MEYRATH, Thierry; MINCU, Clara-Ioana & PERUCCA, Antonella
Esch-sur-Alzette (Luxembourg)

Training mathematical thinking with the inclusive card game UNO

The card game UNO, that celebrated 50 years in 2021, is a great game with a remarkable didactical potential. Its attractive and clear cards are suitable learning material for pupils. It is well-known that UNO and its cards can be used to train number recognition (or, more generally, elementary mathematical concepts), so our investigation aims to describe non-elementary mathematical concepts that can be taught building on this game. We assume that the reader is familiar with the UNO game: in case, the game rules can be found online (Mattel, 2023).

For simplicity, we suppose that there are only 2 players, and we remove the Reverse cards so that the card deck conveniently has precisely 100 cards.

UNO as a model of inclusive games

We support the choice of the game UNO for educational purposes with a number of reasons that show that it is very inclusive:

- *UNO is logistically easy to play.* The game is meant for 2 to 10 players, that play until the end of the game (but they may choose to quit the game without jeopardizing it). Players may speak different languages because they only need to call UNO! or select a color. One does not necessarily need a card table. The game can also be played with traditional packs of cards (as the card game Mau-Mau) or with the App UNO!.
- *UNO is a simple game that allows for different players' levels*. The basic game rules are extremely simple. Since luck plays a big role, unexperienced players have a chance to win. Moreover, alliances among players do not play a big role. Finally, the game outcome is not known in advance, as players who are almost winning may fail to do so.
- *UNO can be adapted to meet special needs.* Speaking/hearing impairments are not an issue for this game, and there can be special adjustments for drawing/holding/playing cards. A fraction of color-blind people can play as the cards use very distinct colors. Moreover, there are special UNO card decks with Color ADD symbols to distinguish colors, or with the tactile writing system Braille.

Mathematical reflections

An example of binary relation. We say that the card B is connected to the card A if it can be played right after the card A. So, the Wild cards are connected to all cards, while the other cards are only connected to the cards of the same color or of the same type. Apart from the Wild cards, the connect relation is symmetric. As any card is connected to itself, the connect relation is reflexive. The game is spiced up by the fact that the connect relation is not transitive: if the cards A, B, and C are played in a row, then it could be that C is not connected to A.

Probability estimates. As our card deck has 100 cards, it is easy to count favorable cards with respect to the full deck and express probabilities in percentage. For example, the probability of drawing a yellow card from the full deck is 23%. It still makes sense to refer to the full deck in an early round of the game or provided that the draw pile is reasonably balanced (for example between red and green, if these colors have been played roughly the same number of times). Notice that many players intuitively spot and remember strong unbalances during the game.

Some UNO algebra. We now analyze a UNO game that has just ended. For simplicity, we suppose that no Wild cards have appeared in the game. We say that a round consists in either of the following actions: play a card, draw a card, draw&play a card. Recall that each player is given 7 cards at the beginning of the game, and there is an initial card in the discard pile. Then we look for relations between the following quantities:

R = number of rounds in the game

P = number of play rounds

D = number of draw rounds

Dp = number of drawn & play rounds

Rem = number of cards remaining in the hand of the losing player

Disc = number of cards in the discard pile minus 1

We have the following relations: R = P + D + Dp ; Disc = P + Dp ;

P + Rem = 14 + D. The first relation is because in each round we can either play, draw, or draw&play. The second relation is because a player adds one card to the discard pile if either they play or draw&play. For the third relation, consider that the number of cards that were played (namely, P+Dp) are the initial 14 cards plus the cards that have been drawn during the game (namely, D+Dp) minus the cards remaining in the hand of the losing player. Note that from the above quantities we cannot infer the individual contributions of the two players, as the following example of triples (Dp,D,P) shows, because in both games we have Dp=5, D=3, P=11, Disc=16, R=19, Rem=6:

First Game: Player 1 (3,0,7) and Player 2 (2,3,4)

Second Game: Player 1 (1,1,8) and Player 2 (4,2,3).

Further investigations. Possible directions of investigations are, for example: analyzing the game scenario in which both players have only one card in their hand; investigating when it is best to play a Wild Draw 4 card (defining "emergency game situations"); investigating when it is best to change colors (considering some game scenarios).

Moreover, by looking at game statistics on a large scale, one could try to assess how much advantage the first player has, and then possibly introduce the following rule: if the second player finishes the cards right after the first player, then there is a draw.

Remark that from the game analysis it becomes evident that it is quite a challenge to express common sense (i.e., reasonable choices made during the game) in a mathematically precise way. Moreover, the necessary simplifications only make sense in some game contexts and any consideration is only valid up to a certain extent (contrarily to what some internet websites say, claiming to reveal the very best strategy for any game).

Some challenges for pupils

The 0 cards. It looks twice as hard to get rid of the 0 cards because there are only four of them rather than eight. Is this really the case?

- Consider some 0 card (respectively, 1 card). Count the remaining color cards in the full deck after which the given card can be played.
- Compute, referring to the full deck, the probability of being able to play a given 0 card (respectively, 1 card) after a color card has been played.

Solution: There are 22 cards of the correct color, and there are 3 (respectively, 6) cards of the correct type from a different color. This gives 25 (respectively, 28) favorable cards.

If the color card has the same color of your card (25% chance), then you can play. Else (75% chance), all depends on whether the color card is a 0 (1/23 chance) or a 1 (2/23 chance). Summing up the two contributions ("probability of a scenario × probability inside the given scenario") we get

25%+75% × 1/23 ≈ 28% and 25%+75% × 2/23 ≈ 32% respectively.

Not playing. Can you think of a situation in which the best strategy is not playing a card despite the possibility of doing so?

Solution: For example, if you know that the other player has a yellow card left (if they could not play red, green, or blue), then playing a yellow card makes you lose.

Never ending game. Describe a never-ending UNO game, where after ending the draw pile the cards from the discard pile get shuffled and become the new draw pile.

Solution: One can first play all wild cards, and then all color cards in such a way that the two players are left with 0Green and 0Blue respectively, the card on the discard pile is 1Red, and the draw pile is empty. With a suitable shuffle of the discard pile this game pattern can be repeated infinitely many times.

Conclusive remarks

Since the game UNO is familiar to many (and it is easily accessible to anyone), it can be used for a variety of purposes, for example for teaching anthropology (Supper, 2023). It's precisely this *familiarity* which makes the game particularly suitable for teaching (for the effects of mere exposure, see (Zajonc, 2001)). We have presented a number of mathematical questions; however the pupils could simply be asked to play the game, collect game strategies, and invent variants of the game. *Correct reasoning* is a crucial mathematical skill, and a homework focusing on critical thinking about a game is less likely to trigger math anxiety (Dondio, 2023).

References

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