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THREE ESSAYS ON THE GENERAL EQUILIBRIUM EFFECTS OF HUMAN INTERACTIONS

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Three Essays on the General Equilibrium Effects of Human Interactions

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The overarching theme of this PhD thesis is human mobility and its externalities, particularly in the context of labour and health economics. Through rigorous modelling and analysis, the three chapters of the thesis demonstrate the potential benefits of policies that regulate human mobility.

In the first chapter of my PhD, I examine how language training can improve the functioning of the labour market, with a particular focus on immigrants with high skills who face language barriers. I argue that fully funding the cost of language acquisition for migrants can bring significant benefits to the economy and migrants, but may marginally worsen the labour market performance of low-skilled natives. Using a search and matching framework with two-dimensional skill heterogeneity, I model the effects of a language acquisition subsidy on migrants' labour market integration and its impact on natives' labour market performance. My study finds that subsidizing language acquisition costs may increase the GDP of the German economy by approximately ten billion dollars by decreasing the aggregate unemployment rate and skill mismatch rate and increasing the share of job vacancies requiring high generic skills.

The second chapter of my PhD explores the challenges involved in devising social contact limitation policies as a means of controlling infectious disease transmission. Using an economic-epidemiological model of COVID-19 transmission, I evaluate

the effectiveness of different intervention strategies and their consequences on public health, social welfare and economic outcomes. The findings emphasize the importance of responsiveness in implementing social contact limitations, rather than solely focusing on their stringency, and suggest that early interventions lead to the lowest losses in economy and mental well-being for a given number of life losses. The study has broader implications for managing the societal impact of infectious diseases and highlights the need to continue refining our understanding of these trade-offs and developing adaptable models and policy tools to safeguard public health while minimizing social and economic consequences. Overall, the study offers a robust and versatile framework for understanding and navigating the challenges posed by public health crises and pandemics.

The third chapter of my PhD builds on the economic-epidemiological model developed in Chapter 2 to analyze the multifaceted effects of vaccine hesitancy in controlling the spread of infectious diseases, with a particular focus on the COVID-19 pandemic in Belgium. The study utilizes actual vaccination rates by age group until June 2021 and simulates the following months by incorporating realistic properties such as temporary immunity, age-specific vaccination hesitancy rates, daily vaccination capacity, and vaccine efficacy rate. The baseline scenario with an overall 27.1% vaccine hesitancy rate indicates that current vaccination rates in Belgium are sufficient to control the spread of COVID-19 without imposing social contact limitations. However, hypothetical scenarios with higher disease transmission rates demonstrate the high costs of vaccine hesitancy, resulting in significant losses in labour supply, mental well-being, and life losses.

Throughout this thesis, I have described the costs and benefits induced by mobility, and shown that mobility policies make winners and losers. In Chapter 1, subsidizing the cost of language acquisition for migrants can bring significant benefits to the economy and migrants, but may marginally worsen the labour market performance of low-skilled natives. In Chapter 2, stringent policies alleviate health losses, but they impact economic activity and mental health. In Chapter 3, the health externalities generated by human interactions impose a potential tradeoff between values, namely the freedom to move and the freedom to choose to get vaccinated. In each of these chapters, I quantify these tradeoffs.

Another important insight from this thesis is the need to incorporate behavioural aspects into macro models evaluating the consequences of policies related to human mobility. In the thesis, these aspects include individual investments in language training, decision-making on infection avoidance, social contacts, labour supply, and vaccination decisions. can lead to more effective policies that balance the interests of various stakeholders.

Overall, this thesis contributes to the literature on human mobility by highlighting the potential benefits and challenges associated with it, and the need for nuanced and responsive policymaking that takes into account behavioural aspects and externalities. The insights gained from this thesis can be relevant for future research in economics on topics related to human mobility, public health, and labour market integration.

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1 Migration, Language Proficiency and Training: An Equilibrium Matching Approach

Abstract

In this chapter, I quantify the effects of subsidizing immigrants' language acquisition costs on the labour market performance of both immigrant and native workers. To this end, I extend the canonical Diamond-Mortensen-Pissarides Framework into a search and matching model with bi-dimensional skills: a generic and a language skill, which can also be interpreted as a country-specific skill. This language skill acts as an eligibility constraint to be hired at some jobs. I calibrate my model using data on Polish and Turkish immigrants in Germany. My simulations suggest that language acquisition subsidies improve the labour market performance of immigrant workers and high-skilled native workers, while they are slightly detrimental for low-skilled native workers. High-skilled immigrants are the biggest gainers with an expected income increase of 22% and an unemployment rate decrease from 4.63% to 2.69%. Low-skilled natives are on average only marginally suffering from this subsidy policy, with an expected income loss of 0.07% and an unemployment rate increase from 3.49% to 3.59%. On the aggregate level, fully subsidizing the financial costs of language acquisition may decrease the unemployment rate from 3.50% to 3.36% and increase productivity by 0.25%.

1.1 Introduction

The increase in the inflow of immigrants over the past years is occupying a significant part of the public debate in developing countries. One of the most pressing issues pertains to immigrants' economic impact, both in terms of their integration and their impact on natives' employment and wages. At the heart of these potentially conflicting concerns, language barriers appear to play a crucial role. On one hand, they harm immigrants' employment rate, job quality and earnings. On the other hand, language barriers reduce the competition that natives must face. While policies aiming at fostering the language acquisition process of immigrants can be used as efficient tools to improve the economic integration of immigrants, such programs are generally limited in the level of language proficiency that they allow immigrants to achieve. In this paper, I assess the net impacts of alleviating such barriers on firms and on natives and immigrants of various skill levels.

Evidence shows that immigrants from the upper tail of the skill distribution benefit highly from acquiring proficiency in their host country's language, while the role of language proficiency appears to be either absent or very low in improving the labour market performance of low-skilled immigrants. Further evidence suggests that when the degree of competition between low-skilled immigrants and low-skilled natives is high, immigration may be harmful to the latter. Hence, policies aimed at fostering integration may generate externalities on natives that need to be accounted for.

While the literature about the impact of language on labour market outcomes is overwhelmingly empirical, it lacks a general equilibrium perspective on allocative efficiency, job creation and wages. To fill in this gap, I build a model that deals with the assignment problem of workers to job qualities in an environment where workers possess and firms require two-dimensional skills: (i) a generic skill level (i.e. education), (ii) a country-specific skill level (i.e. language proficiency), and a costly skill upgrade possibility. The framework I propose provides answers to the following research questions: First, how much can an economy benefit from subsidizing immigrants' language acquisition costs? Second, what can be the underlying dynamics that lead to the economic benefits of incentivizing immigrants' language acquisition? Third, which worker groups (e.g. natives vs immigrants, high-skilled vs low-skilled) can benefit or lose in terms of employment rate, expected income, and job quality from incentivizing immigrants' language acquisition?

In my baseline scenario, fully subsidizing immigrants' financial language acquisition costs results in 67% of the high-skilled immigrants and 39% of the low-skilled immigrants learning the host-country language. As a result of this subsidy, my model predicts a 0.25% increase in productivity, which translates into ten billion euros annually for an economy the size of the German economy. The main driving factor behind this productivity gain is an increase in the job creation rate requiring high skills. Assuming a fixed stock of immigrants, I estimate that the productivity gain of this language acquisition subsidy pays off its cost in around eleven years. I predict an increase from 27.5% to 29.2% in the share of job vacancies requiring high skills among all vacancy creations.

I find that the aggregate unemployment rate would decrease from 3.50% to 3.36% in the scenario of fully subsidizing immigrants' financial language acquisition costs. As the effect of the language barrier is milder and immigrants have access to more job vacancies, the skill mismatch rate falls from an initial 6.52% to 4.60%. The unemployment rate among high-skilled immigrants falls drastically from 4.63% to 2.69%. Low-skilled immigrants enjoy an unemployment rate decrease from 6.21% to 5.65%. High-skilled natives see almost no effect with an unemployment rate change from 1.51% to 1.50%. Low-skilled natives face a higher unemployment rate with an increase from 3.49% to 3.59%.

In terms of expected income, high-skilled immigrants are the main gainers of the language acquisition cost subsidy policy. My model predicts a 22.6% increase in their expected income, as a result of both a lower unemployment rate and a higher job quality. This policy creates a minor spillover on the rest of the population. All other worker groups face an expected income change of less than 1%, being negative only the low-skilled natives. Finally, incentivizing immigrants' language acquisition process yields a negative effect on income equality. Indeed, income inequality increases by 1.75%. This result is driven by the fact that only high-skilled immigrants benefit significantly more from this policy than other groups, whose income only increases by less than 1%. As the main gainers are high-skilled immigrants, income inequality between high- and low-skilled worker groups increases. For a given generic skill level, within-group inequality decreases for the high-skilled while there is almost no change for the low-skilled.

The rest of the chapter is organized as follows: Section 2 presents a literature review on motivation and the proposed methodology. Section 3 explains the theoretical framework. Section 4 characterizes the equilibrium. Section 5 explains data sources and parameter choices. Section 6 presents and discusses the simulation results. Finally, Section 7 concludes.

1.2 Literature Review

Borjas (1994) points out that the political discussion on the impact of immigration on the host economy is focused on the following three questions: "How do immigrants perform in the host country's economy? What impact do immigrants have on the employment opportunities of natives? Which immigration policy most benefits the host country?" These three questions are at the heart of this paper. This section provides a literature review of the studies related to these questions with a specific focus on the role of host country language proficiency.

1.2.1 Migrant economic integration and language acquisition

Chiswick (1978) and Carliner (1980) are among the first studies in the literature on immigrants' performance in host countries. They reach the conclusion that immigrants in the US start working with an earning gap, over a period of 10-15 years catch up with the natives and eventually perform better than them in the labour market in terms of earnings. The argument used to explain this finding is that immigrants were self-selected from the upper tail of the skill distribution, whilst they lack US-specific human capital (such as language proficiency) when they arrive, they accumulate US-specific skills over time and become more productive than natives. However, Borjas (1985) shows that these results are highly overestimated for some immigrant groups. Later, Borjas (1987) adapts the Roy (1951) model of self-selection to immigration. He concludes that depending on the factors in source countries, the immigrants are self-selected from the upper- and lower-tails of the skill distributions. Therefore, depending on their skill levels, their performance may exhibit better or worse trajectories in comparison with the natives.

According to the literature, one important factor underlying the wage convergence of some immigrant groups is the acquisition of proficiency in the host country's language. Grenier (1984) demonstrates that language attributes explain one-third of the wage gap of Hispanic white workers in comparison with non-Hispanic workers in the US, and emphasizes the policy need to assist Hispanic Americans in their English learning process. Chiswick (1991) shows that for an additional year of residence in the US, the probability of becoming proficient in English increases only by 3 per cent among the illegal aliens registered in Los Angeles. This finding supports the relevance and importance of policies aiming at fostering the language proficiency acquisition of immigrants.

There is an abundance of recent studies on the impact of host country language proficiency on immigrants' labour market outcomes. By using data from many different countries, they come to a consensus that language proficiency is positively correlated with higher earnings and higher employment probabilities. Dustmann and Fabbri (2003) is among the first studies that find the positive effect of language on earnings and job-finding probabilities, in their study on ethnic minorities in the UK. One possible explanation for the effect on earnings can be that, instead of a direct effect on earnings, language proficiency leads to a higher employment rate and a higher probability of working at better-quality jobs. Once the selection is taken into account, language proficiency does not seem to result in higher earnings (Aldashev et al. (2009)).

Although language proficiency is positively correlated with labour market outcomes, this correlation varies substantially between high- and low-skilled workers. For more educated workers, the return on earnings is more significantly pronounced for Hebrew proficiency in Israel (Berman et al. (2003)), Catalan proficiency in Catalonia (Di Paolo and Raymond (2012)), Spanish proficiency in Spain (Budria and Swedberg (2015)). None of these studies finds a statistically significant effect of language proficiency on earnings for workers with low education. In line with these findings, Hayfron (2001) shows no effect of Norwegian proficiency on the labour market outcomes of potentially low-skilled immigrants from Third World countries. Isphording (2014) comments on this topic as "Language skills are shown to be complementary to further forms of pre-migration acquired human capital, acting as the medium of translation to apply home country knowledge in the host county labour market."

Two papers shed light on language requirements in occupations. Chiswick and Miller (2010) find that, in the US, higher earnings are observed for workers having better English skills and workers working at occupations with greater English requirements, implying a high incentive for a match between workers' English skills and firms' language requirements. This relationship exists both for the natives and for the immigrants. Additionally, Autor et al. (2006) point out that high-skill workers perform different and more interactive (or communicative) tasks compared with low-skill workers.

In summary, language acquisition brings benefits in terms of better wages and better employment probabilities. Language requirements highly differ between jobs requiring high skills and low skills. The effect of language proficiency, thus, varies substantially for high- and low-skilled workers. Therefore, workers evaluate the cost of and expected benefit from language acquisition to decide on their investment in it.

1.2.2 Impact of immigrants on the employment opportunities of natives

This literature consists of theoretical and empirical papers, contrary to the literature on the effects of language proficiency on labour market outcomes which lacks a sound theoretical framework. The evidence is mixed in the literature that studies find either positive or negative effects on natives' labour market performance. These effects depend on whether immigrants are substitutes or complements of natives in the labour market. If immigrants and natives in the same skill group compete for jobs in the corresponding skill level, immigration has a detrimental effect on natives' labour market outcomes. As the intensity of this competition decreases, this detrimental effect tends to disappear and starts to exhibit a positive effect due to the positive externalities the immigrants create. This relationship is referred to as the degree of substitutability between natives and immigrants. The effect of immigration on natives depends on the skill composition of the immigrants and the degree of substitutability between immigrants and natives in each skill group (Card, 2009; Ottaviano and Peri, 2012). Two seminal studies, Borjas (1990) and Card (2001) demonstrate that when low-skilled immigrants are employed as substitutes for low-skilled natives, immigration slightly reduces the employment rates of the low-skilled natives, while there is no visible effect on wages. A study showing a positive effect of immigration is Sussman and Zakai (1998). This study provides evidence of the complementarity between Russian and Israeli physicians in Israel. They demonstrate that Russian physicians could only take low-profile generalist jobs in Israeli hospitals, allowing their Israeli counterparts to promote to better-paying ranks. As a result, Russian high-skilled immigrants ameliorated the labour market outcomes of high-skilled Israeli natives.

Peri and Sparber (2009) propose a general equilibrium model by using a Nested-CES production structure to explain this issue and argue that low-skilled immigrants are more advantageous for jobs requiring manual and physical tasks, whereas the same relationship holds for the low-skilled natives and language-intensive tasks. This study also provides evidence of the complementarity between low-skilled immigrants and low-skilled natives, in the US. It shows that low-skilled immigrants are concentrated in physically intensive occupations in the sectors of agriculture, construction, and personal or household services. On the other hand, they rarely engage in communication-intensive activities like salespersonship or supervision, because they lack relevant country-specific skills, including language skills. They identify two critical factors in the wage formation of low-educated natives, namely: (i) whether immigrants compete for natives' jobs or they take jobs where they are inherently advantageous, and (ii) whether natives change their occupational choices to protect themselves from immigration. When manual tasks are outsourced to low-skilled immigrants and low-skilled natives shift to language-intensive jobs, even low-skilled natives benefit from immigration due to their shift to better jobs within low-skilled jobs. Lewis (2013) argues that this divergence in occupation choices may partly be due to the worse language skills of immigrants. Although this theoretical framework is very relevant and explanatory, it has one significant drawback: it rules out the possibility of skill downgrading because it assumes that all workers work at jobs exactly matching their skills. Muysken et al. (2015) and Amuedo-Dorantes and De la Rica (2011) point out that skill downgrading is substantial for immigrants, in their studies by using German and Spanish data, respectively.

1.2.3 Methodology

Diamond-Mortensen-Pissarides Framework (DMP) is the benchmark model of frictional unemployment (Diamond (1982), Mortensen (1982), and Pissarides (1985)). The baseline DMP Framework has three main components: The number of matches is determined according to a constant returns to scale matching function depending on the number of unemployed workers and the number of open vacancies; firms can enter the market and open vacancies freely with an incurred cost of vacancy creation; the surplus of a match is shared between the worker and the firm by Nash Bargaining. This setting provides a useful tool to analyze the effects of policies on key aggregate labour market outcomes by determining the equilibrium values of the wage rate, the unemployment rate and the number of job vacancies. Since using homogeneous agents as in the baseline model leaves out important labour market outcomes such as job-to-job transitions, this search and matching framework has been extended in various ways, including heterogeneity of workers and/or firms by skill levels.

In this context, Albrecht and Vroman (2002) develop a search and matching framework with two skills on both sides of the market. On-the-job search is not included in their model. They present the conditions for a cross-skill matching equilibrium, where high-skilled agents are willing to take low-skilled jobs, and an ex-post segmented equilibrium with no skill mismatch. One of the early studies that used two-sided skill heterogeneity with on-the-job search is Gautier (2002). With on-the-job search, high-skilled workers' willingness to accept low-quality jobs depends on the relative productivity of jobs and the workers' probability to find a better job. Dolado et al. (2008) explore the phenomenon of on-the-job search in a labour market characterized by heterogeneous jobs and workers. The authors use a matching model to analyze how the decision to search for a new job while currently employed affects the distribution of workers across jobs, wage inequality, skill mismatch, as well as the overall efficiency of the labour market. The study finds that on-the-job search can have both positive and negative effects, and suggests that policy interventions aimed at reducing search frictions may improve labour market outcomes.

1.3 The Model

This section describes the setting with two-dimensional skill heterogeneity on both sides of the matching market, first without a skill upgrade mechanism. I start by presenting the model's assumptions.

1.3.1 Environment and Main Assumptions

Time is continuous. The economy is populated by workers with a population size normalized to one. Workers have two skill dimensions: a generic skill dimension and a host-country language proficiency dimension, which can both be either low or high. Skill endowments of workers are denoted by $(g^i, s^i) \in \{l, h\} \times \{l, h\}$ [1] with the g representing the generic skill and s representing host-country language proficiency, the superscript i relates to the individual i's skills. Firms open job vacancies by announcing minimum skill requirements at each skill dimension. Skill requirements are denoted by $(g^j, s^j) \in \{l, h\} \times \{l, h\}$, where the superscript j relates to firm j's skill requirements for the job it advertised. A worker has to satisfy the minimum skill requirements announced by a firm, $g^i \succsim g^j$ and $s^i \succsim s^j$, in order to be productive. There is free entry for firms to create job vacancies. All workers and firms are risk-neutral, live infinitely and discount the future at the same discounting rate, r, lower than one.

Production takes place when a firm and a worker match. Produced output is shared between the firm and the worker in a match. Workers' share or their wages is determined via a wage bargaining mechanism. Unemployed workers receive a flow utility that can be interpreted as home production or leisure gain. Workers are free to apply to any job they prefer over their current employment status. The matching mechanism is random. The number of matches in a job market, characterized by the minimum skill requirements, is assumed to have a closed-from matching function that depends on the number of job vacancies and the number of job applicants in this market.

Job destruction in the economy occurs via a) an exogenous channel: at each

[1] l (h) stands for low-skill (high-skill) with a preference relation $h \succsim l$.

job market, exogenous job destruction arrives at a Poisson rate, and b) an endogenous channel: some workers quit their jobs when they find a job yielding a higher lifetime utility.

1.3.2 Population dynamics and matching rates

The population consists of employed and unemployed workers. Workers endowed with high skills in both dimensions are eligible to work in all sectors. I use the following notation for the population shares of workers with high-high skill endowments working at each of the four sectors: $\pi^{hh,hh}, \pi^{hh,hl}, \pi^{hh,lh}, \pi^{hh,ll}$. Workers with high generic skills and low language proficiency endowments are eligible to work in two sectors: hl and ll. Therefore, they are $\pi^{hl,hl}, \pi^{hl,ll}$. Similarly, there are two possibilities for the workers with low generic skill and high language proficiency endowments: lh and ll. The population shares of these workers are $\pi^{lh,lh}, \pi^{lh,ll}$. Workers with low-skill endowments at both dimensions can only work at the ll -sector: $\pi^{ll,ll}$. Finally, there are four types of unemployed workers according to their skill endowments: $\pi^{hh,u}, \pi^{hl,u}, \pi^{lh,u}, \pi^{ll,u}$.

Without a skill upgrade mechanism, cumulative population shares of four worker groups, $\gamma^{(g^i,s^i)}$, $\forall (g^i, s^i) \in \{l,h\} \times \{l,h\}$, are constant, where the sum of γ's is equal to 1 for all t:

$$\begin{aligned} \gamma^{(g^i,s^i)} &= \pi_t^{(g^i,s^i),u} + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ s^i \succsim s^{\tilde{j}}}} \pi_t^{(g^i,s^i),(g^{\tilde{j}},s^{\tilde{j}})} \\ 1 &= \gamma^{hh} + \gamma^{hl} + \gamma^{lh} + \gamma^{ll} \end{aligned} \tag{1}$$

The population dynamics of employed workers can be explained as follows. At each time t, the population share of each worker-sector match, $\pi_t^{(g^i,s^i),(g^j,s^j)}$, changes by an outflow rate and an inflow rate. Outflow occurs via two channels. First, some matches at each sector face a job destruction shock at a sector-specific constant rate, $\chi^{(g^j,s^j)}$, $\forall (g^j, s^j) \in \{l,h\} \times \{l,h\}$, leaving the firm with an open vacancy and the worker unemployed. Second, some workers employed at (g^j, s^j) find a more preferred job, $(g^{\tilde{j}}, s^{\tilde{j}})$, via on-the-job search and vacate their positions. Inflow occurs simply via matches between job seekers and open job vacancies in each sector, that is the job finding rate[2], $p^{(g^j,s^j)}$, times the sum of the workers that find (g^j, s^j) preferable over $(g^{\tilde{j}}, s^{\tilde{j}})$, $\sum_{(g^j,s^j)\succsim(g^{\tilde{j}},s^{\tilde{j}})} \pi^{(g^i,s^i),(g^{\tilde{j}},s^{\tilde{j}})}$. As a result, the following equation represents the population dynamics:[3]

$$\dot{\pi}_t^{(g^i,s^i),(g^j,s^j)} = -\Big(\chi^{(g^j,s^j)} + \sum_{(g^j,s^j)\precsim(g^{\tilde{j}},s^{\tilde{j}})} p^{(g^{\tilde{j}},s^{\tilde{j}})}\Big)\pi_t^{(g^i,s^i),(g^j,s^j)} + p^{(g^j,s^j)} \sum_{(g^j,s^j)\succsim(g^{\tilde{j}},s^{\tilde{j}})} \pi^{(g^i,s^i),(g^{\tilde{j}},s^{\tilde{j}})} \tag{2}$$

The number of worker-firm matches implies the number of the unemployed as $\gamma^{(g^i,s^i)}$'s are constant.

A worker with skills (g^i, s^i) employed at sector $(g^{\tilde{j}}, s^{\tilde{j}})$ would be willing to apply a sector (g^j, s^j) if and a) skills of the worker satisfy the minimum skill requirements of this sector, $g^i \succsim g^j$ and $s^i \succsim s^j$, and b) a) this sector offers them a higher asset value, $(g^j, s^j) \succsim (g^{\tilde{j}}, s^{\tilde{j}})$. Therefore, the number of workers that would be job seekers at each sector, $\Pi_t^{(g^j,s^j)}$ can be expressed by the following equation:

$$\Pi_t^{(g^j,s^j)} = \sum_{\substack{g^i\succsim g^j \\ s^i\succsim s^j}} \Big(\gamma^{(g^i,s^i)} - \sum_{(g^j,s^j)\precsim(g^{\tilde{j}},s^{\tilde{j}})} \pi_t^{(g^j,s^j),(g^{\tilde{j}},s^{\tilde{j}})}\Big) \tag{3}$$

Here, the first summation stands for the first condition. Only workers that satisfy the skill requirements can be job seekers at sector (g^j, s^j). The terms within parentheses express that among workers with skills (g^i, s^i) only the unemployed ones and those that are employed at less preferred sectors $(g^{\tilde{j}}, s^{\tilde{j}})$ would be job-seekers at sector (g^j, s^j).

Job seekers and firms with open job vacancies match randomly. I assume the following standard Cobb-Douglas function, which has been widely used in the literature (Blanchard and Diamond [1989], Petrongolo [2001]), as the matching function giving the number of matches at jobs with minimum skill requirements (g^j, s^j) at time t,

[2]The job finding rates are given by the equation (5).

[3]The whole set of equations governing the time derivatives of the population shares of each worker-firm match is given in Appendix 1.8.1 as the equation set (21).

$M_t^{(g^j,s^j)}$.

$$M_t^{(g^j,s^j)} = A^{(g^j,s^j)}(\Pi_t^{(g^j,s^j)})^{\alpha}(v_t^{(g^j,s^j)})^{1-\alpha} \tag{4}$$

where $A^{(g^j,s^j)}$ is a scale parameter, $\alpha_{(g^j,s^j)}$ stands for the elasticity of the number of matches with respect to the number of job seekers and $v_t^{(g^j,s^j)}$ is the number of open vacancies at sector (g^j, s^j).

Job finding rate at sector (g^j, s^j) at time t, $p_t^{(g^j,s^j)}$, is simply the matching rate divided by the number of applicants:

$$p_t^{(g^j,s^j)} = A^{(g^j,s^j)}(\Pi_t^{(g^j,s^j)})^{\alpha-1}(v_t^{(g^j,s^j)})^{1-\alpha} \tag{5}$$

In the same fashion, worker finding rates, $q_t^{(g^j,s^j)}$, are matching rates divided by the number of vacancies for each sector:

$$q_t^{(g^j,s^j)} = A^{(g^j,s^j)}(\Pi_t^{(g^j,s^j)})^{\alpha}(v_t^{(g^j,s^j)})^{-\alpha_j} \tag{6}$$

1.3.3 Asset values of firms

In case of a match between a worker endowed with skills (g^i, s^i) and a firm with skill requirements (g^j, s^j), production takes place. Each firm pays a fraction of the produced output, $y^{(g^i,s^i),(g^j,s^j)}$, to their worker as wage, $w_t^{(g^i,s^i),(g^j,s^j)}$. Each match at sector (g^j, s^j) may end through the exogenous job destruction channel at rate $\chi^{(g^j,s^j)}$ or the endogenous channel of losing their worker to another sector due to on-the-job search. Therefore, the asset value of a filled vacancy is the remainder of the output after wage payoff, $(y^{(g^i,s^i),(g^j,s^j)} - w_t^{(g^i,s^i),(g^j,s^j)})$, minus the expected value loss due to a possible end of a match, plus the time derivative of the asset value, $\dot{J}_t^{(g^i,s^i),(g^j,s^j)}$:[4]

[4]The whole set of equations governing the asset values of firms are given in Appendix 1.8.1 as the equation set (22).

$$
\begin{aligned}
rJ_t^{(g^i,s^i),(g^j,s^j)} &= y^{(g^i,s^i),(g^j,s^j)} - w_t^{(g^i,s^i),(g^j,s^j)} + \dot{J}_t^{(g^i,s^i),(g^j,s^j)} \\
&\quad - (\chi^{(g^j,s^j)} + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ s^i \succsim s^{\tilde{j}} \\ (g^j,s^j) \precsim (g^{\tilde{j}},s^{\tilde{j}})}} p^{(g^{\tilde{j}},s^{\tilde{j}})}) J_t^{(g^i,s^i),(g^j,s^j)}
\end{aligned} \tag{7}
$$

Holding a job vacancy incurs a fixed vacancy holding cost, $k^{(g^j,s^j)}$. We let $V_t^{(g^j,s^j)}$ denote the asset value of holding a job vacancy. A firm with a job vacancy with a skill requirement (g^j, s^j), matches with a worker at rate $q_t^{(g^j,s^j)}$, and a match yields an asset value increase from $V_t^{(g^j,s^j)}$ to $J_t^{(g^i,s^i),(g^j,s^j)}$ in case of a match with a worker with skill endowments (g^i, s^i). The asset value gain of firms at sector hh is straightforward, $(J_t^{hh,hh} - V_t^{hh})$, as they can only match with hh-type workers. Thus, their expected gain is the probability of matching with a worker times the asset value gain, $(J_t^{hh,hh} - V_t^{hh})$. As the other sectors allow for matches with workers over-qualified with at least one skill requirement, their expected asset value gain in case of a match depends on the probabilities of matching with each worker type and the corresponding asset values of filled jobs, $J_t^{(g^i,s^i),(g^j,s^j)}$. Therefore, the asset values of holding job vacancies are:[5]

$$
\begin{aligned}
rV_t^{(g^j,s^j)} =& - k^{(g^j,s^j)} + \dot{V}_t^{(g^j,s^j)} + q_t^{(g^j,s^j)} \\
& \left(\left(\sum_{\substack{g^{\tilde{i}} \succsim g^j \\ s^{\tilde{i}} \succsim s^j}} J^{(g^{\tilde{i}},s^{\tilde{i}}),(g^j,s^j)} \sum_{(g^{\tilde{j}},s^{\tilde{j}}) \precsim (g^j,s^j)} \frac{\pi_t^{(g^{\tilde{i}},s^{\tilde{i}}),u} + \sum_{(g^{\tilde{j}},s^{\tilde{j}}) \precsim (g^j,s^j)} \pi_t^{(g^{\tilde{i}},s^{\tilde{i}}),(g^{\tilde{j}},s^{\tilde{j}})}}{\Pi_t^{(g^j,s^j)}} \right) - V_t^{(g^j,s^j)} \right)
\end{aligned} \tag{8}
$$

Here, the term inside the second summation gives the share of workers with skills $(g^{\tilde{i}}, s^{\tilde{i}})$ among all job-seekers for a job with skill requirements (g^j, s^j). A worker of $(g^{\tilde{i}}, s^{\tilde{i}})$ skills would be willing to relocate to a (g^j, s^j)-job when being unemployed or working at a less preferred job, $(g^{\tilde{j}}, s^{\tilde{j}}) \precsim (g^j, s^j)$. As the search is random, this share is equal to the probability of finding a $(g^{\tilde{i}}, s^{\tilde{i}})$-worker. The first summation simply

[5]The whole set of equations governing the asset values of holding a vacancy are given in Appendix 1.8.1 as the equation set (23).

sums worker types satisfying (g^j, s^j) skill requirements.

1.3.4 Asset values of workers

Unemployed workers engage in job search and can form a match at a job finding rate $p_t^{(g^j,s^j)}$ with a firm holding a job vacancy at sector (g^j, s^j) in case they are eligible to seek a job at this sector. An unemployed worker with skill endowments (g^i, s^i) that matches with a firm with skill requirements (g^j, s^j) enjoys an asset value increase from $U_t^{(g^i,s^i)}$ to $E_t^{(g^i,s^i),(g^j,s^j)}$. Furthermore, they benefit from an unemployment benefit at a rate b. Therefore, the asset value of an unemployed worker is the value of the unemployment benefit, plus the time derivative of the asset value of staying unemployed, plus the expected asset value gain from finding a job. The following set of equations gives the asset values of the unemployed workers in the absence of skill upgrade possibility:[6]

$$rU_t^{(g^i,s^i)} = b + \dot{U}_t^{(g^i,s^i)} + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ s^i \succsim s^{\tilde{j}}}} p_t^{(g^{\tilde{j}},s^{\tilde{j}})} \left(E_t^{(g^i,s^i),(g^{\tilde{j}},s^{\tilde{j}})} - U_t^{(g^i,s^i)} \right) \tag{9}$$

I introduce the skill upgrade mechanism as follows. Immigrant workers are all assumed to be endowed with low language skills and may opt for acquiring high language skills by paying a cost, c, for this investment. Whereas native workers (hh- and lh-types) are already endowed with high language skills, and thus have no maximization problem. In order to take into account the dual nature of language acquisition costs, i.e. a monetary cost and a heterogeneous effort component, I assume this cost has a mean of μ and is distributed normally with a standard deviation σ, $c \sim \mathcal{N}(\mu, \sigma)$. The benefit of investing in language acquisition, in other words upskilling from hl- to hh-type or upskilling from ll- to lh-type, is that they then become eligible for jobs requiring a high language skill. Thus, it is optimal for an immigrant worker that is subject to a cost of c to upgrade their language skills if its benefit exceeds its cost, c. The equations governing this mechanism for unemployed workers can be written as

[6] The whole set of equations governing the asset values of unemployed workers without skill acquisition are given in Appendix 1.8.1 as the equation set (24).

follows: [7]

$$rU_t^{(g^i,h)} = b + \dot{U}_t^{(g^i,h)} + \sum_{\substack{g^i \gtrsim g^{\tilde{j}} \\ s^{\tilde{j}}}} p_t^{(g^{\tilde{j}},s^{\tilde{j}})} \left(E_t^{(g^i,h),(g^{\tilde{j}},s^{\tilde{j}})} - U_t^{(g^i,h)} \right) \tag{10}$$

$$rU_t^{(g^i,l)}(c) = \max \left\{ b + \dot{U}_t^{(g^i,l)} + \sum_{g^i \gtrsim g^{\tilde{j}}} p_t^{(g^{\tilde{j}},l)} \left(E_t^{(g^i,l),(g^{\tilde{j}},l)} - U_t^{(g^i,l)} \right), -c + rU_t^{(g^i,h)}(c) \right\} \tag{11}$$

The asset values of the employed workers, $E_t^{(g^i,s^i),(g^j,s^j)}$, can be written in the following way. A worker with skill endowments (g^i, s^i) employed at sector (g^j, s^j) earns a wage at rate $w_t^{(g^i,s^i),(g^j,s^j)}$. At each instant, they face the risk of exogenous job destruction at rate $\chi^{(g^j,s^j)}$. In case of exogenous job destruction, their asset value decreases from $E_t^{(g^i,s^i),(g^j,s^j)}$ to the asset value of being unemployed $U_t^{(g^i,s^i)}$. Those seeking a job at a sector (g^j, s^j) providing a higher asset value may form a match at probability $p_t^{(\tilde{g^j,s^j})}$ and consequently enjoy a value increase from $E_t^{(g^i,s^i),(g^j,s^j)}$ to $E_t^{(g^i,s^i),(\tilde{g^j,s^j})}$. Therefore, the asset value of an employed worker is equal to the sum of their wage, expected asset value gain through successful on-the-job search, the time derivative of their asset value, minus the expected value loss due to the probability of facing exogenous job destruction:[8]

$$\begin{aligned} rE_t^{(g^i,s^i),(g^j,s^j)} &= w_t^{(g^i,s^i),(g^j,s^j)} + \dot{E}_t^{(g^i,s^i),(g^j,s^j)} - \chi^{(g^j,s^j)} (E_t^{(g^i,s^i),(g^j,s^j)} - U_t^{(g^i,s^i)}) \\ &+ \sum_{\substack{g^i \gtrsim g^{\tilde{j}} \\ s^i \gtrsim s^{\tilde{j}} \\ (g^j,s^j) \precsim (g^{\tilde{j}},s^{\tilde{j}})}} p_t^{(g^{\tilde{j}},s^{\tilde{j}})} \left(E_t^{(g^i,s^i),(g^{\tilde{j}},s^{\tilde{j}})} - E_t^{(g^i,s^i),(g^j,s^j)} \right) \end{aligned} \tag{12}$$

The language acquisition mechanism is the same for employed workers as the unemployed workers. Here, the benefit of acquiring a high language skill is becoming eligible to conduct an on-the-job search at jobs requiring a high language skill. High-skilled immigrants become eligible for jobs requiring high skills in both skill dimensions, and jobs that do not require a high generic skill but a high language skill. The low-skilled immigrants can only become eligible to work at the latter. Thus, equation sets

[7] The whole set of equations governing the asset values of unemployed workers with skill acquisition are given in Appendix 1.8.1 as the equation set (25).

[8] The whole set of equations governing the asset values of employed workers without skill acquisition are given in Appendix 1.8.1 as the equation set (26).

(11)-(12) characterize the asset values of the employed workers:[9]

$$rE_t^{(g^i,h),(g^j,s^j)} = w_t^{(g^i,h),(g^j,s^j)} + \dot{E}_t^{(g^i,h),(g^j,s^j)} - \chi^{(g^j,s^j)}(E_t^{(g^i,h),(g^j,s^j)} - U_t^{(g^i,h)}) + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ s^{\tilde{j}} \\ (g^j,s^j) \precsim (g^{\tilde{j}},s^{\tilde{j}})}} p_t^{(g^{\tilde{j}},s^{\tilde{j}})} \Big(E_t^{(g^i,h),(g^{\tilde{j}},s^{\tilde{j}})} - E_t^{(g^i,h),(g^j,s^j)} \Big) \tag{13}$$

$$rE_t^{(g^i,l),(g^j,l)}(c) = \max \Big\{ w_t^{(g^i,l),(g^j,l)} + \dot{E}_t^{(g^i,l),(g^j,l)} - \chi^{(g^j,l)}(E_t^{(g^i,l),(g^j,l)} - U_t^{(g^i,l)}) + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ g^j \precsim g^{\tilde{j}}}} p_t^{(g^{\tilde{j}},l)} \Big(E_t^{(g^i,l),(g^{\tilde{j}},l)} - E_t^{(g^i,l),(g^j,l)} \Big), -c + rE_t^{(g^i,h),(g^j,l)}(c) \Big\} \tag{14}$$

1.3.5 Wage determination

The total surplus in a match is the sum of workers' gain and firms' gain from forming a match, that is $(E_t^{(g^i,s^i),(g^j,s^j)} - U_t^{(g^i,s^i)}) + (J_t^{(g^i,s^i),(g^j,s^j)} - V_t^{(g^j,s^j)})$. This surplus is shared between a worker and a firm according to a wage bargaining mechanism à la Pissarides (1994). The assumption in this model is that this surplus is shared according to workers' bargaining powers, $\delta^{(g^i,s^i),(g^j,s^j)} \in (0,1)$, and firms' bargaining powers, $(1-\delta^{(g^i,s^i),(g^j,s^j)}) \in (0,1)$ when a match is formed and re-negotiated continuously when the match is kept active. The following equation gives the surplus sharing mechanism for all eligible $(g^i,s^i),(g^j,s^j)$ pairs:

$$(1 - \delta^{(g^i,s^i),(g^j,s^j)})(E_t^{(g^i,s^i),(g^j,s^j)} - U_t^{(g^i,s^i)}) = \delta^{(g^i,s^i),(g^j,s^j)} J_t^{(g^i,s^i),(g^j,s^j)} \tag{15}$$

[9]The whole set of equations governing the asset values of employed workers with skill acquisition are given in Appendix 1.8.1 as the equation set (27).

1.4 Equilibrium

In this section, I provide the definition of equilibrium and explain the derivation strategy for finding it.

1.4.1 Equilibrium without skill upgrade

Definition 1.1 *A steady-state equilibrium is a set of a) unemployed workers' asset values,* $U^{(g^i,s^i)}$*, b) employed workers' lifetime values,* $E^{(g^i,s^i),(g^j,s^j)}$*, c) asset values of filled vacancies,* $J^{(g^i,s^i),(g^j,s^j)}$*, d) asset values of open vacancies,* $V^{(g^j,s^j)}$*, e) population shares of worker-firm matches and unemployed workers,* $\pi^{(g^i,s^i),(g^j,s^j)}$*, f) vacancy creation rates* $v^{(g^j,s^j)}$*, g) wage levels,* $w^{(g^i,s^i),(g^j,s^j)}$ *that satisfy the equation sets (1)-(9), (12), (15) and a vector of 1) population shares of worker-firm matches,* $\hat{\pi}^{(g^i,s^i),(g^j,s^j)}$*, 2) vacancy creation rates,* $\hat{v}^{(g^j,s^j)}$*, and 3) wage levels* $\hat{w}^{(g^i,s^i),(g^j,s^j)}$ *that give a solution to the system of equation sets (1)-(9), (12), and (15), the free entry conditions,* $V^{(g^j,s^j)} = 0$*, and the steady-state conditions* $\dot{\pi}^{(g^i,s^i),(g^j,s^j)} = \dot{U}^{(g^i,s^i)} = \dot{E}^{(g^i,s^i),(g^j,s^j)} = \dot{J}^{(g^i,s^i),(g^j,s^j)} = \dot{V}^{(g^j,s^j)} = 0$.

At the steady-state, population shares of a) matches between worker types and sectors, and b) unemployed workers of each type are constant, $\dot{\pi}^{(g^i,s^i),(g^j,s^j)} = 0$, leading to the following set of equations:

$$\left(\chi^{(g^j,s^j)} + \sum_{\substack{g^i \succsim g^{\tilde{j}} \\ s^i \succsim s^{\tilde{j}}}} \hat{p}_t^{(g^{\tilde{j}},s^{\tilde{j}})}\right)\hat{\pi}_t^{(g^i,s^i,g^j,s^j)} = \hat{p}_t^{(g^j,s^j)} \sum_{(g^j,s^j)\succ(g^{\tilde{j}},s^{\tilde{j}})} \left(\hat{\pi}_t^{(g^i,s^i,g^{\tilde{j}},s^{\tilde{j}})} + \hat{\pi}_t^{(g^i,s^i,u)}\right) \quad (16)$$

The free entry conditions and the steady-state conditions, $\dot{V}_t^{(g^j,s^j)}$, yield the following set of equations:

$$k^{(g^j,s^j)} = \hat{q}_t^{(g^j,s^j)} \sum_{(g^j,s^j)\succsim(g^{\tilde{j}},s^{\tilde{j}})} \sum_{\substack{g^{\tilde{i}}\succsim g^j \\ s^{\tilde{i}}\succsim s^j}} \frac{\hat{\pi}_t^{(g^{\tilde{i}},s^{\tilde{i}},g^j,s^j)} + \hat{\pi}_t^{(g^{\tilde{i}},s^{\tilde{i}},u)}}{\hat{\Pi}_t^{(g^j,s^j)}} \hat{J}_t^{(g^{\tilde{i}},s^{\tilde{i}},g^j,s^j)} \tag{17}$$

The following equation set that wraps up the equilibrium characterization is obtained as follows. Combining the equation sets (7)-(9) and (12) gives relationships between the asset values of workers and firms. Combining these with equation (15), which represents the wage bargaining mechanism, and imposing the free entry condition eliminates the workers' asset values. Finally, the steady-state conditions $\dot{U}^{(g^i,s^i)} = \dot{E}^{(g^i,s^i),(g^j,s^j)} = \dot{J}^{(g^i,s^i),(g^j,s^j)} = \dot{V}^{(g^j,s^j)} = 0$ yield the following nine equations relating firms' asset values, $\hat{J}^{(g^i,s^i),(g^j,s^j)}$, only to steady-state population shares of the employed, $\hat{\pi}^{(g^i,s^i),(g^j,s^j)}$, and the vacancy creating rates:

$$\begin{aligned}(1-\delta^{(g^i,s^i,g^j,s^j)})(y^{(g^i,s^i,g^j,s^j)} - b) &= (r+\chi^{(g^j,s^j)})\hat{J}_t^{(g^i,s^i,g^j,s^j)} \\ &+ \sum_{\substack{g^j\succsim g^{\tilde{j}} \\ s^j\succsim s^{\tilde{j}}}} \frac{1-\delta^{(g^i,s^i,g^j,s^j)}}{1-\delta^{(g^i,s^i,g^{\tilde{j}},s^{\tilde{j}})}} \delta^{(g^i,s^i,g^{\tilde{j}},s^{\tilde{j}})} \hat{p}_t^{(g^{\tilde{j}},s^{\tilde{j}})} \hat{J}_t^{(g^i,s^i,g^{\tilde{j}},s^{\tilde{j}})}\end{aligned} \tag{18}$$

Therefore, the equation sets (16)-(18) characterize the 22x22 equation-unknown system at the equilibrium:[10] where the unknowns are a) nine population shares of steady-state worker-firm matches, $\hat{\pi}^{(g^i,s^i),(g^j,s^j)}$, b) four vacancy creation rates, $\hat{v}^{(g^j,s^j)}$, and c) nine asset values of filled vacancies $\hat{J}^{(g^i,s^i),(g^j,s^j)}$. From this point on, the vector $\overrightarrow{\hat{\mathcal{S}}} = [\hat{\pi}^{(g^i,s^i),(g^j,s^j)}, \hat{J}^{(g^i,s^i),(g^j,s^j)}, \hat{v}^{(g^j,s^j)}]'$ denotes these 22 steady-state values. Subsequently, the wage rates can also be obtained from the solution of this 22x22 equation-unknown system, and they are given in Appendix 1.8.1.

[10] The whole sets of equations governing the equilibrium are given in Appendix 1.8.1 as the equation sets (28)-(30).

1.4.2 Equilibrium with skill upgrade

In this subsection, I present the procedure to obtain the equilibrium with skill upgrade as no tractable solutions exist. This iterative procedure consists of six steps.

1. The equilibrium is characterized without a skill upgrade possibility, as presented by the equation sets (16)-(18) and denoted by $\overrightarrow{\hat{\mathcal{S}}}$. This steady-state is stored as $\overrightarrow{\mathcal{S}^0} = \overrightarrow{\hat{\mathcal{S}}}$ implying the 0-th iteration step.

2. By using equation sets (9) and (12), employed workers' asset values (nine values), and unemployed workers' asset values (four values) are obtained and stored as the vector $\overrightarrow{\mathcal{W}}^0$.

3. Workers that are endowed with low skills in both skill dimensions can be a) employed at jobs requiring ll-skills, or b) unemployed. Workers that are endowed with high generic skills and low language skills can be c) employed at jobs requiring hl-skills, d) jobs requiring ll-skills, or e) unemployed. As there is no condition asserting the same level of language acquisition benefits for these cohorts, one should expect different rates of language acquisition (i.e. between unemployed hl-workers and between hl-workers employed at hl-jobs). As normal distribution with a mean μ and a standard deviation of σ is assumed for the language acquisition cost distribution, the cumulative distribution function of the benefit of language acquisition gives the fractions of workers who find it infeasible to invest. Thus, the rates of language acquisition can be written as follows:

This is done by using the following equations:

$$\begin{aligned}\phi^{hl,hl(\iota)} &= 1 - \Phi[\hat{w}^{hh,hl(\iota)} - \chi^{hl}(\hat{E}^{hh,hl(\iota)} - \hat{U}^{hh(\iota)}) + \hat{p}^{hh(\iota)}(\hat{E}^{hh,hh(\iota)} - \hat{E}^{hh,hl(\iota)}) \\ &\qquad - \hat{w}^{hl,hl(\iota)} + \chi^{hl}(\hat{E}^{hl,hl(\iota)} - \hat{U}^{hl(\iota)}) \leq \mu]\end{aligned} \tag{19a}$$

$$\begin{aligned}\phi^{hl,ll(\iota)} &= 1 - \Phi[\hat{w}^{hh,ll(\iota)} - \chi^{ll}(\hat{E}^{hh,ll(\iota)} - \hat{U}^{hh(\iota)}) + \hat{p}^{lh(\iota)}(\hat{E}^{hh,lh(\iota)} - \hat{E}^{hh,ll(\iota)}) \\ &\qquad + \hat{p}^{hl(\iota)}(\hat{E}^{hh,hl(\iota)} - \hat{E}^{hh,ll(\iota)}) + \hat{p}^{hh(\iota)}(\hat{E}^{hh,hh(\iota)} - \hat{E}^{hh,ll(\iota)}) \\ &\qquad - \hat{w}^{hl,ll(\iota)} + \chi^{ll}(\hat{E}^{hl,ll(\iota)} - \hat{U}^{hl(\iota)}) - \hat{p}^{hl(\iota)}(\hat{E}^{hl,hl(\iota)} - \hat{E}^{hl,ll(\iota)}) \leq \mu]\end{aligned} \tag{19b}$$

$$\begin{aligned}\phi^{ll,ll(\iota)} &= 1 - \Phi[\hat{w}^{lh,ll(\iota)} - \chi^{ll}(\hat{E}^{lh,ll(\iota)} - \hat{U}^{lh(\iota)}) + \hat{p}^{lh(\iota)}(\hat{E}^{lh,lh(\iota)} - \hat{E}^{lh,ll(\iota)}) \\ &\qquad - \hat{w}^{ll,ll(\iota)} + \chi^{ll}(\hat{E}^{ll,ll(\iota)} - \hat{U}^{ll(\iota)}) \leq \mu]\end{aligned} \tag{19c}$$

$$\begin{aligned}\phi^{hl,u(\iota)} &= 1 - \Phi[\hat{p}^{ll(\iota)}(\hat{E}^{hh,ll(\iota)} - \hat{U}^{hh(\iota)}) + \hat{p}^{lh(\iota)}(\hat{E}^{hh,lh(\iota)} - \hat{U}^{hh(\iota)}) \\ &\qquad + \hat{p}^{hl(\iota)}(\hat{E}^{hh,lh(\iota)} - \hat{U}^{hh(\iota)}) + \hat{p}^{hh(\iota)}(\hat{E}^{hh,hh(\iota)} - \hat{U}^{hh(\iota)}) \\ &\qquad - \hat{p}^{hl(\iota)}(\hat{E}^{hl,hl(\iota)} - \hat{U}^{hl(\iota)}) - \hat{p}^{ll(\iota)}(\hat{E}^{hl,ll(\iota)} - \hat{U}^{hl(\iota)}) \leq \mu]\end{aligned} \tag{19d}$$

$$\begin{aligned}\phi^{ll,u(\iota)} &= 1 - \Phi[\hat{p}^{ll(\iota)}(\hat{E}^{lh,ll(\iota)} - \hat{U}^{lh(\iota)}) + \hat{p}^{lh(\iota)}(\hat{E}^{lh,lh(\iota)} - \hat{U}^{lh(\iota)}) \\ &\qquad - \hat{p}^{ll(\iota)}(\hat{E}^{ll,ll(\iota)} - \hat{U}^{ll(\iota)}) \leq \mu]\end{aligned} \tag{19e}$$

where $\phi^{(g^i,s^i),(g^j,s^j)(\iota)}$ stands for the fraction of workers with skill endowments (g^i, s^i) and employment status (g^j, s^j) investing in language acquisition at the iteration step ι with the first step being $\iota = 0$.

4. hl-type (ll-type) workers become hh-type (lh-type) while keeping their employment status. The corresponding population share changes when the language acquisition rates are as denoted in equation set (19) can be represented as the vector,

$\overrightarrow{\pi}^{(\iota+1)}$:

$$\overrightarrow{\pi}^{(\iota+1)} = \begin{bmatrix} \pi^{hh,hh(\iota+1)} \\ \pi^{hh,hl(\iota+1)} \\ \pi^{hh,lh(\iota+1)} \\ \pi^{hh,ll(\iota+1)} \\ \pi^{hl,hl(\iota+1)} \\ \pi^{hl,ll(\iota+1)} \\ \pi^{lh,lh(\iota+1)} \\ \pi^{lh,ll(\iota+1)} \\ \pi^{ll,ll(\iota+1)} \\ \pi^{hh,u(\iota+1)} \\ \pi^{hl,u(\iota+1)} \\ \pi^{lh,u(\iota+1)} \\ \pi^{ll,u(\iota+1)} \end{bmatrix} = \begin{bmatrix} \pi^{hh,hh(\iota)} \\ \pi^{hh,hl(\iota)} \\ \pi^{hh,lh(\iota)} \\ \pi^{hh,ll(\iota)} \\ \pi^{hl,hl(\iota)} \\ \pi^{hl,ll(\iota)} \\ \pi^{lh,lh(\iota)} \\ \pi^{lh,ll(\iota)} \\ \pi^{ll,ll(\iota)} \\ \pi^{hh,u(\iota)} \\ \pi^{hl,u(\iota)} \\ \pi^{lh,u(\iota)} \\ \pi^{ll,u(\iota)} \end{bmatrix} + \begin{bmatrix} 0 \\ \phi^{hl,hl(\iota)}\pi^{hl,hl(\iota)} \\ 0 \\ \phi^{hl,ll(\iota)}\pi^{hl,ll(\iota)} \\ -\phi^{hl,hl(\iota)}\pi^{hl,hl(\iota)} \\ -\phi^{hl,ll(\iota)}\pi^{hl,ll(\iota)} \\ 0 \\ \phi^{ll,ll(\iota)}\pi^{ll,ll(\iota)} \\ -\phi^{ll,ll(\iota)}\pi^{ll,ll(\iota)} \\ \phi^{hl,u(\iota)}\pi^{hl,u(\iota)} \\ -\phi^{hl,u(\iota)}\pi^{hl,u(\iota)} \\ \phi^{ll,u(\iota)}\pi^{ll,u(\iota)} \\ -\phi^{ll,u(\iota)}\pi^{ll,u(\iota)} \end{bmatrix} \tag{20}$$

5. However, $\overrightarrow{\pi}^{(\iota+1)}$ leads to a different steady-state than the initial population changes before then language acquisition. At this step, a new steady-state, $\overrightarrow{\mathcal{S}^{\iota+1}}$, is obtained by using the new initial population shares in the equation (20).

6. New values of workers' asset values, $\overrightarrow{\mathcal{W}}^{(\iota+1)}$ are computed by using the equation sets (9) and (12). If the Euclidean distance between the workers' asset values at iteration ι and iteration $\iota+1$ is close enough to zero, $\|\overrightarrow{\mathcal{W}}^{(\iota+1)} - \overrightarrow{\mathcal{W}}^{(\iota)}\| < \varepsilon$, the steady-state at iteration step ι, $\overrightarrow{S}^{(\iota)}$, is the approximate steady-state with costly language acquisition. Else, the iterative procedure repeats itself between steps three and six.

1.5 Data and calibration

In this section, I explain the parameter choices for the simulations conducted to estimate the effects of subsidizing immigrants' language acquisition costs provided at different rates. To this end, I utilize a set of exogenous parameter values used in the search and matching literature, and three data sources. The corresponding parameter value choices are presented in Table 1.

Table 1: Parameter value choices

Parameter	Symbol	Value
Interest rate	r	1/300
Elasticity of vacancies in the matching function	α	0.5
Bargaining powers of job-seekers	$\delta^{(g^i,s^i),(g^j,s^j)}$	0.5
Job separation rates at jobs requiring high generic skills	χ^{hh}, χ^{hl}	0.1
Vacancy creation costs at jobs requiring high generic skills	k^{hh}, k^{hl}	0.5
Unemployment benefit	b	0.4
Productivity at jobs requiring low generic skills	y^l	1
Productivity at jobs requiring high generic skills	y^h	1.5
Population share of high-skilled natives	π_0^{hh}	23.56%
Population share of high-skilled immigrants	π_0^{hl}	5.04%
Population share of low-skilled natives	π_0^{lh}	56.03%
Population share of low-skilled immigrants	π_0^{ll}	15.37%
Mean language acquisition cost	μ	7.2
Matching efficiency parameter	A	1.8682
Job separation rates at jobs requiring low generic skills	χ^{lh}, χ^{ll}	0.089
Vacancy creation costs at jobs requiring low generic skills	k^{lh}, k^{ll}	0.47
Standard deviation of language acquisition cost	σ	2.4323

The first set of parameter values is chosen to be in line with the widely-used values in the literature. The interest rate, r, is set to 1/300, which corresponds to approximately one month[11]. The elasticity of the number of matches with respect to the number of job seekers, α, in the matching function, is set to 0.5 (Petrongolo

[11] r=1/300 corresponds to an annual interest rate of approximately 2.5%

and Pissarides (2001)).[12] Following this elasticity choice, equal bargaining powers, $\delta^{(g^i,s^i),(g^j,s^j)} = 0.5$, are given to workers and firms to satisfy the Hosios efficiency condition (Hosios (1990)). As job separation rates and vacancy creation costs may differ for high- and low-skilled workers, I use fixed values (0.1 and 0.5 respectively [Petrongolo and Pissarides (2001)]) and leave the corresponding values for the low-skilled to be calibrated. The productivity of jobs requiring a low generic skill is normalized to 1, and 50% more productivity is assumed for jobs requiring a high generic skill. For the unemployment benefit, I use a standard 0.4 rate as in Gautier (2002).

The second set of parameters is related to population composition in terms of migrant status, education level and language proficiency attainment. I employ the survey conducted by Diehl et al. (2016). This survey is part of the *The SCIP project ("Causes and Consequences of Socio-Cultural Integration Processes among New Immigrants in Europe")* with a focus on migrants' socio-economic integration trajectories. It is conducted in two waves in 2010/2011 and 2012/2013 with one and a half years in between. The corresponding panel data contains observations on the educational level and German proficiency of Polish and Turkish immigrants in Germany. 1089 individuals start with no knowledge of German. 13.12% of university graduates and 9.77% of individuals with an education level lower than a university degree respond to the question *"How well would you say you speak German?"* as *well* or *very well* in the second wave. To assign a value to mean financial language acquisition cost, I use Goethe Institut language course fees in Germany to attain a B2-level German for an absolute beginner. Finally, I use OECD Labour Force Statistics data for Germany (2012), to assign values for population shares by education and migration status, unemployment rates by education and migration status, and skill mismatch rate.

The remaining four parameters are the matching efficiency parameter in the matching function, job separation rates and vacancy creation costs at jobs requiring low generic skills and the standard deviation of language acquisition cost. I choose values for these parameters that yield equilibrium unemployment and language acquisition rates as close as possible to data. To this end, I minimize the squared weighted difference between the target and simulated values of unemployment rates,

[12] Estimations of this elasticity yield values in the following range: $\alpha \in (0.4, 0.7)$.

$\min \sum[\hat{\pi}^i(u^i_{data} - \hat{u}^i)]^2$, by constraining the equilibrium language acquisition rates to stay in the vicinity of 0.99 times to 1.01 times of their actual values.

The following table presents the target unemployment and language acquisition rates when $A = 1.8682, \chi^{lh} = \chi^{lh} = 0.089, k^{lh} = k^{ll} = 0.47, \sigma = 2.4323$.

Table 2: Unemployment and language acquisition rates - Target vs simulation

Parameter	Target	Simulation	Target data source
Unemployment rate	3.50%	3.50%	OECD
Skill mismatch rate	6.00%	6.52%	OECD
Unemployment rate among high-skilled natives	1.40%	1.51%	OECD
Unemployment rate among high-skilled immigrants	4.80%	4.63%	OECD
Unemployment rate among low-skilled natives	3.53%	3.49%	OECD
Unemployment rate among low-skilled immigrants	6.18%	6.21%	OECD
High-skilled immigrant language acquisition rate	13.12%	13.22%	GESIS
Low-skilled immigrant language acquisition rate	9.77%	9.74%	GESIS

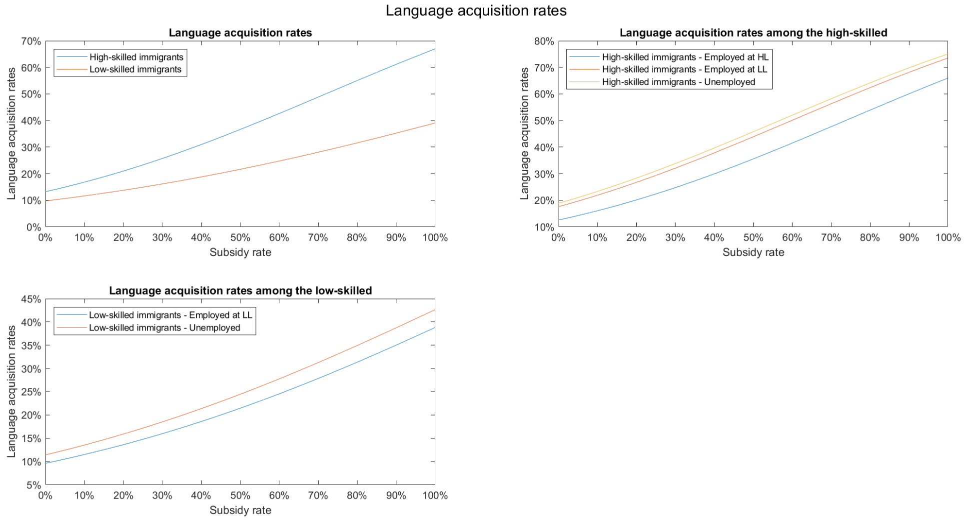
1.6 Results

In this section, I simulate the effects of language acquisition subsidies on the economic performance of immigrants directly, and native workers indirectly. I start with the case of not having a language acquisition subsidy by using the baseline parameter values presented in the previous section. Then, I simulate the model for different subsidy rates for each percentage point between providing no subsidy and a 100% subsidy for the financial costs of language acquisition.

1.6.1 Language acquisition

Figure 1 and Table 3 present the simulated language acquisition rate changes for immigrant workers for each percentage increase in language acquisition subsidies. Simulations start with 13.2% of the high-skilled immigrants and 9.7% of the low-skilled immigrants acquiring language proficiency in the absence of language acquisition subsidies. By using this benchmark scenario, the model generates language acquisition rates of 67.0% and 39.1% for high- and low-skilled immigrants, respectively, when the financial cost of language acquisition is fully subsidized, as shown in the upper-left part of Figure 1. This difference comes from the fact that the expected gain of language acquisition is higher for high-skilled immigrants due to two factors. First, language acquisition decreases the unemployment probability for high-skilled immigrant workers more than for their low-skilled counterparts. It enables high-skilled immigrant workers to be recruited at two more types of firms, with skill requirements being hh and lh, while it enables low-skilled workers to be recruited at only one more firm type, with an lh requirement. Second, the model generates a minute difference between the wage rates for job vacancies requiring lh and ll skills, leading to a minute gain in expected income for a low-skilled immigrant worker given that they find employment. At the same time, the model generates a significant difference between the wage rates for job vacancies requiring hh and hl skills, leading to a significant gain of language acquisition for a high-skilled immigrant worker given that they find employment.

Figure 1: Language acquisition rates by skills and employment statuses



The model also generates different language acquisition rates among high- and low-skilled immigrant workers with different employment statuses. The upper-right part of Figure 1 shows that unemployed high-skilled immigrants acquire language proficiency the most, followed by high-skilled immigrants employed at jobs requiring low and high generic skills, respectively. Language acquisition enables unemployed high-skilled immigrants to be recruited at all job types instead of only jobs requiring hl and ll skills. It enables high-skilled immigrants employed at a job requiring a low generic skill to find a better-paying job at three job types (hh, hl, and lh) instead of only one job type (hl). It enables high-skilled immigrants employed at a job requiring a high generic skill to find a better-paying job at one job type (hh) instead of no improvement possibility. The lower-left part of Figure 1 shows that unemployed low-skilled immigrants acquire a high language skill more than employed low-skilled immigrants. The intuition behind this difference is the same as the intuition behind the difference between high-skilled immigrants that are unemployed and employed at ll-sector.

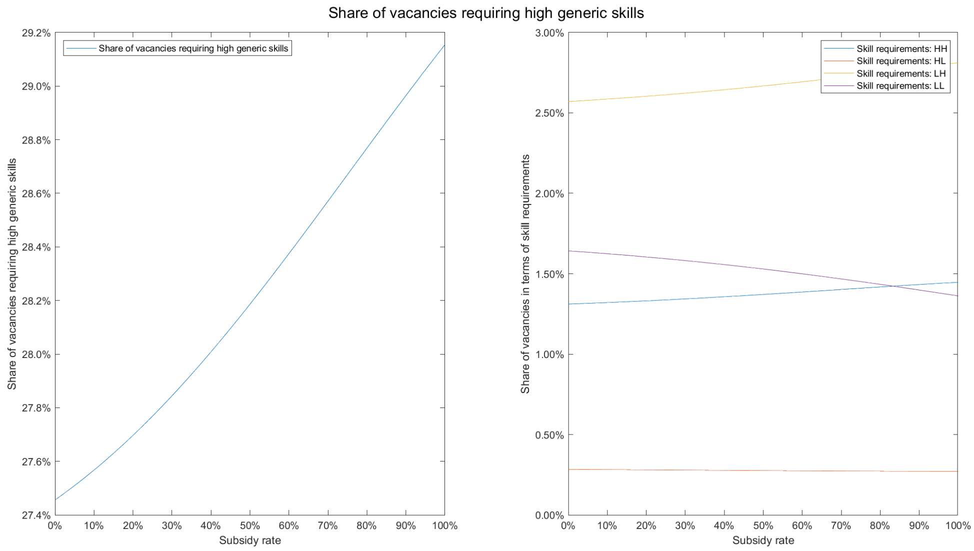
Table 3: Simulated effects of language acquisition subsidies on language acquisition rates

Subsidy rates	0	25%	50%	75%	100%
High-skilled immigrants	13.2%	23.3%	36.7%	51.9%	67.0%
Low-skilled immigrants	9.7%	14.9%	21.7%	29.8%	39.1%
High-skilled immigrants initially employed at hl-jobs	12.6%	22.4%	35.6%	50.8%	66.0%
High-skilled immigrants initially employed at ll-jobs	17.6%	29.4%	43.9%	59.4%	73.5%
Unemployed high-skilled immigrants	18.9%	31.0%	45.8%	61.2%	75.1%
Low-skilled immigrants initially employed at ll-jobs	9.6%	14.8%	21.5%	29.6%	38.8%
Unemployed low-skilled immigrants	11.4%	17.2%	24.5%	33.1%	42.7%

1.6.2 Vacancy creation

Figure 2 presents the effects of language acquisition subsidies on vacancy creation rates. The left part of the figure depicts the shares of vacancies for highly productive jobs among all job vacancies. I find that the share of highly productive job vacancies increases from 27.4% to 29.2% as a result of subsidizing the financial costs of immigrants' language acquisition fully. This is due to the fact that in the absence of a language subsidy, 13.2% of high-skilled immigrant workers choose to learn the local language, resulting in only 84.70% of highly skilled workers (all high-skilled natives and 13.2% of high-skilled immigrants) being proficient in the local language. These workers can move to a highly productive job irrespective of the language skill requirement (a job requiring hh and hl skill sets). Therefore, when matched with a high-skilled worker, a firm requiring a low generic skill face the risk of losing 84.70% of its worker to either of hh- or hl-sectors, while 15.30% of its worker is constrained to move to only the hl-sector. Then, the share of high-skilled workers with language proficiency among all high-skilled workers increases from 84.70% to 94.18% as more high-skilled immigrant workers (from 13.2% to 67.0%) improve their language skills as a result of subsidizing language acquisition costs of immigrants fully. This change creates a higher hazard of losing a high-skilled worker to a better-paying job, lowering the incentive of creating a job vacancy with a low-skill requirement.

Figure 2: Vacancy creation rate changes



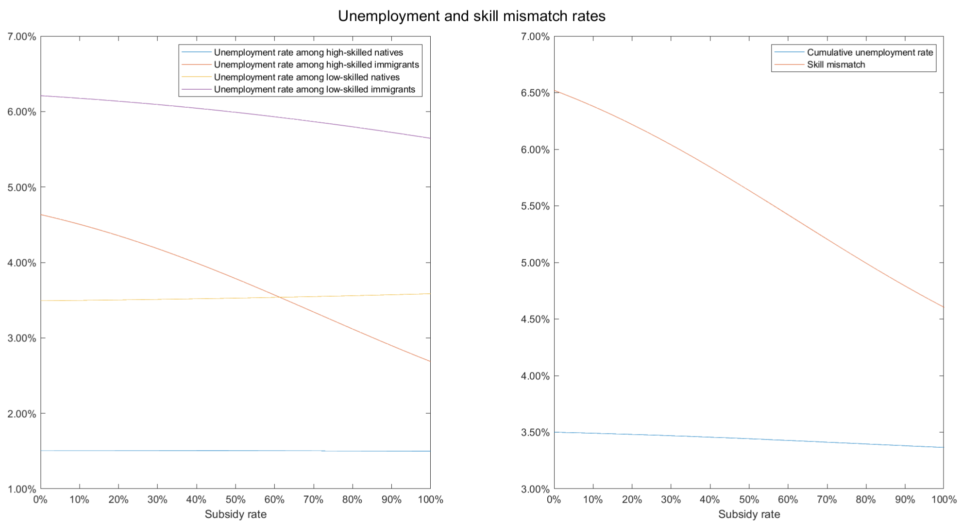
The right part of Figure 2 illustrates the decomposition of vacancy creation rate changes in each sector. Vacancy creation with high skill requirements at both skill dimensions (v^{hh}) increases from 1.31% to 1.45% of the population. This is driven by the increase in the pool of workers that the hh-sector can recruit. Vacancy creation with a high generic and a low language skill requirement slightly decreases from 0.28% to 0.27% of the population. Although the pool of workers this sector can attract remains the same, the hazard of losing a worker becomes higher through two channels. First, more immigrant workers acquire high language skills making them eligible to match with a vacancy in the hh-sector. Second, the vacancy creation rate increase at the hh-sector further exacerbates the hazard of losing any worker to the hh-sector.

Vacancy creation with a low generic and a high language skill increases from 2.57% to 2.81% of the population. There are two factors with opposite effects on this change. As more immigrant workers acquire language skills, the number of workers an lh-firm can recruit increases. On the other hand, the probability of losing a high-skilled immigrant to a highly productive job increases as the vacancy creation rate at the hh-sector increase and also as high-skilled immigrant workers that newly acquire language proficiency gains eligibility to move to the hh-sector. The vacancy creation rate in the sector requiring only low skills at both skill dimensions decreases from 1.64% to 1.36% of the population. The pool of workers it can attract remains the same, the whole workforce. The exit rate from this sector increase for high-skilled workers same as the lh-sector. Additionally, a higher number of low-skilled immigrants gaining eligibility to apply for the lh-sector, and a higher vacancy creation rate at the lh-sector further reduce the rate of vacancy creation in this sector. Overall, my simulations produce a vacancy creation rate increase from 5.81% to 5.89% of the population following fully subsidizing immigrants' financial language acquisition costs.

1.6.3 Unemployment

Figure 3 and Table 4 depict the effects of language acquisition subsidies on unemployment rates. The left side of the figure contains unemployment rate changes for natives and immigrants in terms of their generic skill levels. Language acquisition translates into significantly lower unemployment rates for immigrants as they become eligible to work in more and better-paying jobs. My simulations generate an unemployment rate decrease from 4.63% to 2.69% for high-skilled and from 6.21% to 5.65% for low-skilled immigrant workers. [13]

Figure 3: Unemployment and skill mismatch rates



Improving immigrants' language skills has mixed indirect effects on native work-

[13] Population shares of a) employed workers by skill endowments and employment statuses, b) and unemployed workers by skill endowments are given in Appendix 1.8.2 as Figure 8 and Figure 9, respectively.

ers. A higher vacancy creation rate helps alleviate the unemployment rate of high-skilled natives. At the same time, more immigrants become eligible to compete with high-skilled natives. My model implies that the positive effect of more job vacancy creation slightly exceeds the negative effect of higher competition. In consequence, high-skilled natives enjoy a slightly lower unemployment rate (1.51% to 1.50%). Low-skilled natives suffer from a higher unemployment rate from 3.49% to 3.59%. This increase is caused by two factors: lower job-finding probability and higher competition. Financing immigrants' language acquisition results in a lower overall vacancy creation rate at the low-skill segment of job vacancies (4.21% to 4.17%), decreasing the probability of job finding. Language acquisition further increases competition for jobs requiring a low generic and a high language skill, exacerbating the unemployment rate of low-skilled natives.

The right side of the figure shows the simulated effects of language acquisition subsidies on the aggregate unemployment rate and skill mismatch rate. The above-mentioned unemployment rate changes imply an unemployment rate decrease from 3.50% to 3.36% when all financial costs of language acquisition are subsidized. As this subsidy significantly improves the allocation of immigrant workers to job vacancies skill mismatch rate also decreases significantly from 6.52% to 4.60%.

Table 4: Simulated effects of language acquisition subsidies on unemployment rates

Subsidy rate	0	25%	50%	75%	100%
Unemployment rate	3.50%	3.48%	3.44%	3.40%	3.36%
Skill mismatch rate	6.52%	6.13%	5.64%	5.10%	4.60%
Unemployment rate among high-skilled natives	1.51%	1.51%	1.50%	1.50%	1.50%
Unemployment rate among high-skilled immigrants	4.63%	4.27%	3.79%	3.23%	2.69%
Unemployment rate among low-skilled natives	3.49%	3.51%	3.53%	3.55%	3.59%
Unemployment rate among low-skilled immigrants	6.21%	6.12%	5.99%	5.83%	5.65%

1.6.4 Expected income

Figure 4 shows simulation results for the effects of language acquisition subsidies on wage rates for each possible match type depending on workers' skills and firms' skill requirements. The left (right) part of the figure shows expected wage rate changes for jobs requiring a high (low) generic skill. I observe an increase in wage levels up to 0.45% for jobs requiring a high generic skill and a decrease in wage levels up to a loss of 1.32% for jobs requiring a low generic skill. The intuition is the following: Highly productive vacancy supply increases for a constant vacancy demand as high-skilled labour supply remains constant. This results in firms enjoying a lower part of the output created by a match. As the vacancy supply of jobs requiring only a low-generic skill decrease with a constant size of the population being eligible to work at this skill segment, a higher part of the output remains at firm. An increase in overall productivity also drags all wages slightly upward.

Figure 4: Wage rate changes

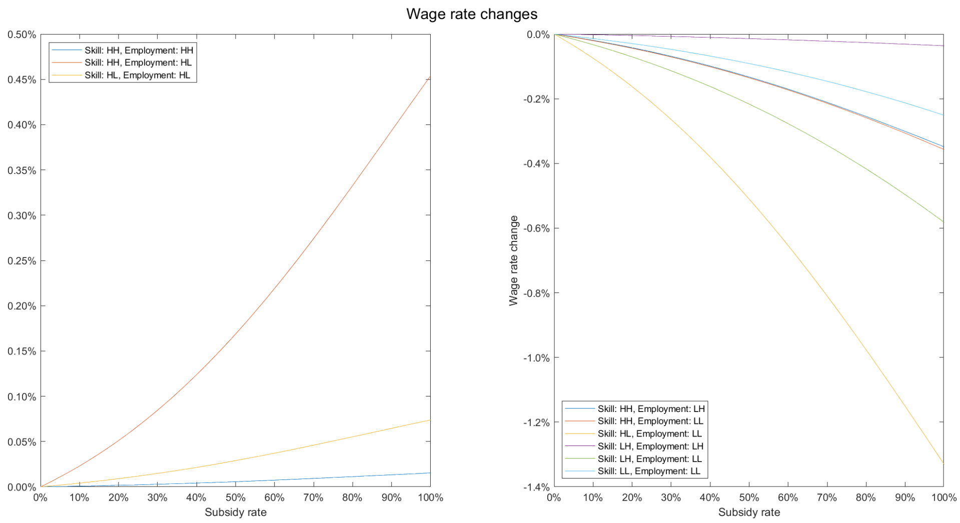
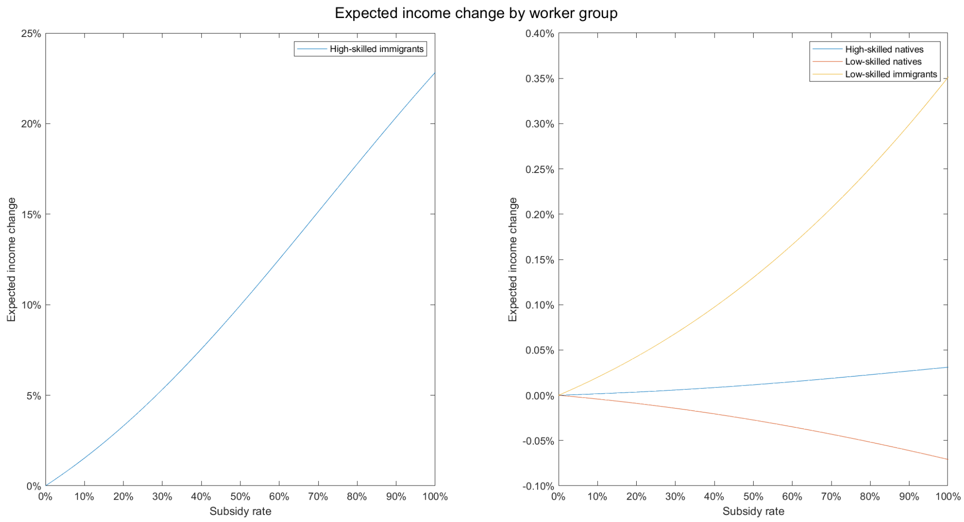


Figure 5 illustrates the simulated effects of language acquisition subsidies on high- and low-skilled immigrant and native workers. The main winners of the subsidy scheme are high-skilled immigrants with an expected income increase of 22.8%. This increase is due to a lower unemployment rate (4.63% to 2.69%), and a significantly higher proportion of high-skilled immigrants (13.2% to 69.0%) becoming eligible at hh-sector. [14] A higher employment rate and a lower expected wage rate given employment resulted in a slight 0.35% increase in the expected income of low-skilled immigrants. High-skilled natives enjoy slight increases in their employment rate and expected wage given employment, together with a slight decrease in job quality due to higher competition. These effects translate into a 0.03% increase in their expected income. The only losers of this subsidy scheme are low-skilled natives. Their expected

[14] My model generates that a match between a hh-type worker and a hh-type job offers a wage 41.4% higher than a match between a hl-type worker and a hl-type job. The simulated wage rates offered for each worker-firm match can be found in Appendix 1.8.2 as Table 5.

income is simulated to decrease slightly by 0.07%. This decrease is caused by a lower employment rate, a lower wage rate given employment and alleviated by a slightly better job quality due to the increase in job vacancy creation rate at lh-sector.

Figure 5: Expected income changes by worker group



1.6.5 Aggregate outcomes

Figure 6 shows the effects of language acquisition subsidies on aggregate production and the time needed to pay off their cost. Here, the aggregate subsidy cost is calculated as the number of individuals that improve their language skills times the amount of financial support per person, which is the mean language acquisition cost, μ, multiplied by the subsidy rate. Simulations imply that subsidizing language acquisition costs fully is optimal. A full subsidy may bring a 0.25% increase in production, with paying off its cost in less than eleven years.

Figure 6: Impact on production

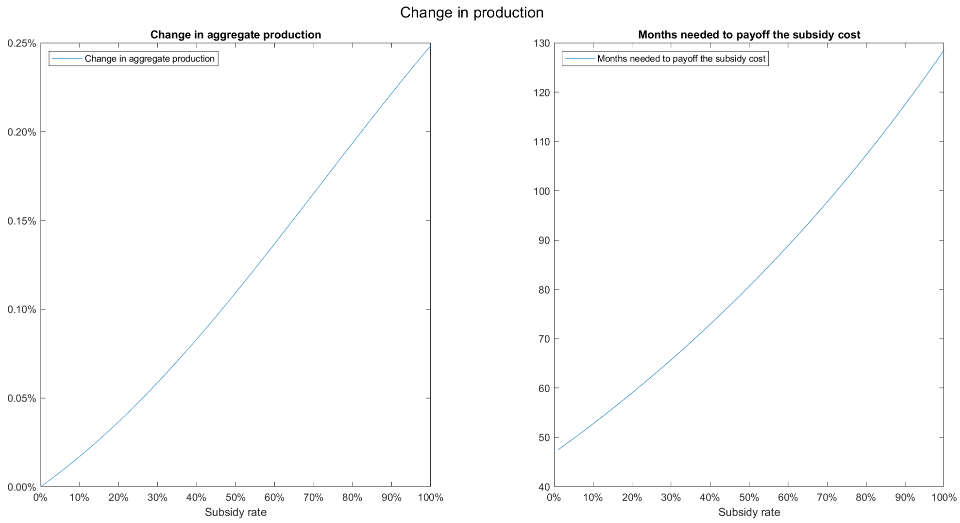
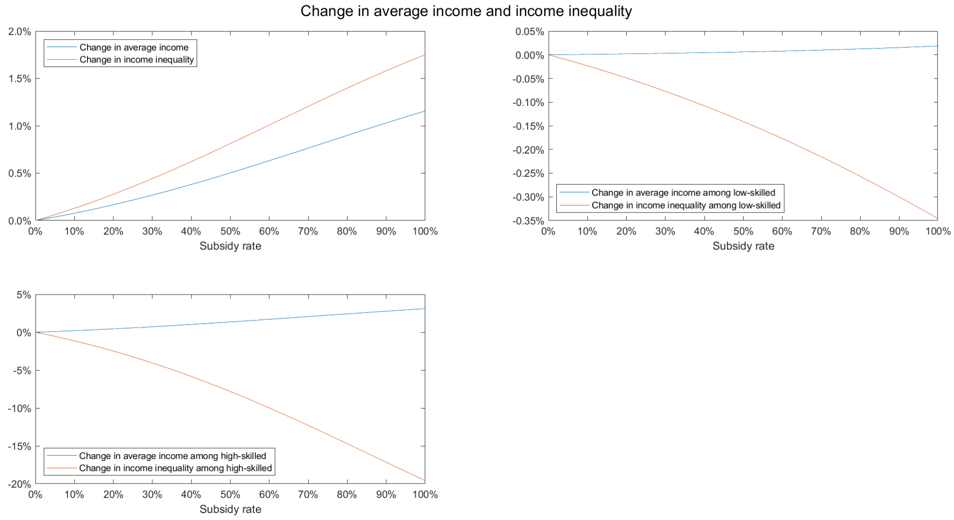


Figure 7 depicts changes in average income and income inequality. Average income increases by 1.16% accompanied by a deterioration of income equality. Income inequality increases by 1.75% when inequality is expressed in terms of the coefficient of variation of income. The reason behind this increase is that the average income of the high-skilled increases by 3.12% while the average income of the low-skilled almost remains unchanged with a 0.02% increase. However, language acquisition subsidies are simulated to decrease income inequality among high- and low-skilled workers. Income inequality within high-skilled workers decreases by 19.6%. This decrease is attributed to high-skilled immigrants' expected income increase by 22.8% while high-skilled natives enjoy only a 0.03% increase in their expected income, implying a narrower gap between these two cohorts. The gap between income levels of low-skilled natives and immigrants also shrinks as a result of natives slightly losing their income by 0.07% while immigrants enjoy a 0.35% increase. This simulated result implies a 0.35% decrease in income inequality among low-skilled workers.

Figure 7: Average income and income inequality



1.7 Conclusion

Economists have a consensus on the benefits of high-skilled migration for both the migrants and their destination countries. Language barriers appear to be one of the most prominent factors hampering the labour market integration of immigrants, in particular the high-skilled. In this study, I argue that fully funding the financial costs of migrants' language acquisition may bring significant gains for the economy, migrants, and many natives. However, these gains can be achieved to the extent of marginally worsening the labour market performance of low-skilled natives. Still, it is clear that the net aggregate effect of the language training subsidy is positive. While introducing redistributive policies goes beyond the scope of this study, it could be an interesting avenue for future research.

I modelled the labour market by using a search and matching framework of the Diamond-Mortensen-Pissarides type with two-dimensional skill heterogeneity on both sides of the labour market. My model contains a costly skill upgrade mechanism with which migrants can acquire host-country language proficiency, making them eligible for a higher set of jobs. I simulate the effects of a policy of subsidizing the language acquisition cost of migrants on migrants' labour market integration and the effects of this integration on natives' labour market performance. By using data on a subset of migrants in Germany, I forecast that fully subsidizing the language acquisition costs of immigrants may bring an additional approximately ten billion dollars to the GDP of an economy the size of the German economy. This gain is mainly driven by three channels: a) a simulated aggregate unemployment rate decrease from 3.50% to 3.36%, b) a skill mismatch rate decrease from 6.52% to 4.60%, and c) an increase in the share of job vacancies requiring high generic skills from 27.4% to 29.2% among all job vacancies. As expected, high-skilled migrants enjoy the highest gain from this subsidy policy as they overcome the language barrier and become eligible to work at more jobs in the high generic skill segment. Low-skilled migrants enjoy a lower unemployment rate as they become eligible to be hired for more vacancies. High-skilled natives enjoy minor gains both in terms of expected income and employment rate. The main factor underlying these increases is the higher vacancy creation rate at the high generic skill segment due to firms finding it more attractive to create vacancies with high generic skills. This betterment in the skill composition of new vacancies and the increase in competition for jobs at the low generic skill segment resulted in slightly worse labour market outcomes for low-skilled natives.

This study has two main caveats. First, the economy in this model is a closed economy in the sense that it treats the stock of migrants as given. Such a policy might indeed impact the inflow and outflow of migrants and I do not account for the impact that the language subsidy might have on potential variations in the volume and composition of immigrants. Second, the skill composition in the labour market is the only endogenous factor affecting job vacancy creation rates. This limitation rules out any impact of other potential constraints that may affect the vacancy creation rates.

1.8 Appendix

1.8.1 Equations

Population dynamics The time derivatives of the population shares of each worker-firm match are given in the equation set (21). The first superscript denotes the hired worker's skill endowment, while the latter denotes the firm's skill requirements:

$$\dot{\pi}_t^{hh,hh} = -\chi^{hh}\pi_t^{hh,hh} + p_t^{hh}(\pi_t^{hh,hl} + \pi_t^{hh,lh} + \pi_t^{hh,ll} + \pi_t^{hh,u}) \tag{21a}$$

$$\dot{\pi}_t^{hh,hl} = -(\chi^{hl} + p_t^{hh})\pi_t^{hh,hl} + p_t^{hl}(\pi_t^{hh,lh} + \pi_t^{hh,ll} + \pi_t^{hh,u}) \tag{21b}$$

$$\dot{\pi}_t^{hh,lh} = -(\chi^{lh} + p_t^{hh} + p_t^{hl})\pi_t^{hh,lh} + p_t^{lh}(\pi_t^{hh,ll} + \pi_t^{hh,u}) \tag{21c}$$

$$\dot{\pi}_t^{hh,ll} = -(\chi^{ll} + p_t^{hh} + p_t^{hl} + p_t^{lh})\pi_t^{hh,ll} + p_t^{ll}\pi_t^{hh,u} \tag{21d}$$

$$\dot{\pi}_t^{hl,hl} = -\chi^{hl}\pi_t^{hl,hl} + p_t^{hl}(\pi_t^{hl,ll} + \pi_t^{hl,u}) \tag{21e}$$

$$\dot{\pi}_t^{hl,ll} = -(\chi^{ll} + p_t^{hl})\pi_t^{hl,ll} + p_t^{ll}\pi_t^{hl,u} \tag{21f}$$

$$\dot{\pi}_t^{lh,lh} = -\chi^{lh}\pi_t^{lh,lh} + p_t^{lh}(\pi_t^{lh,ll} + \pi_t^{lh,u}) \tag{21g}$$

$$\dot{\pi}_t^{lh,ll} = -(\chi^{ll} + p_t^{lh})\pi_t^{lh,ll} + p_t^{ll}\pi_t^{lh,u} \tag{21h}$$

$$\dot{\pi}_t^{ll,ll} = -\chi^{ll}\pi_t^{ll,ll} + p_t^{ll}\pi_t^{ll,u} \tag{21i}$$

Firms The asset values of firms are given in the equation set (22). The first superscript denotes the hired worker's skill endowment, while the latter denotes the firm's skill requirements:

$$rJ_t^{hh,hh} = y^{hh,hh} - w_t^{hh,hh} + \dot{J}_t^{hh,hh} - \chi^{hh} J^{hh,hh} \tag{22a}$$

$$rJ_t^{hh,hl} = y^{hh,hl} - w_t^{hh,hl} + \dot{J}_t^{hh,hl} - (\chi^{hl} + p_t^{hh}) J_t^{hh,hl} \tag{22b}$$

$$rJ_t^{hh,lh} = y^{hh,lh} - w_t^{hh,lh} + \dot{J}_t^{hh,lh} - (\chi^{lh} + p_t^{hl} + p_t^{hh}) J_t^{hh,lh} \tag{22c}$$

$$rJ_t^{hh,ll} = y^{hh,ll} - w_t^{hh,ll} + \dot{J}_t^{hh,ll} - (\chi^{ll} + p_t^{lh} + p_t^{hl} + p_t^{hh}) J_t^{hh,ll} \tag{22d}$$

$$rJ_t^{hl,hl} = y^{hl,hl} - w_t^{hl,hl} + \dot{J}_t^{hl,hl} - \chi^{hl} J_t^{hl,hl} \tag{22e}$$

$$rJ_t^{hl,ll} = y^{hl,ll} - w_t^{hl,ll} + \dot{J}_t^{hl,ll} - (\chi^{ll} + p_t^{hl}) J_t^{hl,ll} \tag{22f}$$

$$rJ_t^{lh,lh} = y^{lh,lh} - w_t^{lh,lh} + \dot{J}_t^{lh,lh} - \chi^{lh} J_t^{lh,lh} \tag{22g}$$

$$rJ_t^{lh,ll} = y^{lh,ll} - w_t^{lh,ll} + \dot{J}_t^{lh,ll} - (\chi^{ll} + p_t^{lh}) J_t^{lh,ll} \tag{22h}$$

$$rJ_t^{ll,ll} = y^{ll,ll} - w_t^{ll,ll} + \dot{J}_t^{ll,ll} - \chi^{ll} J_t^{ll,ll} \tag{22i}$$

Vacancies The asset values of holding a vacancy are given in the equation set (23). The superscript denotes the firm's skill requirements:

$$rV_t^{hh} = -k^{hh} + \dot{V}_t^{hh} + q_t^{hh}(J_t^{hh,hh} - V_t^{hh}) \tag{23a}$$

$$\begin{aligned} rV_t^{hl} &= -k^{hl} + \dot{V}_t^{hl} \\ &+ \frac{q_t^{hl}}{\Pi_t^{hl}}[(\pi_t^{hh,lh} + \pi_t^{hh,ll} + \pi_t^{hh,u}) J_t^{hh,hl} + (\pi_t^{hl,ll} + \pi_t^{hl,u}) J_t^{hl,hl} - \Pi_t^{hl} V_t^{hl}] \end{aligned} \tag{23b}$$

$$rV_t^{lh} = -k^{lh} + \dot{V}_t^{lh} + \frac{q_t^{lh}}{\Pi_t^{lh}}[(\pi_t^{hh,ll} + \pi_t^{hh,u}) J_t^{hh,lh} + (\pi_t^{lh,ll} + \pi_t^{lh,u}) J_t^{lh,lh} - \Pi_t^{lh} V_t^{lh}] \tag{23c}$$

$$rV_t^{ll} = -k^{ll} + \dot{V}_t^{ll} + \frac{q_t^{ll}}{\Pi_t^{ll}}[\pi_t^{hh,u} J_t^{hh,ll} + \pi_t^{hl,u} J_t^{hl,ll} + \pi_t^{lh,u} J_t^{lh,ll} + \pi_t^{ll,u} J_t^{ll,ll} - \Pi_t^{ll} V_t^{ll}] \tag{23d}$$

Unemployed workers without skill acquisition The asset values of unemployed workers without skill acquisition are given in the equation set (24). The superscript denotes the worker's skill endowment:

$$
\begin{aligned}
rU_t^{hh} &= b + \dot{U}_t^{hh} + p_t^{hh}(E_t^{hh,hh} - U_t^{hh}) \\
&\quad + p_t^{hl}(E_t^{hh,hl} - U_t^{hh}) + p_t^{lh}(E_t^{hh,lh} - U_t^{hh}) + p_t^{ll}(E_t^{hh,ll} - U_t^{hh})
\end{aligned} \tag{24a}
$$

$$
rU_t^{hl} = b + \dot{U}_t^{hl} + p_t^{hl}(E_t^{hl,hl} - U_t^{hl}) + p_t^{ll}(E_t^{hl,ll} - U_t^{hl}) \tag{24b}
$$

$$
rU_t^{lh} = b + \dot{U}_t^{lh} + p_t^{lh}(E_t^{lh,lh} - U_t^{lh}) + p_t^{ll}(E_t^{lh,ll} - U_t^{lh}) \tag{24c}
$$

$$
rU_t^{ll} = b + \dot{U}_t^{ll} + p_t^{ll}(E_t^{ll,ll} - U_t^{ll}) \tag{24d}
$$

Unemployed workers with skill acquisition The asset values of unemployed workers with skill acquisition are given in the equation set (25). The superscript denotes the worker's skill endowment:

$$
\begin{aligned}
rU_t^{hh} &= b + \dot{U}_t^{hh} + p_t^{hh}(E_t^{hh,hh} - U_t^{hh}) \\
&\quad + p_t^{hl}(E_t^{hh,hl} - U_t^{hh}) + p_t^{lh}(E_t^{hh,lh} - U_t^{hh}) + p_t^{ll}(E_t^{hh,ll} - U_t^{hh})
\end{aligned} \tag{25a}
$$

$$
rU_t^{hl}(c) = \max_{i \in \{hl,hh\}} \{b + \dot{U}_t^{hl} + p_t^{hl}(E_t^{hl,hl} - U_t^{hl}) + p_t^{ll}(E_t^{hl,ll} - U_t^{hl}), -c + rU_t^{hh}\} \tag{25b}
$$

$$
rU_t^{lh} = b + \dot{U}_t^{lh} + p_t^{lh}(E_t^{lh,lh} - U_t^{lh}) + p_t^{ll}(E_t^{lh,ll} - U_t^{lh}) \tag{25c}
$$

$$
rU_t^{ll}(c) = \max_{i \in \{ll,lh\}} \{b + \dot{U}_t^{ll} + p_t^{ll}(E_t^{ll,ll} - U_t^{ll}), -c + rU_t^{lh}(c)\} \tag{25d}
$$

Employed workers without skill acquisition The asset values of employed workers without skill acquisition are given in the equation set (26). The first superscript denotes the hired worker's skill endowment, while the latter denotes the firm's skill requirements:

$$rE_t^{hh,hh} = w_t^{hh,hh} + \dot{E}_t^{hh,hh} - \chi^{hh}(E_t^{hh,hh} - U_t^{hh}) \tag{26a}$$

$$rE_t^{hh,hl} = w_t^{hh,hl} + \dot{E}_t^{hh,hl} - \chi^{hl}(E_t^{hh,hl} - U_t^{hh}) + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,hl}) \tag{26b}$$

$$\begin{aligned} rE_t^{hh,lh} &= w_t^{hh,lh} + \dot{E}_t^{hh,lh} - \chi^{lh}(E_t^{hh,lh} - U_t^{hh}) + p_t^{hl}(E_t^{hh,hl} - E_t^{hh,lh}) \\ &\quad + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,lh}) \end{aligned} \tag{26c}$$

$$\begin{aligned} rE_t^{hh,ll} &= w_t^{hh,ll} + \dot{E}_t^{hh,ll} - \chi^{ll}(E_t^{hh,ll} - U_t^{hh}) + p_t^{lh}(E_t^{hh,lh} - E_t^{hh,ll}) \\ &\quad + p_t^{hl}(E_t^{hh,hl} - E_t^{hh,ll}) + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,ll}) \end{aligned} \tag{26d}$$

$$rE_t^{hl,hl} = w_t^{hl,hl} + \dot{E}_t^{hl,hl} - \chi^{hl}(E_t^{hl,hl} - U_t^{hl}) \tag{26e}$$

$$rE_t^{hl,ll} = w_t^{hl,ll} + \dot{E}_t^{hl,ll} - \chi^{ll}(E_t^{hl,ll} - U_t^{hl}) + p_t^{hl}(E_t^{hl,hl} - E_t^{hl,ll}) \tag{26f}$$

$$rE_t^{lh,lh} = w_t^{lh,lh} + \dot{E}_t^{lh,lh} - \chi^{lh}(E_t^{lh,lh} - U_t^{lh}) \tag{26g}$$

$$rE_t^{lh,ll} = w_t^{lh,ll} + \dot{E}_t^{lh,ll} - \chi^{ll}(E_t^{lh,ll} - U_t^{lh}) + p_t^{lh}(E_t^{lh,lh} - E_t^{lh,ll}) \tag{26h}$$

$$rE_t^{ll,ll} = w_t^{ll,ll} + \dot{E}_t^{ll,ll} - \chi^{ll}(E_t^{ll,ll} - U_t^{ll}) \tag{26i}$$

Employed workers with skill acquisition The asset values of employed workers with skill acquisition are given in the equation set (27). The first superscript denotes the hired worker's skill endowment, while the latter denotes the firm's skill requirements:

$$rE_t^{hh,hh} = w_t^{hh,hh} + \dot{E}_t^{hh,hh} - \chi^{hh}(E_t^{hh,hh} - U_t^{hh}) \tag{27a}$$

$$rE_t^{hh,hl} = w_t^{hh,hl} + \dot{E}_t^{hh,hl} - \chi^{hl}(E_t^{hh,hl} - U_t^{hh}) + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,hl}) \tag{27b}$$

$$\begin{aligned} rE_t^{hh,lh} &= w_t^{hh,lh} + \dot{E}_t^{hh,lh} - \chi^{lh}(E_t^{hh,lh} - U_t^{hh}) \\ &+ p_t^{hl}(E_t^{hh,hl} - E_t^{hh,lh}) + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,lh}) \end{aligned} \tag{27c}$$

$$\begin{aligned} rE_t^{hh,ll} &= w_t^{hh,ll} + \dot{E}_t^{hh,ll} - \chi^{ll}(E_t^{hh,ll} - U_t^{hh}) + p_t^{lh}(E_t^{hh,lh} - E_t^{hh,ll}) \\ &+ p_t^{hl}(E_t^{hh,hl} - E_t^{hh,ll}) + p_t^{hh}(E_t^{hh,hh} - E_t^{hh,ll}) \end{aligned} \tag{27d}$$

$$rE_t^{hl,hl}(c) = \max_{i\in\{hl,hh\}}\{w_t^{hl,hl} + \dot{E}_t^{hl,hl} - \chi^{hl}(E_t^{hl,hl} - U_t^{hl}), -c + rE_t^{hh,hl}\} \tag{27e}$$

$$\begin{aligned} rE_t^{hl,ll}(c) &= \max_{i\in\{hl,hh\}}\{w_t^{hl,ll} + \dot{E}_t^{hl,ll} - \chi^{ll}(E_t^{hl,ll} - U_t^{hl}) + p_t^{hl}(E_t^{hl,hl} - E_t^{hl,ll}), \\ &- c + rE_t^{hh,ll}\} \end{aligned} \tag{27f}$$

$$rE_t^{lh,lh} = w_t^{lh,lh} + \dot{E}_t^{lh,lh} - \chi^{lh}(E_t^{lh,lh} - U_t^{lh}) \tag{27g}$$

$$rE_t^{lh,ll} = w_t^{lh,ll} + \dot{E}_t^{lh,ll} - \chi^{ll}(E_t^{lh,ll} - U_t^{lh}) + p_t^{lh}(E_t^{lh,lh} - E_t^{lh,ll})\} \tag{27h}$$

$$rE_t^{ll,ll}(c) = \max_{i\in\{ll,lh\}}\{w_t^{ll,ll} + \dot{E}_t^{ll,ll} - \chi^{ll}(E_t^{ll,ll} - U_t^{ll}), -c + rE_t^{lh,ll}(c)\} \tag{27i}$$

Equilibrium without skill acquisition The equation sets (28)-(30) characterize the equilibrium without skill acquisition:

$$\chi^{hh}\hat{\pi}_t^{hh,hh} = \hat{p}_t^{hh}(\hat{\pi}_t^{hh,hl} + \hat{\pi}_t^{hh,lh} + \hat{\pi}_t^{hh,ll} + \hat{\pi}_t^{hh,u}) \tag{28a}$$

$$(\chi^{hl} + \hat{p}_t^{hh})\hat{\pi}_t^{hh,hl} = \hat{p}_t^{hl}(\hat{\pi}_t^{hh,lh} + \hat{\pi}_t^{hh,ll} + \hat{\pi}_t^{hh,u}) \tag{28b}$$

$$(\chi^{lh} + \hat{p}_t^{hh} + \hat{p}_t^{hl})\hat{\pi}_t^{hh,lh} = \hat{p}_t^{lh}(\hat{\pi}_t^{hh,ll} + \hat{\pi}_t^{hh,u}) \tag{28c}$$

$$(\chi^{ll} + \hat{p}_t^{hh} + \hat{p}_t^{hl} + \hat{p}_t^{lh})\hat{\pi}_t^{hh,ll} = \hat{p}_t^{ll}\hat{\pi}_t^{hh,u} \tag{28d}$$

$$\chi^{hl}\hat{\pi}_t^{hl,hl} = \hat{p}_t^{hl}(\hat{\pi}_t^{hl,ll} + \hat{\pi}_t^{hl,u}) \tag{28e}$$

$$(\chi^{ll} + \hat{p}_t^{hl})\hat{\pi}_t^{hl,ll} = \hat{p}_t^{ll}\hat{\pi}_t^{hl,u} \tag{28f}$$

$$\chi^{lh}\hat{\pi}_t^{lh,lh} = \hat{p}_t^{lh}(\hat{\pi}_t^{lh,ll} + \hat{\pi}_t^{lh,u}) \tag{28g}$$

$$(\chi^{ll} + \hat{p}_t^{lh})\hat{\pi}_t^{lh,ll} = \hat{p}_t^{ll}\hat{\pi}_t^{lh,u} \tag{28h}$$

$$\chi^{ll}\hat{\pi}_t^{ll,ll} = \hat{p}_t^{ll}\hat{\pi}_t^{ll,u} \tag{28i}$$

$$k^{hh} = \hat{q}_t^{hh} \hat{J}_t^{hh,hh} \tag{29a}$$

$$k^{hl} = \frac{\hat{q}_t^{hl}}{\hat{\Pi}_t^{hl}}[(\hat{\pi}_t^{hh,lh} + \hat{\pi}_t^{hh,ll} + \hat{\pi}_t^{hh,u})\hat{J}_t^{hh,hl} + (\hat{\pi}_t^{hl,ll} + \hat{\pi}_t^{hl,u})\hat{J}_t^{hl,hl}] \tag{29b}$$

$$k^{lh} = \frac{\hat{q}_t^{lh}}{\hat{\Pi}_t^{lh}}[(\hat{\pi}_t^{hh,ll} + \hat{\pi}_t^{hh,u})\hat{J}_t^{hh,lh} + (\hat{\pi}_t^{lh,ll} + \hat{\pi}_t^{lh,u})\hat{J}_t^{lh,lh}] \tag{29c}$$

$$k^{ll} = \frac{\hat{q}_t^{ll}}{\hat{\Pi}_t^{ll}}[\hat{\pi}_t^{hh,u}\hat{J}_t^{hh,ll} + \hat{\pi}_t^{hl,u}\hat{J}_t^{hl,ll} + \hat{\pi}_t^{lh,u}\hat{J}_t^{lh,ll} + \hat{\pi}_t^{ll,u}\hat{J}_t^{ll,ll}] \tag{29d}$$

$$\begin{aligned}(1-\delta^{hh,hh})(y^{hh,hh}-b) &= (r + \chi^{hh} + \delta^{hh,hh}\hat{p}_t^{hh})\hat{J}_t^{hh,hh} \\ &+ \frac{1-\delta^{hh,hh}}{1-\delta^{hh,hl}}\delta^{hh,hl}\hat{p}_t^{hl}\hat{J}_t^{hh,hl} + \frac{1-\delta^{hh,hh}}{1-\delta^{hh,lh}}\delta^{hh,lh}\hat{p}_t^{lh}\hat{J}_t^{hh,lh} \\ &+ \frac{1-\delta^{hh,hh}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll}\end{aligned} \tag{30a}$$

$$\begin{aligned}(1-\delta^{hh,hl})(y^{hh,hl}-b) &= (r + \chi^{hl} + \delta^{hh,hl}\hat{p}_t^{hl} + \hat{p}_t^{hh})\hat{J}_t^{hh,hl} \\ &+ \frac{1-\delta^{hh,hl}}{1-\delta^{hh,lh}}\delta^{hh,lh}\hat{p}_t^{lh}\hat{J}_t^{hh,lh} + \frac{1-\delta^{hh,hl}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll}\end{aligned} \tag{30b}$$

$$\begin{aligned}(1-\delta^{hh,lh})(y^{hh,lh}-b) &= (r + \chi^{lh} + \delta^{hh,lh}\hat{p}_t^{lh} + \hat{p}_t^{hl} + \hat{p}_t^{hh})\hat{J}_t^{hh,lh} \\ &+ \frac{1-\delta^{hh,lh}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll}\end{aligned} \tag{30c}$$

$$(1-\delta^{hh,ll})(y^{hh,ll}-b) = (r + \chi^{ll} + \delta^{hh,ll}\hat{p}_t^{ll} + \hat{p}_t^{lh} + \hat{p}_t^{hl} + \hat{p}_t^{hh})\hat{J}_t^{hh,ll} \tag{30d}$$

$$(1-\delta^{hl,hl})(y^{hl,hl}-b) = (r + \chi^{hl} + \delta^{hl,hl}\hat{p}_t^{hl})\hat{J}_t^{hl,hl} + \frac{1-\delta^{hl,hl}}{1-\delta^{hl,ll}}\delta^{hl,ll}\hat{p}_t^{ll}\hat{J}_t^{hl,ll} \tag{30e}$$

$$(1-\delta^{hl,ll})(y^{hl,ll}-b) = (r + \chi^{hl} + \hat{p}_t^{hl} + \delta^{hl,ll}\hat{p}_t^{ll})\hat{J}_t^{hl,ll} \tag{30f}$$

$$(1-\delta^{lh,lh})(y^{lh,lh}-b) = (r + \chi^{lh} + \delta^{lh,lh}\hat{p}_t^{lh})\hat{J}_t^{lh,lh} + \frac{1-\delta^{lh,lh}}{1-\delta^{lh,ll}}\delta^{lh,ll}\hat{p}_t^{ll}\hat{J}_t^{lh,ll} \tag{30g}$$

$$(1-\delta^{lh,ll})(y^{lh,ll}-b) = (r + \chi^{ll} + \hat{p}_t^{lh} + \delta^{lh,ll}\hat{p}_t^{ll})\hat{J}_t^{lh,ll} \tag{30h}$$

$$(1-\delta^{ll,ll})(y^{ll,ll}-b) = (r + \chi^{ll} + \delta^{ll,ll}\hat{p}_t^{ll})\hat{J}_t^{ll,ll} \tag{30i}$$

Equilibrium wage levels The equilibrium wage levels for each worker-firm match are given in the equation set (31). The first superscript denotes the hired worker's skill endowment, while the latter denotes the firm's skill requirements:

$$\hat{w}^{hh,hh} = \delta^{hh,hh}y^{hh,hh} + (1-\delta^{hh,hh})b + \delta^{hh,hh}\hat{p}_t^{hh}\hat{J}_t^{hh,hh} + \frac{1-\delta^{hh,hh}}{1-\delta^{hh,hl}}\delta^{hh,hl}\hat{p}_t^{hl}\hat{J}_t^{hh,hl} + \frac{1-\delta^{hh,hh}}{1-\delta^{hh,lh}}\delta^{hh,lh}\hat{p}_t^{lh}\hat{J}_t^{hh,lh} + \frac{1-\delta^{hh,hh}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll} \tag{31a}$$

$$\hat{w}^{hh,hl} = \delta^{hh,hl}y^{hh,hl} + (1-\delta^{hh,hl})b + \delta^{hh,hl}\hat{p}_t^{hl}\hat{J}_t^{hh,hl} + \frac{1-\delta^{hh,hl}}{1-\delta^{hh,lh}}\delta^{hh,lh}\hat{p}_t^{lh}\hat{J}_t^{hh,lh} + \frac{1-\delta^{hh,hl}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll} \tag{31b}$$

$$\hat{w}^{hh,lh} = \delta^{hh,lh}y^{hh,lh} + (1-\delta^{hh,lh})b + \delta^{hh,lh}\hat{p}_t^{lh}\hat{J}_t^{hh,lh} + \frac{1-\delta^{hh,lh}}{1-\delta^{hh,ll}}\delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll} \tag{31c}$$

$$\hat{w}^{hh,ll} = \delta^{hh,ll}y^{hh,ll} + (1-\delta^{hh,ll})b + \delta^{hh,ll}\hat{p}_t^{ll}\hat{J}_t^{hh,ll} \tag{31d}$$

$$\hat{w}^{hl,hl} = \delta^{hl,hl}y^{hl,hl} + (1-\delta^{hl,hl})b + \delta^{hl,hl}\hat{p}_t^{hl}\hat{J}_t^{hl,hl} + \frac{1-\delta^{hl,hl}}{1-\delta^{hl,ll}}\delta^{hl,ll}\hat{p}_t^{ll}\hat{J}_t^{hl,ll} \tag{31e}$$

$$\hat{w}^{hl,ll} = \delta^{hl,ll}y^{hl,ll} + (1-\delta^{hl,ll})b + \delta^{hl,ll}\hat{p}_t^{ll}\hat{J}_t^{hl,ll} \tag{31f}$$

$$\hat{w}^{lh,lh} = \delta^{lh,lh}y^{lh,lh} + (1-\delta^{lh,lh})b + \delta^{lh,lh}\hat{p}_t^{lh}\hat{J}_t^{lh,lh} + \frac{1-\delta^{lh,lh}}{1-\delta^{lh,ll}}\delta^{lh,ll}\hat{p}_t^{ll}\hat{J}_t^{lh,ll} \tag{31g}$$

$$\hat{w}^{lh,ll} = \delta^{lh,ll}y^{lh,ll} + (1-\delta^{lh,ll})b + \delta^{lh,ll}\hat{p}_t^{ll}\hat{J}_t^{lh,ll} \tag{31h}$$

$$\hat{w}^{ll,ll} = \delta^{ll,ll}y^{ll,ll} + (1-\delta^{ll,ll})b + \delta^{ll,ll}\hat{p}_t^{ll}\hat{J}_t^{ll,ll} \tag{31i}$$

1.8.2 Figures and tables

Figure 8: Population shares of employed workers

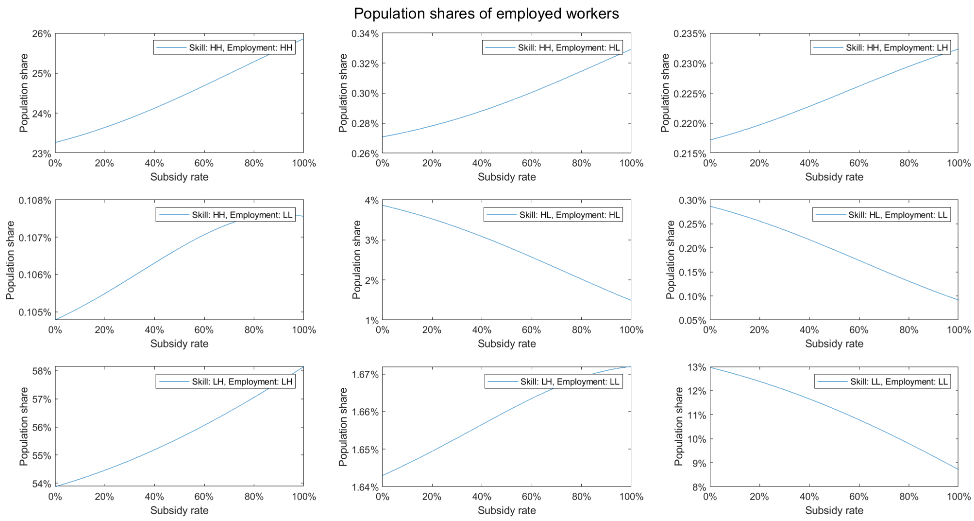


Figure 9: Population shares of unemployed workers

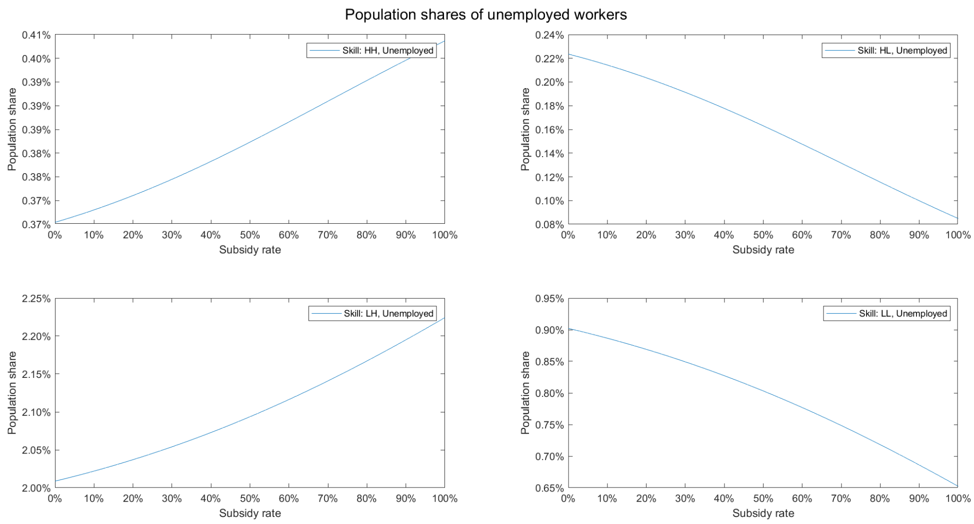


Table 5: Wage rates offered at each possible worker-firm match

Skill endowment	Skill requirement	Wage before subsidy	Wage after subsidy
High-High	High-High	1.4687	1.4689
High-High	High-Low	1.1267	1.1318
High-High	Low-High	0.80341	0.80061
High-High	Low-Low	0.75815	0.75544
High-Low	High-Low	1.0385	1.0393
High-Low	Low-High	0.83181	0.82073
Low-High	Low-High	0.97463	0.97427
Low-High	Low-Low	0.80446	0.79978
Low-Low	Low-Low	0.96315	0.96073

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2 A Versatile Epidemiological and Economic Model for Analysing Public Health Crises and Pandemics: Insights from COVID-19

Abstract

This study analyses the effects of social contact limitation policies against an unexpected epidemic on economic productivity and individuals' mental well-being besides epidemic evolution. We develop a rich economic-epidemiological model with heterogeneous agents, a government imposing social contact limitations, and individuals endogenously deciding on their number of daily social contacts. Agents' behaviour is determined by balancing the risk of infection with the need for social contacts and labour income. We combine various data sources from Belgium to parameterize individual utility, social contacts, economic sectors and infection dynamics. We replicate the first fifteen months of COVID-19 in Belgium and simulate the following months for a wide set of combinations of social contact limitations without a pharmaceutical solution. We find that the timing of social contact limitations matters significantly more than their stringency, with the earliest interventions resulting in the lowest losses in economy and mental well-being for a given number of life losses.

2.1 Introduction

The recent COVID-19 pandemic has highlighted the importance of understanding the complex interplay between public health, social well-being, and economic factors during times of crisis. While the specific context of COVID-19 provides valuable insights, the need for a versatile and adaptable framework to analyse and respond to future pandemics or public health emergencies is paramount.

In this paper, we present a comprehensive model that integrates epidemiological dynamics, endogenous labour supply and social contact decisions among heterogeneous agents. We use this framework to assess the impact of non-pharmaceutical interventions (NPIs) on life losses, social welfare and economic activity. Although our model is calibrated and fitted to match real infection data from the COVID-19 pandemic, with its flexible structure, it can be adapted to study the spread and impact of other infectious diseases, such as seasonal influenza or emerging infectious diseases, helping to inform disease control strategies and resource allocation.

In the aftermath of the COVID-19 pandemic, examining its multifaceted consequences is paramount for preparing for future pandemics. In particular, the tradeoff between protecting lives on the one hand and safeguarding economic activity and social interactions on the other hand, raises a crucial question: "What is the optimal way to set up non-pharmaceutical interventions when they are the only solution available?" Social contact limitations, implemented as a measure to contain the spread of the virus, have indeed had notable repercussions both on the labour market and on mental well-being. The restrictions placed on workplaces have led to a decline in individual income and, consequently, a decrease in macro-level economic activity. Moreover, these limitations have also indirectly constrained the labour supply of parents through school closures and caused job losses due to the closure of a variety of “non-essential” activities. The psychological impact of social contact limitations cannot be overlooked, as individuals have experienced heightened feelings of loneliness and depression, consequently affecting their mental well-being (De Pedraza et al. [2020], Foa et al. [2020]). The optimal timing and strategy for implementing and lifting such restrictions require meticulous assessment, as extended lockdowns are unsustainable from this perspective too. An important dimension of COVID-19 that is also apparent in many viruses is its heterogeneous impact across age groups, with older individuals being more susceptible to severe illness and mortality. This variation led to differing levels of risk tolerance among the population, complicating the imposition of social contact limitations. The effectiveness of these measures varied based on the specific sub-groups and the contexts in which they were applied, such as schools, workplaces, or public gatherings. This chapter offers insights into optimal strategies, in terms of responsiveness and stringency, for mitigating the adverse effects of social contact limitations, while maintaining the objective of safeguarding public health. The development of adaptable policy tools as in our research may allow policymakers to rapidly assess the potential impact of various interventions and make informed decisions based on specific contexts.

This chapter contributes to the literature in four important ways. First, by using micro-data, we empirically estimate the relationship between economic decisions, infection avoidance, and mental well-being. Second, we present an empirically relevant utility function (in terms of income, social contacts, and infection avoidance) that allows endogenous and heterogeneous contact reductions depending on age and sector. Third, we incorporate the specific impact of social contact limitations on parents' labour market outcomes and mental well-being, which was in neglected economic-epidemiological models. Finally, we simulate the effects of responsiveness and stringency of social contact limitations on the economy, health, and mental well-being. With the interdisciplinary nature of our model, we make it relevant to researchers and professionals across various fields, including public health, economics, and social sciences, promoting a more holistic understanding of the factors influencing public health crises and their consequences.

To comprehensively study the impact of epidemics on society, we develop a tractable and rich economic-epidemiological model that allows us to realistically simulate the effects of an unexpected epidemic on health, economy and mental well-being. Our model incorporates three main building blocks: an epidemiological component, an economic component, and a utility maximization problem depending on daily social contacts. Our primary focus is on identifying policies that reduce life losses with a minimal impact on labour supply and mental well-being. To do so, we estimate a utility function (which accounts for mental well-being), and calibrate our model for the first fourteen months of COVID-19 in Belgium, at the time when vaccines were introduced. Our simulations thus allow us to test the impact of various NPI scenarios if vaccines had remained unavailable. We find that the responsiveness of restrictions matters much more than their stringency, with the earliest interventions saving most lives. With social contact limitations imposed early, looser stringency brings significant gains in terms of labour supply and life satisfaction with a minimal effect on life losses. These results suggest that it is possible to minimize the tension between protecting the people and protecting the economy.

The rest of the chapter is organized as follows: In Section 2, we give a brief overview of related literature. In section 3, we present our model. Section 4 describes our data sources and the variables we use in our analysis. Section 5 presents our empirical results and calibration. In Section 6, we present and discuss our simulation results. We provide our conclusion in Section 7.

2.2 Related Literature

To provide accurate assessments of the response of an economy to the emergence of the COVID-19 pandemic, researchers have conducted analyses by combining economic models and epidemiological SIR models.

Early papers such as Alvarez et al. (2020), Eichenbaum et al. (2020a), Jones et al. (2020) analyzed the impact of policies imposed uniformly on the whole population and/or economic decisions uniformly taken by all individuals.

Alvarez et al. (2020) developed a simple planning model to analyze the optimal timing and duration of COVID-19 lockdowns. The authors suggest that a short but intense lockdown can be more effective than a longer but less stringent one in reducing the spread of the virus while minimizing economic costs. In the macroeconomic-SIR model of Eichenbaum et al. (2020a), individuals reduce their consumption and work activities to decrease the likelihood of contracting the virus. While this helps to mitigate the impact of the epidemic, it also contributes to a larger economic downturn. In a dynamic macroeconomic model, Jones et al. (2020) find that a combination of social distancing and remote work can be highly effective in reducing the spread of the virus while minimizing the economic costs of lockdowns.

Policies targeting different groups of the population are found to significantly outperform uniform policies. Acemoglu et al. (2020) studied targeted lockdowns where different age groups have varying infection, hospitalization and mortality rates. They find that stricter policies on the elderly may markedly alleviate economic losses and life losses. Brotherhood et al. (2020) emphasize the importance of age-specific policies in controlling the spread of the virus. The authors show that targeting testing towards the most vulnerable groups, such as the elderly, can be an effective way to reduce the overall spread of the virus and its impact on the economy. Additionally, the authors highlight the importance of age-specific policies, such as social distancing measures targeted towards specific age groups, in reducing the number of infections and deaths. Rampini (2020) divided the population into a young and an old group with different health specificities and rates of labour force participation. He finds that lifting interventions sequentially (for the younger group first and the older group later on) can substantially reduce mortality, demands on the health care system, and the economic cost of interventions.

While the economic-epidemiological literature mainly focuses on the relationship between economic decisions and health outcomes, the mental well-being of individuals was also significantly affected during lockdowns and due to social contact reductions. The findings of Ammar et al. (2020) reveal psychosocial strain due to home confinements with an approximately 71% reduction in social activity leading to a life satisfaction loss of 16%. De Pedraza et al. (2020) concludes that personal well-being is negatively impacted by increasing COVID-19 cases and deaths, extended lockdowns, significant limitations on public life, and an economic decline. On the other hand, Foa et al. (2020) find that due to their impact on reducing life losses and disease incidence, lockdowns during pandemics have an overall positive effect on subjective well-being with a one-month lockdown reducing the negative effect of the pandemics on subjective well-being by 9%.

It is estimated that parents' mental well-being and labour market performance of parents are affected more than people without children during COVID-19 lockdowns. Huebener et al. (2021) show that satisfaction with life decreased more for individuals with children than for other individuals, together with severe adverse effects on parental labour market outcomes during COVID-19 lockdowns in Germany. Fuchs-Schündeln et al. (2020) document that 26% of the German workforce has children aged 14 or younger, and only due to school and child-care closures, 11 % of all workers (42 per cent of all parents) were directly affected leading to an 8% loss in working hours.

2.3 Model

In this section, we describe our model's building blocks. The spread of the epidemic is represented via an extended SEIRD model exploiting contact matrices in the population. We partition our population into 176 individual types depending on several factors, such as employment status or age. Each individual type maximizes utility, which depends on income, number of social contacts, and risk of getting infected. To curb the spread of the epidemic, the policymaker imposes a set of limitations on social contact. These social contact limitations also constrain labour supply and affect life satisfaction on top of their effects on disease transmissions.

2.3.1 Model environment

Social contacts

The epidemic spreads via social contact between contagious and susceptible individuals. We assume four contact locations: home, school, workplace, and other (e.g.: public transport, public events, private gatherings, shopping places, parks,...). In the absence of mobility restrictions, all individuals are free to engage in social contact at home and other locations. On top of contacts at these two locations, some individual types engage in school contacts and some others engage in workplace contacts.

Age groups

We partition our population into 16 age groups (0-4, 5-9, ..., 70-74, 75+). The effects of the epidemic (i.e. mortality and susceptibility rates) vary across age groups.

Employment statuses

We further partition each age group into 11 individual statuses. Individuals are either employed or non-working. By supplying labour, most workers generate social contacts in the workplace. In the first step, we partition employed workers into four categories. When teleworking is possible, individuals can work without generating contacts. The labour supply of workers active in essential sectors is exogenously fixed. In the absence of policy intervention, non-essential workers supply labour endogenously. Other workers are associated with sectors that provide service for contacts at other locations. They supply endogenous labour under some constraints. Since a fraction of workers is also parents, their labour supply may be constrained by school closures, which transfer the responsibility of caring for children during working hours. Hence, we partition each of these four employed worker categories into two sub-categories: workers impacted by school closures and those not impacted. We assume a fraction of each category is subject to childcare, meaning that any decision on the number of school contacts affects their labour supply. Workers without children are not directly affected by decisions related to school contacts. We name the corresponding eight employment types as $remote \,\&\, parent$, $remote \,\&\, childfree$, $essential \,\&\, parent$, $essential \,\&\, childfree$, $non-essential \,\&\, parent$, $non-essential \,\&\, childfree$, $other \,\&\, parent$, and $other \,\&\, childfree$.

The non-working population is decomposed into three individual types. None of these types supplies labour. The first type is the students, who generate contacts at school. The other two types are the *unemployed* and the *nilf* (not in the labour force). *nilf* contains people who are not students, employed, or unemployed (seeking for a job).

2.3.2 Epidemic evolution

Our model is in discrete time. When the epidemic emerges, a small fraction of each individual type becomes exposed to it.

Health statuses

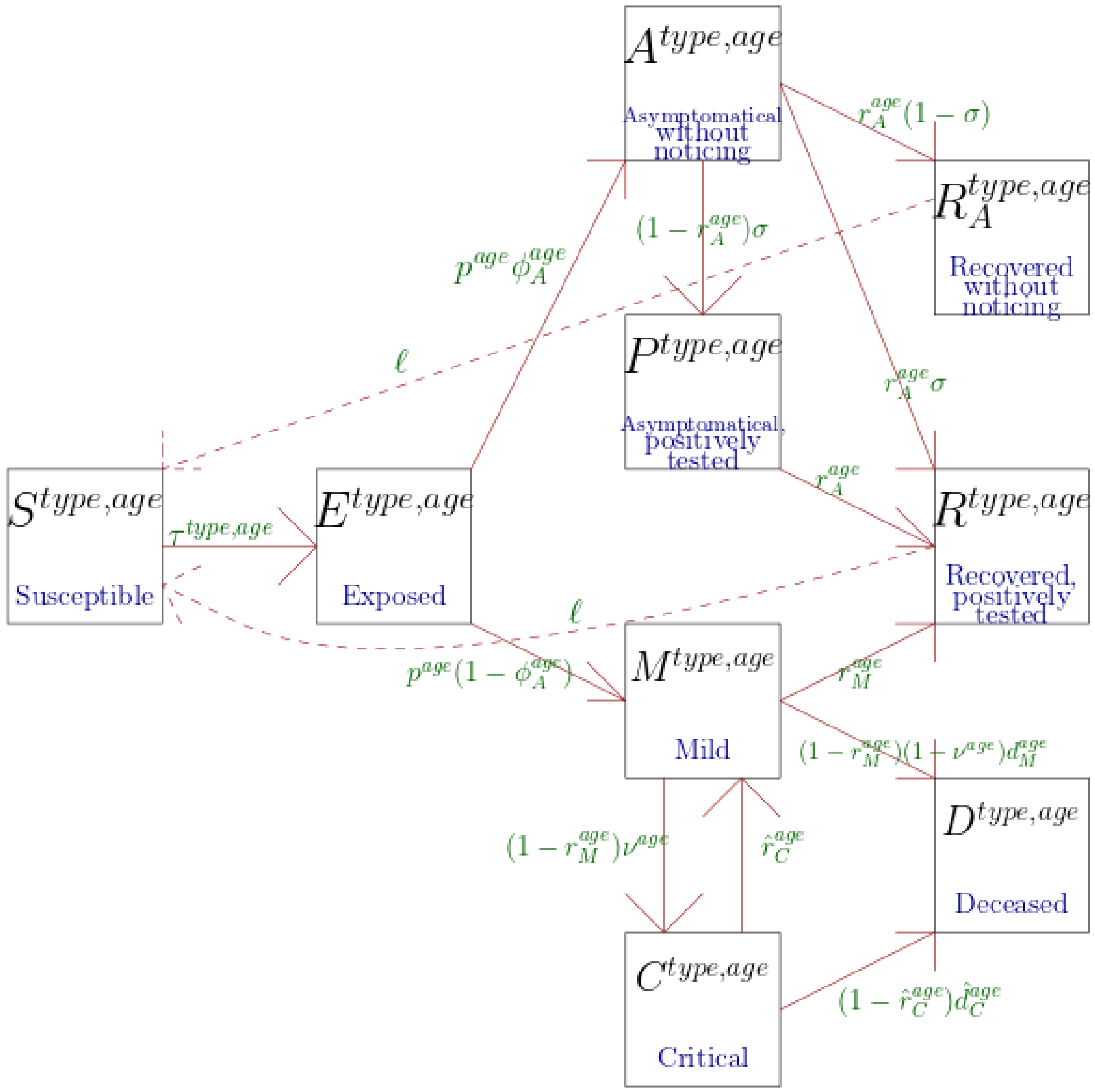
The epidemiological model involves several health statuses. Each individual can experience one of five main health statuses: 1. *Susceptible* individuals are those who have either not been exposed to the virus or have lost immunity after recovery. We assume all individuals are susceptible to the epidemic before its outbreak. 2. *Exposed* individuals are those who are in an incubation period. They have been exposed to the disease, but are not yet contagious. 3. The *infected* status follows the incubation period. At this stage, individuals become contagious and can transmit the disease to susceptible individuals with whom they enter into contact. 4. If an infected individual recovers, they enter into the phase of the *recovered.* At this phase, they are immune to a new infection until immunity vanishes. 5. The individuals who do not recover from the infection become *deceased.*

There are four subgroups within the group of *infected* individuals. 1. contagious Individuals without displaying any disease symptoms are *asymptomatically infected.* We further partition asymptomatic individuals into two groups based on the level of information they possess regarding their infection: 1a. *Positively tested asymptomatically* infected are those that learn their infection upon testing. 1b. Asymptomatically infected without noticing are those that are contagious without realizing it. 2. Individuals who develop symptoms are symptomatically infected. We further partition symptomatic individuals into two groups: 2a. Mildly infected individuals have mild symptoms without developing a need to be admitted to an intensive care unit (ICU). 2b. Critically infected need intensive care.

Among the individuals who recovered from an infection, some were not aware that they had been infected, and they thus ignore the fact that they have gained immunity. We thus also partitioned recovered individuals into two subgroups, whose behaviour may differ: 1. Individuals who recovered from an asymptomatic infection without realizing their infection and recovery, 2. Individuals who realized that they have recovered either from a symptomatic infection or an asymptomatic infection with a positive test result.

The following diagram (Figure 10) summarizes the health statuses mentioned above and the evolution pattern of the disease. Here, parameters coloured in green denote the transition probabilities between each health status.

Figure 10: Flowchart of the epidemiological model



Epidemiological model

The following set of equations describes the dynamics of the epidemic's evolution. First, we let $i \in \{\text{employment,age}\}$ denote the individual type in terms of employment status and age.

S_t^i denotes the number of susceptible individuals of type i at time t, whereas T_t^i denotes the number of new infections. $R_{A,t}^i$ stands for the number of individuals who are immune after recovering from an asymptomatic infection without realizing their infection and recovery. R_t^i stands for the number of individuals who are immune after realising that they recovered from either a symptomatic infection or an asymptomatic infection with a positive test result. Immune individuals lose immunity each day by the daily immunity loss probability, ℓ.

$$S_{t+1}^i = S_t^i - T_t^i + \ell(R_{A,t}^i + R_t^i) \tag{1}$$

E_t^i represents the number of exposed individuals at time t. Newly infected individuals, in other words, individuals who are exposed to the disease, at time t enter into an incubation period $t+1$. In this period they develop no symptoms and they are not contagious yet. At each period, they leave the incubation period at a probability p and become contagious.

$$E_{t+1}^i = E_t^i(1-p) + T_t^i \tag{2}$$

As common in this literature, we assume that upon becoming contagious, individuals either never develop symptoms and become asymptomatically infected, or they develop symptoms immediately. We model a testing policy by which asymptomatic individuals get randomly tested. This policy allows testing a fraction σ_t of the asymptomatic population at time t. When they are tested, infected individuals become aware of their infection, which decreases the pool of unaware infected individuals A. Here, A_t^i represents the number of asymptomatically infected individuals at time t whose infection is not revealed by random testing. ϕ_A is the probability of not developing symptoms. Asymptomatically infected individuals recover at a rate r_A.

$$A^i_{t+1} = [A^i_t(1 - r_A) + p\phi_A E^i_t](1 - \sigma_t) \tag{3}$$

Conversely, the number of asymptomatically infected individuals that know their infection, P^i_t, increases thanks to σ_t. A fraction of the group A^i_{t+1} learn their infection at time t upon random testing by a probability, σ_t. Those that already know their infection recover at the asymptomatic recovery rate, r_A.

$$P^i_{t+1} = [A^i_t(1 - r_A) + p\phi_A E^i_t]\sigma_t + P^i_t(1 - r_A) \tag{4}$$

Those who develop symptoms become mildly infected, M. Mildly infected may recover at a rate $r_M \geq r_A$, develop a need for a bed at an intensive care unit at a rate ν or directly lose their lives at a rate d^{age}_M. A mildly infected individual enters the critical infection phase, C, upon developing a need for the intensive care unit. They are either discharged from the ICU to a normal hospital bed at a probability $\hat{r}_C$, or they lose their lives at a probability, $\hat{d}^{age}_C$, which is higher than the mortality rate when being mildly infected, $\hat{d}^{age}_C \geq d^{age}_M$.

$$M^i_{t+1} = M^i_t(1 - r_M)(1 - \nu)(1 - d^{age}_M) + E^i_t p(1 - \phi_A)^{age} + C^i_t \hat{r}^{age}_C \tag{5}$$

$$C^i_{t+1} = C^i_t(1 - \hat{r}_C)(1 - \hat{d}^{age}_C) + M^i_t(1 - r_M)\nu \tag{6}$$

D^i_t denotes the stock of the deceased individuals at time t. Two sources for the inflow to the status, D, are those who lose their lives when mildly infected and when critically infected.

$$D^i_{t+1} = D^i_t + M^i_t d^{age}_M + C^i_t \hat{d}^{age}_C \tag{7}$$

Equations (8) and (9) give the evolution of the numbers of immune individuals, without realizing their immunity, $R^i_{A,t}$, and by knowing their immunity, R^i_t. Each day a fraction ℓ of these individuals lose their immunity, decreasing the pool of individuals at these health statuses. Daily new recoveries from A, P, M, and C increase the number of immune individuals.

$$R^i_{A,t+1} = R^i_{A,t}(1-\ell) + A^i_t r_A (1-\sigma_t) \tag{8}$$

$$R^i_{t+1} = R^i_t(1-\ell) + P^i_t r_A + M^i_t r_M + \hat{r}_C C^i_t \tag{9}$$

Pop^i_t stands for the population size at time t. It is simply the population size minus the number of life losses at time t.

$$Pop^i_{t+1} = Pop^i_t - D^i_t \tag{10}$$

$Quar^i_t$ is the number of quarantined individuals of type i at time t. We assume the following: Positively tested asymptomatically infected, P is kept under home quarantine. Mildly infected, M, either doesn't develop a need for hospitalization, at a probability ρ, or they need to be hospitalized. M are also kept under home quarantine when they are not hospitalized.

$$Quar^i_t = P^i_t + \rho M^i_t \tag{11}$$

$Hosp^i_t$ is the number of hospitalized individuals of type i at time t. All critically infected and a fraction of mildly infected are hospitalized.

$$Hosp^i_t = (1-\rho)M^i_t + C^i_t \tag{12}$$

Act denotes the active population who are free to have social contacts other than home. Act^i_t is simply the non-quarantined and non-hospitalized population.

$$Act^i_t = Pop^i_t - Quar^i_t - Hosp^i_t \tag{13}$$

We assume everyone who develops symptoms at time t is tested. The remaining testing capacity is allocated randomly to individuals without disease symptoms. Here,

δ_t and σ_t are testing capacity and random testing probability at time t.

$$\sigma_t = \frac{\delta_t - \sum_i p(1-\phi_A)E^i_{t-1}}{\sum_i (S^i_t + E^i_t + A^i_t + R^i_{A,t} + R^i_t)} \tag{14}$$

ICU capacity is limited. When ICU capacity is overwhelmed, some individuals with a need for ICU cannot be admitted to the ICU. For simplicity, we assume no prioritization of ICU capacity for any individual type. When the ICU capacity, χ, is overwhelmed, individuals admitted at the ICU recover at the regular recovery rate, r_C. We assume individuals with a denied ICU admission recover at a lower rate than the regular recovery rate of those who are admitted, $\bar{r}_C < r_C$. Thus, the average recovery rate for individual type i is the weighted average of recovery rates of individuals admitted and denied at the ICU:

$$\hat{r}_{C,t} = \begin{cases} r_C & \text{if } \sum_i C^i_t \leq \chi \\ \frac{\chi r_C + (\sum_i C^i_t - \chi)\bar{r}_C}{C^i_t} & \text{if } \sum_i C^i_t > \chi \end{cases} \tag{15}$$

We assume an individual has a higher mortality rate, $\bar{d}^{age}_C > d^{age}_C$, when denied an ICU bed. Then, the average mortality rate for individual type i can be written as the weighted average of mortality rates of individuals admitted and denied at the ICU:

$$\hat{d}^{age}_{C,t} = \begin{cases} d^{age}_C & \text{if } \sum C^i_t \leq \chi \\ \frac{\chi d^{age}_C + (\sum_i C^i_t - \chi)\bar{d}^{age}_C}{C^i_t} & \text{if } \sum_i C^i_t > \chi \end{cases} \tag{16}$$

Disease transmission

Disease transmission occurs when a susceptible individual (S) engages in social contact with an infected individual (A, P, M, or C) at one of the contact locations (home, school, workplace, other). Following and extending Prem et al. (2017), We use the following expression for the number of new exposures to the disease at $location \in \{work, school, other\}$, $T^i_{location,t}$:

$$T^i_{location,t} = S^i_t \pi^{age}_t \sum_{i'} \mu^{i,i'}_{location,t} \underbrace{\frac{\pi_A A^{i'}_t}{Act^{i'}_t}}_{\text{fraction of contagious}} \tag{17}$$

Here, the term inside the summation sign is the effective number of contagious individuals a member of type i meets from type i' at time t. $\mu^{i,i'}_{location,t}$ is the number of daily social contacts at $location \in \{work, school, other\}$ between these two individual types. A susceptible individual of type i can only meet active individuals (neither hospitalized nor at home quarantine), $Act^{i'}_t$. Among $Act^{i'}_t$, only the asymptomatically infected without a test result, $A^{i'}_t$, can spread the disease. Therefore, the fraction $A^{i'}_t / Act^{i'}_t$ gives the probability of meeting with the disease. π_A is the relative contagiousness of asymptomatic infection to symptomatic infection. As anyone is free to meet with anyone, the sum of these effective numbers of contagious individuals from all groups gives the total effective number of contagious individuals type i can meet. Finally, π^{age}_t is the susceptibility coefficient representing the fact that each age group has different per contact probabilities of entering into the incubation period. π^{age}_t is time-dependent to reflect the transmission rate difference before and after masks were used and social distancing rules were implemented. We assume t_{mask} is the day masks become obligatory.

$$\pi^{age}_t = \begin{cases} \pi^{age}_0, & \text{if} t < t_{mask} \\ \pi^{mask} * \pi^{age}_0, & \text{if} t \geq t_{mask} \end{cases} \tag{18}$$

The number of new exposures to the disease at *home* is different than other locations as quarantined patients, $Quar^{i'}_t$, are also a source of transmission. Here the contagious among i', $(Act^{i'}_t + Quar^{i'}_t)$, also contains the asymptomatically infected with a test result, $P^{i'}_t$, and the non-hospitalized mildly infected $\rho M^{i'}_t$.

$$T^i_{home,t} = S^i_t \pi^{age}_t \sum_{i'} \mu^{i,i'}_{home,t} \underbrace{\frac{\pi_A (A^{i'}_t + P^{i'}_t) + \rho M^{i'}_t}{Act^{i'}_t + Quar^{i'}_t}}_{\text{effective fraction of contagious}}$$

The number of total infections of individual type i, $T^i_{location,t}$ can be written as the sum of newly exposed at $location \in \{home, school, workplace, other\}$: [15]

$$T_t^i = T_{home,t}^i + T_{school,t}^i + T_{work,t}^i + T_{other,t}^i \tag{19}$$

As all members of type i are identical, the probability of exposure to the disease is the newly exposed within type i at $location \in \{home, work, school, other\}$ divided by the number of susceptible within i:

$$\tau_{location,t}^i = T_{location,t}^i / S_t^i \tag{20}$$

2.3.3 Non-pharmaceutical interventions

The policymaker intervenes by imposing non-pharmaceutical interventions (NPIs) that limit social contact at three contact locations; namely schools, workplaces and other locations; to curb the spread of the epidemic.

Quarantined individuals have no contacts other than home contacts. Let j stand for a sub-group of the population depending on health status, employment type, and age, $j \in \{health, employment, age\}$. For $location \in \{school, work, other\}$, social contacts of quarantined individuals, $health \in \{P, M, C\}$ are:

$$\mu_{location,t}^{j,j'} = 0 \tag{21}$$

All remaining limitations are applicable if health status is $health \in \{S, E, A, R_A, R\}$

School closures

The policymaker dictates the number of school contacts at time t. Here, $\mu_{s,0}^{i,i'}$ is the number of school contacts between individuals of types i and i' before the

[15] We assume the probability of being exposed to the disease at more than one location on the same day is zero for simplification.

epidemic. $\lambda_{s,t}$ is the fraction of pre-disease school contacts to be held at time t. For $employment \in \{student\}$:

$$\mu_{s,t}^{j,j'} = \lambda_{s,t}\mu_{s,0}^{j,j'} \tag{22}$$

This limitation acts as a cap on workplace contacts of non-remote employed workers that have children and whose labour supply may be constrained by school closures. For $employment \in \{essential \,\&\, parent, non{-}essential \,\&\, parent, other \,\&\, parent\}$:

$$\mu_{w,t}^{j,j'} \leq \lambda_{s,t}\mu_{w,0}^{j,j'} \tag{23}$$

We assume school closures also affect remote workers that are subject to childcare although less so than essential workers. Here, ζ is the partial effect of school closures on the labour supply of remote workers. For $employment \in \{remote \,\&\, parent\}$:

$$L_t^j = (1-\zeta) + \zeta\lambda_{s,t} \tag{24}$$

Workplace contacts

The policymaker sets a cap on workplace contacts by imposing workplace closures. Essential workers are exempt from this limitation. Non-essential and other workers can now engage in workplace contacts at a maximum rate that is determined by the policymaker as a fraction of the pre-epidemic workplace contacts, $\mu_{w,0}^{j,j'}$. $\lambda_{w,t}$ is the maximum fraction of pre-disease workplace contacts that can be held at time t. For $employment \in \{non{-}essential \,\&\, parent,\ non{-}essential \,\&\, childless,\ other \,\&\, parent,$ $other \,\&\, childless\}$:

$$\mu_{w,t}^{j,j'} \leq \lambda_{w,t}\mu_{w,0}^{j,j'} \tag{25}$$

Other location limitations

The policymaker sets a cap on other contacts for all individuals by imposing workplace closures. Here, $\mu_{o,0}^{j,j'}$ is the number of other contacts between i and i' before the epidemic. $\lambda_{o,t}$ is the maximum fraction of pre-disease other contacts that can be held at time t.

$$\mu_{o,t}^{j,j'} \leq \lambda_{o,t}\mu_{o,0}^{j,j'} \tag{26}$$

We assume a minimum level of other contacts, $\lambda_{o,min}$, would be present in any case (e.g. public transport), and we associate *other workers* with the rest of other contacts (i.e. sectors providing leisure activities). Therefore, for $employment \in \{other\&parent,\ other\ \&\ childless\}$:

$$\mu_{w,t}^{j,j'} \leq \frac{\lambda_{o,t} - \lambda_{o,min}}{1 - \lambda_{o,min}}\mu_{w,0}^{j,j'} \tag{27}$$

2.3.4 Life satisfaction maximization

At the beginning of each day, individuals in the population observe the state of the epidemic and optimally decide on the number of social contacts they would like to have. This optimization is done by maximizing a utility function, which depends on three parameters we empirically found to be correlated with life satisfaction: income, a measure of social isolation and the risk of getting infected. For simplicity, individuals update their behaviour every day based on the evolution of the situation by maximizing for one period. First, we present the sociality (as a measure of social isolation) and income functions.

We define sociality as an individual's number of daily social contacts. Here, $\mu_{location,0}^{i}$'s are the pre-epidemic sum of daily social contacts at each contact location. $\kappa_{location,t}^{i}$'s are the fractions of social contacts kept at time t. $\kappa_t^i = (\kappa_{home,t}^i, \kappa_{school,t}^i,$

$\kappa^i_{work,t}, \kappa^i_{other,t})$ is the vector of social contacts at all locations.

$$sociality(\kappa^i_t) = \kappa^i_{home,t}\mu^i_{home,0} + \kappa^i_{school,t}\mu^i_{school,0} + \kappa^i_{work,t}\mu^i_{work,0} + \kappa^i_{other,t}\mu^i_{other,0} \tag{28}$$

We assume income is immediately consumed and is proportional to daily workplace contacts for non-remote workers. Remote workers supply full labour unless they are affected by school closures.

$$L^i_t = \begin{cases} 1, & \text{if type} \in \{\text{remote \& childless}\} \\ 1 - (1-\zeta)\lambda_{s,t}, & \text{if type} \in \{\text{remote \& parent}\} \\ \kappa^i_{w,t}, & \text{else} \end{cases} \tag{29}$$

We assume the non-working population enjoys a basic income and the working population is compensated at the same rate when they are not able to supply labour due to social contact limitations. The rest of the income is paid as wage, ω. We assume wage to be unity, to equalize the income at full labour supply to 1. Income function:

$$income(L^i_t) = \begin{cases} b(1 - L^i_t) + \omega L^i_t, & \text{if type} \notin \{\text{nilf, student, unemployed}\} \\ b, & \text{if type} \in \{\text{nilf, student, unemployed}\} \end{cases} \tag{30}$$

The risk of disease exposure creates a disincentive to form social contacts. Individuals observe the daily state of the epidemic and assess the probability of being susceptible and the risk of disease exposure per contact. [16] Any infected individual, $health \in \{P, M, C\}$, knows that the probability of being susceptible is zero. Individuals at a health state $health \in \{S, E, A, R_A, R\}$ know that if they are susceptible a contagious person may transmit the disease to them. They observe the number of active cases, corresponding to $health \in \{P, M, C\}$, and calculate the probability of being susceptible. We assume this probability to be the number of susceptible at each type divided by the number of individuals belonging to the set of individuals at $health \in \{S, E, A, R_A, R\}$. Furthermore, since the risk of dying following infection is

[16] A second disincentive might exist because of the risk of contaminating others. However, we found no correlation between having an acquaintance diagnosed with COVID-19 and life satisfaction.

age-specific, it implies that an older individual would be more hesitant to form social contacts than a younger individual.

In light of the argumentation given above, we define the maximization problem of expected daily utility in equation (31), which we explain in Appendix 2.8.1. Here, $\alpha_s, \alpha_i, \alpha_e$ represent how much individuals value having social interactions, income and facing the risk of being exposed to the virus. δ^{age} is the age-specific probability of death upon entering the *exposed* health phase, $\pi_{S,t}$ is the probability of being susceptible when an individual is not informed by their health status ($health \in \{S, E, A, R_A, R\}$), and $\tau(\kappa_t^i)$ is the daily probability of entering into the *exposed* phase at the next day, which depends on the number of social contacts at day t.

$$\max_{\kappa_{h,t}^i, \kappa_{w,t}^i, \kappa_{o,t}^i} \mathbb{E}(LS_t^i) = \Big\{ \alpha_s ln(\text{sociality}_t^i(\kappa_t^i)) + \alpha_i ln(\text{income}_t^i(\kappa_{w,t}^i)) \\ + \alpha_e \underbrace{\delta^{age}}_{\text{mortality rate by age,}} \underbrace{\pi_{S,t}}_{\text{prob. of being susceptible,}} \underbrace{\tau(\kappa_t^i)}_{\text{exposure prob.}} \Big\} \tag{31}$$

subject to non-pharmaceutical interventions given in equations (22)-(27).

2.4 Data

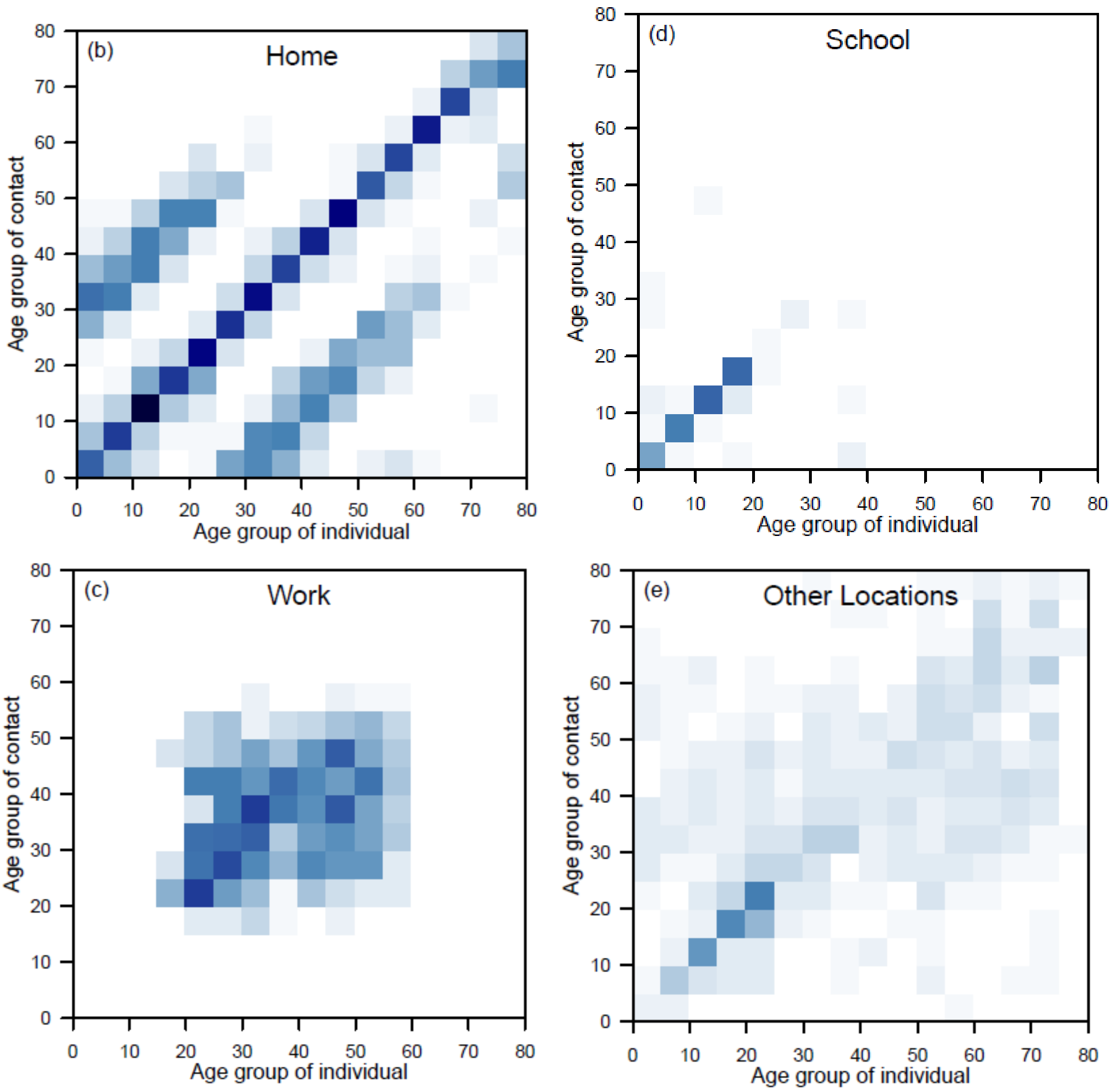
We use several data sources from Belgium to simulate and analyze the effects of NPIs on the economy, life satisfaction, and COVID-19 evolution.

Social contact matrices

Prem et al. (2017) provide estimations of daily social contacts among sixteen age intervals (0-4, 5-9, ... 70-74, 75+) at four contact locations (home, school, workplace, other) for 152 countries by using contact diaries from the POLYMOD survey[17]. We use their estimations for the number of contacts at each contact location among each age group in Belgium in 2017. A heat map of daily contacts at each location is given below (Figure 11) with darker points indicating more contacts. The highest numbers of contacts are as follows: 1. At home, 2.51 contacts per day between age groups 10-14 and 10-14. 2. At school, 4.20 contacts per day between age groups 10-14 and 10-14. 3. At work, 1.75 contacts per day between age groups 30-34 and 35-39. 4. At other locations, 2.58 contacts per day between age groups 20-24 and 20-24.

[17]A survey conducted with 7290 participants in eight European countries to examine mixing patterns among populations to address the increasing use of mathematical models in determining the effects of interventions on infectious diseases transmitted through respiratory or close contact (Mossong et al. [2008]).

Figure 11: Daily social contacts in Belgium



We assume the minimum daily home contact cannot fall below 1.3 as the average household size in Belgium is 2.3.

Sectoral composition of employed workers

Our source for Belgian labour force composition is the following study: *Fana et al. (2020), The COVID confinement measures and EU labour markets*. Fana et al. (2020) decomposes the Belgian labour force into five categories:

1. Essential and fully active (27%): food production, utilities, health and all the other

sectors

2. Active via telework (30%): education, most of the public administration, finance, insurance and telecommunications

3. Mostly essential and partly active (18%): mostly retail and manufacturing of chemicals and paper, which remain to some extent active even in the strict confinement situation

4. Mostly non-essential (17%): the majority of manufacturing not previously mentioned, as well as some machine and computer repair activities and construction

5. Closed (8%): Hotels, restaurants and accommodation, estate and travel agencies, plus leisure and recreation services

We directly take the sizes of the first, second, and fifth categories as the sizes of essential workers, remote workers, and other workers in our model. We assign the total size of the third and fourth categories as the size of non-essential workers in our model.

We use the estimations of Fuchs-Schündeln et al. (2020) to give values to the sizes of the employed workers affected by school closures due to the need for childcare. By using German data, Fuchs-Schündeln et al. (2020) estimate that 11 per cent of workers are affected by childcare, which translates into 8 per cent of foregone working hours[18]. We assume that age groups between the ages of 25-54 (76.7% of the employed) have children of school age and distribute the 11% subject to childcare over the employed between the ages of 25-54 (11%/76.7%=14.3%). We finally assign the remaining labour supply loss (8%-70%x11% = 0.3%) to remote workers (0.3%/(30%x11%) = 9.1%) when all non-remote workers which are subject to childcare lose their labour supply in case of full school closure. Table 6 summarizes these values.

[18]We use these estimations for Germany as we weren't able to find a similar study for Belgium.

Table 6: Labour force composition in Belgium

Parameter	**Value**
Share of remote workers	30%
Share of essential workers	27%
Share of non-essential workers	35%
Share of workers providing labour supply to activities related with other location contacts	8%
Share of workers constrained by childcare (ages 25-54)	14.3%
Labour supply loss of teleworkers constrained by childcare	9.1%

Demography

We use OECD demographics data for several demographic values of Belgium. We use data from 2017 as it is the year of the social contact matrices we employ. The total population in 2017 is 11,349,081. To determine the sizes of the type *student*, we use school enrolment data for Belgium in 2017 from OECD. To determine the sizes of the *employed*, *unemployed*, and *nilf* (not in the labour force), we use the shares of each group within the population above the age of 15, by assuming no one works up to the age of 14. We then partition the employed population by applying the sectoral composition shares presented above to each age group beginning with the age of 15. Table 7 presents the resulting population decomposition.

Table 7: Decomposition of the population of Belgium into individual types

Age	Nilf	School	Unemp	Rem C	Rem NC	Ess C	Ess NC	Non C	Non NC	Oth C	Oth NC
0-4	311579	313169	0	0	0	0	0	0	0	0	0
5-9	8206	657350	0	0	0	0	0	0	0	0	0
10-14	7813	625879	0	0	0	0	0	0	0	0	0
15-19	0	581003	10967	0	10925	0	13110	0	9469	0	2913
20-24	27578	332099	58582	0	76677	0	92012	0	66453	0	20447
25-29	22705	103891	61581	23664	141369	28397	169642	20509	122520	6310	37698
30-34	53859	52359	44422	24707	147600	29648	177120	21413	127920	6588	39360
35-39	46145	53931	36444	26250	156821	31500	188185	22750	135911	7000	41819
40-44	80122	18218	32310	25878	154595	31053	185514	22427	133982	6901	41225
45-49	101269	19509	32147	27153	162211	32583	194653	23532	140583	7241	43256
50-54	139180	20258	32382	26774	159949	32129	191938	23204	138622	7140	42653
55-59	210834	19347	34053	0	154012	0	184814	0	133477	0	41070
60-64	473791	17212	9886	0	57270	0	68724	0	49634	0	15272
65-69	564572	15194	547	0	9116	0	10939	0	7900	0	2431
70-74	470590	12299	60	0	3410	0	4092	0	2955	0	909
75+	973953	25074	0	0	2635	0	3162	0	2284	0	703

COVID-19 disease characteristics

We gather several data sources and estimations in order to parameterize disease characteristics. The incubation period is 5.1 days à la Lauer et al. (2020). Therefore, the daily probability of leaving the incubation period and becoming contagious is 1 / 5.1 = 0.1961. 40% of cases become asymptomatic as estimated by Ma et al. (2021). We assume a daily immunity loss probability is 0.0003 implying that 95% of the recovered individuals stay immune after six months (Dan et al. [2021]). The expected recovery duration is 6.5 and 9.5 days for asymptomatic and symptomatic infection (Rhee et al. [2021], Rees et al. [2020]), respectively. The average length of stay at the ICU is set to 17.5 days à la Zeleke et al. (2021). We use age-specific mortality rates from Verity et al. (2020), in Appendix 2.8.3 in Table 16. We assume an average duration of 7 days before death in the ICU (Auld et al. [2020], Larsson et al. [2021]). Thus, one-seventh of the age-specific mortality rates give daily mortality rates. From Sciensano (2023), we calculated that the mortality rate outside ICU was 11.9% of that in the ICU until the last day of our calibrated period, 19/04/2021. We assume, when a patient is denied an ICU bed due to the ICU capacity overwhelming, they do not recover until they are admitted to the ICU. For the unlikely case of ICU capacity being overwhelming, we assume the mortality rate is three times more when a patient is denied an ICU bed. From Sciensano (2023), we fathom approximately 69.77% of the symptomatic patients did not require hospitalization. Thus, this is the probability of not developing a need for hospitalization when mildly infected, ρ. An asymptomatically infected individual is assumed to be 75% as contagious as a person with symptoms (Cevik et al. [2021])[19]. We use the actual number of ICU beds in Belgium dedicated to COVID-19 patients, corresponding to 15.9 beds per 100,000 people (Berger et al. [2022]). Finally, each age group has different probabilities of entering into the incubation period upon contact (Davies et al. [2020]), leading to age-specific susceptibility rates given in Appendix 2.8.3 in Table 16.

Table 8: COVID-19 disease characteristics

Parameter	**Symbol**	**Value**	**Phase**
Prob. of becoming contagious	p	0.1961	E
Prob. of not showing symptoms	ϕ_A	0.4000	E
Prob. of developing need for critical care	ν	0.0571	M
Immunity loss prob.	ℓ	0.0003	R_A,R
Prob. of recovery when asymptomatically infected	r_A	0.1538	A,P
Prob. of recovery when symptomatically infected	r_M	0.1050	M
Prob. of discharge from critical care	r_C	0.1429	C
Prob. of death when critical care is needed	d_C	Age-specific	C
Prob. of death when critical care is not needed	d_M	$0.119d_C$	M
Prob. of discharge from critical care (overwhelmed)	$\hat{r}_C$	0	C ($\geq \chi$)
Prob. of death (overwhelmed)	$\hat{d}_C$	$3d_C$	C ($\geq \chi$)
Ratio of the mildly infected under home quarantine	ρ	0.6977	M
Relative contagiousness of the asymptomatic	π_A	0.7500	A
Critical care capacity per 100,000 people	χ	15.9	
Base susceptibility rate	π^{age}	Age-specific	S

[19] This value should be approached cautiously as estimations vary significantly.

Disease evolution in Belgium

We use the COVID-19 monitoring data set from the Belgian Institute of Health (Sciensano) for the daily numbers of active cases, ICU occupancy rates, life losses, and COVID-19 tests. We use data from the period between 02/03/2020[20] and 19/04/2021.

Utility function

We use the *WageIndicator Survey of Living and Working in Coronavirus Times*. This survey contains information on several outcomes such as income, life satisfaction, workload changes, or feelings of loneliness. The survey is conducted in 143 countries between March 2020 and March 2021.

Table 9 presents the descriptive statistics from *WageIndicator Survey of Living and Working in Coronavirus Times*. We modify the data in several ways. We discard observations a) from countries with less than ten observations and b) from observations without a valid gender response. We create a categorical variable named *income loss*. *Income loss* has the value a) 2 if a respondent expects job loss or their workplace to go out of business, b) 1 if a respondent expects only less income, and c) 0 if a respondent expects neither less income nor job loss or their workplace to go out of business. The survey contains two questions on a) being ever tested for COVID-19, and b) having recovered in case of a positive test result. We lump the responses to two questions into a categorical variable with five values, named *test result*. The values from 0 to 4 are a) has never tested for COVID-19, b) has tested positive and not yet recovered c) has tested positive and recovered, c) is waiting for a test result, and d) has ever tested but without any positive test result. The survey contains a question *I feel lonely in this corona crisis* with values between 1 and 5, with 5 corresponding to the loneliest case. We create a *sociality* parameter that is the opposite of feeling lonely, with 5 corresponding to the highest level of being social. We further use a set of variables, such as *doing regular exercise* or *living with partner*, that can affect life satisfaction.

[20] 02/03/2020 is the day Belgium started to announce new positive COVID-19 cases every day. Only a single positive COVID-19 case was announced on 04/02/2020 before 02/03/2020.

Table 9: Descriptive statistics

	No of observations	Mean	Std. dev.	Min	Max
Life satisfaction	16998	6.05	2.47	1	10
Male	17004	1.38	0.484	1	2
Age	17004	42.3	12.3	10	90
Weekly Cases per 1000	17004	0.957	1.12	0	10.01
Health problems	16999	2.10	0.759	1	5
Tertiary education	17004	0.544	0.498	0	1
Workload increased	13889	0.314	0.464	0	1
Workload decreased	13889	0.340	0.474	0	1
Can work remotely	13889	0.344	0.475	0	1
Without work due to Covid-19	13889	0.363	0.481	0	1
Income loss	10848	0.549	0.580	0	2
Sociality	17000	3.17	1.31	1	5
Has an acquaintance tested positive	16243	0.188	0.391	0	1
Test result	16999	0.735	1.52	0	4
Lives with partner	17001	0.619	0.486	0	1
Lives with children	17001	0.467	0.499	0	1
Lives with other	17001	0.254	0.435	0	1
Cares about pets	17001	0.298	0.457	0	1
Under self isolation	17000	0.567	0.496	0	1
Does regular exercise	17000	2.83	1.33	1	5
N	17004				

Non-pharmaceutical interventions

Oxford COVID-19 Government Response Tracker (OxCGRT) provides a comprehensive data set on the list of policies implemented against the spread of COVID-19. We extract the following timeline given in Table 10 as the implementation timeline of social contact limitations in Belgium.

Table 10: Timeline of NPIs in Belgium

Date	Workplaces	Schools	Other locations
14/03/2020		Closed	Mildly restricted
18/03/2020	Restricted		Strictly restricted
11/05/2020	Relaxed		
08/06/2020			Mildly restricted
29/07/2020			Strictly restricted
01/09/2020		Opened	
30/09/2020			Mildly restricted
02/11/2020	Restricted	Closed	Strictly restricted
16/11/2020		Opened	
01/12/2020	Relaxed		
29/03/2021		Closed	
19/04/2021		Opened	

Here, we assume workplaces are restricted when the policy "*require closing (or work from home) for all-but-essential workplaces (eg grocery stores, doctors)*" was implemented. Although during the summer of 2020, no school closure policies were in place, we let the schools open on 01/09/2020 because schools were closed due to the summer holiday. We assumed other location contacts were strictly limited when the policies "*require cancelling of public events*" and "*restrictions on gatherings of 10 people or less*" were together in place. When one of these policies on public events or gatherings was relaxed, we assumed other location contacts were mildly restricted.

2.5 Empirical results and calibration

In this subsection, we explain our calibration strategy for two sets of parameters: a) the effects of being social (α_s), income (α_i), and disease exposure (α_e) on life satisfaction; b) two disease evolution parameters, namely the susceptibility rate multiplier[21], and the initial effectiveness of wearing facial masks, π_{mask}. We use estimated coefficients from OLS estimations for the first set of parameters. For the second set of parameters, we choose parameter values that minimize the distance between data on daily ICU occupancy rates and our simulations.

We combine social contact limitation data from *Oxford COVID-19 Government Response Tracker*, disease evolution data from *Sciensano*, and data from *WageIndicator Survey of Living and Working in Coronavirus Times* on perceptions of life satisfaction, loneliness, the risk of income loss, infection status, and several personal characteristics. We run OLS estimations to determine the factors affecting life satisfaction. We present our estimation results in Table 11. The first and the third estimations are done with contact limitations included in the estimation.[22] The first and the second estimations contain perceptions of workload change. The entire set of the estimated determinants of life satisfaction including all control variables[23] can be found in Appendix 2.8.3 as Table 17. The OLS estimations explain 27.0% to 27.3% of the variation in life satisfaction.

[21]We use age-specific susceptibility rates from Chinese data, then multiply these base susceptibility rates with a calibrated value.

[22]1. School closures, 2. Workplace closures, 3. Cancellations of public events, 4. Restrictions on gatherings, 5. Public transport closures, 6. Stay-at-home requirements, 7. Restrictions on the internal move, 8. International travel controls

[23]gender, age, health problems, tertiary education, working from home, living with partner/children/others, care for pets, feeling under self-isolation, doing regular exercise, self-suffered fever/diarrhoea

Table 11: OLS estimations for the determinants of life satisfaction

	(1)	(2)	(3)	(4)
Income loss = 1	-0.482***	-0.494***	-0.486***	-0.498***
Income loss = 2	-1.229***	-1.242***	-1.239***	-1.253***
Sociality = 2	0.760***	0.763***	0.774***	0.774***
Sociality = 3	1.167***	1.164***	1.180***	1.179***
Sociality = 4	1.330***	1.330***	1.350***	1.350***
Sociality = 5	1.472***	1.468***	1.492***	1.489***
Test result = Positive, still infected	0.0212	0.0254	0.0316	0.0153
Test result = Positive, recovered	0.405**	0.404**	0.379**	0.370**
Test result = Waiting	-0.0725	-0.0542	-0.0977	-0.0976
Test result = Negative	0.166***	0.167***	0.151***	0.138**
Weekly cases per thousand people	-0.0198	-0.0241	-0.0724	-0.119**
Weekly cases square per million people	0.0194	0.0199	0.0240*	0.0307**
Has an acquaintance tested positive	0.0240	0.0222	0.0186	0.0129
Constant	8.623***	6.588***	7.891***	6.238***
Contact limitation dummies	Yes	No	Yes	No
Workload change opinions	Yes	Yes	No	No
Control variables	Yes	Yes	Yes	Yes
Observations	10417	10422	10417	10422
Adjusted R^2	0.273	0.272	0.271	0.270

* p<0.10, ** p<0.05, *** p<0.01

We find that the *income loss* categorical variable we constructed is statistically significantly correlated with *life satisfaction* even at 1% significance level. An expectation of facing an income loss is found to lower life satisfaction by between 0.482 and 0.498 points on a scale of 10 compared with the case of expecting no change in income. When an individual expects a job loss or their workplace to go out of business, life satisfaction is expected to decrease by between 1.229 and 1.253 points in comparison with the case of no income loss.

We find a positive correlation between life satisfaction and being social at a statistical significance level of less than 1%. The difference between feeling most and least social is estimated to account for 1.468 to 1.492 life satisfaction points. We further observe a decreasing return for this correlation. In the model including all variables, we estimate that life satisfaction increases by 0.760 points between feeling least social and the sociality score one step above. For each sociality level increase, we observe 0.407, 0.163, and 0.142 point increases in life satisfaction.

Our findings for the impact of COVID-19 test results on life satisfaction are as follows: We find no significant correlation between life satisfaction and being still infected after a positive test result or waiting for a test result. Note that these two answers correspond to only 0.51% and 0.53% of the observations we use in our OLS models. Our models point out a positive correlation between life satisfaction and recovery following a positive test result, which is reported by 1.42% of the respondents in our OLS model. The impact is estimated to be between 0.370 and 0.405 points at a 5% statistical significance level in comparison with having never been tested. The estimated coefficient of having all test results negative (17.14% of the answers) is between 0.138 and 0.166 points with a statistical significance level of at least 5%.

One more parameter in *WageIndicator Survey of Living and Working in Coronavirus Times*, *has an acquaintance tested positive*, could be correlated to life satisfaction, and thus be included in the utility function, as the probability of being asymptomatically contagious and the corresponding risk of transmitting the disease might deter individuals from engaging in social contact. However, our estimations yield no statistically significant correlation between having a positively tested acquaintance and life satisfaction.

Table 12 presents the calibrated values for the three coefficients representing the effects of having social interactions (α_s), income (α_i), and disease exposure (α_e) in the utility function; and two disease evolution parameters, the susceptibility rate multiplier, and the initial effectiveness of wearing facial masks, π_{mask}.

Table 12: Calibrated coefficients

Parameter	**Symbol**	**Value**
Effect of ln(sociality) on life satisfaction	α_s	0.5685
Effect of ln(income) on life satisfaction	α_i	1.3512
Effect of disease exposure on life satisfaction	α_e	-2.2726 $\times 10^7$
Susceptibility rate multiplier		1.1
Initial effectiveness of wearing facial masks	π_{mask}	91.9%

We calibrate α_s by using the coefficients we estimated in OLS estimations for the effect of sociality on life satisfaction. Concretely, we seek the best fit between the impact of sociality on expected utility function and the impact of sociality on life satisfaction in OLS estimations. We first assume "Sociality $=$ 5" and "Sociality $=$ 1" correspond to the maximum and the minimum possible numbers of daily social contact, respectively. In the social contact matrices we employ (Prem et al. [2017]), individuals within the age group 30-34 engage in the highest number of daily social contact, 18.7589. We assume this number corresponds to the case of "Sociality $=$ 5". In our model, an individual who is neither hospitalized nor under home quarantine has only their home contacts and 10% of their other contacts during the most stringent lockdown. The age group 70-74 is endowed with the lowest "home contacts plus 10% of other contacts" value among the age groups in our social contact matrices. They are endowed with 1.1239 and 2.2115 daily contacts at home and other locations. Thus, we assume the minimum possible daily contacts, corresponding to "Sociality $=$ 1", is $1.1239 + 0.1 \times 2.2115 = 1.3450$.

As we normalize pre-epidemic contact rates to 1, we normalize the maximum possible daily social contacts, 18.7589, to 1, and the minimum, 1.3450, to $1.3450/18.7589 = 0.0717$. We then assume the number of daily contacts increases equally between each sociality value, leading to a vector of $\overrightarrow{soc}$ = [0.0717 0.3038 0.5359 0.7679 1]' for "Sociality $=$ 1" to "Sociality $=$ 5". As the maximum possible number of daily contacts is normalized to 1, its impact on the utility function is $\ln(1) = 0$. As the estimated coefficient of "Sociality = 5" gives the impact of social contacts on life satisfaction relative to "Sociality = 1", we normalize it to 0, and the remaining coefficients accordingly to reach a vector of $\overrightarrow{soc}_{OLS}$ = [-1.48025 -0.7125 -0.30775 -0.14025 0]', which is the average values of the estimated effects of sociality on life satisfaction. Here, we are seeking a coefficient, α_s, that gives closest values between $\alpha_s ln(\overrightarrow{soc})$ (the impact of sociality in the expected utility function) and $\overrightarrow{soc}_{OLS}$ (the estimated impact of sociality on life satisfaction). We find that $\alpha_s = 0.5685$ yields the closest values ([-1.4981 -0.6773 -0.3547 -0.1501 0]') to the estimated impact of sociality on life satisfaction with $R^2 = 0.9975$.

To calibrate α_i, we follow the procedure in the preceding paragraph. We assume the pre-epidemic income level, 1, corresponds to the case of "Income loss = 0", the minimum possible income level, $b = 0.4$, to "Income loss = 2", and $(1 + b)/2 = 0.7$ to "Income loss = 1". Here, our target vector is [-1.24075 -0.49 0]', as we estimated an average of -1.24075 and -0.49 points (out of ten) impact on life satisfaction in cases of "Income loss = 2" and "Income loss = 1", in comparison with the benchmark case "Income loss = 0". Here, we find that $\alpha_i = 1.3512$, accompanied by $\iota_i = -0.0028$, gives the closest correspondence and a vector of [-1.2409 -0.4847 -0.0028]' to the natural logarithm of income instead of the target [-1.24075 -0.49 0]'.

For the calibration of the impact of the risk of infection on utility, α_e, we use the following strategy. Having recovered after having tested positive adds 0.3895 points on average (the average of four estimations) to life satisfaction in comparison with having never been tested. In the utility function, the difference between the impact of these two cases is a) not having a *risk of being exposed to the virus* term (health status R) in case of a positive test result followed by recovery, and b) having a *risk of being exposed to the virus* term (unknown health status with possibilities of S, E, A, R, R_A) in case of having never been tested. Therefore, we aim to match a 0.3895-point increase in utility from eliminating the expected risk of being exposed to the virus. The daily probability of becoming exposed to the virus depends on the following factors: a) the daily probability of being at the susceptible, S, phase given that an individual does not know their health status (S, E, A, R, R_A), $\pi_{S,t} = \frac{S_t}{S_t+E_t+A_t+R_t+R_{A,t}}$, b) the daily probability of entering into the incubation period by having social contact with a contagious individual, $\tau(\kappa_t^i)$. Furthermore, as mortality rates are different among age groups, given exposure to the virus, each age group has different incentives to alter their social contacts as their fear of death is proportional to their age-specific mortality rate. Thus, as given in the maximization problem (31), mortality probability upon exposure, δ^{age}, also affects infection risk avoidance proportionally.[24] We use the median values of the probability of being susceptible, $\pi_{S,t} = 0.9735$, and the daily probability of entering into the incubation period by having social contact with a contagious individual, $\tau(\kappa_t^i) = 0.0001331$, that correspond to the disease evolution data between the 02/03/2020[25] and the last day of our calibration, 19/04/2021. We then use the median value of the mortality probability upon exposure, $\delta^{age} = 0.0001322$, and multiply these three values with α_e to reach -0.3895, the impact of the risk of infection on life satisfaction. The corresponding α_e is -2.2726 $\times$ 10^7. This calibrated α_e leads to a -0.3895 point impact for the risk of becoming infected on life satisfaction for the age group with the median mortality rate, 30-34, a -0.0068 point impact for the youngest age group, 0-4, and a -45.25 point impact for the oldest age group, 75+.

We calibrate the two disease evolution parameters, the susceptibility rate multiplier, and the initial effectiveness of wearing facial masks, π_{mask}, by minimizing the distance between daily ICU occupancy data and our simulated ICU occupancy values, that is we use the two values that minimize $\min \sum_t [(ICU_t^{data} - \sum_i C_t^i)]^2$.[26] The resulting daily ICU occupancy and life loss simulations can be seen in Figures 12 and 13, respectively. Finally, our simulations start with an initial exposure rate of 1/1,000,000 among each type-age group.

[24]Derivation of the mortality probability upon exposure, δ^{age}, is given in Appendix 2.8.2.

[25]The first COVID-19 case in Belgium was recorded on 04/02/2020. Then, no cases were reported until 02/03/2020, on which 16 new infections were recorded, followed by an exponential growth of positive cases.

[26]For computational feasibility, we considered a simple grid search over the subsets of [0.5, 0.6, ..., 1.4, 1.5] for the susceptibility rate multiplier, and [0.85, 0.851, ..., 0.949, 0.95] for π_{mask}.

Figure 12: Critical care occupancy data vs simulated critical care occupancy

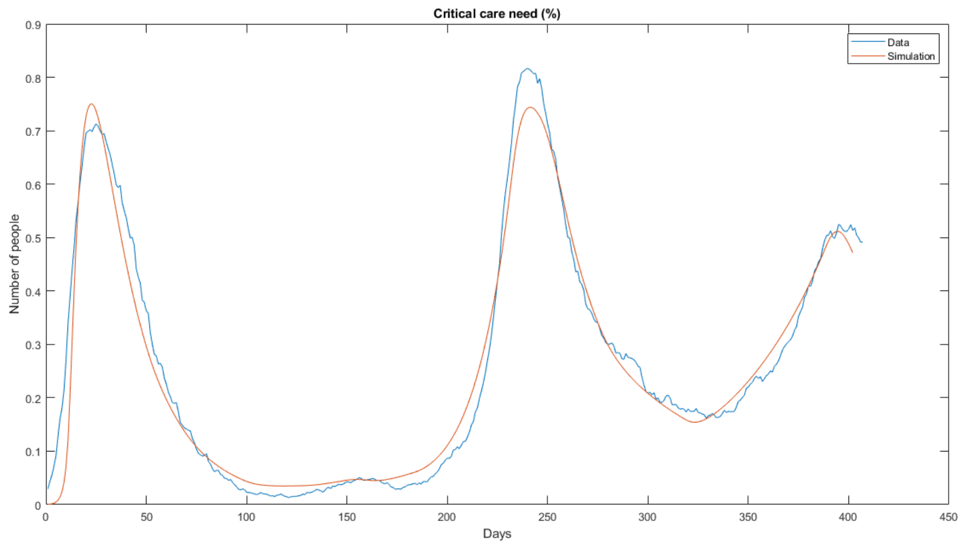
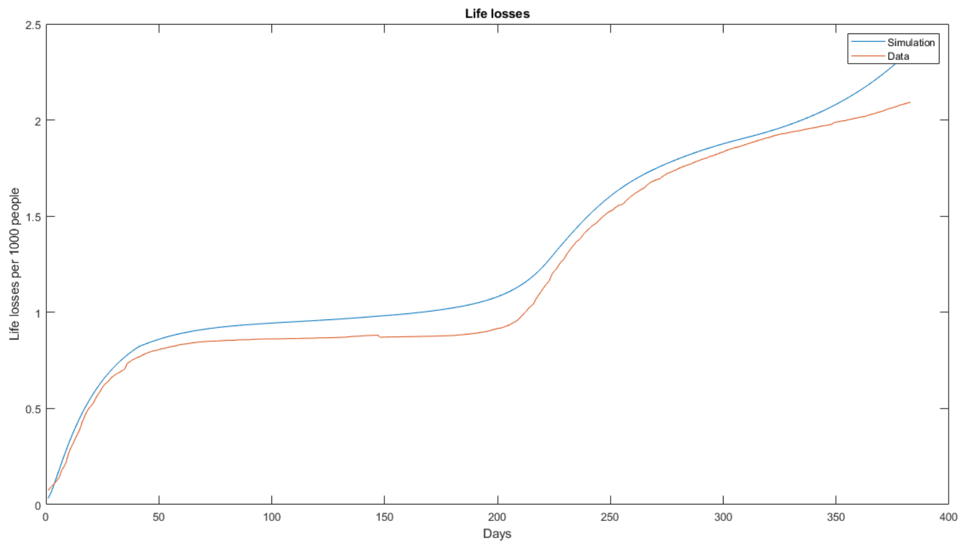


Figure 13: Confirmed death data vs simulated number of deaths



2.6 Simulations

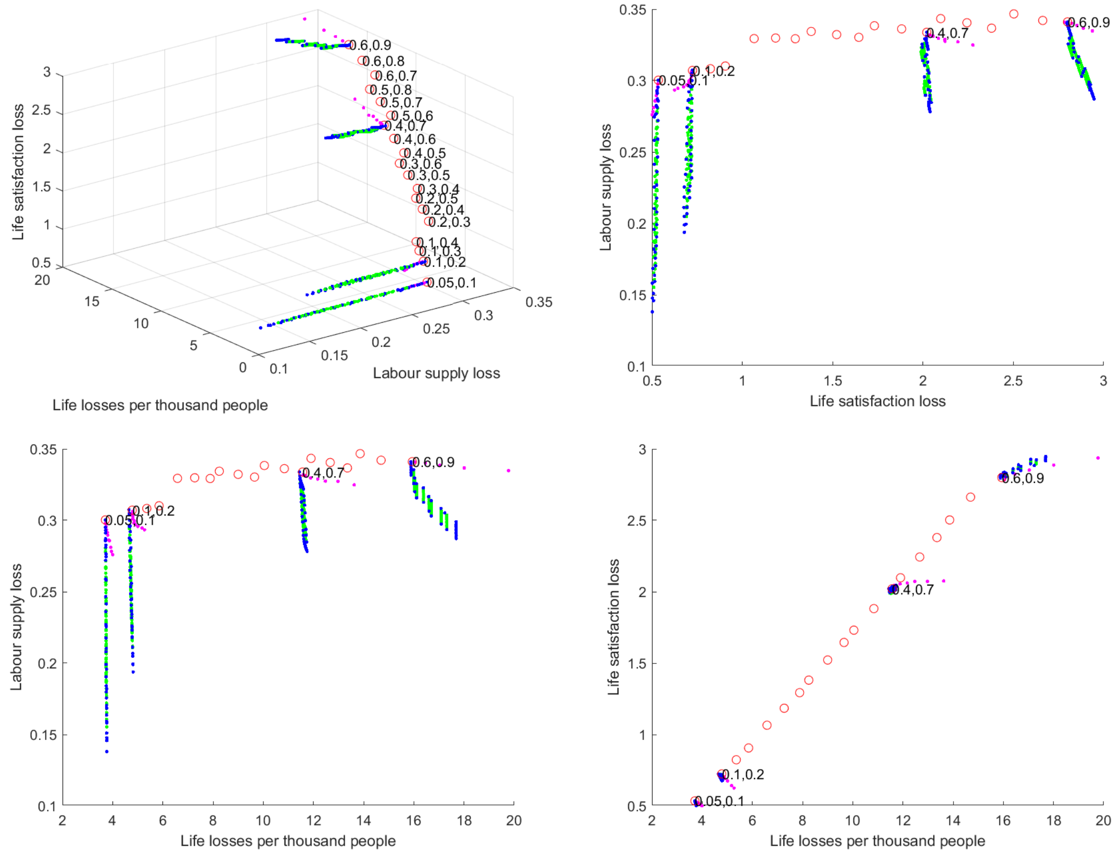
In this section, we simulate various NPI scenarios and study their impacts on three outcomes: the number of life losses, economic costs (measured by the expected loss of supplied labour), and social welfare loss (measured by the expected loss of utility, proxied thanks to our regressions on life satisfaction). Our simulations start on 19/04/2021, the last day of the period we calibrated by using actual contact limitations. We simulate our model for two years following this date. NPI scenarios are characterized by two dimensions: (i) responsiveness and (ii) stringency.

By responsiveness, we mean that NPIs are not in place on a permanent basis and that a criterion is used to decide when to (re)introduce or interrupt them. Since NPIs induce economic costs, for instance in terms of loss of labour supply, they should be implemented only when necessary. The criterion we use is the ICU bed occupancy rate, so NPIs are imposed when ICU occupancy exceeds an upper threshold, and are lifted when ICU occupancy falls below a lower threshold. The most responsive approach we consider would be to impose NPIs when the ICU occupancy rate reaches only 10% and to lift them when the ICU occupancy rate falls below 5%. The least responsive approach we consider would be to impose NPIs only when ICU is used at 90% of its capacity and to lift NPIs when occupancy falls below 60%.

By stringency, we mean that NPIs can restrict mobility with various intensity levels. Mobility restrictions can be applied in three distinct contact locations: workplaces, schools, and "other locations". The stringency of NPIs in these three locations is captured by λ_w, λ_s and λ_o, respectively. These λ parameters represent the share of daily contacts (relative to pre-epidemic contacts) that are allowed when NPIs are implemented, so the smaller the value of λ, the more stringent the policy. For instance, $\lambda_s = 0$ means that schools are fully closed. We assume that schools and workplaces can be fully closed (λ_w and λ_s can go as low as 0), while for "other places", we set the lowest possible contact rate at 0.1. In our simulations, for all locations, we consider 10% increments up to no limitation.

Figure 14 provides a summary of the effects of NPIs' responsiveness and stringency on life losses per thousand people, average labour supply loss, and an average loss of life satisfaction. The upper-left figure simultaneously describes these effects on all three outcomes. On this upper-left figure, the point of origin would describe a situation with minimum life losses, labour supply loss and life satisfaction loss. The other three figures are two-dimensional and pertain to the three possible pairs of outcomes (labour supply and life satisfaction, labour supply and life losses, and life satisfaction and life losses).

Figure 14: Summary of the effects of NPIs' stringency and responsiveness



In each of these graphs, the red circles represent "lockdown outcomes" that result from NPIs with maximal stringency: schools and workplaces are closed and access to other locations is limited to its strict minimum ($\lambda_w = 0$, $\lambda_s = 0$, $\lambda_o = 0.1$). What distinguishes these red circles from each other is their level of responsiveness. The corresponding lower and upper threshold on ICU occupancy is displayed next to each red circle.

Let us now describe how the three outcomes of interest are impacted by a change in NPI responsiveness, considering that when NPIs are imposed, we use the lockdown as a stringency benchmark. First, NPI responsiveness has a strong preserving effect on lives and social welfare. For instance, the least responsive approach (60%-90% thresholds) leads to about sixteen deaths per thousand people and an expected loss of life satisfaction of 2.7 (on a scale from zero to ten), whereas a responsive approach (10%-20%) would lead to only five deaths per thousand people and an expected loss of life satisfaction of only 0.75. This is rather intuitive since intense lockdowns would in this case be imposed as soon as ICUs are being slightly used. One may thus expect that this strategy has severe economic costs. It, therefore, appears counterintuitive that NPI responsiveness also has a mild protective effect on labour supply. For instance, the least responsive scenario (60%-90%) implies a labour supply loss of 33%, whereas the most responsive scenario (5%-10%) implies a 30% loss. This phenomenon can be explained by two factors: a) although the frequency of NPIs decreases as they become less responsive, it takes a lot of time for the reproduction rate of the disease to decrease and for the occupancy rate to fall below the threshold to lift the NPIs, leading to a small difference between the time spent under NPIs between most and least responsive policies, b) as the risk of infection is higher under a less responsive scenario, more workers would prefer to endogenously reduce their labour supply to avoid that risk, leading to exacerbation of labour supply loss. In short, responsiveness may slightly benefit the economy and is for sure critical in preserving lives and social welfare.

Second, let us analyse the impact of NPI stringency on the three outcomes. More specifically, let us depart from the lockdown under four levels of responsiveness (the four red circles corresponding to ICU thresholds of 5%-10%, 10%-20%, 40%-70%, and 60%-90%), and study how decreasing stringency in these four cases affects the outcomes. The blue, green and magenta trails of dots that depart from each of these four red circles represent outcomes that result from various decreases in NPI stringency, holding the level of responsiveness constant. Blue and green dots represent outcomes obtained by gradually opening workplaces and/or at "other places", but keeping schools closed[27]. Blue dots are the boundary cases of stringency [28], and green dots are the

[27]Their impact is discussed in detail with examples in Table 13 and Table 14.

[28]Boundary cases are as follows: a) Other contacts are limited to the minimum possible rate: $\lambda_o = 0.1$ and workplace contacts change by 10% increments: $\lambda_w \in \{0, 0.1, \ldots, 1\}$, b) Other contacts

remaining cases. Magenta dots instead represent outcomes when opening schools to various degrees, while keeping workplaces and other locations closed.[29]

A first important observation is that opening workplaces and other locations strongly reduces labour supply losses while it has virtually no impact on life losses and life expectancy. Under the most reactive approach (5%-10%), the labour supply was at 70% (relative to the pre-pandemic level). Allowing workplaces to operate fully resulted in an increase to 80.9%, while additionally relaxing other locations led to a labour supply level of 86.2%. These gains were accompanied by minor increases in life losses, with 0.16, and 0.28 deaths per 1000 people, respectively. The statement that opening workplaces and other locations reduces labour supply losses at close to zero costs in terms of life and welfare outcomes is valid for all levels of responsiveness, except for the least reactive approach (60%-90%). When policies are sluggish, a trade-off appears between labour and life and social welfare outcomes: the benefits of reopening workplaces and other locations on the labour market come at the cost of more life losses and lower social welfare.

This notion of a trade-off between economic costs and life and welfare costs is even more pronounced when it comes to opening schools: school openings improve labour supply (notably by relaxing working parents' constraints) but increase life losses and decrease social welfare. One exception is worth noting: when policies are very reactive, reopening schools can also be beneficial to social welfare, though it is still detrimental to life outcomes.

Table 13 discusses in detail the impact of different NPI stringency levels of the workplace and other contacts (blue and green dots) on life, labour supply, and life satisfaction losses in an early intervention case. With 5% and 10% ICU occupancy thresholds to lift and impose NPIs, allowing workplaces to operate fully resulted in a labour supply level increase from 70.0% to 80.9%. Allowing other location contacts

are not restricted: $\lambda_o = 1$ and workplace contacts change by 10% increments: $\lambda_w \in \{0, 0.1, \ldots, 1\}$, c) Non-essential workplace contacts are not allowed: $\lambda_w = 0$ and other contacts change by 10% increments: $\lambda_o \in \{0.1, 0.2, \ldots, 1\}$, d) Workplace contacts are not restricted: $\lambda_w = 1$ and other contacts change by 10% increments: $\lambda_o \in \{0.1, 0.2, \ldots, 1\}$.

[29]For illustrative purposes, for the least responsive case (60%-90%), we only display school contact limitations $\lambda_s \in \{0, 0.1, \ldots, 0.5\}$ as allowing higher contacts at schools yielded huge increases in life losses.

increased labour supply from 70.0% to 72.5%. Imposing no restrictions on workplace and other contacts led to a labour supply increase to 86.2%. These gains were accompanied by small increases in life losses, with 0.16, 0.93 and 0.28 death per day over a simulation period of 730 days, respectively. For life satisfaction, we observe that the effect of being more social, and having a higher income through a higher labour supply exceeds the effect of facing a slightly higher risk of exposure to the epidemic. As stringency of limitations has a limited effect on life losses, looser limitations on workplace and other contacts may alleviate losses in labour supply and life satisfaction without causing a high impact on the number of deaths when limitations are imposed frequently at low ICU occupancy levels.

Table 13: Impact of NPI stringency for an early intervention case (5% to lift and 10% to impose)

	Policy 1	Policy 2	Policy 3	Policy 4
Allowed contacts at workplaces	0	100%	0	100%
Allowed contacts at schools	0	0	0	0
Allowed contacts at other locations	10%	10%	100%	100%
Life satisfaction (normalized)	0	0.0095	0.0223	0.0329
Mean active labour supply	70.0%	80.9%	72.5%	86.2%
Extra life losses per day	0	0.16	0.93	0.28

Table 14 describes in detail the impact of NPI stringency in a late intervention case. With 60% and 90% ICU occupancy thresholds to lift and impose NPIs, opening up workplaces, other locations, and both locations brought labour supply gains from 65.9% to 70.1%, 76.8%, and 71.3%, respectively. However, in this case of late intervention, these gains were realized with significant increases in deaths, between 15 and 28 deaths per day on average. In all three cases, life satisfaction was also affected negatively.

Table 14: Impact of NPI stringency for a late intervention case (60% to lift and 90% to impose)

	Policy 1	Policy 2	Policy 3	Policy 4
Cap on non-essential workplace contacts	0	100%	0	100%
School contacts	0	0	0	0
Cap on other contacts	10%	10%	100%	100%
Life satisfaction (normalized)	0	-0.0198	-0.1191	-0.1475
Mean active labour supply	65.9%	70.1%	66.8%	71.3%
Extra life losses per day	0	15.58	18.66	28.06

Table 15 presents a summary of the impact of four example policy sets differing in stringency and responsiveness. In figures 15-19, we present an overlook of the factors underlying our simulated results by using these example policy sets. Policy 1 is the policy set with the most responsive (imposed at 5% and lifted at 10% ICU occupancy) and stringent (all non-essential work and school contacts are forbidden, 10% of other contacts are allowed) limitations. From Policy 1 to Policy 2, stringency changes (workplaces are open, schools are closed, 50% of other contacts are allowed) while timing remains the same. From Policy 1 to Policy 3, timing changes (imposed at 60% and lifted at 90% ICU occupancy) while stringency remains the same. From Policy 1 to Policy 4, both stringency changes while timing remains the same.

Table 15: Comparison of the impact of NPI stringency and responsiveness

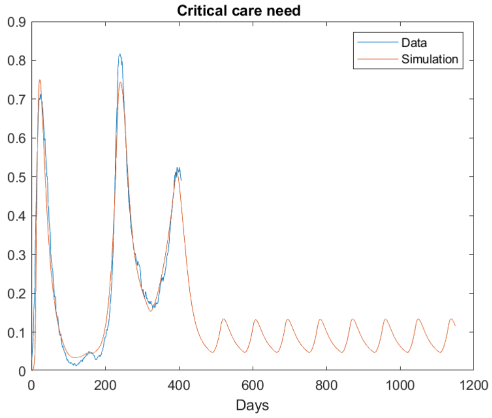
	Policy 1	Policy 2	Policy 3	Policy 4
Cap on non-essential workplace contacts	0	100%	0	100%
School contacts	0	0	0	0
Cap on other contacts	10%	50%	10%	50%
Criteria to impose (in terms of ICU occupancy)	5%	5%	60%	60%
Criteria to lift (in terms of ICU occupancy)	10%	10%	90%	90%
Life satisfaction (normalized)	0	0.0151	-2.2642	-2.3261
Mean active labour supply	70.0%	83.0%	65.9%	68.8%
Life losses per 1000	3.71	3.74	15.93	16.39

To conclude, the main two takeaways from this simulation exercise are the following. First, increasing the responsiveness of NPIs is beneficial to all three outcomes, though the impact on labour supply is milder. Second, decreasing the NPIs' intensity always improves labour supply, and the effect on social welfare and life outcomes depends on the nature of the NPIs and the level of responsiveness of policies. When policies are responsive, opening schools can improve social welfare (though it increases life losses), while opening workplaces and other locations is neutral both in terms of social welfare and life losses. When policies are sluggish, opening any of the locations has adverse impacts on life losses and social welfare. These results suggest that responsiveness is crucial for two reasons: (i) it improves all outcomes holding stringency constant, and (ii) it mitigates the adverse impacts of a reduction of NPI stringency, and by doing so, it allows the implementation of measures that minimize economic costs.

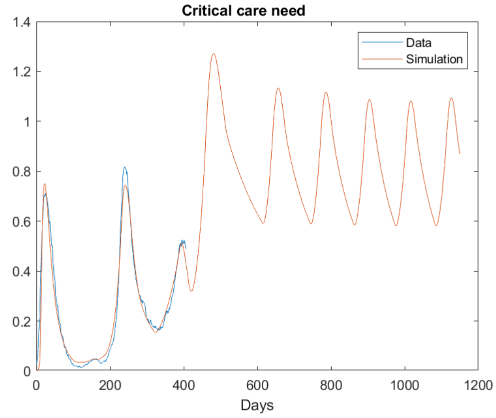
Figure 15 shows the effects of NPIs on critical care needs. Here, blue curves represent actual ICU occupancy rates in Belgium until 19/04/2021, and orange curves represent our simulation results. Before 19/04/2021, these results were based on the NPI stringency that had been actually applied, whereas after this date we apply different NPI scenarios. The upper-left figure represents the rate of critical care occupancy when the most responsive strategy (based on ICU thresholds of 5%-10%) with lockdown levels of stringency is applied (Policy 1 in Table 7). When NPIs are applied, only the minimum possible rate of other contacts is allowed while no school or non-essential workplace contacts are allowed. This leads to a fast drop in the ICU occupancy need, and consequently, there are frequent phases with strict lockdowns and without NPIs. As NPIs are imposed at a low ICU occupancy rate, ICU need never exceeds 13.4%, a rate slightly higher than 10%.

Figure 15: Critical care need

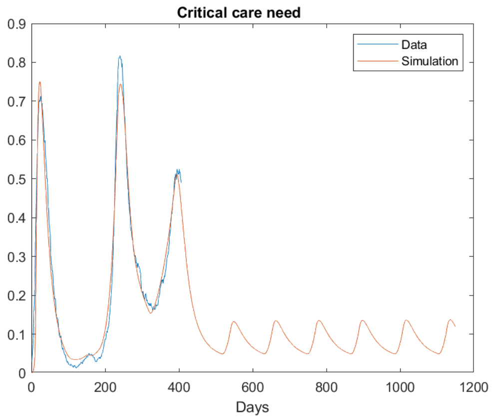
(a) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



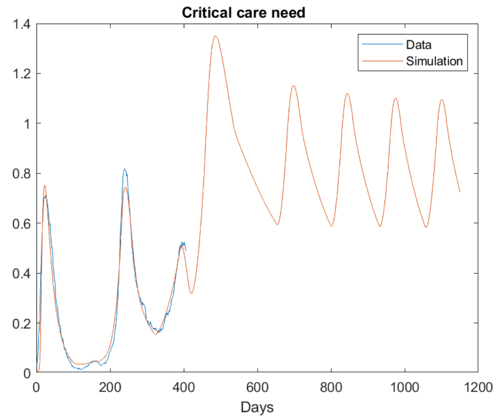
(b) Responsiveness: 60%-90%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(c) Responsiveness: 5%-10%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



(d) Responsiveness: 60%-90%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



The lower-left part of the figure represents the effects of a different set of NPIs where the responsiveness of NPIs is maintained at its highest level, but NPI stringency is reduced. In this case, NPIs are looser with workplaces kept open, schools kept closed, and half of the other location pre-epidemic contacts are allowed during the NPI phases (Policy 2 in Table 7). Under this NPI scenario, as a lower fraction of social contacts is restricted, a longer time is needed to lift NPIs. Critical care needs never exceed 13.7% under this scenario.

On the upper-left to the upper-right part of Figure 15, we maintain NPI stringency but consider instead the lowest level of responsiveness (60%-90% ICU thresholds). In this case, NPIs are imposed most stringently when 90% ICU occupancy is reached and lifted when the ICU occupancy rate drops to 60% (Policy 3 in Table 7).

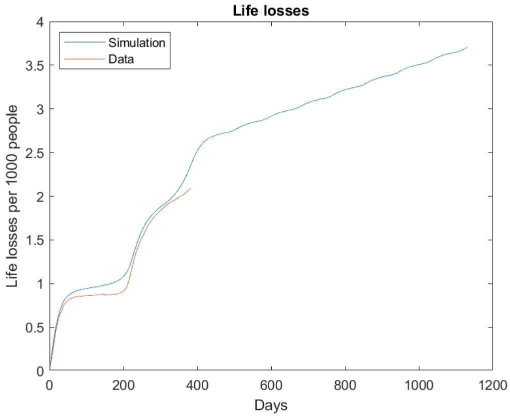
Under this scenario, ICU need cannot be kept under control until it surpasses 100%. This is because although contacts outside the home are limited to the minimum possible rate, the number of contagious individuals is still at a high rate, causing the per-contact exposure rate to stay high. This effect is further exacerbated as there is also a high number of individuals already in the incubation period becoming infected during the NPI phase. Our simulations yielded a maximum 127% ICU need rate.

The lower-right part of Figure 15 corresponds to Policy 4 in Table 7. Here, NPIs are imposed at 90% and lifted at 60% of ICU occupancy. The stringency of NPIs is the same as in the lower-left part of the figure. Here, observe a longer recovery duration, and longer periods with ICU capacity overwhelmed with a maximum 137% ICU need rate.

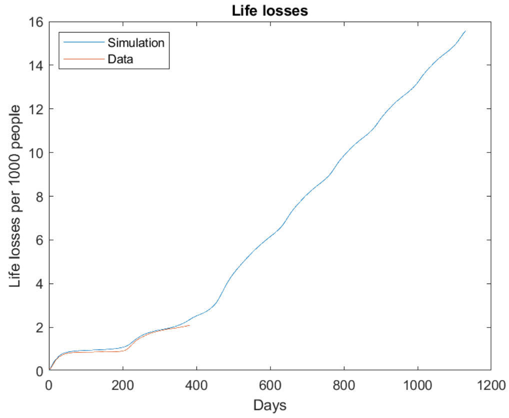
Figure 16 displays the impact of our four representative NPI policies on life losses. The ordering of policies is the same as it is in Figure 15. We obtain 3.71 deaths per thousand people since the beginning of the epidemic when we simulated epidemic evolution with the most stringent and responsive NPIs. Our simulations imply that stringency plays a small role in life losses. With our example policy of keeping workplaces open, and schools closed, and allowing half of the pre-epidemic other location contacts, we obtain 3.74 deaths per thousand people, accounting for a difference of one more death every second day. However, changing the responsiveness of NPIs resulted in a much bigger change in life losses. When we maintain the highest level of stringency while changing responsiveness from the thresholds of 5%-10% to 60%-90%, we obtain 15.93 deaths per thousand people, accounting for 188 more deaths each day on average. Loosening these NPIs slightly (from Policy 3 to Policy 4 in Table 13) exacerbated this situation by adding 7 more deaths daily.

Figure 16: Life losses

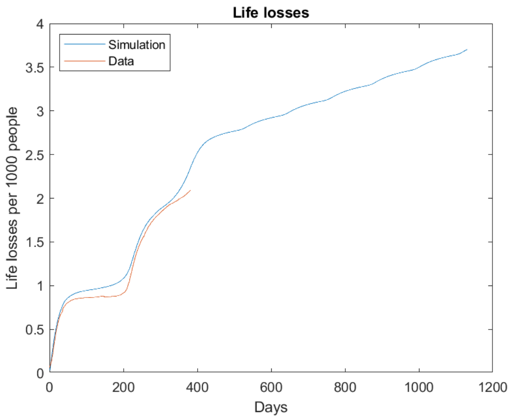
(a) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(b) Responsiveness: 60%-90%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(c) Responsiveness: 5%-10%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



(d) Responsiveness: 60%-90%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$

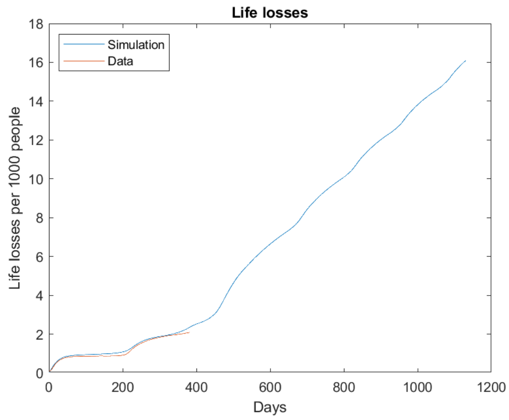
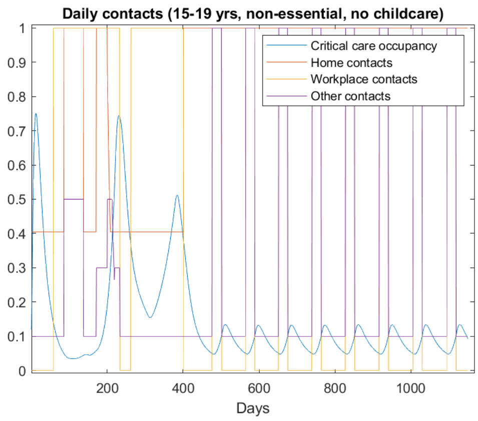


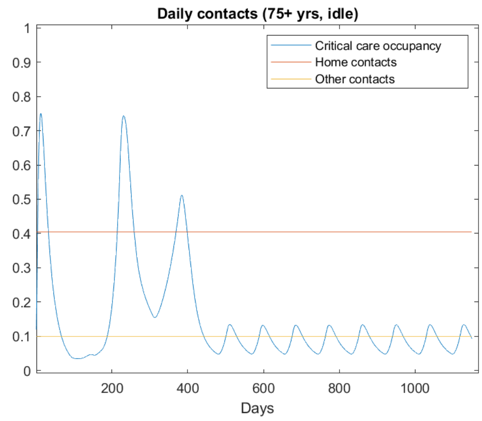
Figure 17 presents optimal daily social contact rates endogenously determined under given NPIs. In each figure, home contacts are given in red, workplace contacts in yellow, and other contacts in purple. These contacts are given as ratios with respect to their pre-epidemic rates, which are 1. Individuals optimize their social contacts at each contact location by observing the number of infected people they may get in contact with. For illustrative purposes, as NPIs are imposed according to ICU needs, we use ICU need (in blue) to represent the severity of the epidemic. The first row of the figure comprises daily contact rate changes for example young (non-essential workers within the age interval 15-19) and an example old (not in the labour force within the age interval 75+) individual group when the most stringent and most responsive NPIs are in place. At one extreme, young individuals tend to engage in as many social contacts as they can. This simulated result is due to their very low mortality rate (d_C^{15-19}=0.000148). At the other extreme, our model generates a result that the elderly tend to decrease their social contacts even when they are allowed to have them. This is driven by their mortality rate is approximately 700 times larger than the example young group at the age interval 15-19 (d_C^{75+}=0.1061). This suggests that our endogenization of exposure behaviour is properly calibrated. For illustrative purposes, we present the impact of NPIs on social contact rates via an individual group in between (within the age interval 35-39, non-essential workers, not constrained by childcare), with a mortality rate, d_C^{35-39}=0.00146.

Figure 17: Optimal daily social contacts for example individual groups

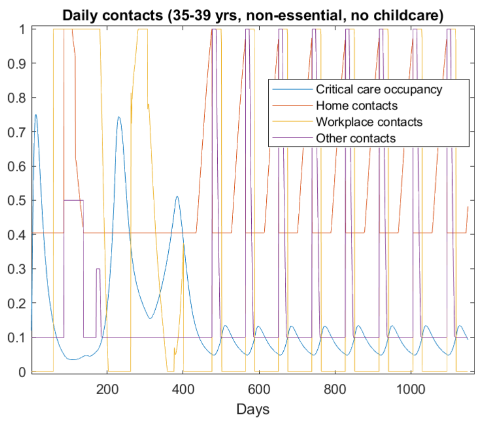
(a) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



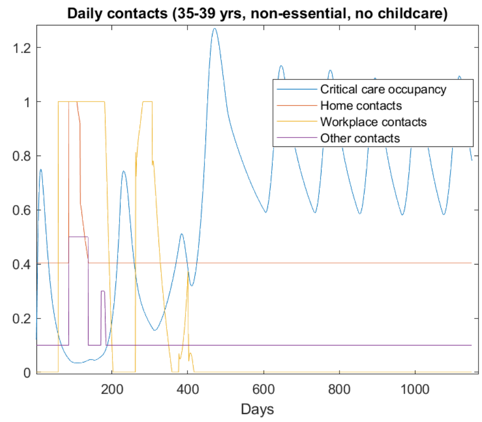
(b) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



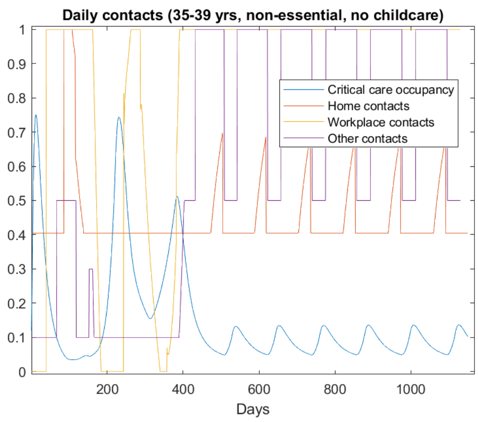
(c) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



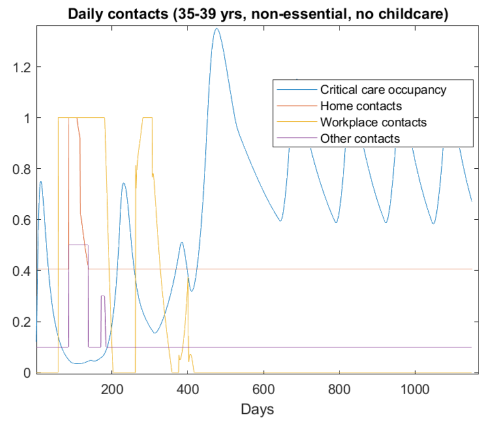
(d) Responsiveness: 60%-90%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(e) Responsiveness: 5%-10%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



(f) Responsiveness: 60%-90%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



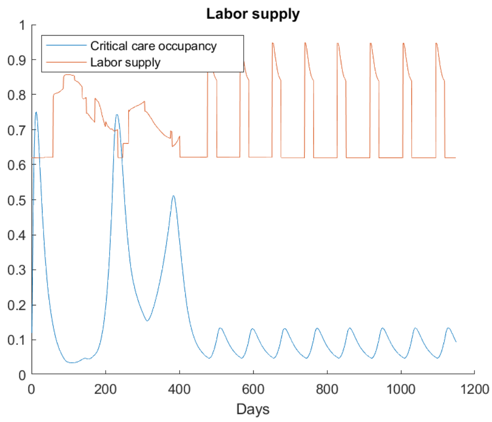
The second and third rows of Figure 17 illustrate social contact changes of the sample individual group (within the age interval 35-39, non-essential workers, not constrained by childcare) for four sets of policies we compare. The middle-left part of the figure stands for the effects of the most stringent and responsive NPIs. Our model generates that this averagely cautious individual group engages in as many social contacts as they are allowed to when the disease incidence rate is very low, corresponding to an ICU occupancy of less than 5%. Then, they start to gradually cut back on home contacts first, as home contacts cause a higher probability of disease exposure due to not wearing protective gear. Then, they cut back on other contacts, mainly driven by the fact that they do not generate income. Finally, around an incidence rate corresponding to a 35% ICU occupancy, they start endogenously decreasing their labour supply. For younger (older) age groups, we observe similar effects at higher (lower) incidence rates.

Stringency changes from the most stringent to a policy of allowing non-essential work contacts and half of the other location contacts, when we move from the middle-left to lower-left parts of Figure 17. Here, as the disease incidence rate is kept at very low levels, we observe no incentives to endogenously reduce work or other contacts. We observe an endogenous reduction in home contacts due to a trade-off between avoiding infection risk and enjoying social interactions. Timing changes from the most responsive to the least responsive as we move from the left to right parts of the middle and low rows of Figure 17. Even though workplace contacts are allowed, we predict that workers might voluntarily decrease their work contacts, exacerbating the labour supply loss due to NPIs. We observed reductions in home and other contacts with a negative impact on life satisfaction.

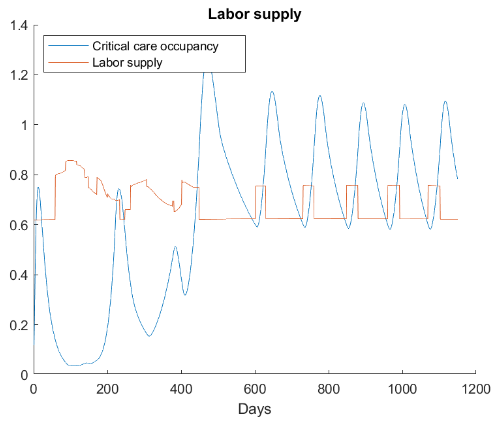
Figure 18 depicts the labour supply streams for the four example policies we compare. Orange curves represent labour supply and blue curves represent ICU needs. The upper-left part of the figure represents the impact of the most stringent and most responsively imposed NPIs on labour supply. When NPIs are in place, all non-essential work, and most of the work related to other location contacts are abandoned. Further, due to the childcare effect, school closures decrease labour supply in the essential and remote sectors. As a result, our model generates labour supply losses of almost 40% during lockdown phases. Under the scheme of implementing the most responsive NPIs, labour supply frequently diminishes to a level slightly above 60%. When restrictions are lifted, labour supply approaches, but does not reach 100%, due to the fact that the labour supply of some parents is constrained by school closures, as well as because non-essential workers endogenously reduce their labour supply. the indirect effect of endogenous labour supply reduction. We observe a slight gradual decrease in labour supply as infection risk increases during the phases in which NPIs are lifted. Overall, we simulated a 31% loss in labour supply under this scenario.

Figure 18: Labour supply

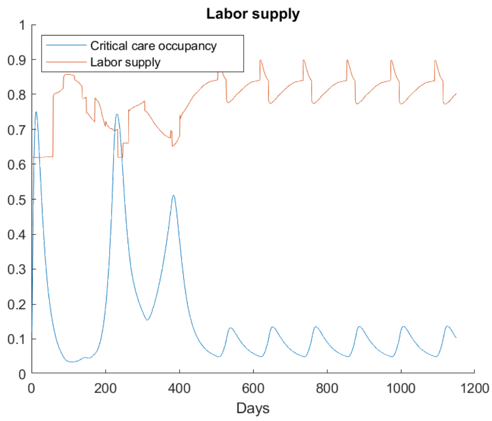
(a) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



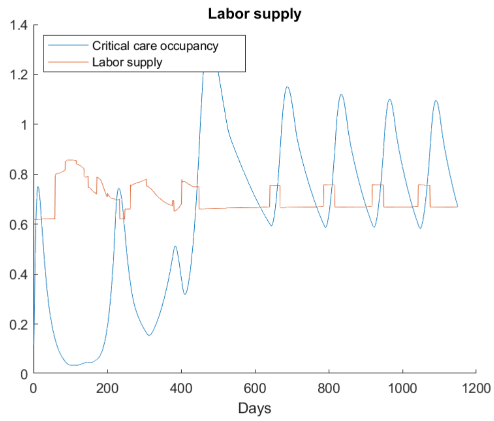
(b) Responsiveness: 60%-90%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(c) Responsiveness: 5%-10%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



(d) Responsiveness: 60%-90%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$

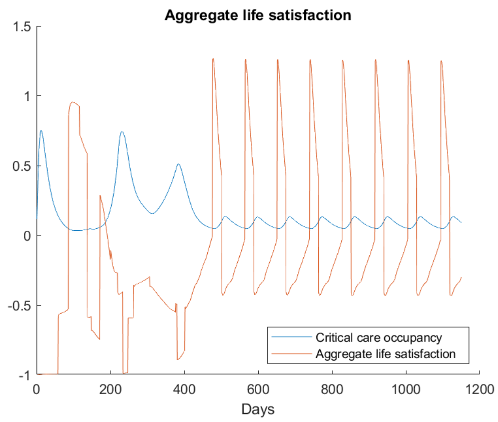


The lower-left part of the figure stands for the policy with all pre-epidemic work and half of the other contacts allowed. Here, a slightly higher time is spent under the NPIs. However, as work and other contacts are partially allowed, labour supply diminishes to the vicinity of 80%, with a gradual increase in labour supply as the risk of infection decreases. We simulated a 14.8% aggregate labour supply loss under this scenario. The upper-right part of the figure stands for the most stringent policy imposed very late, at 90% ICU occupancy. In this case, labour supply is limited to the same level as in the upper-left part during the NPI phases. NPI phases last longer as recovery to a low ICU occupancy takes longer when there are many more infected individuals. During the short-lived phases of no NPIs, labour supply does not reach its maximum allowed rate because of endogenous reduction in work contacts when the risk of infection is too high. Aggregate labour supply loss was 34.1% under this scenario. Even when the NPIs are less stringent by keeping this timing (the lower-right part of the figure), labour supply remained lower than what it used to be under the policy depicted in the lower-left part of the figure, due to the risk of infection staying too high. In this case, aggregate labour supply loss was 28.7%.

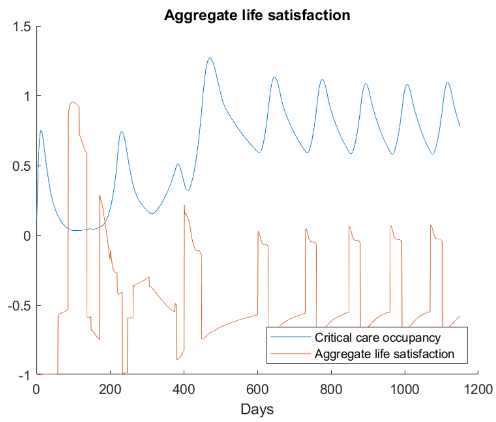
Figure 19 shows the daily life satisfaction levels our model generated for the four example policies we compare in the same order as in Figure 15. Orange curves represent life satisfaction and blue curves represent ICU needs. In all parts of the figure, we observe the same expected pattern at the instant NPIs are lifted and re-imposed. When NPIs are imposed, contacts and income are reduced causing a negative jump in the value of life satisfaction. From then on, life satisfaction gradually increases mainly due to the gradual decrease in the risk of infection. When NPIs are lifted, contacts and income increase, leading to a positive jump in life satisfaction. As the risk of infection increases following the relaxation of NPIs, life satisfaction gradually decreases.

Figure 19: Life satisfaction

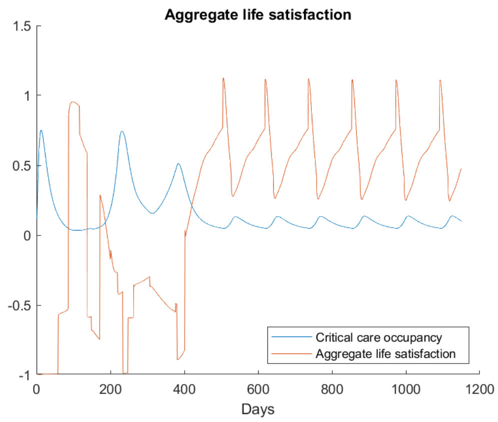
(a) Responsiveness: 5%-10%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



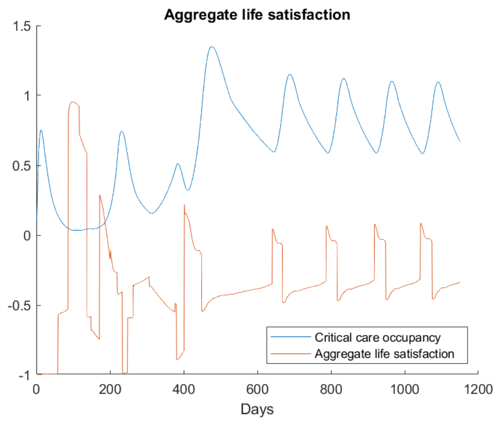
(b) Responsiveness: 60%-90%
Stringency: $\lambda_w = 0$, $\lambda_o = 0.1$, $\lambda_s = 0$



(c) Responsiveness: 5%-10%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



(d) Responsiveness: 60%-90%
Stringency: $\lambda_w = 1$, $\lambda_o = 0.5$, $\lambda_s = 0$



Between two sets of NPIs with the same timing, the environment is almost identical at the instant NPIs are lifted. Therefore, we observe almost identical levels of life satisfaction between the upper-left and the lower-left, and between the upper-right and the lower-right parts of Figure 19. However, in the left column of the figure, life satisfaction levels at this instant are higher as the risk of infection is lower, and as an effect of this endogenous contact reduction is lower. As we move from the upper parts to the lower parts of the figure, timing remains the same with stringency becoming looser. As stringency loosens, more contacts and more labour supply are allowed during the NPI phases. During these phases, our model generates lower life satisfaction loss compatible with this intuition.

Overall, by keeping stringency constant, we found that life satisfaction decreases as we wait longer to implement NPIs. When timing is kept constant, a lower stringency increases life satisfaction when NPIs are implemented at low levels of disease incidence. When stringency is lower, NPIs remain in place for a longer time. However, disease evolution is under control. Thus, the risk of infection does not affect life satisfaction significantly. Moreover, with fewer restrictions on social contacts and labour supply, life satisfaction is higher during the NPI phases. A lower stringency has a negative effect on life satisfaction when NPIs are implemented at high levels of disease incidence. In this case, the disease cannot be kept under control as ICU need exceeds ICU capacity, and it takes much longer to reach the threshold to lift NPIs, leading to a longer time spent with a high life satisfaction loss. The benefit of loosening restrictions on social contacts and labour supply falls short of the loss of life satisfaction.

2.7 Conclusion

In the absence of vaccines or effective medical treatments, governments must resort to non-pharmaceutical interventions (NPIs) to mitigate the spread of infectious diseases. Among the various NPIs, social contact limitations play a pivotal role in controlling transmission. Despite their efficacy in preserving lives, these measures can also inflict negative consequences on economic productivity and individuals' mental well-being. This underscores the intricate balance and challenges involved in devising social contact limitation policies.

In our study, we sought to address the overarching research question: "Given the full range of NPIs, which policy combinations, in terms of stringency and responsiveness, offer an optimal balance between the adverse effects on health, economy, and mental well-being?" Our findings underscore the importance of responsiveness in implementing social contact limitations, rather than solely focusing on their stringency. The most effective strategies involve early interventions, as these lead to the lowest losses in economy and mental well-being for a given number of life losses.

This research has broader implications for managing the societal impact of infectious diseases beyond the specific case of COVID-19. By understanding the complex interplay between policy design, economic productivity, mental well-being, and the progression of a pandemic, we can better inform the development of NPI strategies for future outbreaks. As the threat of emerging infectious diseases persists, it is crucial to continue refining our understanding of these trade-offs and to develop adaptable models and policy tools that will enable us to safeguard public health while minimizing the social and economic consequences of such crises.

To conduct this study we developed an economic-epidemiological model of COVID-19 transmission that interrelates pandemic evolution, labour supply and mental well-being. By incorporating key features such as SEIRD models, social contact matrices by age, and varying degrees of NPI responsiveness and stringency, our model provides a valuable tool for policymakers and researchers to evaluate the effectiveness of different intervention strategies and their consequences on public health and economic outcomes. Furthermore, the model's adaptability allows for the exploration of various scenarios, taking into account differences in population demographics, sector-specific labour supply, and the nature of the crisis at hand.

In our model environment, disease transmission occurs via social contacts between sub-groups of the population, in terms of age and employment status, at four contact locations: a) home, b) schools, c) workplaces, and d) other locations. We model the evolution of the pandemic by an extended version of SEIRD models. NPIs determine a) the maximum ratio of the pre-pandemic daily number of social contacts at each contact location except for home, and b) the timing of these social contact limitations. The timing or responsiveness of NPIs depends on a) an upper intensive care unit (ICU) occupancy rate to impose them, and b) a lower ICU occupancy rate to lift them. Individuals observe the state of the pandemic and NPIs and maximize a mental well-being function that we obtained through econometric estimation and that interrelates the age-specific propensity to avoid infection, income, and the daily number of social contacts. Income comes from economic activity, which takes place either remotely without social contacts or at different types of workplaces through social contacts. Income is affected by workplace closures for non-remote workers and school closures for a proportion of the population that we assume to be affected by childcare needs. By combining data from several sources we replicated the first 15 months of the COVID-19 pandemic in Belgium and simulated the rest of the pandemic by using combinations of policy sets differing in stringency and timing. Our simulations implied that responsiveness matters most for the NPIs and strict lockdowns are not necessary when they are imposed for a short duration at a very early stage of each pandemic wave.

The interdisciplinary nature of our study makes it relevant to researchers and professionals across various fields, including public health, economics, and social sciences, promoting a more holistic understanding of the factors influencing public health crises and their consequences. However, our findings should be taken into consideration with the caveats of our model. We use constant disease specificities, such as mortality rate or transmission rate, after the period that we have replicated. The emergence of new disease variants would alter the quantitative results of each NPI simulation, although qualitatively we still would end up favouring early interventions. Furthermore, we assumed labour supply can be mobilized immediately after NPIs are lifted. However, this approach leaves out any changes in labour demand, which would make the policy of imposing frequent social contact limitations more difficult.

In conclusion, our work offers a robust and versatile framework that can be utilized to better understand and navigate the challenges posed by public health crises and pandemics, transcending the specific case of COVID-19 and providing valuable insights for future events.

2.8 Appendix

2.8.1 Utility function

We define the expected daily utility or daily life satisfaction of an individual of type i in case of survival as:

$$LS^i_{survival,t} = Constant^i \times (sociality^i_t(\kappa^i_t))^{\alpha_s} \times (income^i_t(\kappa^i_{w,t}))^{\alpha_i}$$

We define the expected daily utility or daily life satisfaction of an individual of type i in case of survival as:

$$LS^i_{death,t} = Constant^i \times (sociality^i_t(\kappa^i_t))^{\alpha_s} \times (income^i_t(\kappa^i_{w,t}))^{\alpha_i} \times \hat{\Delta}$$

where $\hat{\Delta}$ is the effect of ending up with death due to disease exposure at time t.

Taking the natural logarithm of the daily life satisfaction functions gives the following expected life satisfaction expression:

$$\mathbb{E}(LS^i_t) = ln(Constant^i) + \alpha_s ln(sociality^i_t(\kappa^i_t)) + \alpha_i ln(income^i_t(\kappa^i_{w,t})) + p^{\Delta}_t(\kappa^i_t) ln(\hat{\Delta})$$

where p^{Δ}_t is the daily probability of ending up with death due to disease exposure at time t. We let $\alpha_e = ln(\hat{\Delta})$ for simplicity.

We regard the constant term as the impact of individual-specific attributes or behaviours, such as age, gender, or doing regular exercise, that are statistically significantly correlated with *life satisfaction* and not related to social contacts. p^{Δ}_t, the daily probability of ending up with death due to disease exposure at time t, depends on three factors: a) an age-specific mortality rate, d^{age}_C (given entry into the *exposed* phase, older people die with a higher probability than younger people), b) the daily probability of being susceptible for the individuals with an unknown health status (S, E, A, R, R_A), $\pi_{S,t} = \frac{S_t}{S_t+E_t+A_t+R_t+R_{A,t}}$ (only the susceptible individuals may enter into the *exposed* phase), and c) the probability of exposure to the disease through social contact, $\tau(\kappa_t^i)$. Therefore, the expected life satisfaction maximization problem can be written as:

$$\max_{\kappa_{h,t}^i,\kappa_{w,t}^i,\kappa_{o,t}^i} \mathbb{E}(LS_t^i) = \Big\{\alpha_s ln(\text{sociality}_t^i(\kappa_t^i)) + \alpha_i ln(\text{income}_t^i(\kappa_{w,t}^i)) + \alpha_e \underbrace{\delta^{age}}_{\text{mortality rate by age,}} \underbrace{\pi_{S,t}}_{\text{prob. of being susceptible,}} \underbrace{\tau(\kappa_t^i)}_{\text{exposure prob.}} \Big\} \quad (32)$$

for the individuals with an unknown health status (S, E, A, R_A). Individuals under home quarantine or at the hospital cannot maximize their life satisfaction as they are enforced to have a fixed number of contacts.

2.8.2 Mortality probability upon exposure

The mortality probability upon exposure, δ^{age}, is derived as follows. With a probability ϕ_A, infection remains asymptomatic, only leading to recovery. Now, let δ_M^{age} and δ_C^{age} represent the mortality probabilities while being mildly and critically infected, respectively. A person may die, recover, or enter into the *critically infected* phase when they are at the *mildly infected* phase. When a person is critically infected, they may die or re-enter the *mildly infected* phase. Thus,

$$\delta_M^{age} = \Big\{p_{M,death}^{age}.1 + p_{M,recovery}^{age}.0 + p_{ICU}^{age}[p_{discharge}^{age}\delta_M^{age} + \delta_C^{age}]\Big\} \quad (33)$$

$\delta_M^{age} = p_M^{age} + p_{ICU}^{age}[p_{discharge}^{age}\delta_M^{age} + \delta_C^{age}]$, where p_{ICU}^{age}, $p_{discharge}$, and $p_{M,recovery}$, p_M^{age} are the probabilities of becoming critically infected while mildly infected, becoming mildly infected while critically infected, and the probability of recovery and death without developing an ICU need. These probabilities can be written in terms of daily probabilities of each event they depend on:

$$p_{ICU}^{age} = \frac{\nu}{\nu + r_M + d_M^{age}}$$

$$p_{M,death}^{age} = \frac{d_M^{age}}{\nu + r_M + d_M^{age}}$$

$$p^{age}_{discharge} = \frac{r_C}{r_C + d^{age}_C}$$

$$\delta^{age}_C = \frac{d^{age}_C}{r_C + d^{age}_C}$$

Now, we plug these four equations into (33):

$$\delta^{age}_M = \frac{d^{age}_M}{\nu + r_M + d^{age}_M} + \frac{\nu}{\nu + r_M + d^{age}_M}\left[\frac{r_C}{r_C + d^{age}_C}\delta^{age}_M + \frac{d^{age}_C}{r_C + d^{age}_C}\right] \tag{34}$$

Simplifying equation (33), yields:

$$\delta^{age}_M = \frac{\nu d^{age}_M r_C + \nu d^{age}_M d^{age}_C + r_M r_C d^{age}_M + r_M d^{age}_M d^{age}_C + r_C d^{age}_M d^{age}_C + d^{age}_M d^{age}_M d^{age}_C}{r_M r_C + r_M d^{age}_C + r_C d^{age}_M + d^{age}_M d^{age}_C} \tag{35}$$

Finally, as $\delta^{age} = \phi_A \times 0 + (1 - \phi_A)\delta^{age}_M$,

$$\delta^{age} = (1-\phi_A)\frac{\nu d^{age}_M r_C + \nu d^{age}_M d^{age}_C + r_M r_C d^{age}_M + r_M d^{age}_M d^{age}_C + r_C d^{age}_M d^{age}_C + d^{age}_M d^{age}_M d^{age}_C}{r_M r_C + r_M d^{age}_C + r_C d^{age}_M + d^{age}_M d^{age}_C} \tag{36}$$

Plugging the values in Table 8 into equation (35) yields:

$$\delta^{age} = 0.6\frac{0.00224 d^{age}_C + 0.021669(d^{age}_C)^2 + 0.014161(d^{age}_C)^3}{0.015005 + 0.0295 d^{age}_C + 0.119(d^{age}_C)^2} \tag{37}$$

2.8.3 Tables and figures

Table 16 shows age-specific mortality rates and base susceptibility rates.

Table 16: Age-specific disease characteristics

Age group	**Base susceptibility rate**	**Mortality rate**
0-4	40%	0.0026%
5-9	40%	0.0026%
10-14	38%	0.0148%
15-19	38%	0.0148%
20-24	79%	0.06%
25-29	79%	0.06%
30-34	86%	0.146%
35-39	86%	0.146%
40-44	80%	0.295%
45-49	80%	0.295%
50-54	82%	1.25%
55-59	82%	1.25%
60-64	88%	3.99%
65-69	88%	3.99%
70-74	74%	8.61%
75+	74%	10.61%

Table 17 presents OLS estimations including the estimated coefficients of all regressors.

Table 17: Dependent variable: Life satisfaction

	(1)	(2)	(3)	(4)
Male	0.0858**	0.0786*	0.0939**	0.0895**
Age	-0.0259**	-0.0257**	-0.0270**	-0.0264**
Age square per thousand people	0.338**	0.337**	0.342**	0.340**
Weekly cases per thousand people	-0.0198	-0.0241	-0.0724	-0.119**
Weekly cases square per million people	0.0194	0.0199	0.0240*	0.0307**
Health problems	-0.542***	-0.534***	-0.544***	-0.538***
Tertiary education	0.225***	0.222***	0.232***	0.230***
Workload increased	-0.145***	-0.146***	-0.144***	-0.147***
Workload decreased	-0.143***	-0.151***	-0.143***	-0.152***
Home office	0.207***	0.205***	0.212***	0.207***
Without work	-0.225***	-0.229***	-0.229***	-0.229***
Income loss = 1	-0.482***	-0.494***	-0.486***	-0.498***
Income loss = 2	-1.229***	-1.242***	-1.239***	-1.253***
Sociality = 2	0.760***	0.763***	0.774***	0.774***
Sociality = 3	1.167***	1.164***	1.180***	1.179***
Sociality = 4	1.330***	1.330***	1.350***	1.350***
Sociality = 5	1.472***	1.468***	1.492***	1.489***
Has an acquaintance tested positive	0.0240	0.0222	0.0186	0.0129
Test result = Positive, still infected	0.0212	0.0254	0.0316	0.0153
Test result = Positive, recovered	0.405**	0.404**	0.379**	0.370**
Test result = Waiting	-0.0725	-0.0542	-0.0977	-0.0976
Test result = Negative	0.166***	0.167***	0.151***	0.138**
Lives with partner	0.279***	0.283***	0.280***	0.281***
Lives with children	-0.0147	-0.0147	-0.0174	-0.0192
Lives with others	-0.0663	-0.0692	-0.0668	-0.0702
Cares for pets	-0.116**	-0.119**	-0.124***	-0.130***
Under self isolation	-0.0313	-0.0310	-0.0167	-0.00238
Does regular exercise	0.220***	0.220***	0.218***	0.217***
Self suffered fever	0.0923	0.0863	0.0991	0.0999
Self suffered diarrhea	-0.0557	-0.0592	-0.0548	-0.0609
Constant	8.623***	6.588***	7.891***	6.238***
Observations	10417	10422	10417	10422
Adjusted R^2	0.273	0.272	0.271	0.270

* p<0.10, ** p<0.05, *** p<0.01

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3 Social, economic and health costs of vaccine hesitancy

Abstract

In this study, we provide a tractable and rich framework that allows us to realistically simulate the effects of vaccine hesitancy on the economy, life losses and mental well-being during an epidemic. Our model is based on the extensive model we present in Chapter 2 with several properties related to vaccination incorporated, such as temporary immunity, age-specific vaccination hesitancy rates, a daily vaccination capacity, and a limited vaccine efficacy rate. We calibrate our model to replicate the first fifteen months of COVID-19 in Belgium. Then, we simulate scenarios with varying rates of willingness to get vaccinated and disease transmission. The baseline scenario with a 27.1% vaccine hesitancy rate (Kessels et al (2021)) predicts the COVID-19 pandemic would be under control without social contact limitations distorting economic productivity and mental well-being. However, in our scenarios with higher disease transmission rates, our baseline vaccination rates imply high costs of vaccine hesitancy in the economy, health and mental well-being. We further present upper thresholds of vaccine hesitancy rates for each disease transmission rate to avoid social contact limitations without overwhelming the health system.

3.1 Introduction

Vaccination has historically played a crucial role in curbing the spread of infectious diseases worldwide. Vaccines reduce infection risk, decrease illness severity, and alleviate stress induced by potential infection. Widespread vaccination also generates positive externalities: it curtails virus transmission, facilitates herd immunity, and lessens the healthcare burden. Moreover, broad immunization reduces the need for non-pharmaceutical interventions, which carry economic and social welfare costs through constrained social interactions.

Nevertheless, the recent pandemic triggered by the novel coronavirus has brought the issue of vaccine hesitancy to the forefront. This phenomenon, characterized by the reluctance or refusal to receive vaccination, has been identified by the World Health Organization as one of the top ten global health threats even prior to the pandemic. Vaccine hesitancy has not only contributed to the resurgence of contagious diseases but also jeopardizes the ability of governments to effectively tackle emerging health crises.

In light of these concerns, this chapter investigates the health, economic, and well-being costs of vaccine hesitancy. We aim to model the effects of vaccine hesitancy against COVID-19 on disease progression, labour supply, and mental well-being, three areas that critically shape the overall public health outcomes of the pandemic. To this end, we extend the model presented in Chapter 2 by introducing vaccine technology and studying its impact under different hesitancy scenarios as well as different transmission rates. The potential emergence of more contagious virus strains underscores the importance of achieving higher levels of vaccine acceptance. This chapter seeks to provide quantitative measures of this relationship, enabling a better understanding of the impact of vaccine uptake on controlling the spread of infectious diseases.

This chapter integrates a vaccination scheme into the economic-epidemiological model we introduced in Chapter 2, incorporating actual vaccination rates by age group in Belgium up until June 2021. The baseline scenario, with a vaccine hesitancy rate of 27.1% (Kessels et al. [2021]), indicates that current vaccination rates in Belgium can control COVID-19 spread while maintaining social distancing rules and protective gear. However, hypothetical scenarios with higher transmission rates show the significant costs of vaccine hesitancy, with hospital capacities, overwhelmed until vaccination rates exceed 80%. This results in significant life losses, reduction in supply and mental well-being in the absence of social contact limitations.

We organize the rest of the chapter as follows: Section 2 presents our model. Section 3 presents a description of our data sources. We present and discuss our simulations in Chapter 4. Chapter 5 concludes.

3.2 Model

We define our model in this section. We incorporate a vaccination scheme into our economic-epidemiological model in Chapter 2. This vaccination scheme has several properties listed below.

Vaccine efficacy We define *vaccine efficacy* as the probability of becoming protected from the disease upon vaccination. We make the following assumptions. If a susceptible individual, at the health phase S, is vaccinated, vaccination is effective by a probability, $\xi = \hat{\xi} \in [0, 1]$. Thus, $\hat{\xi}$ of the susceptible individuals enter into the recovered phase, R, at the next time step. (1-$\hat{\xi}$) of the susceptible remain at S. If an individual is at one of the following health phases, E, A, P, M, C, meaning that they are already exposed to the disease and are not recovered yet, vaccination is ineffective. These individuals follow the disease evolution path, and either become immune upon recovery or decease. If an individual gets exposed to the disease on their vaccination day, we count them already in the disease evolution phase, and thus vaccination is ineffective. If an individual is at the health phase, R_A, meaning that they are already immune without noticing following an asymptomatic infection, vaccination is 100% effective. This assumption is to rule out the unrealistic case of immediately losing immunity due to an inefficient vaccine.

Daily vaccination capacity Vaccines are not available until they are invented at a certain time after the beginning of the epidemic, t_{vac}. Inspired by daily vaccination data, we assume the number of available daily vaccines, $\hat{V}_t$, fluctuates during the first days of vaccination. Then, they reach a maximum vaccination capacity, V_{max}, at time $t_{vac,max}$. The number of available vaccines can be written as below:

$$V_t = \begin{cases} 0 & \text{if } t < t_{vac} \\ \hat{V}_t \in (0, V_{max}) & \text{if } t_{vac} \leq t < t_{vac,max} \\ V_{max} & \text{if } t_{vac,max} \leq t \end{cases} \tag{1}$$

Vaccine hesitancy We assume some individuals among each age group are vaccine-hesitant, meaning that they are not willing to get vaccinated. $\eta^{age} \in [0,1]$ represents the fraction of vaccine-hesitant individuals for each age group, $age \in \{0-4, 5-9, \ldots, 70-74, 75+\}$. At each t, the total number of vaccinated individuals among age group age cannot exceed $(1-\eta^{age}) \sum_{type} Pop_t^{type,age}$.

Vaccine allocation to age groups We assume individuals with a revealed infection, namely the positively tested asymptomatically infected (P), mildly infected (M), and critically infected (C), are ineligible for vaccination. We further assume that individuals at the recovered with noticing health status, R, are also ineligible for vaccination as they are already immune.

The rest of the population is unaware of their health status. These individuals are susceptible (S), exposed (E), asymptomatically infected (A), and recovered without noticing (R_A). We let these individuals be eligible for vaccination, as the remaining individuals are either already infected or know they are immune. Vaccines are randomly administered to people at these health phases, as the statuses are unknown. For the sake of brevity, we let U_t^i denote the number of individuals within individual group i that are eligible for vaccination. Therefore, U_t^i is:

$$U_t^i = S_t^i + E_t^i + A_t^i + R_{A,t}^i \tag{2}$$

For $t \geq t_{vac,max}$, we allocate the daily vaccination capacity by prioritizing the age groups by mortality rates in descending order.[30] In other words, we first let the oldest age group be vaccinated. Then, if there are available vaccines left, we let the next oldest age group be vaccinated. We follow this procedure until all vaccines are used or until everyone except for the vaccine-hesitant individuals is vaccinated. The equations governing this procedure can be given as follows:

The oldest age group is accessible to all available vaccines. Thus, the number

[30]For $t < t_{vac,max}$, we use actual age-specific vaccination data.

of available vaccines for this group, V_t^{75+}, is:

$$V_t^{75+} = V_{max} \tag{3}$$

We let β_t^{age} denote the number of vaccines administered to the age group age. If the individuals within the age group age eligible for vaccination, $(\sum\limits_{type} U_t^{type,age})$, are fewer than the number of vaccine-hesitant individuals within the age group age, $\eta^{age} \sum\limits_{type} Pop_t^{type,age}$, no vaccination is performed for the age group age. If there are individuals willing to get vaccinated, $(\sum\limits_{type} U_t^{type,age} > \eta^{age} \sum\limits_{type} Pop_t^{type,age})$, and there are sufficient vaccines for everyone, everyone is vaccinated. If there are individuals willing to get vaccinated, but the number of available vaccines is not sufficient to vaccinate everyone, all available vaccines are used for this age group. Subsequently, the remaining vaccines are administered to the next oldest age group. This procedure is used until either all vaccines are used or there is no individual left willing to get vaccinated. Therefore, the number of daily vaccines administered to the age group age:

$$\beta_t^{age} = \begin{cases} 0 & \text{if } \left(\sum\limits_{type} U_t^{type,age} - \eta^{age} \sum\limits_{type} Pop_t^{type,age}\right) \leq 0 \\ \left(\sum\limits_{type} U_t^{type,age} - \eta^{age} \sum\limits_{type} Pop_t^{type,age}\right) & \\ & \text{if } 0 < \left(\sum\limits_{type} U_t^{type,age} - \eta^{age} \sum\limits_{type} Pop_t^{type,age}\right) \leq V_t^{age} \\ V_t^{age} & \text{if } V_t^{age} < \left(\sum\limits_{type} U_t^{type,age} - \eta^{age} \sum\limits_{type} Pop_t^{type,age}\right) \end{cases} \tag{4}$$

and the number of vaccines available for the age group age[31]:

$$V_t^{age} = \left(V_{max} - \sum_{age' > age} \beta_t^{age'}\right) \; \forall age \in [0-4, \ldots, 75+] \tag{5}$$

Lastly, we distribute vaccines proportionately to the individual types within

[31] $\{0-4\} < \{5-9\} < \ldots < \{70-74\} < \{75+\}$.

each age group, $age \in \{0-4, \ldots, 75+\}$:

$$\frac{\beta_t^{type,age}}{\beta_t^{age}} = \frac{U_t^{type,age}}{\sum\limits_{type'} U_t^{type',age}} \quad \forall age \in \{0-4, \ldots, 75+\} \tag{6}$$

The effect of vaccination on infection dynamics β_t^i is the number of vaccines administered on the individual group i at time t. Vaccines are administered randomly over the individuals at health phases S_t^i, E_t^i, A_t^i, and $R_{A,t}^i$. Thus, the probability of a vaccine being administered to each of these groups is equal to the share of the corresponding group among U_t^i. Infected individuals (T_t^i, E_t^i, A_t^i) follow the infection path and become immune upon recovery. Vaccines administrated on S_t^i, and $R_{A,t}^i$ can result in immunity. For the sake of brevity, we define the probabilities, $\psi_{S\setminus T,t}^i$ and $\psi_{R_A,t}^i$. $\psi_{S\setminus T,t}^i$ is the fraction of the susceptible that do not become exposed to the disease, $(S_t^i - T_t^i)$, among individuals eligible for vaccination within individual group i at time t. $\psi_{R_A,t}^i$ is the fraction of the recovered without noticing, $(R_{A,t}^i)$, among individuals eligible for vaccination within individual group i at time t.

$$\psi_{S\setminus T,t}^i = \frac{S_t^i - T_t^i}{S_t^i + E_t^i + A_t^i + R_{A,t}^i}$$
$$\psi_{R_A,t}^i = \frac{R_{A,t}^i}{S_t^i + E_t^i + A_t^i + R_{A,t}^i}$$

A fraction, $\psi_{R_A,t}^i$, of β_t^i are administered on the recovered without noticing among i. As we assume all these vaccines are efficient, to rule out the unrealistic case of losing immunity because of the vaccine being inefficient, $\beta_t^i \psi_{R_A,t}^i$ of the recovered without noticing among i enter into the recovered with noticing health phase at time $t+1$. An ℓ fraction of the rest of the individuals at this health phase, $(R_{A,t}^i - \beta_t^i \psi_{R_A,t}^i)$ lose immunity and switch to S at $t+1$. Furthermore, there is an influx from asymptomatically infected as some of them recover without their infection revealed by random testing, $r_A A_t^i (1-\sigma_t)$.

$$R_{A,t+1}^i = (1-\ell)(R_{A,t}^i - \beta_t^i \psi_{R_A,t}^i) + r_A A_t^i (1-\sigma_t) \tag{7}$$

A fraction ℓ of the individuals that are recovered with noticing lose immunity, and shift to S at time $t+1$. There is an influx from the symptomatically infected, P, M, and C, upon recovery. Newly vaccinated from A become aware of their immunity, $\beta_t^i \psi_{R_A,t}^i$. Furthermore, the amount of the susceptible individuals that are vaccinated and not exposed to the disease at time t, $\beta_t^i \psi_{S\setminus T,t}^i$, become immune by probability ξ. Therefore, the number of the recovered among i at time $t+1$ can be written as follows:

$$R_{t+1}^i = (1-\ell)R_t^i + r_A P_t^i + r_M M_t^i + \hat{r}_C C_t^i + \xi \beta_t^i \psi_{S\setminus T,t}^i + \beta_t^i \psi_{R_A,t}^i \tag{8}$$

The evolution equation of the susceptible individuals can be written by taking into account the following health phase changes: T_t^i of them get exposed to the disease and move to E. $\xi \beta_t^i \psi_{S\setminus T,t}^i$ of them are vaccinated with efficient vaccines and thus move to the health phase, R. If not vaccinated, a fraction, ℓ, of the recovered without noticing, $R_{A,t}^i - \beta_t^i \psi_{R_A,t}^i$, lose immunity. Lastly, a fraction, ℓ, of the recovered with noticing, R_t^i, also lose immunity.

$$S_{t+1}^i = S_t^i - T_t^i - \xi \beta_t^i \psi_{S\setminus T,t}^i + \ell(R_t^i + R_{A,t}^i - \beta_t^i \psi_{R_A,t}^i) \tag{9}$$

We use the rest of the epidemiological model in Chapter 2 as vaccines are assumed to be ineffective when an individual is already infected. We further keep the economic and life satisfaction components of the model as vaccines have no direct effect on these outcomes.

3.3 Data

Actual daily vaccination data

We assume an individual is immune against COVID-19 by a probability of 95%, $\hat{\xi} = 0.95$, as found by Polack et al. (2020) for the efficacy rate of the Pfizer-BioNTech vaccine after the second dose. We assume immunity is realized after when the single dose of Johnson & Johnson or the second dose of other available vaccines is administered.

We use the actual vaccination data from the COVID-19 monitoring data set from the Belgian Institute of Health (Sciensano). This data set contains comprehensive information on daily vaccination doses for age groups 0-17, 18-34, 35-44, 45-54, 55-64, 65-74, and 75+. The first Johnson & Johnson vaccines are administered before the first of the second doses of other vaccine brands. These first vaccines were administered on 29/12/2020 in Flanders on ten people within four different age groups between the ages 18 and 64. We take this day, 29/12/2020, as the vaccine invention day, t_{vac}. We use our calibrated vaccine evolution from Chapter 2 until t_{vac}. Then, we proceed with using actual vaccine data until 7/6/2021, which corresponds to $t_{vac,max}$ in the model. To simulate the future evolution of vaccination after $t_{vac,max}$ we assume a daily vaccination capacity as the maximum seven-day moving average of second dose vaccines per day: $V_{max} = 84049$.

Vaccine hesitancy data

Kessels et al. (2021) provide a survey of the Belgian population on their willingness to get vaccinated against COVID-19. The survey took place in October 2020 and contains the answers of 2.060 individuals from Belgium. We count the respondents who said they might definitely get vaccinated or probably get vaccinated as individuals willing to get vaccinated. We count the other respondents who said they would probably not get vaccinated or definitely not vaccinated as hesitant to get vaccinated. Overall, 72.9% of the population is willing and 27.1% of the population is hesitant to get vaccinated according to this survey.

Table 18: Vaccination willingness by age

Age group	**Willingness**
18-24	69.9%
25-34	64.9%
35-44	64.5%
45-54	69.4%
55-64	80%
65-80	86.7%

Table 18 presents the age-specific rates of willingness to get vaccinated. By following these values, we set vaccine hesitancy rates as $\eta^{0-4} = \eta^{5-9} = \eta^{10-14} = \eta^{15-19} = \eta^{20-24} = 0.301$, $\eta^{25-29} = \eta^{30-34} = 0.351$, $\eta^{35-39} = \eta^{40-44} = 0.355$, $\eta^{45-49} = \eta^{50-54} = 0.306$, $\eta^{55-59} = \eta^{60-64} = 0.200$, and $\eta^{65-69} = \eta^{70-74} = \eta^{75+} = 0.133$.

3.4 Simulations

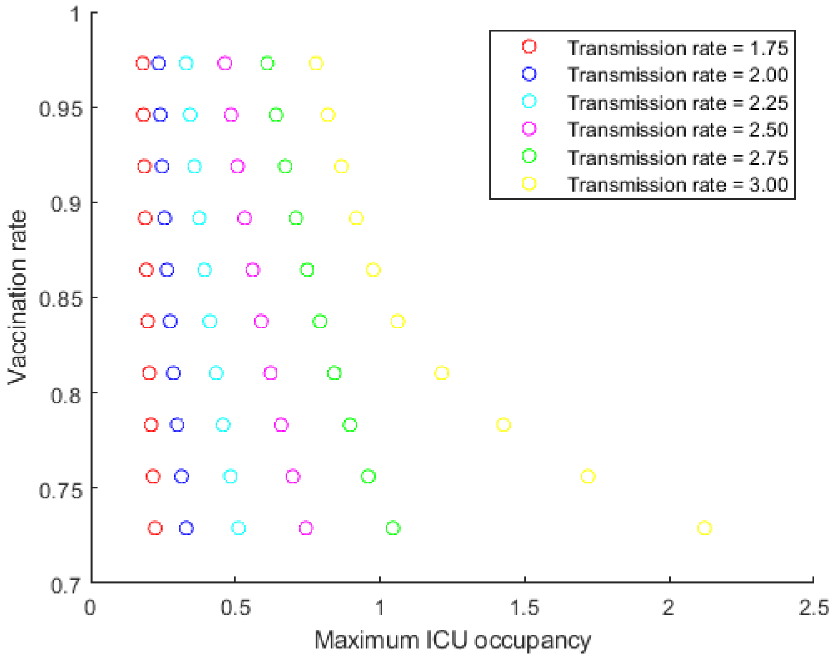
In this section, we present and discuss our simulation results. We simulate our environment for different disease transmission and vaccine willingness rates, with and without NPIs. From these simulations, we obtain losses in life, labour supply, and life satisfaction for each scenario. We run simulations for each combination of disease transmission and vaccine willingness rates for the following 365 days after $t_{vac,max} = 7/6/2021$, the last day we use actual vaccination data.

As the Delta-variant was the dominant COVID-19 variant at the starting day of our simulations, we use a baseline disease transmission rate of 1.75 times the initial transmission rate per contact (Campbell et al. [2021]). We use several different hypothetical disease transmission rates to observe the effect of transmission rate changes on losses in lives, labour supply, and life satisfaction. the transmission rates we used are 2, 2.25, 2.5, 2.75, and 3 and times the initial rate.

We use a baseline overall vaccine willingness rate of 72.1%, with rates varying between 64.9% and 86.7% for different age intervals (Kessels et al. (2021)). A vaccine hesitancy rate of 72.1% implies 27.9% of the population is hesitant to get vaccinated. We run our simulations with this baseline hesitancy rate and different hypothetical hesitancy rates down to 2.79% by decrements of 2.79% from 27.9%. The hypothetical hesitancy rates we use correspond to cases with vaccine hesitancy among each group reduced by 10%, 20%, ..., 90%.

Figure 20 presents the predicted effects of different vaccine hesitancy and disease transmission rates on the maximum ICU need without any NPIs when masks are in use. Here, the vertical axis shows different vaccination rates. The horizontal axis corresponds to the maximum ICU need. Each colour represents a different disease transmission rate between 1.75 and 3 times the initial rate, as indicated in the graph legend.

Figure 20: Maximum ICU need for different vaccination and transmission rates



Our simulations yield that our baseline vaccination rate of 72.9% is sufficient to keep the spread of COVID-19 under control without ending up in ICU capacity overwhelming. In the scenario of a transmission rate of 2.75 times the initial rate, the baseline vaccination rate results in ICU capacity overwhelming. This capacity overwhelming can be avoided without imposing NPIs in case 10% of the vaccine-hesitant individuals (10%×27.9%=2.79% of the population) are convinced of vaccination. According to our simulations, when the transmission rate is 3 times the initial rate, approximately 50% of vaccine-hesitant individuals have to be convinced to avoid ICU overwhelming without NPIs. In this case, the vaccination rate corresponds to 86.05%.

Figure 21 shows an overview of the predicted impact of vaccination rates on life losses, labour supply and life satisfaction. For transmission rates at or slightly above that of the Delta-variant, increasing the baseline vaccination rate bring small benefits in the three outcomes of interest. If the transmission rate increases (e.g. looser social distancing rules, new variants), differences in vaccine hesitancy levels yield significant differences in health outcomes, economic outcomes and life satisfaction, as explained in Table 19 below in detail.

Figure 21: Life losses, labour supply loss, and life satisfaction levels for different vaccination and transmission rates

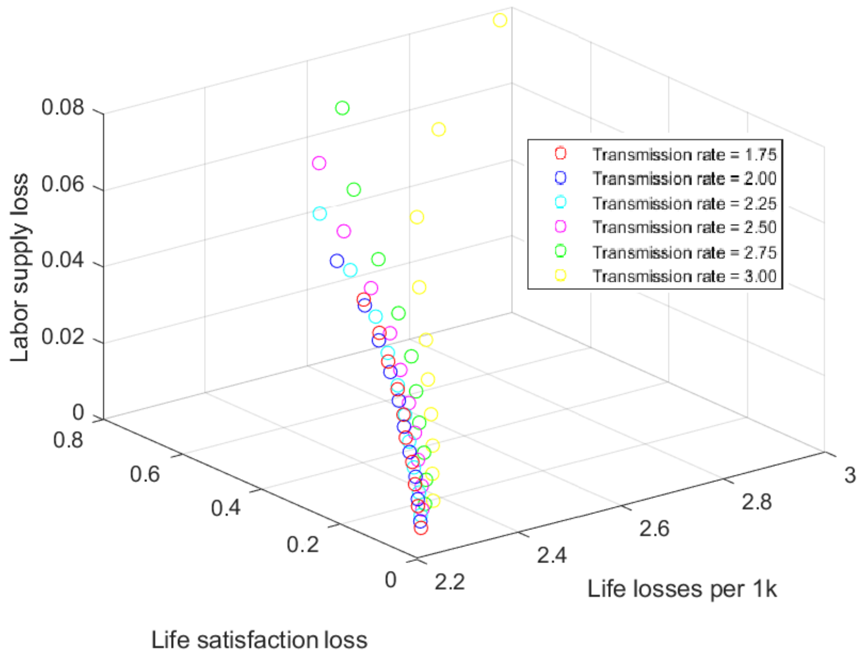


Table 19 displays the social, economic, and health costs of vaccine hesitancy for a set of our simulations. The first column stands for different vaccine hesitancy rates between the baseline rate, 27.9% and the scenario of only one-tenth of the vaccine-hesitant population within each age group remaining vaccine-hesitant, corresponding to a 2.79% vaccine hesitancy rate. In the next columns, we present the predicted effects of vaccine hesitancy on life satisfaction (columns 2-4), labour supply (columns 5-7), and life losses (columns 8-10). For comparison, we consider three scenarios of disease transmission rates: a) a low transmission rate scenario with the Delta-variant rate, $\sigma = 0.1418$ ("Transmission rate = 1.75" in Figure 21), without NPIs, shown in columns 2-5-8, b) a high transmission rate scenario with a transmission rate three times that of the initial rate, $\sigma = 0.243$ ("Transmission rate = 3.00" in Figure 21), without NPIs, shown at columns 3-6-9, c) the same high transmission rate scenario, $\sigma = 0.243$, with NPIs, shown at columns 4-7-10. In the last case, we present the results obtained from an example policy set (most stringent restrictions: schools and non-essential workplaces kept closed, other contacts kept at a minimum 10% pre-epidemic rate) imposed at 50% and lifted at 20% ICU occupancy.

Table 19: Social, economic, and health costs of vaccine hesitancy

	Life satisfaction			Labour supply			Life losses		
	$\sigma = 0.1418$	$\sigma = 0.243$		$\sigma = 0.1418$	$\sigma = 0.243$		$\sigma = 0.1418$	$\sigma = 0.243$	
Hesitancy	Free	Free	NPI	Free	Free	NPI	Free	Free	NPI
2.79%	2.590	2.479	2.240	99.4%	99.0%	99.0%	25192	25816	25428
5.58%	2.586	2.474	2.356	99.2%	98.6%	98.2%	25213	26212	25487
8.37%	2.583	2.467	2.350	99.0%	98.1%	97.4%	25236	26698	25565
11.16%	2.579	**2.457**	2.339	98.8%	**97.6%**	96.5%	25262	**27368**	25654
13.95%	2.574	2.441	2.324	98.7%	97.1%	95.6%	25292	28029	25760
16.74%	2.569	2.414	2.297	98.5%	96.5%	94.5%	25328	28917	25897
19.53%	2.563	2.362	2.244	98.3%	95.6%	92.9%	25371	29921	26088
22.32%	2.555	2.278	2.161	98.2%	94.5%	91.0%	25423	31012	26285
25.11%	2.544	2.142	2.025	98.0%	93.4%	89.0%	25490	32276	26679
27.90%	2.530	1.801	**1.684**	97.8%	92.3%	**86.8%**	25579	33873	**27229**

At the low disease transmission rate scenario, $\sigma = 0.1418$, we observe only small gains from reducing the vaccine-hesitancy rate. Even in the case of convincing 90% of the vaccine-hesitant population (row 4), the benefit of reducing vaccine hesitancy is 1.06 fewer deaths per day (column 8), an increase in the average labour supply over a one-year period from 97.8% to 99.4% (column 5), and a 0.060 point increase in the value of the life satisfaction function we utilize (column 2). Although with Delta-variant disease characteristics, a 27.9% vaccine-hesitancy rate appears to be sufficient for a life without a need for NPIs, a higher transmission rate may bring a need to convince many individuals to get vaccinated. With a 27.9% vaccine-hesitancy rate (the bottom row), our model predicts 22.7 more deaths per day, or 8294 more deaths overall (columns 8 and 9), accompanied by an average labour supply fall from 97.8% to 86.8%, and a life satisfaction loss by 0.729 points between the low and high transmission rate cases of $\sigma = 0.1418$ and $\sigma = 0.243$. In the case of convincing 90% of the vaccine-hesitant population, an increase in the transmission rate translates into 724 deaths (2.0 per day), a labour supply loss decrease from 99.4% to 99.0%, and a life satisfaction loss by 0.111 points.

At the high disease transmission rate scenario ($\sigma = 0.243$), the baseline vaccine hesitancy rate of 27.9% appears to be significantly costly. According to our simulations, the benefit of convincing only 10% of the vaccine-hesitant individuals (2.79% of the population, bottom two rows) is equivalent to saving 1597 lives (4.4 lives per day, column 9), an increase from 92.3% to 94.3% in labour supply (column 6), and a 0.341 point increase in life satisfaction (column 3). As vaccination rates increase, the benefit of convincing the same amount of individuals follows a decreasing returns-to-scale pattern as expected. Between hesitancy rates of 5.58% and 2.79%, we observe differences of 396 deaths (1.1 per day), 0.4% labour supply, and 0.005 life satisfaction points. When the two extreme vaccine-hesitancy scenarios (fourth and the bottom rows) are compared, the cost of having a 27.9% instead of a 2.79% vaccine hesitancy rate is 8057 lives (22.1 per day), 6.7% of overall labour supply, and 0.678 life satisfaction points.

The combination of 27.9% and a high transmission rate ($\sigma = 0.243$) results in an ICU need of around 240% of the ICU capacity, accompanied by high costs in lives, labour supply, and life satisfaction, as also implied by Figure 20 and Figure 21. In such a case, it is reasonable to assume that governments would prefer to impose NPIs. In columns 4, 7, and 10, we present the impact of an example NPI set for the high transmission rate scenario ($\sigma = 0.243$). This NPI set consists of the most stringent restrictions (schools and non-essential workplaces kept closed, other contacts kept at a minimum 10% pre-epidemic rate) imposed and lifted at 50% and 20% ICU occupancy, respectively. We observe that, under a vaccine-hesitancy rate of 27.9%, NPIs may bring down life losses until a level (27229 life losses since the beginning of the epidemic) that can be reached without NPIs when the vaccine-hesitancy rate is around 11% (27368 lives with 11.16% vaccine hesitancy). Here, when NPIs are imposed, the cost of not being able to decrease the vaccine-hesitancy rate from 27.9% to 11.16% is equivalent to 10.8% of annual labour supply, and 0.773 life satisfaction points.[32] Without any NPIs, the cost is 6505 lives (17.8 per day), 5.3% of annual labour supply, and 0.656 life satisfaction points.

[32]Corresponding values are given as bold in the seventh and the bottom rows.

3.5 Conclusion

The COVID-19 pandemic has emphasized the vital role vaccination plays in controlling infectious diseases and mitigating their societal and economic consequences. This study builds on Chapter 2's economic-epidemiological model, which integrates pandemic evolution, labour supply, and mental well-being, to analyze the multifaceted effects of vaccine hesitancy. Applied to Belgium's COVID-19 pandemic, the framework is flexible enough to be adapted to other public health crises, enabling policymakers and researchers to evaluate vaccine hesitancy implications in various contexts.

We use actual vaccination rates by age group in Belgium until June 2021. Then, we simulate the following months by letting individuals get vaccinated by using several realistic properties such as temporary immunity incorporated as a daily probability of immunity loss, age-specific vaccination hesitancy rates, a daily vaccination capacity, and a vaccine efficacy rate lower than 100%. Our baseline scenario with an overall 27.1% vaccine hesitancy rate (Kessels et al (2021)) yields results in line with reality. Current vaccination rates in Belgium would be sufficient even in the presence of the more lethal Delta variant to keep the spread of COVID-19 under control by keeping social distancing rules and protective gears without imposing social contact limitations that hamper economic productivity and constrain social life. However, in our hypothetical scenarios with higher disease transmission rates, our baseline vaccination rates indicate high costs of vaccine hesitancy. In the worst-case scenario, we considered, with a disease transmission rate 50% of the initial COVID-19 transmission rate at the beginning of the pandemic, hospital capacities would be overwhelmed until overall vaccination rates corresponding to an overall vaccination rate slightly higher than 80% were reached, resulting in high life losses, and significant losses in labour supply and mental well-being at the absence social contact limitations.

Our findings underscore the importance of incorporating behavioural aspects, such as individual decision-making on infection avoidance and social contacts, in modelling vaccine hesitancy consequences. This approach enables us to understand the factors influencing vaccination decisions and public health outcomes.

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