training set. The rest of the snapshots (12.5%) are kept
576 and used for the validation of the method. For both cases, the data are scaled
577 in the range of [0, 1] using min-max normalization.

578 The number of modes used for the POD basis are selected after checking
579 the energy retained by the modes and the resulting reconstruction error. The
580 total retained energy for the full dataset and the single reactor dataset, is
581 shown in Fig. 8a and Fig. 8c respectively, whereas the reconstruction error
582 as a function of the basis size is shown in Fig. 8b and Fig. 8d respectively.

583 The full reactor dataset requires at least 50 POD modes to capture more
584 than 95% of the energy of the system, with a corresponding reconstruction
585 error (RMSE) of 0.0059. The single reactor dataset, is accurately represented
586 by 15 POD modes that reflect more than 98 % of the energy with a recon-
587 struction error (RMSE) of 0.004. Eventually, for the immediate comparison
588 of the results, the same basis size is considered, equal to 15 POD modes. The
589 corresponding retained energy and error are shown in Table 4.

Table 4: Number of POD modes selected for each case, along with the corresponding retained energy and reconstruction error.

Case	# POD modes	Energy retained	Recon. error (RMSE)
Full dataset	15	81.69%	0.0373
Single reactor	15	98.70%	0.0040
Single reactor	5	82.74%	0.0456

590 After selecting the size of the POD basis for each case, the mask ele-
591 ments for Gappy POD are obtained using the greedy algorithm described in

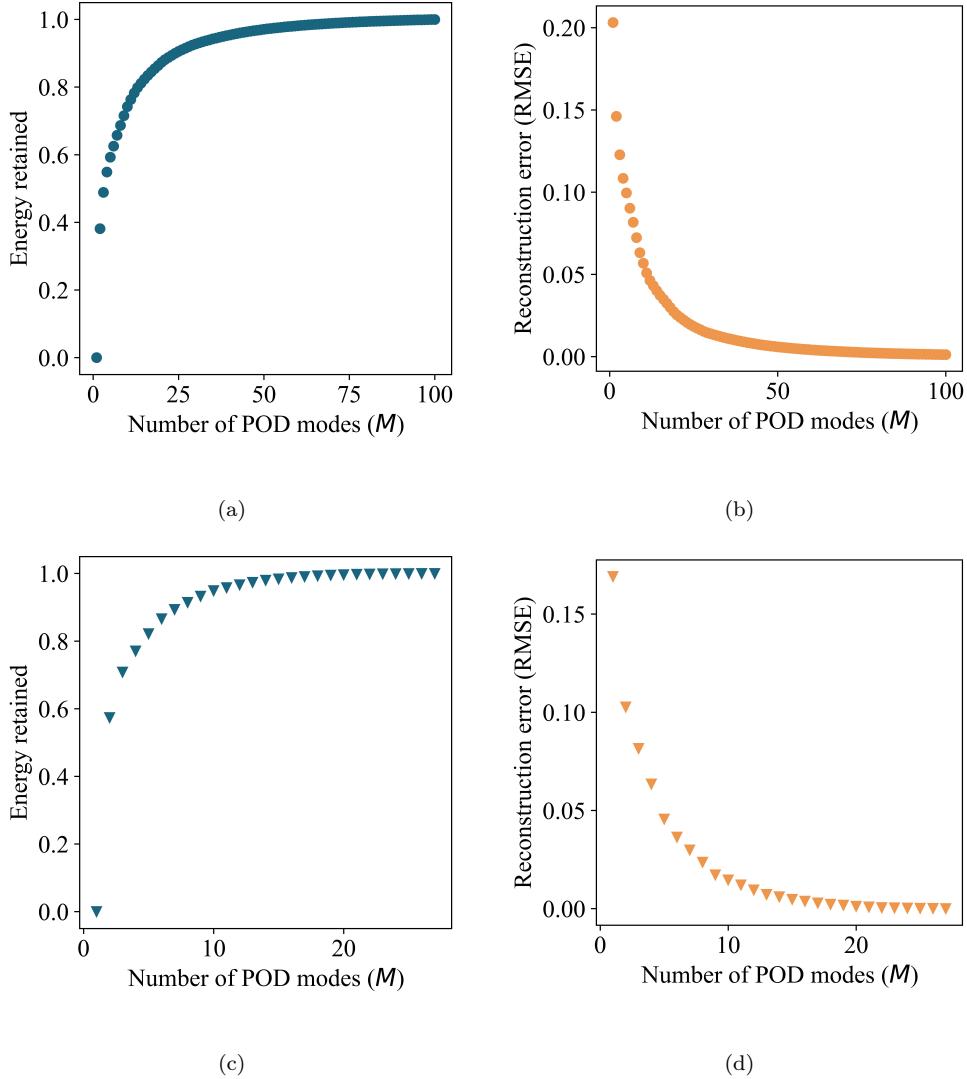


Figure 8: The energy retained (in blue) and the reconstruction error (in orange) of the POD approximation using M modes. (a), (b): Energy and reconstruction error for the full dataset. Only the first 100 modes are shown. (c), (d): Energy and reconstruction error for the single reactor case.

592 Section 4.4. It should be noted that the mask length should be greater or
593 equal to the size of the POD basis. For all three cases, we allow one mask
594 element more than the size of the POD basis. It should be noted that in
595 all cases the mask elements acquired consist of all the quantities of interest
596 (velocity magnitude, pressure, precursor concentration, water concentration)
597 discussed in Section 4.2.

598 After acquiring the mask elements, the RMSE between the Gappy POD
599 approximation and the test set, along with the RMSE between the Gappy
600 POD approximation and the POD reconstruction, can be calculated. Specif-
601 ically, for the case of the full dataset, the RMSE between the Gappy POD
602 approximation and the test set is 0.0648 while the RMSE between the Gappy
603 POD approximation and the POD reconstruction is 0.0512 (cf. Fig. 9). For
604 the case of the single reactor, the RMSE between the Gappy POD approxi-
605 mation and the test set is 0.0099 while the RMSE between the Gappy POD
606 approximation and the POD reconstruction is 0.0064. If we choose to make a
607 comparison using the number of POD modes with the same retained energy
608 and reconstruction error, we choose 5 POD modes (82.74% retained energy
609 and 0.046 reconstruction error) and 6 mask elements for the single reactor
610 case. Then, the RMSE between the Gappy POD approximation and the test
611 set is 0.0474 while the RMSE between the Gappy POD approximation and
612 the POD reconstruction is 0.0143.

613 The performance of the method, is linked to how well the dataset is
614 spanned by the selected POD vectors, generally implying that a larger POD
615 basis is beneficial for the results. Nevertheless, since the ambition of this
616 approach is to select only a few measurements as mask elements, it is more

617 beneficial to work with the smallest possible number of POD vectors.

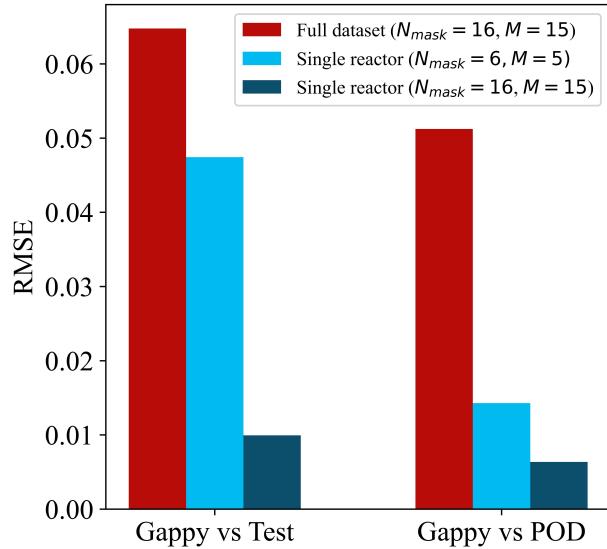


Figure 9: On the left: Error between the Gappy POD approximation and the snapshots of the test set for all cases. On the right: Error between the Gappy POD approximation and the POD approximation for all cases. It is evident that the single reactor case shows the lowest errors. This is probably due to the lower variance observed in the dataset of the single reactor when compared with the full dataset. For the case of the single reactor, using a smaller POD basis (5 modes instead of 15) leads to an increase in both errors.

618 6. Conclusions

619 This work presents an overview of the implementation of equation-based
620 and machine-learning methods in industrial-scale deposition applications.
621 The challenges associated with the complexity of the process and the charac-
622 teristics of real production data are discussed and the methods to overcome
623 them are presented.

624 In the equation-based approach, a reduced model is presented and vali-
625 dated with production measurements of the coating thickness. The simplifi-
626 cations introduced and the pertinent assumptions upon which they are based
627 are discussed, along with the results. The trade-off between the computa-
628 tional cost associated with the CFD model and the physical insight obtained,
629 is discussed and compared to the ML approach. Coating thickness predic-
630 tions are possible with an average error of 6%. In addition, the CFD model,
631 predicts the distributions of velocity, and reactive species, illuminating thus,
632 the mechanisms that contribute to the final product. Furthermore, it can be
633 used to predict the thickness achieved in parts of the reactor where there are
634 no measurements. Moreover, the CFD approach also allows extrapolating for
635 different process conditions and different inlet reactant concentrations. For
636 the 7-disk CFD approach, the results of Table 3, show that appropriate selec-
637 tion of the 7-disk “building blocks” for the simulations is of high importance
638 for the accuracy of the prediction.

639 The ML approach is discussed in detail, as far as the possible specific
640 methods are concerned. The suitability of each is assessed, based on the data
641 available. Eventually the best performing ML method, XGBoost, is able to
642 deliver accurate and time-efficient coating thickness predictions, but cannot
643 provide insight into the transport of species that determines the coating
644 thickness.

645 The implementation of Gappy POD for this specific application, shows
646 how data-driven methods and CFD results can be intertwined to provide
647 further insight on the important quantities of interest inside the reactor. By
648 further analysis of the resulting mask elements, we can explore the hypo-

649 thetical scenario of sensor placement inside such reactors. Furthermore, we
650 can reconstruct entire snapshots from a few measurements inside the reactor,
651 reducing in this way the computational cost of the problem.

652 It should be noted that the strategy employed here is not exclusive to
653 CFD modeling. The same workflow could still be implemented in other
654 applications, regardless of the equation-based modeling approach used. The
655 only limiting factor would be the amount and type of available data for the
656 application.

657 Another important observation is that specific *combinations* of inputs
658 can lead to the same outputs. This merits further investigation, due to its
659 importance in the actual production process, which is the topic of future
660 work.

661 To conclude, it is clear that each individual approach is a valuable tool in
662 studying a complex process offering different advantages: physical insight and
663 extrapolation abilities in CFD and time-efficient, accurate predictions in ML.

664 It is therefore worth investing the effort in each one of them, and ultimately,
665 in merging them in a hybrid approach with additional benefits. Ideally, the
666 resulting model could combine high accuracy, time-efficient predictions, and
667 excellent extrapolation ability, moving in this way toward a digital twin of
668 the process.

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676 **References**

677 Agarwal, P., Tamer, M., Sahraei, M.H., Budman, H., 2020. Deep Learning for
678 Classification of Profit-Based Operating Regions in Industrial Processes.
679 Ind. Eng. Chem. Res. 59, 2378–2395. doi:10.1021/acs.iecr.9b04737.

680 Alonso, A.A., Frouzakis, C.E., Kevrekidis, I.G., 2004a. Optimal sensor place-
681 ment for state reconstruction of distributed process systems. AIChE Jour-
682 nal 50, 1438–1452. doi:10.1002/aic.10121.

683 Alonso, A.A., Kevrekidis, I.G., Banga, J.R., Frouzakis, C.E., 2004b. Optimal
684 sensor location and reduced order observer design for distributed process
685 systems. Computers & Chemical Engineering 28, 27–35. doi:10.1016/
686 S0098-1354(03)00175-3.

687 Aviziotis, I.G., Duguet, T., Vahlas, C., Boudouvis, A.G., 2017. Combined
688 Macro/Nanoscale Investigation of the Chemical Vapor Deposition of Fe
689 from Fe(CO)5. Advanced Materials Interfaces 4, 1601185. doi:10.1002/
690 admi.201601185.

691 Azadi, P., Winz, J., Leo, E., Klock, R., Engell, S., 2022. A hybrid dy-
692 namic model for the prediction of molten iron and slag quality indices of a
693 large-scale blast furnace. Computers & Chemical Engineering 156, 107573.
694 doi:10.1016/j.compchemeng.2021.107573.

695 Bar-Hen, M., Etsion, I., 2017. Experimental study of the effect of coating
696 thickness and substrate roughness on tool wear during turning. *Tribology*
697 *International* 110, 341–347. doi:10.1016/j.triboint.2016.11.011.

698 Blakseth, S.S., Rasheed, A., Kvamsdal, T., San, O., 2022. Deep neural
699 network enabled corrective source term approach to hybrid analysis and
700 modeling. *Neural Networks* 146, 181–199. doi:10.1016/j.neunet.2021.
701 11.021.

702 Boyes, H., Watson, T., 2022. Digital twins: An analysis framework and
703 open issues. *Computers in Industry* 143, 103763. doi:10.1016/j.compind.
704 2022.103763.

705 Cai, H., Feng, J., Yang, Q., Li, W., Li, X., Lee, J., 2020. A virtual metrology
706 method with prediction uncertainty based on Gaussian process for chemical
707 mechanical planarization. *Computers in Industry* 119, 103228. doi:10.
708 1016/j.compind.2020.103228.

709 Carvajal Soto, J.A., Tavakolizadeh, F., Gyulai, D., 2019. An online machine
710 learning framework for early detection of product failures in an Industry
711 4.0 context. *International Journal of Computer Integrated Manufacturing*
712 32, 452–465. doi:10.1080/0951192x.2019.1571238.

713 Cheimarios, N., Koronaki, E.D., Boudouvis, A.G., 2012. Illuminating
714 nonlinear dependence of film deposition rate in a CVD reactor on op-
715 erating conditions. *Chemical Engineering Journal* 181–182, 516–523.
716 doi:10.1016/j.cej.2011.11.008.

717 Cho, J., Mountziaris, T.J., 2013. Onset of flow recirculation in vertical
718 rotating-disc chemical vapor deposition reactors. *AIChE Journal* 59, 3530–
719 3538. doi:10.1002/aic.14179.

720 Creighton, J.R., Parmeter, J.E., 1993. Metal CVD for microelectronic ap-
721 plications: An examination of surface chemistry and kinetics. *Critical*
722 *Reviews in Solid State and Materials Sciences* 18, 175–237. doi:10.1080/
723 10408439308242560.

724 Dai, W., Mohammadi, S., Cremaschi, S., 2022. A hybrid modeling framework
725 using dimensional analysis for erosion predictions. *Computers & Chemical*
726 *Engineering* 156, 107577. doi:10.1016/j.compchemeng.2021.107577.

727 Dalzochio, J., Kunst, R., Pignaton, E., Binotto, A., Sanyal, S., Favilla, J.,
728 Barbosa, J., 2020. Machine learning and reasoning for predictive main-
729 tenance in Industry 4.0: Current status and challenges. *Computers in*
730 *Industry* 123, 103298. doi:10.1016/j.compind.2020.103298.

731 Deng, J., Sierla, S., Sun, J., Vyatkin, V., 2022. Reinforcement learning
732 for industrial process control: A case study in flatness control in steel
733 industry. *Computers in Industry* 143, 103748. doi:10.1016/j.compind.
734 2022.103748.

735 Deshpande, S., Lengiewicz, J., Bordas, S.P.A., 2022. Probabilistic deep learn-
736 ing for real-time large deformation simulations. *Computer Methods in Ap-*
737 *plied Mechanics and Engineering* 398, 115307. doi:10.1016/j.cma.2022.
738 115307.

739 Du, H., Jiang, Y., 2019. Backup or Reliability Improvement Strategy for
740 a Manufacturer Facing Heterogeneous Consumers in a Dynamic Supply
741 Chain. *IEEE Access* 7, 50419–50430. doi:10.1109/access.2019.2911620.

742 Endo, H., Kuwana, K., Saito, K., Qian, D., Andrews, R., Grulke, E.A., 2004.
743 CFD prediction of carbon nanotube production rate in a CVD reactor.
744 *Chemical Physics Letters* 387, 307–311. doi:10.1016/j.cplett.2004.01.
745 124.

746 Everson, R., Sirovich, L., 1995. Karhunen–Loève procedure for gappy data.
747 *J. Opt. Soc. Am. A* 12, 1657. doi:10.1364/JOSAA.12.001657.

748 Fotiadis, D.I., Boekholt, M., Jensen, K.F., Richter, W., 1990. Flow and heat
749 transfer in CVD reactors: Comparison of Raman temperature measure-
750 ments and finite element model predictions. *Journal of Crystal Growth*
751 100, 577–599. doi:10.1016/0022-0248(90)90257-L.

752 Gakis, G., Koronaki, E., Boudouvis, A., 2015. Numerical investigation of
753 multiple stationary and time-periodic flow regimes in vertical rotating disc
754 CVD reactors. *Journal of Crystal Growth* 432, 152–159. doi:10.1016/j.
755 jcrysgr.2015.09.026.

756 Galvis, L., Offermans, T., Bertinetto, C.G., Carnoli, A., Karamujić, E., Li,
757 W., Szymańska, E., Buydens, L.M.C., Jansen, J.J., 2022. Retrospective
758 quality by design r(QbD) for lactose production using historical process
759 data and design of experiments. *Computers in Industry* 141, 103696.
760 doi:10.1016/j.compind.2022.103696.

761 Gkinis, P., Aviziotis, I., Koronaki, E., Gakis, G., Boudouvis, A., 2017. The
762 effects of flow multiplicity on GaN deposition in a rotating disk CVD reac-
763 tor. *Journal of Crystal Growth* 458, 140–148. doi:10.1016/j.jcrysgro.
764 2016.10.065.

765 Gkinis, P., Koronaki, E., Skouteris, A., Aviziotis, I., Boudouvis, A., 2019.
766 Building a data-driven reduced order model of a chemical vapor deposition
767 process from low-fidelity CFD simulations. *Chemical Engineering Science*
768 199, 371–380. doi:10.1016/j.ces.2019.01.009.

769 Hastie, T., Tibshirani, R., Friedman, J., 2009a. Additive Models, Trees, and
770 Related Methods, in: Hastie, T., Tibshirani, R., Friedman, J. (Eds.), *The*
771 *Elements of Statistical Learning: Data Mining, Inference, and Prediction*.
772 Springer, New York, NY. Springer Series in Statistics, pp. 295–336. doi:10.
773 1007/978-0-387-84858-7_9.

774 Hastie, T., Tibshirani, R., Friedman, J., 2009b. Boosting and Additive
775 Trees, in: Hastie, T., Tibshirani, R., Friedman, J. (Eds.), *The*
776 *Elements of Statistical Learning: Data Mining, Inference, and Prediction*.
777 Springer, New York, NY. Springer Series in Statistics, pp. 337–387.
778 doi:10.1007/978-0-387-84858-7_10.

779 He, Z., Tran, K.P., Thomassey, S., Zeng, X., Xu, J., Yi, C., 2021. A deep
780 reinforcement learning based multi-criteria decision support system for op-
781 timizing textile chemical process. *Computers in Industry* 125, 103373.
782 doi:10.1016/j.compind.2020.103373.

783 Hochauer, D., Mitterer, C., Penoy, M., Puchner, S., Michotte, C., Martinz,

784 H., Hutter, H., Kathrein, M., 2012. Carbon doped α -Al₂O₃ coatings grown
785 by chemical vapor deposition. *Surface and Coatings Technology* 206, 4771–
786 4777. doi:10.1016/j.surfcoat.2012.03.059.

787 Hürkamp, A., Gellrich, S., Ossowski, T., Beuscher, J., Thiede, S., Herrmann,
788 C., Dröder, K., 2020. Combining Simulation and Machine Learning as
789 Digital Twin for the Manufacturing of Overmolded Thermoplastic Com-
790 posites. *JMMP* 4, 92. doi:10.3390/jmmp4030092.

791 Iqbal, R., Maniak, T., Doctor, F., Karyotis, C., 2019. Fault Detection and
792 Isolation in Industrial Processes Using Deep Learning Approaches. *IEEE*
793 *Trans. Ind. Inf.* 15, 3077–3084. doi:10.1109/tii.2019.2902274.

794 James, G., Witten, D., Hastie, T., Tibshirani, R., 2021a. Statistical
795 Learning. Springer US, New York, NY. pp. 15–57. doi:10.1007/
796 978-1-0716-1418-1_2.

797 James, G., Witten, D., Hastie, T., Tibshirani, R., 2021b. Tree-Based
798 Methods. Springer US, New York, NY. pp. 327–365. doi:10.1007/
799 978-1-0716-1418-1_8.

800 Jo, T., Koo, B., Kim, H., Lee, D., Yoon, J.Y., 2019. Effective sensor
801 placement in a steam reformer using gappy proper orthogonal decom-
802 position. *Applied Thermal Engineering* 154, 419–432. doi:10.1016/j.
803 applthermaleng.2019.03.089.

804 Kagermann, H., 2015. Change Through Digitization—Value Creation in the
805 Age of Industry 4.0, in: Albach, H., Meffert, H., Pinkwart, A., Reichwald,

806 R. (Eds.), Management of Permanent Change. Springer Fachmedien, Wies-
807 baden, pp. 23–45. doi:10.1007/978-3-658-05014-6_2.

808 Kalaboukas, K., Kiritsis, D., Arampatzis, G., 2023. Governance framework
809 for autonomous and cognitive digital twins in agile supply chains. Com-
810 puters in Industry 146, 103857. doi:10.1016/j.compind.2023.103857.

811 Kathrein, M., Schintlmeister, W., Wallgram, W., Schleinkofer, U.,
812 2003. Doped CVD Al₂O₃ coatings for high performance cutting tools.
813 Surface and Coatings Technology 163–164, 181–188. doi:10.1016/
814 s0257-8972(02)00483-8.

815 Kerfriden, P., Goury, O., Rabczuk, T., Bordas, S.P.A., 2013. A partitioned
816 model order reduction approach to rationalise computational expenses in
817 nonlinear fracture mechanics. Computer Methods in Applied Mechanics
818 and Engineering 256, 169–188. doi:10.1016/j.cma.2012.12.004.

819 Kim, A., Oh, K., Jung, J.Y., Kim, B., 2018. Imbalanced classification of man-
820 ufacturing quality conditions using cost-sensitive decision tree ensembles.
821 International Journal of Computer Integrated Manufacturing 31, 701–717.
822 doi:10.1080/0951192x.2017.1407447.

823 Kim, D., Kang, P., Cho, S., Lee, H.j., Doh, S., 2012. Machine learning-based
824 novelty detection for faulty wafer detection in semiconductor manufactur-
825 ing. Expert Systems with Applications 39, 4075–4083. doi:10.1016/j.
826 eswa.2011.09.088.

827 Kleijn, C.R., Dorsman, R., Kuijlaars, K.J., Okkerse, M., van Santen, H.,
828 2007. Multi-scale modeling of chemical vapor deposition processes for thin

829 film technology. *Journal of Crystal Growth* 303, 362–380. doi:10.1016/j.jcrysgro.2006.12.062.

830

831 Kleijn, C.R., Hoogendoorn, C.J., 1991. A study of 2- and 3-D transport
832 phenomena in horizontal chemical vapor deposition reactors. *Chemical
833 Engineering Science* 46, 321–334. doi:10.1016/0009-2509(91)80141-K.

834 Koronaki, E., Gkinis, P., Beex, L., Bordas, S., Theodoropoulos, C., Boudou-
835 vis, A., 2019. Classification of states and model order reduction of
836 large scale Chemical Vapor Deposition processes with solution multiplicity.
837 *Computers & Chemical Engineering* 121, 148–157. doi:10.1016/j.compchemeng.2018.08.023.

838

839 Koronaki, E.D., Evangelou, N., Psarellis, Y.M., Boudouvis, A.G., Kevrekidis,
840 I.G., 2023. From partial data to out-of-sample parameter and observation
841 estimation with Diffusion Maps and Geometric Harmonics. doi:10.48550/
842 arXiv.2301.11728, arXiv:arXiv:2301.11728.

843 Koronaki, E.D., Gakis, G.P., Cheimarios, N., Boudouvis, A.G., 2016. Ef-
844 ficient tracing and stability analysis of multiple stationary and periodic
845 states with exploitation of commercial CFD software. *Chemical Engineer-
846 ing Science* 150, 26–34. doi:10.1016/j.ces.2016.04.043.

847 Łepicka, M., Grądzka-Dahlke, M., 2019. The initial evaluation of perfor-
848 mance of hard anti-wear coatings deposited on metallic substrates: Thick-
849 ness, mechanical properties and adhesion measurements – a brief re-
850 view. *REVIEWS ON ADVANCED MATERIALS SCIENCE* 58, 50–65.
851 doi:10.1515/ram-s-2019-0003.

852 Liu, S.S., Xiao, W.D., 2015. CFD–PBM coupled simulation of silicon CVD
853 growth in a fluidized bed reactor: Effect of silane pyrolysis kinetic models.
854 Chemical Engineering Science 127, 84–94. doi:10.1016/j.ces.2015.01.
855 026.

856 Ma, Y., Zhu, W., Benton, M.G., Romagnoli, J., 2019. Continuous control
857 of a polymerization system with deep reinforcement learning. Journal of
858 Process Control 75, 40–47. doi:10.1016/j.jprocont.2018.11.004.

859 Malley, S., Reina, C., Nacy, S., Gilles, J., Koohbor, B., Youssef, G., 2022. Pre-
860 dictability of mechanical behavior of additively manufactured particulate
861 composites using machine learning and data-driven approaches. Comput-
862 ers in Industry 142, 103739. doi:10.1016/j.compind.2022.103739.

863 McKinney, W., 2010. Data Structures for Statistical Computing in Python,
864 in: Python in Science Conference, Austin, Texas. pp. 56–61. doi:10.25080/
865 Majora-92bf1922-00a.

866 Ozaydin-Ince, G., Coclite, A.M., Gleason, K.K., 2011. CVD of polymeric thin
867 films: Applications in sensors, biotechnology, microelectronics/organic
868 electronics, microfluidics, MEMS, composites and membranes. Rep. Prog.
869 Phys. 75, 016501. doi:10.1088/0034-4885/75/1/016501.

870 Papavasileiou, P., Koronaki, E.D., Pozzetti, G., Kathrein, M., Czettl, C.,
871 Boudouvis, A.G., Mountziaris, T.J., Bordas, S.P.A., 2022. An efficient
872 chemistry-enhanced CFD model for the investigation of the rate-limiting
873 mechanisms in industrial Chemical Vapor Deposition reactors. Chemical

874 Engineering Research and Design 186, 314–325. doi:10.1016/j.cherd.
875 2022.08.005.

876 Perno, M., Hvam, L., Haug, A., 2022. Implementation of digital twins in
877 the process industry: A systematic literature review of enablers and barri-
878 ers. Computers in Industry 134, 103558. doi:10.1016/j.compind.2021.
879 103558.

880 Potdar, K., S., T., D., C., 2017. A Comparative Study of Categorical Vari-
881 able Encoding Techniques for Neural Network Classifiers. IJCA 175, 7–9.
882 doi:10.5120/ijca2017915495.

883 Priore, P., Ponte, B., Puente, J., Gómez, A., 2018. Learning-based scheduling
884 of flexible manufacturing systems using ensemble methods. Computers &
885 Industrial Engineering 126, 282–291. doi:10.1016/j.cie.2018.09.034.

886 Psarellis, G.M., Aviziotis, I.G., Duguet, T., Vahlas, C., Koronaki, E.D.,
887 Boudouvis, A.G., 2018. Investigation of reaction mechanisms in the chem-
888 ical vapor deposition of al from DMEAA. Chemical Engineering Science
889 177, 464–470. doi:10.1016/j.ces.2017.12.006.

890 Raissi, M., Perdikaris, P., Karniadakis, G., 2019. Physics-informed neural
891 networks: A deep learning framework for solving forward and inverse prob-
892 lems involving nonlinear partial differential equations. Journal of Compu-
893 tational Physics 378, 686–707. doi:10.1016/j.jcp.2018.10.045.

894 Rasheed, A., San, O., Kvamsdal, T., 2020. Digital Twin: Values, Challenges
895 and Enablers From a Modeling Perspective. IEEE Access 8, 21980–22012.
896 doi:10.1109/access.2020.2970143.

897 Saxena, A., Saad, A., 2007. Evolving an artificial neural network classifier
898 for condition monitoring of rotating mechanical systems. *Applied Soft*
899 *Computing* 7, 441–454. doi:10.1016/j.asoc.2005.10.001.

900 Schierling, M., Zimmermann, E., Neuschütz, D., 1999. Deposition kinetics of
901 Al_2O_3 from $\text{AlCl}_3\text{-CO}_2\text{-H}_2\text{-HCl}$ gas mixtures by thermal CVD in a
902 hot-wall reactor. *J. Phys. IV France* 09, Pr8–85–Pr8–91. doi:10.1051/jp4:
903 1999811.

904 Spencer, R., Gkinis, P., Koronaki, E., Gerogiorgis, D., Bordas, S., Boudouvis,
905 A., 2021. Investigation of the chemical vapor deposition of Cu from copper
906 amidinate through data driven efficient CFD modelling. *Computers &*
907 *Chemical Engineering* 149, 107289. doi:10.1016/j.compchemeng.2021.
908 107289.

909 Susto, G.A., Schirru, A., Pampuri, S., McLoone, S., Beghi, A., 2015. Machine
910 Learning for Predictive Maintenance: A Multiple Classifier Approach.
911 *IEEE Trans. Ind. Inf.* 11, 812–820. doi:10.1109/tii.2014.2349359.

912 Topka, K.C., Vergnes, H., Tsirros, T., Papavasileiou, P., Decosterd, L., Di-
913 allo, B., Senocq, F., Samelor, D., Pellerin, N., Menu, M.J., Vahlas, C.,
914 Caussat, B., 2022. An innovative kinetic model allowing insight in the
915 moderate temperature chemical vapor deposition of silicon oxynitride films
916 from tris(dimethylsilyl)amine. *Chemical Engineering Journal* 431, 133350.
917 doi:10.1016/j.cej.2021.133350.

918 Tulsyan, A., Garvin, C., Ündey, C., 2018. Advances in industrial bio-
919 pharmaceutical batch process monitoring: Machine-learning methods for

920 small data problems. *Biotechnology and Bioengineering* 115, 1915–1924.
921 doi:10.1002/bit.26605.

922 Urcun, S., Rohan, P.Y., Skalli, W., Nassoy, P., Bordas, S.P.A., Sciumè, G.,
923 2021. Digital twinning of Cellular Capsule Technology: Emerging outcomes
924 from the perspective of porous media mechanics. *PLOS ONE* 16, e0254512.
925 doi:10.1371/journal.pone.0254512.

926 Wang, R., Cheung, C.F., Wang, C., Cheng, M.N., 2022. Deep learning char-
927 acterization of surface defects in the selective laser melting process. *Com-
928 puters in Industry* 140, 103662. doi:10.1016/j.compind.2022.103662.

929 Willcox, K., 2006. Unsteady flow sensing and estimation via the gappy proper
930 orthogonal decomposition. *Computers & Fluids* 35, 208–226. doi:10.1016/
931 j.compfluid.2004.11.006.

932 Wu, H., Yu, Z., Wang, Y., 2019. Experimental study of the process failure
933 diagnosis in additive manufacturing based on acoustic emission. *Measure-
934 ment* 136, 445–453. doi:10.1016/j.measurement.2018.12.067.