Mark S. Zschocke – 519 888 4567 x33369 – mszschoc@uwaterloo.ca
Benny Mantin – 519 888 4567 x32235 – bmantin@uwaterloo.ca
Beth Jewkes – 519 888 4567 x33279 – emjewkes@uwaterloo.ca

Address of all three authors:
University of Waterloo
Department of Management Sciences
200 University Avenue West
Waterloo, Ontario, Canada
N2L 3G1

THE EFFECT OF COMPETITION ON R&D PORTFOLIO INVESTMENTS

ABSTRACT

Although project portfolio management has been an active research area over the past 50 years, budget allocation models that consider competition are sparse. Faced with the competition, firms contemplating budget allocation for their project portfolio cannot limit their attention to the returns from their projects' target markets, as is the case for monopoly firms, but must also anticipate the competitive effects on these returns. Assuming firms allocate their budgets between projects offering incremental innovation targeting a mature market and projects offering radical innovation targeting an emerging market, we show that while the monopoly firm bases its budget allocation decision solely on the marginal returns of the markets, competing firms—as they take into account their counterparts' investment decisions—need to also consider the projects' average returns from their respective markets. This drives competing firms into incrementalism: faced with competition, firms invest larger portions of their budgets into projects targeting mature markets. This effect is amplified as the number of competing firms increases and firms allocate an even greater share of their budget into projects targeting a mature market. We further demonstrate the effects that changes to firm's individual budgets as well as to market characteristics have on firms' budget allocation decision.

Keywords: Project Portfolio Management, Resource Allocation, R&D Investments, Game Theory, Competition

Original manuscript submitted: June 2010; 1st revision submitted: February 2011; 2nd revision submitted: October 2011; 3rd revision submitted: January 2012; Paper accepted: March 2012

THE EFFECT OF COMPETITION ON R&D PORTFOLIO INVESTMENTS

1. INTRODUCTION

Most firms regularly face project portfolio management decisions – how to allocate a limited budget and other resources among a range of projects. It has been recognized that any type of formal project portfolio management process is better than ad-hoc decision-making (Cooper et al. 2004) and a variety of approaches for supporting the project portfolio management decision have been proposed. Though tools that support the monopoly project portfolio management decision have been developed (see Cooper et al. 2001 for a review), a fundamental complexity of this decision that has largely been ignored is the effect of competition.

In this paper, we consider a stylized model of firms' project portfolio decisions when their portfolio consists of two types of projects: exploitative and explorative projects. Exploitative projects rely on existing market opportunities and offer only incremental innovation for a mature market, whereas explorative projects offer radical changes that aim to step into new opportunities in emerging markets. The former market provides considerable returns for small investments (such as adding minor features to an existing product), but relatively poor returns for large investments as the market quickly becomes saturated. On the other hand, the latter market requires substantial investment in order to achieve high returns (such as a new product accompanied by a comprehensive marketing campaign). When deciding how to distribute its budget over these projects, the firm needs to consider the specific market parameters as well as the size of its budget. An established result for such a project portfolio management decision faced by a monopoly is to invest in the two markets to the level that both offer identical marginal returns (Loch and Kavadias 2002). Is this still the optimal decision under competition? Will competition drive firms to jointly develop the emerging market more aggressively or to defend the mature market? How does the size of firms' budgets influence this decision? We aim to answer these challenging questions in this paper.

Firms that ignored or misjudged the effect of competition in their project selections have paid a steep price. For example, DuPont focused too much of its estimated $2B annual budget on projects aimed at improving existing lines of businesses, without considering the opportunity it was providing for competitors that focused more on innovative projects (BusinessWeek 2003). Other firms made project portfolio management decisions by paying careful attention to their competitors' actions. For example, by the end of the 1990s PepsiCo carefully considered Coca Cola Company's investment decisions in its search for new markets, thereby increasing PepsiCo's international revenue and returns dramatically (Yoffie 2004). Another example is the Niagara Wine Region of Canada where wineries have benefited from other wineries offering competing wine tasting services. Although this has increased competition in the region, the number of wineries in a region is a key driver of increasing tourism, thereby expanding the overall market (Getz and Brown 2006). It is evident, then, that optimal management of project portfolios needs to account for competitive effects.

We find that in contrast to a monopoly, which bases its decision solely on the marginal returns of the two markets, duopoly firms consider the marginal returns as well as the average returns from these markets. This is an important result, as it dramatically alters firms' investment decisions. The inclusion of the average return into the decision making process, induces firms who are faced with competition to invest larger proportions of their budget into exploitative projects (which target the mature market) compared to a monopoly firm. Put differently, competition drives firms into incrementalism, as they direct more funds into the mature markets rather than into developing the emerging markets. This is an undesirable outcome, as the difference in investment strategy implies that duopoly firms receive lower total returns compared to a monopoly. The intuition is simple: firms get engaged in a seemingly unnecessary competition over their share of the mature market, resulting in overinvestment into this market and underinvestment into the emerging market, and eventually leading to lower combined returns. This focus on incrementalism can also lead to underperformance in the long run (Chao et al. 2009).

Firms' focus on incrementalism is amplified as the number of competitors increases. Specifically, extending our duopoly model into an oligopoly (assuming identical budgets, fixed total budget), we find that as the number of competing firms increases, firms invest an even larger portion of their budget into the project targeting the mature market. That is, increementalism is a direct outcome of competition and it is exacerbated by the number of firms in the market.

We further demonstrate the effect of firms' budgets size on their investment allocation decision. For example, we show that as a firm's budget increases, both in a monopoly and a duopoly, the proportion of the budget invested into the mature market is non-increasing. Endowed with a (sufficiently) small budget, a firm will invest fully into the mature market as this market guarantees large returns for small investments. However, as the firm's budget increases, it has greater incentive to shift investment into the emerging market and to gain from the large return available with large investments. We also demonstrate that firms are very sensitive to the size of their competitor's budget if it is small—a small increase in the competitor's budget can result in a large shift of the share of investment from the emerging into mature market. This is a strong defense reaction aimed at protecting the average return from the mature market. However, if their competitor's budget is sufficiently high, changes to their competitor's budget affects firms' resource allocation decision only marginally. We also illustrate that firms may continue to invest significant resources into the emerging market even if returns from this market become highly uncertain. In addition, we show how the effect of changes to the rate with which markets become saturated depends on the firms' budget sizes.

2. Relevant Literature

There is extensive literature on project portfolio management including the development of both qualitative and quantitative tools and methods that can help decision makers faced with the complexities of the project portfolio management problem. Qualitative approaches such as the balanced scorecard (Kaplan and Norton 1992), bubble diagrams (Blau et al. 2004), and the strategic buckets method (Cooper et al. 2001) aim to combine financial data with qualitative aspects for a more complete characterization of individual projects. By contrast, the quantitative research stream aims to capture all key complexities mathematically, for example by considering the risk of individual projects (Graves and Ringuest 1991), the sequencing of projects (Kavadias and Loch 2003), or substitution and cannibalization effects between projects (Chen et al. 2008). However, sophisticated mathematical models rely heavily on financial data (Cooper et al. 2001) and are often not robust (Gupta and Mandakovic 1992). The lack of transparency and the complexity of these models make decision-makers hesitant to fully trust the model recommendations, which has led to low adoption rate of these methods (Cooper et al. 2004). Indeed, the need for attention to project portfolio and resource allocation decisions has been highlighted by Krishnan and Loch (2005) in a retrospective look at *Production and Operations Management* articles on new product development.

In the research noted so far, competitive forces are only captured implicitly or in passing. Yet the need for such consideration has been recognized broadly (Bower and Gilbert 2005; Hauser et al. 2006). To date, the field of quantitative tools and methods for project portfolio management which consider competition remains understudied – notable exceptions follow below.

Gibson and Ohlmann (2009) considered competitive actions within a multidimensional knapsack problem (MKP) framework. In their model, multiple decision-makers make sequential decisions on how to allocate resources over indivisible objects; once a decision-maker chooses a particular object (or project), this object is no longer available. Such a framework is applicable to a limited number of scenarios such as a sports draft. Zhu and Weyant (2003), who explicitly modeled competitive actions in a real options framework, focused on the timing of the decision to pursue a project and the quantity to produce. Their model only considers a single project within a single market, while the comprehensive project portfolio management problem deals with the allocation of resources between multiple projects that may target different markets. Chao et al. (2009) developed a principal-agent model where a CEO (the principal) oversees managers (the agents) who allocate resources between projects of relatively incremental innovation and projects of more radical innovation. However, they do not consider external competition.

The competitive budget allocation problem is also studied in the field of R&D races. Ali et al. (1993) developed a model in which two competing firms, each of whom has a portfolio of two projects to choose from, decide in which project to invest while recognizing that their competitor's allocation decision will have an impact on their returns. However, in their model, both projects target the same market while our model considers investments into two separate markets. Furthermore, we allow firms to distribute their budget over both markets, rather than fully commit to one of the markets. Gerchak and Parlar (1999) also focused on R&D races and developed a model that considers more than two projects. In their case, firms allocate their budget over the range of available projects in a continuous manner. They assume a "winner takes all" framework where investments into a project increase the likelihood of securing a market, thereby excluding the competition from that market. Such a framework is applicable for instance such as patent races (e.g. between pharmaceutical companies), where the first firm to secure the patent secures the full market. Finally, Selove (2010) proposed a dynamic investment model in which duopoly firms compete in two market segments and decide in which segment to invest. Selove assumed that market returns are increasing and that due to small random fluctuations, each firm will initially achieve a higher return in different markets. His framework offers an explanation for why firms focus on different markets and continue to invest in markets where they have already established their presence. As described by Selove, there are markets that provide increasing returns due to, for example, reputation effects or learning curves; however, many markets provide decreasing returns and this is the focus of our work.

Although some of the aforementioned studies have addressed aspects of the project portfolio management decision under competition, models that consider the project portfolio management decision under competition with multiple projects that target separate markets are sparse. The research to date has analyzed the single market scenario or focused on the special case of the winner takes all, as it occurs in patent or R&D races. In Section 3, we introduce a stylized model for a resource allocation problem faced by a monopoly, followed by a resource allocation model for a duopoly setting in Section 4.

3. THE MONOPOLY BENCHMARK

Consider a monopoly faced with two project investment opportunities: one targeting a mature market and another targeting an emerging market. Endowed with an investment budget of B_M, all of which is to be invested, the monopoly needs to decide what share $r_M \in [0,1]$ of its budget to allocate to the project targeting the mature market, where the remaining $1 - r_M$ of the budget is invested in the project targeting the emerging market. Firms generally have limited budgets to some extent, but our assumption of a given budget is particularly applicable in R&D settings where firms have a fixed budget to invest over a particular time horizon. To add further insight, we later explore how firms' investment decisions would change if their budget was increased or decreased. Assuming that project investments can be scaled according to the available budget (Loch and Kavadias 2010), our focus is on how a monopoly allocates a given budget between the two projects.

The mature and emerging markets offer returns for an investment amount x according to functions $f(x)=ax^{\alpha}$ and $g(x)=bx^{\beta}$, respectively, which follow the Inada conditions (Inada 1963): (i) zero investment into a market results in zero returns from that market, $f(0), g(0)=0$; (ii) increasing investment into a market always results in higher total returns, $f', g'>0$; (iii) the marginal returns are decreasing, $f'', g''<0$; and (iv) the functions are continuously differentiable and the limit of the derivative towards zero is positive infinity and the limit towards positive infinity is zero.

The parameters a and b define the market potential of the mature and emerging markets, respectively: the greater the market potential, the greater the returns from that market at any investment level. We characterize the emerging market as the market with the greater market potential, $b > a$, however, the emerging market is also more risky than the well-understood mature market. To capture this uncertainty, we let p, $p \in (0,1)$, denote the probability that the emerging market has the expected return function $g(\cdot)$ and assume that with probability $1-p$ the emerging market provides no returns.

The parameters α and β, $\alpha, \beta \in (0,1)$, represent the degree of homogeneity of the mature and emerging markets' return functions, respectively. In the context of this resource allocation problem, we use these parameters to define the marginal productivity of R&D investment of the respective markets. A low marginal productivity implies that the market provides significant returns for small investments but then experiences diminishing returns quickly. By contrast, markets with high marginal productivity may initially provide lower returns but continue to provide significant returns for large investments. We assume that the emerging market has a higher marginal productivity than the mature market, $\alpha < \beta$, since the mature market provides less opportunities for significant product or service improvements or new product developments.

As depicted in Figure 1 (for $\alpha = 0.1$, $\beta = 0.5$, $a = 1$, $b = 2$, $p = 0.6$), these assumptions generally imply that the mature market provides larger returns than the emerging market for small investments. For example, a small product change to an existing product may provide some quick returns in a mature market in which the firm has already established a customer base, whereas this product change would not see large returns in an emerging market without significant investment into marketing of the new product. By contrast, significant investment into a radically new product may lead to disappointing returns in a mature market, whereas such a new product may help develop the emerging market and lead to very high returns. Although a large investment into the emerging market can lead to the highest possible return for a firm, investment into an emerging market is not without risk. We base our descriptions of projects of incremental versus radical innovations on previous work with similar frameworks (see Figure 2 in Chao and Kavadias 2008), but we are not attempting to fully capture the characteristics of a mature versus an emerging market. Instead, this terminology is intended to facilitate the discussion.

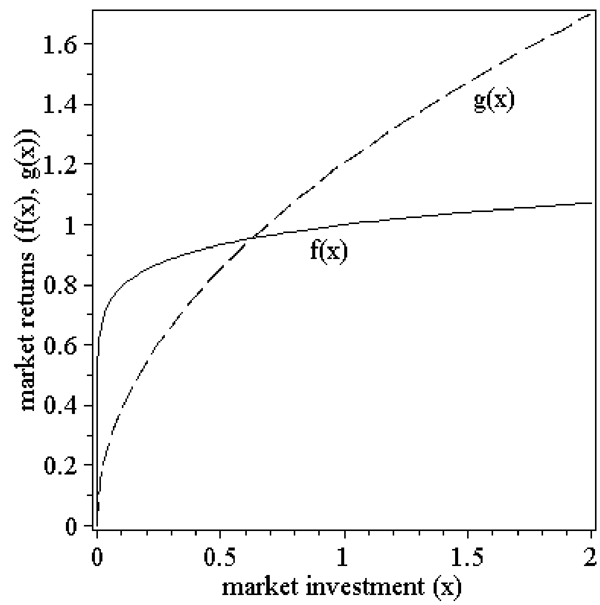


Figure 1: Market returns of mature and emerging markets;
$\alpha = 0.1, \beta = 0.5, a = 1, b = 2, p = 0.6$

The risk neutral monopoly aims to maximize its total expected return $E[\pi_M] = a(r_M B_M)^{\alpha} + pb((1-r_M)B_M)^{\beta}$. Under the Inada conditions, the optimal budget allocation occurs when the marginal returns from all markets coincide (Loch and Kavadias 2002). Specifically, the optimal allocation, r_M^*, is the value of r_M that solves the first order condition of the monopoly's expected return, $E[\pi_M]$:

$$\alpha a(r_M B_M)^{\alpha-1} = p\beta b((1-r_M)B_M)^{\beta-1} . \quad (1)$$

Clearly, the optimal budget allocation depends on the particular market parameters of $f(\cdot)$ and $g(\cdot)$. However, it is not obvious how the size of the budget influences the share of the budget being invested into the respective markets. We have the following result.

Proposition 1*: The monopoly's optimal budget allocation into the mature market is decreasing in its budget, i.e.,* $\frac{\partial r_M^*}{\partial B_M} < 0$.

All proofs are provided in the appendix.

Proposition 1 states that if the monopoly increases (decreases) its budget, it allocates a greater (smaller) share of its budget to the project targeting the emerging market. Indeed, larger investment funds make the emerging market more lucrative compared to the mature market in which large investments experience diminishing returns. However, increasing investment into the emerging market also increases risk exposure since the returns from the emerging market are uncertain. The following proposition characterizes how the market parameters affect the budget allocation decision.

Proposition 2*: If* $f(x) = ax^{\alpha}$ *and* $g(x) = \begin{cases} bx^{\beta} & w.p. \quad p \\ 0 & w.p. \quad 1-p \end{cases}$*, the monopoly's optimal budget allocation* r_M^* *is such that if* $\frac{\alpha a}{p\beta b} = B_M^{\beta-\alpha}$ *(>, <), then* $r_M^* = \frac{1}{2}$ *(>, <, respectively).*

Proposition 2 implies that the market potential parameters a and b make the respective markets more attractive independent of the other market characteristics and the available investment budget. Similarly, the greater the probability that the emerging market will reach its market potential, the more attractive this market becomes. We discuss the effect of market uncertainty in more detail in Section 4.2. The marginal productivity of markets, α and β, affect the rate at which the markets offer returns and their impact on the allocation decision thus depends on the budget available to the monopoly. For larger budgets (smaller budgets), either increasing (decreasing) β or decreasing (increasing) α, makes the emerging market more (less) attractive. Intuitively, an emerging market initially requires more investment than a mature market to achieve a certain return; only for larger investments does the emerging market outperform the mature market. Therefore, if the budget is smaller than a particular threshold, then the mature market is more attractive, and once the budget exceeds this threshold, the emerging market becomes more attractive.

So far, we have focused on the allocation decision of a monopoly wherein the firm needs to account for the parameters of the market return functions and the available investment budget. However, in many instances firm are competing with other firms and project returns further depend on the actions of their competitors. The next section studies the resource allocation problem in a duopoly setting and highlights the effect of competition.

4. Effect of Competition – The Duopoly Case

In this section we study a duopoly setting where both firms are considering investments into projects targeting either the mature or the emerging market. Each Firm n, $n \in \{1,2\}$, decides what share, r_n, of its budget, B_n, to invest into the mature market. As in the monopoly case, we assume that duopoly firms fully invest their budgets. We assume that the products or services of each firm that are targeting a particular market are considered perfect substitutes in that market, i.e., the total market returns depend on the total investment of both firms and follow the previously defined return function $f(\cdot)$ and $g(\cdot)$. The returns achieved by an individual firm from a particular market are proportional to its investment into that market compared to that of its competitor (conceptually, this is similar to Parlar and Weng (2006) where firms' market shares are proportional to the prices they set). At any given investment level, additional investment (including competitive investment) into either market always reduces the average returns obtained from that market because both markets are associated with diminishing returns. Given the mature market's low rate of marginal productivity, diminishing returns are particularly significant in that market. The proportional allocation of returns implies that firms trade off defending their returns in markets where their competitor is investing heavily versus opportunistically taking market share in markets where their competitor is not investing heavily. Formally, the expected return of Firm n is:

$$E[\pi_n] = \frac{r_n B_n}{r_n B_n + r_{-n} B_{-n}} a\left(r_n B_n + r_{-n} B_{-n}\right)^{\alpha} + \frac{(1-r_n) B_n}{(1-r_n) B_n + (1-r_{-n}) B_{-n}} pb\left((1-r_n) B_n + (1-r_{-n}) B_{-n}\right)^{\beta}, \quad (2)$$

where r_{-n} denotes the share of the budget that the other firm invests into the mature market[1]. To obtain the optimal investment decision, we set the first order condition of (2) with respect to r_n to zero:

$$\frac{a\left(r_n B_n + r_{-n} B_{-n}\right)^{\alpha-2}}{pb\left(B_n(1-r_n) + B_{-n}(1-r_{-n})\right)^{\beta-2}} = \frac{\left((1-r_{-n}) B_n B_{-n} + (1-r_n)\beta (B_n)^2\right)}{\left(r_{-n} B_n B_{-n} + r_n \alpha (B_n)^2\right)}. \quad (3)$$

While (3) cannot be solved for r_n in the general case, we can now compare the allocation decision of the monopoly with that of the duopoly firms. Given the functional form of $f(\cdot)$ and $g(\cdot)$, we have the following result.

Theorem 1*: If* $f(x) = ax^{\alpha}$ *and* $g(x) = \begin{cases} bx^{\beta} & w.p. \quad p \\ 0 & w.p. \quad 1-p \end{cases}$ *with* $\alpha < \beta$*, and*

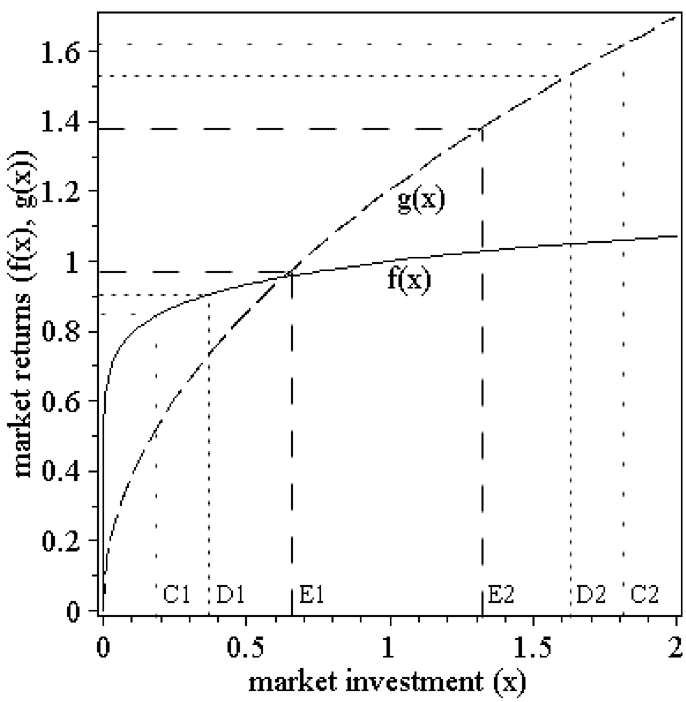
$B_n \le B_{M,}$*, then* $r_n^* > r_M^*$, $n \in \{1,2\}$.

This theorem says that competition alters budget allocation decisions in a clear direction. Under competition, firms overinvest in project offering incremental innovation by targeting the mature market. That is, duopoly firms shift a greater share of their budget from the emerging market to the mature market, which offers significant returns for low levels of investment, but with quickly diminishing returns. Given that a monopoly optimizes its resource allocation in a Pareto efficient manner, the strategic interactions between firms induce them to reach decisions that are not Pareto efficient and obtain lower returns. In effect, firms are trapped in a Prisoner's Dilemma where the returns of both firms could be increased if both firms invested more heavily into the emerging market.

The intuition behind this result is best understood by recognizing that while a monopoly can exclusively focus on marginal returns from the two markets, duopoly firms also consider average returns (see Proposition A1 in the appendix for a proof of this general result). We illustrate with an example. Consider Figure 2 (where $\alpha = 0.1$, $\beta = 0.5$, $a = 1$, $b = 2$, $p = 0.6$, $B_M = 2$, $B_1 = 1$ and $B_2 = 1$), where the monopoly's optimal investment decision, $r_M^* = 0.095$, implies an investment of 0.19 (labeled C1 in Figure 2) into the mature market and 1.81 (C2) into the emerging market, leading to equal marginal returns from both markets (i.e., the slopes at these points are identical). However, given the characteristics of the return functions, the mature market provides a greater average return under this allocation. Although this does not affect the monopoly's decision, average returns are important to duopoly firms due to the proportional allocation of returns.

To understand this result, assume first that duopoly firms invest their budget according to the monopoly's budget allocation, $r_1^* = r_2^* = r_M^*$, implying an investment of 0.095 by both Firm 1 and Firm 2. This leads to the highest possible combined duopoly return. However, Firm 1 can increase its return by investing more into the mature market. For example, if Firm 1 triples its mature market investment to $3r_M^*$, as shown in Figure 2, there are two key effects: first, the total investment into the mature market increases from 0.19 (C1) to 0.38 (D1), thereby increasing the returns from the mature market, while the total return from the emerging market decreases. Given the Inada conditions and the fact that a monopoly invests at equal marginal returns, the total loss of returns from the emerging market is greater than the total gain in returns from the mature market. This reduces the total combined returns, but Firm 1's share from the return from the mature market increases from 0.5 to 0.75, while Firm 1's share from the emerging market return shrinks from 0.5 to only 0.44. I.e., by sacrificing a 6% share of the emerging market, Firm 1 has gained an increase of 25% in its share of the mature market. Although Firm 1 has relinquished a part of the market that is providing larger returns (the emerging market), the relative sizes of the lost and gained market shares and the relative sizes of the two markets (and thus the average returns in those markets) lead to higher total returns for Firm 1.

This strategy is anticipated by the other firm which, in turn, decides also to shift a greater share of its own budget into this market. As firms shift more resources into the mature market, both the marginal and average returns from the mature market decline until an equilibrium is reached (E1 and E2 in Figure 2). As depicted in Figure 2, both firms invest more heavily into the mature market than the monopoly, with $r_1^* = r_2^* = 0.338$, ultimately decreasing the combined returns of both firms. The nature of competition in this setting engages firms in an arms race over the market with greater return per budget unit.



$C1 = r_M^* B_M$ $\quad$ $D1 = 3r_M^* B_1 + r_M^* B_2$ $\quad$ $E1 = r_1^* B_1 + r_2^* B_2$

$C2 = (1 - r_M^*) B_M$ $\quad$ $D2 = (1 - 3r_M^*) B_1 + (1 - r_M^*) B_2$ $\quad$ $E2 = (1 - r_1^*) B_1 + (1 - r_2^*) B_2$

Figure 2: Resource allocation of monopoly versus duopoly firms;
$\alpha = 0.1$, $\beta = 0.5$, $a = 1$, $b = 2$, $p = 0.6$, $B_M = 2$, $B_1 = 1$ and $B_2 = 1$

Overinvestment into mature markets has been previously documented (Dankbaar 1998) and has managerial implications for managers faced with such a project portfolio management problem. Although duopoly firms could achieve their highest possible return if they both invested as a monopoly would, ignoring competitive effects is costly if the other firm acts strategically. Consequently, managers need to recognize that in a competitive setting their firm should invest more heavily into the mature market than a monopoly would, acting in the same market alone. The reason is that duopoly firms must consider both marginal as well as average returns.

4.1. BUDGET CONSIDERATIONS

As demonstrated in the previous subsection, the presence of competition affects firms' budget allocation between two markets. In this subsection, we explore how firms' budgets affect their own as well as their competitor's resource allocation decisions. Given the functional form of $f(\cdot)$ and $g(\cdot)$, we have the following result.

> ***Proposition 3**: If* $f(x) = ax^{\alpha}$, $g(x) = \begin{cases} bx^{\beta} & w.p. \quad p \\ 0 & w.p. \quad 1-p \end{cases}$, *$\alpha < \beta$, then firm n's optimal budget allocation into the mature market is decreasing in its own budget, i.e.,* $\frac{\partial r_n^*}{\partial B_n} \leq 0$ *and increasing in its competitor's budget, i.e.,* $\frac{\partial r_n^*}{\partial B_{-n}} \geq 0$. *Furthermore, if* $B_n = B_{-n}$, *then* $r_n^* = r_{-n}^*$.

This proposition states that as in the monopoly benchmark, an increase in a firm's own budget decreases its investment into projects of incremental innovation which target the mature market. However, increases in its competitor's budget increases its investment into those exploitative projects. To demonstrate the impact of the size of firms' budgets on the resource allocation decisions further, consider the instance illustrated in Figure 3. In this figure, the optimal investment decision of a monopoly (r_M^*) is contrasted with that of two duopoly firms (r_1^* of Firm 1 and r_2^* of Firm 2). We fix Firm 1's budget to 1 and let Firm 2's budget vary from 0 to 3. For comparison, the budget of the monopoly is set to be equal to the combined budget of the two firms in the duopoly case. Alternatively, if the monopoly's budget was fixed to the size of Firm 1, the line labeled r_M^* would be perfectly horizontal. The case where Firm 2's budget is zero corresponds to the instance where Firm 1 is a monopoly, in which case $r_1^* = 0.1$. This figure further supports our insights from the previous subsection, as one can notice that competition significantly alters the investment decision of Firm 1, even if Firm 2 has a very small budget. Hence, accounting for the presence of competition is critical. Refining this observation, we notice that the share of budget being invested into the mature market is very sensitive to changes in the competitor's budget when it is quite small; however, when the competitor's budget is large, changes to the size of the competitor's budget have a marginal effect on r_1^*. Furthermore, if the firms' budgets are identical, i.e., $B_2 = B_1 = 1$ in Figure 3, then both firms invest the same share of their budget into the mature market.

Intuitively, the greater the budget of Firm 2, the more significant the threat is to Firm 1's return in the mature market and the greater the need for Firm 1 to defend this market by increasing its own investment into that market. But, at the same time, the mature market quickly becomes saturated as investment increases, leading to poor marginal returns. These counterbalancing factors lead to the tempered response by Firm 1.

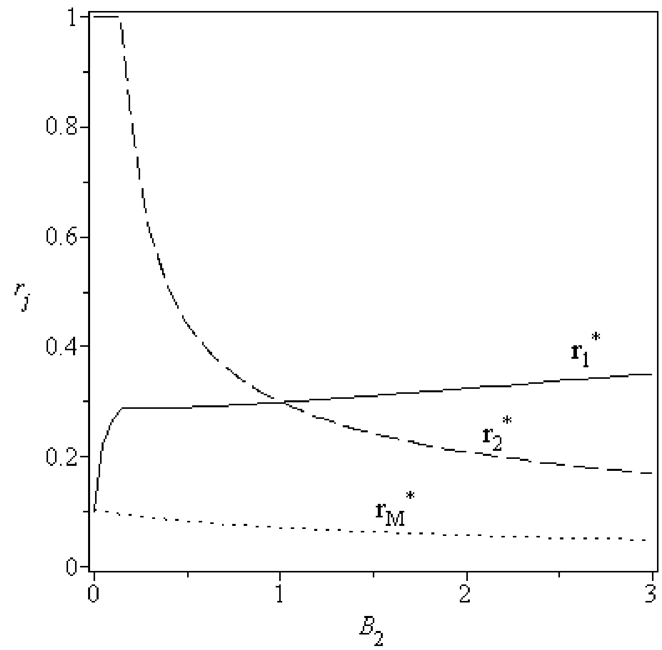


Figure 3: Optimal resource allocation into the mature market as a function of B_2;
$\alpha = 0.1, \beta = 0.6, a = 0.8, b = 2, p = 0.5, B_1 = 1, B_M = B_1 + B_2$

These findings can help guide managers as they contemplate their competitive resource allocation decision. We have shown that competition is an important consideration and should lead firms to increase their investment into the mature market. If the competitor is quite small, the actual size of the competitor's budget impacts the allocation decision critically. However, if the competitor's budget is sufficiently large, the actual size has less of an impact. Hence, facing a large competitor, it is sufficient for managers to respond to competition by defending markets with high average returns without having to spend significant resources gaining competitive intelligence regarding the exact size of the competitor's budget. Changes to a firm's own budget have an even more pronounced effect on its investment strategy. With a very limited budget, the choice is clear: fully invest in the mature market. A gradual increase in a firm's budget leads it to significantly increase its investment into the emerging market (as the mature market becomes saturated). Increasing investment into the emerging market, when faced with budget increases, maximizes the firms' expected returns, but, given the uncertainty associated with the emerging market, this also increases the firms' risk exposure. Consequently, a firm contemplating an increase in its investment budget needs to recognize that an increase in its budget can have two sources of risk: the additional leverage (if borrowing is required to achieve the budget increase) and the additional risk exposure that results from increased investment into the emerging market. The next subsection considers the effect of market uncertainty more closely.

4.2. Effect of Market Uncertainty

We have characterized the mature market as one that has known investment return projections while the emerging market has uncertain returns. Intuitively, we expect an increase in market uncertainty to reduce investment into radical projects targeting the emerging market, and vice versa. Recalling that $1 - p$ is the probability that the emerging market does not provide any returns, we thus expect firms to reduce their investment into the emerging market as this probability increases. Stated differently, we expect r to decrease as p increases. Indeed, Figure 4, which depicts the behavior of r for both monopoly and duopoly firms as a function of p, demonstrates that firms continue to invest a large proportion of their budget into the emerging market even if there is a significant probability that there will be no returns from that market. As the probability of the emerging market achieving its anticipated market potential decreases, firms initially shift resources to the mature market at an increasing rate. However, if this probability is sufficiently small, the duopoly firms shift resources to the mature market at a decreasing rate.

The monopoly, which generally invests a greater share of its budget into the emerging market, is particularly slow to increase investment into the mature market unless the probability of receiving no returns in the emerging market becomes extreme. For example, in the case presented in Figure 4, even if $p = 0.1$, it is still optimal for a monopoly to invest 0.7 of its budget into the emerging market. In contrast, the duopoly firm with the smaller budget (Firm 2) quickly increases its investment into the mature market as the probability of receiving no returns in the emerging market increases.

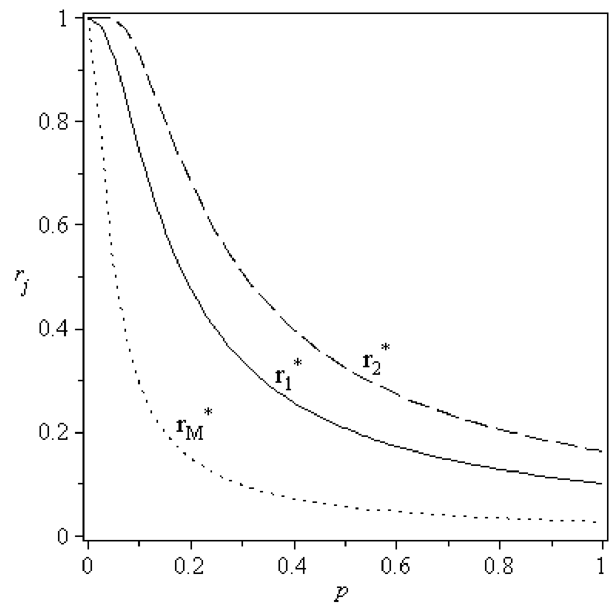


Figure 4: Optimal resource allocation into the mature market as a function of p;
$\alpha = 0.1$, $\beta = 0.6$, $a = 0.8$, $b = 2$, $B_M = 3$, $B_1 = 2$, $B_2 = 1$

The insights gained from Figure 4 are based on a particular set of market parameters. However, these findings are robust. For example, a reduction in a, the market potential of the mature market, decreases the proportion of budget the firm invests into the mature market at any given budget level; however, the relative effects of budget increases still hold. This result implies that risk neutral decision-makers maintain high investment levels into the emerging market to avoid saturating the mature market even in the presence of substantial market uncertainty. Only if it becomes highly likely that the emerging market will provide no returns, should firms completely avoid the emerging market.

4.3. Differences in Marginal Productivity

Recall that α and β, $\alpha < \beta$, define the marginal productivity of the mature and emerging markets, respectively (technically, they represent the degree of homogeneity of the return functions of these markets). A low marginal productivity implies that firms quickly experience diminishing returns for their investments into this market, while a high marginal productivity requires a large investment to obtain the large returns of this market. As the marginal productivity increases, the returns from a market become more linear. In the extreme case, as the marginal productivity of the emerging market, β, approaches 1, the emerging market provides linear returns for any amount of investment.

Intuitively, one would expect that firms would take advantage of an increase in β by decreasing r. This is demonstrated in Figure 5, which depicts the allocation decisions, r, for both monopoly and symmetric duopoly firms as a function of the marginal productivity of the emerging market. Indeed, when the total of firms' budgets is (relatively) high—1.25 implying 0.625 for each duopoly firm—firms monotonically shift their investment from the mature market into the emerging market as β, the emerging market's marginal productivity, increases. The effect of competition emerges as an important factor—the monopoly responds faster to an increase in marginal productivity in the emerging market and shifts significant resources to the emerging market even when such increase is relatively minor.

However, changing β does not always imply shifting resources from the mature into the emerging market. Since the returns from the emerging market follow the expression $g(x)=bx^{\beta}$, $\beta < 1$, the size of investment into the emerging market plays an important role. Namely, if the budget dedicated to the emerging market is *high*, then a *high* marginal productivity is preferred, while if the dedicated budget for this market is *low*, then a *low* marginal productivity is preferred. Thus, if firms have small budgets, they may react differently to changes in the marginal productivity. With small budgets, firms may actually shift resources away from the emerging market if the marginal productivity of this market increases. In Figure 5, the small budget duopoly firms (B_1=B_2=0.025) monotonically increase investment into the mature market as the marginal productivity of the emerging market increases. The small budget monopoly

(B_M=0.05=B_1+B_2), initially reduces investment into the mature market as the marginal productivity of the emerging market increases, but, as the returns in the emerging market become more linear, the monopoly also shifts more of its resources to the mature market. This reveals that the firms are more likely to shift resources from the emerging into the mature market as the difference in the two markets' marginal productivities increases, if their budgets are sufficiently small.

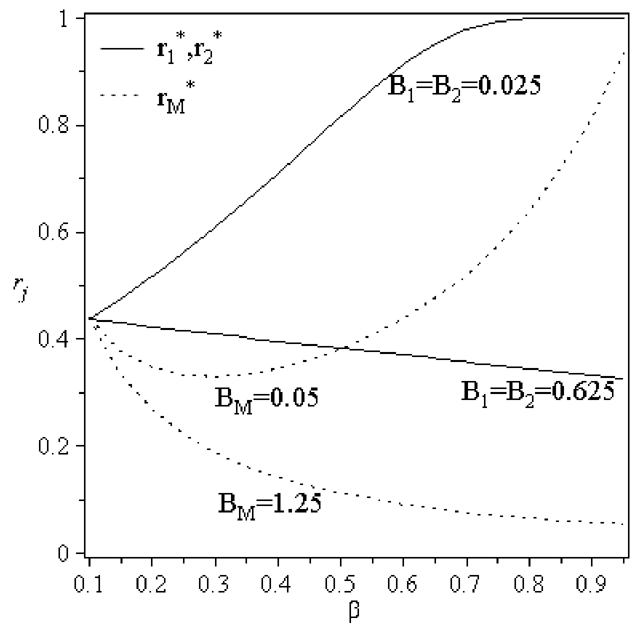


Figure 5: Optimal resource allocation into the mature market for different budgets as a function of β; α = 0.1, a = 0.8, b = 2, p = 0.5

5. OLIGOPOLIES

We recognize that, in practice, the number of firms competing in markets may well exceed two. In this section, we study the oligopoly setting with N identical firms in the sense that all face the same investment decision and have the same budget, with $B_1 = B_2 = \ldots = B_N \equiv B_O^N$, where the subscript O denotes the oligopoly case and the subscript N denotes the number of firms participating in the oligopoly. Each Firm n, $n = 1,\ldots,N$, is seeking to maximize its return by deciding what share r_i^N of its budget to allocate to the mature market:

$$E[\pi_n] = \frac{r_i^N}{\sum_{i=1}^{N} r_i^N} a\left(B_O^N \sum_{i\in N} r_i^N\right)^{\alpha} + \frac{1-r_i^N}{N-\sum_{i=1}^{N} r_i^N} pb\left(B_O^N\left(N-\sum_{i\in N} r_i^N\right)\right)^{\beta}, \; n = 1, \ldots, N, \tag{4}$$

where the first term is the return of Firm n from the mature market and the second term is the return of Firm n from the emerging market. Taking the first order conditions of (4) and imposing symmetry on the firms' decisions yields, after rearranging,

$$(N-1+\alpha)a\left(Nr_O^{N*}B_O^N\right)^{\alpha-1} = (N-1+\beta)pb\left(N\left(1-r_O^{N*}\right)B_O^N\right)^{\beta-1}. \tag{5}$$

We have the following result.

***Proposition 4**: If $f(x) = ax^{\alpha}$ and $g(x) = \begin{cases} bx^{\beta} & w.p. \quad p \\ 0 & w.p. \quad 1-p \end{cases}$, the oligopoly firms' optimal budget allocation r_O^* is such that if $\frac{(N-1+\alpha)a}{(N-1+\beta)pb} = \left(NB_O^N\right)^{\beta-\alpha}$ (>, <), then $r_O^* = \frac{1}{2}$ (>, <, respectively).*

This result extends the findings from the monopoly case and subsumes Proposition 2 (by setting N = 1). The market potential parameters a and b make the respective markets more attractive independent of the marginal productivity of the markets and the available investment budget. As before, for larger budgets either increasing β or decreasing α makes the emerging market more attractive. Conversely, for smaller budgets either increasing β or decreasing α makes the mature market more attractive.

The following theorem characterizes the effect the number of competing firms has on budget allocation decisions.

***Theorem 2**: Let* $f(x) = ax^{\alpha}$, $g(x) = \begin{cases} bx^{\beta} & w.p. \quad p \\ 0 & w.p. \quad 1-p \end{cases}$, *and* $\alpha < \beta$. *As the number of competing firms increases from N1 to N2, while the total budget of all firms is kept constant such that* $N1 \cdot B_O^{N1} = N2 \cdot B_O^{N2}$, *then* $r_O^{N2*} > r_O^{N1*}$.

This result directly extends Theorem 1 for the special case where firms have identical budgets: the more firms compete, the more heavily these firms invest into the mature market. Managers need to anticipate that investment into the mature markets will intensify as the number of competitors increases. Consequently, to protect their returns from the mature market, firms should increase their own share of investment into such markets.

Proposition 4 characterizes the condition that drives firms to invest a majority of their budget into the mature market. Since competition leads to even higher investment into the mature market, firms may end up investing their budget almost entirely into the mature market. This, of course, is the optimal decision for each firm individually, but detrimental to the combined returns of firms in an oligopoly. Depending on the market parameters, this dynamic may also prevent firms from diversifying their budget over multiple markets, which may contradict some firms' overarching goals of distributing their investments over multiple markets and further building a presence in emerging markets.

6. CONCLUSIONS AND FUTURE RESEARCH

We have studied and demonstrated how competition affects firms' resource allocation decisions. Firms, in our setting, consider investment into two types of projects: exploitative (those targeting mature markets) and explorative (those targeting emerging markets). In the benchmark case, monopoly firms simply need to consider the marginal returns of the markets in order to achieve the optimal decision. However, faced with competition, we find that firms need to account for the average returns of these markets as well. This has led to our main insight: competition drives firms to invest more heavily into exploitative projects which offer incremental innovation by targeting mature markets which have a lower marginal productivity than emerging markets. Incrementalism is an outcome of competition. This incrementalism is further exacerbated as the number of competing firms increases. Specifically, firms invest into the mature market to a greater degree the greater the number of competitors. To refine our insights, we have further looked into the effect of individual firms' budgets and market characteristics. For example, we find that overinvestment into mature markets persists even when the counterpart firm is endowed with a small budget, and that as a firm increases its own budget, it will invest more heavily into radical projects that target the emerging market. This increase in investment into the emerging market (as their own budget increases) only partially mitigates the incrementalism effect of competition, as firms continue to overinvest into the mature market to protect their returns from this market.

With this work we hope to lay the groundwork for future research. There are many additional complexities of the project portfolio management decision with competition that could be modeled and which may alter some of our findings. For example, some markets may provide increasing returns (Selove 2010) or may even provide s-shaped returns (Savin and Terwiesch 2005), which could lead to additional investment strategies. Furthermore, firms may not only differ with respect to the size of their budget but their investment into a market may also influence market demand in different ways. For example, a firm such as Apple may increase market demand to a greater degree that another firm in an emerging technology market such as the tablet market. Our work does also not consider the timing of firms' investment decisions. Project portfolio decisions are often made continuously in dynamic environments (Chao and Kavadias 2008; Zhu and Weyant 2003), where time can influence firms' costs and reduce uncertainty associated with some markets. Another opportunity in the project portfolio management problem with competition is analytical work on large project portfolios, where projects are targeting a large number of markets. Such a framework dramatically increases the complexity of the problem, both from a modeling perspective and from the decision-maker's perspective.

ENDNOTE

[1] The assumption that total market returns depend on the combined investment of both firms holds in R&D settings where firms' investments are targeted at creating value. For example, in the tablet market, created singlehandedly by Apple through its iPad, the additional investments of other firms into developing and marketing their own tables increased the total size of the tablet market further, rather than just redistributing the existing market returns. In contrast, if firms' investments are focused on, for example, negative advertising against their competitor's products, this assumption does not hold. For completeness, numerical analysis was also undertaken for an alternative competition model, where the expected return of Firm *n* is:

$$E[\pi_n] = \frac{r_n B_n}{r_n B_n + r_{-n} B_{-n}} a\left(r_n B_n + v r_{-n} B_{-n}\right)^{\alpha} + \frac{(1-r_n)B_n}{(1-r_n)B_n + (1-r_{-n})B_{-n}} pb\left((1-r_n)B_n + w(1-r_{-n})B_{-n}\right)^{\beta},$$

were v and w define the effect of competitive investment on the markets' total returns. With these additional parameters, the impact of firms' investments on total market returns can be modeled in more detail and adjusted for different levels of competitive intensity. However, numerical analysis confirms that our key findings still hold in this setting.

APPENDIX: PROOFS

Proof of Proposition 1: We want to show that if the monopoly budget changes from B_M to $\overline{B_M} > B_M$, then the optimal budget allocation changes from r_M^* to $\overline{r_M}^* < r_M^*$ (and vice versa).

Substituting $\overline{r_M}^* = \theta r_M^*$ and $\overline{B_M} = \lambda B_M$ into (1), we get

$\alpha a\left(\theta r_M^* \lambda B_M\right)^{\alpha-1} = p\beta b\left(\left(1-\theta r_M^*\right)\lambda B_M\right)^{\beta-1}$, which can be rearranged to:

$$\frac{f'\left(r_M^* B_M\right)}{(\theta\lambda)^{1-\alpha}} = -\frac{g'\left(\frac{1}{\theta}B_M - r_M^* B_M\right)}{(\theta\lambda)^{1-\beta}}. \tag{A1}$$

We want to show that if $\alpha < \beta$ and $\lambda > 1$, then $\theta < 1$. By contradiction: if we had $\theta \geq 1$, then $\frac{1}{(\theta\lambda)^{1-\alpha}} < \frac{1}{(\theta\lambda)^{1-\beta}}$. Given (A1), this implies $f'\left(r_M^* B_M\right) > -g'\left(\frac{1}{\theta}B_M - r_M^* B_M\right)$ which for $\theta \geq 1$ cannot hold under (1) and the Inada condition $\frac{\partial g^2}{\partial r_M^{\,2}} < 0$. Following the same arguments, α < β and $\lambda < 1$ necessitate $\theta > 1$. □

Proof of Proposition 2: The monopoly's optimal decision is the allocation r_M^* that solves (1), $\alpha a\left(r_M^* B_M\right)^{\alpha-1} = \beta p b\left(\left(1-r_M^*\right)B_M\right)^{\beta-1}$ or $\frac{\alpha a}{\beta p b} = \frac{\left(\left(1-r_M^*\right)B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}}$. Thus, if $\frac{\alpha a}{\beta p b} = B_M^{\;\beta-\alpha}$, we have $\frac{\left(\left(1-r_M^*\right)B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}} = B_M^{\;\beta-\alpha}$ or $r_M^{\;*} = \frac{1}{2}$. Similarly, if $\frac{\alpha a}{\beta p b} > B_M^{\;\beta-\alpha}$ (<), we have $\frac{\left(\left(1-r_M^*\right)B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}} > B_M^{\;\beta-\alpha}$ (<, respectively) or $r_M^{\;*} > \frac{1}{2}$ (<, respectively). □

Proof of Theorem 1: We know that $\frac{\partial r_M^*}{\partial B_M} < 0$ (Proposition 1) and we can also show that $\frac{\partial r_n^*}{\partial B_{-n}} > 0$ (see proof of Proposition 3). I.e., starting from identical investment if the monopoly firm and duopoly Firm *n* had the same budget and both faced no competition, $r_M^{\;*}$ decreases if the monopoly's budget increases and $r_n^{\;*}$ increases as competition increases for Firm n. Consequently, if $B_n \leq B_M$, then $r_n^{\;*} > r_M^{\;*}$ for any $B_{-n} > 0$.

Proof of Proposition 3: Comparing the response functions of Firm n and Firm $-n$, using (3), we get

$$\frac{\left(r_{-n}B_nB_{-n}+r_n\alpha(B_n)^2\right)}{\left((1-r_{-n})B_nB_{-n}+(1-r_n)\beta(B_n)^2\right)}=\frac{\left(r_nB_nB_{-n}+r_{-n}\alpha(B_{-n})^2\right)}{\left((1-r_n)B_nB_{-n}+(1-r_{-n})\beta(B_{-n})^2\right)}. \tag{A2}$$

Next, we solve (A2) for r_n:

$$r_n^*=\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{2B_n}+\frac{1}{2}\right)-\sqrt{\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{2B_n}+\frac{1}{2}\right)^2-\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}r_{-n}-\left(\frac{B_{-n}}{B_n}\right)^2\left(r_{-n}-r_{-n}{}^2\right)}.$$

Consequently, if $\frac{\partial}{\partial B_n}\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}r_{-n}+\left(\frac{B_{-n}}{B_n}\right)^2\left(r_{-n}-r_{-n}{}^2\right)\right)\le 0,$ then $\frac{\partial r_n^*}{\partial B_n}\le 0$ and if $\frac{\partial}{\partial B_{-n}}\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}r_{-n}+\left(\frac{B_{-n}}{B_n}\right)^2\left(r_{-n}-r_{-n}{}^2\right)\right)\ge 0,$ then $\frac{\partial r_{-n}^*}{\partial B_n}\ge 0$.

So we want to prove that $\frac{\partial}{\partial B_n}\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}r_{-n}+\left(\frac{B_{-n}}{B_n}\right)^2\left(r_{-n}-r_{-n}{}^2\right)\right)\le 0$, or:

$$\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}+\left(\frac{B_{-n}}{B_n}\right)^2-\left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right)\frac{\partial}{\partial B_n}r_{-n}\le\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{(B_n)^2}r_{-n}+2\frac{(B_{-n})^2}{(B_n)^3}\left(r_{-n}-r_{-n}{}^2\right), \tag{A3}$$

and that $\frac{\partial}{\partial B_{-n}}\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}r_{-n}+\left(\frac{B_{-n}}{B_n}\right)^2\left(r_{-n}-r_{-n}{}^2\right)\right)\ge 0$, or:

$$\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}+\left(\frac{B_{-n}}{B_n}\right)^2-\left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right)\frac{\partial}{\partial B_{-n}}r_{-n}\ge-\frac{1-\alpha\beta}{\beta-\alpha}\frac{1}{B_n}r_{-n}-2\frac{B_{-n}}{(B_n)^2}\left(r_{-n}-r_{-n}{}^2\right). \tag{A4}$$

We now examine two cases.

Case 1: if $\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}+\left(\frac{B_{-n}}{B_n}\right)^2-\left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right)>0$, then (A3) and (A4) become:

$$\frac{\partial}{\partial B_n}r_{-n}\le\frac{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{(B_n)^2}r_{-n}+2\frac{(B_{-n})^2}{(B_n)^3}\left(r_{-n}-r_{-n}{}^2\right)}{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n}+\left(\frac{B_{-n}}{B_n}\right)^2-\left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}}\quad\text{and} \tag{A5}$$

$$\frac{\partial}{\partial B_{-n}} r_{-n} \geq \frac{-\frac{1-\alpha\beta}{\beta-\alpha}\frac{1}{B_n} r_{-n} - 2\frac{B_{-n}}{(B_n)^2}\left(r_{-n} - r_{-n}^{\ 2}\right)}{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}}. \tag{A6}$$

This proposition states that $\frac{\partial}{\partial B_n} r_{-n} \geq 0$ and $\frac{\partial}{\partial B_{-n}} r_{-n} \leq 0$, and thus (A5) and (A6) can only hold if $\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right) > 0$ for all $\alpha, \beta \in (0,1)$, $\alpha < \beta$, $r_{-n} \in [0,1]$, which was the original condition for this case.

Case 2: if $\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right) < 0$, then (A3) and (A4) become:

$$\frac{\partial}{\partial B_n} r_{-n} \geq \frac{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{(B_n)^2} r_{-n} + 2\frac{(B_{-n})^2}{(B_n)^3}\left(r_{-n} - r_{-n}^{\ 2}\right)}{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}} \text{ and} \tag{A7}$$

$$\frac{\partial}{\partial B_{-n}} r_{-n} \leq \frac{-\frac{1-\alpha\beta}{\beta-\alpha}\frac{1}{B_n} r_{-n} - 2\frac{B_{-n}}{(B_n)^2}\left(r_{-n} - r_{-n}^{\ 2}\right)}{\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}}. \tag{A8}$$

This proposition states that $\frac{\partial}{\partial B_n} r_{-n} \geq 0$ and $\frac{\partial}{\partial B_{-n}} r_{-n} \leq 0$, and thus (A7) and (A8) hold for all $\left(\frac{1-\alpha\beta}{\beta-\alpha}\frac{B_{-n}}{B_n} + \left(\frac{B_{-n}}{B_n}\right)^2 - \left(\frac{B_{-n}}{B_n}\right)^2 2r_{-n}\right) < 0$, $\alpha, \beta \in (0,1)$, $\alpha < \beta$, $r_{-n} \in [0,1]$, which was the original condition for this case.

Lastly, we prove the case where $B_n = B_{-n} = B_D$. The equation describing the optimal allocation of Firm *n*, (3), can be rearranged to

$$\frac{a\left(r_n B_n + r_{-n} B_{-n}\right)^{\alpha-1}}{b\left(B_n(1-r_n) + B_{-n}(1-r_{-n})\right)^{\beta-1}} = \frac{\left((1-r_{-n})B_{-n} + (1-r_n)\beta B_n\right)}{B_n(1-r_n) + B_{-n}(1-r_{-n})} \cdot \frac{r_n B_n + r_{-n} B_{-n}}{r_{-n} B_{-n} + r_n \alpha B_n},$$

which for $B_n = B_{-n} = B_D$ becomes:

$$\frac{a\left((r_n + r_{-n})B_D\right)^{\alpha-1}}{b\left((2 - r_n - r_{-n})B_D\right)^{\beta-1}} = \frac{(1-r_{-n}) + (1-r_n)\beta}{(1-r_n) + (1-r_{-n})} \cdot \frac{r_n + r_{-n}}{r_{-n} + r_n\alpha}.$$

This is the response function for Firm n. We insert the response function for Firm $-n$ to get: $\frac{((1-r_n)+(1-r_{-n})\beta)(r_n+r_{-n})}{((1-r_n)+(1-r_{-n}))(r_n+r_{-n}\alpha)}=\frac{((1-r_{-n})+(1-r_n)\beta)(r_{-n}+r_n)}{((1-r_{-n})+(1-r_n))(r_{-n}+r_n\alpha)}$ which has two solutions for r_n, $r_n=r_{-n}$ and $r_n=\frac{(\alpha\beta-1)}{\alpha-\beta}+1-r_{-n}$. Since we have $r_n+r_{-n}\le 2$, the second solution can only hold if $\frac{(\alpha\beta-1)}{\alpha-\beta}+1\le 2$, which simplifies to $\alpha\beta-\alpha+\beta\ge 1$ (for $\alpha<\beta$). Let $h=\alpha\beta-\alpha+\beta$. We have $\frac{\partial h}{\partial\alpha}<0$ and $\frac{\partial h}{\partial\beta}>0$. Given the feasible region of α and β, $\max(h)<1$, which implies that $\alpha\beta-\alpha+\beta<1$. Consequently, the symmetric solution $r_n=r_{-n}=r_D$ is the only feasible solution if $B_n=B_{-n}=B_D$. □

Proof of Proposition 4: This proof follows the same arguments as in the proof of Proposition 1. □

Proof of Theorem 2: From (5), the optimal budget allocation of firms in an oligopoly of size *N1* is the value of r_O^{N1*} that satisfies $\frac{a\left(N1\cdot r_O^{N1*}B_O^{N1}\right)^{\alpha-1}}{pb\left(N1\left(1-r_O^{N1*}\right)B_O^{N1}\right)^{\beta-1}}=\frac{(N1-1+\beta)}{(N1-1+\alpha)}$ and that of firm in an oligopoly of size *N2* is the value of r_O^{N2*} that satisfies

$\frac{a\left(N2\cdot r_O^{N2*}B_O^{N2}\right)^{\alpha-1}}{pb\left(N2\left(1-r_O^{N2*}\right)B_O^{N2}\right)^{\beta-1}}=\frac{(N2-1+\beta)}{N2(L-1+\alpha)}$. Since $N2>N1$ and $\alpha<\beta$, one can show that

$\frac{(N1-1+\beta)}{(N1-1+\alpha)}>\frac{(N2-1+\beta)}{(N2-1+\alpha)}$. Consequently, $\frac{a\left(N1\cdot r_O^{N1*}B_O^{N1}\right)^{\alpha-1}}{pb\left(N1\left(1-r_O^{N1*}\right)B_O^{N1}\right)^{\beta-1}}>\frac{a\left(N2\cdot r_O^{N2*}B_O^{N2}\right)^{\alpha-1}}{pb\left(N2\left(1-r_O^{N2*}\right)B_O^{N2}\right)^{\beta-1}}$,

which simplifies to $\left(\frac{r_O^{N1*}}{r_O^{N2*}}\right)^{\alpha-1}>\left(\frac{1-r_O^{N1*}}{1-r_O^{N2*}}\right)^{\beta-1}$. Since $\alpha<\beta$, we have $r_O^{N2*}>r_O^{N1*}$. □

The following proposition compares the optimal resource allocation of a monopoly with that of duopoly firms if the two markets offer returns according to the general functions $f(\cdot)$ and $g(\cdot)$, which both follow the Inada conditions.

> ***Proposition A1****: When optimizing its resource allocation over two markets with general return functions that follow the Inada conditions, a monopoly*

considers only marginal returns, whereas duopoly firms also account for the average return per budget allocation unit of the two markets.

Proof of Proposition A1: Using the general return functions $f(\cdot)$ and $g(\cdot)$, which both follow the Inada conditions, the optimal allocation of a monopoly, r_M^*, is the value of r_M that solves the first order condition of the monopoly's return, $\pi_M = f(r_M B_M) + g((1-r_M)B_M)$:

$$f'(r_M B_M) = -g'((1-r_M)B_M). \tag{A9}$$

The return for the duopoly firms is:

$$\pi_n = \frac{r_n B_n}{r_n B_n + r_{-n} B_{-n}} f(r_n B_n + r_{-n} B_{-n}) + \frac{(1-r_n)B_n}{(1-r_n)B_n + (1-r_{-n})B_{-n}} g((1-r_n)B_n + (1-r_{-n})B_{-n}). \tag{A10}$$

To derive analytical results for the general case, we impose symmetry such that the firms are identical, i.e., $B_1 = B_2 \equiv B_D$ and such that $r_1 = r_2 \equiv r_D$, $f_1'(x) = f_2'(x) = f'(x)$, and $g_1'(x) = g_2'(x) = g'(x)$. Under symmetry, the first order condition of (A10) is:

$$\frac{f(2r_D B_D)}{2r_D} + f'(2r_D B_D) = \frac{g(2(1-r_D)B_D)}{2(1-r_D)} - g'(2(1-r_D)B_D),$$

i.e., the duopoly firms' optimal budget allocation depends on the marginal market return functions, $f'(2r_D B_D)$ and $g'(2(1-r_D)B_D)$, as well as on the average returns per allocation percentage, $\frac{f(2r_D B_D)}{2r_D}$ and $\frac{g(2(1-r_D)B_D)}{2(1-r_D)}$. If $\frac{f(2r_D B_D)}{2r_D} \neq \frac{g(2(1-r_D)B_D)}{2(1-r_D)}$, then we have $f'(2r_D B_D) \neq -g'(2(1-r_D)B_D)$. However, a monopoly with a budget equal to the combined budget of the duopoly firms, $B_M = 2B_D$, invests according to (A9), $f'(r_M 2B_D) = -g'((1-r_M)2B_D)$. Consequently, if $\frac{f(2r_D B_D)}{2r_D} \neq \frac{g(2(1-r_D)B_D)}{2(1-r_D)}$, then $r_D \neq r_M$. □

ACKNOWLEDGMENTS

The authors would like to thank the Department Editor, the Senior Editor, and two anonymous referees, for their insightful comments and important suggestions which have led to a significant improvement in the positioning and exposition of our results. The authors are grateful for the feedback provided by Mahmut Parlar, Xuan Zhao, Dave Fuller, and Brian Cozzarin. This research was supported by Natural Sciences and Engineering Research Council of Canada (NSERC) and Mathematics of Information Technology And Complex Systems (MITACS) grants.

REFERENCES

Ali, A., M.U. Kalwani, D. Kovenock. 1993. Selecting product development projects: pioneering versus incremental innovation strategies. *Management Science*, 39 (3), 255-274.

Blau, G.E., J.F. Pekny, V.A. Varma, P.R. Bunch. 2004. Managing a portfolio of interdependent new product candidates in the pharmaceutical industry. *Journal of Product Innovation Management*, 21, 227-245.

Bower, J.L., C.G. Gilbert. 2005. *From Resource Allocation to Strategy*, Oxford University Press, New York, NY.

BusinessWeek. 2003. *DuPont tries to unclog a pipeline*, 27 January, 103-104.

Chao, R.O., S. Kavadias, C. Gaimon. 2009. Revenue Driven Resource Allocation: Funding Authority, Incentives, and New Product Development Portfolio Management. *Management Science*, 55 (9), 1556-1569.

Chao, R.O., S. Kavadias. 2008. A Theoretical Framework for Managing the New Product Development Portfolio: When and How to Use Strategic Buckets. *Management Science*, 54 (5), 907-921.

Chen, Y., A.J. Vakharia, A. Alptekinoglu. 2008. Product Portfolio Strategies. *Production and Operations Management*, 17 (6), 587-598.

Cooper, R.G., S.J. Edgett, E.J. Kleinschmidt. 2001. *Portfolio Management for New Products*, 2nd Edition, Perseus Publishing.

Cooper, R.G., S.J. Edgett, E.J. Kleinschmidt. 2004. Benchmarking Best NPD Practices – II. *Research Technology Management*, 47 (1), 50-59.

Dankbaar, B. 1998. On the over-investment in automotive technology. *International Journal of Technology Management*, 16 (7), 631-640.

Gerchak, Y., M. Parlar. 1999. Allocating resources to research and development projects in a competitive environment. *IIE Transactions*, 31, 827-834.

Getz, D., G. Brown. 2006. Critical success factors for wine tourism regions: a demand analysis. *Tourism Management*, 27, 146-158.

Gibson, M.R., J.W. Ohlmann, M. J. Fry. 2009. An agent-based stochastic ruler approach for a stochastic knapsack problem with sequential competition. *Computers & Operations Research*, 37 (3), 598-609.

Graves, S.B., J.L. Ringuest. 1991. Evaluating competing R&D investments. *Research Technology Management*, 34 (4), 32-36.

Gupta, D.K., T. Mandakovic. 1992. *Contemporary approaches to R&D project selection, a literature survey*. D.F. Kocaogly, ed. Management of R&D and Engineering. Elsevier Publishers, 67-86.

Hauser, J., G.J. Tellis, A. Griffin. 2006. Research on innovation: a review and agenda for Marketing Science. *Marketing Science*, 25(6), 687-717.

Inada, K. 1963. On a Two-Sector Model of Economic Growth: Comments and a Generalization. *The Review of Economic Studies*, 30 (2), 119-127.

Kaplan, R., D. Norton. 1992. The balanced scorecard – measures that drive performance. *Harvard Business Review*, 70, 71-79.

Kavadias, S., C.H. Loch. 2003. Optimal Project Sequencing with Recourse at a Scarce Resource. *Production and Operations Management*, 12 (4), 433-444.

Krishnan, V., C.H. Loch. 2005. A Retrospective Look at Production and Operations Management Articles on New Product Development. *Production and Operations Management*, 14 (4), 433-441.

Loch, C.H., S. Kavadias. 2010. *Creating Rather than Optimizing Portfolios*. Presentation at Annual INFORMS Conference, Austin, TX.

Loch, C.H., S. Kavadias. 2002. Dynamic portfolio Selection of NPD programs using marginal returns. *Management Science*, 48 (10) 1227-1241.

Parlar, M., Z.K. Weng. 2006. Coordinating price and production decisions in the presence of price competition. *European Journal of Operational Research*, 170, 211-227.

Savin, S., C. Terwiesch. 2005. Optimal product launch times in a duopoly: balancing life-cycle revenues with product cost. *Operations Research*, 53 (1), 26-47.

Selove, M. 2010. Three essays on strategic and tactical issues in product design. Doctoral dissertation, Massachusetts Institute of Technology, MA.

Yoffie, D.B. 2004. *Cola wars continue: Coke and Pepsi in the twenty-first century*, Harvard Business School Publishing, Boston, MA.

Zhu, K., J. Weyant. 2003. Strategic exercise of real options: Investment decisions in technological systems. *Journal of Systems Science and Systems Engineering*, 12 (3), 257-278.