Practical weight-constrained conditioned portfolio optimisation using risk aversion indicator signals

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Outline

1. Context
   - Portfolio optimisation with conditioning information

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2. Empirical study
   - Description
   - Results

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3. Extras

Problem context

- Discrete-time optimisation
- Minimise portfolio variance for a given expected portfolio mean
- Postulate that there exists some relationship $\mu(s)$ between a signal s and each asset return r observed at the end of the investment interval:
$$r_t = \mu(s_{t-1}) + \epsilon_t,$$
with $E[\epsilon_t | s_{t-1}] = 0$.
- How do we optimally use this information in an otherwise classical (unconditional mean / unconditional variance) portfolio optimisation process?

Problem history

- Hansen and Richard (1983): functional analysis argument suggesting that unconditional moments should enter the optimisation even when conditioning information is known
- Ferson and Siegel (2001): closed-form solution of unconstrained mean-variance problem using unconditional moments
- Chiang (2008): closed-form solutions to the benchmark tracking variant of the Ferson-Siegel problem
- Basu et al. (2006), Luo et al. (2008): empirical studies covering conditioned optima of portfolios of trading strategies

Possible signals

Taken from a continuous scale ranging from purely macroeconomic indices to investor sentiment indicators. Indicators taking into account investor attitude may be based on some model or calculated in an ad-hoc fashion. Examples include

- short-term treasury bill rates (Fama and Schwert 1977);
- CBOE Market Volatility Index (VIX) (Whaley 1993) or its European equivalents (VDAX etc.);
- risk aversion indices using averaging and normalisation (UBS Investor Sentiment Index 2003) or PCA reduction (Coudert and Gex 2007) of several macroeconomic indicators;

Possible signals (2)

- global risk aversion indices (GRAI) (Kumar and Persaud 2004) based on a measure of rank correlation between current returns and previous risks;
- option-based risk aversion indices (Tarashev et al. 2003);
- sentiment indicators directly obtained from surveys (e.g. University of Michigan Consumer Sentiment Index)

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Aim

- Carry out backtests executing constrained-weight conditioned optimisation strategies with different settings

Data set

- 11 years of daily data, from January 1999 to February 2010 (2891 samples)
- Risky assets: 10 different EUR-based funds commercialised in Luxembourg chosen across asset categories (equity, fixed income) and across Morningstar style criteria
- Risk-free proxy: EURIBOR with 1 week tenor
- Signals: VDAX, volatility of bond index, PCA-based indices built using both 2 and 4 factors and estimation window sizes of 50, 100 and 200 points, Kumar and Persaud currency-based GRAI obtained using 1 month and 3 month forward rates

Individual backtest

- Rebalance Markowitz-optimal portfolio alongside conditioned optimal portfolio, both with and without the availability of a risk-free proxy asset, over the 11-year period
- Assume lagged relationship $\mu(s)$ between signal and return can be represented by a linear regression
- Use kernel density estimates for signal densities
- Estimate the above using a given rolling window size (15 to 120 points)
- Use direct collocation method for numerical problem solutions

Individual backtest(2)

- Obtain efficient frontier for every date and choose portfolio based on quadratic utility functions with risk aversion coefficients between 0 and 10
- Compare Sharpe ratios (ex ante), additive observed returns (ex post), observed standard deviations (ex post), maximum drawdowns / drawdown durations (MD/MDD) of both strategies

Set of backtests

- Decide on one baseline case - VDAX index, 60 point estimation window, weights constrained to allow for long investments only
- Vary these parameters to check both for robustness of strategy results and whether results can be further improved while staying with a linear regression model for the relationship between signal and returns

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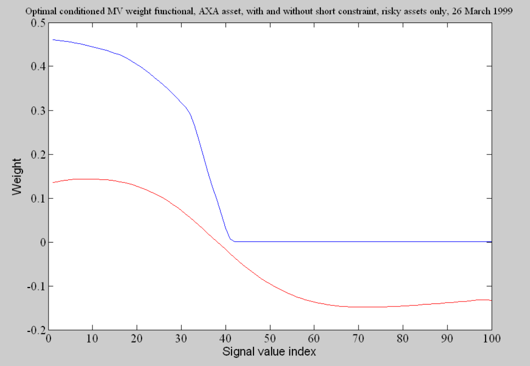
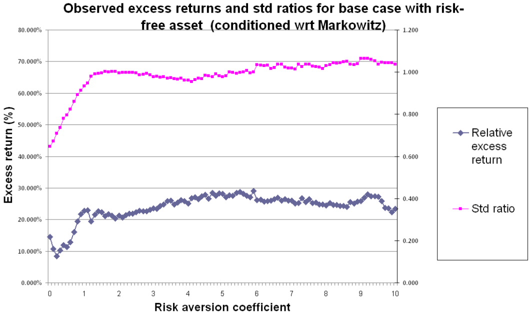
Typical optimal weight functionals with and without weights constraints



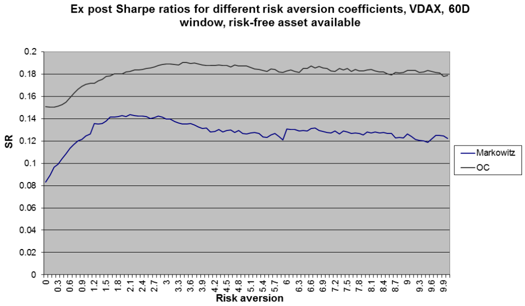
- Constrained optimal weights are not simply a truncated version of the unconstrained optimal (Ferson-Siegel) weights

Base case (with risk free asset): ex post observed relative excess additive returns, standard deviation ratios



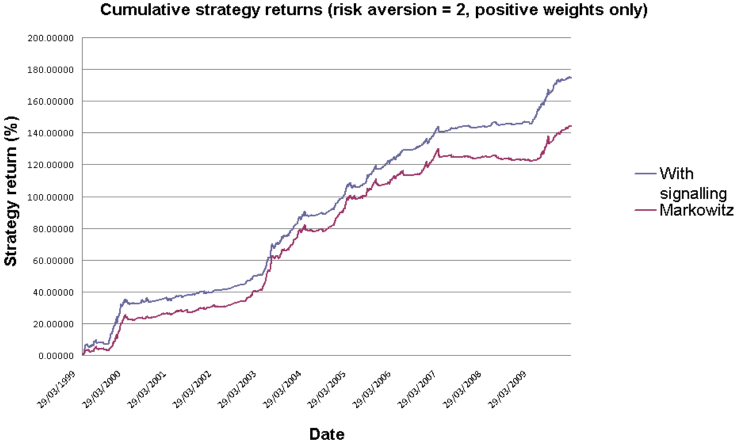
- General observation: stable outperformance by conditioned strategy with (at worst) a slight increase in risk

Base case (with risk free asset): ex post observed Sharpe ratio relative improvements

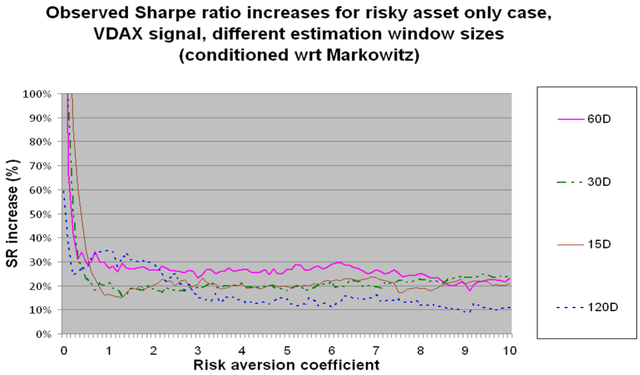


- (Ex ante) average Sharpe ratios: Markowitz - 0.391, using signal - 0.529 (using business daily returns and volatilities)
- Observation on previous lside confirmed: advantageous trade-off remains ex post and for any level of risk aversion

Base case (with risk free asset): Time path of additive strategy returns for $\lambda = 2$

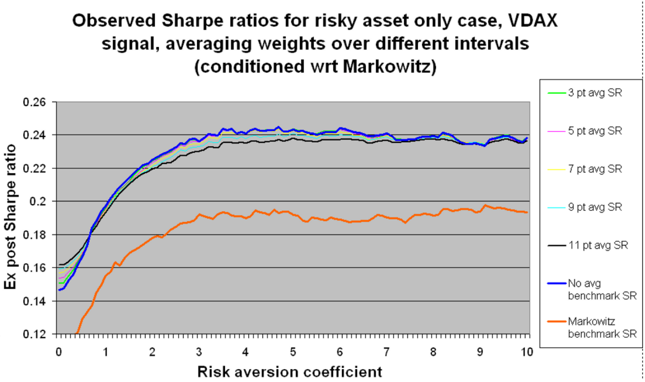


Ex post results for different estimation window sizes



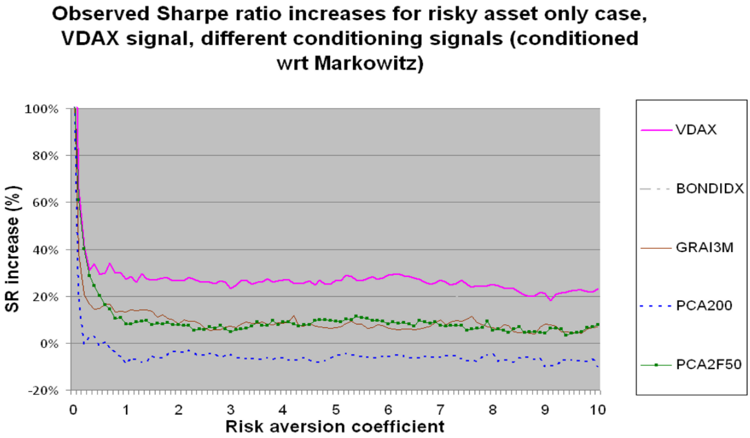
- Excess returns (and standard deviations) larger as window sizes increase
- Trade-off between statistical quality of estimates and impact of conditional nonstationarities

Ex post results for weight averages over different numbers of signal points



- Negligible changes in excess returns, slight chages in standard deviations: little risk attached to signal observations

Ex post results for different signals



- Best results seen for baseline VDAX signal, averaging seems to detract from signal power

Robustness check using least risky (money market) asset weight for risk neutral investor

Average money market asset weights observed for the case with no risk-free asset:

	Markowitz	Conditioned	Difference
All data	0.1295	0.2661	0.1366
01/06/07 to 01/03/09	0.3872	0.6334	0.2462

The conditioned strategy generally achieves the required ex ante unconditional return using a smaller risky investment, and manages this particularly well during the sample crisis period

Summary

- Backtesting using a number of different settings shows robust outperformance of the Markowitz strategy given a useful signal is exploited
- In any case,the present strategy shares the characteristics of the Markowitz approach and, as such, the consistently observed improvements reported make it interesting to practitioners investing within the mean-variance framework
- The gap between ex ante and (less good) ex post results as well as the results for lagged signals suggest that use of a signal-return relationship model that captures both autocorrelation and heteroscedasticity is likely to lead to further improvements

Conditioned optimisation use of unconditional moments

This choice of moments is standard and can be justified in three different ways:

- Hansen and Richard (1983): the unconditional conditioned efficient frontier contains the corresponding conditional conditioned efficient frontier as a proper subset in general
- Investment managers applying conditioned portfolio optimisation will be evaluated by agents who take an unconditioned (Markowitz) view on performance (Ferson and Siegel (2001))
- Empirical studies (Chiang (2008)) provide statistical evidence that the use of conditional moments in conditioned optimisation underperforms with respect to the use of unconditional moments

Expressions for unconditional expected return and variance given conditioning information

These are obtained as expectation integrals over the signal domain. If a risk-free asset with return r_f is available, they are

$$E(P) = E\Big[u'(s)(\mu(s) - r_f e)\Big] = E\Big[I_1(u,s)\Big]$$

$$\sigma^2(P) = E\Big[u'(s)\Big[(\mu(s) - r_f e)(\mu(s) - r_f e)' + \Sigma_\epsilon^2\Big]u(s)\Big] - \mu_P^2$$

$$= E\Big[I_2(u,s)\Big] - \mu_P^2,$$

for an expected unconditional return of μ_P.

Optimal control formulation

$$\begin{array}{ll} \text{Minimise} & J_{[s^-,s^+]}(x,u) = \int_{s^-}^{s^+} l_2(u,s)p_s(s)ds \text{ as } s^- \to -\infty, s^+ \to \infty \\ \text{subject to} & \dot{x}(s) = l_1(u,s)p_s(s)\ \forall s \in (s^-, s^+) \\ & \lim_{s \to -\infty} x(s) = x_-, (\lim_{s \to \infty} x(s) = x_+), \\ \text{and} & u(s) \in U\ \forall s \in (s^-, s^+) \end{array}$$

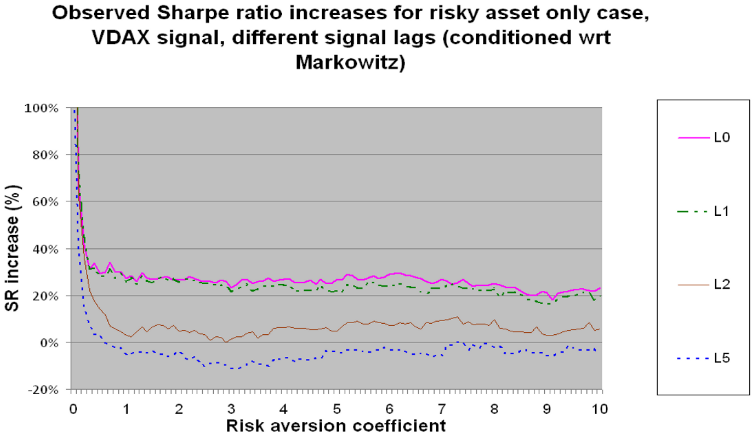
where $U \subseteq \mathbb{R}^n$, $x(s) \in \mathbb{R}^m$ and l_1 as well as l_2 are continuous and differentiable in both u and s.

Necessity and sufficiency results generalised

- The Pontryagin Minimum Principle (PMP) and Mangasarian sufficiency theorem are shown to continue holding if the control problem domain corresponds to the full real axis: the corresponding optimal control problems are well-posed.
- The PMP is then used to show that the given optimal control formulation of the conditioned mean-variance problem generalises classical (Ferson and Siegel; Markowitz) problem expressions.

The optimal control formulation is thus well-posed and the more practically relevant (e.g. constrained weight) variants of the conditioned problem are now open for (numerical) solution.

Ex post results for different signal lags



- Signal adds value for at least two additional lags