



Journées SL2R

Reims, 6-7 avril 2023

Jeudi 6 avril

13h30-14h00 Accueil

14h00-14h50 Valentin OVSIENKO (Université de Reims-Champagne Ardenne)

Shadows of numbers : supergeometry with a human face. In this elementary and accessible to everybody talk I will explain an attempt to apply supersymmetry and supergeometry to arithmetic. The following general idea looks crazy. What if every integer sequence has another integer sequence that follows it like a shadow? I will demonstrate that this is indeed the case, though perhaps not for every integer sequence, but for many of them. The main examples are those of the Markov numbers and Somos sequences. In the second part of the talk, I will discuss the notions of supersymmetric continued fractions and the modular group, and arrive at yet a more crazy idea that every rational and every irrational has its own shadow.

15h00-15h50 Khalid KOUFANY (Université de Lorraine)

Poisson transforms for spinors on the real hyperbolic space. Let (τ, V_τ) be a spinor representation of $\text{Spin}(n)$ and let (σ, V_σ) be a spinor representation of $\text{Spin}(n-1)$ that occurs in the restriction $\tau|_{\text{Spin}(n-1)}$. We consider the real hyperbolic space $H^n(\mathbb{R})$ as the rank one homogeneous space $\text{Spin}_0(1, n)/\text{Spin}(n)$ and the spinor bundle $\Sigma H^n(\mathbb{R})$ over $H^n(\mathbb{R})$ as the homogeneous bundle $\text{Spin}_0(1, n) \times_{\text{Spin}(n)} V_\tau$.

In this talk we will show which eigenspinors of the algebra of invariant differential operators acting on $\Sigma H^n(\mathbb{R})$ are Poisson transforms of L^p -sections of the bundle $\text{Spin}(n) \times_{\text{Spin}(n-1)} V_\sigma$ over the boundary $S^{n-1} \simeq \text{Spin}(n)/\text{Spin}(n-1)$ of $H^n(\mathbb{R})$, for $1 < p < \infty$.

(Joint work with Salem Bensaïd and Abdelhamid Boussejra.)

16h00-16h30 Pause

16h30-17h20 Oliver BRAMMEN (Ruhr-Universität Bochum)

Harmonic analysis on harmonic manifolds. In the 1930's and 40's Copson and Ruse tried to establish harmonic analysis on general Riemannian manifolds. They failed in their attempt but discovered a necessary condition encoded in the geometry of the manifold : harmonicity. This sparked nearly a century of research leading to the eventual development of harmonic analysis on harmonic manifolds. In this talk I will introduce the Abel transform and its dual on non flat harmonic manifolds and sketch the construction of the Fourier transform on harmonic manifolds of rank one by comparing it to the euclidian case. Furthermore I will present some applications of these techniques, in particular the characterization of solutions of the shifted wave equation.

19h30- Dîner au Restaurant Côté Cuisine, 43 boulevard Foch.

Vendredi 7 avril

9h00-9h50 Hong-Wei ZHANG (Ghent University)

Dispersive equations on symmetric and locally symmetric spaces. A challenge in harmonic analysis on non-compact symmetric spaces of higher rank is that the Plancherel density involved in the inverse Fourier transform is not a differential symbol in general. In this talk, we will share recent progress in overcoming this difficulty and how to use it to establish pointwise wave kernel estimates. Next, we will briefly show their applications in the study of non-linear dispersive equations. Finally, we will discuss, on a locally symmetric space, how the geometric properties of the discrete group affect these results.

10h00-10h30 Pause

10h30-11h20 Guenda PALMIROTTA (Université de Luxembourg)

How to solve invariant systems of differential equations on $SL(2, \mathbb{R})$? In the Euclidean case, it is well-known, by Malgrange and Ehrenpreis, that linear differential operators with constant coefficients are solvable. However, what happens if we genuinely extend this situation and consider systems of linear invariant differential operators, are they still solvable? In the case of \mathbb{R}^n (for some positive integer n), the question has been answered positively, mainly by Hörmander. We will show that this remains still true for Riemannian symmetric spaces of non-compact type $X = G/K$, in particular for hyperbolic planes. More precisely, we will present a possible strategy to solve this problem by using the Fourier transform and its Paley-Wiener(-Schwartz) theorem for (distributional) sections of vector bundles over $\mathbb{H}^2 = SL(2, \mathbb{R})/SO(2)$. (This work was part of my doctoral dissertation supervised by Martin Olbrich.)

11h30-12h20 Robert YUNCKEN (Université de Lorraine)

Crystallizing Functions on Compact Lie Groups. The theory of crystal bases, due to Kashiwara and Lusztig, is a means of simplifying the representation theory of semisimple Lie algebras by passing through quantum groups. Specifically, varying the parameter q of a quantized enveloping algebra, we pass from the classical theory at $q = 1$ through the Drinfeld-Jimbo algebras at $0 < q < 1$ to the crystal limit at $q = 0$. At this point, the main features of the representation theory (matrix coefficients, Clebsch-Gordan coefficients, branching rules) crystallize into purely combinatorial data described by crystal graphs. In this talk, we will describe what happens to the $*$ -algebra of functions on a compact semisimple Lie group under the crystallization process, leading to a higher-rank graph algebra. (This is joint work with Marco Matassa.)