Skill, Scale, and Value Creation in the Mutual Fund Industry

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ABSTRACT

We develop a flexible and bias-adjusted approach to jointly examine skill, scalability, and value added across individual funds. We find that skill and scalability (i) vary substantially across funds, and (ii) are strongly related as great investment ideas are difficult to scale up. The combination of skill and scalability produces a value added that (i) is positive for the majority of funds, and (ii) approaches its optimal level after an adjustment period possibly due to investors' learning. These results are consistent with theoretical models in which funds are skilled and able to extract economic rents from capital markets.

The academic literature on mutual funds has largely focused on performance, i.e., whether investors earn positive alphas when they buy mutual fund shares.[1] However, we know far less about value creation, i.e., whether funds extract value from capital markets through their investment decisions. Active funds create value when they trade based on superior information (stock picking, factor timing), or when they provide liquidity to absorb selling pressure (see Pedersen (2015, ch. 3) for a discussion).

The study of value creation, or value added, is pioneered by Berk and van Binsbergen (2015) (BvB hereafter). They define the fund value added as the product of its gross alpha and size. As such, the value added is similar to the economic rent of a firm defined as the markup price of its product times the quantity sold. BvB show that the gross alpha gives a distorted view of

[1]A non-exhaustive list of papers on performance includes Barras, Scaillet, and Wermers (2010), Carhart (1997), Elton et al. (1993), Harvey and Liu (2018), Jensen (1968), Kosowski et al. (2006), Roussanov, Ruan, and Wei (2020), and Wermers (2000).

value creation—measuring the value added across funds, they find that its variation is mostly driven by cross-sectional differences in fund size.

In this paper, we provide the first fund-level analysis of skill, scalability, and value added. Put simply, we propose an alternative decomposition of the value added—instead of focusing on size and gross alpha as in BvB, we focus on skill and scalability. Our analysis builds on the premise that the value created by each fund ultimately depends on (i) skill, i.e., the fund's ability to identify profitable investment ideas, and (ii) scalability, i.e., the fund's exposure to scale constraints when it grows in size. The extensive panel evidence documented by Zhu (2018) confirms the presence of such diseconomies of scale.

Our joint analysis of skill, scalability, and value added contributes to the literature in several ways. First, we quantify how many funds create value and assess whether they do so with more profitable or scalable ideas. Second, we examine whether funds create more value over time as investors learn about skill and scalability. Third, we measure how far the fund value added is from its optimal level determined by skill and scalability. Finally, we examine whether the industry delivers negative alphas to investors because it is populated by unskilled funds or by funds that scale their ideas too far.

To address these issues, we develop a new estimation approach to infer the entire cross-sectional distributions of skill, scalability, and value added. Our fund-level approach is key to incorporate the suspected vast heterogeneity in skill and scalability across funds and determine how many of them are able to create value. The estimation of each distribution is flexible and bias-adjusted. It is flexible because we use a nonparametric approach which does not require to specify the shape of the true distribution. This flexibility is essential because misspecification risk is large—a joint specification of skill, scalability, and value added is a daunting challenge for which theory offers little guidance. The estimation is also adjusted for the Error-in-Variable (EIV) bias that largely distorts the shape of the estimated distribution. This bias arises because of estimation noise, i.e., we can only use as inputs the estimated fund measures instead of the true (unobservable) ones.[2] Our EIV bias adjustment rests on a theoretical asymptotic analysis and allows us to conduct proper statistical inference in a large set of funds.

In our baseline specification, we follow Berk and Green (2004) and model the gross alpha of each fund as $\alpha_{i,t} = a_i - b_i q_{i,t-1}$, where $q_{i,t-1}$ is the lagged fund size (in real terms). Skill is measured with a_i—the gross alpha on the first dollar invested in the fund. Scalability is measured with b_i—the regression slope that captures the fund's sensitivity to diseconomies of scale. Finally, the value added is given by $va_i = E\left[\alpha_{i,t} q_{i,t-1}\right] = E\left[(a_i - b_i q_{i,t-1}) q_{i,t-1}\right]$. This parametrization provides (i) an explicit link between skill, scalability, and the value added, and (ii) a simple expression of its optimal value to make normative statements on value creation.[3] To compute these measures, we use the four-factor model of Cremers, Petajisto, and Zitzewitz (2012). This

[2]This bias is reminiscent of the well-known EIV bias in the two-pass regression in which we use the estimated betas to estimate risk premia (e.g., Shanken (1992)).

[3]To address concerns regarding the validity of our baseline specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$, we conduct an extensive analysis using (i) daily fund returns, (ii) additional variables that capture changes in the fund's economic conditions, and (iii) a new formal specification test (see Section V).

index-based model is constructed from the SP500 and Russell indices which are tradable and widely used as benchmarks by mutual funds.

Our analysis of US equity funds over the period 1975-2019 uncovers several new insights about skill and scalability. First, mutual fund skill is widespread and economically large—the skill coefficient is positive for 83.1% of the funds and equal to 3.0% per year on average. Second, funds are highly sensitive to diseconomies of scale—on average, a one standard deviation increase in size reduces the gross alpha by 1.3% per year. Third, the skill and scale coefficients vary substantially, both in the whole population and within fund groups. The cross-sectional volatility of a_i and b_i is typically much larger than the mean—a heterogeneity that contradicts the extensively-used panel regression that imposes a constant scale coefficient b across funds (e.g., Chen et al. (2004), Yan (2008), Zhu (2018)). Fourth, the skill and scale coefficients are strongly positively correlated. In other words, great investment ideas are difficult to scale up.

These results shape the cross-sectional distribution of the value added. In line with their investment skills, 60.0% of the funds create value over the sample period. This result helps to reconcile the seemingly puzzling evidence in BvB showing that va_i is positive on average, whereas the majority of funds destroy value. This surprising discrepancy disappears with the EIV bias adjustment. The unadjusted distribution is plagued by estimation noise which distorts the fund proportion estimators and largely inflates the tail probabilities. Our improved estimation has therefore implications for the debate on the size of the active industry (e.g., Cochrane (2013), Greenhood and Scharfstein (2013)). It shows that the proportion of funds that are unambiguously too large—those that destroy value—is not as high as initially thought.

Our results shed new light on the value creation process. With correlated skill and scale coefficients, the most valuable funds are not those with the best investment ideas. Instead, their investment strategies balance skill and scalability—their skill (scale) coefficients are slightly above (below) average. We observe similar trade-offs among funds with different levels of liquidity and turnover—two key determinants of the fund investment strategy (Pastor, Stambaugh, and Taylor (2020)). For instance, small cap funds buy more illiquid stocks which generate both higher mispricing and trading costs. With this particular combination of skill and scalability—high a_i and high b_i—, we find that small cap funds create more value than large cap funds. Finally, funds directly sold to investors are more skilled and generate higher value added than broker sold funds. This result is consistent with the view that direct sold funds are more incentivized to generate superior returns (Del Guercio and Reuter (2014)).

Next, we examine the dynamics of the value added as the fund gets older. This analysis is important to capture the potential impact of investors' learning—as discussed by Pastor and Stambaugh (2012), investors need time to learn about skill and scalability and optimize their fund allocation. To this end, we compare (i) the standard measure va_i computed over the entire fund's life with (ii) the last subperiod measure computed over the last decile of the fund's observations and denoted by $va_i(10)$. The difference between the two measures is economically large—on average, the gap between $va_i(10)$ and va_i equals \$3.5M per year (\$5.4M vs \$1.9M) In particular, the standard measure is driven down during the early years when fund size is particularly low. These results suggest that uncertainty about skill and scalability is an important source of short-term capital misallocation across funds.

We also find that the value added approaches optimality as the fund gets older. We examine the equilibrium predictions of the Berk and Green model in which funds are skilled and set fees to maximize profits, or, equivalently, the value added. In line with these predictions, we find that the last subperiod value added represents more than 50% of the optimal level $va_i^* = \frac{a_i^2}{4b_i}$. In contrast, Zhu (2018) finds that the ratio of actual to optimal value added is below 1%. This striking difference emphasizes two elements that are essential for uncovering the ability of the Berk and Green model to fit the data. First, we focus on the last subperiod measure ($va_i(10)$ instead of va_i) to control for the investors' learning process. Second, we account for the heterogeneity in skill and scalability across funds—a feature that cannot be captured with the extensively-used panel specification.

Finally, an important question for investors is whether they benefit, at last partially, from the value created by funds. Our new estimation approach combined with the index-based version of the four-factor model produces a more optimistic performance evaluation than previous studies. However, we still find that 62.9% of the funds exhibit negative net alphas. We find evidence consistent with the view that unskilled funds exploit disadvantaged investors among the worst performing funds. However, unskilled funds only represent 16.9% of the population and thus cannot fully explain why many funds exhibit both a positive value added and negative alpha. An explanation in the context of the Berk and Green model is that investors tend to overestimate fund skill. In this case, funds still maximize the value added but fees remain too high. An alternative explanation is that information frictions prevent investors from searching for cheaper funds (Roussanov, Ruan, and Wei (2020)). If these mechanisms are at play, the number of underperforming funds may decrease over time as investors sharpen their evaluation of skill and benefit from technological advances that reduce information frictions.

The remainder of the paper is as follows. Section I presents our framework for measuring skill, scalability, and value added. Section II describes our nonparametric approach. Section III presents the mutual fund dataset. Section IV contains the empirical analysis, and Section V concludes. The internet appendix provides additional information regarding the methodology, the construction of the database, and the empirical results.

I. Measuring Skill, Scalability, and Value Added

A. Skill and Scalability

We consider a population of n funds over T observations, where we denote each fund by the subscript i ($i = 1, ..., n$) and each observation by the subscript t ($t = 1, ..., T$). To define our measures of skill and scalability, we use the linear model proposed by Berk and Green (2004). For each fund, the total benchmark-adjusted revenue from active management is given by $TR_{i,t} = a_i q_{i,t-1}$, where $q_{i,t-1}$ denotes the lagged fund size measured in real terms. The total cost of trading is modeled as a convex function of fund size, i.e., $TC_{i,t} = b_i q_{i,t-1}^2$. Taking the difference between $TR_{i,t}$ and $TC_{i,t}$ and dividing by $q_{i,t-1}$, we obtain the following formulation:

$$\alpha_{i,t} = a_i - b_i q_{i,t-1}, \tag{1}$$

in which the gross alpha $\alpha_{i,t}$ varies linearly with the lagged fund size.

We measure skill using the coefficient a_i which captures the profitability of the fund's investment ideas. This coefficient is equal to the gross alpha of the first dollar (when $q_{i,t-1} = 0$). As such, it can be interpreted as a "paper" return that is unencumbered by the drag of real world implementation (Perold and Salomon (1991)). We measure scalability using the coefficient b_i which captures the fund's sensitivity to diseconomies of scale. This coefficient determines how the gross alpha changes when the fund deploys more capital on its investment ideas.

A key feature of our approach is that we allow both a_i and b_i to be fund-specific. To do so, we treat a_i and b_i not as fixed parameters, but as random realizations from their cross-sectional distributions $\phi(a)$ and $\phi(b)$. Our approach contrasts with previous studies which rely on a panel specification and impose the restriction that the scale coefficient is constant across funds, i.e., $b_i = b$ (e.g., Chen et al. (2004), Pastor, Stambaugh, and Taylor (2015), Yan (2008), Zhu (2018)). Whereas this pooling assumption (if correct) helps to reduce estimation errors, it is unclear from an economic perspective why diseconomies of scale should be identical across all funds. Consistent with this view, we find that the panel specification is strongly rejected in the data (as discussed in Section IV.A).

B. Value Added

We next turn to the measurement of the economic value created by the fund based on its skill and scale coefficients. To this end, we follow BvB and use the concept of value added. It is defined as the average product of the fund gross alpha and size: $va_i = E[\alpha_{i,t} q_{i,t-1}]$. Replacing $\alpha_{i,t}$ with $a_i - b_i q_{i,t-1}$, we obtain

$$va_i = a_i E[q_{i,t-1}] - b_i E[q_{i,t-1}^2] = a_i \operatorname*{plim}_{T\to\infty} \overline{q}_i - b_i \operatorname*{plim}_{T\to\infty} \overline{q}_{i,2}, \tag{2}$$

where $\overline{q}_i = \frac{1}{T}\sum_{t=1}^{T} q_{i,t-1}$ and $\overline{q}_{i,2} = \frac{1}{T}\sum_{t=1}^{T} q_{i,t-1}^2$ denote the time-series averages of the (real) fund size and its squared value, and plim denotes the limit in probability.

The value added provides an intuitive measure that is strongly rooted in economics. If the fund has bargaining power over investors, va_i is identical to the dollar profits of a monopolist, measured as the markup price of its product multiplied by the total quantity sold. As such, it departs from the gross alpha which does not control for the impact of size on value creation. Put differently, using the gross alpha is akin to measuring the monopolist's profits with the markup price, regardless of how much quantity is sold.

The standard measure in Equation (2) captures the average value added by the fund over its entire lifetime. As such, it may differ from the value created at different stages of the fund's lifecycle. For one, investors may need time to learn about skill and scalability and allocate the right amount of capital to each fund (Pastor and Stambaugh (2012)). To examine this issue, we split the return history of each fund into S subperiods and measure the value added in each subperiod s as

$$va_i(s) = a_i\overline{q}_i(s) - b_i\overline{q}_{i,2}(s), \tag{3}$$

where $va_i(s)$ is conditioned on the realized averages of the fund size and its squared value denoted by $\overline{q}_i(s)$ and $\overline{q}_{i,2}(s)$ $(s = 1, ..., S)$.[4] Using Equation (3), we can then measure the last subperiod value added $va_i(S) = a_i\overline{q}_i(S) - b_i\overline{q}_{i,2}(S)$ to examine whether the value added grows larger as the fund gets older.[5]

Our approach for measuring value added is fund-specific. As both va_i and $va_i(s)$ inherit the randomness of a_i and b_i, we also treat them as random realizations from the cross-sectional distributions $\phi(va)$ and $\phi(va(s))$. Importantly, our approach is based on the linear specification $\alpha_{i,t} = a_i - b_iq_{i,t-1}$ and thus departs from the approach of BvB which does not impose any functional form on the gross alpha. This extra parametrization is necessary to examine how the skill and scale coefficients drive the value added. It is also required to make normative statements about the value added—that is, we can follow Berk and Green (2004) and use Equation (1) to obtain a simple, closed-form expression of the optimal value added (as discussed in Section IV.C).

[4]Working conditionally on the realized values $\overline{q}_i(s)$ and $\overline{q}_{i,2}(s)$ for each subperiod is similar to the approach used on short panels of assets when estimating ex post risk premia with fixed T and large n (Shanken (1992), Raponi, Robotti, and Zaffaroni (2020)).

[5]Averaging over the last subperiod S instead of using the final value of fund size allows us to smooth out short-term fluctuations in size caused by nonfundamental liquidity or sentiment shocks (e.g., Ben-Rephael, Kandel, and Wohl (2012)).

C. Remarks about the Specification

Our baseline specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ in Equation (1) calls for some comments. First, the estimation of a_i and b_i does not require to model the determinants of skill and scalability across funds. For instance, skill can vary because some funds are run by extremely talented managers or benefit from a high speed of information dissemination within their family (Cicci, Jaspersen, and Kempf (2017)). Similarly, the scale coefficient can vary because some funds trade more efficiently or follow specific strategies.[6] In this case, we can simply interpret a_i and b_i as fund-specific functions of the characteristics of the family/manager and the fund strategy such as liquidity and turnover (e.g., Pastor, Stambaugh, and Taylor (2020)).[7]

Second, Equation (1) may omit variables that are useful in capturing the time-variation in the skill and scale coefficients. For instance, skill could depend on the fund's economic environment including the levels of industry competition and aggregate mispricing (e.g., Hoberg, Kumar, and Prabhala (2018), Pastor, Stambaugh, and Taylor (2015, 2018)). Furthermore, the scale coefficient may vary with fund size if the relation between the gross alpha and size is nonlinear. To formalize this intuition, suppose that we have

[6] For instance, Dimensional Fund Advisors (DFA) highlights its ability to minimize the costs of trading small-cap stocks by buying large share blocks from forced sellers (Cohen (2002)). Its scale coefficient should therefore reflect its unique trading approach and its specific strategy (small-cap stocks).

[7] It may still be informative to learn about the determinants of a_i and b_i. If we impose a common panel structure across funds, we can then examine how the different fund characteristics explain the cross-sectional variation in a_i and b_i (see Section IV.A).

$\alpha_{i,t} = a_{i,t} - b_{i,t}q_{i,t-1} = (a_i + a'_{i,z}z_{i,t-1}) - (b_i + b'_{i,z}z_{i,t-1})q_{i,t-1}$, where $z_{i,t-1}$ is a demeaned vector of variables that drive the dynamics of $a_{i,t}$ and $b_{i,t}$. In this case, using Equation (1) is problematic because it omits the vector of variables $p_{i,t-1} = (z'_{i,t-1}, z'_{i,t-1}q_{i,t-1})'$. As a result, the estimated values of a_i and b_i could be biased and lead to wrong conclusions regarding the prevalence of skilled funds in the population or the magnitude of diseconomies of scale.

To address this issue, we conduct an extensive analysis presented in Section IV. First, we use daily fund returns to estimate $a_{i,t}$ and $b_{i,t}$ over short-time windows. This procedure allows us to gauge the extent of time-variation in $a_{i,t}$ and $b_{i,t}$ without having to identify the additional variables $z_{i,t-1}$ (Lewellen and Nagel (2006)). Second, we consider several extensions of Equation (1) which explicitly account for the impact of industry competition and changing aggregate mispricing. Third, we develop a new formal specification test of the linear function $\alpha_{i,t} = a_i - b_i q_{i,t-1}$. This test borrows from the strategy of a Hausman test (Hausman (1978)) and is based on a comparison with the model-free alpha estimate of BvB. In short, this analysis reveals that the empirical results are not driven by the omission of important variables in Equation (1).

II. Methodology

A. Motivation for the Nonparametric Approach

We now describe our novel nonparametric approach for estimating the cross-sectional distribution ϕ of each measure $m_i \in \{a_i, b_i, va_i, va_i(s)\}$. Our nonparametric approach imposes minimal structure on the true density ϕ and thus departs from a standard parametric/Bayesian approach which requires a full specification of the shape of ϕ. The choice between a parametric and nonparametric approach involves the usual trade-off between efficiency and misspecification. If the structure imposed by the parametric approach is correct, the estimated distribution is more precise. However, it can be heavily biased if the imposed structure is wrong.

The analysis of skill, scalability, and value added favors a nonparametric approach because misspecification risk is large. Whereas theory predicts that performance clusters around zero, it offers no such guidance here. In principle, we can gain parametric flexibility by using normal mixture models (e.g., Harvey and Liu (2018)). However, determining the correct number of mixtures is difficult because (i) the parameters are estimated with significant noise (Yan and Cheng (2019)), (ii) the numerical optimization of the likelihood is non-standard (van der Vaart (1998, p. 74)), and (iii) the statistical inference is technically involved (Chen (2017)). Misspecification risk further increases because we jointly study four measures (a_i, b_i, va_i, $va_i(s)$). Therefore, a parametric/Bayesian approach involves the daunting task of correctly specifying a multivariate distribution whose marginals are potentially mixtures of distributions with different supports.

In addition to its robustness to misspecification, the nonparametric approach brings several benefits. First, its implementation is simple and fast. In contrast, parametric and Bayesian approaches require sophisticated and computer-intensive Gibbs sampling and Expectation Maximization (EM) methods (e.g., Harvey and Liu (2018), Jones and Shanken (2005)). Second, it provides a unified framework for estimating both the distribution ϕ and its various characteristics (moments, proportions, quantiles). Third, it comes with a full-fledged inferential theory. We derive the asymptotic properties of each estimated quantity (distribution and characteristics) as the number of funds n and the number of observations T grow large (simultaneous double asymptotics with $n, T \to \infty$). We can therefore conduct proper statistical inference guided by econometric theory.

B. Estimation Procedure

B.1. Estimation of the Fund Measure

Our nonparametric estimation of the density ϕ consists of three main steps. To begin, we need to estimate the measure m_i for each fund. Using Equation (1), we write the fund gross return (before fees) over the risk-free rate as

$$r_{i,t} = \alpha_{i,t} + \beta_i' f_t + \varepsilon_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}, \tag{4}$$

where f_t is a K_f-vector of benchmark excess returns, and $\varepsilon_{i,t}$ is the error term. We interpret Equation (4) as a random coefficient model (e.g., Hsiao (2003)) in which the skill and scale coefficients a_i and b_i are random realizations from a continuum of funds. Under this sampling scheme, we can invoke cross-sectional limits to infer the density of each measure m_i.[8]

Two remarks are in order here. First, we assume for the inferential theory that the (real) fund size is asymptotically stationary as the time index t

[8]Gagliardini, Ossola, and Scaillet (2016) use a similar sampling scheme for testing the arbitrage pricing theory (see also Gagliardini, Ossola, and Scaillet (2020) for a review of the literature).

grows large (see Potscher and Prucha (1989)). Under this assumption, $q_{i,t-1}$ can therefore perfectly trend up during the early years when the impact of investors' learning is strong.[9] Theoretically, long-run size stationarity is consistent with any model that features diseconomies of scale at the fund or industry level. For instance, Pastor and Stambaugh (2012) show that fund size converges to an equilibrium regardless of the investors' risk attitude and bargaining power over funds. Empirically, we also find little evidence that the (real) size keeps trending up. Over the second half of the fund's lifecycle, the relative size variation, $\frac{\overline{q}_i(s+1)-\overline{q}_i(s)}{\overline{q}_i(s)}$, has a median value of -3.4% (versus 43.1% for the first half).

Second, it is well known that the estimated coefficients in Equation (4) are subject to a small sample bias (Stambaugh (1999)). Whereas this bias vanishes asymptotically (for large T), it may impact funds with short return histories. The small-sample bias arises because fund size is endogenous in the time-series—its innovation $\varepsilon_{q_i,t}$ tends to be positively correlated with the return innovation $\varepsilon_{i,t}$ (i.e., a positive return comes with a higher fund size). To remove this bias, we follow Amihud and Hurvich (2004) and Avramov, Barras, and Kosowski (2013) and include a proxy for the size innovation $\varepsilon^c_{q_i,t}$

[9] We can illustrate the concept of asymptotic stationarity with an AR(1) model $x_t = \rho x_{t-1} + \varepsilon_t$ initialized at a given value $x_0 \neq 0$ (with $|\rho| < 1$). This process is not stationary for the early dates t. However, it is asymptotically stationary because x_t becomes independent of the initialization value x_0 as t grows large. The burn-in period for this process is the analog of the learning period in our setting.

(see the appendix (Section I.A)). We then rewrite Equation (4) as

$$r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \psi_i \varepsilon^c_{q_i,t} + \upsilon_{i,t}, \tag{5}$$

and estimate the coefficients for each fund separately. Importantly, our fund-by-fund analysis is not based on a panel specification, where $r_{i,t} = a_i - bq_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}$ (with a common b). Therefore, Equation (5) is immune to the incidental parameter bias that affects the estimated pooled coefficient $\hat{b}$ (Scott and Neyman (1948), Nickel (1981)). Whereas this bias also arises from the time-series endogeneity in fund size, it departs from the small sample bias along two key dimensions: (i) it requires a different adjustment based on recursive demeaning (e.g., Hjalmarsson (2010), Zhu (2018)), and (ii) it must always be controlled for, even asymptotically (for large T and n).[10]

From Equation (5), we compute the coefficients $\hat{\gamma}_i = \left(\hat{a}_i, \hat{b}_i, \hat{\beta}_i', \hat{\psi}_i\right)'$ as

$$\hat{\gamma}_i = \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_{t=1}^{T} I_{i,t} x_{i,t} r_{i,t}, \tag{6}$$

where $I_{i,t}$ is an indicator variable equal to one if $r_{i,t}$ is observable (zero otherwise), $T_i = \sum_{t=1}^{T} I_{i,t}$ is the number of observations, $x_{i,t} = (1, -q_{i,t-1}, f_t', \varepsilon^c_{q_i,t})'$ is the vector of explanatory variables, and $\hat{Q}_{x,i} = \frac{1}{T_i}\sum_{t=1}^{T} I_{i,t} x_{i,t} x_{i,t}'$ is the estimated matrix of the second moments of $x_{i,t}$. We can then infer each of

[10]The intuition for this result is that the time series information about the incidental (fund-specific) parameters a_i and β_i in a sample of size T stops accumulating when the cross-sectional dimension n becomes large (see Lancaster (2000) for a review).

the four measures as

$$\begin{aligned}
\text{Skill coefficient} &: \quad \hat{m}_i = \hat{a}_i, \\
\text{Scale coefficient} &: \quad \hat{m}_i = \hat{b}_i, \\
\text{Value added} &: \quad \hat{m}_i = \widehat{va}_i = \hat{a}_i \overline{q}_i - \hat{b}_i \overline{q}_{i,2}, \\
\text{Subperiod value added} &: \quad \hat{m}_i = \widehat{va}_i(s) = \hat{a}_i \overline{q}_i(s) - \hat{b}_i \overline{q}_{i,2}(s). \qquad (7)
\end{aligned}$$

Our econometric framework accounts for the unbalanced nature of the mutual fund sample. We follow Gagliardini, Ossola, and Scaillet (2016) and introduce a formal fund selection rule $\mathbf{1}_i^{\chi}$ equal to one if the following two conditions are met (zero otherwise):

$$\mathbf{1}_i^{\chi} = \mathbf{1}\left\{CN_i \leq \chi_{1,T}, T/T_i \leq \chi_{2,T}\right\}, \qquad (8)$$

where $CN_i = \sqrt{eig_{max}\left(\hat{Q}_{x,i}\right)/eig_{\min}\left(\hat{Q}_{x,i}\right)}$ is the condition number of the matrix $\hat{Q}_{x,i}$ defined as the ratio of the largest to smallest eigenvalues eig_{max} and eig_{min}, and $\chi_{1,T}$, $\chi_{2,T}$ denote the two threshold values. The total number of selected funds is therefore equal to $n_{\chi} = \sum_{i=1}^{n} \mathbf{1}_i^{\chi}$. The first condition $\{CN_i \leq \chi_{1,T}\}$ excludes funds for which the time-series regression is subject to multicollinearity problems (Belsley, Kuh, and Welsch (2004), Greene (2008)). The second condition $\{T/T_i \leq \chi_{2,T}\}$ excludes funds for which the sample size is too small. Both thresholds $\chi_{1,T}$ and $\chi_{2,T}$ increase with the sample size T—with more return observations, we estimate the fund coefficients with greater accuracy which allows for a less stringent selection rule.

B.2. Kernel Density Estimation

In the next step, we estimate the density function ϕ using a standard nonparametric approach based on kernel smoothing.[11] We compute the estimated density $\hat{\phi}$ at a given point m as

$$\hat{\phi}(m) = \frac{1}{n_\chi h} \sum_{i=1}^{n} \mathbf{1}_i^\chi K\left(\frac{\hat{m}_i - m}{h}\right), \tag{9}$$

where K is a symmetric kernel function and h is the vanishing bandwidth—similar to the length of the histogram bars, the smoothing parameter h determines how many observations around point m we use for estimation. Because the choice of K is not a crucial aspect of nonparametric density estimation, we favor simplicity and use the standard Gaussian kernel $K(u) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{u^2}{2})$ (see Silverman (1986)). For the bandwidth, we choose the optimal value h^* that minimizes the Asymptotic Mean Integrated Squared Error (AMISE) of $\hat{\phi}(m)$.[12] By minimising the AMISE, we explicitly control for the trade-off between the bias and the variance of $\hat{\phi}(m)$ over its entire support. Therefore, we avoid overfitting the data by choosing a bandwidth that is too small.

The following proposition examines the asymptotic properties of $\hat{\phi}(m)$ as the number of funds n and the number of observations T grow large. To derive these properties, we impose that n is larger than T to capture the large cross-sectional dimension of the mutual fund population.

[11] See, for instance, Ait-Sahalia (1996) and Ait-Sahalia and Lo (1998) for applications in finance.

[12] The AMISE is defined as the integrated sum of the leading terms of the asymptotic variance and squared bias of the estimated density $\hat{\phi}(m)$.

PROPOSITION 1: *As* $n, T \to \infty$ *such that* $\frac{T}{n} = o(1)$ *and* $h \to 0$ *such that* $nh \to \infty$ *and* $\sqrt{nh}(h^2 T + (1/T)^{\frac{3}{2}}) \to 0$, *we have*

$$\sqrt{nh}\left(\hat{\phi}(m) - \phi(m) - bs(m)\right) \Rightarrow N\left(0, K_1\phi(m)\right), \tag{10}$$

where $\Rightarrow$ *denotes convergence in distribution. The bias term* $bs(m)$ *is the sum of two components,*

$$bs_1(m) = \frac{1}{2}h^2 K_2 \phi^{(2)}(m), \tag{11}$$

$$bs_2(m) = \frac{1}{2T}\psi^{(2)}(m), \tag{12}$$

where $K_1 = \int K(u)^2 du$, $K_2 = \int u^2 K(u) du$ *(under a Gaussian kernel, we have* $K_1 = \frac{1}{2\sqrt{\pi}}$ *and* $K_2 = 1$*). The functions* $\phi^{(2)}(m)$ *and* $\psi^{(2)}(m)$ *are the second derivative of the density* $\phi(m)$ *and the second derivative of the function* $\psi(m) = \omega(m)\phi(m)$ *with* $\omega(m) = E[S_i | \, m_i = m]$*. The term* S_i *is the asymptotic variance of the estimated centered measure* $\sqrt{T}(\hat{m}_i - m_i)$ *equal to* $\underset{T\to\infty}{plim}(\frac{T}{T_i^2}\sum_{t,s=1}^{T} I_{i,t}I_{i,s}u_{i,t}u_{i,s})$*. For each measure, the term* $u_{i,t}$ *is given by*

$$\begin{aligned}
\textit{Skill coefficient} &: \; u_{i,t} = e_1' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t}, \\
\textit{Scale coefficient} &: \; u_{i,t} = e_2' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t}, \\
\textit{Value added} &: \; u_{i,t} = E[q_{i,t-1}]e_1' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t} + a_i(q_{i,t-1} - E[q_{i,t-1}]) \\
&\qquad - E[q_{i,t-1}^2]e_2' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t} - b_i(q_{i,t-1}^2 - E[q_{i,t-1}^2]), \\
\textit{Subperiod value added} &: \; u_{i,t} = \overline{q}_i(s)e_1' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t} - \overline{q}_{i,2}(s)e_2' Q_{x,i}^{-1} x_{i,t}\upsilon_{i,t},
\end{aligned} \tag{13}$$

where e_1 (e_2) *is a vector with one in the first (second) position and zeros* *elsewhere and* $Q_{x,i} = E[x_{i,t}x'_{i,t}]$. *Finally, as* $\frac{n^{\frac{2}{5}}}{T} \to \infty$, *the optimal bandwidth is given by*

$$h^* = \left(\frac{K_2}{K_1} \int \phi^{(2)}(m)\psi^{(2)}(m)dm \right)^{-\frac{1}{3}} \left(\frac{n}{T} \right)^{-\frac{1}{3}}. \tag{14}$$

Proof. See the appendix (Sections I.B and I.C). □

Proposition 1 reveals three important insights. First, $\hat{\phi}(m)$ is a biased estimator of $\phi(m)$, which implies that we can improve the estimation by adjusting for the two bias terms in Equations (11)-(12). The first component $bs_1(m)$ is the smoothing bias which is standard in nonparametric density estimation (e.g., Silverman (1986), Wand and Jones (1995)). The second component $bs_2(m)$, which is referred to as the Error-In-Variable (EIV) bias, is non-standard in nonparametric statistics. It arises because of estimation noise—that is, we can only estimate $\phi(m)$ using as inputs the estimated fund measures instead of the true ones ($\hat{m}_i$ instead of m_i).

Second, the EIV bias is the key driver of the total bias. The smoothing bias $bs_1(m)$ becomes negligible in a population of several thousand funds (as n grows large, we have h and $bs_1(m)$ go to zero). On the contrary, the EIV bias $bs_2(m)$ remains large because it depends on the number of observations T. In other words, $bs_2(m)$ arises from the estimation noise contained in $\hat{m}_i$ which does not vanish even in a large fund population.

Third, noisier estimated fund measures do not translate into a noisier estimation of the density $\phi(m)$. The estimation error in $\hat{m}_i$ only changes the magnitude of the EIV bias, but not the variance of $\hat{\phi}(m)$ (as shown in Equation (10)). As long as we correctly adjust for the EIV bias, we can therefore estimate $\phi(m)$ with the same asymptotic accuracy even if the sampling variation in $\hat{m}_i$ gets larger.

B.3. Adjustment for the Error-in-Variable Bias

Our final step is to adjust the standard density estimator $\hat{\phi}(m)$ for the EIV bias. To do so, we apply a Gaussian reference model to compute the EIV bias (Equation (12)), as well as the smoothing bias and optimal bandwidth (Equations (11) and (14)).[13] Under this model, the fund measure m_i and the log of the asymptotic variance $s_i = \log(S_i)$ follow a bivariate normal distribution where $m_i \sim N(\mu_m, \sigma_m^2)$, $s_i \sim N(\mu_s, \sigma_s^2)$, and $corr(m_i, s_i) = \rho$.

Using a Gaussian reference model is appealing for several reasons discussed in more detail in the appendix (Section III). First, the computations are straightforward because they are all available in closed form. Second, the bias terms are precisely estimated because of parsimony—they only depend on the five parameters $\theta = (\mu_m, \sigma_m, \mu_s, \sigma_s, \rho)'$. Third, the closed-form expressions shed light on the determinants of the bias. Finally, several compelling arguments suggest that the bias inferred from the reference model provides a good approximation of the true bias (i.e., $bs^r(m) \approx bs(m)$).

These benefits are not shared by a fully nonparametric estimation of the bias via the second-order derivatives $\phi^{(2)}$ and $\psi^{(2)}$. Estimating these derivative terms is notoriously difficult and generally leads to large estimation errors (e.g., Bhattacharya (1967), Wand and Jones (1995, ch. 2)). Similarly,

[13] A Gaussian reference model underlies the celebrated Silverman rule for the optimal choice of the bandwidth in standard non-parametric density estimation without the EIV problem. This rule gives $h^* = 1.06\sigma n^{-\frac{1}{5}}$, where σ is the standard deviation of the observations (Silverman (1986)).

the standard bootstrap usually underestimates the bias in curve estimation problems (Hall (1990), Hall and Kang (2001)). The design of resampling techniques suitable for our unbalanced setting with an EIV problem is a difficult and still open question.

The results obtained with the Gaussian reference model as the number of funds n and the number of observations T grow large are reported in the next proposition.

PROPOSITION 2: *As* $n, T \to \infty$ *such that* $\frac{T}{n} = o(1)$ *and* $h \to 0$ *such that* $nh \to \infty$ *and* $\sqrt{nh}(h^2T + (1/T)^{\frac{3}{2}}) \to 0$, *the two bias terms are equal to*

$$bs_1^r(m) = \left[\frac{1}{2}K_2h^2\frac{1}{\sigma_m^2}(\bar{m}_1^2 - 1)\right]\frac{1}{\sigma_m}\varphi(\bar{m}_1), \tag{15}$$

$$bs_2^r(m) = \left[\frac{1}{2T}\exp(\mu_s + \frac{1}{2}\sigma_s^2)\frac{1}{\sigma_m^2}(\bar{m}_2^2 - 1)\right]\frac{1}{\sigma_m}\varphi(\bar{m}_2), \tag{16}$$

where $\bar{m}_1 = \dfrac{m - \mu_m}{\sigma_m}$, $\bar{m}_2 = \dfrac{m - \mu_m - \rho\sigma_m\sigma_s}{\sigma_m}$, *and* $\varphi(x) = \dfrac{1}{\sqrt{2\pi}}\exp(-\dfrac{1}{2}x^2)$ *is the density of the standard normal distribution. The optimal bandwidth* h^* *is given by*

$$h^* = \left[\frac{K_2}{K_1 2\sqrt{\pi}}\frac{3}{4\sigma_m^5}\left(\frac{\rho^4\sigma_s^4}{12} - \rho^2\sigma_s^2 + 1\right)\exp\left(\mu_s + \frac{1}{2}\sigma_s^2(1 - \frac{\rho^2}{2})\right)\right]^{-\frac{1}{3}}\left(\frac{n}{T}\right)^{-\frac{1}{3}}. \tag{17}$$

Proof. See the appendix (Section I.D) □

The EIV bias adjustment in Equation (16) changes the shape of the standard estimated density $\hat{\phi}(m)$ in two ways. First, it removes probability mass from the tails of $\hat{\phi}(m)$. This adjustment is required because estimation noise produces large values of $\hat{m}_i$ which then inflate the tail probabilities (i.e., $\hat{\phi}(m)$ is too flat). Second, the bias adjustment can be asymmetric by removing more mass from the left tail of $\hat{\phi}(m)$. This asymmetry arises when the correlation ρ is positive—an empirical regularity for each measure $m_i \in \{a_i, b_i, va_i, va_i(s)\}$.[14] In this case, funds with a positive measure m_i also exhibit a higher estimation variance S_i. As a result, it is not unusual for these funds to exhibit a low estimated value $\hat{m}_i$ (i.e., the left tail of $\hat{\phi}(m)$ is too thick).

Using the results in Proposition 2, we can now compute the bias-adjusted density $\tilde{\phi}(m)$. We first estimate the parameter vector θ using the estimated quantities $\hat{m}_i$ and $\hat{s}_i$ $(i = 1, ..., n_\chi)$. To compute $\hat{s}_i = \log(\hat{S}_i)$, we use the standard variance estimator of Newey and West (1987):

$$\hat{S}_i = \frac{T}{T_i^2} \sum_{t=1}^{T} I_{i,t} \hat{u}_{i,t}^2 + 2 \sum_{l=1}^{L} \left(1 - \frac{l}{L+1}\right) \left[\frac{T}{T_i^2} \sum_{t=1}^{T-l} I_{i,t} I_{i,t+l} \hat{u}_{i,t} \hat{u}_{i,t+l}\right], \quad (18)$$

where $\hat{u}_{i,t}$ is computed from the expressions in Equation (13) for the chosen fund measure, and L is the number of lags to capture potential serial correlation. Then, we plug the elements of the estimated vector $\hat{\theta}$ into Equations (15)-(17) to compute the bias terms $\widehat{bs}_1^r(m)$, $\widehat{bs}_2^r(m)$, and the optimal bandwidth h^*. Finally, we remove the bias terms from the standard kernel density

[14] As noted by Jones and Shanken (2005), the sign of ρ is related to the concentration of the fund's holdings. For instance, if skilled funds tend to only hold a few stocks, the idiosyncratic variance increases and the correlation ρ between the true skill a_i and estimation variance S_i is positive.

$\hat{\phi}(m)$ to obtain

$$\tilde{\phi}(m) = \hat{\phi}(m) - \widehat{bs}_1^r(m) - \widehat{bs}_2^r(m). \quad (19)$$

With the bias-adjusted density $\tilde{\phi}(m)$ at hand, we can compute the characteristics of the distribution (moments, proportions, and quantiles) via numerical integration. For instance, the proportion of funds with a negative m_i is given by the cdf estimate $\tilde{\pi}^- = \int_{-\infty}^{0} \tilde{\phi}(u)du$. Among all the characteristics, the estimated average $\hat{M}$ is the only one that is immune to the EIV bias—given that the two bias functions $\widehat{bs}_1^r(m)$, $\widehat{bs}_2^r(m)$ scaled by m integrate to zero, we have $\tilde{M} = \int \tilde{\phi}(u)udu = \int \hat{\phi}(u)udu = \hat{M}$ (see the appendix (Section I.C)).[15]

III. Data Description

A. *Mutual Fund Data and Benchmark Model*

We conduct our analysis on the entire population of open-end actively managed US equity funds.[16] We collect monthly data on net returns and size, as well as annual data on fees, turnover, and investment objectives from

[15] As an alternative to the numerical approach, we can directly derive analytically the asymptotic properties of the characteristics. Similar to the density, we show in the appendix (Section II) that the different estimators are normally distributed and suffer from the EIV bias (except the mean).

[16] We exclude index funds and ETFs because our baseline model $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ is likely to be misspecified for them. First, the skill of passive funds, if any, are operational (e.g., securities lending programs) and thus very different from that of active funds (Crane and Crotty (2018)). Second, passive funds may actually benefit from economies of scale if a portion of their operational costs are fixed.

the CRSP database between January 1975 and December 2019. It allows us to construct the time-series of the gross return and size for the entire population and different fund groups (small/large cap, low/high turnover, broker/direct sold). To compute the real value of size, we follow BvB and compute its value in terms of January 1, 2000 dollars. The appendix (Section V) provides additional information on the construction of the mutual fund dataset.

To estimate Equation (5), we use the model of Cremers, Petajisto, and Zitzewitz (2012) which includes the vector $f_t = (r_{m,t}, r_{smb,t}, r_{hml,t}, r_{mom,t})'$, where $r_{m,t}$, $r_{smb,t}$, $r_{hml,t}$, and $r_{mom,t}$ capture the gross excess returns of the market, size, value, and momentum factors.[17] The distinguishing feature of this model is to proxy for the market factor using the SP500, and use index-based versions of the size and value factors obtained from the Russell indices. Contrary to the model of Carhart (1997), it therefore correctly assigns a zero alpha to the SP500 and Russell 2000—two indices that are widely used as benchmarks by mutual funds.

To obtain reliable estimates of each fund measure m_i, we apply the fund selection rule in Equation (8) and require that (i) the condition number of the matrix $\hat{Q}_{x,i}$ is below 15 (as in Gagliardini, Ossola, and Scaillet (2016)), and (ii) the minimum number of monthly observations is above 60. Whereas the original sample of funds includes all dead funds, this selection rule may introduce a survivorship bias if unskilled funds ($a_i < 0$) disappear early. An

[17]We use the gross factor returns (instead of the net returns) to exclude diversification services and only focus on the value created by the funds from their investment ideas (see BvB for a discussion of diversification services).

offsetting effect is that it mitigates the reverse survivorship bias that arises when skilled funds ($a_i > 0$) disappear after unexpectedly low returns (Linnainmaa (2013)). Our analysis in the appendix (Section VI.C) reveals that any biases introduced by the selection rule are minimal—the results remain largely unchanged when changing the minimum number of observations from 60 to 12.

B. Summary Statistics

Table I reports summary statistics for our final sample of 2,321 funds. We construct an equal-weighted portfolio of all existing funds at the start of each month. In Panel A, we report the first four moments of the portfolio gross excess returns. In the entire population, the portfolio achieves a mean–volatility tradeoff similar to that of the aggregate stock market (9.3% and 15.2% per year). It also exhibits a negative skewness (-0.7) and a positive kurtosis (5.3). The results are similar across fund groups, except for the small cap portfolio which produces higher mean and volatility.

In Panel B, we report the estimated portfolio betas for the four factors. We find that small cap funds are heavily exposed to the size factor (0.79), which is also the case for high turnover funds (0.46). Finally, Panel C reports additional characteristics which include the average number of funds in the portfolio and the time-series average of the median fund size, fees, and turnover. Consistent with intuition, small cap funds manage a smaller asset base—the median size is equal to $135M versus $256M for large cap funds. We also find that high turnover funds trade very aggressively. The median turnover reaches 112% per year versus 28% for low turnover funds.

Finally, funds sold by brokers charge higher expenses than funds directly sold to investors (similar to Del Guercio and Reuter (2014) and Sun (2020)).

Please insert Table I here

IV. Empirical Results

A. *Analysis of Skill and Scalability*

A.1. Magnitude of the Skill and Scale Coefficients

We begin our empirical analysis by examining the cross-sectional distributions of the skill and scale coefficients. For each fund, we estimate a_i and b_i and use our nonparametric approach to infer the densities $\phi(a)$ and $\phi(b)$. To describe their properties, we compute the bias-adjusted estimates of the moments (mean, variance, skewness, kurtosis), the proportions of funds with negative and positive coefficients, and the quantiles at 5% and 95%. We also compute the standard deviation of each estimator (see the appendix (Section I.E)). The results for $\phi(a)$ and $\phi(b)$ are shown in Table II.

Panel A reveals that the vast majority of funds in the population are skilled—on average, the skill coefficient equals 3.0% per year and is positive for 83.1% of the funds in the population. These results resonate with the numerical analysis of Berk and Green (2004) who find a similar proportion (80%) based on a calibration of their model. In short, the empirical evidence suggests that most funds are able to generate profitable ideas and identify undervalued stocks.

A small number of funds are unskilled and thus unable to generate profitable investment ideas ($a_i < 0$). This result is a priori surprising because funds always have the option to invest passively such that $a_i = 0$. One possible explanation proposed by Berk and van Binsbergen (2019) is that such funds, referred to as charlatans, trade actively in order to mislead investors about their true skill levels.

Turning to the analysis of Panel B, we find that 82.4% of the funds in the population experience diseconomies of scale. The magnitude of the scale coefficient is typically large—on average, a one standard deviation increase in size reduces the gross alpha by 1.3% per year. Equivalently, a $100M increase in size lowers the gross alpha by 0.2% per year.[18] Overall, the results are largely consistent with the literature that emphasizes the importance of diseconomies of scale in the mutual fund industry.

At the same time, a minority of funds seem to benefit from economies of scale ($b_i < 0$)—a view that is inconsistent with the Berk and Green model. The low economic magnitude of this effect suggests that it is due to estimation noise. Even among the funds with negative and highly significant $\hat{b}_i$ (at the 5% level), all of them are classified as false discoveries, i.e., their true coefficient b_i equals zero (see Barras, Scaillet, and Wermers (2010)). With a cluster of values for $\hat{b}_i$ around zero, it could be more difficult to perfectly control for estimation noise—whereas the EIV bias adjustment produces a

[18]To obtain this upper bound, we use the Jensen inequality: $E[\frac{b_i}{\sigma_{qi}}] < \frac{E[b_i]}{E[\sigma_{qi}]} = \frac{\bar{b}}{\bar{\sigma}_q}$, where $\bar{b}$ is the average size coefficient, and $\bar{\sigma}_q$ is the average volatility of fund size (i.e., time-series volatility averaged across funds). With $\bar{b} = 1.3\%$ and $\bar{\sigma}_q = \$655$M, we obtain $\frac{\bar{b}}{\bar{\sigma}_q}100 = 0.2\%$.

10%-drop in the proportion of negative-scale funds, it does not bring it down to zero.

Alternatively, the existence of these funds may signal that the linear model $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ is misspecified and omits relevant variables $p_{i,t-1}$. To address this issue, we develop a new test of the null hypothesis $H_{0,i}$ that the linear specification is correct. As discussed in the appendix (Section VI.A), this test does not require that we identify $p_{i,t-1}$—instead, we can reject $H_{0,i}$ based on any variable $w_{i,t-1}$ that is correlated with $p_{i,t-1}$. For the selection of $w_{i,t-1}$, we either use (i) the total industry size and aggregate turnover (Pastor, Stambaugh, and Taylor (2015, 2018)) to capture changes in the fund's economic environment, or (ii) $q_{i,t-1}^2$ and $q_{i,t-1}^3$ to capture nonlinearities in fund size. We find that the rejections of $H_{0,i}$ are limited in numbers—they represent less than 15% of all tests (at the 5% level), and one third of these rejections are false discoveries ($H_{0,i}$ is rejected whereas it is true). While not perfect, the linear specification seems to hold for the vast majority of funds.

Please insert Table II here

A.2. Variation in the Skill and Scale Coefficients

An important insight from Table II is the substantial variation in the skill and scale coefficients. Some funds have stellar investment skills—5% of them exhibit levels above 8.9% per year, which is three times larger than the average. Similarly, funds largely differ in their sensitivity to diseconomies of scale. This large cross-sectional variation is at odd with the panel regression approach that imposes a constant scale coefficient b across all funds. To formally test the validity of the panel approach, we consider the linear specification $\alpha_{i,t} = a_i - bq_{i,t-1}$ used in previous work (e.g., Chen, Hong, Huang, and Kubik (2004), Pastor et al. (2015), Yan (2008), Zhu (2018)). We also examine the log specification $\alpha_{i,t} = a_i - b\log(q_{i,t-1})$ based on the assumption that a relative (instead of absolute) size change has the same effect on all funds (e.g., Yan (2008), Zhu (2018)). Then, we test the null hypothesis of homogeneous coefficients $H_0 : b_1 = b_2 = ... = b$ (see the appendix (Section VI.B)). Consistent with the large heterogeneity observed in Panel B, we reject H_0 with probability one for each specification (size and log size).[19]

The observed variation in the skill and scale coefficients is potentially driven by the specific strategies followed by the funds. To examine this issue, we repeat the analysis among funds with different levels of liquidity (small/large cap) and turnover (low/high turnover)—two key determinants of the fund investment strategy (Pastor, Stambaugh, and Taylor (2020)). Table II confirms that the skill and scale coefficients vary considerably across all four investment categories. The average values of a_i and b_i vary between 1.7% and 4.6% per year, and between 0.9% and 1.8% per year. We also find that direct sold funds generate more profitable ideas than broker sold funds—the average skill coefficient equals 3.3% per year and is positive for

[19]The large variation in scalability also helps to understand the statistically weak and conflicting evidence on the effect of size using panel information (e.g., Chen, Hong, Huang, and Kubik (2004), Pastor, Stambaugh, and Taylor (2015), Zhu (2018)). When b_i varies across funds, the standard deviation of the estimated pooled $\hat{b}$ is inflated and its t-statistic decreases (Pesaran and Yagamata (2008)). Therefore, $\hat{b}$ may not be statistically significant even if most funds have a positive b_i (as shown in Panel B).

88.4% of the funds (versus 2.9% and 82.7% for broker sold funds). This result resonates with that of Del Guercio and Reuter (2014) who suggest that managers of direct sold funds face a more elastic investors' demand curve and have therefore more incentives to generate superior returns.

However, Table II still reveals a substantial cross-fund variation within each fund group. In other words, forming groups is not sufficient to absorb the large heterogeneity in a_i and b_i. To quantify this effect, we run a panel regression of $\hat{a}_i$ and $\hat{b}_i$ on 18 dummies that capture the fund's profile (small/medium/large cap, low/medium/high turnover, and broker/direct sold). Whereas these variables contribute the variation in skill and scalability across funds, their explanatory power is modest (the R^2 is equal to 8.7% and 7.3%).

A.3. Correlated Skill and Scale Coefficients

Another insight from our cross-sectional analysis is that the skill and scale coefficients are strongly correlated. In the entire population, the pairwise correlation between $\hat{a}_i$ and $\hat{b}_i$ is equal to 0.78. Put differently, great investment ideas are difficult to scale up.[20]

Part of this correlation is explained by the fund's investment style—that

[20]This correlation is cross-sectional and is therefore not driven by the fund-level correlation between $\hat{a}_i$ and $\hat{b}_i$. To verify this point, we run a simple simulation where we impose that the cross-sectional correlation between a_i and b_i is equal to zero (see the appendix (Section IV.C) for details). Consistent with theory, we find that the correlation between $\hat{a}_i$ and $\hat{b}_i$ is positive at the fund level, but the correlation between $\hat{a}_i$ and $\hat{b}_i$ across funds equals zero.

is, a_i and b_i are correlated because they both depend on the characteristics of the fund strategy such as liquidity and turnover. As illustrated in Figure 1 (Panels A and B), small cap funds have both higher skill and scale coefficients than large cap funds. These results are consistent with the difference in liquidity between the two groups. Illiquidity tends to increase the mispricing of small cap stocks (higher a_i)—as noted by Hong, Lim, and Stein (2018), these stocks are largely untouched by mutual funds. At the same time, illiquidity increases the cost of trading small cap stocks (higher b_i).

We document a similar pattern for high versus low turnover funds (Panels C and D). By rebalancing their portfolio more often, high turnover funds are able to exploit more investment opportunities (higher a_i). However, they also incur higher trading costs (higher b_i), possibly as a result of excessive trading (e.g., Dow and Gorton (1997)). Overall, these results imply that the ability of funds to create value depends on the trade-off between skill and scalability—a point we examine in more detail below.

Please insert Figure 1 here

B. Analysis of the Value Added

B.1. Magnitude of the Value Added

We now turn to the cross-sectional distribution of the value added. In our analysis below, we only focus on the value created by the funds from exploiting their investment skills (i.e., we exclude diversification services (see BvB for a discussion)). For each fund, we estimate va_i as a function of a_i

and b_i and use our approach to infer the density $\phi(va)$. We then report the bias-adjusted distribution characteristics in Table III.

The majority of funds are able to extract value from capital markets through their investment decisions. We find that 60% of them produce a positive value added which, on average, equals \$1.9M per year. In contrast, BvB and Zhu (2018) find that the majority of funds destroy value. This difference is important for the debate on the size of the finance industry (e.g., Cochrane (2013)). Funds that destroy value are unambiguously too large. If their proportion is large, it raises the bar for any theory that tries to rationalize the actual size of the fund industry.

Our empirical analysis highlights the importance of the EIV bias adjustment. Figure 2 shows that the bias-adjusted distribution $\tilde{\phi}(va)$ departs markedly from the unadjusted distribution $\hat{\phi}(va)$. In other words, the noise contained in the estimated values $\widehat{va}_i$ materially distorts the shape of $\hat{\phi}(va)$. For instance, it implies that only 48.4% of the funds create value (similar to the values reported in BvB and Zhu (2018)). It also produces a gap between the quantiles at 5% and 95% that is 1.4 times larger than the one reported in Table III (\$27.1M versus \$38.1M).[21]

Among the minority of funds that destroy value ($va_i < 0$), we can distinguish between (i) unskilled funds ($a_i < 0$) and (ii) skilled funds ($a_i > 0$) that grow too large to maintain revenues below costs. Combining the results in Tables II and III, we find that the relative importance of unskilled funds only

[21]The estimated average can be compared with previous studies because it is immune to the EIV bias. BvB find an annual average of \$2.0M net of diversification services, which is very close to ours (\$1.9M).

equals 42.3% (16.9/40.0%). It implies that the majority of value-destroying funds could ultimately create value if they scale down their size.

Please insert Table III and Figure 2 here

B.2. Skill, Scalability, and Value Added

Our framework explicitly links the value added to the skill and scale coefficients. We can therefore examine the importance of each coefficient in the value creation process. Because a_i and b_i are strongly correlated, the most valuable funds are not necessarily those with the best ideas. As these funds typically face tighter scale constraints, they could be dominated by funds that are able to scale up less profitable ideas.

To examine this issue, we sort $\hat{a}_i$ and $\hat{b}_i$ for each fund into deciles to create a scoring system from 1 to 10 (1=lowest value, 10=highest value). We then identify the most valuable funds as those with highly positive and significant $\widehat{va}_i$ using a 5%-significance level (247 funds). We find that the median skill and scalability scores are equal to 7 and 4. In addition, only 14.6% (18.2%) of the funds achieve the best skill (scalability) score of 10 (1). Therefore, the most valuable funds are those able to strike a balance between skill and scalability.

Our previous analysis reveals that different types of funds exhibit specific combinations of skill and scalability. To determine which combination produces the highest value, we re-estimate $\phi(va)$ within each fund group. Table III shows that the small cap group creates more value than the large cap group—we observe a positive difference for both the average ($4.0M versus -

\$0.6M) and the proportion of value-creating funds (66.6% versus 47.0%). We obtain qualitatively similar results when comparing low and high turnover funds. Interestingly, both small cap and low turnover funds create more value, but rely on a very different skill-scalability combination: (i) high investment skills for small cap funds, (ii) high scalability for low turnover funds. Finally, direct sold funds exhibit a higher value added than broker sold funds as they take advantage of a more attractive skill-scalability combination (i.e., higher a_i and similar b_i).

Overall, the empirical evidence shows that active funds are skilled and able to create value through their investment activities. However, the effect of active management on welfare is a priori ambiguous. On the one hand, the expertise of the active funds enables them to increase their returns at the expense of their trading counterparts and, possibly, their clients. This point reflects the view that finance is engaged in rent-seeking, socially wasteful activities (see, for instance, Greenhood and Scharfstein (2013) and Tobin (1984) for a discussion). On the other hand, active funds perform the socially valuable function of making prices more informative, thus allowing for an improvement of the allocation decisions made by capital providers, managers, employees, or regulators (see Bond, Edman, and Goldstein (2012) for a literature review). Measuring the social value of active management therefore requires to determine the relative importance of these two effects (e.g., Kurlat (2019)).

B.3. Last Subperiod Value Added

The standard measure va_i considered so far captures the value created by the fund over its entire life. To the extent that size varies over time, it may therefore not provide a precise measure of the value created by the fund when it gets older.

To address this issue, we begin by examining the dynamics of size across funds. We split the total observations of each fund in 10 subperiods ($S = 10$). For each subperiod s ($s = 1, ..., 10$), we then compute the difference $\Delta q_{i,s} = \overline{q}_i(s) - \overline{q}_i$, where $\overline{q}_i(s)$ and $\overline{q}_i$ denote the averages over subperiod s and the full sample. Figure 3 plots the median value of $\Delta q_{i,s}$ for each subperiod. We find that the size is generally substantially below its average when the fund is young. In subperiod 1, the median size gap equals -\$134M which represents -78% of the average fund size. Then, the size reaches its maximum value in subperiod 7 before falling back close to $\overline{q}_i$—in the last subperiod, the median size gap is a mere -\$13M (-13% in relative terms).

Please insert Figure 3 here

Motivated by these results, we turn to the analysis of the last subperiod value added denoted by $va_i(10)$. For each fund, we estimate $va_i(10)$ as a function of a_i and b_i and use our approach to infer the density $\phi(va(10))$. We then report the bias-adjusted distribution characteristics in Table IV.

The difference between the two measures is economically large. On average, the last subperiod value added equals \$5.4M per year—a gap of \$3.5M vis-a-vis the standard measure va_i. In addition, the proportion of funds with a positive value added is higher (70.5% versus 60.0%). This increase arises because some of the large funds see a reduction of their size as they get older. As a result, their value added is negative on average ($va_i < 0$), but positive during the last subperiod ($va_i(10) > 0$). Finally, we document the same patterns across all fund groups—for one, the last subperiod value added among large cap funds is \$4.3M higher than the standard measure.

An intuitive explanation for the difference between the two measures is the presence of learning effects. Investors need time to learn about skill and scalability and allocate the right amount of capital to each fund (e.g., Berk and Green (2004), Pastor and Stambaugh (2012)). To formalize this intuition, suppose that the size over the last subperiod is constant and equal to its average $E[q_{i,t-1}]$. In this case, we have $va_i(10) - va_i = a_i E[q_{i,t-1}] - b_i E[q_{i,t-1}]^2 - a_i E[q_{i,t-1}] - b_i E[q_{i,t-1}^2] = b_i V[q_{i,t-1}]$, where the variance term $V[q_{i,t-1}]$ captures changes in fund size as investors update their priors about a_i and b_i. Combined with the strong diseconomies of scale in Table II, these size fluctuations create a large wedge between $va_i(10)$ and va_i. Overall, the empirical evidence points to uncertainty about skill and scalability as an important source of short-term capital misallocation.

Please insert Table IV here

C. Equilibrium Considerations

C.1. Actual versus Optimal Value Added

Our analysis so far shows that most funds create value—especially once we focus on the last part of their return history. However, it does not imply that the size of the fund industry is consistent with a rational model of fund capital allocation. To address this issue, we examine the equilibrium predictions of the Berk and Green model.

In this model, we have (i) a set of skilled funds in scarce supply, and (ii) a large number of rational investors that compete for performance. Because funds are in a strong bargaining position, they can maximize profits π_i under the constraint that investors break even and pay fees $f_{e,i}$ equal to the gross alpha α_i. As a result, profit maximization corresponds to value added maximization, i.e., $\pi_i = f_{e,i} q_i = \alpha_i q_i = va_i$.

Using the linear specification $\alpha_i = a_i - b_i q_i$ (Equation (4)) and taking the first order condition $\frac{\partial va_i}{\partial q_i} = 0$, we obtain a simple expression for the optimal value added:

$$va_i^* = a_i q_i^* - b_i \left(q_i^*\right)^2 = \frac{a_i^2}{4b_i}, \tag{20}$$

where $q_i^* = \frac{a_i}{2b_i}$ is the optimal active size. Using Equation (20), we can compute $\widehat{va}_i^*$ as $\frac{\hat{a}_i^2}{4\hat{b}_i}$ and compare it with the actual value added observed in the data.

Our comparison analysis requires that va_i^* is positive. To meet this condition, we only select funds with sufficiently high estimated values $\widehat{va}_i^*$ or, more formally, funds for which we reject the null hypothesis

$$H_{i,0} : va_i^* = 0. \tag{21}$$

We begin by computing the fund t-statistic $\hat{t}_i$ as $\frac{\widehat{va}_i^*}{\hat{\sigma}_{va_i^*}}$, where the standard deviation $\hat{\sigma}_{va_i^*}$ equals $\sqrt{\frac{1}{T}\hat{S}_i}$, and the asymptotic variance $\hat{S}_i$ is obtained from Equation (18) with $u_{i,t} = \frac{2a_i}{4b_i}e_1' Q_{x,i}^{-1} x_{i,t} v_{i,t} - \frac{a_i^2}{4b_i^2} e_2' Q_{x,i}^{-1} x_{i,t} v_{i,t}$. Then, we select all funds for which $\hat{t}_i$ is above the threshold t_γ. This threshold is defined such that the proportion of false discoveries (funds with significant $\widehat{va}_i^*$ whereas $va_i^* = 0$) among the selected funds is equal to γ.[22] For the results presented in Table V, we set γ equal to 10%, 20%, and 30% which guarantees that the vast majority of selected funds satisfy the condition $va_i^* > 0$.

Panel A shows that the optimal value added $\widehat{va}_i^*$ among the selected funds is economically large. On average, it is equal to \$18.6M per year when the false discovery target γ equals 30%. Consistent with intuition, this number increases to \$26.9M when imposing a more stringent fund selection ($\gamma = 10\%$). In contrast, the actual value added obtained with the standard measure va_i is substantially lower, i.e., its average only represents between 9.7% and 25.1% of the optimal value added. Taken at face value, these results imply that funds largely fail to optimally exploit their investment abilities. However, this failure could be particularly severe during the investors' learning process—our previous analysis suggests that uncertainty about skill and scalability can generate substantial capital misallocation.

[22]See Barras, Scaillet, and Wermers (2010, Section III.C) for a description of the False Discovery Rate (FDR) approach applied to the problem of fund selection.

The analysis of the last subperiod value added $va_i(10)$ confirms this view. We find that funds extract on average between 47.0% and 54.8% of the optimal value added. This result is not merely driven by a few funds with extreme skill and scale coefficients. Panels B and C show that the ratio of actual to optimal value added becomes even stronger after trimming the top (bottom) 10% and 20% of funds with the highest (lowest) values of $\hat{a}_i$ ($\hat{b}_i$). In addition, the strong correlation of 0.81 between the estimated values $\widehat{va}_i(10)$ and $\widehat{va}_i^*$ confirms that funds with higher potential for value creation do create more value. In short, we find that the value added approaches its optimal level as funds get older. This result is therefore consistent with the Berk and Green model in which funds are skilled and able to extract economic rents from capital markets.

Please insert Table V here

To shed further light into the temporal adjustment towards optimality, we examine the gap between the actual fund size and the optimal active size measured as $\hat{q}_i^* = \frac{\hat{a}_i}{2\hat{b}_i}$. Across the three sets of selected funds ($\gamma = 10\%$, 20%, and 30%), we measure the following difference in each subperiod s $(s = 1, ..., 10)$: $\Delta q_{i,s}^* = \overline{q}_i(s) - \hat{q}_i^*$. Figure 4 shows that the median value of $\Delta q_{i,s}^*$ is highly negative in subperiod 1, then increases substantially before narrowing down at a level 23% higher than the optimal active size. In theory, a positive size gap $\Delta q_{i,s}^*$ does not necessarily reveal a failure to optimize the value added—as shown by Berk and Green (2004), funds can still do so as long as they passively invest the excess capital $\overline{q}_i(s) - \hat{q}_i^*$.[23] To interpret the remaining negative difference between $\widehat{va}_i(10)$ and $\widehat{va}_i^*$ in Table V, we must therefore understand why funds do not fully follow this passive strategy. One possible reason is that funds are unsure of their own skill and scale coefficients and must learn about them alongside with investors.

Overall, we find that two elements are crucial to uncover the ability of the Berk and Green model to fit the data. First, we need to focus on the last subperiod value added $va_i(10)$ (instead of va_i) to control for the short-term capital misallocation across funds (Table V). Second, it is important to allow for heterogeneity in skill and scalability—a feature that cannot be captured with a panel approach where scalability is constant across funds ($b_i = b$). To illustrate, Roussanov, Ruan, and Wei (2020) use a panel specification to examine the relation between skill and optimal size, and find that its slope is too steep compared to the data. However, we find that funds with high skill generally face tighter scale constraints, which lowers the value of their optimal size. Therefore, accounting for the positive correlation between a_i and b_i flattens the slope of the skill-optimal size relation and improves the fit of the Berk and Green model.

[23] Habib and Johnsen (2016) argue that funds have a preference for a positive gap $\Delta q_{i,s}^*$ because it allows them to mitigate several institutional constraints. Specifically, the Investment Company Act imposes diversification rules that may prevent funds from exhausting their investment opportunities if they are too small. Holding a portion of the portfolio passively managed may also help funds to hide their informed trades and obtain better prices.

Please insert Figure 4 here

C.2. From Value Added to Performance

An important question for investors is whether they benefit, at least partially, from the value created by mutual funds. To address this issue, we measure the net alpha received by investors as $\alpha_i^n = E[\alpha_{i,t}] - E[f_{e,i,t}] = a_i - b_i E[q_{i,t-1}] - E[f_{e,i,t}]$, where $f_{e,i,t}$ is the monthly fund fees. To infer its cross-sectional density $\phi(\alpha^n)$, we apply our nonparametric approach using the following expressions for $\hat{m}_i$ and $u_{i,t}$:

$$\begin{aligned} \hat{m}_i &= \hat{\alpha}_i^n = \hat{a}_i - \hat{b}_i \bar{q}_i - \bar{f}_{e,i}, \\ u_{i,t} &= e_1' Q_{x,i}^{-1} x_{i,t} \varepsilon_{i,t} - E[q_{i,t-1}] e_2' Q_{x,i}^{-1} x_{i,t} \varepsilon_{i,t} \\ &\quad - b_i (q_{i,t-1} - E[q_{i,t-1}]) - (f_{e,i,t} - E[f_{e,i,t}]), \end{aligned} \tag{22}$$

where $\bar{f}_{e,i} = \frac{1}{T_i} \sum_{t=1}^{T} I_{i,t} f_{e,i,t}$ is the sample average of the fund fees.

Of course, we are not the first to estimate the entire net alpha distribution—recent studies use standard parametric approaches to infer $\phi(\alpha^n)$ (e.g., Chen, Cliff, and Zhao (2017), Harvey and Liu (2018)). However, our nonparametric approach potentially brings several advantages discussed in Section II.A. In particular, it is robust to misspecification, simple to apply, and based on a full-fledged asymptotic theory.

Consistent with the literature on performance, Table VI provides limited evidence that investors benefit from the value created by mutual funds.[24] Yet,

[24] Another way to illustrate this point is to compute the total dollar amount of excessive

the performance evaluation is more positive than in most previous studies. In particular, our approach produces an average alpha close to zero (-0.4% per year) and a proportion of positive alpha funds equal to 37.1%. Part of this performance improvement is due to the index-based model of Cremers, Petajisto, and Zitzewitz (2012). For one, the proportion of positive-alpha funds drops to 26.5% with the standard model of Carhart (1997)).

A common explanation for poor performance is that unskilled funds manage to sell their shares to disadvantaged investors. These investors are either ignorant of underperformance (e.g., Gruber (1996), or willing to pay extra fees for financial advice (Del Guercio and Reuter (2014))—a view that is reflected in the strong underperformance of broker versus direct sold funds in Table VI. Consistent with this explanation, we find that the majority of the worst performing funds (those with net alphas below the 25% quantile of $\phi(\alpha^n)$) are unskilled and charge high fees. However, this explanation cannot fully account for the observed underperformance because unskilled funds only represent 16.9% of the population. Put differently, it cannot fully reconcile our two findings that the proportions of funds with positive value added ($va_i > 0$) and negative alphas ($\alpha_i^n < 0$) are both large.

Please insert Table VI here

We can reconcile both findings in the context of the Berk and Green model if we allow investors to have more optimistic views about skill than the funds

fees (measured by α_i^n) paid by investors. Similar to Cooper, Halling, and Yang (2020), we find that the total amount of excessive fees across years and funds is equal to $152 billion.

themselves. We illustrate this point in Figure 5 using a simple example where (i) the fund knows its skill level a_i and sets fees such that it operates at the optimal active size q_i^* ($f_{e,i} = \alpha_i(a_i, q_i^*)$), and (ii) investors believe that skill is higher at a_i^1 and are willing to invest q_i^1 ($q_i^1 > q_i^*$) based on their perceived break-even point $f_{e,i} = \alpha_i(a_i^1, q_i^1)$. Whereas the fund invests the difference $q_i^1 - q_i^*$ passively to keep the value added at its optimal value (i.e., $va(q_i^1) = \frac{a_i q_i^* - b_i \left(q_i^*\right)^2}{q_i^1} q_i^1 = va_i^*$), the alpha is negative (i.e., $f_{e,i} > \alpha_i(a_i, q_i^1)$).

Please insert Figure 5 here

An alternative explanation is that investors face information frictions (search costs) that prevent them from evaluating the entire fund population (Roussanov, Ruan, and Wei (2020)). In this setting, individual funds—including the skilled ones with positive value added—may find it optimal to incur marketing expenses to attract investors with high search costs. These investors are then charged high fees and receive negative alphas as they do not switch to cheaper funds.

If these mechanisms are at play, we should see a progressive reduction in the high proportion of underperforming funds. The net alphas of existing funds should increase as investors sharpen their views on skill and reallocate their capital. In addition, the recent advances in technology should reduce the information frictions faced by investors and give them access to a larger sample of funds to choose from.

D. Additional Results

D.1. Alternative Asset Pricing Models

Our empirical analysis depends on the choice of asset pricing model. To address this issue, we benchmark the fund returns using the four-factor model of Carhart (1997) and the five-factor model of Fama and French (2015) (see the appendix (Section VI.D)). Whereas the skill and scalability distributions remain largely unchanged, we observe two noticeable differences. First, the average skill coefficient among small cap funds drops from 4.6% to 3.3% per year under the Carhart model, consistent with the analysis of Cremers, Petajisto, and Zitzewitz (2012). Second, the proportion of skilled funds decreases from 83.1% to 74.0% with the Fama-French model. This reduction arises because some funds tilt their portfolios towards profitability- and investment-based strategies.

D.2. Analysis based on Daily Fund Returns

Our baseline specification assumes that the regression coefficients remain constant over time. To examine this issue, we repeat our analysis using daily fund returns. This procedure allows us to capture potential changes in the coefficients without explicitly modeling their dynamics (Lewellen and Nagel (2006)). We proceed in two steps. Each year, we first run a regression of the fund return on the factors to extract the daily gross alpha after controlling for short-term variations in factor loadings. Second, we run a regression of the daily gross alpha on lagged size to infer the time-varying skill and scale coefficients. Given the potential persistence over a small window of only one year, we run this regression over each non-overlapping five-year window τ, i.e., we have: $\alpha_{i,t} = a_{i,\tau} - b_{i,\tau} q_{i,t-1}$ (see the appendix (Section VI.E) for details).

In short, we do not observe a large time-variation in the skill coefficient. Testing the null hypothesis of constant skill $H_{0,i,\tau} : \Delta a_{i,\tau} = a_{i,1} - a_{i,\tau} = 0$ for each fund and each window, we only find 11.4% of rejections (at the 5% level) among which 38.7% are false discoveries (funds with significant $\Delta \hat{a}_{i,\tau}$ whereas $\Delta a_{i,\tau} = 0$). Repeating this analysis for the scale coefficient, we find similar results—there are only 9.0% of rejections among which 48.5% are false discoveries.

D.3. Impact of Changes in Economic Conditions

Finally, we extend our baseline specification to capture the impact of changes in economic conditions. First, we control for changes in industry competition using as proxy the ratio of industry size to total market capitalization (as in Pastor, Stambaugh, and Taylor (2015)). Second, we account for potential changes in aggregate mispricing using aggregate fund turnover (as in Pastor, Stambaugh, and Taylor (2018)).

The results reported in the appendix (Section VI.F) show that the skill and scalability distributions remain largely unchanged after including these additional variables. When the industry competition proxy is used alone in the regression, the majority of funds are negatively impacted by an increase in competition. However, its explanatory power substantially weakens when we add fund size and allow for fund-specific scale coefficients (instead of using a panel approach). Consistent with Pastor, Stambaugh, and Taylor (2018), we also find that the majority of funds produce higher returns in times of higher mispricing in capital markets.

V. Conclusion

In this paper, we apply a new approach to study skill, scalability, and value added in the mutual fund industry. For each of these measures, we provide an estimation of the entire distribution across funds. Our approach is nonparametric and thus avoids the challenge of correctly specifying each distribution. In addition to its flexibility, our approach is bias-adjusted, simple to implement, and supported by econometric theory.

Our empirical analysis brings several insights. Most funds are skilled and thus able to extract value from capital markets. Second, the value added distribution is shaped by the strong heterogeneity in the skill and scale coefficients, as well as their strong positive correlation. Third, the value added approaches optimality once we allow for an adjustment period possibly due to investors' learning. This result contributes to the debate on the size of the finance industry (e.g., Cochrane (2013), Greenhood and Scharfstein (2013)). It suggests that a rational model in which skilled funds extract value from capital markets does a good job at explaining the size of active management.

Whereas our paper focuses on mutual funds, our nonparametric approach has potentially wide applications in finance and economics. It provides a new tool for measuring heterogeneity in structural models (Bonhomme and Shaikh (2017)). We can use it to estimate the cross-sectional distribution of any coefficient of interest in a random coefficient model. It is, for instance, the case in asset pricing for capturing the heterogeneity across stocks (risk exposure, commonality in liquidity), in corporate finance for capturing the heterogeneity across firms (investment and financing decisions), and, more recently, in household finance for capturing the heterogeneity in time preference and risk aversion across households (see Calvet et al. (2019)).

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Table I

Summary Statistics of the Mutual Fund Dataset

This table provides summary statistics for all funds in the population, as well as small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or directly sold to investors). Panel A reports the mean (annualized), standard deviation (annualized), skewness, and kurtosis of the gross excess return of an equal-weighted portfolio of all existing funds at the start of each month. Panel B reports the estimated portfolio betas on the market, size, value, and momentum factors using the model of Cremers, Petajisto, and Zitzewitz (2012). Panel C reports the average number of funds in the portfolio, as well as the time-series average of the median fund size ($M), fees (annual), and turnover (annual). The statistics are computed using monthly data between January 1975 and December 2019 and the median size is expressed in terms of January 1, 2000 dollars.

Panel A: Portfolio Gross Excess Return

	Mean	Std. Dev.	Skewness	Kurtosis
All Funds	9.3	15.2	-0.7	5.3
Small Cap	11.4	18.4	-0.6	5.0
Large Cap	9.0	14.6	-0.7	5.1
Low Turnover	9.1	14.2	-0.7	5.6
High Turnover	9.8	16.5	-0.7	5.1
Broker Sold	9.2	15.2	-0.7	5.3
Direct Sold	10.1	15.3	-0.8	5.3

Panel B: Portfolio Betas

	Market	Size	Value	Momentum
All Funds	0.94	0.36	-0.08	0.01
Small Cap	0.98	0.79	-0.14	0.03
Large Cap	0.95	0.17	-0.04	0.01
Low Turnover	0.91	0.27	0.06	-0.02
High Turnover	0.96	0.46	-0.24	0.06
Broker Sold	0.94	0.34	-0.10	0.02
Direct Sold	0.93	0.39	-0.06	0.01

Table I

Summary Statistics of the Mutual Fund Dataset (Continued)

Panel C: Portfolio Characteristics

	Nb. Funds	Size	Fees	Turnover
All Funds	1007	208	1.11	58
Small Cap	209	135	1.28	63
Large Cap	418	256	1.00	54
Low Turnover	338	217	1.05	28
High Turnover	317	185	1.21	112
Broker Sold	467	266.0	1.17	60
Direct Sold	354	157.4	1.07	54

Table II

Distributions of Skill and Scalability

Panel A contains the summary statistics of the distribution of the skill coefficient for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or directly sold to investors). It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, the proportions (%) of funds with a negative and positive skill coefficient, and the quantiles (annualized) at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the scale coefficient. To ease interpretation, we standardize the scale coefficient for each fund so that it corresponds to the change in gross alpha for a one standard deviation change in size.

Panel A: Skill Coefficient

	Moments				Proportions		Quantiles	
	Mean	Std. Dev.	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	3.0 (0.1)	4.1 (0.2)	1.6 (0.7)	23.4 (6.0)	16.9 (0.8)	83.1 (0.8)	-2.2 (0.1)	8.9 (0.2)
Small Cap	4.6 (0.2)	4.5 (0.4)	1.8 (1.1)	18.3 (10.6)	11.5 (1.3)	88.5 (1.3)	-1.8 (0.3)	11.2 (0.3)
Large Cap	1.7 (0.1)	2.9 (0.2)	1.7 (0.6)	15.8 (2.9)	23.1 (1.3)	76.9 (1.3)	-2.1 (0.2)	6.1 (0.2)
Low Turnover	2.5 (0.2)	3.3 (0.3)	0.0 (0.8)	13.9 (2.8)	17.0 (1.3)	83.0 (1.3)	-1.9 (0.2)	7.3 (0.2)
High Turnover	3.4 (0.2)	4.9 (0.4)	2.0 (0.9)	22.0 (6.3)	18.7 (1.4)	81.3 (1.4)	-2.8 (0.2)	10.7 (0.3)
Broker Sold	2.9 (0.2)	4.2 (0.4)	2.0 (1.1)	26.5 (9.5)	17.3 (1.2)	82.7 (1.2)	-2.1 (0.2)	9.2 (0.2)
Direct Sold	3.3 (0.2)	3.2 (0.2)	0.9 (0.5)	9.2 (1.8)	11.6 (1.1)	88.4 (1.1)	-1.1 (0.2)	8.4 (0.2)

Table II

Distributions of Skill and Scalability (Continued)

Panel B: Scale Coefficient

	Moments				Proportions		Quantiles	
	Mean	Std. Dev.	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.3 (0.1)	1.7 (0.1)	1.6 (0.7)	16.7 (11.0)	17.6 (0.8)	82.4 (0.8)	-0.9 (0.1)	3.9 (0.1)
Small Cap	1.6 (0.1)	1.7 (0.1)	0.0 (0.8)	7.1 (6.6)	16.3 (1.5)	83.7 (1.5)	-1.1 (0.1)	4.5 (0.1)
Large Cap	0.9 (0.1)	1.3 (0.1)	1.2 (0.6)	10.4 (4.7)	21.8 (1.3)	78.2 (1.3)	-0.9 (0.1)	3.0 (0.1)
Low Turnover	0.9 (0.1)	1.1 (0.1)	0.2 (0.5)	5.7 (2.7)	18.7 (1.4)	81.3 (1.4)	-0.7 (0.1)	2.8 (0.1)
High Turnover	1.8 (0.1)	2.1 (0.2)	1.0 (0.6)	9.7 (4.3)	17.3 (1.3)	82.7 (1.3)	-1.1 (0.1)	5.1 (0.2)
Broker Sold	1.4 (0.1)	1.8 (0.1)	0.9 (0.5)	10.4 (1.6)	17.6 (1.2)	82.4 (1.2)	-0.9 (0.1)	4.2 (0.1)
Direct Sold	1.4 (0.1)	1.4 (0.1)	1.0 (0.4)	8.3 (2.0)	13.3 (1.2)	86.7 (1.2)	-0.6 (0.1)	3.6 (0.1)

Table III

Distribution of the Value Added

This table contains the summary statistics of the distribution of the value added using all the fund's observations (entire period) for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or directly sold to investors). It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, the proportions (%) of funds with a negative and positive value added, and the quantiles (annualized) at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. The value added of each fund is expressed in $M in terms of January 1, 2000 dollars.

	Moments				Proportions		Quantiles	
	Mean	Std. Dev.	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.9 (0.3)	13.6 (1.1)	5.6 (0.8)	68 (8.3)	40.0 (1.0)	60.0 (1.0)	-6.7 (0.3)	20.4 (0.4)
Small Cap	4.0 (0.5)	10.6 (1.1)	3.6 (0.7)	27.6 (6.1)	33.4 (1.9)	66.6 (1.9)	-5.8 (0.4)	21.7 (0.7)
Large Cap	-0.6 (0.4)	10.9 (1.6)	5.6 (1.8)	86.7 (16.2)	53.0 (1.6)	47.0 (1.6)	-10.5 (0.4)	10.5 (0.4)
Low Turnover	5.9 (0.8)	22.9 (2.2)	3.9 (0.6)	32.1 (4.3)	34.2 (1.7)	65.8 (1.7)	-11.1 (0.7)	43.8 (1.4)
High Turnover	-0.8 (0.3)	8.7 (1.1)	1.0 (3.0)	54.3 (29.6)	57.0 (1.7)	43.0 (1.7)	-10.5 (0.4)	8.7 (0.4)
Broker Sold	0.8 (0.4)	11.8 (1.5)	4.4 (1.9)	71.4 (12.1)	44.5 (1.5)	55.5 (1.5)	-8.5 (0.4)	16.0 (0.5)
Direct Sold	3.7 (0.7)	17.9 (1.9)	4.2 (0.8)	39.4 (6.3)	35.6 (1.7)	64.4 (1.7)	-7.3 (0.5)	30.7 (0.9)

Table IV

Distribution of the Last Subperiod Value Added

This table contains the summary statistics of the distribution of the value added over the last decile of the fund's observations (last subperiod) for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or directly sold to investors). It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, the proportions (%) of funds with a negative and positive value added, and the quantiles (annualized) at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. The value added of each fund is expressed in $M in terms of January 1, 2000 dollars.

	Moments				Proportions		Quantiles	
	Mean	Std. Dev.	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	5.4 (0.5)	22.7 (1.4)	4.1 (0.6)	42.0 (5.3)	29.5 (0.9)	70.5 (0.9)	-6.7 (0.4)	42.1 (0.8)
Small Cap	8.3 (1.1)	28.2 (2.9)	3.6 (0.6)	27.9 (4.8)	32.3 (1.8)	67.7 (1.8)	-15.6 (0.9)	57.3 (1.9)
Large Cap	3.6 (0.5)	16.2 (1.4)	3.0 (1.0)	32.3 (4.6)	26.2 (1.4)	73.8 (1.4)	-4.5 (0.4)	32.6 (1.0)
Low Turnover	8.9 (1.3)	36.3 (3.4)	3.7 (0.6)	31.0 (3.0)	32.4 (1.7)	67.6 (1.7)	15.9 (1.0)	64.7 (2.0)
High Turnover	2.9 (0.5)	14.4 (1.6)	0.4 (1.7)	38.1 (5.5)	32.6 (1.7)	67.4 (1.7)	-5.9 (0.5)	22.9 (0.7)
Broker Sold	5.1 (0.6)	18.5 (1.6)	3.0 (1.2)	34.2 (10.9)	26.0 (1.4)	74.0 (1.4)	-5.9 (0.5)	38.2 (1.0)
Direct Sold	7.7 (1.0)	28.1 (2.7)	2.9 (1.0)	30.6 (4.8)	31.6 (1.6)	68.4 (1.6)	-13.3 (0.8)	58 (1.9)

Table V
Optimal Versus Actual Value Added

Panel A shows the means (annualized) of the actual and optimal value added and its ratio (%). This analysis is based on the funds with significant estimated optimal value added. We define the significance threshold such that the False Discovery Rate (FDR) among the selected funds (i.e., the proportion of funds with a true optimal value equal to zero) equals 10%, 20%, and 30%. In Panels B and C, we repeat the analysis after removing the funds with estimated skill coefficients in the top 10% (20%), or with estimated scale coefficients in the bottom 10% (20%). Figures in parentheses denote the estimated standard deviation of the mean. The value added of each fund is expressed in $M in terms of January 1, 2000 dollars.

Panel A: No Trimming on the Estimated Skill and Scale Coefficients

	Fund Selection					
	FDR=10%		FDR=20%		FDR=30%	
	Mean	Ratio	Mean	Ratio	Mean	Ratio
Optimal Value Added	26.9 (2.1)		21.7 (1.4)		18.6 (1)	
Actual Value Added						
Entire Period	6.7 (0.8)	25.1	4.3 (0.5)	19.7	1.8 (0.3)	9.7
Last Subperiod	14.7 (1.9)	54.8	10.2 (1.3)	46.8	8.7 (0.9)	47.0

Panel B: 10% Trimming on the Estimated Skill and Scale Coefficients

	Fund Selection					
	FDR=10%		FDR=20%		FDR=30%	
	Mean	Ratio	Mean	Ratio	Mean	Ratio
Optimal Value Added	27.5 (2.5)		21.8 (1.6)		18.7 (1)	
Actual Value Added						
Entire Period	7.7 (1.1)	28.1	4.8 (0.6)	21.9	1.9 (0.3)	10.3
Last Subperiod	17.3 (2.1)	63.0	10.9 (1.5)	49.8	9.2 (0.9)	49.1

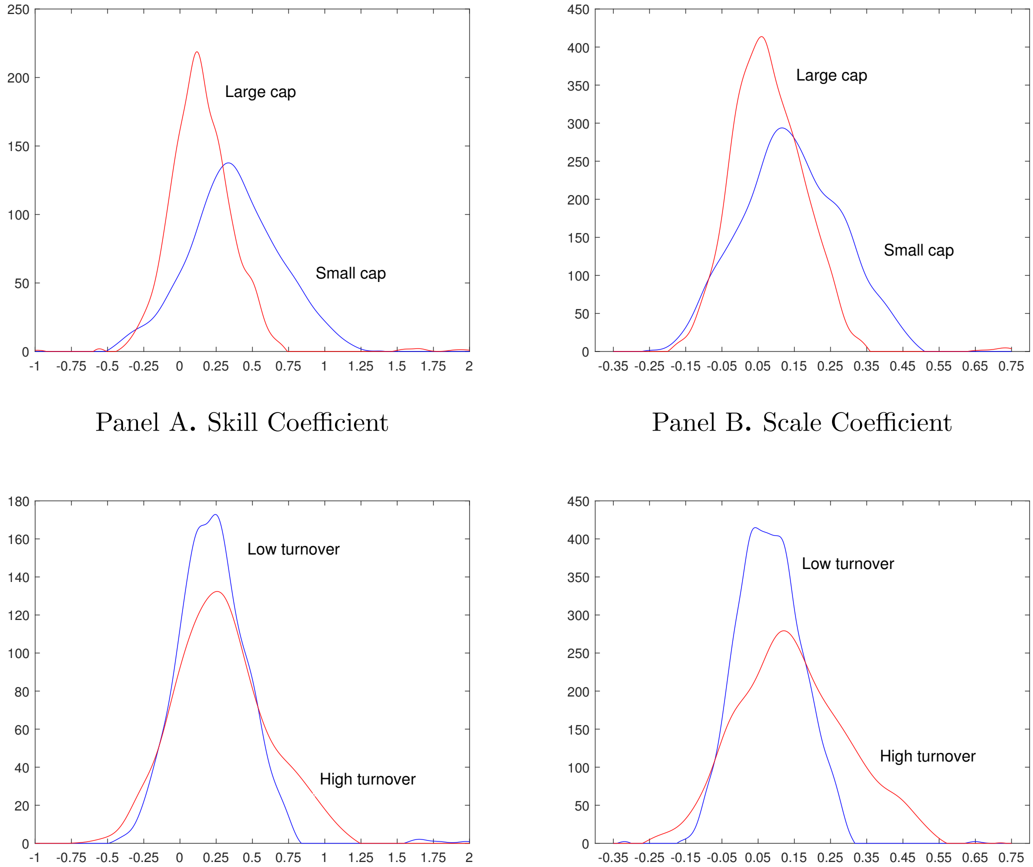
Panel C: 20% Trimming on the Estimated Skill and Scale Coefficients

	Fund Selection					
	FDR=10%		FDR=20%		FDR=30%	
	Mean	Ratio	Mean	Ratio	Mean	Ratio
Optimal Value Added	29 (3.3)		21.6 (1.8)		18.2 (1.1)	
Actual Value Added						
Entire Period	8.7 (0.7)	30.1	4.5 (0.4)	20.9	0.6 (1)	3.2
Last Subperiod	18.2 (2.4)	62.9	10.8 (1.6)	50.2	8.7 (0.9)	47.9

Table VI
Distribution of the Net Alpha

This table shows the summary statistics of the distribution of the net alpha for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or directly sold to investors). It reports the mean (annualized), standard deviation (annualized), skewness, kurtosis, the proportions (%) of funds with a negative and positive net alpha, and the quantiles (annualized) at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator.

	Moments				Proportions		Quantiles	
	Mean	Std. Dev.	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	-0.4 (0.1)	1.4 (0.1)	-0.3 (0.3)	5.7 (1.2)	62.9 (1)	37.1 (0.9)	-2.7 (0.1)	1.9 (0.1)
Small Cap	0.5 (0.1)	1.9 (0.2)	0.8 (0.6)	9.0 (3.7)	40.4 (1.9)	59.6 (1.9)	-2.2 (0.1)	3.5 (0.2)
Large Cap	-0.9 (0.1)	1.1 (0.1)	-0.5 (0.4)	6.2 (1.8)	79.8 (1.3)	20.2 (1.3)	-2.5 (0.1)	0.8 (0.1)
Low Turnover	-0.1 (0.1)	1.3 (0.1)	0.1 (0.2)	3.5 (0.9)	53.8 (1.8)	46.2 (1.8)	-2.2 (0.1)	2.1 (0.1)
High Turnover	-0.8 (0.1)	1.6 (0.1)	-0.6 (0.4)	6.3 (1.9)	69.2 (1.6)	30.8 (1.6)	-3.3 (0.1)	1.8 (0.1)
Broker Sold	-0.7 (0.1)	1.3 (0.1)	-0.2 (0.2)	4.0 (1.0)	67.6 (1.4)	32.4 (1.4)	-2.7 (0.1)	1.5 (0.1)
Direct Sold	0.1 (0.1)	1.4 (0.1)	-0.3 (0.4)	5.6 (2.0)	50.5 (1.8)	49.5 (1.8)	-2.1 (0.1)	2.4 (0.1)



Panel A. Skill Coefficient

Panel B. Scale Coefficient

Panel C. Skill Coefficient

Panel D. Scale Coefficient

Figure 1. Distributions of Skill and Scalability across Fund Groups. Panels A and B plot the distributions of the monthly skill and scale coefficients across small/large cap funds. Panels C and D provide the same information for low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover). We adjust all the estimated distributions for the error-in-variable (EIV) bias using our nonparametric approach.

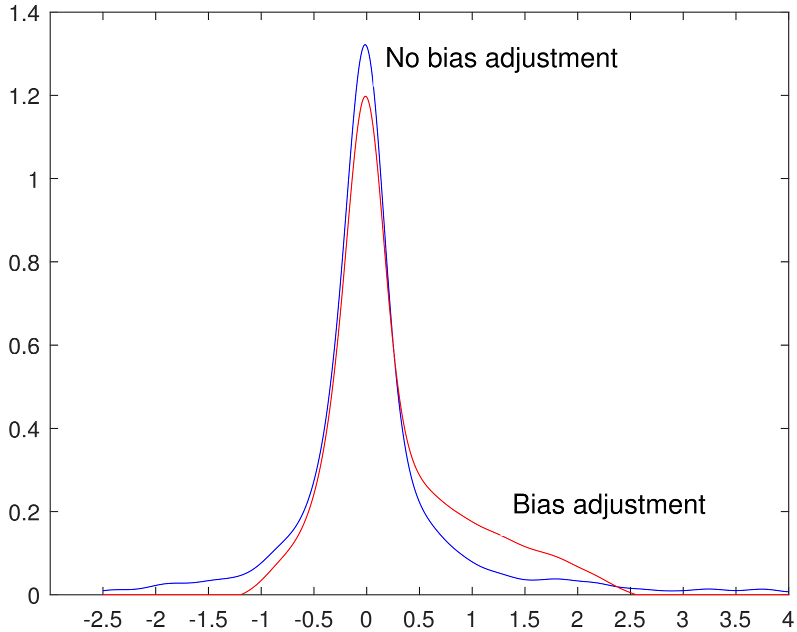


Figure 2. Impact of the Error-in-Variable Bias on the Distribution of the Value Added. This figure plots the distribution of the monthly value added ($M) for all funds in the population with and without the adjustment for the error-in-variable (EIV) bias using our nonparametric approach. The value added of each fund is expressed in terms of January 1, 2000 dollars.

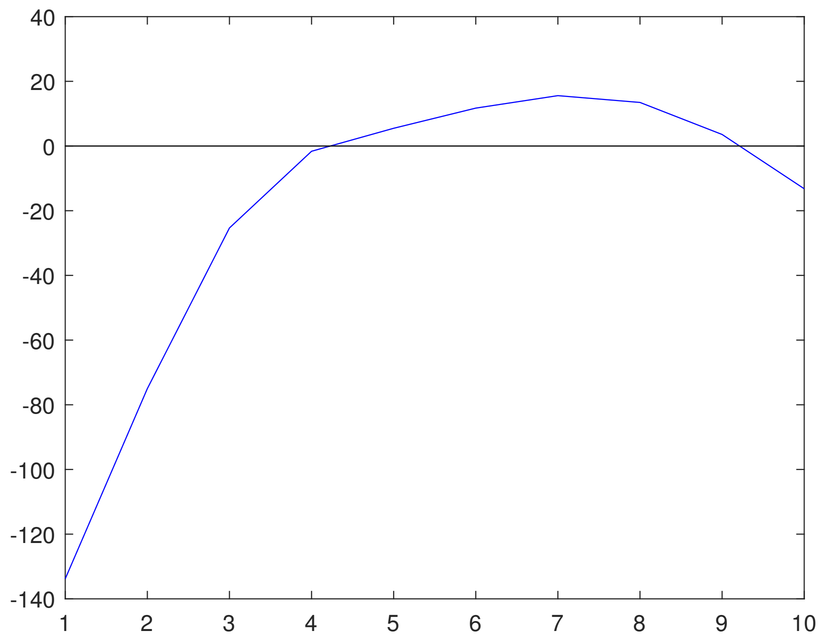


Figure 3. Variation in Fund Size over Time. This figure plots the cross-sectional median difference between the average fund size in each subperiod and its full-sample average. The subperiods are obtained by splitting the total number of fund's observations in ten groups. The size of each fund is expressed in $M in terms of January 1, 2000 dollars.

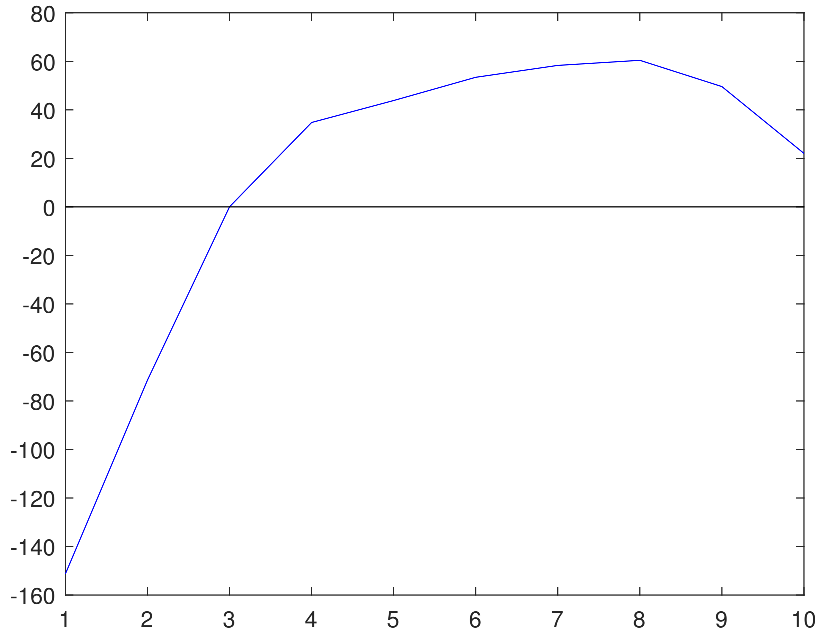


Figure 4. Difference Between the Actual and Optimal Active Fund Size over Time. This figure plots the cross-sectional median difference between the average fund size in each subperiod and its optimal active size predicted by the Berk and Green model. The subperiods are obtained by splitting the total number of fund's observations in ten groups. This analysis is based on the sample of funds with significant estimated optimal value added (i.e., funds for which we reject the null hypothesis that the true optimal value added equals zero). The size of each fund is expressed in \$M in terms of January 1, 2000 dollars.

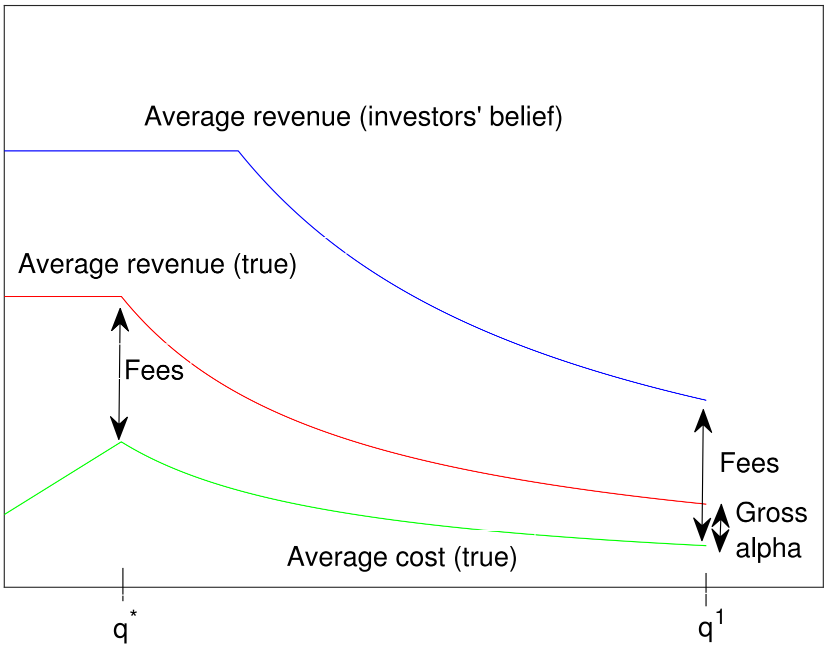


Figure 5. Example of a Fund with Positive Value Added and Negative Alpha. This figure plots the average revenue and cost functions of a fund under the Berk and Green model. The fund knows its average revenue and cost functions (denoted by true) and sets fees such that the fund size is equal to the optimal active size q^*. Investors have optimistic beliefs about skill and, given the level of fees, are willing to invest q^1. To keep the value added unchanged at its optimal level va^*, the fund invests the difference between q^1 and q^* passively (which explains the drop in the true average revenue and cost functions past q^*). Whereas this strategy maximizes the value added, it still produces a negative alpha, i.e., the difference between the gross alpha and fees is negative at q^1.

Internet Appendix for "Skill, Scale, and Value Creation in the Mutual Fund Industry"*

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This appendix is divided in six sections. Section I provides a description of the methodology for estimating the kernel density and the distribution characteristics (moments, proportion, and quantile). It also contains the proofs of the propositions discussed in the paper. Section II examines the asymptotic properties of the estimated distribution characteristics under an alternative analytical approach (as opposed to the numerical approach used in the paper). Section III provides a detailed discussion of the Error-in-Variable (EIV) bias. Section IV describes our extensive Monte-Carlo analysis. It also presents simulation results under the assumption that skill and scalability are uncorrelated. Section V describes the construction of the data set and different fund groups. Finally, Section VI explains the construction of our new formal specification test. It also reports additional empirical results on (i) the validity of the panel specification, (ii) the impact of survivorship and reverse survivorship bias, (iii) the use of alternative asset pricing models, (iv) the analysis based on daily fund returns, and (v) the introduction of variables that capture changes in the economic conditions.

I Methodology

A Estimation Procedure

To begin the presentation of the methodology, we explain how to estimate the measure m_i for each fund, where $m_i \in \{a_i, b_i, va_i, va_i(s)\}$. The estimation procedure explicitly controls for the small-sample bias in the time-series regression $r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}$. This bias, which disappears asymptotically, arises because the mutual fund error term $\varepsilon_{i,t}$ is positively correlated with the innovation in size $\varepsilon_{q_i,t}$, i.e., $\varepsilon_{i,t} = \psi_i \varepsilon_{q_i,t} + \upsilon_{i,t}$, where ψ_i is positive. Specifically, $\varepsilon_{q_i,t}$ denotes the size innovation projected onto the space spanned by the factors f_t: $\varepsilon_{q_i,t} = e_{q_i,t} - \beta_{q_i}' x_t$, where $x_t = (1, f_t')'$ and $e_{q_i,t}$ is the innovation of the size regression $q_{i,t} = \theta_{q_i} + \rho_{q_i} q_{i,t-1} + e_{q_i,t}$. Failing to adjust for the small-sample bias produces values for the skill and scale coefficients that are too high.[1]

As noted by Amihud and Hurvich (2004), adding the regressor $\varepsilon_{q_i,t}$ eliminates the small-sample bias. To see this point, we can replace $\varepsilon_{i,t}$ with $\psi_i \varepsilon_{q_i,t} + \upsilon_{i,t}$ to obtain

$$r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \psi_i \varepsilon_{q_i,t} + \upsilon_{i,t}, \tag{A1}$$

and verify that strict exogeneity holds, i.e., $E[\upsilon_i | X_i] = 0$, where $\upsilon_i = (\upsilon_{i,1}, ..., \upsilon_{i,T})'$, X_i

[1] Using the analysis of Stambaugh (1999), we have $E[\hat{b}_i^{wb} - b_i] = -E[\hat{\rho}_{q_i} - \rho_{q_i}]\psi_i > 0$ and $E[\hat{a}_i^{wb} - a_i] = E[\hat{b}_i^{wb} - b_i]E[q_{i,t-1}] > 0$, where $\hat{a}_i^{wb}$ and $\hat{b}_i^{wb}$ denote the estimators of a_i and b_i without the small-sample bias correction.

is the $T_i \times (K_f+3)$ matrix of the available observations of $x_{i,t} = (1, -q_{i,t-1}, f_t', \varepsilon_{q_i,t})'$, and K_f is the number of factors. Of course, we do not observe the true projected innovation $\varepsilon_{q_i,t}$. Therefore, we use the procedure proposed by Amihud and Hurvich (2004) and Avramov, Barras, and Kosowski (2013) to compute a proxy for $\varepsilon_{q_i,t}$ denoted by $\varepsilon^c_{q_i,t}$.

This four-step procedure is applied to each fund i individually ($i = 1, .., n$). First, we run the size regression to obtain the estimated coefficients $\hat{\theta}_{q_i}$ and $\hat{\rho}_{q_i}$. Second, we compute the adjusted size innovation as

$$e^c_{q_i,t} = q_{i,t} - (\hat{\theta}^c_{q_i} + \hat{\rho}^c_{q_i} q_{i,t-1}), \tag{A2}$$

where the second-order coefficients corrected for the small-sample bias are given by $\hat{\rho}^c_{q_i} = \min(\hat{\rho}_{q_i} + (1+3\hat{\rho}_{q_i})/T_i + 3(1+3\hat{\rho}^2_{q_i})/T_i^2, 0.999)$ and $\hat{\theta}^c_{q_i} = (1-\hat{\rho}_{q_i})\overline{q}_i$. Third, we regress $e^c_{qi,t}$ on the factors to obtain $\varepsilon^c_{q_i,t} = e^c_{q_i,t} - \hat{\beta}'_{q_i} x_t$. Finally, we insert $\varepsilon^c_{q_i,t}$ in Equation (A1) to obtain

$$r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \psi_i \varepsilon^c_{q_i,t} + \upsilon_{i,t}. \tag{A3}$$

From this regression, we can obtain estimated values for m_i that are adjusted for the small-sample bias.

B Asymptotic Properties of the Kernel Density

Proof of Proposition III.1 (Asymptotic properties). In this section, we provide a proof of the asymptotic properties of the kernel density $\hat{\phi}(m)$ for each measure m_i. To this end, we initially focus on the skill coefficient, i.e., $m_i = a_i$. We allow for weak serial dependence in the error terms (i.e., temporal mixing). To simplify the presentation and avoid unnecessary technicalities related to spatial mixing conditions, we assume that the error terms are cross-sectionally independent. To further ease the presentation, we do not explicitly include the small-sample bias correction of the previous section because it has no impact on the asymptotic analysis when T becomes large.[2]

[2] The inclusion of the estimated variable $\varepsilon^c_{q_i,t}$ in the set of regressors does not change the asymptotic properties of the nonparametric density kernel estimator because the estimation error in $\varepsilon^c_{q_i,t}$ only affects the higher order terms beyond T^{-1}.

From the OLS estimation of the regression $r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}$, we have:

$$\begin{aligned}
\hat{m}_i &= e_1' \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_{i,t} r_{i,t} = m_i + e_1' \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_{i,t} \varepsilon_{i,t} \\
&= m_i + \frac{1}{\sqrt{T}} \tau_{i,T} e_1' \hat{Q}_{x,i}^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_{i,t} \varepsilon_{i,t} \right) \equiv m_i + \frac{1}{\sqrt{T}} \hat{\eta}_{i,T}, \qquad \text{(A4)}
\end{aligned}$$

where $x_{i,t} = (1, -q_{i,t-1}, f_t')'$. Moreover, let us write

$$\hat{\eta}_{i,T} = \eta_{i,T} + \frac{1}{\sqrt{T}} \hat{v}_{i,T}, \qquad \text{(A5)}$$

where $\eta_{i,T} = \tau_i \frac{1}{\sqrt{T}} \sum_t I_{i,t} u_{i,t}$, $u_{i,t} = e_1' Q_{x,i}^{-1} x_{i,t} \varepsilon_{i,t}$, $\hat{v}_{i,T} = (\tau_{i,T} - \tau_i) \sum_t I_{i,t} e_1' \hat{Q}_{x,i}^{-1} x_{i,t} \varepsilon_{i,t} + \tau_i \sum_t I_{i,t} e_1' (\hat{Q}_{x,i}^{-1} - Q_{x,i}^{-1}) x_{i,t} \varepsilon_{i,t}$, $\tau_i = \text{plim}_{T \to \infty} \tau_{i,T}$, and $\tau_{i,T} = T/T_i$. The term $\hat{\eta}_{i,T}/\sqrt{T}$ corresponds to the estimation error of $\hat{m}_i$. It is equal to the sum of $\eta_{i,T}/\sqrt{T}$ and $\hat{v}_{i,T}/T$, where the second component captures the errors due to estimating the matrix $Q_{x,i}$ and the random sample size T_i. We can write $\hat{\phi}(m) - \phi(m) = I_1 + I_2 + I_3 + I_4$, where:

$$\begin{aligned}
I_1 &= \frac{1}{h} E \left[K \left(\frac{m_i - m}{h} \right) \right] - \phi(m), \\
I_2 &= \frac{1}{h} E \left[K \left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h} \right) \right] - \frac{1}{h} E \left[K \left(\frac{m_i - m}{h} \right) \right], \\
I_3 &= \frac{1}{nh} \sum_i \left\{ K \left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h} \right) - E \left[K \left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h} \right) \right] \right\}, \\
I_4 &= \frac{1}{nh} \sum_i \left[\mathbf{1}_i^{\chi} K \left(\frac{m_i + \eta_{i,T}/\sqrt{T} + \hat{v}_{i,T}/T - m}{h} \right) \right] \\
&\quad - \frac{1}{nh} \sum_i \left[K \left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h} \right) \right]. \qquad \text{(A6)}
\end{aligned}$$

The first term I_1 is the smoothing bias, the second term I_2 is the Error-In-Variable (EIV) bias, and I_3 is the main stochastic term. The remainder term I_4 is associated with $\hat{v}_{i,T}/T$ and is negligible with respect to the others. We now characterize the first three dominating terms.

(i) From standard results in kernel density estimation, the smoothing bias is such that $I_1 = \frac{1}{2} \phi^{(2)}(m) K_2 h^2 + O(h^3)$, with $K_2 = \int u^2 K(u) du$.

(ii) By a Taylor expansion of the kernel function K we have

$$I_2 = \sum_{j=1}^{\infty} \frac{1}{j! T^{j/2} h^{j+1}} E\left[K^{(j)}\left(\frac{m_i - m}{h}\right) (\eta_{i,T})^j \right]. \tag{A7}$$

We can then apply j times partial integration and a change of variable to obtain

$$\begin{aligned} \frac{1}{h^{j+1}} E\left[K^{(j)}\left(\frac{m_i - m}{h}\right) (\eta_{i,T})^j \right] &= \frac{1}{h^{j+1}} \int K^{(j)}\left(\frac{u-m}{h}\right) \psi_{T,j}(u) du \\ &= (-1)^j \frac{1}{h} \int K\left(\frac{u-m}{h}\right) \psi_{T,j}^{(j)}(u) du \\ &= (-1)^j \int K(u) \psi_{T,j}^{(j)}(m + hu) du, \qquad \text{(A8)} \end{aligned}$$

where $\psi_{T,j}(m) = E[(\eta_{i,T})^j | m_i = m]\phi(m)$ for $j = 1, 2, ...$ We have $\psi_{T,1}(m) = 0$ and $\lim_{T\to\infty}\psi_{T,2}(m) = E[S_i|m_i = m]\phi(m) \equiv \psi(m)$ where S_i is equal to $\tau_i^2 \text{plim}_{T\to\infty} \frac{1}{T} \sum_{t,s} I_{i,t} I_{i,s} u_{i,t} u_{i,s}$. By weak serial dependence of the error terms, functions $\psi_{T,j}(m)$ for $j > 2$ are bounded with respect to T. Thus, we get: $I_2 = \frac{1}{2T}\psi^{(2)}(m) + O(1/T^{3/2} + h^2/T)$.

(iii) Let us now consider term I_3. For expository purpose, we treat the factor values f_t as given constants. Then:

$$V[I_3] = \frac{1}{nh^2} V\left[K\left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h}\right) \right]. \tag{A9}$$

From the above arguments, we have $\frac{1}{h} E\left[K\left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h}\right) \right] = \phi(m) + o(1)$ and

$$\begin{aligned} \frac{1}{h} E\left[K\left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h}\right)^2 \right] &= \int K(u)^2 du \frac{1}{h} E\left[\bar{K}\left(\frac{m_i + \eta_{i,T}/\sqrt{T} - m}{h}\right) \right] \\ &= \phi(m) \int K(u)^2 du + o(1), \qquad \text{(A10)} \end{aligned}$$

where $\bar{K}(u) = K(u)^2 / \int K(u)^2 du$. Therefore:

$$V[I_3] = \frac{1}{nh} \phi(m) \int K(u)^2 du + o\left(\frac{1}{nh}\right). \tag{A11}$$

Under regularity conditions, we can apply an appropriate central limit theorem

(CLT) to obtain $\sqrt{nh}I_3 \Rightarrow N\left(0, \phi(m)K_1\right)$, where $K_1 = \int K(u)^2 du$. Grouping the different elements completes the proof.[3] QED

We can apply the same arguments for all the other measures used in the paper: (i) the scale coefficient ($m_i = b_i$), (ii) the value added ($m_i = va_i$), and (iii) the subperiod value added ($m_i = va_i(s)$). The only required change is to use the appropriate definition for $\hat{m}_i$ and $u_{i,t}$ given in the paper.

C Optimal Bandwidth

Proof of Proposition III.1 (Optimal bandwidth) We now prove the remaining part of Proposition III.1 by solving for the optimal bandwidth h^* that minimizes the Asymptotic Mean Integrated Squared Error (AMISE) of the density $\hat{\phi}(m)$. From the arguments in Section B above, we get the asymptotic expansion of the bias $bs(m)$ of the estimator $\hat{\phi}(m)$ with leading terms,

$$bs_1(m) = \frac{1}{2}h^2 K_2 \phi^{(2)}(m), \tag{A12}$$

$$bs_2(m) = \frac{1}{2T}\psi^{(2)}(m). \tag{A13}$$

where $bs_1(m)$ denotes the smoothing bias and $bs_2(m)$ denotes the EIV bias.[4] We also get the asymptotic expansion of the variance of the estimator $\hat{\phi}(m)$ with leading terms $\sigma^2(m) = \frac{1}{nh}\phi(m)K_1$. Combining these elements, we can write the AMISE as

$$\begin{aligned} AMISE(h) &= \int [\sigma^2(u) + bs(u)^2]du = \int [\sigma^2(u) + (bs_1(u) + bs_2(u))^2]du \\ &= \frac{1}{nh}K_1 + \frac{h^4 K_2^2}{4}\int [\phi^{(2)}(u)]^2 du \\ &\quad + \frac{h^2 K_2}{2T}\int \phi^{(2)}(u)\psi^{(2)}(u)du + \frac{1}{4T^2}\int \left[\psi^{(2)}(u)\right]^2 du, \end{aligned} \tag{A14}$$

[3] Okui and Yanagi (2020) also consider a kernel estimator for the density of the mean and autocorrelation of random variables. However, their distributional results differ from our regression-based results aimed at measuring fund skill.

[4] From Equations (A12) and (A13), the integral of $bs_1^r(m)m$ and $bs_2^r(m)m$ is equal to zero if $\phi^{(1)}$ and $\psi^{(1)}$ vanish at the boundary of the support (which is the case in our Gaussian reference model). Hence it implies that the bias adjusted density (Equation (19) in the paper) integrates to one by construction.

where we assume that $\int \phi^{(2)}(u)\psi^{(2)}(u)du \geqslant 0$ so that the AMISE is convex. The optimal bandwidth h^* minimizes the AMISE and solves the equation:

$$\begin{aligned} & -\frac{1}{nh^2} + c_1 h^3 + c_2 \frac{h}{T} = 0 \\ \iff \quad & 1 = c_1 n h^5 + c_2 \frac{nh^3}{T}, \end{aligned} \tag{A15}$$

where $c_1 = K_2^2 \int [\phi^{(2)}(m)]^2 dm / K_1$ and $c_2 = \frac{K_2}{K_1} \int \phi^{(2)}(m)\psi^{(2)}(m)dm$ (with c_1, $c_2 > 0$).

The analytical approximation of the optimal bandwidth h^* depends on the relative increase of n and T. If (i) nh^3/T tends to a nonzero constant and (ii) nh^5 tends to zero, Equation (A15) implies that asymptotically

$$h^* = c_2^{-\frac{1}{3}} \left(\frac{n}{T}\right)^{-\frac{1}{3}} = \left(\frac{K_2}{K_1} \int \phi^{(2)}(m)\psi^{(2)}(m)dm\right)^{-\frac{1}{3}} \left(\frac{n}{T}\right)^{-\frac{1}{3}}. \tag{A16}$$

This solution is admissible (i.e., it satisfies $nh^5 \to 0$) if the sample sizes n and T are such that $n^{2/5}/T \to \infty$ or, put differently, if T is small relative to n. QED

We now consider the asymptotic distribution of the kernel density obtained with the optimal bandwidth h^*. We can check that $\sqrt{nh^*}(h^{*3} + h^{*2}/T + 1/T^{3/2}) = o(1)$ if $n/T^4 \to 0$. Replacing $bs(m)$ with its asymptotic approximation we have:

$$\sqrt{nh^*}\left(\hat{\phi}(m) - \phi(m) - \frac{1}{2}\phi^{(2)}(m)K_2 h^{*2} - \frac{1}{2T}\psi^{(2)}(m)\right) \Rightarrow N\left(0, \phi(m)K_1\right), \tag{A17}$$

where the smoothing bias is negligible and the dominant component is the EIV bias of order $O(1/T)$ (because we have $n^{2/5}/T \to \infty$ and $Th^{*2} \to 0$).

Note that if (i) nh^3/T tends to zero and (ii) nh^5 tends to a nonzero constant, Equation (A15) produces a different optimal bandwidth of the form $h^* \sim c_1^{-\frac{1}{5}} n^{-\frac{1}{5}}$ (i.e., the usual Silverman rule). This solution is admissible (i.e., it satisfies $nh^3/T \to 0$) if the sample sizes n and T are such that $n^{2/5}/T \to 0$ or, put differently, if T is large relative to n.[5]

Our Monte-Carlo analysis in Section IV reveals that given our actual sample size, the optimal bandwidth in Equation (A16) produces the best results. Motivated by these results, we therefore use it in our baseline specification. We also verify that the empirical results are remarkably similar under the two bandwidth choices.

[5] In the special case where $n^{2/5}/T \to \rho$, with $\rho > 0$, the two rates of convergence $n^{-1/5}$ and $(n/T)^{-1/3}$ coincide. Then, Equation (A15) has a solution such that $h^* \sim \bar{c}^{1/5} n^{-1/5}$, where $\bar{c}$ solves the equation $1 = c_1\bar{c} + c_2\rho\bar{c}^{3/5}$. Therefore, the optimal bandwidth remains proportional to $n^{-1/5}$ (similar to the Silverman rule).

D Adjustment of the Density Bias

Proof of Proposition III.2. We now prove the second proposition of the paper which provides closed form expressions for the two bias components $bs_1(m)$ and $bs_2(m)$ and the optimal bandwidth h^*. We use a Gaussian reference model in which m_i and $s_i = \log(S_i)$ follow a bivariate Gaussian distribution with mean parameters μ_m, μ_s, variance parameters σ_m^2, σ_s^2, and correlation parameter ρ.[6] We also use a standard Gaussian kernel $K(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-u^2/2\right)$ with $K_1 = \int K(u)^2 du = \frac{1}{2\sqrt{\pi}}$ and $K_2 = \int u^2 K(u) du = 1$. The constants c_1 and c_2 are given by:

$$c_1 = 2\sqrt{\pi} \int [\phi^{(2)}(u)]^2 du, \tag{A18}$$

$$c_2 = 2\sqrt{\pi} \int \phi^{(2)}(u)\psi^{(2)}(u) du = 2\sqrt{\pi} \int \phi^{(4)}(u)\psi(u) du, \tag{A19}$$

where we use twice partial integration for c_2.

Let us now compute the two integrals appearing in these formulas. We have $\phi(m) = \frac{1}{\sigma_m}\varphi\left(\frac{m-\mu_m}{\sigma_m}\right)$ where $\varphi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-z^2/2\right)$ is the standard Gaussian density. We have

$$\phi^{(1)}(m) = -\frac{1}{\sigma_m}\left(\frac{m-\mu_m}{\sigma_m}\right)\phi(m), \tag{A20}$$

$$\phi^{(2)}(m) = \frac{1}{\sigma_m^2}\left(\left(\frac{m-\mu_m}{\sigma_m}\right)^2 - 1\right)\phi(m). \tag{A21}$$

Therefore, the first integral is equal to

$$\begin{aligned}\int [\phi^{(2)}(u)]^2 du &= \frac{1}{\sigma_m^5}\int (z^2-1)^2 \frac{1}{2\pi}\exp(-z^2) dz = \frac{1}{2\sqrt{\pi}\sigma_m^5}\int (v^2/2-1)^2 \varphi(v) dv \\ &= \frac{3}{8\sqrt{\pi}\sigma_m^5},\end{aligned} \tag{A22}$$

with the changes of variables from u to $z = (u-\mu_m)/\sigma_m$, and from z to $v = \sqrt{2}z$.

We can write the second integral as

$$\begin{aligned}\int \phi^{(4)}(m)\psi(m) dm &= \frac{\exp\left(\mu_s + \frac{1}{2}\sigma_s^2(1-\rho^2)\right)}{\sigma_m^5}\int \varphi^{(4)}(z)\exp(\rho\sigma_s z)\varphi(z) dz \\ &= \frac{\exp\left(\mu_s + \frac{1}{2}\sigma_s^2(1-\rho^2)\right)}{2\sqrt{\pi}\sigma_m^5}\int (v^4/4 - 3v^2 + 3)\exp(\lambda v)\varphi(v) dv,\end{aligned} \tag{A23}$$

[6] The Gaussian marginal density of m_i implies that our reference model nests the standard model underlying the derivation of the Silverman rule for kernel smoothing.

where $\psi(m) = E[\exp(s_i)|m_i = m]\phi(m) = \exp\left(\mu_s + \rho\sigma_s\left(\frac{m-\mu_m}{\sigma_m}\right) + \frac{1}{2}\sigma_s^2(1-\rho^2)\right)\phi(m)$, $\lambda = \rho\sigma_s/\sqrt{2}$ by using the same changes of variables as above, and $\varphi^{(4)}(z) = (z^4 - 6z^2 + 3)\varphi(z)$. To compute the integral in Equation (A23), we can exploit the following equality that applies to a standard Gaussian random variable Z: $\int z^k \exp(\lambda z)\varphi(z)dz = E[Z^k \exp(\lambda Z)] = \frac{\partial^k}{\partial\lambda^k}E[\exp(\lambda Z)]$ with $E[\exp(\lambda Z)] = \exp(\lambda^2/2)$. This yields $\int(v^4/4 - 3v^2 + 3)\exp(\lambda v)\varphi(v)dv = \left(\frac{1}{4}\frac{\partial^4}{\partial\lambda^4} - 3\frac{\partial^2}{\partial\lambda^2} + 3\right)\exp(\lambda^2/2) = \frac{1}{4}(\lambda^4 - 6\lambda^2 + 3)\exp(\lambda^2/2)$. Therefore, we obtain

$$\int \phi^{(4)}(m)\psi(m)dm = \frac{3\exp\left(\mu_s + \frac{1}{2}\sigma_s^2\left(1-\frac{\rho^2}{2}\right)\right)}{8\sqrt{\pi}\sigma_m^5}(\rho^4\sigma_s^4/12 - \rho^2\sigma_s^2 + 1). \tag{A24}$$

Using these results, we obtain the optimal bandwidth

$$h^* = \left[\frac{3(\rho^4\sigma_s^4/12 - \rho^2\sigma_s^2 + 1)}{4\sigma_m^5}\exp\left(\mu_s + \frac{1}{2}\sigma_s^2\left(1-\frac{\rho^2}{2}\right)\right)\right]^{-\frac{1}{3}}(n/T)^{-1/3}, \tag{A25}$$

where $c_2 \geq 0$ when either $\rho^2\sigma_s^2 \leq 6 - 2\sqrt{6}$, or $\rho^2\sigma_s^2 \geq 6 + 2\sqrt{6}$.

Finally, we can use the Gaussian reference model to obtain closed form expressions of the smoothing bias and the EIV bias. Differentiating $\psi(m)$ twice, we obtain[7]

$$\begin{aligned}
\psi^{(2)}(m) &= \exp\left(\mu_s + \rho\sigma_s\left(\frac{m-\mu_m}{\sigma_m}\right) + \frac{1}{2}\sigma_s^2(1-\rho^2)\right)\phi(m) \\
&\quad \times\left\{\left(\frac{\sigma_s\rho}{\sigma_m}\right)^2 - 2\frac{\sigma_s\rho}{\sigma_m^2}\left(\frac{m-\mu_m}{\sigma_m}\right) + \frac{1}{\sigma_m^2}\left[\left(\frac{m-\mu_m}{\sigma_m}\right)^2 - 1\right]\right\} \\
&= \exp\left(\mu_s + \frac{1}{2}\sigma_s^2\right)\frac{1}{\sigma_m^2}\left(\left(\frac{m-\mu_m-\rho\sigma_s\sigma_m}{\sigma_m}\right)^2 - 1\right) \\
&\quad \times\frac{1}{\sigma_m}\varphi\left(\frac{m-\mu_m-\rho\sigma_s\sigma_m}{\sigma_m}\right).
\end{aligned} \tag{A26}$$

Using Equations (A21) and (A26), we can replace $\phi^{(2)}(m)$ and $\psi^{(2)}(m)$ in Equations (A12) and (A13) to obtain the two bias terms under the reference model:

$$bs_1^r(m) = \frac{1}{2}h^2K_2\phi^{(2)}(m) = \left[\frac{1}{2}h^2\frac{1}{\sigma_m^2}(\bar{m}_1^2 - 1)\right]\frac{1}{\sigma_m}\varphi(\bar{m}_1), \tag{A27}$$

$$bs_2^r(m) = \frac{1}{2T}\psi^{(2)}(m) = \left[\frac{1}{2T}\exp(\mu_s + \frac{1}{2}\sigma_s^2)\frac{1}{\sigma_m^2}(\bar{m}_2^2 - 1)\right]\frac{1}{\sigma_m}\varphi(\bar{m}_2), \tag{A28}$$

[7] Alternatively, we can directly derive Equation (A26) by (i) rewriting $\psi(m)$ as a recentered Gaussian density up to a multiplicative constant, i.e., $\psi(m) = \exp\left(\mu_s + \frac{1}{2}\sigma_s^2\right)\frac{1}{\sigma_m}\varphi\left(\frac{m-\mu_m-\rho\sigma_s\sigma_m}{\sigma_m}\right)$, and (ii) differentiating this expression twice.

where $\bar{m}_1 = \frac{m-\mu_m}{\sigma_m}$, and $\bar{m}_2 = \frac{m-\mu_m-\rho\sigma_m\sigma_s}{\sigma_m}$. In our implementation, the parameters of the bivariate Gaussian distribution are estimated by the sample moments of $\hat{m}_i$ and $\hat{s}_i = \log \hat{S}_i$. QED

E Estimators of the Distribution Characteristics

To compute the characteristics of the distribution, we use a numerical approach based on the bias-adjusted density $\tilde{\phi}(m)$. For the moments, we simply use the respective definitions of the standard deviation, skewness, and kurtosis:

$$SD = V^{\frac{1}{2}} = \left(\int \phi(u)(u-M)^2 du\right)^{\frac{1}{2}}, \tag{A29}$$

$$Sk = \frac{\int \phi(u)(u-M)^3 du}{V^{\frac{3}{2}}}, \tag{A30}$$

$$Ku = \frac{\int \phi(u)(u-M)^4 du}{V^2}, \tag{A31}$$

where V denotes the variance of the distribution. To obtain the bias-adjusted estimators $\widetilde{SD}$, $\widetilde{Sk}$, and $\widetilde{Ku}$, we replace $\phi(u)$ with the bias-adjusted density estimator $\tilde{\phi}(u)$ in the above expressions We also compute the mean $\tilde{M}$ as the empirical average of the estimated measures which does not suffer from the EIV bias: $\tilde{M} = \hat{M} = \frac{1}{n_\chi}\sum_i \hat{m}_i \mathbf{1}_i^\chi$.[8] Once we have the bias-corrected estimates, we can approximate the asymptotic variance of the mean, standard deviation, skewness, and kurtosis using the delta method to conduct statistical inference:

(i) For the estimated mean, we have the asymptotic variance:

$$V\left[\tilde{M}\right] = \frac{V}{n}, \tag{A32}$$

which only requires a consistent estimator of the variance of the distribution V.

(ii) For the estimated volatility, we have:

$$V\left[\widetilde{SD}\right] = \frac{E\left[\left((2SD)^{-1}\Psi_2\right)^2\right]}{n}, \tag{A33}$$

[8] The integrals of the bias terms $bs_1^\tau(m)m$ and $bs_2^\tau(m)m$ are equal to zero (footnote 4), which implies that the empirical average is the same as the average obtained via a numerical integration of $\tilde{\phi}(m)$.

where $\Psi_2 = (m_i - E[m_i])^2 - E\left[(m_i - E[m_i])^2\right]$.

(iii) For the estimated skewness, we have:

$$V\left[\widetilde{Sk}\right] = \frac{E\left[\left(SD^{-3}\Psi_3 - \frac{3}{2}SD^{-2}Sk\Psi_2 - 3SD^{-1}\Psi_1\right)^2\right]}{n}, \qquad \text{(A34)}$$

where $\Psi_3 = (m_i - E[m_i])^3 - E\left[(m_i - E[m_i])^3\right]$, and $\Psi_1 = (m_i - E[m_i]) - E\left[(m_i - E[m_i])\right]$ (see Bai and Ng (2005)).

(iv) For the estimated kurtosis $\widetilde{Ku}$, we have:

$$V\left[\widetilde{Ku}\right] = E\left[\left(SD^{-4}\Psi_4 - 2SD^{-2}Ku\Psi_2 - SD^{-1}Sk\Psi_1\right)^2\right], \qquad \text{(A35)}$$

where $\Psi_4 = (m_i - E[m_i])^4 - E\left[(m_i - E[m_i])^4\right]$ (see Bai and Ng (2005)).

We also use a numerical approach to compute the proportion and quantile estimators. We denote the proportion of funds with a measure m_i below the threshold m by $\Phi(m) = P[m_i \leq m]$ and the quantile at any given percentile level $p \in (0,1)$ by $Q(p) = \Phi^{-1}(p)$, where Φ is the cdf. We obtain bias-adjusted estimators of $\Phi(m)$ and $Q(p)$ via a numerical integration of the density, i.e., we have

$$\Phi(m) = \int_{-\infty}^{m} \phi(u)du, \qquad \text{(A36)}$$

$$\int_{-\infty}^{Q(p)} \phi(u)du = p, \qquad \text{(A37)}$$

where we replace $\phi(u)$ with the bias-adjusted density estimator $\tilde{\phi}(u)$ (for the quantile, we use an iterative procedure until Equation (A37) holds). We can then use the bias-corrected estimated proportion and quantile to estimate their asymptotic variances

$$V[\tilde{\Phi}(m)] = \frac{\Phi(m)(1-\Phi(m))}{n}, \qquad \text{(A38)}$$

$$V[\tilde{Q}(p)] = \frac{\frac{p(1-p)}{\phi(Q(p))^2}}{n}, \qquad \text{(A39)}$$

where ϕ is the normal density obtained from the Gaussian reference model.

II Overview of the Analytical Approach

A Moments

An alternative to the numerical approach described above is to estimate the distribution characteristics using an analytical approach. Asymptotically, both approaches (numerical and analytical) are equivalent.

To begin, we consider the estimation of the cross-sectional expectation $E[g(m_i)]$, where g is a given smooth function of m_i. We investigate the convergence properties of the cross-sectional estimator $\frac{1}{n}\sum_{i=1}^{n} g(\hat{m}_i)\mathbf{1}_i^{\chi}$ based on the OLS estimates $\hat{m}_i$ of the non-trimmed assets. The following proposition proves the asymptotic normality of the estimator under the baseline specification $r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}$.

Proposition A.1. As $n, T \to \infty$, such that $n = o(T^3)$,

$$\sqrt{n}\left(\frac{1}{n}\sum_i g(\hat{m}_i)\mathbf{1}_i^{\chi} - E\left[g(m_i)\right] - \mathcal{B}_T\right) \Rightarrow N\left(0, V\left[g(m_i)\right]\right), \tag{A40}$$

where $\mathcal{B}_T = \frac{1}{2T}E[g^{(2)}(m_i)S_i]$ and $V\left[g(m_i)\right]$ is the cross-sectional variance of $g(m_i)$.

Proof of Proposition A.1. Equation (A4) yields the mean value expansion

$$g(\hat{m}_i) = g(m_i) + g^{(1)}(\bar{m}_i)\frac{1}{\sqrt{T}}\hat{\eta}_{i,T} + g^{(2)}(\bar{m}_i)\frac{1}{2T}\hat{\eta}_{i,T}^2, \tag{A41}$$

where $\bar{m}_i$ lies between $\hat{m}_i$ and m_i. Then, we get

$$\begin{aligned}
& \sqrt{n}\left(\frac{1}{n}\sum_i g(\hat{m}_i)\mathbf{1}_i^{\chi} - E\left[g(m_i)\right] - \mathcal{B}_T\right) \\
= \quad & \frac{1}{\sqrt{n}}\sum_i \left(g(m_i) - E\left[g(m_i)\right]\right) - \frac{1}{\sqrt{n}}\sum_i g(m_i)(1-\mathbf{1}_i^{\chi}) + \frac{1}{\sqrt{nT}}\sum_i \mathbf{1}_i^{\chi} g^{(1)}(\bar{m}_i)\hat{\eta}_{i,T} \\
& + \frac{1}{2T}\frac{1}{\sqrt{n}}\sum_i \left(\mathbf{1}_i^{\chi} g^{(2)}(\bar{m}_i)\hat{\eta}_{i,T}^2 - E\left[g^{(2)}(m_i)S_i\right]\right) \\
\equiv \quad & I_{21} + I_{22} + I_{23} + I_{24}.
\end{aligned} \tag{A42}$$

We have $I_{22} = o_p(1)$ and $I_{23} = O_p(1/\sqrt{T}) = o_p(1)$ using similar arguments as in Lemma 2 of Gagliardini, Ossola, and Scaillet (2016). The remainder term $I_{24} = O_p(\sqrt{n/T^3} + \sqrt{n}/T^2 + 1/T)$, which gives $I_{24} = o_p(1)$ if $n = o(T^3)$.[9] Therefore, the asymptotic distribution in Equation (A38) depends on the first term $I_{21} \Rightarrow N(0, V[g(m_i)])$ from the

[9] The condition $n = o(T^3)$ is used to control the remainder term in the Taylor expansion of the function g and the bias term.

standard CLT. QED

The distribution results in Equation (A38) reveal that we have an asymptotic bias $\mathcal{B}_T$ of order $1/T$ which comes from the estimation error of $\hat{m}_i$ (EIV contribution). To compute the bias-adjusted estimated mean, standard deviation, skewness, and kurtosis, we can use an analytical approach (based on the delta method) and replace the unknown moments with consistent estimators based on empirical averages:

(i) The mean is given by $M = E[m_i]$. Therefore, the asymptotic bias $\mathcal{B}_T$ is zero because $g^{(2)}(m) = 0$. For this particular case, we do not need the condition $n = o(T^3)$ for the above proposition to hold.

(ii) The variance is given by $V = E\left[(m_i - E[m_i])^2\right]$. To obtain the bias of the standard deviation $SD = V^{\frac{1}{2}}$, we apply the delta method:

$$\mathcal{B}_T(SD) = (2SD)^{-1}\mathcal{B}_T(E[m_i^2]), \tag{A43}$$

where the asymptotic bias of the second moment is given by

$$\mathcal{B}_T(E[m_i^2]) = \frac{1}{2T}E[2S_i]. \tag{A44}$$

(iii) The skewness is given by $Sk = E\left[(m_i - E[m_i])^3\right] / E\left[(m_i - E[m_i])^2\right]^{3/2}$. Applying the delta method, we obtain

$$\mathcal{B}_T(Sk) = (\nabla_3 Sk)\mathcal{B}_T(E[m_i^3]) + (\nabla_2 Sk)\mathcal{B}_T(E[m_i^2]), \tag{A45}$$

where $\nabla_j Sk$ denotes the derivative of Sk w.r.t. $E[m_i^j]$ and the different terms are given by

$$\begin{aligned} \mathcal{B}_T(E[m_i^3]) &= \frac{1}{2T}E[6m_iS_i], \\ \nabla_3 Sk &= V[m_i]^{-3/2}, \\ \nabla_2 Sk &= -3E[m_i]V[m_i]^{-3/2} + E[m_i^3](\frac{-3}{2})V[m_i]^{-5/2} \\ &\quad + \left\{-3E[m_i^2]E[m_i] + 2E[m_i]^3\right\}(\frac{-3}{2})V[m_i]^{-5/2}. \end{aligned} \tag{A46}$$

(iv) The kurtosis is given by $Ku = E\left[(m_i - E[m_i])^4\right] / E\left[(m_i - E[m_i])^2\right]^2$. Applying the delta method, we obtain

$$\mathcal{B}_T(Ku) = (\nabla_4 Ku)\mathcal{B}_T(E[m_i^4]) + (\nabla_3 Ku)\mathcal{B}_T(E[m_i^3]) + (\nabla_2 Ku)\mathcal{B}_T(E[m_i^2]), \quad \text{(A47)}$$

where the different terms are given by

$$\begin{aligned} \mathcal{B}_T(E[m_i^4]) &= \frac{1}{2T}E[12m_i^2 S_i], \\ \nabla_4 Ku &= V[m_i]^{-2}, \\ \nabla_3 Ku &= -4E[m_i]V[m_i]^{-2}, \\ \nabla_2 Ku &= 6E[m_i]^2 V[m_i]^{-2} + \left\{E[m_i^4] - 4E[m_i^3]E[m_i]\right\}(-2)V[m_i]^{-3} \\ &\quad + \left\{6E[m_i^2]E[m_i]^2 - 3E[m_i]^4\right\}(-2)V[m_i]^{-3}. \quad \text{(A48)} \end{aligned}$$

B Proportion and Quantile

We now turn to the analysis of the proportion estimator inferred from the cumulative distribution function (cdf) and the associated quantile. The proportion estimator is the cross-sectional average of the indicator function $g(\hat{m}_i) = \mathbf{1}\{\hat{m}_i \leq m\}$ based on the OLS estimates $\hat{m}_i$ for the non-trimmed assets, $\hat{\Phi}(m) = \frac{1}{n_\chi}\sum_i \mathbf{1}\{\hat{m}_i \leq m\}\mathbf{1}_i^\chi$, while the quantile estimator is the inverse function $\hat{Q}(p) = \hat{\Phi}^{-1}(p)$.

The next proposition extends Proposition A.1 to the proportion and quantile.

Proposition A.2. As $n, T \to \infty$, such that $n = o(T^3)$,

$$\sqrt{n}\left(\hat{\Phi}(m) - \Phi(m) - \mathcal{B}_T(m)\right) \Rightarrow N\left(0, V[\hat{\Phi}(m)]\right), \quad \text{(A49)}$$

$$\sqrt{n}\left(\hat{Q}(p) - Q(p) + \frac{\mathcal{B}_T(Q(p))}{\phi(Q(p))}\right) \Rightarrow N\left(0, V[\hat{Q}(p)]\right), \quad \text{(A50)}$$

where $\mathcal{B}_T(m) = \frac{1}{2T}\psi^{(1)}(m)$, $V[\hat{\Phi}(m)] = \Phi(m)(1-\Phi(m)$, and $V[\hat{Q}(p)] = \frac{p(1-p)}{\phi(Q(p))^2}$.

Proof of Proposition A.2. The proof builds on our previous analysis. From Equation (A4), we have $E\left[1\{\hat{m}_i \leq m\}\right] = P\left[m_i + \frac{1}{\sqrt{T}}\hat{\eta}_{i,T} \leq m\right]$. By using the results in Gourieroux, Laurent, and Scaillet (2000), Martin and Wilde (2001), and Gagliardini and Gourieroux (2011), we obtain:

$$\begin{aligned} P\left[m_i + \frac{1}{\sqrt{T}}\hat{\eta}_{i,T} \leq m\right] &= \Phi(m) - \frac{1}{\sqrt{T}}\phi(m)E[\hat{\eta}_{i,T}|m_i = m] \\ &\quad + \frac{1}{2T}\frac{d}{dm}(\phi(m)E[\hat{\eta}_{i,T}^2|m_i = m]) + o(1/T). \quad \text{(A51)} \end{aligned}$$

From Equation (A47), the bias expansion is such that: $E[\hat{\Phi}(m)] - \Phi(m) = \mathcal{B}_T(m) +$

$E\left[1\{\hat{m}_i \leq m\}(1-\mathbf{1}_i^{\mathcal{X}})\right] + o(1/T)$. We deduce the asymptotic normality of the proportion estimator by controlling the different terms and applying the CLT. To deduce the asymptotic normality of the quantile estimator, we use the Bahadur expansion for the quantile estimator at level $u \in (0,1)$: $\hat{Q}(p) - Q(p) = -\frac{1}{\phi(Q(p))}\left(\hat{\Phi}(Q(p)) - p\right)$. QED

As in the previous section, we can approximate the asymptotic bias using the Gaussian reference model.[10] With our bivariate Gaussian reference model, the term $\psi^{(1)}(m)$ in the bias is equal to

$$\begin{aligned}
\psi^{(1)}(m) &= \exp\left(\mu_s + \rho\sigma_s\left(\frac{m-\mu_m}{\sigma_m}\right) + \frac{1}{2}\sigma_s^2(1-\rho^2)\right)\phi(m)\left(\frac{\sigma_s\rho}{\sigma_m} - \frac{m-\mu_m}{\sigma_m^2}\right) \\
&= \exp\left(\mu_s + \frac{1}{2}\sigma_s^2\right)\frac{-1}{\sigma_m}\left(\frac{m-\mu_m-\rho\sigma_s\sigma_m}{\sigma_m}\right) \\
&\quad \times \frac{1}{\sigma_m}\varphi\left(\frac{m-\mu_m-\rho\sigma_s\sigma_m}{\sigma_m}\right). \qquad \text{(A52)}
\end{aligned}$$

III Analysis of the EIV Bias Adjustment

In this section, we provide additional information on the EIV bias adjustment obtained with the Gaussian reference model. As explained in the paper, this approach is appealing because the bias adjustment is available in closed form. It is also precisely estimated because of parsimony—it only depends on the five parameters of the normal distribution $\theta = (\mu_m, \sigma_m, \mu_s, \sigma_s, \rho)'$. These benefits are not shared by a fully nonparametric approach in which the bias is estimated via a nonparametric estimation of the second-order derivatives $\phi^{(2)}$ and $\psi^{(2)}$.[11]

An important question is whether the EIV bias obtained with the normal reference model provides a good approximation of the true bias (i.e., whether $bs_2^r(m) \approx bs_2(m)$). Two compelling arguments show that it is the case. First, Proposition III.1 shows that the true bias $bs_2(m)$ is a function of the second-order derivative of the true density ϕ. As long as ϕ peaks around its mean, this derivative takes negative values in the center and positive values in the tails—exactly like the function $bs_2^r(m)$. The two terms $bs_2(m)$ and $bs_2^r(m)$ only differ if ϕ is a mixture of distributions whose components have means

[10] The asymptotic bias takes the same form as the one in Jochmans and Weidner (2018) where they consider n parameters of interest directly drawn from a Gaussian distribution whose measurement errors decrease at a parametric rate $\sqrt{T}$. In their setting, they use other arguments based on the behaviour of the probability integral transform for their proofs. In a different context, Okui and Yanagi (2019) also derive an estimator of the cdf to examine the mean and autocorrelation of random variables.

[11] We can estimate the rth-derivative of a density ϕ by kernel smoothing (Bhattacharya (1967)). The rate of consistency of the derivative estimator equals $\sqrt{nh^{2r+1}}$ and is much slower than the rate $\sqrt{nh}$ for the density estimator. In other words, the higher-order derivatives are imprecisely estimated because the rate of consistency decreases with the derivative's order r.

extremely far away from one another. In this case, we have a trough instead of a peak around the mean.

Second, our extensive Monte-Carlo analysis calibrated on the data reveals that the bias-adjusted density captures the true density well (see Section IV). Our Monte-Carlo analysis resonates with the one performed by Silverman (1986) for the standard non-parametric density estimation without the EIV problem. He shows that the rule of thumb for the bandwidth choice, which relies on a normal reference model, is quite robust to departures from normality.

The reference model allows us to conduct a comparative static analysis of the EIV bias. As shown in Equation (A28), there are three key parameters that determine $bs_2^r(m)$: (i) the variance of the true measure σ_m^2, (ii) the average across funds of the variance of the estimated measure, measured as $\sigma_{\hat{m}}^2 = \frac{1}{T}E[S_i] = \frac{1}{T}\exp(\mu_s + \frac{1}{2}\sigma_s^2)$, and (iii) the correlation ρ between the true measure and estimation variance.

A higher value of σ_m^2 reduces the magnitude of the EIV bias because it makes the cross-sectional variation of the estimated measure more aligned with that of the true measure (i.e., the relative importance of m_i over noise increases). On the contrary, a higher value of $\sigma_{\hat{m}}^2$ makes the EIV bias more severe because the estimated measure becomes more volatile (i.e., the relative importance of noise over m_i increases). Finally, a higher value of $|\rho|$ keeps the shape of the bias unchanged, but creates asymmetry.

In Figure A1, we quantify these changes for the skill coefficient a_i. To begin, we compute $bs_2^r(m)$ in the benchmark case where the parameters of the reference model are obtained from our sample. The mean μ_m is set equal to 0.24% per month, the variance terms σ_m^2 and $\sigma_{\hat{m}}^2$ are equal to $\frac{0.0017}{100}$ and $\frac{0.0011}{100}$, and the correlation ρ reaches 0.21. Plugging these parameter values in Equation (A28), we find that the EIV bias adjustment requires a transfer of probability mass from the tails to the center equal to 15%. This proportion is obtained by integrating $bs_2^r(m)$ over the area for which $bs_2^r(m)$ takes negative values.

Next, we sequentially increase the values of (i) σ_m^2 from $\frac{0.0017}{100}$ to $\frac{0.0037}{100}$, (ii) $\sigma_{\hat{m}}^2$ from $\frac{0.0011}{100}$ to $\frac{0.0031}{100}$, and (iii) ρ from 0.21 to 0.44. We find that changes in the variance terms have a significant impact on the shape of the EIV bias. Panel A shows that increasing σ_m^2 reduces the transfer of probability from 15% to just 7%, while Panel B shows that increasing $\sigma_{\hat{m}}^2$ implies an increase in probability transfer from 15% to 26%. Finally, Panel C shows that increasing ρ implies that 87% of the probability transfer (0.13/0.15) is at the right of the mean (versus 75% in the baseline case (0.10/0.15)).

Please insert Figure A1 here

IV Monte-Carlo Simulations

A The Setup

We now conduct a Monte-Carlo analysis to evaluate the finite-sample properties of the estimated skill and scale distributions obtained with our nonparametric approach. We consider a hypothetical population of n funds with T return observations ($n = 1,000$, 2,500, 5,000, and 10,000; $T = 100$, 250, 500, and $1,000$). To model the fund return $r_{i,t}$ and its lagged size $q_{i,t-1}$, we use the baseline specification

$$r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}, \tag{A53}$$

along with an AR(1) model for the log size $lq_{i,t-1} = log(q_{i,t-1})$ to ensure the positivity of fund size,

$$lq_{i,t} = \theta_{lq_i} + \rho_{lq} lq_{i,t-1} + e_{lq_i,t}, \tag{A54}$$

where f_t is the vector of four factors (market, size, value, and momentum), $\theta_{lq_i} = \mu_{lq_i}(1 - \rho_{lq})$, and $\mu_{lq_i} = E[lq_{i,t-1}]$. The residual terms $\varepsilon_{i,t}$ and $e_{lqi,t}$ are drawn from a bivariate normal: $\varepsilon_{i,t} \sim N(0, \sigma^2_{\varepsilon_i})$, $e_{lq_i,t} \sim N(0, \sigma^2_{e_{lq_i}})$, where $\sigma^2_{e_{lq_i}} = (1 - \rho^2_{lq})\sigma^2_{lq_i}$ and $\sigma^2_{lq_i}$ is the variance of $lq_{i,t-1}$. We also account for the positive correlation between the fund residual and the innovation in fund size by setting $corr(\varepsilon_{i,t}, e_{lqi,t})$ equal to ρ.

To determine the values for the fund-specific parameters $\{a_i,\, b_i,\, \beta_i',\, \mu_{lq_i},\, \sigma^2_{lq_i}\}$, we randomly draw from the estimated vectors observed in our sample $\{\hat{a}_i,\, \hat{b}_i,\, \hat{\beta}_i',\, \hat{\mu}_{lq_i},\, \hat{\sigma}^2_{lq_i}\}$. This approach allows us to maintain the correlation structure between the different parameters, in particular between the skill coefficient, the scale coefficient, and the size parameters: $\mu_{lq_i} = \mu_{lq_i}(a_i, b_i)$, $\sigma^2_{lq_i} = \sigma^2_{lq_i}(a_i, b_i)$.[12] The remaining parameters are calibrated using the median values in the data, which yields $\rho_{lq} = 0.97$, $\rho = 0.20$, and $\sigma^2_{\varepsilon_i} = 0.0167^2$.

To reproduce the salient features of the skill and scale distributions, we rescale the estimated values of $\hat{a}_i$ and $\hat{b}_i$ to match the cross-sectional volatility reported in Table II of the paper (4.1% and 1.7% per year for a_i and b_i). The true distributions of the skill and scale coefficients are both non normal (the skewness is equal to 0.7 and 0.9, and the kurtosis is equal to 11.7 and 12.1). Therefore, our Monte-Carlo setting allows us to

[12] In particular, we capture the strong correlation between the skill and scale coefficients. Interestingly, this correlation has implications for modeling the prior distributions of a_i and b_i in an empirical Bayes setting. For instance, Pastor and Stambaugh (2012) elicit the joint prior distribution of a_i and b_i by setting their correlation equal to zero. Therefore, investors in their model believe that the variance of $\alpha_{i,t}$ is higher than the one inferred from an empirical Bayes prior. This initial belief implies a lower allocation to active funds which could persist for a long time.

examine the properties of the estimators when the Gaussian reference model (used for the EIV bias adjustment) differs from the true distributions.

Conditional on the values $\{\hat{a}_i, \hat{b}_i, \hat{\beta}_i', \hat{\mu}_{lq_i}, \hat{\sigma}^2_{lq_i}\}$ taken by each fund, we examine the properties of the estimators. For each iteration s ($s = 1, ..., 500$), we build the return and size time-series of each fund as follows. First, we draw the initial value of $lq_{i,0}(s)$ from its unconditional distribution: $lq_{i,0}(s) \sim N(\mu_{lq_i}, \sigma^2_{lq_i})$. Second, we draw the vector $f_1(s)$ from the realized values in the sample, and the innovations $\varepsilon_{i,1}(s)$ and $e_{lq_i,1}(s)$ from the bivariate normal. Third, we construct the fund gross return and log size at time 1 as

$$\begin{aligned} r_{i,1}(s) &= a_i - b_i q_{i,0}(s) + \beta_i' f_1(s) + \varepsilon_{i,1}(s), \\ lq_{i,1}(s) &= \theta_{lq,i} + \rho_{lq} lq_{i,0}(s) + e_{lq_i,1}(s), \end{aligned} \tag{A55}$$

where $q_{i,0}(s) = \exp(lq_{i,0}(s))$. Fourth, we repeat the two previous steps for each time t ($t = 2, ..., T$), we obtain the entire time-series for the fund gross return and size: $r_{i,1}(s), ..., r_{i,T}(s)$, $q_{i,0}(s), ..., q_{i,T-1}(s)$. Fifth, we apply our nonparametric approach to compute the bias-adjusted density $\tilde{\phi}(s)$ and a set of several distribution characteristics that include the mean, volatility, skewness, and the proportion of funds with a positive measure $\pi^+ = 1 - \Phi(0)$. Finally, we repeat the entire procedure across all S iterations.

To assess the performance of the bias-adjusted density $\tilde{\phi}$, we compute the Mean Integrated Squared Error (MISE) defined as

$$MISE = \int [\sigma^2(m) + bs(m)^2] dm, \tag{A56}$$

where the bias and variance functions are given by

$$bs(m) = \frac{1}{S} \sum_{s=1}^{S} \tilde{\phi}(m; s) - \phi(m), \tag{A57}$$

$$\sigma^2(m) = \frac{1}{S} \sum_{s=1}^{S} \left(\tilde{\phi}(m; s) - \frac{1}{S} \sum_{s=1}^{S} \tilde{\phi}(m; s) \right)^2 . \tag{A58}$$

For the moment/proportion estimator $\tilde{\varphi}$ ($\tilde{\varphi} = \tilde{M}$, $\widetilde{SD}$, $\widetilde{Sk}$, $\tilde{\pi}^+$), we compute the Mean Squared Error (MSE) as

$$MSE(\tilde{\varphi}) = \sigma^2(\tilde{\varphi}) + bs^2(\tilde{\varphi}), \tag{A59}$$

where the bias and the variance terms are given by

$$bs(\tilde{\varphi}) = \frac{1}{S}\sum_{s=1}^{S}\tilde{\varphi}(s) - \varphi, \tag{A60}$$

$$\sigma^2(\tilde{\varphi}) = \frac{1}{S}\sum_{s=1}^{S}\left(\tilde{\varphi}(s) - \frac{1}{S}\sum_{s=1}^{S}\tilde{\varphi}(s)\right)^2. \tag{A61}$$

B Main Results

In Table AI, we report the MISE and its two components (integrated squared bias and variance) for the skill distribution. In Panel A, we compute the MISE of the bias-adjusted density $\tilde{\phi}(m)$ for the baseline choice of the optimal bandwidth $h^* \sim c_2^{-1/3}(n/T)^{-1/3}$ (shown in Equation (A16)). In Panel B, we repeat the analysis for the alternative choice of the optimal bandwidth under which $h^* \sim c_1^{-1/5}n^{-1/5}$. Finally, Panel C reports the MISE of the estimated density $\hat{\phi}(m)$ obtained with the standard approach which does not adjust for the bias.

Our analysis reveals two main insights. First, accounting for the EIV bias improves the estimation of the true distribution $\phi(m)$. To illustrate, we consider the scenario where $n = 2,500$ and $T = 250$, which is representative of our actual sample after trimming (i.e., $\sum_{i=1} \mathbf{1}_i^{\chi} \approx 2,500$ and $\frac{1}{n_\chi}\sum_{i=1} \mathbf{1}_i^{\chi} T_i \approx 250$) . We find that the MISE of $\hat{\phi}(m)$ is nearly two times larger than the level observed for $\tilde{\varphi}(m)$ with the baseline bandwidth (4.96 vs 9.06). Second, our nonparametric approach yields a stronger performance under the baseline choice for the optimal bandwidth—in all scenarios, using the alternative bandwidth choice produces a higher MISE.

In Table AII, we examine the performance of the moment and proportion estimators for the skill distribution. Panel A shows the MSE and its two components (bias and standard deviation) of each bias-adjusted estimator obtained via a numerical integration of $\tilde{\phi}(m)$ (using the baseline bandwidth). Panel B reports the same statistics for the bias-adjusted estimators obtained with the analytical approach described in Section II. For comparison, Panel C reports the bias-unadjusted estimators (obtained via a numerical integration of $\hat{\phi}(m)$).

The results show that the bias-adjusted estimators perform better when the numerical integration is used. In most cases, it produces a lower MSE than the one obtained with the analytical formulas. We also find that the unadjusted estimators are markedly biased. When $n = 2,500$ and $T = 250$, the bias for the volatility and the unadjusted proportion is equal to 1.08% per year and -5.52%, respectively. In contrast, our nonparametric approach reduces the bias for all quantities. Overall, these findings highlight the importance of controlling for the bias.

Next, we turn to the analysis of the scalability distribution. Tables AIII and AIV report the MISE of the estimated density and the MSE of the moment and proportion estimators. Similar to the skill coefficient, we find that $\tilde{\varphi}(m)$ outperforms $\hat{\phi}(m)$. If $n = 2,500$ and $T = 250$, the difference in MISE between the two estimated densities is equal to 11.47 (28.58 vs 17.01). The bias adjustment is also important for the other estimators. For instance, the standard approach underestimates the proportion of funds with a positive scale coefficient by 6.8% (in absolute terms).

To sum up, the Monte Carlo analysis yields three main insights. First, the EIV bias has a notable impact on the different estimators and thus cannot be ignored. Second, the baseline choice for the optimal bandwidth produces a lower MISE for the bias-adjusted density. Third, the numerical approach generally outperforms the analytical approach. These results justify the use of the optimal bandwidth in Equation (A16) and the numerical approach for the empirical analysis of the paper.

Please insert Tables AI to AIV here

C Simulations with Uncorrelated Skill and Scalability

The asymptotic distribution of the OLS estimators $\hat{a}_i$ and $\hat{b}_i$ implies that they are correlated at the fund level. If, for simplicity, we omit the factors f_t, we have

$$\sqrt{T}\begin{bmatrix} \hat{a}_i - a_i \\ \hat{b}_i - b_i \end{bmatrix} \Rightarrow N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{E[q_{i,t-1}^2]}{V[q_{i,t-1}]}\sigma_{\varepsilon_i}^2 & \frac{E[q_{i,t-1}]}{V[q_{i,t-1}]}\sigma_{\varepsilon_i}^2 \\ \frac{E[q_{i,t-1}]}{V[q_{i,t-1}]}\sigma_{\varepsilon_i}^2 & \frac{1}{V[q_{i,t-1}]}\sigma_{\varepsilon_i}^2 \end{bmatrix}\right), \tag{A62}$$

where $cov(\sqrt{T}\hat{a}_i, \sqrt{T}\hat{b}_i) = \frac{E[q_{i,t-1}]}{V[q_{i,t-1}]}\sigma_{\varepsilon_i}^2 > 0$. Therefore, one concern is that the strong cross-sectional correlation between $\hat{a}_i$ and $\hat{b}_i$ observed in the data is mechanically driven by the fund-level correlation between $\hat{a}_i$ and $\hat{b}_i$. To address this concern, we consider a world where a_i and b_i are uncorrelated across funds. Consistent with this assumption, we show that the cross-sectional correlation between $\hat{a}_i$ and $\hat{b}_i$ is equal to zero (even though the fund-level correlation between $\hat{a}_i$ and $\hat{b}_i$ is positive).

We consider a simple modification of the Monte-Carlo setup in which a_i and b_i are uncorrelated across funds. We draw the true coefficients a_i and b_i of each fund i ($i = 1, ..., 2,500$) independently from the estimated vectors observed in our sample (rescaled to match the cross-sectional volatility in Table II of the paper). If a_i and b_i are positive, we assume that the average size $E[q_{i,t-1}]$ is equal to $\frac{a_i}{2b_i}$ (as in the model of Berk and Green (2004), and that $\sigma_{lq_i}^2$ is proportional to μ_{lq_i} (by a factor k calibrated on the data). These two assumptions provide a simple way to model the link that exists between skill, scale, and size. Specifically, we have $\mu_{lq,i} + \frac{1}{2}\sigma^2_{lq_i} = \log\left(\frac{a_i}{2b_i}\right) \Rightarrow \left(1+\frac{1}{2}k\right)\mu_{lq,i} = \log\left(\frac{a_i}{2b_i}\right) \Rightarrow \mu_{lq,i} = \log\left(\frac{a_i}{2b_i}\right) / \left(1+\frac{1}{2}k\right)$. With a log-normally distributed size, we can then compute the parameters of the asymptotic distribution in Equation (A60) as $E[q^2_{i,t-1}] = e^{2\mu_{lq,i}+2k\mu_{lq,i}}$ and $V[q_{i,t-1}] = E[q^2_{i,t-1}] - (E[q_{i,t-1}])^2$. Otherwise, if a_i and b_i are negative, we measure $E[q_{i,t-1}]$, $E[q^2_{i,t-1}]$, and $V[q_{i,t-1}]$ as the median values among funds for which $\hat{a}_i$ or $\hat{b}_i$ are negative.

For each iteration s ($s = 1, ..., 500$), we draw $[\hat{a}_i(s), \hat{b}_i(s)]'$ from the asymptotic distribution of each fund in Equation (A60). We then compute the average fund-level correlation (FLC) between $\hat{a}_i$ and $\hat{b}_i$ as

$$FLC(\hat{a}_i, \hat{b}_i) = \frac{1}{n}\sum_i \left(\frac{1}{S}\sum_s (\hat{a}_i(s) - a_i)\left(\hat{b}_i(s) - b_i\right)\right), \tag{A63}$$

and the average cross-sectional correlation (CSC) as

$$CSC(\hat{a}_i, \hat{b}_i) = \frac{1}{S}\sum_s \left(\frac{1}{n}\sum_i (\hat{a}_i(s) - \bar{a}(s))\left(\hat{b}_i(s) - \bar{b}(s)\right)\right), \tag{A64}$$

where $\bar{a}(s) = \frac{1}{n}\sum_i \hat{a}_i(s)$ and $\bar{a}(s) = \frac{1}{n}\sum_i \hat{b}_i(s)$. Consistent with the theoretical predictions, we find that $FLC(\hat{a}_i, \hat{b}_i)$ is equal to 0.18, whereas $CSC(\hat{a}_i, \hat{b}_i)$ is essentially equal to zero (i.e., $CSC(\hat{a}_i, \hat{b}_i)$ =0.00004).

V Mutual Fund Dataset

A Construction of the Dataset

We now provide additional information on the construction of the mutual fund dataset. To begin, we collect monthly data on net returns and size, as well as annual data on fees, turnover, and investment objectives from the CRSP database between January 1975 and December 2019 (540 observations). We measure the monthly gross return of each fund as the sum of its monthly net return and fees. The net return is computed as a value-weighted average of the net returns across all shareclasses using their beginning-of-month total net asset values. The monthly fees are defined as the value-weighted average of the most recently reported annual fees across shareclasses divided by 12. We eliminate the monthly gross return observation when (i) the monthly net return is below -100% or above 100%, or when (ii) the monthly fees are below 2.5 bps (0.3% per year) or above 83 bps (10% per year). We measure fund size by taking the sum of the beginning-of-month net asset values across all shareclasses. We apply a linear interpolation to fill in missing observations when funds report size on a quarterly basis. We also adjust size for inflation by expressing all numbers in January 1, 2000 dollars (see Berk and van Binsbergen (2015)). Finally, we correct for reporting errors for the TNA.[13]

We apply a set of filters before conducting the empirical analysis. First, we remove all funds that are classifed as passive or closed for more than a third of the observations using (i) the index fund indicator (letter B, D, or E), (ii) the ETF indicator (letter F or N), (iii) and the closed fund indicator (letter N). Therefore, our sample focuses on open-end, actively managed funds with a well-defined equity style (as described below), and a weight invested in equities above 80%. Second, we eliminate funds if they are tiny, i.e., if their size is below minimum size of $15 million for more than a third of the observations (similar to Chen et al. (2004), Pastor, Stambaugh, and Taylor (2015)). Third, we delete the following-month return after a missing return observation because CRSP fills this with the cumulated return since the last nonmissing return. Fourth, we run a correlation analysis to eliminate duplicates, i.e., funds for which the return correlation is above 0.99 (using a minimum of 12 monthly observations).

To benchmark each fund, we use the four-factor model of Cremers, Petajisto, and Zitzewitz (2012) which includes the vector $f_t = (r_{m,t}, r_{smb,t}, r_{hml,t}, r_{mom,t})'$, where $r_{m,t}$, $r_{smb,t}$, $r_{hml,t}$, and $r_{mom,t}$ capture the excess returns of the market, size, value, and momentum factors. This model departs from the traditional model of Carhart (1997) in two respects: (i) the market factor is proxied by the excess return of the SP500 (instead of the CRSP index), and (ii) the size and value factors are index-based and measured as the return difference between the Russell 2000 and SP500, and between the Russell 3000 Value and Russell 3000 Growth. Because the index-based returns for size and value are not available between January 1975 and December 1978, we replace them with the values of the size and value factors obtained from Ken French's website (focusing on the period January 1979-December 2018 does not change our main results). For the momentum factor, we use data obtained from Ken French's website.

The motivation for using this model is that it correctly assigns a zero alpha to the SP500 and Russell 2000. Both indices cover about 85% of the total market capitalization and are widely used as benchmarks by mutual funds. On the contrary, the Carhart model fails to price these indices—for one, the Russell 2000 has a negative alpha of -2.4% per

[13]For instance, we find more than 1,500 observations in CRSP for which the TNA of a given shareclass jumps (or is reduced) by a factor higher than 3 in a given month before going right back to the same value the following month.

year over the period 1980-2005 (Cremers, Petajisto, and Zitzewitz (2012)). Therefore, small cap funds that use this index as a benchmark are likely to be classified as unskilled under the Carhart model.

We obtain our final universe of funds after applying the selection rule in Equation (8) of the paper. We follow Gagliardini, Ossola, and Scaillet (2016) and select funds for which the condition number of the matrix $\hat{Q}_{x,i}$ is below 15 and the number of monthly observations is above 60. These selection criteria produce a final universe of 2,427 funds. To apply our nonparametric approach, we compute the asymptotic variance of each fund measure using a lag of three months ($L = 3$). To mitigate the impact of outliers on the vector $\hat{\theta}$ of estimated parameters in the reference model, we also exclude the values for $\hat{m}_i$ and $\hat{s}_i$ whose cross-sectionally standardized values are above 10.

B Construction of the Fund Groups

To classify funds into the small cap and large cap groups, we proceed as follows. At the start of each month, we classify each fund in different style groups using the style information provided by Lipper. If this information is missing, we use the investment objectives reported by Strategic Insight, Wiesenberger, and CRSP in a sequential manner. Table AV provides the list of the 32 styles across the different data providers which are used for forming our final universe of equity funds. In addition, it shows the mapping between the 32 styles and the small/large cap dimensions. A value of: (i) 1 refers to a small cap fund, (ii) 2 refers to a mid cap fund, and (iii) 3 refers to a large cap fund. A fund is included in a given group (small cap, large cap) if its style corresponds to that of the group for the majority of its monthly observations.

Please insert Table AV here

For the turnover groups, we sort funds in three categories (low, medium, and high turnover) based on their average monthly turnover. To measure the monthly turnover of each fund, we follow Pastor, Stambaugh, and Taylor (2018) and use the most recently observed ratio of min(buys,sells) on fund size.

Finally, we construct the set of broker and direct sold funds using the procedure proposed by Del Guercio and Reuter (2014) and Sun (2020). At the start of each month, we only select shareclasses that are sold to retail investors. We consider each shareclass as direct sold if it charges no front or back load and has an annual distribution fee (12b-1 fees) of no more than 0.25% per year. Otherwise, we consider it as broker sold. Aggregating across shareclasses, the fund is then considered as broker sold (direct sold) for that particular month if at least 75% of its assets are broker sold (direct sold). A fund is included in a given group (broker sold, direct sold) if it belongs to it for the majority of its monthly observations.

VI Additional Results

A Derivation of the Specification Test

In this section, we derive a new specification test to confirm the validity of our empirical results. Our objective is to test the null hypothesis $H_{0,i}$ that our baseline linear specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ is correct for each fund. Our specification test follows the strategy of a Hausman test which evaluates the difference beween two consistent estimators under the null hypothesis of well specification (Hausman (1978)). In our context, we compare the linear estimator of the gross alpha $\hat{\alpha}_{i,t}$ with its model-free version proposed by Berk and van Binsbergen (2015) and denoted by $\hat{\alpha}_{i,t}^{bvb}$. Whereas the two estimators converge to the same quantity under the null hypothesis $H_{0,i}$, they converge to different quantities under the alternative hypothesis of misspecification.

We consider our baseline model

$$r_{i,t} = a_i - b_i q_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}, \tag{A65}$$

and want to test this specification against the extended time-series regression model

$$r_{i,t} = a_i - b_i q_{i,t-1} + c_i' p_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}, \tag{A66}$$

where $p_{i,t-1}$ is a vector of variables that are omitted in our baseline specification and orthogonal to the factors f_t and error $\varepsilon_{i,t}$. To ease the presentation, we do not explictly include the small-sample bias correction which has no impact on the asymptotic analysis (see Section I.B).

The linear estimator of the gross alpha under Equation (A65) is

$$\hat{\alpha}_{i,t} = \hat{a}_i - \hat{b}_i q_{i,t-1}, \tag{A67}$$

where $\hat{a}_i = e_1'\hat{\gamma}_i$, $\hat{b}_i = e_2'\hat{\gamma}_i$, $\hat{\gamma}_i = \hat{Q}_{x,i}^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_{i,t}r_{i,t}$, $\hat{Q}_{x,i} = \frac{1}{T_i}\sum_t I_{i,t}x_{i,t}x_{i,t}'$, $x_{i,t} = (1, -q_{i,t-1}, f_t')'$, and e_1 (e_2) is a vector with one in the first (second) position and zeros elsewhere. The model-free estimator of the gross alpha is given by

$$\hat{\alpha}_{i,t}^{bvb} = r_{i,t} - \hat{\beta}_i' f_t, \tag{A68}$$

where $\hat{\beta}_i = E_2'\tilde{Q}_x^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t r_{i,t}$, E_2 is a selection matrix that selects the lower K_f-subvector of coefficients, $\tilde{Q}_{x,i} = \frac{1}{T_i}\sum_t I_{i,t}x_t x_t'$, and $x_t = (1, f_t')'$.

We build the difference $\Delta\hat{\alpha}_{i,t} = \hat{\alpha}_{i,t}^{bvb} - \hat{\alpha}_{i,t}$ when $I_{i,t} = 1$ and let $\Delta\hat{\alpha}_i$ be the $T_i \times 1$ vector of such differences for fund i at dates with $I_{i,t} = 1$. We then select a $p \times 1$ vector of variables $w_{i,t-1}$ and regress it onto $x_{i,t}$ to obtain the residuals $\tilde{w}_{i,t-1}$. Let us consider the auxiliary time-series regression:

$$\Delta\hat{\alpha}_{i,t} = \tilde{w}_{i,t-1}'d_i + e_{i,t}, \tag{A69}$$

where d_i is the parameter vector and $e_{i,t}$ is the error term. The R^2 of this regression is

$$R_i^2 = 1 - \frac{\Delta\hat{\alpha}_i' M_{\tilde{W}_i}\Delta\hat{\alpha}_i}{\Delta\hat{\alpha}_i'\Delta\hat{\alpha}_i} = \frac{\Delta\hat{\alpha}_i' P_{\tilde{W}_i}\Delta\hat{\alpha}_i}{\Delta\hat{\alpha}_i'\Delta\hat{\alpha}_i} \tag{A70}$$

where $P_{\tilde{W}_i} = \tilde{W}_i(\tilde{W}_i'\tilde{W})_i^{-1}\tilde{W}_i' = I_{T_i} - M_{\tilde{W}_i'}$ and $\tilde{W}_i$ is the $T_i \times p$ matrix of the available values for $\tilde{w}_{i,t-1}$.[14]

We use the quantity $T_i R_i^2$ as the test statistic for the null hypothesis $H_{0,i}$. We now derive the asymptotic distribution under Equation (A65) or (A66). Suppose first that Equation (A65) holds in the data, i.e., the linear specification with lagged size is correctly specified. Then $\hat{\alpha}_{i,t}^{bvb} = a_i - b_i q_{i,t-1} + \varepsilon_{i,t} - f_t'(\hat{\beta}_i - \beta_i)$ and

$$\Delta\hat{\alpha}_{i,t} = \hat{\alpha}_{i,t}^{bvb} - \hat{\alpha}_{i,t} = \varepsilon_{i,t} - (\hat{a}_i - a_i) + (\hat{b}_i - b_i)q_{i,t-1} - f_t'(\hat{\beta}_i - \beta_i) = \varepsilon_{i,t} - x_{i,t}'(\tilde{\gamma}_i - \gamma_i), \tag{A71}$$

where $\tilde{\gamma}_i = (\hat{a}_i, \hat{b}_i, \hat{\beta}_i')'$ is a consistent estimator of γ_i. Hence, we have in vector notation: $\Delta\hat{\alpha}_i = \varepsilon_i - X_i(\tilde{\gamma}_i - \gamma_i)$, where X_i is the $T_i \times (K_f + 2)$ matrix of the available values for $x_{i,t}$. Using $\tilde{W}_i'X_i = 0$ and assuming conditional homoscedasticity for the error term $\varepsilon_{i,t}$, we obtain:

$$T_i R_i^2 = \frac{(\frac{1}{\sqrt{T_i}}\varepsilon_i'\tilde{W}_i)(\frac{1}{T_i}\tilde{W}_i'\tilde{W}_i)^{-1}(\frac{1}{\sqrt{T_i}}\tilde{W}_i'\varepsilon_i)}{\frac{1}{T_i}\Delta\hat{\alpha}_i'\Delta\hat{\alpha}_i} \Rightarrow \chi^2(p), \tag{A72}$$

which holds because we have $plim\frac{1}{T_i}\Delta\hat{\alpha}_i'\Delta\hat{\alpha}_i = plim\frac{1}{T_i}\varepsilon_i'\varepsilon_i = \sigma_i^2$, $\frac{1}{\sqrt{T_i}}\tilde{W}_i'\varepsilon_i \Rightarrow N(0, \sigma_i^2 Q_{\tilde{W}_i})$ and $Q_{\tilde{W}_i} = plim\frac{1}{T_i}\tilde{W}_i'\tilde{W}_i = plim\frac{1}{T_i}\sum_t I_{i,t}\tilde{w}_{i,t-1}\tilde{w}_{i,t-1}'$.

Suppose now that the linear model is misspecified and data are generated according to the model in Equation (A66). Consider the linear projection of $p_{i,t-1}$ onto the constant and $q_{i,t-1}$, with residual $\tilde{p}_{i,t-1}$, and let $c_i'p_{i,t-1} = \xi_i + \rho_i q_{i,t-1} + c_i'\tilde{p}_{i,t-1}$ (by our assumption, $\tilde{p}_{i,t-1}$ is also the residual in the regression of $p_{i,t-1}$ onto $x_{i,t}$). By plugging into Equation

[14] Note that Equation (A67) does not include the constant, and the R^2 is defined accordingly. Including a constant and modifying the definition of R^2 does not change the behaviour of the test statistic under the null hypothesis and its consistency under the alternative.

(A66), we get $r_{i,t} = a_i^* - b_i^* q_{i,t-1} + c_i' \tilde{p}_{i,t-1} + \beta_i' f_t + \varepsilon_{i,t}$, with pseudo-true parameter values $a_i^* = a_i + \xi_i$ and $b_i^* = b_i - \rho_i$. Then, we have

$$\hat{\alpha}_{i,t}^{bvb} = a_i^* - b_i^* q_{i,t-1} + c_i' \tilde{p}_{i,t-1} + \varepsilon_{i,t} - f_t'(\hat{\beta}_i - \beta_i), \tag{A73}$$

$$\hat{\alpha}_{i,t} = \hat{a}_i^l - \hat{b}_i^l q_{i,t-1}, \tag{A74}$$

where $\hat{a}_i^l = e_1'(X_i'X_i)^{-1}X_i'(X_i\gamma_i^* + \varepsilon_i + \tilde{P}_i c_i) = a_i^* + e_1'(X_i'X_i)^{-1}X_i'(\varepsilon_i + \tilde{P}_i c_i)$, $\hat{b}_i^l = b_i^* + e_2'(X_i'X_i)^{-1}X_i'(\varepsilon_i + \tilde{P}_i c_i)$, $\gamma_i^* = (a_i^*, b_i^*, \beta_i')'$, and $\tilde{P}_i$ is the matrix of the observations of the variables $\tilde{p}_{i,t-1}$ when $I_{i,t} = 1$. Combining Equations (A73)-(A74), we have

$$\Delta\hat{\alpha}_{i,t} = \varepsilon_{i,t} - (\hat{a}_i^l - a_i^*) + (\hat{b}_i^l - b_i^*)q_{i,t-1} - f_t'(\hat{\beta}_i - \beta_i) + c_i'\tilde{p}_{i,t-1}, \tag{A75}$$

or, in vector notation, $\hat{\Delta}\alpha_i = \varepsilon_i - X_i(\tilde{\gamma}_i - \gamma_i^*) + \tilde{P}_i c_i$, where $\tilde{\gamma}_i$ is a consistent estimator of γ_i^*. Then, we have:

$$plim\ \frac{1}{T_i}\tilde{W}_i'\Delta\hat{\alpha}_i = plim\ \frac{1}{T_i}\tilde{W}_i'\varepsilon_i + (plim\ \frac{1}{T_i}\tilde{W}_i'\tilde{P}_i)c_i = Q_{\tilde{W}_i}\Lambda_i c_i, \tag{A76}$$

where $\Lambda_i = plim\ (\frac{1}{T_i}\tilde{W}_i'\tilde{W}_i)^{-1}\frac{1}{T_i}\tilde{W}_i'\tilde{P}_i$ is the regression coefficient matrix of $\tilde{p}_{i,t-1}$ onto $\tilde{w}_{i,t-1}$, i.e., Λ_i is the coefficient vector associated with $z_{i,t-1}$ in a regression of $p_{i,t-1}$ onto $x_{i,t}$ and $w_{i,t-1}$: $p_{i,t-1} = \delta_i' x_{i,t} + \Lambda_i' w_{i,t-1} + \eta_{i,t}$, where $\eta_{i,t}$ is the regression error with variance V_{η_i}. By using $plim\ \tilde{\gamma}_i - \gamma_i^* = 0$, we have

$$\begin{aligned} plim\ \frac{1}{T_i}\Delta\hat{\alpha}_i'\Delta\hat{\alpha}_i &= plim\frac{1}{T_i}[\varepsilon_i + \tilde{P}_i c_i]'[\varepsilon_i + \tilde{P}_i c_i] = \sigma_i^2 + c_i'(plim\ \frac{1}{T_i}\tilde{P}_i'\tilde{P}_i)c_i \\ &= \sigma_i^2 + c_i'(\Lambda_i' Q_{\tilde{W}_i}\Lambda_i + V_{\eta_i})c_i, \end{aligned} \tag{A77}$$

which implies that

$$plim\ R_i^2 = \frac{c_i'(\Lambda_i' Q_{\tilde{W}_i}\Lambda_i)c_i}{\sigma_i^2 + c_i'(\Lambda_i' Q_{\tilde{W}_i}\Lambda_i + V_{\eta_i})c_i}. \tag{A78}$$

Based on Equation (A78), we know that the test based on $T_i R_i^2$ is consistent, namely $T_i R_i^2$ diverges in large samples and the power of the test approaches one asymptotically under misspecification. As long as $\Lambda_i c_i \neq 0$, the vector $w_{i,t-1}$ captures the effect of the omitted component $p_{i,t-1}' c_i$ and is therefore informative about the source of misspecification of the linear specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$. If the omitted variables $p_{i,t-1}$ coincide with the chosen $w_{i,t-1}$ (i.e., $p_{i,t-1} = w_{i,t-1}$), we automatically obtain that $\Lambda_i c_i \neq 0$ if $|corr(p_{i,t-1}' c_i, q_{i,t-1})| < 1$, and the consistency of the test follows.

Applying this theoretical framework, we consider two sets of variables for $w_{i,t}$. First, we include the ratio of industry size to total market capitalization to capture changes in industry competition, and aggregate turnover to capture changes in the level of mispricing in capital markets (see Pastor, Stambaugh, and Taylor (2015, 2018)). Second, we include higher order terms of fund size ($q^2_{i,t-1}$ and $q^3_{i,t-1}$) to capture nonlinearities in the relationship between the gross alpha and fund size. For each fund, we then test the null hypothesis $H_{0,i}$ that the linear specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ is correct.

For the first set of variables, we reject the null hypothesis only 13.0% of the times at the 5%-significance level. In other words, $T_i R_i^2$ is larger than the 95%-quantile of the $\chi^2(p)$ distribution for only 12.7% of the funds. Furthermore, we find that 29.3% of these funds can be classified as false discoveries ($H_{0,i}$ is rejected whereas it is true) using the approach proposed by Barras, Scaillet, and Wermers (2010). Turning to the analysis of the second set of variables, we obtain similar results—we reject $H_{0,i}$ for 14.1% of the funds (at the 5% level), among which more than 26.8% are false discoveries.

B Validity of the Panel Approach

In this section, we formally test whether the panel approach that imposes a constant scale coefficient across funds ($b_i = b$) is consistent with the data. To this end, we use the test of slope homogeneity developed by Pesaran and Yamagata (2008) for large panels. The null hypothesis is H_0: $b_i = b$ for $i = 1, ..., n$ against the alternative hypothesis H_1: $b_i \neq b_j$ for a non-zero fraction of pairwise slopes for $i \neq j$.

We denote by r_i and q_i the T_i-vectors of the fund gross excess returns and lagged fund sizes and by Z_i the $T_i \times (K_f + 1)$ matrix of available values for $x_t = (1, f_t')'$. The idea of the test is to investigate the dispersion of individual slope estimates from a suitable pooled estimate. We define the weighted sum of squared deviations:

$$\hat{S} = \sum_i (\hat{b}_i - \hat{b}_{WFE})^2 \frac{q_i' M_i q_i}{\hat{\sigma}_i^2}, \tag{A79}$$

where $M_i = I_{T_i} - Z_i (Z_i' Z_i)^{-1} Z_i'$ is the projection matrix, I_{T_i} is the $T_i \times T_i$ identity matrix, $\hat{b}_i = (q_i' M_i q_i)^{-1} q_i' M_i r_i$ is the estimated scale coefficient of each fund, $\hat{b}_{WFE} = \left(\sum_i \frac{q_i' M_i q_i}{\hat{\sigma}_i^2}\right)^{-1} \left(\sum_i \frac{q_i' M_i r_i}{\hat{\sigma}_i^2}\right)$ is the weighted fixed effect pooled estimate, $\hat{\sigma}_i^2$ is the variance estimate defined as $\frac{(r_i - \hat{b}_{FE} q_i)' M_i (r_i - \hat{b}_{FE} q_i)}{T_i - K - 1}$, and $\hat{b}_{FE} = (\sum_i q_i' M_i q_i)^{-1} \sum_i q_i' M_i r_i$ is the standard fixed effect pooled estimate. Pesaran and Yamagata (2008) show that under the null hypothesis H_0 the test statistic

$$\hat{\Delta} = \sqrt{n}\left(\frac{\frac{1}{n}\hat{S} - 1}{\sqrt{2}}\right) \tag{A80}$$

is asymptotically distributed as a standard Gaussian random variable when $n, T \to \infty$ such that $\sqrt{n}/T_{\min}^2 \to 0$ with $T_{\min} = \min_{1\leqslant i\leqslant n} T_i$. Therefore, we can build the chi-square test statistic $\hat{\Delta}^2$ which is asymptotically distributed as a chi-square random variable χ_1^2 with one degree of freedom.[15]

We examine two specifications for the panel regression: (i) the linear specification $\alpha_{i,t} = a_i - bq_{i,t-1}$, and (ii) the log specification $\alpha_{i,t} = a_i - b\log(q_{i,t-1})$.[16] We also conduct the test in the entire population and within each group (small/large cap, low/high turnover, broker/direct sold). Examining each fund group separately allows us to determine whether grouping funds into well-defined categories absorbs the heterogeneity. Our results reveal that the test of homogeneity is always strongly rejected, i.e., for each specification (size, log size), we reject H_0 with probability one. Furthermore, the null hypothesis of homogeneous coefficients is also rejected with probability one in each fund group. Therefore, forming groups is not sufficient to absorb the large heterogeneity in a_i and b_i.

C Survivorship and Reverse Survivorship Bias

In this section, we examine the impact of the survivorship and reverse survivorship bias. Our empirical analysis does not require that the funds remain alive until the end of the sample in 2019. In other words, our original sample includes all both living and dead funds. However, our fund selection rule requires that each fund has a minimum of 60 monthly observations ($T_{\min} = 60$) to be included in our final sample. Our results could therefore be subject to a survivorship bias if unskilled funds (a_i<0) disappear early. To examine this issue, we repeat our analysis across different thresholds for $T_{\min}$ ranging from 12 to 60. Panel A of Table AVI shows that our main results are not driven by the survivorship bias—the skill distribution remains largely unchanged as $T_{\min}$ changes from 60 to 12.

[15] The requirement on the relative rate between n and T_i, namely $n = o(T_{\min}^4)$ for the asymptotic validity of the testing procedure is weak and matches the time-series and cross-sectional sample sizes in our application since $n_\chi = 2,321$ is much smaller that $T_{\min}^4$ =60^4 = 12,960,000.

[16] In the logarithmic specification, the intercept loses its interpretation as a first-dollar alpha (i.e, it corresponds to the alpha when $q_{i,t-1} = 1$ instead of 0). In addition, the intercept depends on the measurement unit (e.g, \$1 or \$1M). The invariance to size denomination is an advantage of the linear specification.

It is a priori tempting to choose $T_{\min} = 12$ (instead of 60) to mitigate the survivorship bias and offer an improved estimation of the skill distribution. However, reducing $T_{\min}$ may not be optimal because it potentially increases the severity of the reverse survivorship bias (i.e., the reported skill could be biased downwards). The reverse survivorship bias arises because some skilled funds (a_i>0) may perform unexpectedly poorly and disappear early. For these funds, the estimated skill is lower than the true level because it is computed based on unusually low return observations (Linnainmaa (2013)). By reducing $T_{\min}$, we increase the likelihood of including these funds in the sample.

To examine this issue, we compare the skill distributions among the disappearing funds for $T_{\min} = 60$ and 108. The assumption is that unskilled funds tend to disappear early (during the first five years). In this case, the difference between the two distributions captures the impact of the reverse survivorship bias, i.e., it should decrease with $T_{\min}$ as we exclude a larger number of skilled funds that disappear after unexpected poor performance. Panel B shows that the difference in the proportion of skilled funds equals 4.1% as $T_{\min}$ decreases from 108 to 60. This number represents 85% of the proportion difference when reducing $T_{\min}$ from 60 to 12 in Panel A. This back-of-the-envelope calculation suggests that the reduction in skill observed for $T_{\min} = 12$ is mainly due to the reverse survivorship bias. Motivated by these results, we therefore choose $T_{\min} = 60$ in our baseline analysis.

Please insert Table AVI here

D Alternative Asset Pricing Models

Our empirical results potentially depend on the choice of the asset pricing model. To address this issue, we repeat our analysis using the four-factor model of Carhart (1997) which contains the same factors as the model of Cremers, Petajisto, and Zitzewitz (2012) except that the market, size, and value factors are not computed from tradable indices. We also consider the five-factor model of Fama and French (2015) which includes the market, size, value, profitability, and investment factors.[17]

Table AVII shows that the skill and scalability distributions remain qualitatively unchanged with the Carhart model. The skill coefficient is equal to 2.4% per year on average, and is positive for 78.5% of the funds (vs 3.0% and 83.1% for the baseline results). The scale coefficient is, on average, equal to 1.3% per year and 80.8% of the funds face diseconomies of scale (vs 1.3% and 82.4% for the baseline results). We observe the

[17] The size and value factors in the five-factor model are similar to ones used in the Carhart model.

main difference for the small cap group in which the skill coefficient drops from 4.6% to 3.3% on average. This sharp reduction arises because the Carhart model assigns a negative alpha to the Russell 2000 index and therefore penalizes the performance of small cap funds (consistent with the analysis by Cremers, Petajisto, and Zitzewitz (2012)). Next, Table AVIII reports the results obtained with the five-factor Fama-French model. Under this model, the scalability distribution remains largely unchanged but the proportion of funds with positive skill decreases from 83.1% to 74.0%. This reduction suggests that some funds achieve positive returns partly because they implement profitability- and investment-based strategies.

Please insert Tables AVII and AVIII here

E Analysis based on Daily Fund Returns

Our baseline specification $\alpha_{i,t} = a_i - b_i q_{i,t-1}$ assumes that the skill and scale coefficients remain constant over time. To examine the stability of these coefficients, we conduct an extensive analysis using daily fund returns. This procedure allows us to capture potential changes in the coefficients without explicitly modeling their dynamics (see Lewellen and Nagel (2006)).

To conduct this analysis, we use the daily fund return CRSP database available between January 1999 and December 2019. The CRSP database only reports the daily net return and Net Asset Value (NAV) of each shareclass, but not its daily size. To address this issue, we compute the number of shares for each shareclass at the start of the month. We can then compute the daily size of each shareclass within the month as the product between its daily NAV and the number of shares. We match the fund identifier across the daily and monthly databases to maintain our selection of open-end, actively managed funds with a well-defined equity style. We measure the daily gross return of each fund as the sum of the daily net return and fees. The daily net return is computed as the value-weighted average of the daily net returns across all shareclasses. The daily fees are defined as the value-weighted average of the most recently reported annual fees across shareclasses divided by 21·12.

We can summarize our estimation procedure in two steps. First, we run the following time-serie regression for each year a ($a = 1999, ..., 2019$):

$$r_{i,t} = \alpha_{i,a} + \beta'_{i,a} f_t + e_{i,t}, \tag{A81}$$

where $r_{i,t}$ is the fund daily gross excess return and f_t is the vector of daily factor excess returns in the model of Cremers, Petajisto, and Zitzewitz (2012). Using Equation (A81), we can extract the daily gross alpha of the fund after controling for short-term variations in factor loadings (i.e., $\beta_{i,a}$ is allowed to change on an annual basis):

$$\alpha_{i,t} = \alpha_{i,a} + e_{i,t}. \tag{A82}$$

Second, we run a regression of the daily gross alpha on lagged size to infer the time-varying skill and scale coefficients. Given the potential persistence over a small window of only one year, we estimate the regression over a non-overlapping window τ of five years:

$$\alpha_{i,t} = a_{i,\tau} - b_{i,\tau} q_{i,t-1} + \varepsilon_{i,t}, \tag{A83}$$

where each of the windows covers the years 1999-2004, 2005-2009, 2010-2014, and 2015-2019 (i.e., $\tau = 1, ..., 4$).

To examine the stability of the skill coefficient, we take the first window $\tau = 1$ as the benchmark and test the null hypothesis of constant skill $H_{0,i,\tau} : \Delta a_{i,\tau} = a_{i,1} - a_{i,\tau} = 0$ (for $\tau = 2, 3,4$). Overall, there is little evidence of time-variation in the skill coefficient. Using a 5%-significance threshold, we find that $H_{0,i,\tau}$ is only rejected for (i) 10.7% of the funds for $\tau = 2$, (ii) 9.6% of the funds for $\tau = 3$, and (iii) 14.1% of the funds for $\tau = 4$. We also uncover a substantial fraction of false discoveries among these funds ($H_{0,i,\tau}$ is rejected whereas it is true)—this fraction ranges between 30.1% and 45.5% across the three windows ($\tau = 2, 3,4$) using the approach proposed by Barras, Scaillet, and Wermers (2010).

Repeating this analysis for the scale coefficient, we test the null hypothesis of constant scale $H_{0,i,\tau} : \Delta b_{i,\tau} = b_{i,1} - b_{i,\tau} = 0$ (for $\tau = 2, 3,4$). The results are similar to those obtained for the skill coefficient. Using a 5%-significance threshold, we find that $H_{0,i,\tau}$ is only rejected for (i) 9.0% of the funds for $\tau = 2$, (ii) 7.8% of the funds for $\tau = 3$, and (iii) 10.3% of the funds for $\tau = 4$. Among the rejected funds, the proportion of false discoveries ($H_{0,i,\tau}$ is rejected whereas it is true) ranges between 41.2% and 56.1%.

We also find a remarkable similarity between the skill and scalability distributions measured at the daily and monthly frequencies. Specifically, we measure the annual fund skill and scale levels from daily data as $a_i^d = (\frac{1}{\tau} \sum_\tau a_{i,\tau}) 21 \cdot 12$ and $b_i^d = (\frac{1}{\tau} \sum_\tau b_{i,\tau}) 21 \cdot 12$, and examine the characteristics of the two cross-sectional distributions. We then conduct our baseline monthly analysis over the same period as the one covered by the CRSP daily database (1999-2019). The daily analysis reveals that 82.4% of the funds have a positive skill coefficient which, on average, equals 4.7% per year (vs 77.8% and 3.6% for the monthly analysis). For the scale coefficient, these numbers obtained at the daily frequency are equal to 79.5% and 1.3% per year (vs 76.8% and 1.4% for the monthly analysis).

F Impact of Changes in Economic Conditions

We now extend our baseline specification to capture the impact of changes in economic conditions. We consider two alternative specifications motivated by the recent mutual fund literature. First, we examine whether the gross alpha changes with the level of industry competition using

$$\alpha_{i,t} = a_i - a_{i,sw} sw_{t-1} - b_i q_{i,t-1}, \tag{A84}$$

where sw_{t-1} is defined as the demeaned ratio of industry size to total market capitalization (as in Pastor, Stambaugh, and Taylor (2015)). Second, we account for potential changes in aggregate mispricing using an extended version of Equation (A84)

$$\alpha_{i,t} = a_i - a_{i,sw} sw_{t-1} + a_{i,tu} tu_{t-1} - b_i q_{i,t-1}, \tag{A85}$$

where tu_{t-1} is defined as the demeaned aggregate turnover across all funds (as in Pastor, Stambaugh, and Taylor (2018)). Under both specifications, we can still interpret a_i as the alpha on the first dollar when industry competition and aggregate mispricing are equal to their average levels (i.e., $sw_{t-1} = tu_{t-1} = 0$).

The results in Table AIX show that adding the industry variable sw_{t-1} leaves the skill and scalability distributions largely unchanged. For instance, we find that 82.3% and 82.6% of the funds exhibit positive skill and scale coefficients (vs 83.1% and 82.4% for the baseline results). When the variable sw_{t-1} is used alone in the regression (i.e., $\alpha_{i,t} = a_i - a_{i,sw} sw_{t-1}$), the majority of the funds respond negatively to an increase in industry size (51.0% of the funds have a positive coefficient $a_{i,sw}$). However, this result is overturned when we include lagged size, i.e., only 46.6% of the funds have a positive coefficient $a_{i,sw}$.[18] One possible explanation for this result is that sw_{t-1} may not capture changes in industry competition with sufficient granularity (see Hoberg, Kumar, and Prabhala (2020) for a discussion).

Table AX also shows that the empirical evidence on skill and scalability remains largely unchanged under the extended model in Equation (A85). In this case, the average levels of the skill and scale coefficients are equal to 3.4% and 1.6% per year (vs 3.0% and 1.3% for the baseline results). In addition, the proportions of funds with

[18] Therefore, this result departs from the evidence in Pastor, Stambaugh, and Taylor (2015) obtained with a panel approach in which fund scale is assumed to be constant ($b_i = b$).

positive skill and scale coefficients equal 80.8% and 80.6% (vs 83.2% and 82.4% for the baseline results). Consistent with Pastor, Stambaugh, and Taylor (2018), we find that the majority of funds produce higher returns in times of higher mispricing in capital markets. The proportion of funds with a positive coefficient $a_{i,tu}$ is equal to 60.8%.

Please insert Tables AIX to AX here

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Table AI
Properties of the Estimated Density: Skill Coefficient

Panel A shows the Mean Integrated Squared Error (MISE) and its two components (integrated squared bias and variance) for the bias-adjusted skill density under the baseline choice for the optimal bandwidth across different values for the number of funds and the number of monthly observations. Panel B repeats the analysis under the alternative choice of the optimal bandwidth. For comparison, Panel C reports the same information for the bias-unadjusted density.

Panel A: Bias Adjustment (Baseline Choice for Optimal Bandwidth)

MISE

n\T	100	250	500	750	1000
1000	20.50	5.24	1.91	1.16	0.83
2500	20.62	4.96	1.59	0.84	0.60
5000	20.76	4.82	1.39	0.69	0.45
7500	20.14	4.73	1.35	0.64	0.40
10000	20.40	4.65	1.29	0.57	0.37

Bias^2

n\T	100	250	500	750	1000
1000	19.92	4.50	1.26	0.56	0.29
2500	20.32	4.59	1.24	0.54	0.32
5000	20.58	4.59	1.18	0.50	0.27
7500	20.00	4.55	1.18	0.50	0.26
10000	20.29	4.51	1.16	0.46	0.26

Variance

n\T	100	250	500	750	1000
1000	0.58	0.74	0.65	0.60	0.53
2500	0.30	0.37	0.36	0.30	0.29
5000	0.18	0.23	0.21	0.19	0.18
7500	0.14	0.18	0.17	0.14	0.14
10000	0.11	0.14	0.13	0.11	0.11

Panel B: Bias Adjustment (Alternative Choice for Optimal Bandwidth)

MISE

n\T	100	250	500	750	1000
1000	21.76	5.66	2.06	1.24	0.83
2500	21.58	5.33	1.75	0.91	0.62
5000	21.52	5.13	1.55	0.76	0.47
7500	20.85	5.01	1.48	0.70	0.41
10000	21.02	4.92	1.42	0.63	0.39

Bias^2

n\T	100	250	500	750	1000
1000	21.53	5.33	1.74	0.90	0.52
2500	21.46	5.18	1.59	0.77	0.47
5000	21.46	5.05	1.46	0.67	0.38
7500	20.80	4.95	1.42	0.64	0.34
10000	20.99	4.87	1.37	0.58	0.33

Variance

n\T	100	250	500	750	1000
1000	0.23	0.33	0.32	0.34	0.32
2500	0.12	0.14	0.15	0.14	0.15
5000	0.06	0.08	0.09	0.09	0.09
7500	0.05	0.06	0.07	0.06	0.07
10000	0.03	0.05	0.05	0.05	0.05

Panel C: No Bias Adjustment

MISE

n\T	100	250	500	750	1000
1000	29.59	9.80	3.64	2.17	1.38
2500	28.87	9.06	3.08	1.56	1.01
5000	28.38	8.61	2.73	1.28	0.75
7500	27.68	8.31	2.55	1.15	0.63
10000	27.76	8.18	2.43	1.04	0.58

Bias^2

n\T	100	250	500	750	1000
1000	29.46	9.59	3.41	1.91	1.13
2500	28.80	8.96	2.96	1.44	0.89
5000	28.34	8.54	2.66	1.21	0.68
7500	27.65	8.26	2.50	1.10	0.57
10000	27.73	8.14	2.38	1.00	0.53

Variance

n\T	100	250	500	750	1000
1000	0.13	0.21	0.23	0.26	0.25
2500	0.07	0.10	0.12	0.11	0.12
5000	0.04	0.06	0.07	0.07	0.08
7500	0.03	0.05	0.06	0.05	0.06
10000	0.03	0.04	0.04	0.04	0.04

Table AII
Properties of the Estimated Moments and Proportion:
Skill Coefficient

Panel A shows the Mean Squared Error (MSE) and its two components (bias and standard deviation) of the bias-adjusted estimators (mean and volatility (annualized), skewness, and proportion of funds with a positive skill measure) based on a numerical integration of the bias-adjusted density (under the baseline bandwidth choice) across different values for the number of funds and the number of monthly observations. Panel B reports the same information for the bias-adjusted estimators obtained with the analytical approach. For comparison, Panel C reports the same information for the bias-unadjusted estimators (obtained by integrating the bias-unadjusted density).

Panel A: Bias Adjustment (Numerical Integration)

Mean

MSE

n\T	100	250	500	750	1000
1000	1.37	0.21	0.07	0.04	0.03
2500	1.31	0.21	0.06	0.03	0.02
5000	1.35	0.20	0.05	0.02	0.02
7500	1.28	0.21	0.05	0.02	0.02
10000	1.30	0.20	0.05	0.02	0.01

Bias

n\T	100	250	500	750	1000
1000	1.15	0.43	0.23	0.14	0.12
2000	1.14	0.45	0.23	0.16	0.12
3000	1.16	0.44	0.22	0.15	0.11
4000	1.13	0.45	0.22	0.15	0.11
5000	1.14	0.44	0.22	0.15	0.11

Standard Deviation

n\T	100	250	500	750	1000
1000	0.22	0.15	0.12	0.12	0.12
2000	0.14	0.08	0.08	0.07	0.07
3000	0.09	0.06	0.05	0.05	0.05
4000	0.08	0.05	0.05	0.04	0.05
5000	0.06	0.04	0.04	0.04	0.04

Volatility

MSE

n\T	100	250	500	750	1000
1000	1.93	0.07	0.03	0.03	0.03
2500	1.88	0.03	0.01	0.01	0.01
5000	1.77	0.02	0.01	0.00	0.00
7500	1.66	0.02	0.00	0.00	0.00
10000	1.65	0.02	0.00	0.00	0.00

Bias

n\T	100	250	500	750	1000
1000	1.36	0.18	0.03	0.01	-0.00
2500	1.36	0.15	0.02	0.01	0.01
5000	1.32	0.13	0.03	0.01	0.01
7500	1.28	0.13	0.02	0.02	0.01
10000	1.28	0.13	0.02	0.01	0.01

Standard Deviation

n\T	100	250	500	750	1000
1000	0.28	0.19	0.16	0.16	0.16
2500	0.18	0.10	0.09	0.08	0.08
5000	0.13	0.08	0.06	0.06	0.06
7500	0.11	0.06	0.06	0.06	0.06
10000	0.08	0.07	0.05	0.05	0.05

Skewness

MSE

n\T	100	250	500	750	1000
1000	0.29	0.13	0.11	0.10	0.09
2500	0.18	0.06	0.05	0.04	0.04
5000	0.09	0.03	0.03	0.03	0.02
7500	0.06	0.03	0.02	0.02	0.02
10000	0.06	0.02	0.02	0.02	0.02

Bias

n\T	100	250	500	750	1000
1000	-0.01	-0.01	0.03	0.01	0.01
2000	0.14	0.01	0.03	0.04	0.04
3000	0.16	0.03	0.03	0.02	0.01
4000	0.11	0.02	0.04	0.02	0.02
5000	0.13	0.04	0.04	0.03	0.01

Standard Deviation

n\T	100	250	500	750	1000
1000	0.54	0.37	0.32	0.31	0.30
2000	0.41	0.24	0.22	0.20	0.20
3000	0.26	0.18	0.17	0.16	0.16
4000	0.23	0.17	0.15	0.14	0.14
5000	0.21	0.14	0.12	0.13	0.13

Proportion with Positive Skill Measure

MSE

n\T	100	250	500	750	1000
1000	21.85	4.06	1.46	1.47	0.95
2500	21.47	2.38	0.72	0.50	0.47
5000	19.89	1.94	0.49	0.32	0.29
7500	19.17	1.53	0.39	0.22	0.19
10000	19.23	1.59	0.28	0.16	0.13

Bias

n\T	100	250	500	750	1000
1000	-4.40	-1.39	-0.48	-0.49	-0.13
2500	-4.53	-1.26	-0.44	-0.22	-0.18
5000	-4.41	-1.25	-0.48	-0.23	-0.17
7500	-4.34	-1.12	-0.45	-0.23	-0.12
10000	-4.36	-1.19	-0.35	-0.20	-0.11

Standard Deviation

n\T	100	250	500	750	1000
1000	1.57	1.46	1.11	1.11	0.97
2500	0.99	0.89	0.72	0.67	0.66
5000	0.68	0.63	0.50	0.52	0.51
7500	0.60	0.53	0.43	0.41	0.42
10000	0.49	0.42	0.40	0.34	0.35

Table AII
Properties of the Estimated Moments and Proportion:
Skill Coefficient (Continued)

Panel B: Bias Adjustment (Analytical Approach)

Mean

MSE

n\T	100	250	500	750	1000
1000	1.37	0.21	0.07	0.04	0.03
2500	1.31	0.21	0.06	0.03	0.02
5000	1.35	0.20	0.05	0.02	0.02
7500	1.28	0.21	0.05	0.02	0.02
10000	1.30	0.20	0.05	0.02	0.01

Bias

n\T	100	250	500	750	1000
1000	1.15	0.43	0.23	0.14	0.12
2500	1.14	0.45	0.23	0.16	0.12
5000	1.16	0.44	0.22	0.15	0.11
7500	1.13	0.45	0.22	0.15	0.11
10000	1.14	0.44	0.22	0.15	0.11

Standard Deviation

n\T	100	250	500	750	1000
1000	0.22	0.15	0.12	0.12	0.12
2500	0.14	0.08	0.08	0.07	0.07
5000	0.09	0.06	0.05	0.05	0.05
7500	0.08	0.05	0.05	0.04	0.05
10000	0.06	0.04	0.04	0.04	0.04

Volatility

MSE

n\T	100	250	500	750	1000
1000	1.14	0.06	0.03	0.03	0.03
2500	1.22	0.04	0.01	0.01	0.01
5000	1.15	0.03	0.01	0.00	0.00
7500	1.07	0.03	0.01	0.00	0.00
10000	1.09	0.03	0.00	0.00	0.00

Bias

n\T	100	250	500	750	1000
1000	1.02	0.17	0.04	0.02	0.01
2500	1.09	0.17	0.04	0.02	0.02
5000	1.06	0.16	0.05	0.02	0.01
7500	1.03	0.16	0.04	0.03	0.02
10000	1.04	0.17	0.04	0.02	0.02

Standard Deviation

n\T	100	250	500	750	1000
1000	0.31	0.19	0.16	0.16	0.16
2500	0.18	0.10	0.09	0.08	0.08
5000	0.13	0.08	0.06	0.06	0.06
7500	0.11	0.06	0.06	0.06	0.06
10000	0.08	0.07	0.05	0.05	0.05

Skewness

MSE

n\T	100	250	500	750	1000
1000	1.60	0.90	0.42	0.23	0.18
2500	1.63	0.81	0.34	0.20	0.14
5000	1.63	0.78	0.30	0.16	0.11
7500	1.53	0.76	0.30	0.16	0.11
10000	1.51	0.77	0.30	0.16	0.09

Bias

n\T	100	250	500	750	1000
1000	1.19	0.88	0.55	0.37	0.29
2500	1.24	0.88	0.54	0.39	0.31
5000	1.26	0.87	0.53	0.37	0.28
7500	1.23	0.86	0.53	0.37	0.29
10000	1.22	0.87	0.53	0.38	0.28

Standard Deviation

n\T	100	250	500	750	1000
1000	0.42	0.36	0.34	0.31	0.30
2500	0.31	0.21	0.22	0.21	0.21
5000	0.18	0.16	0.16	0.16	0.16
7500	0.16	0.14	0.14	0.14	0.14
10000	0.15	0.12	0.12	0.13	0.12

Proportion with Positive Skill Measure

MSE

n\T	100	250	500	750	1000
1000	14.48	3.49	1.47	1.43	0.93
2500	15.50	1.89	0.82	0.55	0.50
5000	14.14	1.65	0.53	0.33	0.30
7500	13.54	1.26	0.44	0.26	0.19
10000	13.88	1.37	0.34	0.20	0.16

Bias

n\T	100	250	500	750	1000
1000	-3.46	-1.11	-0.37	-0.39	0.05
2500	-3.80	-1.01	-0.45	-0.24	-0.14
5000	-3.70	-1.11	-0.50	-0.24	-0.15
7500	-3.63	-0.99	-0.50	-0.28	-0.14
10000	-3.69	-1.09	-0.42	-0.27	-0.15

Standard Deviation

n\T	100	250	500	750	1000
1000	1.58	1.50	1.15	1.13	0.96
2500	1.04	0.93	0.79	0.70	0.69
5000	0.68	0.65	0.53	0.52	0.53
7500	0.60	0.52	0.44	0.42	0.42
10000	0.49	0.43	0.40	0.35	0.37

Table AII
Properties of the Estimated Moments and Proportion:
Skill Coefficient (Continued)

Panel C: No Bias Adjustment

Mean

MSE

n\T	100	250	500	750	1000
1000	1.37	0.21	0.07	0.04	0.03
2500	1.31	0.21	0.06	0.03	0.02
5000	1.35	0.20	0.05	0.02	0.02
7500	1.28	0.21	0.05	0.02	0.02
10000	1.30	0.20	0.05	0.02	0.01

Bias

n\T	100	250	500	750	1000
1000	1.15	0.43	0.23	0.14	0.12
2500	1.14	0.45	0.23	0.16	0.12
5000	1.16	0.44	0.22	0.15	0.11
7500	1.13	0.45	0.22	0.15	0.11
10000	1.14	0.44	0.22	0.15	0.11

Standard Deviation

n\T	100	250	500	750	1000
1000	0.22	0.15	0.12	0.12	0.12
2500	0.14	0.08	0.08	0.07	0.07
5000	0.09	0.06	0.05	0.05	0.05
7500	0.08	0.05	0.05	0.04	0.05
10000	0.06	0.04	0.04	0.04	0.04

Volatility

MSE

n\T	100	250	500	750	1000
1000	11.00	1.18	0.25	0.12	0.07
2500	11.28	1.17	0.23	0.10	0.06
5000	11.17	1.15	0.24	0.10	0.05
7500	10.99	1.15	0.23	0.10	0.05
10000	11.06	1.16	0.23	0.09	0.05

Bias

n\T	100	250	500	750	1000
1000	3.31	1.07	0.48	0.31	0.22
2500	3.35	1.08	0.47	0.30	0.22
5000	3.34	1.07	0.48	0.30	0.22
7500	3.31	1.07	0.48	0.31	0.22
10000	3.32	1.08	0.48	0.30	0.22

Standard Deviation

n\T	100	250	500	750	1000
1000	0.27	0.16	0.15	0.15	0.15
2500	0.16	0.09	0.08	0.08	0.08
5000	0.12	0.07	0.06	0.06	0.06
7500	0.09	0.06	0.06	0.05	0.05
10000	0.07	0.06	0.05	0.05	0.05

Skewness

MSE

n\T	100	250	500	750	1000
1000	0.10	0.09	0.08	0.08	0.08
2500	0.06	0.06	0.05	0.04	0.04
5000	0.03	0.06	0.04	0.03	0.03
7500	0.04	0.06	0.04	0.03	0.02
10000	0.04	0.05	0.03	0.02	0.02

Bias

n\T	100	250	500	750	1000
1000	-0.19	-0.21	-0.14	-0.12	-0.09
2500	-0.15	-0.21	-0.15	-0.10	-0.07
5000	-0.14	-0.22	-0.15	-0.12	-0.09
7500	-0.17	-0.22	-0.15	-0.11	-0.08
10000	-0.17	-0.21	-0.15	-0.11	-0.10

Standard Deviation

n\T	100	250	500	750	1000
1000	0.25	0.22	0.25	0.26	0.26
2500	0.18	0.13	0.16	0.17	0.18
5000	0.11	0.10	0.12	0.13	0.13
7500	0.09	0.09	0.11	0.11	0.12
10000	0.08	0.08	0.09	0.10	0.11

Proportion with Positive Skill Measure

MSE

n\T	100	250	500	750	1000
1000	115.98	33.21	9.56	5.57	2.44
2500	118.94	31.23	9.49	4.31	2.52
5000	118.33	32.04	9.36	4.10	2.37
7500	117.45	30.67	9.30	4.16	2.23
10000	118.59	31.50	8.85	4.10	2.22

Bias

n\T	100	250	500	750	1000
1000	-10.69	-5.61	-2.90	-2.10	-1.25
2500	-10.87	-5.52	-2.99	-1.96	-1.44
5000	-10.86	-5.63	-3.02	-1.96	-1.45
7500	-10.83	-5.52	-3.02	-2.00	-1.44
10000	-10.88	-5.60	-2.95	-2.00	-1.45

Standard Deviation

n\T	100	250	500	750	1000
1000	1.31	1.31	1.07	1.08	0.93
2500	0.86	0.85	0.75	0.68	0.68
5000	0.54	0.58	0.50	0.50	0.52
7500	0.50	0.48	0.41	0.40	0.40
10000	0.42	0.38	0.38	0.33	0.36

Table AIII
Properties of the Estimated Density:
Scale Coefficient

Panel A shows the Mean Integrated Squared Error (MISE) and its two components (integrated squared bias and variance) for the bias-adjusted scale density under the baseline choice for the optimal bandwidth across different values for the number of funds and the number of monthly observations. Panel B repeats the analysis under the alternative choice of the optimal bandwidth. For comparison, Panel C reports the same information for the bias-unadjusted density.

Panel A: Bias Adjustment (Baseline Choice for Optimal Bandwidth)

	MISE					Bias^2					Variance				
n\T	100	250	500	750	1000	100	250	500	750	1000	100	250	500	750	1000
1000	77.11	18.04	5.35	2.95	2.08	75.98	16.46	3.95	1.69	0.95	1.13	1.58	1.40	1.26	1.14
2500	75.05	17.01	4.63	2.15	1.55	74.51	16.29	4.03	1.64	1.05	0.53	0.72	0.61	0.52	0.50
5000	75.13	17.03	4.09	1.89	1.18	74.54	16.66	3.80	1.63	0.89	0.59	0.37	0.29	0.26	0.29
7500	74.82	16.78	4.07	1.79	1.04	74.12	16.57	3.87	1.60	0.85	0.70	0.21	0.20	0.19	0.19
10000	74.13	16.61	4.03	1.71	1.02	73.90	16.41	3.89	1.56	0.88	0.23	0.19	0.14	0.15	0.14

Panel B: Bias Adjustment (Alternative Choice for Optimal Bandwidth)

	MISE					Bias^2					Variance				
n\T	100	250	500	750	1000	100	250	500	750	1000	100	250	500	750	1000
1000	91.92	20.42	5.85	3.16	2.12	90.72	19.42	5.14	2.45	1.45	1.21	1.00	0.71	0.71	0.67
2500	87.96	19.27	5.24	2.43	1.68	87.38	18.76	4.93	2.17	1.41	0.58	0.51	0.31	0.26	0.27
5000	86.18	18.79	4.63	2.18	1.32	85.45	18.55	4.47	2.03	1.16	0.73	0.24	0.17	0.15	0.17
7500	84.71	18.29	4.57	2.05	1.17	83.90	18.16	4.44	1.94	1.05	0.80	0.13	0.13	0.12	0.12
10000	83.13	17.98	4.48	1.95	1.15	82.88	17.85	4.38	1.85	1.06	0.24	0.13	0.09	0.10	0.09

Panel C: No Bias Adjustment

	MISE					Bias^2					Variance				
n\T	100	250	500	750	1000	100	250	500	750	1000	100	250	500	750	1000
1000	97.17	31.16	10.69	5.84	3.60	96.81	30.78	10.20	5.32	3.08	0.36	0.38	0.48	0.52	0.51
2500	92.06	28.58	9.41	4.25	2.66	91.89	28.39	9.20	4.06	2.44	0.18	0.19	0.22	0.20	0.22
5000	89.26	27.34	8.39	3.80	2.14	88.94	27.25	8.27	3.68	2.01	0.32	0.09	0.12	0.12	0.14
7500	87.56	26.59	8.14	3.52	1.86	87.13	26.52	8.05	3.43	1.76	0.43	0.07	0.09	0.09	0.10
10000	85.48	26.25	7.87	3.36	1.77	85.42	26.20	7.80	3.29	1.70	0.06	0.05	0.07	0.08	0.07

Table AIV
Properties of the Estimated Moments and Proportion: Scale Coefficient

Panel A shows the Mean Squared Error (MSE) and its two components (bias and standard deviation) of the bias-adjusted estimators (mean and volatility (annualized), skewness, and proportion of funds with a positive scale measure) based on a numerical integration of the bias-adjusted density (under the baseline bandwidth choice) across different values for the number of funds and the number of monthly observations. Panel B reports the same information for the bias-adjusted estimators obtained with the analytical approach. For comparison, Panel C reports the same information for the bias-unadjusted estimators (obtained by integrating the bias-unadjusted density).

Panel A: Bias Adjustment (Numerical Integration)

Mean

MSE

n\T	100	250	500	750	1000
1000	1.63	0.12	0.02	0.01	0.01
2500	1.48	0.12	0.02	0.01	0.01
5000	1.49	0.12	0.02	0.01	0.01
7500	1.40	0.11	0.02	0.01	0.01
10000	1.43	0.11	0.02	0.01	0.01

Bias

n\T	100	250	500	750	1000
1000	1.22	0.32	0.14	0.09	0.07
2000	1.20	0.34	0.14	0.10	0.08
3000	1.21	0.34	0.14	0.09	0.07
4000	1.18	0.33	0.14	0.10	0.07
5000	1.19	0.33	0.14	0.10	0.07

Standard Deviation

n\T	100	250	500	750	1000
1000	0.38	0.11	0.06	0.06	0.06
2000	0.21	0.06	0.04	0.03	0.03
3000	0.16	0.05	0.03	0.02	0.02
4000	0.14	0.04	0.02	0.02	0.02
5000	0.10	0.04	0.02	0.02	0.02

Volatility

MSE

n\T	100	250	500	750	1000
1000	14.87	0.12	0.01	0.01	0.00
2500	16.29	0.19	0.00	0.00	0.00
5000	22.82	0.09	0.00	0.00	0.00
7500	32.68	0.07	0.00	0.00	0.00
10000	16.64	0.07	0.00	0.00	0.00

Bias

n\T	100	250	500	750	1000
1000	3.54	0.28	-0.02	-0.02	-0.03
2500	3.80	0.29	-0.03	-0.03	-0.02
5000	4.10	0.27	-0.03	-0.03	-0.02
7500	4.32	0.25	-0.03	-0.03	-0.02
10000	3.94	0.25	-0.03	-0.03	-0.02

Standard Deviation

n\T	100	250	500	750	1000
1000	1.52	0.22	0.08	0.07	0.06
2500	1.37	0.32	0.05	0.04	0.04
5000	2.45	0.13	0.03	0.03	0.03
7500	3.74	0.08	0.03	0.03	0.03
10000	1.05	0.09	0.03	0.03	0.02

Skewness

MSE

n\T	100	250	500	750	1000
1000	47.90	4.04	0.17	0.10	0.10
2500	109.91	19.89	0.10	0.05	0.05
5000	206.97	10.49	0.04	0.02	0.02
7500	406.00	3.89	0.04	0.02	0.02
10000	254.93	7.25	0.03	0.02	0.01

Bias

n\T	100	250	500	750	1000
1000	1.85	0.02	0.01	-0.00	0.02
2000	4.50	1.02	0.10	0.06	0.05
3000	5.55	1.30	0.08	0.05	0.05
4000	6.36	0.94	0.10	0.07	0.06
5000	7.69	0.82	0.11	0.08	0.05

Standard Deviation

n\T	100	250	500	750	1000
1000	6.67	2.01	0.41	0.32	0.31
2000	9.47	4.34	0.30	0.22	0.21
3000	13.27	2.96	0.18	0.14	0.14
4000	19.12	1.73	0.18	0.11	0.11
5000	13.99	2.57	0.13	0.10	0.10

Proportion with Positive Skill Measure

MSE

n\T	100	250	500	750	1000
1000	65.64	4.91	1.55	1.51	1.16
2500	62.50	2.05	0.68	0.41	0.49
5000	59.38	1.01	0.20	0.28	0.28
7500	56.14	0.77	0.23	0.18	0.17
10000	55.74	0.78	0.26	0.19	0.14

Bias

n\T	100	250	500	750	1000
1000	-7.45	-0.99	-0.05	-0.27	-0.10
2500	-7.65	-0.80	0.07	0.19	0.11
5000	-7.42	-0.70	0.11	0.13	0.08
7500	-7.32	-0.67	0.13	0.17	0.13
10000	-7.33	-0.66	0.22	0.14	0.12

Standard Deviation

n\T	100	250	500	750	1000
1000	3.20	1.98	1.24	1.20	1.07
2500	1.98	1.19	0.82	0.61	0.69
5000	2.07	0.72	0.44	0.51	0.53
7500	1.60	0.57	0.46	0.38	0.40
10000	1.40	0.59	0.46	0.41	0.36

Table AIV
Properties of the Estimated Moments and Proportion: Scale Coefficient (Continued)

Panel B: Bias Adjustment (Analytical Approach)

Mean

MSE n\T	100	250	500	750	1000
1000	1.63	0.12	0.02	0.01	0.01
2500	1.48	0.12	0.02	0.01	0.01
5000	1.49	0.12	0.02	0.01	0.01
7500	1.40	0.11	0.02	0.01	0.01
10000	1.43	0.11	0.02	0.01	0.01

Bias n\T	100	250	500	750	1000
1000	1.22	0.32	0.14	0.09	0.07
2000	1.20	0.34	0.14	0.10	0.08
3000	1.21	0.34	0.14	0.09	0.07
4000	1.18	0.33	0.14	0.10	0.07
5000	1.19	0.33	0.14	0.10	0.07

Standard Deviation n\T	100	250	500	750	1000
1000	0.38	0.11	0.06	0.06	0.06
2000	0.21	0.06	0.04	0.03	0.03
3000	0.16	0.05	0.03	0.02	0.02
4000	0.14	0.04	0.02	0.02	0.02
5000	0.10	0.04	0.02	0.02	0.02

Volatility

MSE n\T	100	250	500	750	1000
1000	6.75	0.08	0.01	0.00	0.00
2500	7.26	0.15	0.00	0.00	0.00
5000	11.97	0.06	0.00	0.00	0.00
7500	8.80	0.05	0.00	0.00	0.00
10000	6.48	0.05	0.00	0.00	0.00

Bias n\T	100	250	500	750	1000
1000	2.10	0.21	0.03	0.02	0.01
2500	2.24	0.26	0.03	0.01	0.01
5000	2.45	0.22	0.03	0.01	0.01
7500	2.00	0.22	0.03	0.01	0.01
10000	2.24	0.22	0.03	0.01	0.01

Standard Deviation n\T	100	250	500	750	1000
1000	1.54	0.18	0.07	0.07	0.06
2500	1.49	0.28	0.05	0.04	0.04
5000	2.44	0.12	0.03	0.03	0.03
7500	2.20	0.06	0.03	0.03	0.03
10000	1.21	0.09	0.03	0.02	0.02

Skewness

MSE n\T	100	250	500	750	1000
1000	78.69	3.59	0.48	0.24	0.17
2500	117.73	9.57	0.45	0.21	0.14
5000	452.01	16.92	0.38	0.17	0.10
7500	549.80	4.15	0.40	0.18	0.10
10000	367.33	4.94	0.40	0.18	0.10

Bias n\T	100	250	500	750	1000
1000	3.19	0.21	-0.05	-0.03	-0.02
2500	3.37	0.24	-0.05	-0.04	-0.03
5000	3.44	0.23	-0.07	-0.05	-0.03
7500	3.41	0.22	-0.06	-0.05	-0.03
10000	3.43	0.22	-0.06	-0.04	-0.02

Standard Deviation n\T	100	250	500	750	1000
1000	8.28	1.88	0.69	0.48	0.42
2500	10.31	3.08	0.67	0.46	0.37
5000	20.98	4.11	0.62	0.41	0.32
7500	23.20	2.03	0.63	0.42	0.32
10000	18.86	2.21	0.63	0.42	0.31

Proportion with Positive Skill Measure

MSE n\T	100	250	500	750	1000
1000	41.51	4.31	1.79	1.44	1.41
2500	45.02	1.68	0.84	0.51	0.51
5000	45.71	0.66	0.30	0.30	0.33
7500	47.16	0.38	0.29	0.18	0.18
10000	47.51	0.41	0.36	0.18	0.14

Bias n\T	100	250	500	750	1000
1000	-5.61	0.15	0.35	-0.06	0.10
2500	-6.34	0.20	0.34	0.29	0.14
5000	-6.42	0.18	0.31	0.14	0.09
7500	-6.60	0.20	0.28	0.15	0.09
10000	-6.72	0.21	0.36	0.13	0.08

Standard Deviation n\T	100	250	500	750	1000
1000	3.17	2.07	1.29	1.20	1.18
2500	2.20	1.28	0.85	0.66	0.70
5000	2.12	0.79	0.45	0.53	0.56
7500	1.91	0.58	0.46	0.39	0.42
10000	1.56	0.61	0.48	0.41	0.37

Table AIV
Properties of the Estimated Moments and Proportion:
Scale Coefficient (Continued)

Panel C: No Bias Adjustment

Mean

MSE

n\T	100	250	500	750	1000
1000	1.63	0.12	0.02	0.01	0.01
2500	1.48	0.12	0.02	0.01	0.01
5000	1.49	0.12	0.02	0.01	0.01
7500	1.40	0.11	0.02	0.01	0.01
10000	1.43	0.11	0.02	0.01	0.01

Bias

n\T	100	250	500	750	1000
1000	1.22	0.32	0.14	0.09	0.07
2000	1.20	0.34	0.14	0.10	0.08
3000	1.21	0.34	0.14	0.09	0.07
4000	1.18	0.33	0.14	0.10	0.07
5000	1.19	0.33	0.14	0.10	0.07

Standard Deviation

n\T	100	250	500	750	1000
1000	0.38	0.11	0.06	0.06	0.06
2000	0.21	0.06	0.04	0.03	0.03
3000	0.16	0.05	0.03	0.02	0.02
4000	0.14	0.04	0.02	0.02	0.02
5000	0.10	0.04	0.02	0.02	0.02

Volatility

MSE

n\T	100	250	500	750	1000
1000	24.88	0.89	0.09	0.03	0.02
2500	26.67	1.00	0.09	0.03	0.01
5000	34.43	0.90	0.09	0.03	0.01
7500	46.26	0.87	0.09	0.03	0.01
10000	26.96	0.87	0.09	0.03	0.01

Bias

n\T	100	250	500	750	1000
1000	4.77	0.93	0.30	0.17	0.11
2500	4.99	0.96	0.30	0.16	0.11
5000	5.30	0.94	0.29	0.16	0.11
7500	5.52	0.93	0.30	0.16	0.11
10000	5.09	0.93	0.29	0.17	0.11

Standard Deviation

n\T	100	250	500	750	1000
1000	1.45	0.18	0.07	0.06	0.06
2500	1.31	0.30	0.04	0.04	0.04
5000	2.51	0.11	0.03	0.03	0.03
7500	3.97	0.06	0.03	0.02	0.02
10000	1.01	0.07	0.02	0.02	0.02

Skewness

MSE

n\T	100	250	500	750	1000
1000	29.55	1.90	0.13	0.10	0.09
2500	67.08	13.11	0.08	0.06	0.04
5000	147.57	3.22	0.06	0.05	0.03
7500	293.19	0.88	0.06	0.04	0.02
10000	159.53	2.10	0.05	0.03	0.02

Bias

n\T	100	250	500	750	1000
1000	1.18	-0.26	-0.23	-0.20	-0.14
2500	3.06	0.46	-0.20	-0.17	-0.12
5000	3.98	0.45	-0.22	-0.18	-0.13
7500	4.57	0.20	-0.21	-0.17	-0.13
10000	5.46	0.15	-0.21	-0.17	-0.14

Standard Deviation

n\T	100	250	500	750	1000
1000	5.31	1.35	0.28	0.25	0.26
2500	7.60	3.59	0.21	0.17	0.17
5000	11.48	1.74	0.11	0.11	0.11
7500	16.50	0.92	0.12	0.08	0.09
10000	11.39	1.44	0.09	0.08	0.08

Proportion with Positive Skill Measure

MSE

n\T	100	250	500	750	1000
1000	164.79	49.68	13.90	7.38	3.97
2500	163.81	46.76	13.13	5.01	2.98
5000	160.98	46.56	12.92	5.51	2.98
7500	163.14	47.12	13.06	5.37	2.85
10000	163.64	47.03	12.63	5.48	2.87

Bias

n\T	100	250	500	750	1000
1000	-12.75	-6.91	-3.55	-2.48	-1.65
2500	-12.76	-6.79	-3.54	-2.15	-1.59
5000	-12.67	-6.80	-3.57	-2.30	-1.64
7500	-12.76	-6.85	-3.59	-2.29	-1.64
10000	-12.78	-6.85	-3.53	-2.31	-1.66

Standard Deviation

n\T	100	250	500	750	1000
1000	1.52	1.41	1.13	1.10	1.12
2500	0.95	0.83	0.76	0.61	0.66
5000	0.58	0.57	0.40	0.48	0.54
7500	0.48	0.42	0.40	0.36	0.40
10000	0.43	0.41	0.43	0.37	0.35

Table AV
Fund Style Classification

This table provides the list of 32 styles across the different data providers of style information (Wiesbenberger, Strategic Insight, Lipper, Policy CRSP). For each style, it also shows the mapping between each style and the growth/value (GV) and small/large cap (SL) dimensions. A value of 1 refers to growth or small cap. A value of two refers to neutral fund in terms of GV or SL dimension. Finally, a value of 3 refers to value or large cap.

Wiesenberger	Symbol	Name	Style GV	Style SL
1	G	Growth	1	
2	GCI	Growth and current income	3	
3	G-I	Income	3	
4	IEQ	Equity income	3	
5	LTG	Long-term growth	1	
6	MCG	Maximum capital gains	1	
7	SCG	Small-cap growth	1	1

Strategic Insight	Symbol	Name	Style GV	Style SL
8	AGG	Aggressive growth	1	
9	GMC	Equity mid-cap		2
10	GRI	Growth and income	3	
11	GRO	Growth	1	
12	ING	Income and growth	3	
13	SCG	Small-cap		1

Table AV
Fund Style Classification (Continued)

Lipper	Symbol	Name	Style GV	Style SL
14	CA	Capital appreciation	1	
15	G	Growth	1	
16	GI	Growth and income	3	
17	LCCE	Large-cap core	2	3
18	LCGE	Large-cap growth	1	3
19	LCVE	Large-cap value	3	3
20	MC	Mid-cap		2
21	MCCE	Mid-cap core	2	2
22	MCGE	Mid-cap growth	1	2
23	MCVE	Mid-cap value	3	2
24	MLCE	Multi-cap core	2	
25	MLGE	Multi-cap growth	1	
26	MLVE	Multi-cap value	3	
27	MR	Micro-cap		1
28	SCCE	Small-cap core	2	1
29	SCGE	Small-cap growth	1	1
30	SCVE	Small-cap value	3	1
31	SG	Small-cap		1

Policy CRSP	Symbol	Name	Style GV	Style SL
32	CS	Common stock		

Table AVI
Impact of Survivorship and Reverse Survivorship Bias

Panel A contains the summary statistics of the distributions of the skill and scale coefficients for all funds in the population across different thresholds for the minimum number of return observations (ranging from 12 to 60 monthly observations). It reports the first four moments, the proportions of funds with a negative and positive skill coefficient, and the quantiles at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the subpopulation of funds that disappear during the sample period across two thresholds for the minimum number of return observations (60 and 108 monthly observations). This analysis provides a rough estimate of the magnitude of the reverse survivorship bias.

Panel A: All Selected Funds

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
Min. Observations=12								
Skill Coefficent	2.7 (0.2)	5.4 (0.4)	0.1 (1.2)	37.6 (8.1)	21.8 (0.8)	78.2 (0.8)	-3.2 (0.2)	9.5 (0.2)
Scale Coefficient	1.3 (0.1)	1.9 (0.1)	1.2 (0.9)	26 (7.3)	20 (0.8)	80 (0.8)	-1.1 (0.1)	4.2 (0.1)
Min. Observations=36								
Skill Coefficent	2.8 (0.1)	4.8 (0.3)	1.3 (0.9)	28.8 (5.3)	19.9 (0.8)	80.1 (0.8)	-2.8 (0.1)	9.2 (0.2)
Scale Coefficient	1.3 (0.1)	1.9 (0.1)	1.8 (0.9)	25.6 (8.8)	19.2 (0.8)	80.8 (0.8)	-1 (0.1)	4.1 (0.1)
Min. Observations=60								
Skill Coefficent	3 (0.1)	4.1 (0.2)	1.6 (0.7)	23.4 (6)	16.9 (0.8)	83.1 (0.8)	-2.2 (0.1)	8.9 (0.2)
Scale Coefficient	1.3 (0.1)	1.7 (0.1)	1.6 (0.7)	16.7 (11)	17.6 (0.8)	82.4 (0.8)	-0.9 (0.1)	3.9 (0.1)

Panel B: Selected Funds that Disappear During the Sample Period

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
Min. Observations=60								
Skill Coefficent	2.5 (0.1)	4.9 (0.3)	2.2 (0.7)	27 (6)	25.8 (0.9)	74.2 (0.9)	-3.6 (0.2)	9.9 (0.2)
Scale Coefficient	1.4 (0.1)	2.1 (0.1)	2.1 (0.7)	21.5 (7.3)	23.2 (0.9)	76.8 (0.9)	-1.4 (0.1)	4.6 (0.1)
Min. Observations=108								
Skill Coefficent	2.4 (0.1)	3.3 (0.1)	0.8 (0.3)	8 (1.1)	21 (0.8)	79 (0.8)	-2.4 (0.1)	7.8 (0.1)
Scale Coefficient	1.3 (0.1)	1.6 (0.1)	1.3 (0.3)	9.9 (1.5)	19.2 (0.8)	80.8 (0.8)	-0.9 (0.1)	3.7 (0.1)

Table AVII
Distributions of Skill and Scalability
Four-Factor Model

Panel A contains the summary statistics of the distribution of the skill coefficient for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or funds directly sold to investors) based on the four-factor model of Carhart (1997). It reports the first four moments, the proportions of funds with a negative and positive skill coefficient, and the quantiles at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the scale coefficient. To ease interpretation, we standardize the scale coefficient for each fund so that it corresponds to the change in gross alpha for a one standard deviation change in size.

Panel A: Skill Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	2.4 (0.1)	3.9 (0.3)	1.8 (1.1)	30.4 (15.4)	21.5 (0.8)	78.5 (0.8)	-2.6 (0.1)	8.1 (0.1)
Fund Groups								
Small Cap	3.3 (0.2)	4.2 (0.5)	1.9 (2.5)	27.9 (41)	19.4 (1.6)	80.6 (1.6)	-3 (0.3)	9.8 (0.3)
Large Cap	1.6 (0.1)	2.7 (0.2)	1.2 (0.6)	13.5 (3.7)	24.5 (1.4)	75.5 (1.4)	-2.1 (0.2)	5.9 (0.2)
Low Turnover	2.1 (0.2)	3.1 (0.2)	-0.2 (0.7)	13.4 (2)	18.7 (1.4)	81.3 (1.4)	-2 (0.2)	6.7 (0.2)
High Turnover	2.6 (0.2)	5 (0.5)	2.1 (1.3)	26.3 (15.4)	24.9 (1.5)	75.1 (1.5)	-3.7 (0.2)	9.8 (0.3)
Broker Sold	2.4 (0.2)	4.1 (0.4)	2.6 (1.8)	37.7 (25.9)	22.2 (1.3)	77.8 (1.3)	-2.7 (0.2)	8.3 (0.2)
Direct Sold	2.6 (0.1)	2.9 (0.2)	0.4 (0.5)	8.5 (1.8)	14.1 (1.2)	85.9 (1.2)	-1.4 (0.2)	7.4 (0.2)

Panel B: Scale Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.3 (0.1)	1.7 (0.1)	1.5 (0.7)	16 (9.8)	19.2 (0.8)	80.8 (0.8)	-1 (0.1)	3.8 (0.1)
Fund Groups								
Small Cap	1.5 (0.1)	1.7 (0.1)	-0.1 (1)	8.4 (9.9)	18.4 (1.5)	81.6 (1.5)	-1.2 (0.1)	4.3 (0.1)
Large Cap	0.9 (0.1)	1.4 (0.1)	1.7 (0.6)	13.5 (4)	24.6 (1.4)	75.4 (1.4)	-1 (0.1)	3 (0.1)
Low Turnover	0.8 (0.1)	1.1 (0.1)	0.4 (0.3)	4.7 (1.3)	21.6 (1.5)	78.4 (1.5)	-0.9 (0.1)	2.7 (0.1)
High Turnover	1.7 (0.1)	2.1 (0.2)	1 (0.5)	8.9 (3.7)	18.7 (1.4)	81.3 (1.4)	-1.2 (0.1)	5.1 (0.2)
Broker Sold	1.3 (0.1)	1.8 (0.1)	1.2 (0.5)	11.1 (1.4)	19.5 (1.2)	80.5 (1.2)	-1 (0.1)	4.2 (0.1)
Direct Sold	1.3 (0.1)	1.3 (0.1)	0.6 (0.5)	7.5 (2.3)	15.3 (1.3)	84.7 (1.3)	-0.7 (0.1)	3.4 (0.1)

Table AVIII
Distributions of Skill and Scalability
Five-Factor Model

Panel A contains the summary statistics of the distribution of the skill coefficient for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or funds directly sold to investors) based on the five-factor model of Fama and French (2015). It reports the first four moments, the proportions of funds with a negative and positive skill coefficient, and the quantiles at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the scale coefficient. To ease interpretation, we standardize the scale coefficient for each fund so that it corresponds to the change in gross alpha for a one standard deviation change in size.

Panel A: Skill Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	2.4 (0.1)	4.4 (0.2)	1.4 (0.5)	16.8 (2.3)	26 (0.9)	74 (0.9)	-3.1 (0.1)	8.9 (0.2)
Fund Groups								
Small Cap	3.4 (0.2)	4.3 (0.3)	0.8 (0.5)	7.8 (2.8)	20.2 (1.6)	79.8 (1.6)	-3 (0.3)	10.3 (0.3)
Large Cap	1.5 (0.2)	3.5 (0.2)	1.8 (0.4)	13.3 (2.3)	31.3 (1.5)	68.7 (1.5)	-3 (0.2)	6.8 (0.2)
Low Turnover	1.5 (0.2)	3.9 (0.3)	1.4 (0.8)	18.4 (3.1)	32.8 (1.7)	67.2 (1.7)	-3.2 (0.2)	7.1 (0.2)
High Turnover	3.5 (0.2)	5.1 (0.4)	1 (0.6)	12.8 (2.7)	19.7 (1.4)	80.3 (1.4)	-3.5 (0.3)	11.5 (0.3)
Broker Sold	2.4 (0.2)	4.3 (0.3)	1 (0.6)	13 (2.8)	26.2 (1.4)	73.8 (1.4)	-3.2 (0.2)	8.9 (0.2)
Direct Sold	2.5 (0.2)	3.6 (0.2)	1.1 (0.5)	11.3 (1.7)	21.8 (1.5)	78.2 (1.5)	-2.1 (0.2)	8.1 (0.2)

Panel B: Scale Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.2 (0.1)	1.7 (0.1)	1.1 (0.5)	13.9 (4)	19.9 (0.8)	80.1 (0.8)	-1 (0.1)	3.9 (0.1)
Fund Groups								
Small Cap	1.5 (0.1)	1.7 (0.1)	-0.1 (0.9)	7.3 (8.4)	18.4 (1.5)	81.6 (1.5)	-1.2 (0.1)	4.4 (0.2)
Large Cap	0.9 (0.1)	1.5 (0.1)	1.9 (0.7)	16.9 (4.2)	24.7 (1.4)	75.3 (1.4)	-1.1 (0.1)	3.2 (0.1)
Low Turnover	0.9 (0.1)	1.2 (0.1)	-0.1 (0.5)	6.3 (2.4)	24.4 (1.5)	75.6 (1.5)	-1 (0.1)	2.9 (0.1)
High Turnover	1.6 (0.1)	2.1 (0.2)	1 (0.6)	9.8 (4.3)	18.7 (1.4)	81.3 (1.4)	-1.2 (0.1)	5.1 (0.2)
Broker Sold	1.3 (0.1)	1.8 (0.1)	0.8 (0.5)	10.5 (1.5)	19.7 (1.2)	80.3 (1.2)	-1.1 (0.1)	4.2 (0.1)
Direct Sold	1.2 (0.1)	1.4 (0.1)	-0.4 (0.4)	7.5 (1.4)	16.6 (1.3)	83.4 (1.3)	-0.7 (0.1)	3.4 (0.1)

Table AIX
Distributions of Skill and Scalability
Changes in Industry Competition

Panel A contains the summary statistics of the distribution of the skill coefficient for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or funds directly sold to investors) after including a proxy for industry competition in the set of variables (the ratio of the industry size on the total market capitalization). It reports the first four moments, the proportions of funds with a negative and positive skill coefficient, and the quantiles at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the scale coefficient. To ease interpretation, we standardize the scale coefficient for each fund so that it corresponds to the change in gross alpha for a one standard deviation change in size.

Panel A: Skill Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	3.5 (0.1)	5.1 (0.3)	1.4 (0.9)	28 (6.2)	17.7 (0.8)	82.3 (0.8)	-2.7 (0.2)	10.5 (0.2)
Fund Groups								
Small Cap	5.4 (0.3)	5.9 (0.6)	2.4 (1.2)	25.3 (8.9)	12.1 (1.3)	87.9 (1.3)	-2.3 (0.3)	14.4 (0.4)
Large Cap	2 (0.2)	3.4 (0.2)	1.1 (0.6)	13.2 (1.7)	22.7 (1.3)	77.3 (1.3)	-2.4 (0.2)	7 (0.2)
Low Turnover	3.1 (0.2)	4.3 (0.3)	-0.7 (1)	17.1 (5.5)	18.4 (1.4)	81.6 (1.4)	-2.7 (0.2)	9 (0.2)
High Turnover	3.9 (0.3)	6.2 (0.6)	2.1 (1.1)	26.3 (6.5)	19.3 (1.4)	80.7 (1.4)	-3.4 (0.3)	12.5 (0.3)
Broker Sold	3.4 (0.2)	4.8 (0.4)	2 (1.3)	28.7 (13.8)	17.9 (1.2)	82.1 (1.2)	-2.5 (0.2)	10.6 (0.2)
Direct Sold	3.8 (0.2)	4.2 (0.3)	0.5 (0.5)	9.4 (1)	12.9 (1.2)	87.1 (1.2)	-1.5 (0.2)	10.1 (0.2)

Panel B: Scale Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.6 (0.1)	2.3 (0.1)	1.1 (0.6)	17.3 (4.3)	17.4 (0.8)	82.6 (0.8)	-1.2 (0.1)	5.1 (0.1)
Fund Groups								
Small Cap	2.2 (0.1)	2.6 (0.2)	1.5 (0.5)	11.8 (1.9)	14.8 (1.4)	85.2 (1.4)	-1.3 (0.2)	6.1 (0.2)
Large Cap	1.2 (0.1)	1.6 (0.1)	1.1 (0.6)	10.3 (3.6)	20.4 (1.3)	79.6 (1.3)	-1.1 (0.1)	3.7 (0.1)
Low Turnover	1.3 (0.1)	1.7 (0.1)	0.4 (0.9)	11.6 (6.5)	18.9 (1.4)	81.1 (1.4)	-1 (0.1)	4.1 (0.1)
High Turnover	2.1 (0.1)	2.7 (0.2)	0.8 (0.8)	12.6 (5.4)	18.1 (1.4)	81.9 (1.4)	-1.6 (0.2)	6.2 (0.2)
Broker Sold	1.7 (0.1)	2.3 (0.2)	1 (0.5)	11.2 (1.8)	17.9 (1.2)	82.1 (1.2)	-1.2 (0.1)	5.2 (0.1)
Direct Sold	1.8 (0.1)	2 (0.2)	1.9 (1.4)	20.2 (21)	14.2 (1.2)	85.8 (1.2)	-0.8 (0.1)	4.9 (0.1)

Table AX
Distributions of Skill and Scalability
Changes in Industry Competition and Aggregate Mispricing

Panel A contains the summary statistics of the distribution of the skill coefficient for all funds in the population, small/large cap funds, low/high turnover funds (i.e., bottom or top tercile of funds sorted on turnover), and broker/direct sold funds (i.e., funds that are sold through brokers or funds directly sold to investors) after including a proxy for industry competition and aggregate mispricing in the set of variables (the ratio of the industry size on the total market capitalization and aggregate fund turnover). It reports the first four moments, the proportions of funds with a negative and positive skill coefficient, and the quantiles at 5% and 95%. We compute all cross-sectional estimates by integrating numerically the bias-adjusted density obtained with our nonparametric approach. Figures in parentheses denote the estimated standard deviation of each estimator. Panel B repeats the analysis for the scale coefficient. To ease interpretation, we standardize the scale coefficient for each fund so that it corresponds to the change in gross alpha for a one standard deviation change in size.

Panel A: Skill Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	3.4 (0.2)	5.4 (0.3)	1.4 (0.8)	26.2 (5.6)	19.2 (0.8)	80.8 (0.8)	-3.1 (0.2)	10.8 (0.2)
Fund Groups								
Small Cap	5.4 (0.3)	6.1 (0.6)	2.4 (1)	23.6 (8.2)	13.6 (1.4)	86.4 (1.4)	-2.6 (0.3)	14.5 (0.4)
Large Cap	2.1 (0.2)	3.7 (0.3)	0.6 (0.6)	14 (1.5)	24.3 (1.4)	75.7 (1.4)	-2.8 (0.2)	7.3 (0.2)
Low Turnover	3.1 (0.2)	4.7 (0.4)	0.2 (1.1)	20.1 (5.9)	18.8 (1.4)	81.2 (1.4)	-2.7 (0.2)	9.6 (0.2)
High Turnover	3.8 (0.3)	6.3 (0.6)	2.1 (1)	25.1 (6.8)	21.7 (1.5)	78.3 (1.5)	-3.9 (0.3)	12.6 (0.3)
Broker Sold	3.3 (0.2)	5.1 (0.5)	2.3 (1.4)	32.6 (14.7)	18.9 (1.2)	81.1 (1.2)	-3 (0.2)	10.8 (0.3)
Direct Sold	3.8 (0.2)	4.5 (0.3)	0.7 (0.5)	10.9 (1)	14.3 (1.2)	85.7 (1.2)	-1.9 (0.2)	10.3 (0.3)

Panel B: Scale Coefficient

	Moments				Proportions (%)		Quantiles (Ann.)	
	Mean (Ann.)	Std. Dev. (Ann.)	Skewness	Kurtosis	Negative	Positive	5%	95%
All Funds	1.6 (0.1)	2.4 (0.1)	1.4 (0.5)	17.4 (3.2)	19.4 (0.8)	80.6 (0.8)	-1.3 (0.1)	5.3 (0.1)
Fund Groups								
Small Cap	2.2 (0.2)	2.9 (0.2)	1.4 (0.5)	10.9 (1.9)	17.1 (1.5)	82.9 (1.5)	-1.5 (0.2)	6.4 (0.2)
Large Cap	1.2 (0.1)	1.6 (0.1)	0.7 (0.4)	7.3 (2.1)	21.6 (1.3)	78.4 (1.3)	-1.2 (0.1)	3.7 (0.1)
Low Turnover	1.3 (0.1)	1.8 (0.2)	0.6 (1.3)	14 (14.4)	20.7 (1.4)	79.3 (1.4)	-1.1 (0.1)	4.1 (0.1)
High Turnover	2 (0.1)	2.9 (0.2)	0.9 (0.6)	10.9 (2.9)	20 (1.4)	80 (1.4)	-1.8 (0.2)	6.3 (0.2)
Broker Sold	1.7 (0.1)	2.3 (0.2)	1.2 (0.5)	11.4 (1.5)	19.4 (1.2)	80.6 (1.2)	-1.3 (0.1)	5.2 (0.1)
Direct Sold	1.8 (0.1)	2.1 (0.2)	1.8 (1)	18 (9.9)	15.8 (1.3)	84.2 (1.3)	-1 (0.1)	5.3 (0.2)

Figure A1
Comparative Static Analysis of the EIV Bias
Skill Coefficient

This figure performs a comparative static analysis of the EIV bias function for the skill coefficient. We plot the benchmark curve using the parameters of the Gaussian reference model calibrated on our sample. In Panel A, we plot the new EIV bias function after increasing the variance of the true skill coefficient by 0.002/100. In Panel B, we plot the new EIV bias function after increasing the variance of the estimated skill coefficient by 0.002/100. In Panel C, we plot the new EIV bias function after increasing the correlation between the true skill coefficient and the estimation variance by 50% in relative terms.