Microeconomic Uncertainty and Macroeconomic Indeterminacy[1]

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Abstract

The paper proposes a stylized intertemporal macroeconomic model wherein the combination of decentralized trading and microeconomic uncertainty (taking the form of privately observed and uninsured idiosyncratic shocks) creates an information problem between agents and generates indeterminacy of the macroeconomic equilibrium. For a given value of the economic fundamentals, the economy admits a continuum of equilibria that can be indexed by the sales expectations of firms at the time of investment. The Walrasian allocation is one of these possible equilibria but it is reached only if firms are optimistic enough. With a weaker degree of optimism, equilibrium output, employment and real wages will be lower than in the Walrasian equilibrium. Moreover, the range of possible equilibria will depend positively on the wage elasticity of the labour supply and on the magnitude of the information problem between buyers and sellers (in our case, the variance of the idiosyncratic shocks).

Stochastic simulations performed on a calibrated version of the model show that pure demand expectation shocks may generate business cycle statistics that are not inconsistent with the observed ones.

Keywords: indeterminacy, non-Walrasian economy, business cycle, animal spirits, continuum of equilibria

JEL classification: E10, E24

1 Introduction

Business cycle analysts often associate the success or failure of an economic recovery to investors and/or consumers expectations and degree of confidence. Obviously enough, subjective confidence effects cannot be discussed in standard Walrasian setups wherein the knowledge of the economy's "fundamentals" typically suffices to determine a unique equilibrium (trajectory). As is well-known, the macroeconomic outcome may be affected by "animal spirits" only if there are market imperfections. Various contributions to the business cycle literature have outlined that technological externalities and complementarities, imperfect competition and increasing returns may each cause equilibrium indeterminacy and lead to economic fluctuations purely driven by agents' expectations[1]. Our paper will stress that the very working of a decentralized (competitive) economy, in the absence of a market mechanism comparable to the Walrasian auctioneer, is sufficient to explain that the equilibrium may depend not only on economic fundamentals but also on agents' expectations. In an intertemporal setup, the paper illustrates how decentralised trading may lead to equilibrium indeterminacy when agents have to make decisions that will commit them on a market (at least temporarily) while they are imperfectly informed about their trading opportunities.

The simplest example is a productive capacity choice by a firm that is still uncertain about future purchase orders. In a Walrasian setup, the very question of this uncertainty is almost incongruous: for a firm, uncertainty about future market conditions concerns future output price and not sales volume. When investing, a Walrasian firm indeed knows that at the time of exchange it will be able to realise all (*ex post*) profitable transactions at the market price vector. In a decentralized market where there is no device that always guarantees full information about the existing trading opportunities between buyers and sellers, a firm may end up with some idle productive capacity if *ex post* it receives too few purchase orders in comparison to its optimal productive capacity; it may leave some orders unfilled in the opposite case. Expectations about forthcoming orders then affect the firm's investment decision and thereby the actual level of transactions, low activity levels resulting from low investments due to pessimistic expectations.

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[1]See Farmer and Benhabib (1999) for a survey of business cycle models with such types of imperfections and Schmitt-Grohe (1997) for a comparative study of their quantitative implications.

Our objective is to formalise these intuitions in a model that departs as little as possible from a standard Walrasian intertemporal macroeconomic model. The model economy consists of four competitive markets (labour, capital, final good and intermediate goods) and only differs from a Walrasian model in the working of the intermediate goods market. On that market, an information problem between buyers and sellers[2] exists because intermediate firms experience purely idiosyncratic technological shocks that are uninsured and only privately observed. These shocks imply heterogeneous employment and production decisions at the prevailing competitive prices and wages. In a Walrasian world, those shocks would be inconsequential at the aggregate economy level: *ex post*, final firms would always buy a quantity of intermediate goods such that every intermediate firm sells its optimal production level. This could occur (in the case of centralized trading but also) in the case of decentralized trading if final firms received all the relevant information about every intermediate firm's situation and sent purchase orders accordingly. We depart slightly from this scenario by analysing the case where final firms send purchase orders to intermediate firms without knowing every intermediate firm's optimal production level[3]. At the competitive price, an intermediate firm enjoying a good productivity shock may then receive too few purchase orders, and vice-versa. The possibility of such outcomes makes the intermediate firm's investment choice depend on expected forthcoming purchase orders. These expectation effects are shown to induce indeterminacy of the macroeconomic equilibrium (even with full employment of a fixed labour supply): the economy possesses a continuum of equilibria that can be indexed by the demand expectations of the intermediate firms. The Walrasian allocation[4] is one of these possible equilibria but it is reached only if firms are optimistic enough. With a

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[2]In a decentralized economy, what an agent has to anticipate includes the anticipations and behaviours of the other agents. There will be no such sophistications in the present model that will deal with these coordination problems in a highly stylized way.

[3]The assumption that idiosyncratic technological shocks are at the root of the information problem between buyers and sellers should not be interpreted too literally. The existence of an uninsured microeconomic risk is a key ingredient of our model but the assumption that it bears on productivity is only made for convenience. We present in Appendix 2 an alternative version of our model where idiosyncratic productivity shocks are substituted for idiosyncratic demand shocks: it is shown to have the same properties as in the model of the main text.

[4]In the present model with an idiosyncratic risk for input firms, we call Walrasian the equilibrium in which all agents fulfill their Walrasian transaction plans at the market prices and wages: in particular, each intermediate firms sells its profitable output level conditionally on the microeconomic shock it has experienced.

weaker degree of optimism, equilibrium output, employment and real wages will be lower than in the Walrasian equilibrium. Moreover, the range of possible equilibria will depend on the magnitude of the information problem between buyers and sellers i.e. here, on the variance of the idiosyncratic shocks. Once there is no microeconomic shock, there is no informational problem between agents and the decentralized economy reaches a unique equilibrium, equivalent to the Walrasian outcome.

The idea that equilibrium indeterminacy may follow from the coordination difficulties raised by the very working of a decentralized market is not new. Several theoretical contributions on the subject have been developed independently from (and even before) the stochastic business cycle models with indeterminacy due to imperfect competition, increasing returns or externalities. Among others, Diamond (1982) stresses the effects of decentralised trading in an economy where the Walrasian auctioneer is replaced by a stochastic matching process; Bryant (1983) emphasises coordination problems in a stylised model with complementary intermediate goods and decentralised decision-making; in a non-cooperative game framework, Roberts (1987, 1989) shows that non-Walrasian market institutions regarding price and quantity determination may imply multiple equilibria in which prices and wages are set at their Walrasian levels while quantities vary down from their Walrasian level to zero. Our model (which emphazises the role of imperfect information in a decentralized exchange process) shares a same basic intuition with the above mentioned papers even though it models the interactions between agents in an incomparably less sophisticated way than in Roberts' or in market games. But advantageously, the simplicity of our formulation will allow us to explore its quantitative implications in a stochastic/calibrated business cycle exercice.

The rest of the paper is organised as follows. Section 2 describes the behaviours of agents; it presents the equilibrium conditions and discusses the result of equilibrium indeterminacy, first under a general technological assumption, next under the assumption of a Cobb Douglas production function. In section 3, stochastic simulations performed on a calibrated version of the model show that pure demand expectation shocks may generate business cycle statistics that are not inconsistent with the observed ones. Section 4 concludes.

2   The Economy

We consider an economy where the production of the final good requires two stages. Intermediate good firms use labour and capital to produce an homogeneous good that is the sole input of final goods firms. Both types of firms operate under perfect competition. Intermediate firms experience idiosyncratic technological shocks. These shocks imply heterogeneous employment and production decisions at the competitive prices and wages. We assume a decentralised trading process where final firms have to send purchase orders without full information about the shocks that hit the different input suppliers. An intermediate firm may then run some idle capacities since the orders it receives may fall short of its Walrasian output level.

Since a final goods firm cannot distinguish among intermediate goods producers, it sends the same purchase orders q_t^d to all input firms. We further assume that all input firms expect the same purchase orders. This simplification makes all input firms identical before the realization of the idiosyncratic shocks.

2.1   Behaviours

Intermediate Goods Producers

We assume a continuum of *ex ante* identical firms, uniformly distributed over the unit interval. In each firm, factor productivity is random. With k_t units of capital and ℓ_t units of labour, a firm produces a quantity of output q_t given by:

$$q_t = f(k_t, \ell_t, \theta_t),$$

$$\text{where} \quad \theta_t \in [\theta_{\min}, \ \theta_{\max}], \qquad \text{with} \quad 0 \leq \theta_{\min} < \theta_{\max} < \infty. \tag{1}$$

θ_t is a firm specific productivity shock with distribution function $G(\theta)$. f is concave and increasing ($f_k, f_\ell > 0$, $f_{kk}, f_{\ell\ell} < 0$ and $f_{k\ell} > 0$), with non-increasing returns to scale. Every firm takes the intermediate goods price p_t, the wage rate w_t and the interest rate r_t as given (the final good serves as *numéraire*).

We assume the following sequence of events and decisions. At the beginning of period t, each intermediate goods producer decides the value of the productive capital stock k_t before observing

the factor productivity shock θ_t, and given expectations about output demand q_t^d, the intermediate goods price p_t and the wage rate w_t. The employment and production decisions of period t are taken later, once the realized value of θ_t has been observed by the intermediate firm. We analyse this sequence of decisions backwards, starting with employment at given capital stock.

Optimal labour demand

Let us first consider the case of an intermediate goods firm that receives a sufficient quantity of orders and is not sales-constrained. Given a predetermined capital stock k_t and a realized value of the shock θ_t, the employment decision in period t is then the solution of a standard Walrasian optimization programme:

$$\max_{\ell_t} \; p_t \, f(k_t, \ell_t, \theta_t) \, - \, w_t \, \ell_t.$$

The optimal employment level ℓ_t^w must be such that $f_\ell(k_t, \ell_t^w, \theta_t) = w_t / p_t$. This implies:

$$\ell_t^w \, = \, \ell(k_t, \omega_t, \theta_t) \qquad \text{and} \qquad q_t^w \, = \, q(k_t, \omega_t, \theta_t), \tag{2}$$

where ω_t is the period t real wage for an intermediate good firm (i.e., $\omega_t = w_t / p_t$) and q_t^w is the corresponding profitable output. Functions ℓ and q are increasing in both k_t and θ_t and decreasing in ω_t. Let us denote Π_t^w the operating surplus corresponding to these employment and output levels:

$$\Pi_t^w \, = \, p_t \, \Pi^w(k_t, \omega_t, \theta_t) \qquad \text{where} \quad \Pi^w(k_t, \omega_t, \theta_t) = q_t^w \, - \, \omega_t \, \ell_t^w. \tag{3}$$

Under our assumptions on f, function Π^w is concave in k_t and decreasing in ω_t.

We now turn to the case of an intermediate goods firm receiving a quantity of orders smaller than the profitable productive capacity q_t^w. The demand for labour then coincides with the employment level ℓ_t^d necessary to produce q_t^d, i.e., [5] such that $f(k_t, \ell_t^d, \theta_t) = q_t^d$, or,

$$\ell_t^d \, = \, \ell^d(k_t, q_t^d, \theta_t), \tag{4}$$

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[5]Note that we could introduce the possibility of stocking the unsold production. This would not change our results qualitatively. Indeed, inventories cannot provide full insurance and cannot eliminate the problem at the origin of the equilibrium indeterminacy. Inventories would have two types of effect. On the one hand, inventories would change the supply capacity of the firms: a firm could now sell q_t^w plus inventories. But qualitatively, this would not change anything to the fact that a firm that would receive more orders than its supply capacity (whatever it is $q_t^w + inventories$ or simply q_t^w) would be capacity constrained. On the other hand, if the firm

where function ℓ^d is increasing in q_t^d and decreasing in the other two arguments. Let us denote Π_t^d the operating surplus of a sales-constrained firm, that is:

$$\Pi_t^d = p_t \, \Pi^d \left(k_t, \, q_t^d, \, \omega_t, \, \theta_t \right) \qquad \text{where} \qquad \Pi^d \left(k_t, \, q_t^d, \, \omega_t, \, \theta_t \right) = q_t^d - \omega_t \, \ell_t^d \tag{5}$$

Given our assumptions on f, function Π^d is easily shown to be increasing in q_t^d and k_t, decreasing in ω_t.

As all intermediate goods firms receive the same quantity of orders q_t^d, a given firm will be sales-constrained only in the case where it experiences a productivity shock θ_t sufficiently high to imply $q_t^w = q(k_t, \omega_t, \theta_t) > q_t^d$. Let us denote by $\bar{\theta}_t$ the critical value of the productivity shock such that the corresponding Walrasian production plan matches exactly the quantity of orders received by a firm:

$$\bar{\theta}_t : \qquad q(k_t, \omega_t, \bar{\theta}_t) = q_t^d \, . \tag{6}$$

If θ_t is larger (resp. smaller) than $\bar{\theta}_t$, the firm is (resp. is not) sales-constrained.

Optimal capital stock

The capital stock is chosen before the realized value of the shock is known to the producer. The optimal choice takes into account the fact that depending on the realized value of the shock, the firm may turn out to be sales-constrained (if θ lies in between $\bar{\theta}$ and θ_{max}). Expected profits maximization can thus be written as follows:

$$\max_{k_t} \int_{\theta_{\min}}^{\bar{\theta}_t} p_t \, \Pi^w \left(k_t, \omega_t, \theta \right) \, \mathrm{d}G(\theta) + \int_{\bar{\theta}_t}^{\theta_{\max}} p_t \, \Pi^d \left(k_t, q_t^d, \omega_t, \theta \right) \, \mathrm{d}G(\theta) \, - \, (r_t + \delta) \, k_t. \tag{7}$$

The capital stock k_t is determined by the following first-order optimality condition:

$$\frac{r_t + \delta}{p_t} = \int_{\theta_{\min}}^{\bar{\theta}_t} \Pi_k^w \left(k_t, \omega_t, \theta \right) \, \mathrm{d}G(\theta) + \int_{\bar{\theta}_t}^{\theta_{\max}} \Pi_k^d \left(k_t, q_t^d, \omega_t, \theta \right) \, \mathrm{d}G(\theta) \, , \tag{8}$$

where function Π_k^w (resp. Π_k^d) represents the first partial derivative of function Π^w (resp. Π^d) with respect to k. The first term on the right-hand side of (8) is the expected marginal revenue of capital when productivity is low; it is decreasing in k_t under the assumption of strictly decreasing

received less orders, it could obviously choose to produce more than sales and so reduce the effect of the sales contraint. But a firm that has already accumulated large inventories and faces a severe sales constraint is unlikely to produce at full capacity, even if the inventory cost is limited to the financial capital cost.

returns to scale. The second term on the right-hand side of (8) represents the expected marginal revenue of capital when the firm is sales-constrained. This second term is non-negative and decreasing in k_t, increasing in the demand level q_t^d. The optimality condition (8) thus determines the optimal capital stock as a function of factor costs and sales orders. It can be written more concisely as follows[6]:

$$k_t = \mathcal{K}\left(\frac{r_t + \delta}{p_t}, \omega_t, q_t^d, \bar{\theta}_t, \Theta\right), \tag{9}$$

where Θ summarises the parameters characterising the distribution of the idiosyncratic shocks. Function $\mathcal{K}$ is decreasing in $r_t + \delta$ and increasing in q_t^d. It depends ambiguously on ω_t because a higher real wage increases the marginal return on capital in the case of a sales constraint and reduces it in the other case.

In the limit case where $\bar{\theta}_t \to \theta_{\min}$, the firm chooses so large a capital stock (relatively to q_t^d) that it will always be in a position to serve the demanded quantity[7]. This case may occur when the real wage ω_t is low enough to compensate the under-utilization of the productive capital occurring for all $\theta_t > \theta_{\min}$. The opposite limit case ($\bar{\theta}_t \to \theta_{\max}$) corresponds to a Walrasian investment behaviour and will be examined below (see section 2.2).

Final Goods Producers

To keep the model as simple as possible, we assume that the final good production process uses only intermediate goods. We furthermore assume constant returns to scale and perfect substitutability between all intermediate goods, that is:

$$y_t = \int_0^1 q_{jt}\, dj, \tag{10}$$

where y_t is the final output level and q_{jt} is the quantity of input j used in production. Perfect competition between intermediate good producers implies a unique price $p_{jt} = p_t$, $\forall j$. With the production technology (10), the intermediate goods market equilibrium condition will further imply that p_t be equal to the price of the final good[8], i.e., $p_t = 1$. Whatever its output level,

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[6]Note that the optimal value of the capital stock is determined even in the constant returns-to-scale case, provided the probability of a sales-constraint is strictly positive.

[7]In the limit case where $\theta_{\min} = 0$, this would of course imply that $q_t^d \to 0$.

[8]Assuming decreasing returns would break the equality between the intermediate and final good prices but would not change our results qualitatively.

a final firm will make zero profits and will accept to serve any final demand level. For a firm using (10), the total demand for intermediate goods is thus equal to its output level y_t.

As said before, we assume that the final goods firm sends the same purchase order q_t^d to every intermediate goods producer, without knowing each individual producer's productivity level[9]. For every intermediate good, there is now a positive probability (equal to $\Pr(\theta_t < \bar{\theta}_t)$) that the supply will fall short of the ordered quantity q_t^d. This possibility is taken into account by the final firm when formulating its input demands. Because inputs are perfectly substitutable, the final firm will order a quantity q_t^d of each input such that the total amount q_t eventually received and defined by

$$q_t = \int_{\theta_{\min}}^{\bar{\theta}_t} q(k_t,\, \omega_t,\, \theta)\, \mathrm{d}G(\theta)\ +\ \left[1 - G(\bar{\theta}_t)\right] q_t^d$$

satisfies the production constraint $y_t = q_t$. This gives q_t^d as a function of y_t, $\bar{\theta}_t$ and $q(k_t,\, \omega_t,\, \theta)$:

$$q_t^d = \frac{y_t - \int_{\theta_{\min}}^{\bar{\theta}_t} q(k_t,\, \omega_t,\, \theta)\, \mathrm{d}G(\theta)}{1 - G(\bar{\theta}_t)}. \tag{11}$$

Consumers

A representative infinitely-lived agent consumes, supplies labour, accumulates productive capital and lends it to input firms. Her total revenue coincides with the total gross domestic income: wage and interest rate income, plus the firms' profits[10].

Let us denote c_t the consumption spending in t, n_t the labour supply in t and k_{t+1} the stock of capital accumulated by the consumer at the end of period t. Her optimisation programme can then be written as follows:

$$\max_{\{c_t, n_t\}_{t \geq 1}} \quad \sum_{t=1}^{\infty} \frac{u(c_t) - v(n_t)}{(1 + \rho)^t},$$

$$\text{subject to:} \quad k_{t+1} + c_t = (1 + r_t)\, k_t + D_t + w_t\, n_t, \quad \forall\, t \geq 1,$$

$$\text{and:} \quad \lim_{t \to \infty} R_{1,t+1}\, k_{t+1} \geq 0,\, k_1 \text{ given,}$$

[9]In a centralized or a Walrasian market, the allocation of this total demand across intermediate firms would coincide with the Walrasian output levels of those firms.

[10]As the technological shocks experienced by input firms are purely idiosyncratic, they do not induce any uncertainty for the representative household (or for any household who has diversified perfectly her portfolio choice). For her, there is thus no market incompleteness.

where $\rho > 0$ and $R_{1,t+1}$ is the discount factor associated to period $t+1$ $(R_{1,t+1} = \Pi_{s=2}^{t+1} (1+r_s)^{-1})$ and D_t stands for the total amount of dividends distributed by intermediate firms:

$$D_t = p_t \left[\int_{\theta_{\min}}^{\bar{\theta}_t} \Pi^w (k_t, \omega_t, \theta) \, dG(\theta) + \int_{\bar{\theta}_t}^{\theta_{\max}} \Pi^d \left(k_t, q_t^d, \omega_t, \theta \right) \, dG(\theta) - (r_t + \delta) \, k_t \right]. \quad (12)$$

Functions $u(\cdot)$ and $v(\cdot)$ are assumed to be such that: $u'(\cdot), v'(\cdot) > 0$, $u''(\dot{)} < 0$ and $v''(\cdot) > 0$. The representative consumer's optimal consumption path and labour supply satisfy the usual first-order conditions: $\forall\, t \geq 1$,

$$u'(c_t) = \frac{(1 + r_{t+1})}{1 + \rho} u'(c_{t+1}) \qquad \text{or} \quad c_t = c(c_{t+1}, r_{t+1}), \quad (13)$$

$$v'(n_t) = u'(c_t)w_t \qquad \text{or} \quad n_t = n(w_t, c_t), \quad (14)$$

where given our assumptions on functions u and v, function c is increasing in c_{t+1} and decreasing in r_{t+1} and function n is increasing in w_t and decreasing in c_t.

2.2 General Equilibrium

The market equilibrium conditions are defined as follows.

On the intermediate goods markets, p_t is equal to 1 (hence $\omega_t = w_t$) and an intermediate firm for which $\theta \leq \bar{\theta}_t$ (resp. $\theta > \bar{\theta}_t$) produces $q(k_t, \omega_t, \theta_t)$ (resp. q_t^d), where $\bar{\theta}_t$ and q_t^d are determined by (6) and (11) respectively.

On the final good market, the final good supply is equal to the consumption and investment demands, i.e.,

$$y_t = \int_{\theta_{\min}}^{\bar{\theta}_t} q(k_t, \omega_t, \theta) \, dG(\theta) + \left[1 - G(\bar{\theta}_t) \right] q_t^d = c_t + \Delta k_{t+1} + \delta \, k_t, \quad (15)$$

where c_t satisfies the consumer's optimality conditions.

On the capital market, the demand for capital (9) is equal to the capital stock accumulated by the households:

$$\mathcal{K}(r_t + \delta, \omega_t, q_t^d, \bar{\theta}_t, \Theta) = k_t. \quad (16)$$

Finally, *the labour market equilibrium condition* implies that labour demand be equal to the total workforce n_t:

$$n_t = \int_{\theta_{\min}}^{\bar{\theta}_t} \ell(k_t, \omega_t, \theta_t) \, dG(\theta) + \int_{\bar{\theta}_t}^{\theta_{\max}} \ell^d(k_t, q_t^d, \theta_t) \, dG(\theta). \quad (17)$$

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An intertemporal general equilibrium of this economy is defined by a sequence of vectors of prices $(p_t, r_t, \omega_t)_{t \geq 1}$ and quantities $\left(q_t^d, y_t, c_t, n_t, k_{t+1}\right)_{t \geq 1}$ and of values of $\bar{\theta}_t$ such that, at given predetermined capital stock k_t, the following conditions are satisfied:

- consumers, intermediate and final goods producers behave optimally (see equations (13), (14), (4) and (8), (11) respectively) ;

- on the intermediate goods markets, a proportion $G(\bar{\theta}_t)$ of firms experiences a productivity shock smaller than or equal to $\bar{\theta}_t$ (defined by (6)) and produces $q(k_t, \omega_t, \theta_t)$; a proportion of firms $\left[1 - G(\bar{\theta}_t)\right]$ experiences a higher productivity shock and produces q_t^d;

- there is competitive equilibrium on all the other markets (labour, capital, final goods).

Proposition 1

1. If $G(\theta)$ is a degenerate distribution (no microeconomic uncertainty), the equilibrium is unique and coincides with the Walrasian equilibrium without heterogeneity between intermediate firms.

2. If $G(\theta)$ is a non-degenerate distribution, there is a continuum of equilibria, which can be indexed by intermediate firms' sales expectations.

3. If $G(\theta)$ is a non-degenerate distribution, a Walrasian equilibrium with heterogeneous intermediate firms can be obtained provided sales expectations are sufficiently optimistic.

Proof

1. If the distribution $G(\theta)$ is degenerate, one has $\theta_{\min} = \theta_{\max}$. q_t^d is then necessarily equal to the Walrasian output level q_t^w (see e.g. (6)), which takes the same value for all intermediate goods producers. Equations (13) to (17) then define a standard Walrasian equilibrium.

2. In the general case where $0 \leq \theta_{\min} < \theta_{\max} < \infty$, at given expectations on c_{t+1} and r_{t+1}, there are only six equations to determine the seven unknowns $(r_t, \omega_t, \bar{\theta}_t, q_t^d, y_t, n_t, k_{t+1})$. Otherwise stated there is a vector of factor prices (r_t, ω_t) and a vector of quantities $(y_t, q_t^d, n_t, k_{t+1})$ satisfying the equilibrium conditions (6), (11), (14), (15), (16), (17) for each possible value of $\bar{\theta}_t$ between $\theta_{\min}$ and $\theta_{\max}$. This is true for every period t. Obviously enough, the interval of equilibrium values of each of the endogenous variables $(r_t, \omega_t, q_t^d, y_t, n_t, k_{t+1})$ is larger, the larger the interval of admissible values for $\bar{\theta}_t$, i.e., the larger the measure of $[\theta_{\min}, \theta_{\max}]$.

   One can easily check that the stationary equilibrium is also indeterminate.

3. In the presence of idiosyncratic uncertainty, the Walrasian equilibrium can be thought as coming from a centralized trading process on the intermediate goods market. It would then be possible to achieve *ex post* a perfect match between demands and supplies[11]. Each intermediate firm would then produce and sell its Walrasian output level $q(k_t, \omega_t, \theta)$ and aggregate output would be uniquely determined by:

$$y_t^w = \int_{\theta_{\min}}^{\theta_{\max}} q(k_t^w, \omega_t^w, \theta) \, \mathrm{d}G(\theta) \tag{18}$$

   with a Walrasian wage rate satisfying:

$$n(\omega_t^w, c_t) = \int_{\theta_{\min}}^{\theta_{\max}} \ell(k_t^w, \omega_t^w, \theta) \, \mathrm{d}G(\theta) \tag{19}$$

   and a demand for capital k_t^w such that:

$$r_t + \delta = \int_{\theta_{\min}}^{\theta_{\max}} \Pi_k^w (k_t^w, \omega_t^w, \theta) \, \mathrm{d}G(\theta). \tag{20}$$

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[11]Equivalently, we could assume that the final good firm has perfect information about the profitable productive capacity of each individual intermediate goods producer. Input orders would then be adjusted in such a way that $q_j^d \equiv q_j^w$, $\forall j$.

The same equilibrium values are also obtained in our setup provided sales expectations are sufficiently optimistic to imply $\bar{\theta}_t \geq \theta_{\max}$.

QED

The idiosyncratic shocks introduced in point 2 of proposition 1 imply heterogeneous employment and production decisions at the prevailing competitive prices and wages. In a Walrasian world, those shocks would be inconsequential at the aggregate economy level: *ex post*, final firms would always buy from each intermediate firm a quantity of intermediate goods equal to its optimal production level. We have departed from this Walrasian scenario by considering the case where final firms send purchase orders to intermediate firms without knowing every intermediate firm's optimal production level. At the competitive price, an intermediate firm enjoying a good productivity shock may then receive too few purchase orders, and vice-versa. The possibility of such outcomes makes the intermediate firm's investment choice depend on expected forthcoming purchase orders. It is these expectation effects that induce indeterminacy of the macroeconomic equilibrium: pessimistic sales expectations reduce the demand for capital, which leads to a lower rental price of capital on the one hand and to a lower labour demand and lower wages on the other hand. Both evolutions lower the domestic income, low activity levels following from low investments due to pessimistic expectations.

Points 1 and 3 of Proposition 1 show that the market organization we assumed here is not *per se* incompatible with the realization of the Walrasian equilibrium. On the one hand (point 1), this market organization leads to the unique Walrasian equilibrium in the limit case where there are no microeconomic shocks: agents then possess all the relevant information and decentralised trading raises no coordination difficulty. On the other hand (point 3), when there is uncertainty and therefore imperfect information between agents, the same market organization is able to reproduce the Walrasian equilibrium provided that agents be optimistic enough.

2.3   An Example

To gain further insights into the properties of the non-Walrasian economy described so far, let us assume a Cobb-Douglas production function with non-increasing returns to scale. More

specifically, we assume:

$$q_t \;=\; \theta_t \, \ell_t^\alpha \, k_t^\beta \,, \qquad \text{with} \quad \alpha + \beta \le 1 \,. \tag{21}$$

The detailed expressions corresponding to equations (2) and (4) are given in Appendix 1.

The capital demand equation (8) becomes:

$$\frac{r_t + \delta}{\beta} \;=\; \left(\frac{w_t}{\alpha}\right)^{-\alpha/(1-\alpha)} k_t^{-\frac{1-\alpha-\beta}{1-\alpha}} \, h(\bar\theta_t, \Theta) \,, \tag{22}$$

$$\text{where} \qquad h(\bar\theta_t, \Theta) = \left\{ \int_{\theta_{\min}}^{\bar\theta_t} \theta^{\frac{1}{1-\alpha}} \, dG(\theta) \;+\; \left(\bar\theta_t\right)^{\frac{1}{\alpha(1-\alpha)}} \int_{\bar\theta_t}^{\theta_{\max}} \theta^{-1/\alpha} \, dG(\theta) \right\} \,, \tag{23}$$

$$\text{and} \qquad \bar\theta_t = \left(\frac{w_t}{\alpha}\right)^{\alpha} \left(q_t^d\right)^{1-\alpha} k_t^{-\beta}. \tag{24}$$

Because all intermediate goods producers face *ex ante* the same decision problem and the aggregate capital stock is predetermined, (22) is also the capital market equilibrium condition.

In the Cobb-Douglas case, the labor market equilibrium condition (17) similarly become:

$$n_t = \left(\frac{w_t}{\alpha}\right)^{-1/(1-\alpha)} k_t^{\beta/(1-\alpha)} \, h(\bar\theta_t, \Theta) \,, \tag{25}$$

where $h(\bar\theta_t, \Theta)$ and $\bar\theta_t$ keep the same definition as in (23) and (24).

The final output supply is given by

$$y_t \;=\; \left(\frac{w_t}{\alpha}\right)^{-\alpha/(1-\alpha)} k_t^{\beta/(1-\alpha)} \, H(\bar\theta_t, \Theta) \tag{26}$$

$$\text{where} \qquad H(\bar\theta_t, \Theta) = \left\{ \int_{\theta_{\min}}^{\bar\theta_t} \theta^{\frac{1}{1-\alpha}} \, dG(\theta) \;+\; \left(\bar\theta_t\right)^{\frac{1}{1-\alpha}} \int_{\bar\theta_t}^{\theta_{\max}} dG(\theta) \right\} \,, \tag{27}$$

Let us further assume that the consumers' preferences are described by isoelastic functions:

$$u(c_t) \;=\; \frac{c_t^{1-\sigma}}{1-\sigma} \qquad \text{with} \quad \sigma \ge 0 \tag{28}$$

$$v(n_t) \;=\; m\frac{n_t^{1+\eta}}{1+\eta} \qquad \text{with} \quad \eta \ge 0, m \ge 0 \tag{29}$$

The labour supply function n_t is then described by

$$n_t \;=\; \left(\frac{c_t^{-\sigma} w_t}{m}\right)^{1/\eta} \tag{30}$$

Lemma 1

Functions $h(\bar{\theta}_t, \Theta)$ and $H(\bar{\theta}_t, \Theta)$ are increasing in $\bar{\theta}_t$ and such that

$$h(\bar{\theta}_t, \Theta) \; < \; H(\bar{\theta}_t, \Theta), \; \forall \, \bar{\theta}_t < \theta_{\max} \;\; \text{and}$$

$$0 \; \leq \; h(\theta_{\min}, \Theta) \; < \; H(\theta_{\min}, \Theta) \leq \; h(\theta_{\max}, \Theta) \; = \; H(\theta_{\max}, \Theta)$$

Hence, $h(\theta_{\min}, \Theta)/h(\theta_{\max}, \Theta) < 1$; moreover,

$$\frac{H(\theta_{\min}, \Theta)}{h(\theta_{\min}, \Theta)} > \frac{H(\theta_{\max}, \Theta)}{h(\theta_{\max}, \Theta)} = 1$$

Proof

The first-partial derivatives of $h(\bar{\theta}_t, \Theta)$ and $H(\bar{\theta}_t, \Theta)$ with respect to $\bar{\theta}_t$ are respectively:

$$\frac{\partial h(\bar{\theta}_t, \Theta)}{\partial \bar{\theta}_t} \;\; = \;\; \frac{1}{\alpha(1-\alpha)} \; (\bar{\theta}_t)^{\frac{1}{\alpha(1-\alpha)}-1} \int_{\bar{\theta}_t}^{\theta_{\max}} \left(\frac{1}{\theta}\right)^{1/\alpha} \mathrm{d}G(\theta) \; \geq 0;$$

$$\frac{\partial H(\bar{\theta}_t, \Theta)}{\partial \bar{\theta}_t} \;\; = \;\; \frac{1}{(1-\alpha)} \; (\bar{\theta}_t)^{\frac{\alpha}{1-\alpha}} \int_{\bar{\theta}_t}^{\theta_{\max}} \mathrm{d}G(\theta) \; \geq 0$$

$h(\bar{\theta}_t, \Theta)$ and $H(\bar{\theta}_t, \Theta)$ differ only in their second term, the value of which is smaller in $h(\bar{\theta}_t, \Theta)$ than in $H(\bar{\theta}_t, \Theta)$. Indeed,

$$(\bar{\theta}_t)^{\frac{1}{\alpha(1-\alpha)}} \int_{\bar{\theta}_t}^{\theta_{\max}} \theta^{-1/\alpha} \, \mathrm{d}G(\theta) \; = \; (\bar{\theta}_t)^{\frac{1}{1-\alpha}} \int_{\bar{\theta}_t}^{\theta_{\max}} \left(\frac{\theta}{\bar{\theta}_t}\right)^{-1/\alpha} < (\bar{\theta}_t)^{\frac{1}{1-\alpha}} \int_{\bar{\theta}_t}^{\theta_{\max}} \mathrm{d}G(\theta)$$

since

$$\left(\frac{\theta}{\bar{\theta}_t}\right)^{-1/\alpha} = \left(\frac{\bar{\theta}_t}{\theta}\right)^{1/\alpha} < 1, \; \forall \, \theta > \bar{\theta}_t$$

The other results are obvious.

<div align="right">QED</div>

With a Cobb-Douglas production, the aggregate capital/labour ratio (given by the ratio of equations (22) and (25) only depends on relative factor prices:

$$\frac{k_t}{n_t} = \frac{w_t/\alpha}{(r_t + \delta)/\beta} \, . \tag{31}$$

The ratio of equations (26) and (22) and the ratio of equations (26) and (25) give the output-capital and output-labour ratios respectively:

$$\frac{y_t}{k_t} \;\; = \;\; \frac{r_t + \delta}{\beta} \frac{H(\bar{\theta}_t, \Theta)}{h(\bar{\theta}_t, \Theta)} \tag{32}$$

$$\frac{y_t}{n_t} \;\; = \;\; \frac{w_t}{\alpha} \frac{H(\bar{\theta}_t, \Theta)}{h(\bar{\theta}_t, \Theta)} \tag{33}$$

Let us denote by x_{ss} the stationary state value of variable x. In a stationary state, the first order condition on consumption determined the real rate of interest $r_{ss} = \rho$ and consumption is given by the net output level, i.e.,

$$c_{ss} = y_{ss} - \delta k_{ss} = k_{ss}\, g(\bar{\theta}_{ss}, \Theta) \tag{34}$$

where $g(\bar{\theta}_{ss}, \Theta)$ is the stationary consumption-capital ratio given (using (32)) by

$$g(\bar{\theta}_{ss}, \Theta) = \frac{\rho + \delta}{\beta}\, \frac{H(\bar{\theta}_{ss}, \Theta)}{h(\bar{\theta}_{ss}, \Theta)} - \delta. \tag{35}$$

Inserting (30) into the stationary state expression of (25) and substituting the stationary state consumption by (34) allow ones to express the stationary state wage rate as a function of the capital stock and $\bar{\theta}_{ss}$. By using this stationary equilibrium relationship to eliminate the wage rate into (22), one can then write the stationary state value of the capital stock as an increasing function of $\bar{\theta}_{ss}$:

$$k_{ss} = \left(\frac{\alpha}{m}\right)^{\phi_m} \left(\frac{\rho + \delta}{\beta}\right)^{\phi_k} I(\bar{\theta}_{ss}, \Theta) \tag{36}$$

where

$$I(\bar{\theta}_{ss}, \Theta) = \left(g(\bar{\theta}_{ss}, \Theta)\right)^{\phi_g} \left(h(\bar{\theta}_{ss}, \Theta)\right)^{\phi_h}$$

$$\text{with}$$

$$\phi_m = \frac{\alpha}{(1 + \eta)(1 - \beta) - \alpha(1 - \sigma)}$$

$$\phi_k = -\frac{1 + \eta - \alpha}{(1 + \eta)(1 - \beta) - \alpha(1 - \sigma)} < 0$$

$$\phi_g = -\frac{\alpha\sigma}{(1 + \eta)(1 - \beta) - \alpha(1 - \sigma)} < 0$$

$$\phi_h = \frac{(1 + \eta)(1 - \alpha)}{(1 + \eta)(1 - \beta) - \alpha(1 - \sigma)} > 0$$

Proposition 2

a) The set of all possible stationary state values of the capital stock k_{ss} is defined by the following interval:

$$\frac{I(\theta_{\min}, \Theta)}{I(\theta_{\max}, \Theta)}\, k_{ss}^w \leq k_{ss} \leq k_{ss}^w, \tag{37}$$

where k_{ss}^w is the stationary state value of the capital stock at the Walrasian equilibrium obtained from (36) with $\bar\theta_{ss} = \theta_{max}$

$$k_{ss}^w = \left(\frac{\alpha}{m}\right)^{\phi_m} \left(\frac{\rho+\delta}{\beta}\right)^{\phi_k} I(\theta_{max}, \Theta)$$

$$\text{where } I(\theta_{max}, \Theta) = \left(\frac{\rho+\delta}{\beta} - \delta\right)^{\phi_g} \left(\int_{\theta_{min}}^{\theta_{max}} \theta^{1/(1-\alpha)} \, \mathrm{d}G(\theta)\right)^{\phi_h}.$$

(37) and the other equations can be combined to obtain the equilibrium interval for the real wage, output and employment.

b) The interval of indeterminacy is increasing in the wage-elasticity of the labour supply $1/\eta$.

c) The interval of indeterminacy depends ambiguously on the intertemporal elasticity of substitution in consumption, $1/\sigma$.

Proof

a) follows directly from the ratio of the expressions of k_{ss} in (36) respectively for $\bar\theta_{ss} = \theta_{\max}$ and $\bar\theta_{ss} = \theta_{\min}$.

Note in particular that

$$\frac{I(\theta_{\min}, \Theta)}{I(\theta_{\max}, \Theta)} = \left(\frac{g(\theta_{min}, \Theta)}{g(\theta_{max}, \Theta)}\right)^{\phi_g} \left(\frac{h(\theta_{\min}, \Theta)}{h(\theta_{\max}, \Theta)}\right)^{\phi_h} < 1. \tag{38}$$

Indeed, the ratio between $h(\theta_{\min}, \Theta)$ and $h(\theta_{\max}, \Theta)$ is smaller than one (see lemma 1) and raised to a positive power. Moreover, lemma 1 implies that $g(\theta_{min}, \Theta) > g(\theta_{max}, \Theta)$: the ratio between $g(\theta_{min}, \Theta)$ and $g(\theta_{max}, \Theta)$ is thus larger than 1 but raised to a negative power. Hence, (38) is the product of two terms smaller than 1.

b) Obviously enough, the smaller the ratio between $I(\theta_{\min}, \Theta)$ and $I(\theta_{\max}, \Theta)$, the larger the interval of indeterminacy. Let us show that (38) is increasing in η so that the interval of indeterminacy is decreasing in η (or increasing in $1/\eta$):

$$\frac{\partial}{\partial \eta}\left(\frac{I(\theta_{\min}, \Theta)}{I(\theta_{\max}, \Theta)}\right) = \frac{I(\theta_{\min}, \Theta)}{I(\theta_{\max}, \Theta)}\left[\left(\ln \frac{g(\theta_{min}, \Theta)}{g(\theta_{max}, \Theta)}\right)\frac{\partial \phi_g}{\partial \eta} + \left(\ln \frac{h(\theta_{\min}, \Theta)}{h(\theta_{\max}, \Theta)}\right)\frac{\partial \phi_h}{\partial \eta}\right],$$

which is unambiguously positive since lemma 1 implies the positivity of the first log (see 35)

and the negativity of the second one and

$$
\begin{aligned}
\frac{\partial \phi_g}{\partial \eta} &= \frac{\alpha(1-\beta)\sigma}{((1+\eta)(1-\beta)-\alpha(1-\sigma))^2} > 0 \\
\frac{\partial \phi_h}{\partial \eta} &= -\frac{\alpha(1-\alpha)(1-\sigma)}{((1+\eta)(1-\beta)-\alpha(1-\sigma))^2} < 0.
\end{aligned}
$$

c) The first derivative of (38) with respect to σ is equal to

$$
\frac{\partial}{\partial \sigma}\left(\frac{I(\theta_{\min},\Theta)}{I(\theta_{\max},\Theta)}\right) = \frac{I(\theta_{\min},\Theta)}{I(\theta_{\max},\Theta)}\left[\left(\ln \frac{g(\theta_{min},\Theta)}{g(\theta_{max},\Theta)}\right)\frac{\partial \phi_g}{\partial \sigma} + \left(\ln \frac{h(\theta_{\min},\Theta)}{h(\theta_{\max},\Theta)}\right)\frac{\partial \phi_h}{\partial \sigma}\right],
$$

which has an ambiguous sign because the two logs have opposite signs whereas both partial derivatives are negative:

$$
\begin{aligned}
\frac{\partial \phi_g}{\partial \sigma} &= -\alpha\frac{1-\alpha-\beta+\eta(1-\beta)}{((1+\eta)(1-\beta)-\alpha(1-\sigma))^2} < 0 \\
\frac{\partial \phi_h}{\partial \sigma} &= -\frac{\alpha(1-\alpha)(1+\eta)}{((1+\eta)(1-\beta)-\alpha(1-\sigma))^2} < 0.
\end{aligned}
$$

<div align="right">QED</div>

Proposition 2 shows the determinants of the interval of indeterminacy in the particular case of an economy with a Cobb-Douglas technology and isoelastic utility functions. Proposition 2.b is quite intuitive. In the model economy, a wave of optimism/pessimism shifts capital and labour demands upwards/downwards. The flatter the labour supply curve (the larger $1/\eta$), the less a given labour demand shift will affect wages and the more it will affect employment (and thereby capital demand and output). It is worth noting that indeterminacy remains even in the extreme case of an inelastic labour supply ($\eta \to \infty$): in such a case indeed,

$$
\frac{I(\theta_{\min},\Theta)}{I(\theta_{\max},\Theta)} = \left(\frac{h(\theta_{\min},\Theta)}{h(\theta_{\max},\Theta)}\right)^{\frac{1-\alpha}{1-\beta}} < 1.
$$

Employment is then exogenously determined but there is still room for indeterminacy in wages, capital, output and consumption.

Proposition 2.c about the ambiguous impact of $1/\sigma$ is intuitive as well. At given wage, a wave of optimism that increases the labour demand also reduces the labour supply (since consumption increases and leisure is a normal good). Obviously enough, the larger the labour supply contraction at given wage, the smaller the quantitative impact of a labour demand increase: the labour supply shift thus dampens the employment/output fluctuations following from a wave of

optimism/pessimism. But the size of the labour supply shift depends ambiguously on σ. On the one hand for a given increase in consumption, a larger σ implies a larger labour supply shift at given wage. On the other hand, the increase in consumption itself depends negatively on σ: the larger σ (the smaller $1/\sigma$), the more consumers will choose to smooth consumption intertemporally and the less consumption will increase.

3 Demand Shocks and Business Cycle Fluctuations

The model developed in the previous sections implies that pure demand expectation shocks can have real effects. Our objective in this section is to calibrate a model and use numerical simulations to examine the characteristics of the propagation mechanism associated to such stochastic demand shocks. We compare these characteristics to those of a typical RBC model, where fluctuations are triggered by technological shocks, and to those of a model where "animal spirit" effects arise from the multiplicity of equilibrium trajectories around a deterministic steady state (*i.e.* the steady state is a sink).

Two ways of introducing demand expectations effects

The dynamics of an intertemporal general equilibrium model with rational expectations can be illustrated by the following linearized system borrowed from Benhabib-Farmer (1999):

$$
\begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{s}_{t+1} \end{bmatrix} = \Psi \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix} + \Gamma \begin{bmatrix} e_{t+1} \\ u_{t+1} \end{bmatrix}
\tag{39}
$$

A hat over a variable indicates the percentage deviation from the steady state value; c is a control variable (typically consumption) while k and s are two state variables, typically the capital stock and total factor productivity (the Solow residual). e and u represent unanticipated changes (innovations) in c and s. In standard RBC models, matrix Ψ has as many unstable (resp. stable) roots as there are control (resp. state) variables. The model then satisfies the Blanchard-Kahn conditions and has a unique rational expectation equilibrium trajectory, obtained by eliminating the influence of the unstable root. In such a case the control variable is solely a function of the state variables k and s. In other words, expectational errors on consumption are solely a function

18

of the innovations u; there is no room for independent demand expectations effects e. A variety of model though have been constructed where the Blanchard-Kahn conditions are violated. The unique steady state equilibrium is then a sink, which leaves a role for pure demand expectation effects like e (see the survey in Benhabib-Farmer (1999)).

The demand expectation effect appearing in the non-Walrasian model of the previous section is of a different nature. There is one extra state variable (sales orders q^d), whose effect can be represented in the above dynamic system in a way similar to that of the technology shock s and the associated innovation term u. Around any stationary equilibrium corresponding to a given steady state value q^d, stochastic demand fluctuations will generate output fluctuations via their impact on firms and households behaviors. In other words, we obtain demand expectation effects even though the model satisfies the Blanchard-Kahn conditions.

In a case with centralized trading, our model boils down to a standard RBC model with Walrasian properties and no demand expectation effects. We will use numerical simulations to compare a non-Walrasian and a Walrasian version of the model, the former with aggregate demand shocks, the latter with aggregate productivity shocks.

Model specification and calibration

We extend our model in order to introduce the possibility of aggregate technological shocks and endogenise labour supply decisions. We assume a Cobb-Douglas production function, which is an obvious generalization of (21):

$$q_t = A_t \, \theta_t \, (X_t \, \ell_t)^\alpha \, k_t^{1-\alpha} \,, \qquad \text{with} \qquad X_t = \gamma \, X_{t-1} \,, \quad \gamma > 1. \qquad (40)$$

Aggregate productivity is made of two components, a deterministic component X_t assumed to grow at constant rate, and a random component A_t determined by the following autoregressive stochastic process:

$$A_t = \left(\bar{A} \right)^{1-\mu_u} \, (A_{t-1})^{\mu_u} \, e^{u_t} \qquad \text{where} \qquad u_t \sim N(0, \sigma_u^2). \qquad (41)$$

The idiosyncratic productivity shock θ_t is assumed to be distributed uniformly over an interval $[\theta_{min}, \theta_{max}]$ centered around one and such that there can be a 40% difference between the most and the least productive firms.

Consumers' preferences with respect to consumption and labour are described by the isoelastic functions (28) and (29).

For the Walrasian version of the model, we use the same parameter values as in the baseline model of King-Rebelo (1999), except for the disutility of work, where we use the indivisibility assumption of Hansen (1985) and set $\eta = 0$ (instead of -1). The parameter values used to simulate the model are reproduced in table 1. In the non-Walrasian version of the model, the stochastic demand shock is assumed to follow the autoregressive process:

$$q_t^d = \left(\bar{q}^d\right)^{1-\mu_v} \left(q_{t-1}^d\right)^{\mu_v} e^{v_t} \qquad \text{where} \qquad v_t \sim N(0, \sigma_v^2). \qquad (42)$$

A distinctive feature of the non-Walrasian model is its direct implications in terms of capacity utilization. The macroeconomic rate of capacity utilization can be defined as the ratio between the observed and the Walrasian output levels, at given output and factor prices, i.e.,

$$d_t = \frac{y_t}{y_t^w},$$

with y_t^w given by (18). We choose to set $\bar{q}^d$ at 0.80, so as to obtain an aggregate rate of capacity utilization equal to $d = 0.92$ at steady state. The parameters determining the volatility and the persistence of the demand shocks are given the same values as those of the productivity shocks in the Walrasian model (that is $\mu_v = \mu_u$ and $\sigma_v = \sigma_u$).[12] The models are simulated under the assumption of an exogenous deterministic growth of 0.4% on a quarterly basis.

σ	$\rho^{-1}-1$	m	η	γ	α	δ	μ_u	σ_u	$\bar{q}^d$	$\bar{\theta}_{min}$	$\bar{\theta}_{max}$
1	0.984	3.48	0	1.004	0.667	0.025	0.979	0.0072	0.80	0.80	1.20

Table 1: Calibration

Simulation results

The simulation results are summarized in table 2. All variables have been logged and detrended with the HP filter. The first part of the table gives the standard deviations of the main variables

---

[12] A sensitivity analysis (see Appendix 4) makes clear that the simulation results reported in table 2 (column b) do not depend importantly on the calibration we chose for the demand shock.

relative to that of output; the second part of the table gives the contemporaneous correlations with output. Column (a) reproduces the business cycle characteristics of US data reported in King-Rebelo (1999). The second-order moments generated with our non-Walrasian model and demand shocks are given in column (b). For comparison, we reproduce in columns (c) the values obtained with the Walrasian version of our model and aggregate productivity shocks. Except for idiosyncratic shocks, this version of our model corresponds to a basic RBC model. Column (d) gives the simulation results obtained without these idiosyncratic productivity shocks (*i.e.* with $\theta_{min} = \theta_{max} = 1$). Except for the intertemporal elasticity of leisure ($\eta = 0$ instead of -1), the model of column (d) is identical to the baseline RBC model of King-Rebelo (1999). The last column reproduces the results obtained by Farmer-Guo (1994). In the latter model, demand expectations can also have real effects, albeit by a quite different channel than in our non-Walrasian setup. Demand expectation effects in Farmer-Guo (FG hereafter) arise from the multiplicity of admissible equilibrium trajectories (around a unique steady state) generated by the increasing returns to scale assumption, while in our non-Walrasian setup there is a continuum of steady state equilibria but a unique equilibrium trajectory associated to a given steady state. In order to ease the comparisons between columns (b) and (c), we display and comment in Appendix 3 figures comparing the impulse response functions of our model in the cases of aggregate productivity and demand shocks.

Comparing columns (c) and (d) shows that idiosyncratic shocks do not change the cyclical properties of the Walrasian economy in reaction to productivity shocks. In a non-Walrasian setup they imply however that pure demand shocks can generate cyclical properties not too far from those observed in the data (see columns (a) and (b)). A positive demand shock increases investment demand, which stimulates production (and employment) in all firms with idle profitable capacities. At the same time, a higher investment demand raises the real interest rates, which dampens partially the increase in consumption demand following from the income expansion. The higher labour demand leads to a procyclical increase in real wages. With the chosen calibration, investment and employment are more volatile and more correlated with output in the non-Walrasian model than in the standard RBC model, while consumption and wages are made less volatile and less correlated with output. It is worth noting that the same comments apply

	US data	Non-Walrasian	Walrasian	Walrasian*	FG (94)
		(demand shocks)	(RBC+idios. sh.)	(basic RBC)	
	(a)	(b)	(c)	(d)	(e)
Relative standard deviation					
y	1.00	1.00	1.00	1.00	1.00
c	0.74	0.26	0.43	0.43	0.24
i	2.93	4.38	3.30	3.30	5.13
ℓ	0.99	1.39	0.61	0.62	0.83
w	0.38	0.26	0.43	0.43	0.24
d	>1	1.22	-	-	-
Contemporaneous correlation with output					
y	1.00	1.00	1.00	1.00	1.00
c	0.88	0.56	0.93	0.93	0.78
i	0.80	0.98	0.98	0.98	0.99
ℓ	0.88	0.99	0.96	0.96	0.98
w	0.12	0.56	0.93	0.93	0.78
d	>0	1.00	-	-	-

* King-Rebelo (1999)'s baseline model with labor indivisibility *à la* Hansen (1985)

Table 2: Business cycle statistics

to the comparison between the Farmer-Guo model with demand shocks and the standard RBC model, with the non-Walrasian model being perhaps somewhat closer to the data.

The non-Walrasian model with pure demand shocks thus appears to be capable of generating interesting business cycle characteristics. It certainly performs as well as the RBC model or the Farmer-Guo model with increasing returns. Because its main propagation mechanism works through capacity utilization changes, it furthermore implies a strongly volatile and procyclical capacity utilization rate, as suggested by available empirical evidence. The literature on RBC models (a.o. Burnside *et* al. (1996), Fagnart *et* al. (1998)) has already stressed that capital utilization changes can be a strong propagation and amplification mechanism of aggregate tech-

nological shocks. Capacity utilisation may similarly amplify the effects of pure demand shocks. The real effects of a given demand shock will be larger the lower the initial value of the economy's capacity utilization rate. As figure 1 shows, the sensitivity of output to demand shocks depends on the macroeconomic rate of capacity utilization (d): the lower d, the larger the volatility of output relative to that of the demand shock. For a given distribution of shocks, the relative volatility of output is multiplied by almost 2 when capacity utilization decreases from (almost) 100% to 80%.
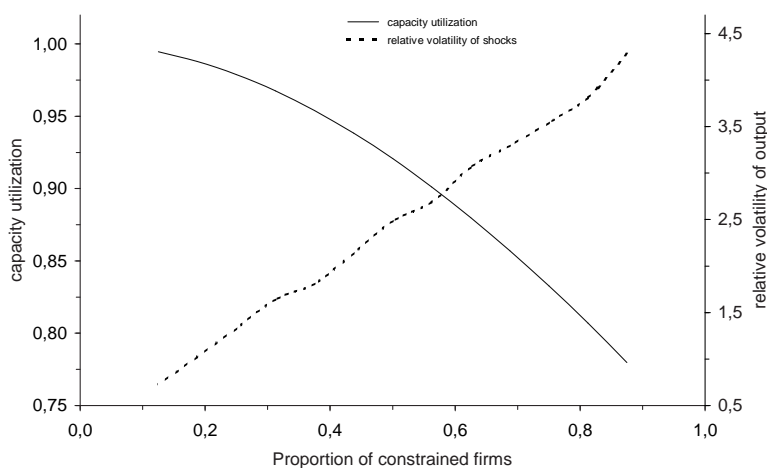
Figure 1: Aggregate capacity utilization and relative output volatility

4 Conclusions

The main result of the paper can be summarized as follows: in an economy where on some markets firms produce and sell on orders, genuine demand expectations effects (in the investment decisions of those firms) may appear and create equilibrium indeterminacy if firms are imperfectly informed about their trading opportunities when investing. Our indeterminacy result is thus rooted in the very working of a decentralized market in the absence of a market institution reproducing the coordinating activity of the Walrasian auctioneer. It does not rely on the existence of externalities, technological complementarities or increasing returns to scale as is the case in many other business cycle models with indeterminacy. A quantitative exploration of the implications of our model shows that it is consistent with an alternative interpretation of business cycle fluctuations, driven by self-fulfilling demand shocks instead of technological

shocks.

We want to stress that the market organization we assumed here is not *per se* incompatible with the realization of the Walrasian equilibrium. On the one hand, this market organization leads to the unique Walrasian equilibrium in the limit case where there are no microeconomic shocks: agents then possess all the relevant information and decentralised trading raises no coordination difficulty. On the other hand, when there is uncertainty (and therefore imperfect information between agents), the same market organization is able to reproduce the Walrasian equilibrium provided that agents be optimistic enough.

The model developed in this paper is admittedly a very stylised one. From a theoretical point of view, the representation of the information problem between agents should clearly be refined. From an empirical point of view, various extensions like a more realistic description of the labour market would be necessary to reproduce more precisely the cyclical properties of actual economies.

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Appendix 1: Firms' behaviour with Cobb Douglas technologies

Assume the Cobb Douglas technology $q_t = \theta_t \, \ell_t^\alpha \, k_t^\beta$, $\qquad \alpha + \beta \leq 1$.

The employment and production decisions corresponding to a Walrasian behaviour are

$$\ell_t^w = \ell(k_t, \omega_t, \theta_t) = \left[\frac{\alpha \, \theta}{\omega_t} k_t^\beta\right]^{\frac{1}{1-\alpha}} \quad \text{and} \quad q_t^w = q(k_t, \omega_t, \theta_t) = \theta_t \left[\frac{\alpha \, \theta_t}{\omega_t} k_t^{\beta/\alpha}\right]^{\frac{\alpha}{1-\alpha}} \tag{43}$$

Π_t^w then becomes

$$\Pi^w(k_t, \omega_t, \theta_t) = (1 - \alpha) \, q_t^w \tag{44}$$

with

$$\Pi_k^w(k_t, \omega_t, \theta_t) = \beta \, \theta_t^{1/(1-\alpha)} \left(\frac{\alpha}{\omega_t}\right)^{\alpha/(1-\alpha)} k_t^{-\frac{1-\alpha-\beta}{1-\alpha}} \tag{45}$$

In the case of a sales contraint, one has

$$\ell_t^d = \ell^d(k_t, q_t^d, \theta_t) = \left[\frac{q_t^d}{\theta_t}\right]^{1/\alpha} (k_t)^{-\beta/\alpha} \tag{46}$$

$$\Pi^d(k_t, q_t^d, \omega_t, \theta_t) = q_t^d - \omega_t \ell_t^d \tag{47}$$

and

$$
\begin{aligned}
\Pi_k^d(k_t, q_t^d, \omega_t, \theta_t) &= -\omega_t \frac{\partial \ell_t^d}{\partial k_t} \\
&= \frac{\beta}{\alpha} \omega_t \left[\frac{q_t^d}{\theta_t}\right]^{1/\alpha} (k_t)^{-(\alpha+\beta)/\alpha}
\end{aligned}
$$

A firm then installs a capital stock level k_t given by the first order optimality condition given in the main text (see (22)).

Appendix 2: Alternative modelling with idiosyncratic demand shock

This appendix presents an alternative version of our model where uncertainty follows from idiosyncratic demand shock instead of idiosyncratic productivity shocks. θ_t is now a technological parameter identical to all intermediate firms, which face idiosyncratic demand uncertainty following from the behaviour of the final firms.

More precisely, we assume that there is a mass N of *ex ante* identical producers of the final good, with N much larger than 1 (which is the mass of intermediate firms). With a quantity

q_t of intermediate good, a final good firm produces q_t units of final good. In order to simplify the presentation, we assume that a final firm orders its inputs to only one intermediate firm and that each intermediate firm is initially uncertain about the number of final firms that will be its customers during a given period: during period t, an intermediate firm receives input orders from ν_t final firms, where ν_t is a random number with distribution function $G(\nu)$ defined over the interval $[\nu_{\min}, \nu_{\max}]$ with $0 \leq \nu_{\min} < \nu_{\max} < N$ and

$$\int_{\nu_{\min}}^{\nu_{\max}} \nu \, d \, G(\nu) = N.$$

Optimal labour demand of intermediate good producers

If each final firm orders an input quantity q_t^d, an input firm thus receives a global order of $q_t^d \nu_t$ and produces a quantity given by $\min(q_t^d \nu_t, q_t^w)$. Variables q_t^w ℓ_t^w, and Π_t^w remain described by (2) and (3). When the input firm is sales constrained ($q_t = q_t^d \nu_t$), its labour demand and operating surplus become

$$\ell_t^d = \ell^d(k_t, q_t^d \nu_t, \theta_t), \tag{48}$$

$$\Pi_t^d = p_t \Pi^d\left(k_t, \, q_t^d \nu_t, \, \omega_t, \, \theta_t\right) \quad \text{where} \quad \Pi^d\left(k_t, \, q_t^d \nu_t, \, \omega_t, \, \theta_t\right) = q_t^d \nu_t - \omega_t \ell_t^d \tag{49}$$

Let us denote by $\bar{\nu}_t$ the critical value of the demand shock such that the quantity of orders received by an intermediate firm matches exactly its Walrasian production plan:

$$\bar{\nu}_t = \frac{q(k_t, \omega_t, \theta_t)}{q_t^d}. \tag{50}$$

An intermediate firm for which ν_t is smaller (resp. larger) than $\bar{\nu}_t$ is (resp. is not) sales-constrained.

Optimal capital stock of intermediate good producers

Expected profits maximization can thus be written as follows:

$$\max_{k_t} \int_{\nu_{min}}^{\bar{\nu}_t} p_t \Pi^d\left(k_t, \, q_t^d \nu, \, \omega_t, \, \theta_t\right) dG(\nu) + \int_{\bar{\nu}_t}^{\nu_{max}} p_t \Pi^w(k_t, \omega_t, \theta_t) dG(\nu) - (r_t + \delta) k_t. \tag{51}$$

The capital stock k_t is determined by the following first-order optimality condition:

$$\frac{r_t + \delta}{p_t} = \int_{\nu_{min}}^{\bar{\nu}_t} \Pi_k^d\left(k_t, \, q_t^d \nu, \, \omega_t, \, \theta_t\right) dG(\nu) + \int_{\bar{\nu}_t}^{\nu_{max}} \Pi_k^w(k_t, \omega_t, \theta_t) dG(\nu), \tag{52}$$

where function Π_k^w (resp. Π_k^d) represents the first partial derivative of function Π^w (resp. Π^d) with respect to k. The first term on the right-hand side of (52) is the expected marginal revenue

of capital when the firm is sales-constrained. This term is non-negative and decreasing in k_t, increasing in the demand level $q_t^d \nu_t$. The second term on the right-hand side of (52) represents the expected marginal revenue of capital when the firm operates at its Walrasian level. Like (8), the optimality condition (52) thus determines the optimal capital stock as a function of factor costs and sales orders: more concisely,

$$k_t = K\left(\frac{r_t + \delta}{p_t}, \omega_t, q_t^d, \bar{\nu}_t, \Xi\right),$$
(53)

where Ξ summarizes the parameters characterising the distribution function of the idiosyncratic demand shock.

Input demand of a final firm

When ν_t, the number of final firms which are customers of a given intermediate firm, is smaller (resp. larger) than $\bar{\nu}_t$, the input orders are all fulfilled (resp. are filled up to a quantity q_t^w). In aggregate, the final output supply Q_t following from the order of a quantity q_t^d by each final firm is thus given by

$$Q_t = \int_{\nu_{min}}^{\bar{\nu}_t} q_t^d \nu \, dG(\nu) + \int_{\bar{\nu}_t}^{\nu_{max}} q(k_t, \omega_t, \theta_t) \, dG(\nu).$$
(54)

At the final good market equilibrium, the final output supply Q_t must match the final output demand y_t. Final good market clearing thus requires that final firms (which make zero profit at any final output level) order an input quantity q_t^d such that

$$q_t^d = \frac{y_t - \int_{\bar{\nu}_t}^{\nu_{max}} q(k_t, \omega_t, \theta_t) \, dG(\nu)}{\int_{\nu_{min}}^{\bar{\nu}_t} \nu \, dG(\nu)}.$$
(55)

Consumers

Except for the definition of D_t, the section describing the consumers's behaviour is identical to the one of the main text.

General equilibrium

The market equilibrium conditions can be defined as in the main text *mutatis mutandis*.

On the intermediate goods markets, p_t is equal to 1 (hence $\omega_t = w_t$) and an intermediate firm for which $\nu \geq \bar{\nu}_t$ (resp. $\nu < \bar{\nu}_t$) produces $q(k_t, \omega_t, \theta_t)$ (resp. $q_t^d \nu$), where $\bar{\nu}_t$ and q_t^d are determined by (50) and (55) respectively.

On the final good market, the final good supply is equal to the consumption and investment demands, i.e.,

$$y_t = \int_{\nu_{min}}^{\bar{\nu}_t} q_t^d \, \nu \, dG(\nu) \; + q(k_t, \omega_t, \theta_t) \; [1 - G(\bar{\nu}_t)] \; = \; c_t + \Delta k_{t+1} + \delta \, k_t, \tag{56}$$

where c_t satisfies the consumer's optimality conditions.

On the capital market, the demand for capital (53) is equal to the capital stock accumulated by the households:

$$K\left(\frac{r_t + \delta}{p_t}, \omega_t, q_t^d, \bar{\nu}_t, \Xi\right) = k_t. \tag{57}$$

Finally, *the labour market equilibrium condition* implies that labour demand be equal to the total workforce n_t:

$$n_t \; = \; \int_{\nu_{\min}}^{\bar{\nu}_t} \ell^d(k_t, q_t^d \, \nu, \theta_t) \, dG(\nu) + \ell(k_t, \omega_t, \theta_t) \int_{\bar{\nu}_t}^{\nu_{\max}} dG(\nu) \,. \tag{58}$$

An intertemporal general equilibrium of this economy is defined by a sequence of vectors of prices $(p_t, r_t, \omega_t)_{t \geq 1}$ and quantities $\left(q_t^d, y_t, c_t, n_t, k_{t+1}\right)_{t \geq 1}$ and of values of $\bar{\nu}_t$ such that, at given predetermined capital stock k_t, the following conditions are satisfied:

- consumers, intermediate and final goods producers behave optimally (see equations (13), (14), (48) and (52), (55) respectively) ;

- on the intermediate goods markets, a proportion $G(\bar{\nu}_t)$ of firms experiences a sales shortage; a proportion of firms $[1 - G(\bar{\nu}_t)]$ produces q_t^w;

- there is competitive equilibrium on all the other markets (labour, capital, final goods).

It is then obvious to see that the model with idiosyncratic demand shocks exhibits the same type of equilibrium indeterminacy as the model of the main text. If $G(\nu)$ is a non-degenerate distribution, there is a continuum of equilibria. Indeed, with $0 \leq \nu_{\min} < \nu_{\max} < N$, at given expectations on c_{t+1} and r_{t+1}, there are only six equations to determine the seven unknowns $(r_t, \omega_t, \bar{\nu}_t, q_t^d, y_t, n_t, k_{t+1})$. Otherwise stated there is a vector of factor prices (r_t, ω_t) and a vector of quantities $(y_t, q_t^d, n_t, k_{t+1})$ satisfying the equilibrium conditions (50), (55), (14), (56), (57), (58) for each possible value of $\bar{\nu}_t$ between $\nu_{\min}$ and $\nu_{\max}$.

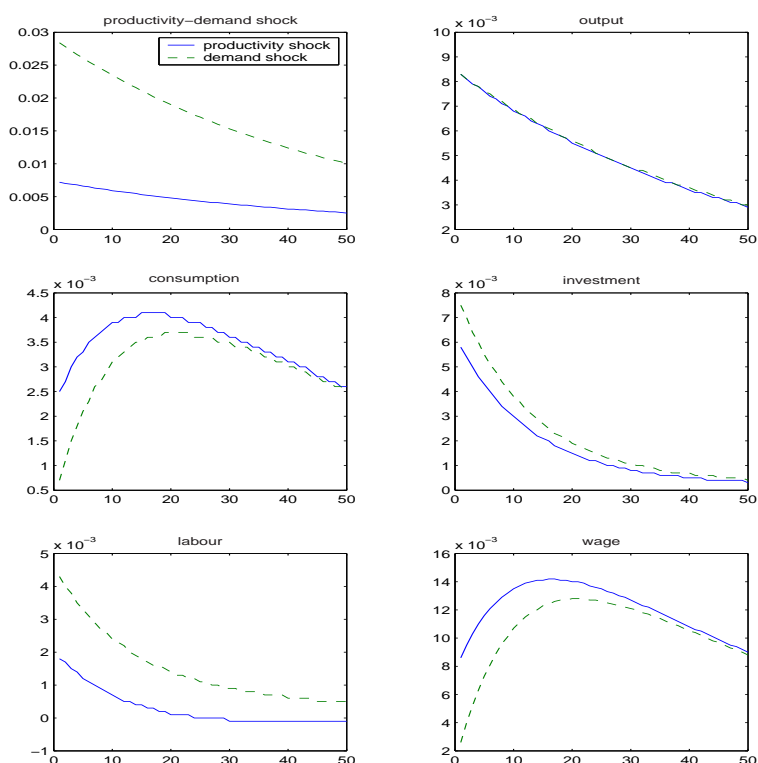**Appendix 3: Impulse response functions of a technological and a pure demand shock**

Figure 2: Impulse response functions to productivity vs demand shocks

The figures above display the impulse response functions (IRF) of a productivity and a pure demand shock (models (c) and (b) respectively). For the sake of comparison, these IRFs have been constructed by calibrating the size of the two shocks so as to produce the same output response in the first period. As the first graphic shows, this requires a demand shock larger than the technological shock because the demand shock only affects the output of the sales constrained firms whereas the technological shock affects all firms. As explained in the main text, investment and labour are more volatile in response to a demand shock whereas real wages (and consumption) are less volatile.

Appendix 4: Sensitivity analysis

This appendix shows how the standard deviation and the correlation with output of employment, investment and wages (and thus consumption) are affected by the volatility and the serial correlation of the demand shocks. The variance of the shock has almost no effet on the simulated moments. Its persistence reduces slightly the volatility and the cross correlation of employment and investment; its effect on the volatility and the cross correlation of the real wage (and thus consumption) is a bit stronger but remains weak.
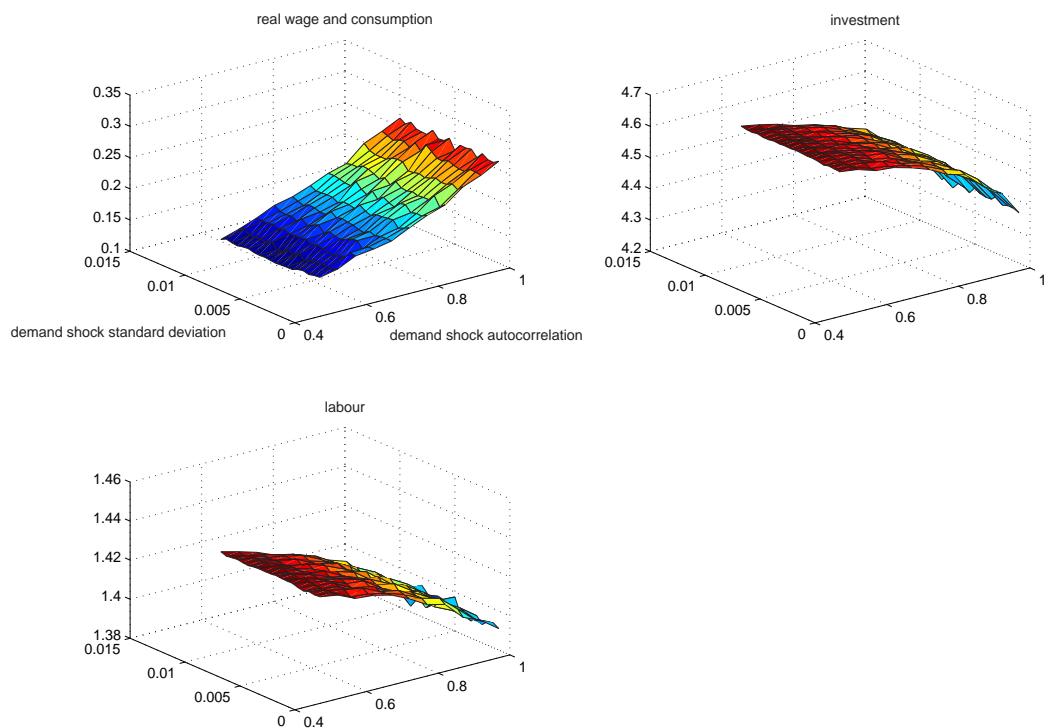

Figure 3: Volatility of investment, labour and the real wage as a function of the volatility and the persistence of the demand shocks
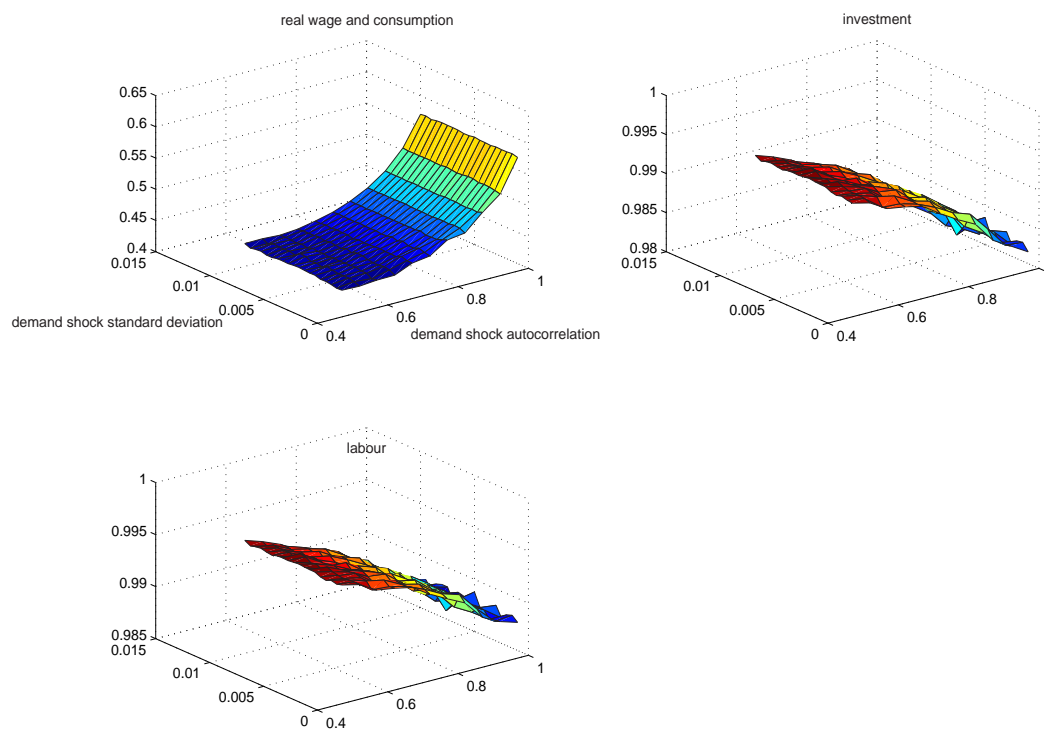
Figure 4: Output correlation of investment, labour and the real wage as a function of the volatility and the persistence of the demand shocks