Currency Areas and Voluntary Transfers[*]

Pierre M. Picard[a], Tim Worrall[b]

[a]*CREA, University of Luxembourg, 6, rue Richard Coudenhove-Kalergi, L-1359 Luxembourg, and CORE, Université Catholique de Louvain, Voie du Roman Pays, 34, 1348 Louvain-la-Neuve, Belgium.*
[b]*School of Economics, University of Edinburgh, 30 Buccleuch Place, Edinburgh, EH8 9JT, UK.*

Abstract

Fiscal integration is recognized as an important issue in determining whether countries establish a common currency area. Fiscal integration between sovereign states is, however, limited by the ability of countries to commit to fiscal transfers. This paper supposes that fiscal transfers between countries must be voluntary and asks how this influences the choice between a currency area and a flexible exchange rate regime. It presents a model with wage rigidity in which, absent transfers, the flexible exchange rate regime is preferred. If there are transfers that equalize consumption, then the choice of exchange rate regime is irrelevant. Nevertheless, the currency area may be preferable if transfers are made voluntarily, because the currency area can sustain greater risk sharing. It is shown that the currency area can be optimal for a plausible set of parameter values. We consider the robustness of the conclusions to some modifications of the model.

Keywords: Optimal Currency Area, Fiscal Union, Limited Commitment, Mutual Insurance
JEL: F12, F15, F31, F33, F45

1. Introduction

Fiscal integration has long been recognized as an important issue in determining whether countries decide to establish a common currency area (see, e.g., Kenen, 1969). Fiscal integration between sovereign states is, however, limited by the ability of countries to commit to fiscal transfers. This paper supposes that fiscal transfers between countries must be *voluntary* and asks how this influences the choice between a currency area and a flexible exchange rate regime. The analysis is relevant to the recent controversy over the refinancing of high deficit countries within the Euro-zone area and the reluctance of the core Euro-zone countries to provide fiscal assistance to peripheral countries.

Our analysis starts from two fundamental premises. First, there are welfare gains to risk sharing between countries. There is much evidence to support the position that risk sharing across countries through international capital markets is highly imperfect. For example, Forni and Reichlin (1999) have shown that there exists a large potential insurable income risk in the EU (about 45%), yet risk diversification is highly incomplete.[1] In what follows, we assume that intercountry transfers are the only means to share risk. The assumption that transfers are the only means to share risk is a strong one, but it allows us to highlight the role played by intercountry transfers and is a good starting point given the evidence that risk diversification is highly incomplete. Second,

---

[1]French and Poterba (1991) find that there is strong home bias in their international asset portfolios preventing international risk sharing (see also, Baxter and Jermann, 1997; Lewis, 1999). For OECD and EU countries Sørensen and Yosha (1998) find that at a three year frequency only about a quarter of the shocks to GDP, at a three year frequency, are smoothed through the use of credit markets.

the concept of a currency area differs from that of a fiscal union. In a fiscal union, intra-country transfers are implemented by constitution, law or governmental decrees or through the tax system. By contrast, a currency area consists of sovereign and independent nations with no ultimate supra-governmental authority. Therefore, we assume that transfers between countries must be voluntary: a country will make a transfer only if it perceives that the long-term benefit of risk sharing offsets the current cost of making the transfer.

To examine this issue of voluntary intercountry transfers, we consider a standard two-country model with labor productivity shocks. Households supply their labor monopsonistically and consume local and foreign goods and money balances. A wage rigidity is introduced by assuming that wages are set optimally by households, but one period in advance and before the outcome of the productivity shock is known. A rigidity of this type is needed for there to be a difference in equilibrium outcomes under different exchange rate regimes. We consider two regimes: a currency area, in which the exchange rate is fixed, and a flexible exchange rate regime, where the local money supply is fixed. The model is intertemporal and countries are allowed to make voluntary transfers contingent on the productivity shock. Transfers are sustained by the threat of returning to a situation without transfers. To keep the model tractable, we focus on a case with two negatively correlated productivity shocks.[2]

A currency area is costly because it closes down the relative price adjustments that come from exchange rate movements, making output and consumption more sensitive to shocks. This cost may turn out to be a benefit if it helps to sustain better risk sharing arrangements between countries in the currency area. The idea that more risk can paradoxically improve risk sharing, because it worsens outside options, is not new. For example, Thomas and Worrall (2007) show that reducing public insurance (increasing risk) can, with limited commitment, crowd-in private insurance, raising welfare, and similar results can be found in Krueger and Perri (2011) and Park (2014), amongst others. However, to our knowledge, this paper is the first to consider the issue in the context of endogenous risk sharing transfers between countries and the choice of exchange rate regime in a standard trade model with shocks to productivity.[3]

The purpose of the paper is not to show that a currency area is optimal, rather, to show that risk sharing through voluntary transfers can be an important factor in determining the optimality of a currency area. We believe our results are important because they qualify the standard result of Mundell (1961) on optimal currency areas. The model embeds three features that are usually seen as inimical to an optimal currency area: wage stickiness, asynchronous business cycles and absence of transaction costs.[4] Crucially, by allowing for voluntary fiscal transfers, there is an additional and endogenous risk sharing mechanism that can counteract the other factors. As we shall show, asynchronous shocks exacerbate the inefficiency caused by wage rigidity and increase the volatility of consumption in a currency area. However, the combination of asynchronous shocks and wage rigidity imply that there are risk-sharing benefits to be derived from intercountry transfers. If a currency area is associated with more risk sharing through the use of intercountry transfers than the flexible exchange rate regime, then the benefits of a currency area may outweigh its costs, reversing Mundell's result. A contribution of this paper is to provide a model in which these issues can be examined and provide conditions where Mundell's result is reversed.

Section 2 presents the baseline model, the preferences and technology, and the condition for the sustainability of transfers. It describes the economic decisions of firms and households as well as the two exchange rate regimes, the flexible exchange rate regime and the currency area, together with the corresponding money supply rules. Section 3 establishes two important preliminary results. First, Proposition 1 shows that absent any transfer, the flexible exchange rate regime dominates the currency area. If, in both regimes, transfers are chosen to equalize

---

[2]Section 6.4 considers a cases with identically and independently distributed shocks.

[3]There may, of course, be other reasons for favouring a currency area. For example, Alesina et al. (1995) and Persson and Tabellini (1996) that have emphasized the role of public goods and externalizes.

[4]That transaction costs can provide a rationale for a currency area is well known (see, e.g., Mundell, 1961, 1973; Bayoumi, 1994; Alesina and Barro, 2002). We examine the implications of transaction costs in Section 6.3.

consumption, then the choice of exchange rate regime is irrelevant: both regimes produce the same outcome. Thus, if the currency area is to be preferred, it is because more risk can be shared in the currency area than the flexible exchange rate regime. The purpose of the paper is to show how this can occur when transfers are voluntary. Secondly, we define a 'fiscal union' as a currency area with the first-best risk sharing transfers and compare this to the flexible exchange rate regime without transfers.[5] Proposition 2 shows that the fiscal union is preferred for a plausible set of parameters. In particular, for sufficiently large shocks, high risk aversion and an elasticity of product substitution that is greater than one, but not too high. In the fiscal union, consumption is perfectly smoothed, which is most desirable when shocks are large and risk aversion is high. The effect of the elasticity of product substitution is more subtle. If the substitutability between local and imported goods is high, then demand is more responsive to price changes. Absent transfers in the flexible exchange rate regime, this leads to undesirable variability in consumption. In the currency area, a higher elasticity of product substitution means more variability in labor, which is compensated for by a higher wage. Thus, a higher elasticity of product substitution leads to greater distortions in the currency area. The net effect on the desirability of the exchange rate regime is ambiguous. In simulations we find that the currency area dominates for an intermediate range of the elasticity of product substitution (the range depends on other parameter values) but that the flexible exchange rate dominates outside this intermediate range.

Having established that a fiscal union can be preferred, Section 4 analyzes how the voluntary nature of transfers affect risk sharing in each exchange rate regime. As in Trionfetti (2018), transfers between countries have a direct impact on consumption and employment but also general equilibrium effects through prices and wages. It is shown that risk-sharing transfers can be supported for lower discount factors in the currency area than in the flexible exchange rate regime (see Proposition 3). This occurs, both because the benefits of risk sharing are greater in the currency area, and because there is a harder landing in the currency area if transfers are reneged upon. Therefore, the currency area provides a stronger incentive to engage in informal insurance. Put differently, the formation of a currency area can be seen as a commitment device that may allow countries to share more risk. As we show, this effect may be so pronounced that, for certain parameter values, the currency area sustains the first-best risk-sharing transfers whereas the flexible exchange rate regime sustains no voluntary transfers (see the Corollary to Proposition 3).

In less extreme cases, transfers are not set at the first-best level but constrained by the requirement that transfers are voluntary. In these cases, the giving country transfers an amount that makes it indifferent between continuing with, and reneging on, the risk sharing scheme. Our results show that the currency area can be preferred for some plausible parameter values of shocks intensity, risk aversion, elasticity of substitution and discount factors This conclusion should nevertheless be applied with caution. The flexible exchange rate regime is preferred for other parameters that might equally be regarded as plausible. Indeed, a crucial parameter for determining which regime is preferred is the 'trade elasticity', i.e., the elasticity of substitution between local and imported goods. Since there is considerable debate about the empirical magnitude of this parameter (see, e.g., Fontagné et al., 2018), the paper does not show that the currency area is optimal, only that the combination of risk sharing and voluntary transfers are important in determining when a currency area is preferred.

The nature of the distortions in the model are discussed in Section 5. There are two types of market distortion in the model. There is an absence of insurance markets to offset the risk caused by the uncertainty in productivity and an imperfection in the labor market. The imperfection in the labor market is two-fold: there is monopsonistic wage setting by labor and there is wage rigidity because the wage is set before the outcome of the productivity shock is known. A fiscal union overcomes the distortion in the insurance market and the flexible exchange rate regime overcomes the wage rigidity in the labor market. Using a simple two state model, it is possible to delineate

---

[5]This is a benchmark case. If the fiscal union does not dominate the flexible exchange rate regime with no transfers, then the currency area could never dominate the flexible exchange rate regime when transfers are voluntary.

the circumstances in which risk sharing motives matter. This is important because Cole and Obstfeld (1991) established that trade itself plays an important role in risk sharing: a bad shock in one country is compensated by a depreciation of the currency, effectively 'importing' the good shock from elsewhere. We confirm that this automatic risk sharing is perfect when the elasticity of substitution between Home and Foreign product is unity (the Cobb-Douglas preference case considered by Cole and Obstfeld). The distortion in the allocation of labor is eliminated in precisely this Cobb-Douglas case. The Cole-Obstfeld model is, therefore, exactly the benchmark in which no transfers are needed and the choice of exchange rate regime is irrelevant (see again, Proposition 1). However, when the elasticity of substitution is strictly larger than one (which we more realistically assume), then trade provides an imperfect risk sharing mechanism and consumption is pro-cyclical, both in a currency area and in the flexible exchange rate regime.

Section 6 considers the robustness of the model to some possible extensions. First, for the sake of analytical tractability, our baseline model assumes money supplies that are fixed and independent of the state of nature. In the context of symmetric countries and shocks, such money supply policies are optimal in the currency area but are not so in the flexible exchange rate regime. Section 6.1 considers a case where in the flexible exchange rate regime and in the absence of transfers, money supply is determined as a Nash equilibrium of competing central banks that maximize domestic household expected utility taking as given the policy of the other central bank and wages in the other country. Second, we discuss the default option in more detail. The baseline model assumes that currency area is maintained following a default. In Section 6.2, we assume, not only that transfers cease following a default, but also that countries adopt a flexible exchange rate system from the period after the default. The differences between the two situations can be thought to be related to whether the fixed exchange rate regime corresponds to a common currency area or to a system of pegged exchange rates. In the former case, there may be very high political and procedural costs of exiting a currency union and our baseline assumption is warranted.[6] By contrast, such costs are lower if the countries operated a pegged exchange rate system. In that case, the assumption that there is reversion to a flexible exchange rate system after a default may be more reasonable. We show that, under this new default assumption, results on the preference for exchange rate systems qualitatively hold but are quantitatively different: a higher degree of risk aversion is required for the pegged exchange rate regime to be preferred to a flexible exchange rate regime.[7] Third, an important advantage of monetary union lies in the reduction of transaction costs in money conversion (Bayoumi, 1994). Section 6.3 considers the presence of transaction costs and show how this favors a currency area. Section 6.4 relaxes the assumption that shocks are perfectly negatively correlated and supposes instead that shocks are independently and identically distributed. It is shown that results are qualitatively unchanged. The intuition for this result is that transfers play a role only when productivity shock realizations are asymmetric across countries. Finally, Section 6.5 considers a case where the flexible exchange regime dominates the currency area with optimal transfers as well as without transfers. Nevertheless, our result that the currency area may preferable when transfers are voluntary still holds.

In summary, our analysis shows that the choice of a currency regime cannot be disentangled from the choice about risk sharing. It suggests that currency areas may, in some circumstances, make redistribution more likely. Empirically, this may be hard to establish. Rose and Engel (2002) find a small but positive relationship between currency areas and risk sharing, but given limited data, the effect is statistically insignificant. Similarly, the examples of long-lived currency areas with a federal structure, like the US, France and Germany, show considerable intra-country risk sharing, which has been implemented after the creation of their national currencies.

---

[6]EU treaties on the Eurozone include articles about central bank and other economic policies. A decision to replace the Euro would have to be preceded by very extensive discussion in government and parliament. (Re-)introducing a national currency requires updating all contracts on wages, bank deposits, bonds, mortgages, taxes, etc. Moreover, the defaulting country is likely to have antagonised its union partners and may no longer be welcomed at the table where other important decisions are made.

[7]Section 6.2 also considers whether the transfers we identify are renegotiation-proof.

By contrast, the recent Eurozone experience is a testament to a concerted resistance to centralize the fiscal authority.[8]

Related literature. The paper relates to the literature on optimal currency areas and risk sharing initiated by Mundell (1973). Kenen (1969) emphasizes the need for interregional transfers within a currency area and Drèze (2000) demonstrates that transfers between regions can be used as a means of insurance against regional income shocks.[9] We build upon the analysis of Devereux (2004) and Ching and Devereux (2003). Devereux (2004) considers a static model with incomplete financial markets and no risk sharing. He demonstrates that a currency area can be desirable because, absent financial markets, both regimes produce inefficient outcomes and the fixed exchange rate can be associated with more stability. A key parameter in his analysis is the elasticity of labor supply: when labor supply is very elastic, a currency area dominates because output becomes highly responsive to demand shocks in a direction that would be chosen by a social planner. Ching and Devereux (2003) consider a similar model where risk is fully shared in the common currency area but where no risk is shared in the flexible exchange rate regime. Our baseline model is similar to Devereux (2004) except that there are productivity rather than preference shocks. However, we consider a repeated version of the model and endogenize risk-sharing transfers by considering transfers that maximize welfare subject to participation constraints.

Our model is also in the tradition of the New Open Economy Macroeconomics. This literature has mostly focused on monetary policies under the assumption of complete financial markets (see, e.g., Obstfeld and Rogoff, 1995; Corsetti et al., 2010). Under this assumption, financial markets offer an important risk-sharing mechanism, so that fiscal transfers between countries are not likely to be relevant. However, a number of papers have considered cases where financial markets are incomplete (see, e.g., Corsetti et al., 2008) and, in particular, Corsetti and Pesenti (2001, 2005) have examined the international transmission of monetary and fiscal policies. Our analysis is in this vein although we assume that there are no financial markets and that transfers are the only means to share risk. Furthermore, we suppose that the ability to share risk is limited by the willingness of countries to make transfers voluntarily.

The model shows that sometimes a currency area may achieve more risk-sharing through transfers. Frankel and Rose (1998) also point out that currency areas may deepen reciprocal trade and hence, endogenously create an optimal currency area even if non existed beforehand. As we have mentioned, Rose and Engel (2002) attempt to empirically address the issue of improved risk sharing in a currency union and DeGrauwe-Mongelli (2005) discuss more generally how financial and labor markets may become more integrated and property rights more uniform, endogenously creating an optimal currency area. These papers show that many aspects of the economy, for example, the synchronization of macroeconomic shocks, financial and commercial integration, and so on, may be endogenous to the type of monetary union or exchange rate system. However, these papers take the fiscal policies as exogenous, whereas we analyze how fiscal policy changes endogenously with the choice of regime.

Four related papers that address currency areas in a similar context to ours are Arellano and Heathcote (2010), Castro and Koumtingué (2014), Farhi and Werning (2017) and Fuchs and Lippi (2006). Arellano and Heathcote (2010) consider full dollarization rather than a currency area and non-contingent debt rather than risk sharing. Nevertheless, the basic mechanism at work is similar to ours: borrowing is limited because default is punished only by exclusion from future borrowing; dollarization has a cost because there is a loss in seigniorage

---

[8]This might be because from its inception, the Euro currency was seen as a political rather than economic project (in particular by its principal advocates, Chancellor Kohl and President Mitterand).

[9]Cooper and Kempf (2004) examine a slightly different trade-off. They have risk sharing within countries but no transfers between countries. In their model a currency area overcomes a cash in advance constraint, allowing consumption to be adapted to taste shocks, but the central monetary authority in a currency area cannot adapt its policy to divergent unemployment shocks in the separate countries. Thus, a currency area is welfare improving when unemployment shocks are sufficiently positively correlated across countries and when taste shocks are sufficiently large.

but the very fact that countries cannot use monetary easing makes the costs of default on borrowing greater and hence, may allow the country to borrow more in international markets. Castro and Koumtingué (2014) also considers risk sharing and limited commitment in examining the optimality of a currency area. They, however, assume that the formation of a union enables full risk sharing and that trade with countries outside of the union is restricted by limited enforcement. Thus, their modeling assumptions are very different from ours. Farhi and Werning (2017) address a similar issue but in a different model. Their model has a non-traded good and sticky prices that generates an aggregate demand externality. In their dynamic model, financial markets are incomplete, but shocks occur only once at the beginning of the first period. In our model, the need for transfers arises because shocks are repeated. Fuchs and Lippi (2006) consider a dynamic policy game where policy has to be coordinated in a monetary union. This provides a tension between co-ordination and flexibility. Although Fuchs and Lippi (2006) consider the intertemporal incentives to leave the monetary union, they do not provide a welfare analysis of the two regimes.

The paper is organized as it follows. Section 2 presents the baseline model, preferences, technology and transfers, and describes the economic decisions of firms and households as well as the two exchange rate regimes. Section 3 presents two preliminary results. Section 4 analyzes how voluntary transfers are sustained and establishes the parameter set for which the currency area sustains larger transfers and yields higher expected utility than the flexible exchange rate regime. The different distortions in the model are discussed in Section 5. Section 6 considers the robustness of the model to some possible extensions. Section 7 concludes. Appendix A provides some standard derivations for the model and proofs of propositions are contained in Appendix B.

2. The Model

The model builds upon a two-country trade model with money demand. Each country, Home and Foreign, has a unit mass of households and produces an imperfectly differentiated good under perfect competition. Households supply imperfectly differentiated labor services $j \in [0, 1]$, and derive utility from the consumption of both goods and money. Money is supplied by central banks in each country in a way described below. Home and Foreign can engage in voluntary intercountry transfers to share risk.

2.1. Preferences, technology and transfers

Countries are ex-ante symmetric with respect to preferences and technology. Productivity is uncertain and varies across countries. Home and Foreign have productivity shocks a_s and a_s^* in the state of nature $s \in \{1, ..., S\}$ (goods are indexed by H or F and Foreign variables are denoted by an $*$). A unit of Home and Foreign output is determined by the production functions $F(\ell(\cdot))/a_s$ and $F(\ell^*(\cdot))/a_s^*$, where $F(\ell(\cdot) \equiv (\int_0^1 (\ell(j))^{\frac{(\theta-1)}{\theta}} \, \mathrm{d}j)^{\frac{\theta}{(\theta-1)}}$ is the common technology function, $\ell(j)$ and $\ell^*(j^*)$ are the Home and Foreign labor services provided by household j and j^*, and $\theta > 1$ is the elasticity of substitution between labor services. As $\theta \to \infty$, labor services become homogenous and the labor market is perfectly competitive. The productivity shock a_s is an inverse measure of productivity.

Each time period $t = 1, 2, \ldots, \infty$ includes the following sequence of three events. First, there is a shock to productivity a_s^t. Second, there is a transfer between countries. Thirdly, local money supply is set by local central banks, firms and households make their decisions on demands and supplies, markets clear and wages for next period are chosen (how households choose the next period wage is explained in Section 2.2).

Let T_s^t denote the transfer received by Home (expressed in Home currency) in state s at date t (how transfers are determined is discussed shortly). Conditional on the productivity shock a_s^t and transfer T_s^t, the central banks implement a monetary policy to supply $m_{0,s}^t$ to each household. The money supply policy can depend on shocks and transfers and will vary depending on the exchange rate regime considered (the details of the monetary policy in the two regimes is explained in Section 2.3 and discussed further in Section 6.1).

Given the productivity shock, transfer and wage set from the previous period, Home firms choose the labor service mix that maximizes profit $\pi_{H,s}^t = p_s^t d_{H,s}^t - \int_0^1 w^{t-1}(j)\ell_s^t(j)\mathrm{d}j$ subject to the production technology, where $d_{H,s}^t$ is the demand for Home output at date t, p_s^t is its price and $w^{t-1}(j)$ is the price for labor service j, which was set at the end of period $t{-}1$. Since the wage is set in advance, it is unresponsive to the current productivity shock. As mentioned in the introduction, a rigidity of this type is needed for there to be a difference in equilibrium outcomes under different exchange rate regimes. There is perfect competition in the product market, so that, profits are zero in equilibrium: $\pi_{H,s}^t = 0$. A similar argument applies to Foreign with demand $d_{F,s}^{*t}$, price p_s^{*t}, wage $w^{*t-1}(j)$ and profits $\pi_{F,s}^t = 0$.

Given the productivity shock, transfer, wage from the previous period and monetary supply $m_{0,s}^t$, each household j chooses its current consumption of Home and Foreign goods $c_{H,s}^t(j)$ and $c_{F,s}^t(j)$, its current consumption of real money balance $m_s^t(j)$ supply $\ell_s^t(j)$ of differentiated labor service, and its wage, $w^t(j)$ applying in the next time period, to maximize the expected discounted utility,

$$\mathbb{E}_0 \sum_{t=0}^\infty \delta^t u\left(c_{H,s}^t(j), c_{F,s}^t(j), \ell_s^t(j), m_s^t(j)\right),$$

subject to the per-period budget constraints:

$$p_s^t c_{H,s}^t(j) + \varepsilon_s^t p_s^{*t} c_{F,s}^t(j) + m_s^t(j) = w^{t-1}(j)\ell_s^t(j) + T_s^t + m_{0,s}^t, \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator at date $t = 0$, $\delta < 1$ is the discount factor and ε_s^t is the nominal exchange rate, defined as the units of Home currency required to purchase one unit of Foreign currency. Product demands are $d_{H,s}^t = \int_0^1 [c_{H,s}^t(j) + c_{H,s}^{*t}(j)]\mathrm{d}j$ and $d_{F,s}^t = \int_0^1 [c_{F,s}^t(j) + c_{F,s}^{*t}(j)]\mathrm{d}j$. Note, that in this specification, households do not save or borrow to smooth consumption and money depreciates within the period. This is not because we believe the role of savings and borrowing is unimportant. Rather, we wish to focus on the role played by transfers in sharing risk.

To further simplify the analysis, we restrict attention to two perfectly anti-correlated and equiprobable states. That is, where the productivity shocks are $(a_1, a_1^*) = (a_G, a_B)$ in state 1 and $(a_2, a_2^*) = (a_B, a_G)$ in state 2, with $a_G < a_B$. We normalize $a_B = 1$ and let $a_G = z \in (0, 1)$. Thus, the Home country has the 'good' (G) productivity in state 1 and the 'bad' (B) productivity in state 2, with the reverse being true for the Foreign country. This simplification brings a number of advantages in addition to tractability. First, the negative correlation in productivity allows us to focus on a case where there is a strong desire to share risk. Second, the standard Mundell argument against a currency area is strongest when productivity levels are negatively correlated. In this example, productivity levels are perfectly anti-correlated, so that, the model is designed in a way that would normally be considered to favor the flexible exchange rate regime. It provides the simplest setting in which the opposing forces of risk sharing and flexibility in exchange rates can be assessed.

Another key advantage of working with two anti-correlated states is that it allows us to consider the situation where transfers depend only on the current state and not on the date or past history of states.[10] In this case, transfers in the two states can be described by two values T_G and T_B where $T_1^t = T_G < 0$, and $T_2^t = T_B > 0$. Similarly, in Foreign, $T_1^{*t} = T_B$ and $T_2^{*t} = T_G$. Since there are no transfers from outside the two countries, $T_G = -\varepsilon_G T_B > 0$, where ε_G is the Home exchange rate when it has the good productivity shock. Given the homogeneity of households, wages are identical across households $(w_s^{t-1}(j) = w_s^{t-1}(j'))$. Since wages are set

---

[10]In principle, transfers are history contingent and not just state contingent. For example, Kocherlakota (1996) and Thomas and Worrall (1988) show that when commitment is limited, transfers are, in general, history dependent as well as state dependent. However, there is convergence of transfers to a steady-state invariant distribution in the long run. With two states and as soon as both states have occurred, optimal voluntary transfers depend only on the current state. The two-state case is widely used in the literature on limited commitment (see, e.g., Alvarez and Jermann, 2001; Kehoe and Levine, 2001) to simplify the analysis.

one period in advance and countries and transfers are symmetric, wages are also set identically across countries $(w_s^{t-1}(j) = w_s^{*t-1}(j^*))$. Since wages are set in advance, they depend on the vector of the anticipated transfers $T = (T_G, T_B)$. With this vector of transfer the same each period, the wage set in advance is the same each period and we write the preset wage as the function $W(T)$. Similarly, since households are identical, consumption and labor supply are identical across households. Therefore, we write the *contemporaneous indirect utility* in state s as a function of the wage and the transfer in that state: $U_s(W(T), T_s)$. For a given vector of anticipated transfers T, the discounted lifetime utility in state s can then be defined as:

$$V_s(W(T), T) \equiv U_s(W(T), T_s) + \frac{\delta}{1-\delta} \mathbb{E}_q U_q(W(T), T_q), \tag{2}$$

where $\mathbb{E}_q$ is the expectation operator across states $q \in \{1, 2\}$.

As explained in the introduction, we require that transfers are *voluntary*. That is, the short run cost of making a transfer must be offset by the long run benefits of future risk sharing. For Home, the short run utility loss of giving a transfer $T_G < 0$ in state 1 is $U_G(W(T), T_G) - U_G(W(T), 0) < 0$, where the comparison is made at the fixed preset wage. By contrast, its short run utility gain of receiving a transfer $T_B > 0$ in state 2 is $U_B(W(T), T_B) - U_B(W(T), 0) > 0$. The long run benefits depend on how a country that has reneged on the agreed transfer is treated. We suppose that after a default all trust is lost and no future transfers are made.[11] Then, the expected net gain for Home in the next period is $\mathbb{E}_s[U_s(W(T), T_s) - U_s(W(0), 0)]$, where the wage next period in the event of default is $W(0)$. Therefore, for the transfer vector T to be voluntary, the following constraint for Home in state 1 must be satisfied:

$$U_G(W(T), T_G) - U_G(W(T), 0) \geq \frac{\delta}{1-\delta} \mathbb{E}_s[U_s(W(T), T_s) - U_s(W(0), 0)].$$

Using the definition of $V_s(W(T), T)$, this can be rewritten as:

$$V_G(W(T), T) \geq U_G(W(T), 0) + \frac{\delta}{1-\delta} \mathbb{E}_s U_s(W(0), 0). \tag{3}$$

This type of *incentive* or *participation constraint* is common in the literature on limited commitment (see, e.g., Thomas and Worrall, 1988). A similar constraint to (3) must also hold for Foreign. Transfers that satisfy these constraints are said to be *sustainable*. We assume that countries agree sustainable transfers T at date $t = 0$ to maximize the utilitarian objective $\mathbb{E}_s[V_s(W, T) + V_s^*(W, T)]$.

To derive the indirect utility function $U_s(W, T_s)$ we suppose that the direct utility of a household has a Cobb-Douglas-CES form:

$$u(c_H, c_F, \ell, m) = \frac{x^{(1-\gamma)} - 1}{1-\gamma} - \frac{\ell^2}{2} \quad \text{where} \quad x = \left(\tfrac{1}{2} c_H^{\frac{\sigma-1}{\sigma}} + \tfrac{1}{2} c_F^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\mu} \left(\frac{m}{P}\right)^{1-\mu}. \tag{4}$$

The term x is a Cobb-Douglas upper tier composite of the real money balance m/P and the lower tier CES composite of the consumption of local and imported goods. The variable P is the local price index. The parameter $\mu \in (0, 1)$ measures the preference for real money balances (as $\mu \to 1$, real money balances are not valued).[12] The parameter σ is the elasticity of substitution between home and imported goods. We assume

---

[11]This is, of course, an extreme assumption, but weaker assumptions, such as exclusion for n periods would result in qualitatively similar results. Although the assumption of return to zero transfers following a default is not efficient, the outcome we identify is renegotiation-proof, but we delay a discussion of this issue until Section 6.2. For the moment, we will also assume that the exchange rate regime remains the same, independently of whether there is default or not. The discussion of alternative assumptions is again delayed to Section 6.2.

[12]Feenstra (1986), for example, demonstrates a functional equivalence between using money balances in the utility function and liquidity costs entering the household budget constraint. Thus, μ might also be considered as a measure of these liquidity costs. Our results also apply as $\mu \to 1$ and our baseline numerical calculations use $\mu = 0.99$.

that $\sigma > 1$: goods are imperfect substitutes. A higher value of σ means that goods are more substitutable and demand will respond more to a change in prices. In the limit where $\sigma \to 1$, the lower tier composite has the Cobb-Douglas form $\sqrt{c_H c_F}$ considered by Cole and Obstfeld (1991). The utility derived from consumption and money balances exhibits constant relative risk aversion with coefficient γ over the upper-tier composite x. The functional form given in (4) is similar to the Cobb-Douglas preferences considered by Blanchard and Kiyotaki (1987) when $\gamma \to 0$ and corresponds to the logarithmic form used by Corsetti and Pesenti (2005) as $\gamma \to 1$, a special case that we also consider at points throughout the text. Finally, utility decreases quadratically with the supply of labor service, which corresponds to a Frisch elasticity of labor supply equal to unity.

2.2. Households' and firms' decisions

In this section, we discuss the choices of contemporaneous consumptions and labor demands for a given transfer T_s and state of nature s, as well as the choice of the wage for a given transfer system T. To simplify the notation, we dispense time superscripts and state subscripts whenever it does not lead to confusion.

For a given transfer T_s, each Home household j chooses the contemporaneous consumptions of local and imported goods $(c_H(j), c_F(j))$ and the money balance $m(j)$ that maximize its utility (given by (4)) subject to its budget constraint (given by (1)). Each firm takes households' wages as given and hires the mix of labor services $\ell(\cdot)$ that minimizes its unit costs within the current period. In each period, the product and labor markets clear. Wages are determined at the end of the period and before the next productivity shock is known. The Home household j sets its wage to maximize its expected utility, anticipating transfers, prices, the exchange rate, labor demand and money supply in the next period. The equilibrium solution is standard and summarized in Table 1 and details can be found in Appendix A.

Table 1 presents economic relationships between the aggregate variables, which are denoted by a capital letter: $C_H \equiv \int_0^1 c_H(j) \mathrm{d}j$, $L \equiv \int_0^1 \ell(j) \mathrm{d}j$, etc. Since households and firms are symmetric and households have unit mass in each country, their consumption and hiring choices are symmetric: $C_H = c_H(j)$, $L = \ell(j)$, ...$\forall j$. Similarly, preset wages are symmetric within the same country so that all $w(j)$ are identical and equal to the cost of a mix of labor services $W \equiv (\int_0^1 (w(j))^{1-\theta} \mathrm{d}j)^{1/(1-\theta)}$. The equilibrium properties reported in Table 1 are standard for a model with CES-Cobb-Douglas preferences: local prices increase with the local inverse productivity and the wage; product demands are isoelastic functions of own prices; local labor demands are proportional to local inverse productivity and product demands; local expenditure comprises the local wage bill, transfer and money supply; expenditures on goods and on money balances are constant shares of total expenditure; the consumption mix of local and imported goods falls with relative prices and money markets clear. There is a trade balance equation, which equates the value of Home imports to the value of Home exports *plus* the transfer to Home. The equilibrium exchange rate ε depends on the inverse productivities and importantly, on the preset wages and transfers. Finally, the preset wage W and W^* satisfy the formulas given in Table 1 (see Appendix A for the derivation). These equations for the preset wages balance the expected marginal disutility of labor with the expected marginal utility from consumption taking account of monopsony power of workers. It can be shown that the wage decreases with the elasticity of substitution the monopsony power of the workers in the labor market as measured by the parameter θ. The formula for the preset wage W in Table 1 is implicit because prices and labor supplies also depend on W. Since consumption X_s and labor L_s depend on the transfer T_s, the wage depends on the vector T of state-contingent transfers. As already noted, symmetry means $W^*(T) = W(T)$.

2.3. Exchange rate regimes

In this section, we consider the two alternative exchange rate regimes for a given vector of transfers T: a currency area with a fixed exchange rate and a flexible exchange rate regime. The money supply policy and the wage and the equilibrium outcomes are different in the two regimes.

Table 1: Contemporaneous decisions for a given productivity pair, (a, a^*), given wage W, given money supplies, M_0 and M_0^*, and given transfer T.

Definitions	Home	Foreign
lower tier composite:	$C^{\frac{\sigma-1}{\sigma}} = \frac{1}{2}C_H^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}C_F^{\frac{\sigma-1}{\sigma}}$	$(C^*)^{\frac{\sigma-1}{\sigma}} = \frac{1}{2}\left(C_H^*\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}\left(C_F^*\right)^{\frac{\sigma-1}{\sigma}}$
upper tier composite:	$X = (C)^\mu \left(\frac{M}{P}\right)^{1-\mu}$	$X^* = (C^*)^\mu \left(\frac{M^*}{P^*}\right)^{1-\mu}$

World Aggregates		
income:	$Y^w = Y + \varepsilon Y^*$	
money demand:	$M^w = M + \varepsilon M^*$	
money supply:	$M_0^w = M_0 + \varepsilon M_0^*$	
transfers:	$T + \varepsilon T^* = 0$	

Equilibrium		
competitive prices:	$p = aW$	$p^* = a^* W^*$
price index:	$P^{1-\sigma} = \left(\frac{1}{2}\right)^\sigma p^{1-\sigma} + \left(\frac{1}{2}\right)^\sigma (\varepsilon p^*)^{1-\sigma}$	$P^* = P/\varepsilon$
product demand:	$D = \mu\left(\frac{1}{2}\right)^\sigma \frac{p^{-\sigma}}{P^{1-\sigma}} E^w$	$D^* = \left(\frac{p}{\varepsilon p^*}\right)^\sigma D.$
labor mkt. clearing:	$L = aD$	$L^* = a^* D^*$
expenditure:	$E = WL + T + M_0$	$E^* \equiv W^* L^* + T^* + M_0^*$
household demand:	$\frac{PC}{\mu} = \frac{M}{1-\mu} = E$	$\frac{P^* C^*}{\mu} = \frac{M^*}{1-\mu} = E^*$
upper tier composite:	$X = \xi \frac{E}{P}$	$X^* = \xi \frac{E^*}{P^*}$
consumption mix:	$\frac{C_H}{C_F} = \left(\frac{p}{\varepsilon p^*}\right)^{-\sigma}$	$\frac{C_H^*}{C_F^*} = \left(\frac{p}{\varepsilon p^*}\right)^{-\sigma}$
money mkt. clearing:	$M = M_0$	$M^* = M_0^*$

trade balance:	$\varepsilon p^* C_F = p C_H^* + T$	
exchange rate:	$\varepsilon = \left(\frac{aW}{a^* W^*}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\vartheta M_0 - T}{\vartheta M_0^* - T^*}\right)^{\frac{1}{\sigma}}$	

preset wage:	$W(T) = \left(\frac{\theta}{\theta-1}\right)\left(\frac{\mathbb{E}_s\left[L_s^2\right]}{\xi \mathbb{E}_s\left[X_s^{-\gamma}\frac{L_s}{P_s}\right]}\right)$	$W^*(T) = \left(\frac{\theta}{\theta-1}\right)\left(\frac{\mathbb{E}_s\left[L_s^{*2}\right]}{\xi \mathbb{E}_s\left[X_s^{-\gamma}\frac{L_s^*}{P_s^*}\right]}\right)$

Identities		
national accounts:	$Y = pD = WL$	$Y^* = p^* D^* = W^* L^*$
	$PC = pD + T$	$P^* C^* = p^* D^* + T^*$
constants:	$\xi = \mu^\mu (1-\mu)^{1-\mu}$	$\vartheta = \frac{\mu}{1-\mu}$

First note that since money demand equals money supply in each country and because the shares of consumption and money demand are constant, we can write labor and consumption as:

$$L_s = \frac{\vartheta M_{0,s} - T_s}{W}, \quad \text{and} \quad X_s = \frac{\xi}{\mu}\frac{\vartheta M_{0,s}}{P_s},$$

where $\vartheta = \mu/(1-\mu)$ measures the preference for consumption over real money balances. It is convenient to write the price index P_s as:

$$P_s = AB_s W, \quad \text{where}$$

$$A \equiv \left(\left(\frac{1}{2}\right)^\sigma a_s^{1-\sigma} + \left(\frac{1}{2}\right)^\sigma (a_s^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \quad b_s \equiv \frac{a_s^{1-\sigma}}{\frac{1}{2}a_s^{1-\sigma} + \frac{1}{2}(a_s^*)^{1-\sigma}}, \quad B_s \equiv \left(\frac{1}{2}b_s + \frac{1}{2}b_s^* \varepsilon_s^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

The term A is a measure of global productivity and is a constant because states are symmetric (i.e. $A = A_G = A_B$). The term b_s is a measure of relative productivity in the Home country with b_s^* defined similarly with $\frac{1}{2}b_s + \frac{1}{2}b_s^* = 1$. The term B_s is a measure of the impact of the exchange rate on local prices.[13] From Table 1 and given that wages are symmetric, the exchange rate is given by:

$$\varepsilon_s = \left(\frac{b_s}{b_s^*}\right)^{-\frac{1}{\sigma}} \left(\frac{\vartheta M_{0,s} - T_s}{\vartheta M_{0,s}^* - T_s^*}\right)^{\frac{1}{\sigma}}. \tag{5}$$

In a currency area, the exchange rate is constant: $\varepsilon_s = 1$. There are two ways to interpret monetary policy in a currency area. In the first, countries retain a separate monetary policy but adjust their money supplies to fix the exchange rate to unity. In the second, countries establish a unitary monetary authority with a common currency, so that the exchange rate is trivially equal to unity. Since money is neutral, we can, without loss of generality, fix the world money supply independently of the state of nature: $M_{0,s}^w \equiv M_0^w$.[14]

First, consider a fixed exchange rate system with separate monetary policies. The exchange rate is maintained to unity with the following simple monetary policy rules:

$$M_{0,s}^c = \tfrac{1}{2}M_0^w b_s + \frac{T_s}{\vartheta} \quad \text{and} \quad M_{0,s}^{c*} = \tfrac{1}{2}M_0^w b_s^* + \frac{T_s^*}{\vartheta}, \tag{6}$$

where we use ^{c} superscript to denote the currency area. Substituting these monetary policy responses into (5) shows that $\varepsilon_s = 1$, fixed independently of the state. The policy $M_{0,s}^c$ means that if productivity is good, then money supply is expanded to match the increased demand. Likewise, the money supply is expanded when the transfer is increased.[15] Second, consider a currency area with a single common currency with money supply of M_0^w where money supply in each country adjusts to match money demand. Money demand adjusts to the productivity shock and transfer. In particular, in equilibrium, Home money demand satisfies $M_s = (1 - \mu)(Y_s + T_s + M_s)$ while the nominal output is $Y_s = (1/2)\vartheta M_0^w b_s$. Combining these two equations shows that $M_s = M_{0,s}^c$ given in (6). Thus, these two ways to interpret monetary policy in a currency area are formally equivalent. Of course, countries may find it easier to exit the currency area when monetary independence is retained than when a common currency has been established. We discuss this further and consider the implications for what happens when there is a default in Section 6.2.

In the flexible exchange rate regime, assume both the world and local money supply is fixed:

$$M_{0,s}^f = M_{0,s}^{f*} = \tfrac{1}{2}M_0^w, \tag{7}$$

where ^{f} denotes the flexible exchange rate regime.[16] Under the money supply in (7), the exchange rate varies with productivity and the transfer. Since $T_s + \varepsilon_s T_s^* = 0$, the exchange rate in equation (5) is an implicit function of T_s. We write $\varepsilon_s(T_s)$, and correspondingly $B_s(T_s)$, to emphasize this dependence.

To simplify notation, and w.l.o.g., we normalize money supply such that $\frac{1}{2}\vartheta M_0^w = 1$. Using this normalization and substituting for the money supply rules given in (6) and (7), the equilibrium values of income, composite consumption and labor supply are:

$$\begin{array}{llll} Y_s^c = b_s, & X_s^c = \frac{\xi}{\mu A}\frac{b_s + T_s}{W^c(T)}, & L_s^c = \frac{b_s}{W^c(T)}, \\ Y_s^f = 1 - T_s, & X_s^f = \frac{\xi}{\mu A}\frac{1}{B_s(T_s)W^f(T)}, & L_s^f = \frac{1 - T_s}{W^f(T)}, \end{array} \tag{8}$$

---

[13]With two states, $b_G \in (1, 2)$ and $b_B \in (0, 1)$. When $\sigma = 1$, $b_s = b_s^* = 1$: relative productivity is constant across countries and states. A higher value of σ produces more variability in the relative productivity of the two countries because demand becomes more responsive to price changes. If the exchange rate is $\varepsilon_s = 1$ (as in the currency area), then $B_s = 1$.

[14]Since money is neutral and countries are symmetric, maintaining a fixed world money supply is the money supply rule that maximizes the equally-weighted joint utility of the two countries.

[15]Note that productivity is high in the good state but the transfer is negative. Although the two effects are offsetting, the former effect is dominant and money supply will not contract in the good productivity state.

[16]This fixed money supply rule is not, in general, optimal in the flexible exchange rate regime, although absent transfers it stabilizes employment. Alternative money supply rules that respond to the shock will be discussed in detail Section 6.1.

where $W^c(T)$ and $W^f(T)$ denote the preset wage in the two regimes. The indirect utility in regime $r \in \{c, f\}$ is

$$U_s^r(W^r, T_s) = \frac{(X_s^r)^{1-\gamma} - 1}{1 - \gamma} - \frac{(L_s^r)^2}{2}.$$

The main similarities and differences between the two exchange rate regimes can be summarized as follows. In the fixed exchange rate regime, there is a direct effect of transfers on consumption X_s^c but no direct effect on labor L_s^c. In the flexible exchange rate regime, there is a direct effect of transfers on labor but no direct effect on consumption. In both regimes, there is an indirect effect of transfers on labor because transfers affect the wage set at the start of the period. Equally, there is an indirect effect of transfers on prices. In the fixed exchange rate regime, this effect occurs through the wage, whereas in the flexible exchange rate regime, there is also an effect on prices caused by the impact of transfers on the exchange rate.

3. Preliminary results

In this section, we first compare the currency area and flexible exchange rate regime with no transfers and when transfers equalize consumption across states. Next, we compare the currency area with consumption equalizing transfers to the flexible exchange rate regime without transfers. We do this without considering whether countries would voluntary make these transfers. That will be considered in the next section.

3.1. Comparing the currency area with the flexible exchange rate regime

It is easily checked that the transfers that would be chosen by a utilitarian social planner to maximize the objective function $\mathbb{E}_s[U_s^c(W, T_s) + U_s^{c*}(W, T_s)]$ satisfy

$$\bar{T}_s = \frac{(b_s^* - b_s)}{2}. \tag{9}$$

We shall refer to the transfers $\bar{T}_s$ as *optimal* transfers. These optimal transfers equalize consumption across states.[17]

Using the wage formula from Table 1, the following proposition that compares the currency area with the flexible exchange rate regime can be established.

Proposition 1. *The preset wage as a function of transfers in the two regimes is given by:*

$$\frac{W^c(T)}{W^c(0)} = \left(\frac{\mathbb{E}_s[b_s^{1-\gamma}]}{\mathbb{E}_s[b_s(b_s+T_s)^{-\gamma}]} \right)^{\frac{1}{1+\gamma}}, \qquad \frac{W^f(T)}{W^f(0)} = \left(\frac{(\mathbb{E}_s[B_s(0)^{\gamma-1}])(\mathbb{E}_s[(1-T_s)^2])}{\mathbb{E}_s[B_s(T_s)^{\gamma-1}(1-T_s)]} \right)^{\frac{1}{1+\gamma}}, \tag{10}$$

where

$$W^c(0) = \left(\kappa_0 \frac{\mathbb{E}_s[b_s^2]}{\mathbb{E}_s[b_s^{1-\gamma}]} \right)^{\frac{1}{1+\gamma}}, \qquad W^f(0) = \left(\kappa_0 \frac{1}{\mathbb{E}_s[B_s(0)^{\gamma-1}]} \right)^{\frac{1}{1+\gamma}}, \tag{11}$$

are the wages in the absence of transfers and

$$\kappa_0 = \left(\frac{\theta}{\theta-1} \right) \left(\frac{1}{\mu} \right) \left(\frac{\xi}{\mu A} \right)^{\gamma-1}.$$

The two exchange rate regimes yield the same allocation if $\sigma = 1$, or if the transfers are optimal (satisfy (9)), in which case $W^c(\bar{T}) = W^f(\bar{T})$. Absent any transfer, the flexible exchange rate regime yields higher welfare than the currency area if and only if

$$\left(\mathbb{E}_s\left[b_s^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} < \frac{\sqrt{\mathbb{E}_s[b_s^2]}}{\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1} \right] \right)^{\frac{1}{\gamma-1}}}. \tag{12}$$

For the two equiprobable state case, this condition is always satisfied.

---

[17]Note that in the flexible exchange rate regime, the optimal transfers do not maximize welfare. We return to this issue in Section 6.5.

There are two cases in which the currency area and flexible exchange rate regime yield the same expected utility. First, when the elasticity of substitution equals unity ($\sigma = 1$). This is the Cobb-Douglas framework analyzed by Cole and Obstfeld (1991). Then, $\varepsilon_s = 1$, $B_s(T_s) = 1$ and $P = AW$ independent of the state. Transfers are not needed, $W^c(0) = W^f(0)$ and the allocation is the same in both regimes. That is, the flexible exchange rate regime and the currency area are indistinguishable and the choice of regime is irrelevant. Second, when transfers are optimal and given by equation (9). In this case, it follows from the exchange rate formula in equation (5) that $\varepsilon_s(\bar{T}_s) = 1$, and from (8) that $Y_s^c = Y_s^f$. Moreover, it can be checked that the wage is the same in both regimes:

$$W^c(\bar{T}) = W^f(\bar{T}) = \left(\kappa_0 \mathbb{E}_s\left[b_s^2\right]\right)^{\frac{1}{1+\gamma}}.$$

Let $\bar{W}$ denote this common wage. It follows that $L_s^c = L_s^f = b_s/\bar{W}$ and $X_s^c = X_s^f = (\xi/\mu)/(A\bar{W})$ independent of the state. With the transfers $\bar{T}$, the allocation in both exchange rate regimes is identical and the upper tier composite consumption is equalized across states and countries.

Consider the case where there are no transfers in either regime. Proposition 1 establishes that $\mathbb{E}_s[U_s^f(W^f(0), 0)] > \mathbb{E}_s[U_s^c(W^c(0), 0)]$ if and only if condition (12) is satisfied. Here we provide an illustration of this result for $\gamma = 1$. In this case, it follows from equation (11) that the preset wages in the two regimes satisfy:

$$\frac{W^c(0)}{W^f(0)} = \sqrt{\mathbb{E}_s\left[b_s^2\right]}.$$

Since, $\mathbb{E}_s[b_s] = 1$, convexity (of the function b^2) implies that $W^c(0) > W^f(0)$, which shows that the wage rigidity distortion is greater in the currency area. Nevertheless, the expected disutility of labor is the same in both regimes:

$$\frac{1}{2}\mathbb{E}_s\left[\left(L_s^f(0)\right)^2\right] = \frac{1}{2}\frac{1}{(W^f(0))^2} = \frac{1}{2}\frac{\mathbb{E}_s\left[b_s^2\right]}{(W^c(0))^2} = \frac{1}{2}\mathbb{E}_s\left[\left(L_s^c(0)\right)^2\right].$$

So, when $\gamma = 1$, the expected utility difference between the currency area and the flexible exchange rate regime without transfers stems only from the difference in consumption. The difference in the expected utility from consumption is

$$\mathbb{E}_s\left[\log\left(X_s^f(0)\right)\right] - \mathbb{E}_s\left[\log\left(X_s^c(0)\right)\right] = -\mathbb{E}_s\left[\log\left(B_s(0)\right)\right] - \log\left(W^f(0)\right) - \mathbb{E}_s\left[\log(b_s)\right] + \log\left(W^c(0)\right).$$

Thus, absent transfers, the flexible exchange rate regime dominates if and only if

$$\log\left(\frac{W^c(0)}{W^f(0)}\right) > \mathbb{E}_s\log\left(B_s(0)\right) + \mathbb{E}_s\left[\log(b_s)\right].$$

This corresponds to condition (12) with $\gamma = 1$. It can be checked that condition (12) always holds for our two state case. Thus, if the currency area is ever preferable, it is because of the risk sharing benefits provided by transfers.

3.2. Fiscal Union

We have shown in Proposition 1 that the rigidities of a currency area lead to lower levels of welfare compared to a flexible exchange rate if there are no transfers. If, on the other hand, transfers are optimal, then the choice of exchange rate regime is irrelevant. A currency area might be preferable if it is associated with more transfers than the flexible exchange rate regime. Let us here define a *fiscal union* as a currency area associated with the optimal transfers $\bar{T}$. In this section, we compare the fiscal union with a flexible exchange rate regime with no transfers. That is, we compare $\mathbb{E}_s[U_s^c(W^c(\bar{T}), \bar{T}_s)]$ with $\mathbb{E}_s[U_s^f(W^f(0), 0)]$.

The fiscal union is a useful benchmark to consider because it corresponds to a situation in which transfers can be legally enforced. A currency area with voluntary transfers cannot do better than this fiscal union. The comparison is important because, if there were no parameter values such that the fiscal union dominates the

flexible exchange rate regime without transfers, then there will be no parameter values for which a currency area ever dominates when transfers are voluntary. It is also an interesting comparison in its own right (Ching and Devereux (2003) make the same comparison, albeit in a slightly different model). It is pertinent when a currency area is combined with institutional mechanisms that enforce fiscal transfers. For example, where a currency area coincides with a legal state, the constitution may provide a legal framework for redistributive fiscal policies.

Using equation (8) and the definition of the optimal transfer from equation (9), consumption and labor in the fiscal union and flexible exchange rate regime without transfers are given by:

$$\bar{X}^c = \frac{\xi}{\mu A}\frac{1}{\bar{W}}, \qquad\qquad \bar{L}^c_s = \frac{b_s}{\bar{W}},$$
$$X^f_s(0) = \frac{\xi}{\mu A}\frac{1}{B_s(0)W^f(0)}, \quad L^f_s(0) = \frac{1}{W^f(0)}.$$

Consumption is state independent in the currency area but state dependent in the flexible exchange rate regime. The reverse is true of labor, and hence, also output.

For the sake of exposition, we present the comparison of $\mathbb{E}_s[U^c_s(W^c(\bar{T}), \bar{T}_s)]$ and $\mathbb{E}_s[U^f_s(W^f(0), 0)]$ for $\gamma = 1$, and Proposition 2 gives the general case. For $\gamma = 1$, it can be checked from Proposition 1 that the preset wages in the two regimes satisfy

$$\frac{\bar{W}}{W^f(0)} = \sqrt{\mathbb{E}_s\left[b^2_s\right]}.$$

Since, $\mathbb{E}_s[b_s] = 1$, $\bar{W} > W^f(0)$. To understand this, recall that labor supply is constant in the flexible exchange rate regime (without transfers) but varies positively with productivity in the fiscal union. The wage is higher in the fiscal union than the flexible exchange rate regime without transfers because a higher wage is needed to compensate for the variability in labor supply. Nevertheless, the expected disutility of labor is the same in both regimes because

$$\frac{1}{2}\mathbb{E}_s\left[\left(\bar{L}^c_s\right)^2\right] = \frac{1}{2}\frac{\mathbb{E}_s\left[b^2_s\right]}{\bar{W}^2} = \frac{1}{2}\frac{1}{(W^f(0))^2} = \frac{1}{2}\mathbb{E}_s\left[\left(L^f_s(0)\right)^2\right].$$

So, when $\gamma = 1$, the expected utility difference between the fiscal union and the flexible exchange rate regime without transfers stems only from the difference in consumption. The difference in the expected utility from consumption is:

$$\mathbb{E}_s\left[\log\left(\bar{X}^c\right)\right] - \mathbb{E}_s\left[\log\left(X^f(0)\right)\right] = -\log\left(\bar{W}\right) + \log\left(W^f(0)\right) + \mathbb{E}_s\left[\log\left(B_s(0)\right)\right].$$

Thus, the fiscal union dominates the flexible exchange rate regime without transfers if and only if

$$\log\left(\frac{\bar{W}}{W^f(0)}\right) < \mathbb{E}_s\log\left(B_s(0)\right). \tag{13}$$

This condition highlights the main trade-off: the left hand side of condition (13) is a measure of the distortion in the fiscal union caused by wage rigidity; the right hand side is a measure of the price and consumption variability caused by exchange rate movements. The fiscal union dominates if the wage rigidity distortion in the fiscal union is not too strong compared to the price volatility in the flexible exchange rate regime. In our two state case, it is always true that inequality (13) holds for large enough shocks.[18] The variability of relative productivity increases with σ and, therefore, the left hand side of (13) increases with σ. Likewise, an increase in σ increases price variability, and hence, increases the right hand side of (13). For low values of σ, this latter effect dominates and hence, the fiscal union is preferable.[19] For larger values of σ, the wage effect dominates at least for small shocks. For the general case, $\gamma \geq 1$, we have the following proposition.

---

[18]Taking the limit as $z \to 0$, $\bar{W}/W^f(0) \to \sqrt{2}$, so that the left hand side of condition (13) is bounded, whereas $B_G(0) \to 1$ and $B_B(0) \to \infty$ as $z \to 0$, so that the right hand side is infinite.

[19]This is true when $\sigma \leq (1/2)(1 + \sqrt{5})$, the golden ratio. See the proof of Proposition 2 for details.

Proposition 2 (Fiscal Union). *The fiscal union dominates the flexible exchange rate regime without transfers if and only if*

$$\frac{\sqrt{\mathbb{E}_s\left[b_s^2\right]}}{\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{\gamma-1}}} < 1. \tag{14}$$

In the two equiprobable state case with productivity parameter z, there exists a unique threshold $\widehat{z} \in (0,1]$ such that for $z < \widehat{z}$, the fiscal union dominates. For, $\gamma > \sigma(\sigma - 1)$, equivalently $\sigma \leq \frac{1}{2}(1 + \sqrt{1 + 4\gamma})$, then $\widehat{z} = 1$.

Condition (14) is the generalization of (13) to the case where $\gamma > 1$. Proposition 2 shows that the fiscal union dominates for sufficiently high risk aversion and weak product substitutability. A higher degree of risk aversion favors the fiscal union because households dislike the variability of consumption in the flexible exchange rate regime. On the other hand, as we have just seen if the product substitutability is high, then the wage ratio $\bar{W}/W^f(0)$ is high for small shocks compared to price variability.

Figure 1 illustrates results in (z, σ)-space for different values of γ with $\theta = 5$ and $\mu = 0.99$ (ignore the shaded areas and the point labeled P for the moment). Each solid curve depicts the locus of points where a fiscal union yields the same expected welfare as a flexible exchange rate regime without transfers for six different values of γ. As Proposition 2 shows, the fiscal union dominates for higher risk aversion coefficient γ and lower elasticity of substitution σ. The area below (above) each locus shows the parameter combinations of z and σ for which the fiscal union dominates (is dominated by) the flexible exchange rate regime. Typical estimates from the literature put the value of relative risk aversion coefficient γ in a range between 2 and 7 and elasticity of product substitution in a range between 2 and 8.[20] The loci are fairly insensitive to the size of the productivity shock z for the region shown in Figure 1. However, the loci are downward sloping and more sensitive for larger values of the shock (smaller values of z) because the variability of consumption increases with the shock size and, hence, increase the desire for more risk sharing.

The above discussion leaves unexplained why the currency area is associated with risk-sharing transfers while a flexible exchange rate regime is not. In the next section, we examine how the adoption of a currency area may enhance risk sharing mechanisms absent a legal framework for redistribution.

4. Sustaining voluntary transfers

In this section, we consider voluntary transfers. As explained in Section 2, sustaining voluntary transfers requires satisfying the participation constraint (3) for the state in which the country is called upon to make a transfer (state 1 for the Home country, when it has a good productivity shock). For the sake of exposition, we repeat the conditions (3) here and emphasize the dependence on the exchange rate regime by including a superscript r where $r \in \{c, f\}$.[21]

$$V_G^r\left(W\left(T\right), T\right) \geq U_G^r(W\left(T\right), 0) + \frac{\delta}{1-\delta}\mathbb{E}_q U_q^r(W(0), 0) \quad \text{for } r \in \{c, f\}, \tag{3}$$

where $V_s^r(W\left(T\right), T)$ is the expected discounted utility in exchange rate regime r as defined in equation (2).

---

[20]Tödter (2008) assesses the coefficient of relative risk aversion in the range $\gamma \in [1.4, 7.1]$. Backus et al. (1992) and Corsetti et al. (2008) use $\gamma = 2$. Estimates of the elasticity of substitution are subject to the "international elasticity puzzle" and vary considerably depending on whether elasticity is measured relative to exchange rate or tariff rate changes. Basu and Fernald (1997) estimate an elasticity of substitution in a range $\sigma \in [4, 6]$. Mendoza (1991) uses an elasticity equal to $\sigma = 3.8$ and Stockman and Tesar (1995) provide an estimate of elasticity of $\sigma = 1.8$. Corsetti et al. (2008) estimate that the short term volatility of real exchange rates is consistent with σ slightly lower than one for traded goods. Fontagné et al. (2018) have estimated $\sigma \in [0.6, 5]$ depending on the measurement used. However, it is natural here to focus on with $\sigma > 1$ because, in this case, consumption in the currency area is pro-cyclical. In the baseline numerical computations presented in Table 2, we use $\sigma = 2$.

[21]The wage function and transfer also depend on the exchange rate regime but we don't use a superscript to denote the dependence to avoid cluttering the notation.
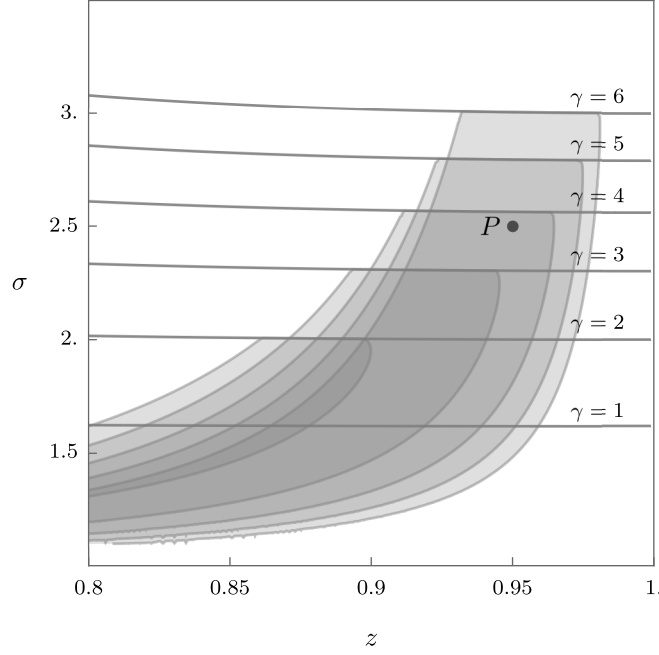
Figure 1: Thick lines show the loci of indifference between the fiscal union and the flexible exchange rate regime with no transfers for $\gamma \in \{1, 2, 3, 4, 5, 6\}$. Other parameter values are $\delta = 0.95$, $\theta = 5$ and $\mu = 0.99$. The shaded regions show the parameter space where the currency area with constrained transfers dominates the flexible exchange rate regime.

4.1. Critical Discount Factors

In this section, we consider the discount factors above which the optimal transfers $\bar{T}$, given in equation (9), can be sustained and the discount factors below which no transfers can be sustained. These critical discount factors are different in the two different exchange rate regimes. The differences help in understanding why the currency area might be better able to sustain voluntary transfers.

First, let $\underline{\delta}^r$ denote the lower critical discount factor in exchange rate regime r. It can be determined by solving (3) for δ and taking the limit as the transfers tend to zero. With $T_G = -T_B \varepsilon_G$ and ε_G given as a function of the transfers by equation (5), transfers can be sustained when:

$$\delta > \underline{\delta}^r := \lim_{T_B \to 0} \frac{U_G^r(W(T), 0) - U_G^r(W(T), -T_B \varepsilon_G)}{U_G^r(W(T), 0) - U_G^r(W(T), -T_B \varepsilon_G) + \mathbb{E}_q \left[U_q^r(W(T), T_q) - U_q^r(W(0), 0) \right]}.$$

Both the numerator and denominator on the right hand side of this equation tend to zero as $T_B \to 0$ and the solution for $\underline{\delta}^r$ can be found using L'Hôpital's rule (see Appendix B for details). Second, let $\bar{\delta}^r$ denote the upper critical discount factor. From (3), the optimal transfers $\bar{T}$ can be sustained provided:

$$V_G^r(\bar{W}, \bar{T}) \geq U_G^r(\bar{W}, 0) + \frac{\delta}{1 - \delta} \mathbb{E}_q U_q^r(W(0), 0).$$

Using this inequality, the critical discount factor above which the optimal transfers are sustainable is

$$\bar{\delta}^r := \frac{U_G^r(\bar{W}, 0) - U_G^r(\bar{W}, \bar{T}_G)}{U_G^r(\bar{W}, 0) - U_G^r(\bar{W}, \bar{T}_G) + \mathbb{E}_q \left[U_q^r(\bar{W}, \bar{T}_q) - U_q^r(W(0), 0) \right]}.$$

The critical discount factors for the two exchange rate regimes are given explicitly in the following proposition for the case where $\gamma = 1$, and both $\mu \to 1$ and $\theta \to \infty$.[22]

---

[22]Results outside these limit cases can be obtained using the same method used in the proof of the proposition.
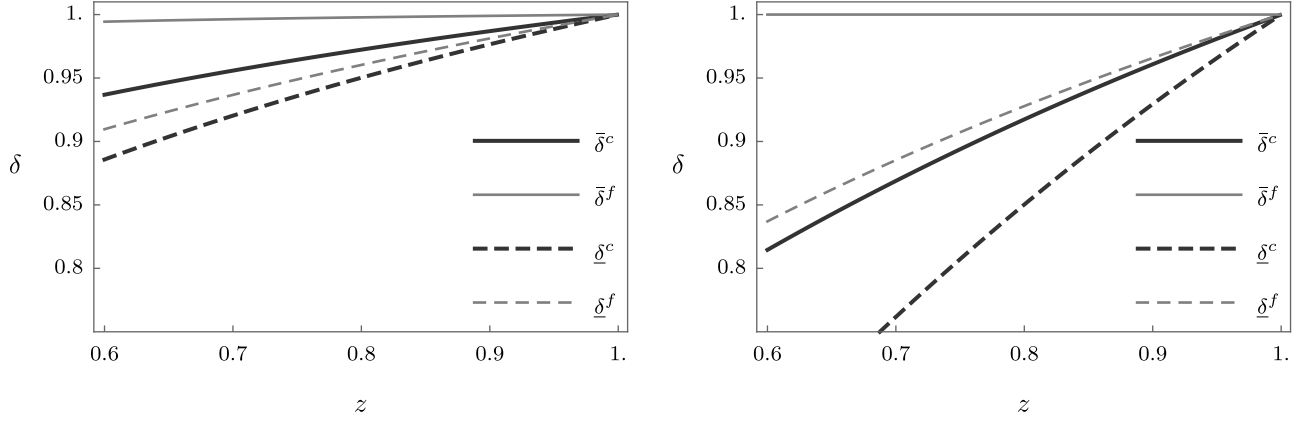
Figure 2: Critical discount factors as a function of the shock z. The left panel has $\sigma = 1.5$. The right panel has $\sigma = 2.5$. Other parameter values: $\gamma = 1$, $\theta = 5$ and $\mu = 0.99$.

Proposition 3 (Critical Discount Factors). *With two anti-correlated states, the critical discount factors for the currency area and flexible exchange are, in the limit as $\mu \to 1$ and $\theta \to \infty$ and for $\gamma = 1$:*

$$\bar{\delta}^c = \frac{\log(b_G)}{\frac{1}{2}\log(b_G) - \frac{1}{2}\log(b_B)}, \qquad\qquad \underline{\delta}^c = b_B,$$

$$\bar{\delta}^f = \frac{\left(b_G^2 - 1\right) - \left(b_G^2 + b_B^2\right)\log(B_G(0))}{\left(b_G^2 - 1\right) - \left(\frac{1}{2}b_G^2 + \frac{1}{2}b_B^2\right)\left(\log\left(\frac{1}{2}b_G^2 + \frac{1}{2}b_B^2\right) + \log(B_G(0)) - \log(B_B(0))\right)}, \qquad \underline{\delta}^f = \frac{b_B^{\frac{1}{\sigma}}}{\frac{1}{2}b_G^{\frac{1}{\sigma}} + \frac{1}{2}b_B^{\frac{1}{\sigma}}}.$$

We have $0 < \underline{\delta}^c < \bar{\delta}^c < 1$ and $\underline{\delta}^c < \underline{\delta}^f < 1$ for $z \in (0,1)$ and $\sigma > 1$.

Since $0 < \underline{\delta}^c < \underline{\delta}^f < 1$, transfers are sustained for lower discount factors in the currency area than in the flexible exchange rate regime. The reason for this is two-fold. First, absent transfers, consumption is more variable in the currency area than in the flexible exchange rate regime. This means that the marginal benefit of a small transfer is greater in the currency area. Second, the future expected utility in the absence of transfers is lower in the currency area regime than the flexible exchange rate regime, providing an additional incentive to sustain a transfer.

Figure 2 plots the critical discount factors against the shock z, for a given set of parameter values of θ, μ γ and σ. The left panel of Figure 2 has $\sigma = 1.5$ and shows $\underline{\delta}^c < \underline{\delta}^f < \bar{\delta}^c < \bar{\delta}^f$. It shows that $\bar{\delta}^f$ is an increasing function of z that lies below one and above $\bar{\delta}^c$.[23] The right hand panel of Figure 2 has $\sigma = 2.5$ and shows $\underline{\delta}^c < \bar{\delta}^c < \underline{\delta}^f < \bar{\delta}^f$. Since $\bar{\delta}^c < \underline{\delta}^f$, for any $\delta \in (\bar{\delta}^c, \underline{\delta}^f)$, the currency area sustains the optimal transfers $\bar{T}$, whereas the flexible exchange rate regime cannot sustain any transfers. For the range of shocks illustrated in the right hand panel of Figure 2, $\bar{\delta}^f = 1$ and the optimal transfers $\bar{T}$ cannot be sustained for any discount factor $\delta < 1$.

We have seen from Figure 1 that a fiscal union can dominate the flexible exchange rate regime for some parameter values. Figure 2 shows that there are parameter values such that the currency area sustains the optimal transfers $\bar{T}$, whereas the flexible exchange rate regime cannot sustain any transfers. Putting these two facts together gives the following corollary.

Corollary. *There exist parameter values such that $\bar{\delta}^c < \underline{\delta}^f$. That is, there exist parameter values, in particular for $\delta \in (\bar{\delta}^c, \underline{\delta}^f)$, for which the optimal transfer $\bar{T}$ can be sustained in the currency area and no transfer can be sustained in the flexible exchange rate regime. For a subset of these parameter values, the fiscal union dominates the flexible exchange rate regime without transfers. For example, such a parameter configuration occurs for small shocks (z close to one), μ close to 1, θ large, and $\gamma > \max\{2\sigma/(\sigma^2 + \sigma - 2), \sigma(\sigma - 1)\}$.*

---

[23]This is consistent with our Proposition 2 and it can be shown that for $\sigma < \frac{1}{2}(1 + \sqrt{1 + 4\gamma})$, $\bar{\delta}^f < 1$.
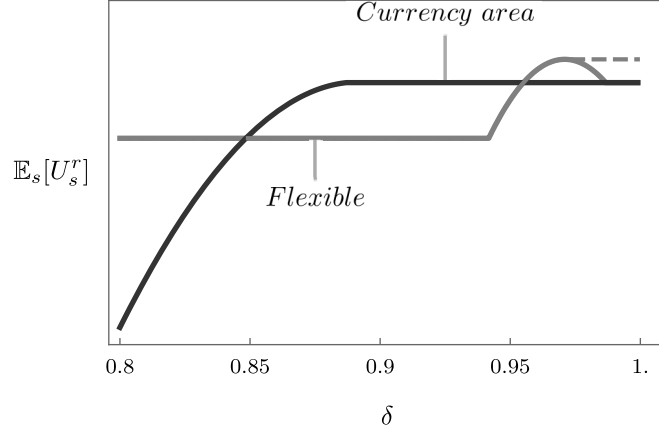
Figure 3: Expected Utility with Constrained Transfers as a Function of the Discount Factor. Parameter values: $\sigma = 2$, $\gamma = 5$, $\theta = 5$, $\mu = 0.99$ and $z = 0.9$.

The Corollary shows that there are parameter configurations where the currency area dominates the flexible exchange rate regime without the need for a mechanism to enforce transfers. For example, when $z = 0.95$, $\sigma = 2.5$ and $\gamma = 4$ (with $\theta = 5$ and $\mu = 0.99$), it can be checked that $\bar{\delta}^c \approx 0.9329$ and $\underline{\delta}^f \approx 0.9727$. Point P in Figure 1 corresponds to $z = 0.95$ and $\sigma = 2.5$ and it can be seen that with $\gamma = 4$, the fiscal union dominates the flexible exchange rate regime without transfers. Thus, with $\delta = 0.95$, this parameter configuration ($z = 0.95$, $\sigma = 2.5$, $\gamma = 4$, $\theta = 5$, $\mu = 0.99$) satisfies the conditions of the Corollary.

The second part of the Corollary, for the case of small shocks, follows from two properties. First, from Proposition 2, the fiscal union is preferable to a flexible exchange rate regime without transfers provided $\gamma > \sigma(\sigma - 1)$. Second, it is possible to take first-order approximations of the critical discount factors around $z = 1$ to show that for z close to one and for μ close to one and θ large that $\bar{\delta}^c < \underline{\delta}^f$ when $\gamma > 2\sigma/(\sigma^2 + \sigma - 2)$. Although the condition in the Corollary is not satisfied for $\gamma = 1$, it is satisfied, for example, when $\sigma = \sqrt{3}$ and $\gamma > 3 - \sqrt{3} \approx 1.268$.

The Corollary qualifies Mundell's statement that currency areas cannot be optimal if shocks are not positively correlated. The Corollary shows that the negative correlation of shocks can give rise to voluntary transfers in the currency area that smooth consumption. The flexible exchange rate regime partially accommodates shocks through movements in the exchange rate but does not sustain additional risk sharing through intercountry transfers.

The Corollary is extreme in considering parameter values such that the currency area sustains optimal transfers and the flexible exchange rate regime does not sustain any transfers. We turn now to intermediate situations where it may be possible to sustain some transfers in either regime and consider whether, in this case, the currency area may dominate, and if so, for what parameter values.

4.2. Constrained transfers

The optimal transfer $\bar{T}$ is chosen when $\delta > \bar{\delta}^r$ and there is no transfer $\delta < \underline{\delta}^r$. When $\delta \in (\underline{\delta}^r, \bar{\delta}^r)$, welfare is maximized by choosing a transfer $\hat{T}^r$ such that the participation constraint (3) is binding. Analytical results are difficult to obtain for the constrained transfer $\hat{T}^r$ but it is possible to derive numerical results to compare the two regimes and find the parameter values for which the currency area dominates the flexible exchange rate regime.

Figure 3 plots the expected utility against the discount factor, in both regimes, for a given set of parameter values. The dark gray curve is the expected utility in the currency area and the light gray curve is the expected utility in the flexible exchange rate regime (ignore the dashed red line for the moment). For low values of the discount factor $\delta < 0.75$ (not shown in the figure), no transfers are sustainable in either regime and the flexible

18

Table 2: Sensitivity Analysis.

	δ	z	γ	σ	θ	μ
Baseline:	0.950	0.900	5.00	2.00	5.00	0.99
Minimum:	0.849	0.890	2.10	1.30	1.10	0.28
Maximum:	0.955	0.965	6.30	2.10	∞	0.99

The table reports the minimum and maximum value of a parameter for which the currency area dominates, keeping other parameters to baseline levels.

exchange rate regime dominates as explained in Proposition 1. For $\delta \in (0.75, 0.845)$, the currency area sustains constrained transfers while the flexible exchange rate regime supports no transfers. In this interval, transfers and expected utility in the currency area rise with δ. For $\delta \in (0.845, 0.940)$, full consumption sharing is sustained in the currency area whereas no transfer is sustained in the flexible exchange rate regime. This is the situation considered in Proposition 2. In the interval $\delta \in (0.935, 0.975)$, transfers are sustained in the flexible exchange rate regime and expected utility associated with this regime increases with δ.[24] Finally, for high enough discount factors, $\delta > 0.99$, the optimal transfers $\bar{T}$ can be sustained in either regime, so that both regimes yield the same welfare, as explained in Proposition 1.

For any parameter values, the constrained transfer $\hat{T}^r$ can be numerically computed for each exchange rate regime. Given these computed values, the expected discounted values for household utility can be calculated. Consider again Figure 1. It plots, for different values of γ, the set of parameters (z, σ) (gray areas) where the currency area dominates the flexible exchange rate regime (the overlap in the regions is illustrated by the increasingly darker shades of gray). Recall that the area below the solid lines shows the parameter values such that the fiscal union dominates the flexible exchange rate regime with no transfers. Taking account of the participation constraint limits the region where the currency area dominates because there may be sustainable transfers in the flexible exchange rate regime and the optimal transfers may not be sustained in the currency area. The gray area, associated with the different values of γ, shows that, for the currency area to dominate, shocks should neither be too small nor too great. If shocks are small (large z), then it becomes difficult to sustain transfers and the benefits of a currency area are limited. If shocks are large (small z), both regimes can sustain risk-sharing transfers and the currency area won't be advantageous. Note that point P is in the shaded area corresponding to $\gamma = 4$.

This analysis extends our qualification of Mundell's statement about optimal currency areas. In this model with negatively correlated shocks, the dominance of the currency area is not limited to the case with optimal transfers in a currency area and zero transfers in the flexible exchange rate regime. It also occurs when transfers are constrained by the participation constraints. For some economic parameters, the currency area yields higher expected utility.

Figure 1 showed that the currency area with voluntary transfers can dominate for some parameter values. Table 2 presents a sensitivity analysis around the baseline parameter values. For each parameter, Table 2 reports the minimum and maximum values of the parameter for which the currency area dominates, keeping the other parameters at the baseline level.

To understand Table 2, recall that consumption and employment volatility is higher in the currency area without transfers. Hence, larger shocks (smaller z) increases the desire for insurance through transfers. A larger elasticity of substitution σ makes demand more responsive to changes in productivity and increases the volatility

---

[24]The hump shape in welfare under flexible exchange rates and the dashed line will be explained in Section 6.5.

in the currency area, which again increases the desire for insurance. Likewise, an increase in γ increases the demand for insurance. However, if the demand for insurance is too strong, then transfers can also be sustained in the flexible exchange rate regime. Equally if the demand for insurance is too weak, then transfers cannot be sustained even in the currency area, Results are not very sensitive to the labor market power parameter θ because an increase in θ reduces the wage by the same proportion in each regime. Similarly, results are not very sensitive to changes in the preference for money balances μ because this impacts consumption in a similar way in both regimes.

The parameter ranges given in Table 2 for which the currency area dominates are quite plausible. As mentioned in the introduction, it is natural to interpret a period as more than annual, say three years. Then, a baseline discount factor $\delta = 0.95$ corresponds to an annual real interest rate of approximately 1.72%. A value of $z = 0.9$ corresponds to an annual standard deviation of the productivity shock of approximately 0.29%. The range of values for γ and σ are within plausible limits that have already been discussed in footnote 20. Our claim here is not that a currency area is optimal in all situations. It can be preferable for plausible parameter values in a simple example. What the analysis shows is that whether a currency area is preferable or not depends on the possibility that voluntary transfers may be more easily sustained in a currency area.

5. Discussion

There are two types of market distortion in the model we have examined. There is an absence of insurance markets to offset the risk caused by the productivity shock and the labor market is imperfect. The imperfection in the labor market is two-fold: there is monopsonistic wage setting and there is wage rigidity because the wage is set before the outcome of the productivity shock is known.

The fiscal union overcomes the distortion in the insurance market and the flexible exchange rate regime overcomes the wage rigidity in the labor market. Table 3 compares the different exchange rate regimes with three benchmarks. They are the optimum of a utilitarian planner and two allocations with ex-post wage setting (when wages are set *after* the productivity realization), one with optimal transfers, and the other without transfers. Distortions are measured by the logarithm of the relative difference using the planner optimum as the reference point. Table 3 reports these distortions in the ratios of domestic and foreign consumption and labor supply. The ratios indicate the volatility of consumption and labor. A pro-cyclical effect, when a local variable increases with higher local productivity (lower z), is denoted by "(pc)" in the table, while a counter-cyclical effect is noted by "(cc)". Also reported is the distortion in the expected disutility of labor supply. For simplicity, we report the expected disutility when $\gamma = 1$.[25] The benchmark cases are reported in columns (2)-(4). Column (2) reports the planner optimum and columns (3) and (4) consider the case of ex-post wage setting with optimal transfers and with no transfer respectively. Columns (5) and (6) report the case with ex-ante wage setting in the flexible exchange rate regime without transfers and the currency area without transfers respectively. Column (7) consider the case with ex-ante wage setting and optimal transfers (by Proposition 1, the outcome is the same in both regimes). Column (8) can be ignored for the moment and will be discussed in Section 6.1.

Column (2) of Table 3 reports the planner optimum that maximizes $\mathbb{E}_s[U_s^f(W, T_s) + U_s^{f*}(W, T_s)]$ with money supply constant and equal in both countries.[26] There is full insurance: consumption is equalized across states and countries. In the first-best outcome, labor is pro-cyclical responding positively to the good shock. It can also be checked that the expected labor disutility, $\mathbb{E}L_s^2/2 = \mu/2$, increases proportionally with the share of goods consumed.

---

[25]The disutility depends on the wage, which depends upon γ.

[26]The choice of money supply is discussed in Sections 6.1 and 6.5.

Table 3: Distortions in consumption and labor supply compared to the first best outcome.

Model	(2)	(3)	(4)	(5)	(6)	(7)	(8)
money supply:	cst.	cst.	cst.	cst.	cst. in area	cst.	counter-cyclical
wage setting:		ex-post	ex-post	ex-ante	ex-ante	ex-ante	ex-ante
exchange rate:	n.a.	irrelevant	irrelevant	flexible	fixed	irrelevant	flexible
transfer:		optimal	none	none	none	optimal	none
Consumption							
$\Delta^{fb}\log\left(C/C^*\right)$:	0	0	$-\frac{\sigma-1}{\sigma}\log z$ (pc)	$-\frac{\sigma-1}{\sigma}\log z$ (pc)	$-(\sigma-1)\log z$ (pc)	0	0
$\log\left(C/C^*\right)$:	0						
Labor							
$\Delta^{fb}\log\left(L/L^*\right)$:	0	0	$\frac{\sigma-1}{1+\sigma}\log z$ (cc)	$\frac{\sigma-1}{1+\sigma}\log z$ (cc)	$-\frac{\sigma(\sigma-1)}{1+\sigma}\log z$ (pc)	$-\frac{\sigma(\sigma-1)}{1+\sigma}\log z$ (pc)	$\frac{2\sigma}{1+\sigma}\log z$ (cc)
$\log\left(L/L^*\right)$:	$-\frac{\sigma-1}{1+\sigma}\log(z)$ (pc)						
Disutility							
$\Delta^{fb}\log\left(\mathbb{E}_s[L_s^2]\right)$:	0	$\log\left(\frac{\theta-1}{\theta}\right)$	$\log\left(\frac{\theta-1}{\theta}\right)$	$\log\left(\frac{\theta-1}{\theta}\right)$	$\log\left(\frac{\theta-1}{\theta}\right)$	$\log\left(\frac{\theta-1}{\theta}\right)$	$\log\left(\frac{\theta-1}{\theta}\right)$
$\log\left(\left(\mathbb{E}_s[L_s^2]\right)\right)$:	$\log(\mu)$						

Distortions are measured relative to the first best: $\Delta^{fb}\log(x) \equiv \log(x) - \log(x^{fb})$, where x^{fb} is the first-best value of x.

Column (3) shows the case with ex-post wage setting and optimal transfers. Ex-post wage setting with optimal transfers removes both the distortion caused by wage rigidity and the distortion caused by lack of insurance markets. In this case, the exchange rate is constant even in the flexible exchange rate regime and hence, the choice of regime is irrelevant. The relative allocation of consumption and labor are the same as in the planner optimum. The only distortion is to the expected disutility of labor supply. This is distorted by $\log((\theta - 1)/\theta)$, which is negative indicating that labor supply is distorted below the first best because of the monopsonistic power of households in the labor market. This distortion goes to zero when workers have no market power ($\theta \to \infty$), in which case the first-best of column (2) is replicated. It is worth noting column (3) represents the outcome that would be achieved by perfect capital or insurance markets that replicate the optimal transfers.

Column (4) reports the allocation with ex-post wage setting but absent transfers or insurance markets. As in column (3), the expected disutility of labor supply is distorted downward because of the monopsony power of workers. For $\sigma > 1$, the distortion in relative consumption is pro-cyclical (recall $\log(z) < 0$) and the distortion in relative labor supply is counter-cyclical. In the bad productivity state, households consume less than in the planner optimum, because of the missing insurance market, but work more to compensate for the fall in productivity. Observe that, labor supplies are equalized across states and countries: volatility fully shifts from labor supply to consumption.

The next two columns consider the alternative exchange rate regime with preset wages and no transfers. Column (5) reports the distortions in the flexible exchange rate regime with a constant money supply. In this case, the exchange rate adjusts to fully compensate for the rigidity in wages. Hence, outcomes and distortions are exactly the same as under ex-post wage setting in column (4). Column (6) reports the distortions in a currency area. The distortions in both consumption and labor supply are pro-cyclical compared to the first best allocation. A comparison of columns (5) and (6) shows that the amplitudes of the consumption and labor supply distortions are larger in the currency area than the flexible exchange rate regime. Therefore, absent transfers the flexible exchange rate regime delivers higher expected welfare than the currency area (see, Proposition 1).

Column (7) reports the case of a fiscal union that has been discussed in Section 3.2. The optimal transfers replicate the insurance from the first best but labor supply is unaffected by these transfers. Comparing columns (5) and (7) illustrates the basic trade-off between the fiscal union and the flexible exchange rate regime: a fiscal union eliminates the distortion in consumption but has a pro-cyclical distortion in labor supply, whereas the flexible exchange rate regime balances a pro-cyclical consumption distortion with counter-cyclical distortion in labor supply. This helps to explain why transfers might be more easily sustained in a currency area. The need for transfers to share risk is greater. Equally, the fall back position in the currency area without transfers is worse than in the flexible exchange rate regime.

Note that the distortion caused by imperfect competition in the labor market, as measured by parameter θ affects all columns (3)-(8) is the same way and has no impact on the distortions in labor and consumption ratios. Importantly, note that all distortions in the labor and consumption ratios vanish as $\sigma \to 1$, the case considered by Cole and Obstfeld (1991). In this case, the terms of trade adjust to exactly offset changes in productivity. As stated by Cole and Obstfeld, "fluctuations in international terms of trade can play an important role in automatically pooling national output risks, since (other things equal) a country's terms of trade are negatively correlated with growth in its export sector". The Cole-Obstfeld model is therefore, precisely the benchmark in which no transfers are required and the choice of exchange rate regime is irrelevant (see again, Proposition 1).

6. Robustness

This section considers the robustness of our conclusions to some modifications of the model.

6.1. Generalized monetary policies

We assumed that the money supply in the flexible exchange rate regime is fixed and independent of the state. As we stated above, this is not necessarily optimal. A natural question to ask is whether allowing the money supply to adjust would substantially change our results. A full analysis in which the transfer between countries and the money supply within each country are both optimized is beyond the scope of this paper.[27] However, it is possible to examine the optimal monetary policy in the absence of transfers. This is relevant when making the comparison between the fiscal union and the exchange rate regime without transfers and when considering what happens in default. In this section, therefore, we briefly consider a situation where, in the absence of transfers, the money supply is set by competing central banks.[28]

Before considering how the money supply is set by central banks, we consider three possible simple money supply rules as benchmarks. A money supply rule is the choice of money in the two states, where Home money supply in the two states is $M_{0,G}$ and $M_{0,B}$, and the money supply in Foreign is $M_{0,G}^*$ and $M_{0,B}^*$. By symmetry, $M_{0,G}^* = M_{0,B}$ and $M_{0,B}^* = M_{0,G}$. Since money is neutral, it is only the ratio $M_{0,G}/M_{0,B}$ that is determined. First, consider the money supply rule we have already assumed, namely, that money supply is constant and $M_{0,G}/M_{0,B} = 1$. The outcome is described by column (5) in Table 3. The policy will not in general be optimal because it does not stabilize consumption. Second, consider the money supply rule where $M_{0,G}/M_{0,B} = b_G/b_B$. This rule stabilizes the exchange rate: $\varepsilon_G = \varepsilon_B = 1$. It is equivalent to a fixed exchange rate regime of column (6) of Table 3 and it stabilizes the price index. This money supply rule is pro-cyclical, completely offsetting the appreciation of the Home exchange rate by expanding the money supply in the good productivity

---

state and conversely contracting the money supply in the bad state. Consumption and labor are pro-cyclical. Third, consider the money supply rule where $M_{0,G}/M_{0,B} = (b_G/b_B)^{1/(1-\sigma)}$. In this case, it can be checked that $M_{0,G}/M_{0,B} = \varepsilon_G = z$. Such a money supply rule is counter-cyclical: it adjusts money supply to increase demand in the low productivity state and reduce demand in the good productivity state. Since $P_B = P_G/\varepsilon_G$, and consumption is proportional to $M_{0,s}/P_s$, this money supply rule equalizes consumption across states. Although it equalizes consumption, it creates counter-cyclical distortions in labor supply as shown in column (8) of Table 3.

We now consider a simple scenario where the money supply is set by the central bank in each country. Assume that the central banks act simultaneously and independently. For simplicity, assume that households have logarithmic utility for consumption ($\gamma = 1$). Furthermore, assume that the objective of the Home central bank is to choose $M_{0,G}$ and $M_{0,B}$ to maximize the expected utility of Home households. In doing so, it takes the money supply of the Foreign central bank as given. It takes the Foreign wage W^* as given but acts as a Stackelberg leader for wage setting in the Home country. That is, it takes account of its policy on wage setting at Home. The Foreign central bank similarly chooses $M_{0,G}^*$ and $M_{0,B}^*$ to maximize the expected utility of Foreign households. It is possible to solve for a symmetric Nash equilibrium of the game between central banks. It can be shown that at the symmetric Nash equilibrium, the relative money supply is determined by the following first-order condition:[29]

$$1 - \left(\frac{M_{0,G}}{M_{0,B}}\right)^2 \left(\frac{b_G}{b_B}\right)^{\frac{1}{\sigma}} \left(\frac{M_{0,G}}{M_{0,B}}\right)^{\frac{\sigma-1}{\sigma}} = \sigma\left(1 - \left(\frac{M_{0,G}}{M_{0,B}}\right)^2\right)\left(1 + \left(\frac{b_G}{b_B}\right)^{\frac{1}{\sigma}} \left(\frac{M_{0,G}}{M_{0,B}}\right)^{\frac{\sigma-1}{\sigma}}\right). \tag{15}$$

It can be checked that the solution satisfies $b_G/b_B > M_{0,G}/M_{0,B} > 1$.[30] That is, the Nash equilibrium money supply rule in the flexible exchange rate regime without transfers is pro-cyclical, $M_{0,G}/M_{0,B} > 1$, but not so pro-cyclical as to stabilize the exchange rate, $M_{0,G}/M_{0,B} < b_G/b_B$. The Nash equilibrium money supply rule partly counteracts the appreciation of the Home exchange rate by expanding the money supply in the good productivity state and likewise, partly mitigates the depreciation of the Home exchange rate by contracting the money supply in the bad productivity state. It is not possible to provide an exact analytical solution to equation (15). It can be solved numerically for parameter values. For example, with $z = 0.9$ and $\sigma = 2$ (baseline parameters from Table 2), the optimal money supply rule has $M_{0,G}/M_{0,B} \approx 1.00962$, that is, a 1% deviation from the fixed money supply rule. It is possible to linearize equation (15) to find an approximation for the Nash equilibrium money supply rule for small shocks (z close to 1):

$$\frac{M_{0,G}}{M_{0,B}} = 1 + \left(\frac{\sigma-1}{4\sigma^2 - 3\sigma + 1}\right)(1-z). \tag{16}$$

The largest deviation from the fixed money supply rule occurs for $\sigma = 1 + \sqrt{2}/2$, and hence, an upper bound on the deviation from a fixed money supply rule is $(4\sqrt{2} - 5)(1-z)/7$. That is, even the largest deviation from the constant money supply rule is less than one-tenth the size of the shock. For example, with $z = 9/10$ (a 10% shock) the approximate deviation from a constant money supply rule is no greater than 0.938%.

We take this analysis as suggesting that our assumption of a constant money supply, whilst not optimal, is also not widely divergent from the outcome with competing central banks. Obviously, the adjustment to the money supply described by equation (15) increases the expected utility of households in the flexible exchange rate regime. Thus, in comparing the fiscal union (currency area with optimal transfers) to the flexible exchange rate without transfers, the ability to adjust money supply will make the flexible exchange rate regime more attractive. Equally, if the money supply is adjusted in default, then this will make sustaining transfers in the

---

[29]It can be checked that the central bank's objective function is single-peaked and there in a unique solution to equation (15). See, the supplementary materials for the derivation of equations (15) and (16).

[30]In the case where $z = 1$ (no shock), $b_G/b_B = 1$ and the solution to equation (15) is a fixed money supply.

flexible exchange rate regime more difficult. In particular, the value of $\bar{\delta}^f$, above which the optimal transfers can be sustained, is increased. Of course, this assumes that money supply is still constant when there are transfers, but it illustrates that the ability of central banks to adjust money supply need not necessarily be advantageous for the flexible exchange rate regime.

6.2. Alternative default assumptions

In this section, we discuss the robustness of the results to alternative specifications of what happens in the event of default.

We have assumed so far that a default on the promised transfer results in a complete breakdown in trust and the absence of transfers thereafter. It is possible to pose the choice of transfers as a repeated game between the counties. In game-theoretic terms the choice of sustainable transfers corresponds to the constrained Pareto-frontier of the set of sub-game perfect equilibrium of a repeated game between the two countries (see, e.g., Thomas and Worrall, 1988). In the stage game, each country announces an acceptable transfer and the transfer is made if there is agreement and not otherwise. Punishment in the repeated game consists of a 'grim trigger strategy' with an infinite punishment phase where there is 'reversion to the Nash equilibrium' of the corresponding stage game (no transfer). In the punishment phase, the defaulting country receives its lowest individually rational payoff from the transfer game.

The assumption of the grim strategy of zero transfers following default is less restrictive than it seems. First, an alternative assumption, say that the punishment phase lasts for a finite length of time, would modify quantitatively, by not qualitatively, our results. Although a shorter punishment phase would reduce the punishment cost, the currency area would remain preferred as long as this cost and the risk sharing benefit is high enough.

Second, the transfers we identify can be sustained as a renegotiation-proof equilibrium. It is true that the threat to return to zero transfers is not credible because countries would have an incentive to renegotiate away from such a punishment. What is needed to avoid renegotiation following a default is that the default payoffs are themselves efficient subject to the participation constraints (on the constrained Pareto-frontier). To establish that the transfers we have identified can be sustained as part of a renegotiation-proof equilibrium requires two things. First, that there is a transfer scheme that gives the defaulting country its lowest individually rational payoff (what it gets in autarky), and second, that the corresponding payoff the other country receives is its maximal payoff given the participation constraints. This is possible because there is a transfer scheme that gives a country the same expected discounted utility it receives from autarky, and the only way to improve this expected discounted utility is to make an increased transfer to the country, either now or at some point in the future, which will, in either case, lower the expected discounted utility of the other country. Hence, the punishment of zero transfers can be replaced by a phase of punishment that is itself constrained Pareto-efficient. Since the right hand side of the participation constraints is unchanged (the defaulter receives their lowest individually rational payoff in either case), the sustainable transfers we have identified are, in fact, renegotiation-proof. The property that renegotiation-proofness has no bite in limited commitment models of this type was first proved by Asheim and Strand (1991) in the context of a wage contracts model and the same property applies in our model too.[31]

We now turn to a discussion of the default option in the case of a currency union. Up to now, the participation constraint (3) assumes that countries defaulting on transfers maintain their participation in the currency union.

---

[31]It is, of course, in general true that renegotiation-proofness does restrict the set of sub-game perfect equilibria in repeated games (see, e.g., Farrell and Maskin, 1989). Renegotiation-proofness is also restrictive in more general models of limited commitment. For example, Thomas and Worrall (1994) consider a model where one of the parties makes an investment. The lowest individually rationally payoff to the investor is not to invest. However, the constrained Pareto-frontier is flat at the point where investing player makes no investment and hence, it is possible to find an efficient point that Pareto-dominates the no-investment outcome. They show how to construct a renegotiation-proof outcome using a fixed point argument to calculate the worst constrained efficient outcome for the investing player.

This is a reasonable assumption if the currency union is strongly embedded and there are high political and procedural costs to leaving the union. By contrast, the fixed exchange rate regime could also correspond a pegged exchange rate system where countries retain separate monetary authorities. In this case, the political and procedural costs of exiting a pegged system may be much smaller.

To model the situation of a pegged exchange rate system, we replace the participation constraint (3) by

$$V_G^c(W^c(T^c), T^c) \geq U_G^c(W^c(T^c), 0) + \frac{\delta}{1-\delta}\mathbb{E}_s U_s^f\left(W^f(0), 0\right). \tag{17}$$

The left hand side of the inequality is expected discounted utility from the transfers in the pegged exchange rate in the good state, when the country is called upon to make a transfer. The right hand side has two terms. The first is the current period utility when the country defaults on its transfer but the wage remains fixed and the exchange rate peg is maintained. The second term is the expected discounted utility with no transfers and a flexible exchange rate regime.[32] That is, the constraint (17) corresponds to a situation in which the exchange rate system reverts to a flexible exchange rate one period after a default in the transfer promises.[33] We know from Proposition 1 that with two symmetric states:

$$\mathbb{E}_s U_s^f\left(W^f(0), 0\right) \geq \mathbb{E}_s U_s^c\left(W^c(0), 0\right),$$

so that, constraint (17) is more stringent than constraint (3). This means that imposing constraint (17) rather than constraint (3) makes it more difficult to sustain transfers in the pegged exchange rate regime relative to a currency union. It does not however, rule out that the pegged exchange rate regime under constraint (17) might be better than the flexible exchange rate regime for some parameter values.

Analogous to Proposition 3, we can calculate the critical discount factor $\bar{\delta}^p$ above which the first-best transfers are sustained, and the discount factor $\underline{\delta}^p$ below which no transfer can be sustained under the constraint (17), the pegged exchange rate regime. In the case where $\theta \to \infty$, $\mu \to 1$ and $\gamma \to 1$, these are given by:

$$\bar{\delta}^p = \frac{\log(b_G)}{\log(b_G) - \frac{1}{2}\left(\log\left(\frac{1}{2}b_G^2 + \frac{1}{2}b_B^2\right) - \log(B_0(G)) - \log(B_0(B))\right)} > \bar{\delta}^c, \quad \underline{\delta}^p = b_B = \underline{\delta}^c.$$

Note that the lower critical discount factor is the same whichever default assumption is made. This is because the lower discount factor depends on the marginal impact of small transfers on expected discounted utility and on the marginal effect of small transfers for determining the preset wage in the first period of default. Since these marginal impacts occur only when the fixed exchange rate is in place, the exchange rate regime one period after the default has taken place is irrelevant for determining the lower critical discount factor. That is, it does not matter whether the constraint is given by (3) or (17). In contrast, sustaining the first-best transfers requires comparing the long run with the first-best transfers and the long run without transfers. Hence, sustaining the first-best transfers will be more difficult if a country reverts to a flexible exchange rate in the period after default ($\bar{\delta}^p > \bar{\delta}^c$).

We can further examine the effect of participation constraint (17) on the results of Corollary to Proposition 3. The Corollary showed that the currency area could be preferred for small shocks and large enough risk aversion because the first-best transfers could be sustained in the currency area whereas no transfers could be sustained in the flexible exchange rate regime. The result also holds for the pegged exchange rate regime, that is, under constraint (17), but the parameters for which it holds are different. To see this, note first that the sustainability condition of the flexible exchange rate is unaltered since (17) applies only to the fixed exchange rate regime. Then, take a first-order approximation of $\bar{\delta}^p$ around $z = 1$ and take the limit $\mu \to 1$ and $\theta \to \infty$ (and for $\gamma \neq 1$).

---

[32]The superscripts on the transfer and the wage are included in (17) to emphasize the dependence on the exchange rate regime.

[33]We have assumed that the fixed exchange rate regime is retained for one period following default because exiting the peg may still involve some political and procedural costs and adjustment may take some time.

Table 4: Sensitivity Analysis.

	δ	z	γ	σ	θ	μ
Baseline:	0.950	0.850	9.00	1.25	5.00	0.99
Minimum:	0.939	0.810	7.50	1.18	2.70	0.25
Maximum:	0.961	0.874	12.60	1.35	10.00	0.99

The table reports the minimum and maximum value of a parameter for which the pegged exchange rate regime (under constraint (17)) dominates the flexible exchange rate regime, keeping other parameters at the baseline levels.

It can then be checked that a fiscal union is sustained if $\bar{\delta}^p < \underline{\delta}^f$, or equivalently, if $\gamma > \sigma(\sigma^2 + 2\sigma - 1)/(\sigma - 1)$. It can be shown that this condition is more restrictive than the one stated in the last line of the Corollary. For example, if $\sigma = \sqrt{3}$, then $\bar{\delta}^p < \underline{\delta}^f$ for z close to one when $\gamma > 6 + 4\sqrt{3} \approx 12.928$. In that case, there exists a $\delta \in (\bar{\delta}^p, \underline{\delta}^f)$ such that no transfers can be sustained in the flexible exchange rate regime while first-best transfers can be sustained in the pegged exchange rate regime and the pegged exchange rate delivers higher expected utility. This is to be contrasted with the much less restrictive condition $\gamma > 3 - \sqrt{3} \approx 1.268$ when there is a currency area and the fixed exchange rate is maintained after a default (under constraint (3)).

We finally examine the choice of regime for larger shocks. Table 4 presents the sensitivity analysis for an economy with a pegged exchange rate (under constraint (17)). Table 4 is analogous to Table 2 (under constraint (3)). However, since (17) is more stringent than (3), the baseline parameters have been changed to illustrate the parameter values where the pegged exchange rate regime dominates the flexible exchange rate regime. Given these baseline parameters, the degree of risk aversion must be at least 7.5 for the pegged exchange rate regime to dominate and the elasticity of substitution has to be at most 1.35. This shows that the pegged exchange rate dominates the flexible exchange rate regime for a less plausible set of parameters than is the case for a currency area. To sum up, what happens after a default is quantitatively important for determining whether a fixed exchange rate regime dominates the flexible exchange rate regime.

6.3. Transaction costs

One advantage of currency areas lies in the elimination of transaction costs in exchanging currencies (Alesina and Barro, 2002; Bayoumi, 1994). In this section, we consider the robustness of the previous analysis by adding iceberg transaction costs. In particular, to receive one unit of the foreign currency, domestic households must purchase $\tau > 1$ units of foreign currency. That is, households pay a transaction cost of $\tau - 1 > 0$ for the exchange of currencies. For simplicity, we assume that government transfers are not subject transaction costs, and maintain the identity $T = -\varepsilon T^*$. Finally, we assume that since the currency area has a common currency, it is free of such transaction costs.

In the presence of transaction costs, the Home consumer price of Foreign good is $\tau \varepsilon p^*$ and the Foreign consumer price of Home good is $\tau p/\varepsilon$.[34] Thus, the price indices, and in particular, the relative price $P/(\varepsilon P^*)$, depend on τ. Hence, the equilibrium exchange rate depends on τ as well as the transfer T. Since symmetry is retained, it follows that $W = W^*$. With these appropriate modifications to prices, transaction costs leave the expressions for consumption and employment unchanged.

It can be shown, as expected, that consumption and welfare fall with transaction costs τ. This increases the range of parameters such that a fiscal union (currency area with optimal transfers) dominates the flexible

---

[34]Transaction costs of this type have a similar affect to allowing for home bias in household preferences.
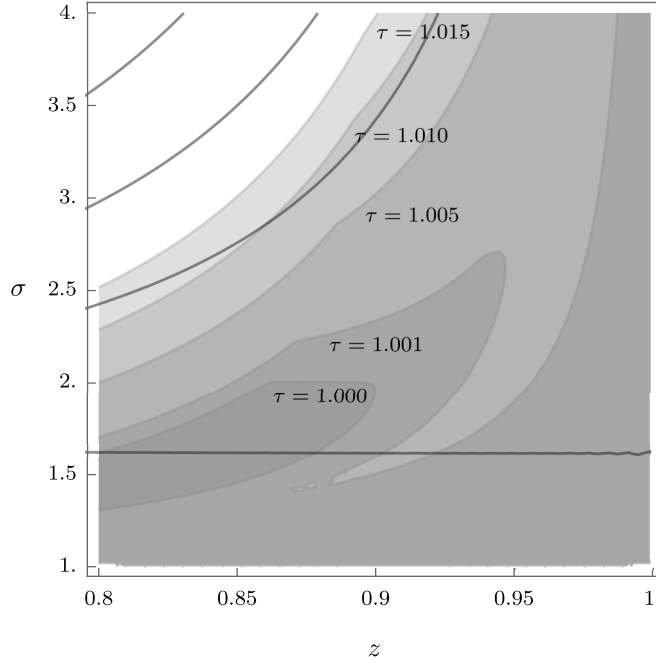
Figure 4: Thick lines show the loci of indifference between the fiscal union and the flexible exchange rate regime with no transfers for transaction costs $\tau \in \{1.000, 1.001, 1.005, 1.010, 1, 015\}$. Other parameter values are $\gamma = 2$, $\delta = 0.95$, $\theta = 5$ and $\mu = 0.99$. The shaded regions shown the parameter space where the currency area with constrained transfers dominates the flexible exchange rate regime.

exchange rate regime with no transfers. The left hand panel of Figure 4 depicts (for $\gamma = 2$) the sets of parameters σ and z where fiscal union dominates for four different values of τ. Comparing this to Figure 1, it can be seen that even for low transaction costs, the fiscal union dominates for higher values of σ than without transaction costs ($\tau = 1$). This is particularly true for small shocks (high z). The impact on sustainability is more nuanced because the participation conditions (3) are expressed in terms of utility differences. One may conjecture that the effect of transaction costs on the participation constraint is small. Indeed, the numerical calculations we have done show that transaction costs have only small effects on the critical discount factors $\underline{\delta}^f$ and $\overline{\delta}^f$.

To summarize, accounting for transaction costs favors the case for a currency area. It does not, however, drastically change the thresholds for sustainability of transfers and overall there is only a small impact on the relative benefits of a currency area compared to a flexible exchange rate regime when transfers are voluntary.

6.4. Independent shocks

The assumption of two equiprobable and anti-correlated states was made both for analytical tractability and because Mundell's argument against a currency area is often couched in terms of lack of business cycle correlation. To consider the robustness to a different shock structure, we briefly present the analysis where the two countries have two symmetric, but independent, productivity shocks. In this case, there are four states. To maintain comparability with the previous analysis, we assume that the productivity shocks are either $a_G = z$ or $a_B = 1$ and assume that each of the four states are equally likely. We assume a completely stationary environment in which there are transfers only if the two countries have different productivity shocks.

Figure 5 recomputes the analysis of Figure 1 to show the parameter values in (z, σ) space for which a currency area dominates the flexible exchange rate regime. Comparing Figures 1 and 5, it can first be seen that the fiscal union (currency area with optimal transfers) dominates the flexible exchange rate regime without transfers for a very similar sets of parameters (see the almost horizontal curves). The gains from insurance and flexible exchange rates are obtained in the states where productivity levels are different across countries. Since those lead to the
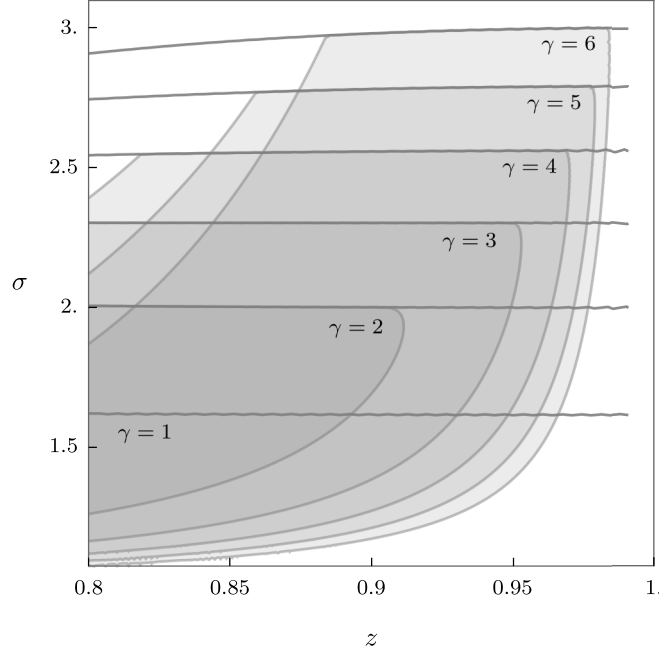
Figure 5: Reproduces Figure 1 when the two shocks are independently distributed. Thick lines show the loci of indifference between the fiscal union and the flexible exchange rate regime with no transfers for $\gamma \in \{1, 2, 3, 4, 5, 6\}$. Other parameter values are $\delta = 0.95$, $\theta = 5$ and $\mu = 0.99$. The shaded regions shown the parameter space where the currency area with constrained transfers dominates the flexible exchange rate regime.

same distinct productivity levels as the two state model, it is not surprising that very similar results are obtained. Put differently, the states where productivity shocks are common across countries do not matter for determining the preference of a fiscal union over a flexible exchange rate regime.

Again, comparing the shaded areas of Figures 1 and 5, it is seen that the sets of parameters for which the currency area with voluntary transfers dominates the flexible exchange rate regime with voluntary transfers are also very similar. Since the critical discount factors (see Section 4) are computed as ratios of net benefits, the impact of states in which the shocks are common across countries is negligible and the critical discount factors are very similar to those computed in Section 4. To summarize, this example suggests that the trade-off between a currency area and a flexible exchange rate regime depends on the demand for risk sharing, not on the particular anti-correlated structure we have assumed in the main part of the text.

6.5. Transfers in the flexible exchange rate regime

In making comparisons between the two regimes, we considered the transfers $\bar{T}$, defined in equation (9), that are optimal in the currency area regime and which equalize consumption. This is a natural benchmark to consider. Proposition 1 showed that in this case, the fixed and flexible exchange rate regimes produce exactly the same outcome.

However, transfers introduce undesirable variability into labor supply in the flexible exchange rate regime. It may be possible to do better in the flexible exchange rate regime. In the two state case, the first-best optimum can be achieved by choosing money supply and transfer jointly. The optimum is achieved by setting $M_{0,G}/M_{0,B} = \varepsilon_G = (b_G/b_B)^{-1/\sigma}$ and setting the transfer received by the country with the bad shock $T_B^f = (1 - \varepsilon_G)/(1 + \varepsilon_G)$.[35] It can be shown that this transfer is less than the optimal transfer given in (9).

---

[35]Here the transfer has been normalized by setting $\vartheta M_{0,B} = 1$.

As stated above, solving jointly for the money supply and transfer in the presence of the default constraint is difficult. However, it is possible to numerically compute the transfer that maximizes the utilitarian objective $\mathbb{E}_s[U_s^f(W, T_s) + U_s^{f*}(W, T_s)]$ with a fixed money supply and show that this maximum is also less than the optimal transfer given in (9). The red line in Figure 3 plots the expected household utility in the flexible exchange rate case but keeping money supply constant. The hump shape of the line reflects the fact that increasing transfers in the flexible exchange rate regime beyond a certain point lowers expected utility. The dashed red line in Figure 3 shows the expected utility in the flexible exchange rate regime when the transfer is at the welfare maximizing level. Because the optimal transfer is below the consumption equalizing transfer, it can be sustained when the discount factor is high. Hence, when discount factors are high, the expected utility from the flexible exchange rate regime is higher than the expected utility in the currency area. Taking this into account, Proposition 1 should be modified: the flexible exchange rate regime strictly dominates the fixed exchange rate regime both without transfers and when transfers are optimal. Nevertheless, as can be seen from the Figure 3, this does not change our principal conclusion that when transfers are constrained, the currency area may dominate for an intermediate range of discount factors.

7. Conclusion

This paper examines the relationship between the formation of a currency area and the use of voluntary transfers between countries. It discusses a model where the trade-off between exchange rate flexibility and formal and informal risk sharing can be analyzed.

The paper establishes the conditions under which a fiscal union (currency area with optimal transfers) dominates a flexible exchange rate regime without transfers. It then explains why a currency area might be associated with better risk sharing than the flexible exchange rate regime: when transfers have to satisfy a participation constraint, transfers are more easily supported in the currency area because the cost of reneging on transfers is higher than in the flexible exchange rate regime. In this sense, the formation of a currency area is a commitment device that allows countries to share more risk. In that case, Mundell's argument about an optimal currency area should be qualified because poorly-correlated business cycles create a demand for risk sharing, which provides the currency area with an advantage. As shown in Section 6, the incentives to share risk and sustain voluntary transfers are strengthened in the presence of transaction costs and remain effective when business cycles are uncorrelated.

It has been shown that the optimality of a currency area depends on structural parameters: the discount factor, the degree of risk aversion, the amplitude of the productivity shock, the elasticity of product substitution (or trade elasticity) and transaction costs. For the currency area to dominate, the discount factor should be high but not too high. A higher risk aversion favours the currency area because it increases the demand for risk sharing. On the other hand, the elasticity of product substitution should not be too high because this leads to more variability in relative productivity and a greater wage distortion in the currency area.

We have found that the currency area dominates for a plausible set of economic parameters but the flexible exchange rate regime also dominates for other parameter values that are not less plausible. The optimality of a currency area therefore depends on the empirical values. Moreover, the extent of transfers between countries depends on parameter values as well as the exchange rate regime. The paper has demonstrated that the nature of risk and the limits on risk sharing are important determinants that should not be ignored in assessing the optimality of a currency area.

Our model is perhaps too limited to be able to fully address the recent Euro-zone experience. For example, our model has no aggregate risk and no moral hazard. Both aggregate risk and moral hazard have been important elements in discussions about the Euro-zone. Nevertheless, our results may shed some light on why the creation of the Euro-zone has led to tensions among the member countries and little appetite to enhance the financial

stability arrangements. Limited commitment means that risk sharing is likely to be far from perfect. In addition, the model predicts that transfers are made at the margin where the country making the transfer is just indifferent to defaulting, inevitably leading to tensions when countries are called upon to make a transfer. Moreover, our results suggest that parameter values matter. Changes in, or uncertainty about, parameter values may lead to frequent re-evaluations of what is optimal. Clearly more research is needed to understand the Euro-zone experience, but we would suggest that any analysis will not only include elements of aggregate risk and moral hazard, but also limited commitment and risk sharing.

References

Alesina, A., Barro, R.J., 2002. Currency unions. The Quarterly Journal of Economics, 117, 409–436. doi:10.1162/003355302753650283.

Alesina, A., Perotti, R., Spolaore, E., 1995. Together or separately? Issues on the costs and benefits of political and fiscal unions. European Economic Review, 39, 751–758. doi:10.1016/0014-2921(95)00082-O.

Alvarez, F., Jermann, U.J., 2001. Quantitative asset pricing implications of endogenous solvency constraints. Review of Financial Studies, 14, 1117–1151. doi:10.1093/rfs/14.4.1117.

Arellano, C., Heathcote, J., 2010. Dollarization and financial integration. Journal of Economic Theory, 145, 944–973. doi:10.1016/j.jet.2009.08.002.

Asheim, G., Strand, J., 1991. Long-term, union-firm contracts. Zeitschrift für Nationalökonomie, 53, 161–184. doi:10.1007/BF01227465.

Backus, D.K., Kehoe, P.J., Kydland, F.E., 1992. International real business cycles. Journal of Political Economy, 100, 745–775. doi:10.1086/261838.

Basu, S., Fernald, J.G., 1997. Returns to scale in U.S. production: Estimates and implications. Journal of Political Economy, 105, 249–283. doi:10.1086/262073.

Baxter, M., Jermann, U.J., 1997. The international diversification puzzle is worse than you think. American Economic Review, 87, 170–180.

Bayoumi, T.A., 1994. A formal model of optimum currency areas. IMF Staff Papers, 42, 537–554. doi:10.2307/3867519.

Blanchard, O.J., Kiyotaki, N., 1987. Monopolistic competition and the effects of aggregate demand. American Economic Review, 77, 647–666.

Castro, R., Koumtingué, N., 2014. On the individual optimality of economic integration. Journal of Monetary Economics, 68, 115–135. doi:10.1016/j.jmoneco.2014.08.001.

Ching, S., Devereux, M.B., 2003. Mundell revisited: a simple approach to the costs and benefits of a single currency area? Review of International Economics, 11, 674–691. doi:10.1111/1467-9396.00410.

Cole, H.L., Obstfeld, M., 1991. Commodity trade and international risk sharing: How much do financial markets matter? Journal of Monetary Economics, 28, 3–24. doi:10.1016/0304-3932(91)90023-H.

Cooper, R., Kempf, H., 2004. Overturning Mundell: Fiscal policy in a monetary union. The Review of Economic Studies, 71, 371–396. doi:10.1111/0034-6527.00288.

Corsetti, G., Dedola, L., Leduc, S., 2008. International risk sharing and the transmission of productivity shocks. Review of Economic Studies, 75, 443–473. doi:10.1111/j.1467-937X.2008.00475.x.

Corsetti, G., Dedola, L., Leduc, S., 2010. Optimal monetary policy in open economies, in: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics. Elsevier. Volume 3, chapter 16, pp. 861–933.

Corsetti, G., Pesenti, P., 2001. Welfare and macroeconomic interdependence. The Quarterly Journal of Economics, 116, 421–445. doi:10.1162/00335530151144069.

Corsetti, G., Pesenti, P., 2005. International dimensions of optimal monetary policy. Journal of Monetary Economics, 52, 281–305. doi:10.1016/j.jmoneco.2004.06.002.

De Grauwe, P., Mongelli, F.P., 2005. Endogeneities of optimum currency areas: what brings countries sharing a single currency closer together? European Cental Bank, European Cental Bank Working Paper, 468, Frankfurt am Main.

Devereux, M.B., 2004. Should the exchange rate be a shock absorber? Journal of International Economics, 62, 359–377. doi:10.1016/S0022-1996(03)00050-3.

Drèze, J.H., 2000. Economic and social securtiy in the twenty-first century, with attention to Europe. Scandinavian Journal of Economics, 102, 327–348. doi:10.1111/1467-9442.00204.

Farhi, E., Werning, I., 2017. Fiscal unions. American Economic Review, 107, 3788–3834. doi:10.1257/aer.20130817.

Farrell, J., Maskin, E., 1989. Renegotiation in repeated games. Games and Economic Behavior, 1, 327–360. doi:10.1016/0899-8256(89)90021-3.

Feenstra R., 1986. Functional equivalence between liquidity costs and the utility of money. Journal of Monetary Economics, 17, 271–291. doi:10.1016/0304-3932(86)90032-2.

Fontagné, L., Martin, P., Orefice, G., 2018. The international elasticity puzzle is worse than you think. Journal of International Economics, 115, 115–129. doi:10.1016/j.jinteco.2018.08.011.

Forni, M., Reichlin, L., 1999. Risk and potential insurance in Europe. European Economic Review, 43, 1237–1256. doi:10.1016/S0014-2921(98)00124-X.

Frankel, J.A., Rose, A.K., 1998. The endogenity of the optimum currency area criteria. The Economic Journal, 108, 1009–1025. doi:10.1111/1468-0297.00327.

French, K.R., Poterba, J.M., 1991. Investor diversification and international equity markets. American Economic Review, 81, 170–180.

Fuchs, W., Lippi, F., 2006. Monetary union with voluntary participation. Review of Economic Studies, 73, 437–457. doi:10.1111/j.1467-937X.2006.00382.x.

Kehoe, T.J., Levine, D.K., 2001. Liquidity constrained markets versus debt constrained markets. Econometrica, 69, 575–598. doi:http://dx.doi.org/10.1111/1468-0262.00206.

Kenen, P.B., 1969. The theory of optimum currency areas: An eclectic view, in: Mundell, R.A., Swoboda, A.K. (Eds.), Monetary Problems of the International Economy. Chicago University Press, Chicago, pp. 41–60.

Kocherlakota, N.R., 1996. Implications of efficient risk sharing without commitment. Review of Economic Studies, 63, 595–610. doi:10.2307/2297795.

Krueger, D., Perri, F., 2011. Public versus private risk sharing. Journal of Economic Theory, 146(3), 920–956. doi:10.1016/j.jet.2010.08.013.

Lewis, K.K., 1999. Trying to explain home bias in equities and consumption. Journal of Economic Literature, 37, 571–608. doi:10.1257/jel.37.2.571.

Mendoza, E.G., 1991. Real business cycles in a small open economy. American Economic Review, 81, 797–818.

Mundell, R.A., 1961. A theory of optimum currency areas. American Economic Review, 51, 657–665.

Mundell, R.A., 1973. Uncommon arguments for common currencies, in: Johnson, H.G., Swoboda, A.K. (Eds.), The Economics of Common Currencies. George Allen and Unwin, London. Proceedings of the Madrid Conference on Optimum Currency Areas, pp. 195–220.

Obstfeld, M., Rogoff, K., 1995. Exchange rate dynamics redux. Journal of Political Economy, 103, 624–660. doi:10.1086/261997.

Park, Y., 2014. Optimal taxation in a limited commitment economy. Review of Economic Studies, 81(2), 884–918. doi:10.2307/43551583.

Persson, T., Tabellini, G., 1996. Federal fiscal constitutions: Risk sharing and moral hazard. Econometrica, 64, 623–646. doi:10.2307/2171864.

Rose, A.K., Engel, C., 2002. Currency unions and international integration. Journal of Money, Credit and Banking, 34, 1067–1089. doi:10.1353/mcb.2002.0058.

Sørensen, B.E., Yosha, O., 1998. International risk sharing and European monetary unification. Journal of International Economics, 45, 211–238. doi:10.1016/S0022-1996(98)00033-6.

Stockman, A.C., Tesar, L.L., 1995. Tastes and technology in a two-country model of the business cycle: Explaining international comovements. American Economic Review, 85, 168–185.

Thomas, J.P., Worrall, T., 1988. Self-enforcing wage contracts. Review of Economic Studies, 55, 541–554. doi:10.2307/2297404.

Thomas, J.P., Worrall, T., 1994. Foreign direct investment and the risk of expropriation. Review of Economic Studies, 61, 81–108. doi:10.2307/2297878.

Thomas, J.P., Worrall, T., 2007. Unemployment insurance under moral hazard and limited commitment: Public versus private provision. Journal of Public Economic Theory, 9(1), 151–181. doi:10.1111/j.1467-9779.2007.00302.x.

Thomas, J.P., Worrall, T., 2018. Dynamic relational contracts under complete information. Journal of Economic Theory, 175, 624–651. doi:10.1016/j.jet.2018.02.004.

Tödter, K.H., 2008. Estimating the uncertainty of relative risk aversion. Applied Financial Economics Letters, 4, 25–27. doi:10.1080/17446540701335474.

Trionfetti, F., 2018. Firm heterogeneity, comparative advantage and the transfer problem. European Economic Review, 108, 246–258. doi:10.1016/j.euroecorev.2018.07.007.

Appendix A

In this appendix, we discuss the contemporaneous choice of consumption and labor supply for a given transfer T_s and state of nature s, as well as the choice of the wage for a given transfer vector T. We discuss these choices in the fixed and flexible exchange rate regimes. To simplify the notation, we dispense with time superscripts and state subscripts whenever it does not lead to confusion. Aggregate variables are denoted by a capital letter: $C_H \equiv \int_0^1 c_H(j)\mathrm{d}j$, etc. The equilibrium solution is summarized in Table 1.

Maximization of household utility (4) with respect to consumption of Home and Foreign goods, and money holdings, subject to the budget constraint (1) gives:

$$c_H(j) = \mu \frac{\left(\frac{1}{2}\right)^\sigma p^{-\sigma}}{P^{1-\sigma}} e(j); \quad c_F(j) = \mu \frac{\left(\frac{1}{2}\right)^\sigma (\varepsilon p^*)^{-\sigma}}{P^{1-\sigma}} e(j) \quad \text{and} \quad m = (1-\mu)e(j),$$

where $e(j)$ is household j's expenditure and

$$P = \left(\left(\tfrac{1}{2}\right)^\sigma p^{1-\sigma} + \left(\tfrac{1}{2}\right)^\sigma (\varepsilon p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

is the Home price index. It can be checked that $P = \varepsilon P^*$. The upper tier composite of consumption and money balances is:

$$x(j) = \xi \frac{e(j)}{P}, \quad \text{where} \quad \xi = \mu^\mu (1-\mu)^{1-\mu}.$$

Expressions for the Foreign country are similarly defined. Aggregate demands for Home and Foreign goods are given by $D = C_H + C_H^*$ and $D^* = C_F + C_F^*$.

Home firms choose the labor input mix that minimizes the cost per unit of output. That is, $\min_\ell \int_0^1 w(j)\ell(j)\mathrm{d}j$ subject to $1 = F(\ell)/a$. Therefore, the firms' demand for labor service per unit of output is $\ell^D(j) = a(w(j)/W)^{-\theta}$ where $W \equiv (\int_0^1 (w(j))^{1-\theta}\,\mathrm{d}j)^{1/(1-\theta)}$ is the Home wage index. The cost per unit of output is $\int_0^1 w(j)\ell^D(j)\mathrm{d}j = aW$. The Home

firm's profit per unit of demand is $p - aW$. Since firms are competitive and all profits are competed away, $p = aW$. Finally, as mentioned in the main text, wages are the same across households: $W = w(j)$.

A household's expenditure $e(j)$ is equal to its earnings $y(j) = w(j)\ell(j)$, plus the transfer T, plus the money endowment m_0. Given the unit mass of households, the aggregate of Home expenditure is $E \equiv WL + T + M_0$ where $L = \int_0^1 \ell(j)\mathrm{d}j = \ell$ and $M_0 = \int_0^1 m_0 \mathrm{d}j = m_0$. National income is $Y = \int_0^1 y(j)\mathrm{d}j = WL$. Since earnings come from production, it also follows that $Y = pD$. Total world income, denominated in the Home currency, is $Y^w \equiv Y + \varepsilon Y^*$, where a superscript ^{w} indicates a world aggregate. Similarly, the world money supply, denominated in the Home currency, is $M_0^w \equiv M_0 + \varepsilon M_0^*$. Since there are no transfers from outside the two countries, $T + \varepsilon T^* = 0$ and world expenditure is $E^w = E + \varepsilon E^* = Y^w + M^w$.

We now turn to the determination of wages (and use the s subscript to emphasize the dependence on the state). Wages are determined simultaneously and independently in each country at the end of the period and before the next productivity shock is known. A Home household j sets its wage to maximize its expected utility, anticipating transfers, prices, the exchange rate, demand and money supply in the next period. Let $w(j)$ denote the wage decided at the end of a specific period, and let $\ell_s(j)$, P_s, $M_{0,s}$, etc. denote the Home variables in state s in the next period. Then, $w(j)$ is chosen to maximize

$$\mathbb{E}_s \left[\frac{1}{1-\gamma} \left(\xi \frac{w(j)\ell_s(j) + T_s + M_{0,s}}{P_s} \right)^{1-\gamma} \right] - \frac{\mathbb{E}_s \left[\ell_s(j)^2 \right]}{2}, \quad \text{where} \quad \ell_s(j) = a_s \left(\frac{w(j)}{W} \right)^{-\theta} d_s,$$

and d_s is the product demand to firms. Since households are identical, we get $w(j) = W$. Aggregating over the households' first-order conditions gives equation for $W(T)$ given in Table 1 in the main text.

Appendix B

This appendix provides proofs of Propositions 1-3. A superscript c denotes the currency area and a superscript f denotes the flexible exchange rate regime. A $^-$ denotes a variable when transfers are optimal and a $^\sim$ denotes a certainty equivalent. We assume symmetry: $A(s) = A$ and $\mathbb{E}_s[b(s)] = 1$. Recall that world money supply is normalized: $\frac{1}{2}\vartheta M_0^w = 1$.

Proof of Proposition 1. From the text, the consumption, labor and prices as a function of the transfer and the wage in the two regimes are:

$$
\begin{aligned}
X_s^c(T,W) &= \frac{\xi}{\mu A} \frac{b_s + T}{W}; & L_s^c(T,W) &= \frac{b_s}{W}; & P_s^c(T,W) &= AW; \\
X_s^f(T,W) &= \frac{\xi}{\mu A} \frac{1}{B_s(T)W}; & L_s^f(T,W) &= \frac{1-T}{W}; & P_s^f(T,W) &= AB_s(T)W.
\end{aligned}
\tag{B.1}
$$

The choice of W satisfies the aggregate first-order condition:

$$W = \left(\frac{\theta}{\theta - 1} \right) \left(\frac{\mathbb{E}_s \left[L_s(T_s, W)^2 \right]}{\xi \mathbb{E}_s \left[X_s(T_s, W)^{-\gamma} \frac{L_s(T_s,W)}{P_s(T_s,W)} \right]} \right). \tag{B.2}$$

Substituting the expressions in (B.1) into (B.2) and solving gives the expressions in equation (10) for the two different regimes.

The optimal transfer is $\bar{T}(s) = \frac{1}{2}(b_s^* - b_s)$ and hence, since $\frac{1}{2}(b_s^* + b_s) = 1$, $b_s + \bar{T}_s = 1$. Equally, $\varepsilon_s(\bar{T}_s) = 1$ and hence, $B_s(\bar{T}_s) = 1$. Substituting into equations (10) gives:

$$W^c(\bar{T}) = W^f(\bar{T}) = \left(\kappa_0 \mathbb{E}_s \left[b_s^2 \right] \right)^{\frac{1}{1+\gamma}}.$$

Denoting $W^c(\bar{T}) = W^f(\bar{T})$ by $\bar{W}$, we have (writing $\bar{X}_s^c$ for $X_s^c(\bar{W}, \bar{T}_s)$ etc.):

$$
\begin{aligned}
\bar{X}_s^c &= \frac{\xi}{\mu A} \frac{1}{\bar{W}}; & \bar{L}_s^c &= \frac{b_s}{\bar{W}}; & \bar{P}_s^c &= A\bar{W}; \\
\bar{X}_s^f &= \frac{\xi}{\mu A} \frac{1}{\bar{W}}; & \bar{L}_s^f &= \frac{b_s}{\bar{W}}; & \bar{P}_s^f &= A\bar{W}
\end{aligned}
$$

Thus, the allocation is the same in both regimes.

Absent any transfers, the ratio of the wages in the two regimes is:

$$\frac{W^c(0)}{W^f(0)} = \left(\frac{\mathbb{E}_s \left[b_s^2 \right] \mathbb{E}_s \left[B_s(0)^{\gamma-1} \right]}{\mathbb{E}_s \left[b_s^{1-\gamma} \right]} \right)^{\frac{1}{1+\gamma}},$$

and the allocation is (writing $X_s^c(0)$ for $X_s^c(W^c(0), 0)$ etc.):

$$X_s^c(0) = \frac{\xi}{\mu A} \frac{b_s}{W^c(0)}; \qquad L_s^c(0) = \frac{b_s}{W^c(0)};$$
$$X_s^f(0) = \frac{\xi}{\mu A} \frac{1}{B_s(0)W^f(0)}; \quad L_s^f(0) = \frac{1}{W^f(0)}.$$

The certainty equivalent values for consumption (for $\gamma > 1$) and labor are:

$$\tilde{X}^c(0) = \frac{\xi}{\mu A} \frac{\left(\mathbb{E}_s\left[b_s^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{W^c(0)}; \qquad \tilde{L}^c(0) = \frac{\sqrt{\mathbb{E}_s\left[b_s^2\right]}}{W^c(0)}$$
$$\tilde{X}^f(0) = \frac{\xi}{\mu A} \frac{\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{1-\gamma}}}{W^f(0)}; \quad \tilde{L}^f(0) = \frac{1}{W^f(0)}.$$

Substituting for the wage ratio $W^c(0)/W^f(0)$, it can be checked that:

$$\left(\frac{\tilde{X}^f(0)}{\tilde{X}^c(0)}\right)^{\frac{1+\gamma}{2}} = \left(\frac{\tilde{L}^f(0)}{\tilde{L}^c(0)}\right)^{\frac{1+\gamma}{1-\gamma}} = \frac{\sqrt{\mathbb{E}_s\left[b_s^2\right]}}{\left(\mathbb{E}_s\left[b_s^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{\gamma-1}}}.$$

Hence, if inequality (12) is satisfied, then $\tilde{X}^f(0) > \tilde{X}^c(0)$ and $\tilde{L}^f(0) < \tilde{L}^c(0)$. Thus, absent any transfers the flexible exchange rate regime yields higher welfare than the currency area provided (12) is satisfied.

For the two state case, let

$$r_0(z, \sigma, \gamma) = \frac{\sqrt{\mathbb{E}_s\left[b_s^2\right]}}{\left(\mathbb{E}_s\left[b_s^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{\gamma-1}}}.$$

It is easy to check that $r_0(z, \sigma, \gamma)$ is increasing in γ. Since, we want to check if $r_0(z, \sigma, \gamma) > 1$, it suffices to check the condition for $\gamma = 1$. This requires:

$$\log\left(\frac{1}{2}\frac{b_G}{b_B} + \frac{1}{2}\frac{b_B}{b_G}\right) > \log\left(B_G(0)\right) + \log\left(B_B(0)\right). \tag{B.3}$$

Since $B_s(0)$ is a weighted power mean of ε_s and 1, $\log(B_G(0)) \leq \frac{1}{2}b_B \log(\varepsilon_G)$ and $\log(B_B(0)) \leq \frac{1}{2}b_G \log(\varepsilon_B)$. Using this property together with the bounds $(x-1)/x < \log(x) < x - 1$, it can be established that

$$\log\left(\frac{1}{2}\frac{b_G}{b_B} + \frac{1}{2}\frac{b_B}{b_G}\right) > 1 - \frac{2z^{1-\sigma}}{z^2+z^\sigma} \quad \text{and} \quad \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1-z^{\sigma-1}}{1+z^{\sigma-1}}\right)(1-z) > \log\left(B_G(0)\right) + \log\left(B_B(0)\right).$$

Taking ratios, condition (B.3) is satisfied when

$$\frac{\sigma}{\sigma-1} > \frac{(1-z)\left(1+z^{2(\sigma-1)}\right)}{\left(1-z^{2(\sigma-1)}\right)}.$$

It can be checked that for $\sigma \leq 2$, the right hand side of this inequality has a maximum of $1/(\sigma-1)$ in the limit as $z \to 1$. Equally, for $\sigma > 2$, it has a maximum of 1 in the limit as $z \to 0$. Thus, the inequality is always satisfied. Hence, we conclude that for $z < 1$, $r_0(z, \sigma, \gamma) > r_0(z, \sigma, 1) > 1$ and the condition in (12) is satisfied for all parameter values. $\square$

Proof of Proposition 2. Evaluating at consumption and labor in the currency area when $T_s = \bar{T}_s$ and at $T_s = 0$ for the flexible exchange rate regime gives:

$$\bar{X}^c = \frac{\xi}{\mu A}\frac{1}{\bar{W}}; \qquad \bar{L}_s^c = \frac{b_s}{\bar{W}};$$
$$X_s^f(0) = \frac{\xi}{\mu A}\frac{1}{B_s(0)W^f(0)}; \quad L^f(0) = \frac{1}{W^f(0)}.$$

The certainty equivalents can therefore be written as:

$$\bar{X}^c = \tilde{X}^f(0)\left(\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{\gamma-1}}\right)\left(\frac{W^f(0)}{\bar{W}}\right); \quad \tilde{L}^c = L^f(0)\left(\sqrt{\mathbb{E}_s\left[b_s^2\right]}\right)\left(\frac{W^f(0)}{\bar{W}}\right);$$

where

$$\frac{W^f(0)}{\bar{W}} = \left(\frac{1}{\mathbb{E}\left[b_s^2\right]\mathbb{E}\left[B_s(0)^{\gamma-1}\right]}\right)^{\frac{1}{1+\gamma}}.$$

Substituting the wage ratio $W^f(0)/\bar{W}$ into the certainty equivalents shows that:

$$\left(\frac{\tilde{X}^c}{\tilde{X}^f(0)}\right)^{\frac{1+\gamma}{2}} = \left(\frac{\tilde{L}^c}{L^f(0)}\right)^{\frac{1+\gamma}{1-\gamma}} = \frac{\left(\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{1}{\gamma-1}}}{\sqrt{\mathbb{E}_s\left[b_s^2\right]}}.$$

It is then easily checked that $\bar{X}^c > \tilde{X}^f(0)$ and $\tilde{L}^c < L^f(0)$ if and only if condition (14) is satisfied. Condition (14) can be rewritten as:

$$\frac{\mathbb{E}_s\left[B_s(0)^{\gamma-1}\right]-1}{(\gamma-1)} > \frac{\left(\sqrt{\mathbb{E}_s\left[b_s^2\right]}\right)^{\gamma-1}-1}{(\gamma-1)}.$$

Taking the limit as $\gamma \to 1$ and using $\bar{W}/W^f(0) = \sqrt{\mathbb{E}_s\left[b_s^2\right]}$ gives condition (13) in the text.

For the two state case, use $b_B = 2 - b_G$ and the definition of $B_s(0)$ to write

$$\chi(b_G) = \tfrac{1}{2} \log\left(\mathbb{E}_s[b_s^2]\right) \quad \text{and} \quad \lambda(b_G) = \tfrac{1}{\gamma-1} \log\left(\mathbb{E}_s[B_s(0)^{\gamma-1}]\right).$$

Condition (14) is satisfied, and the fiscal union dominates, when $\chi(b_G) - \lambda(b_G) < 0$. By definition $b_G \in (1,2)$ and is negatively related to z with $\lim_{z\to 1} b_G = 1$ and $\lim_{z\to 0} b_G = 2$. It can be checked that both $\chi(b_G)$ and $\lambda(b_G)$ are increasing and convex, with limits $\chi(1) = \lambda(1) = 0$, $\chi(2) = \log(2)/2$ and $\lambda(2) = \infty$. It can also be checked that there are limits $\chi'(1) = \lambda'(1) = 0$, $\chi'(2) = 1/2$ and $\lambda'(2) = \infty$. With these properties, there are two cases to consider. First, if $\lambda''(1) \ge \chi''(1)$, then $\chi(b_G) - \lambda(b_G)$ has a maximum of 0 when $b_G = 1$ ($z = 1$). In this case, the fiscal union dominates for all $z < 1$. Second, if the limit $\lambda''(1) < \chi''(1)$, then there is a unique value of b_G, where $\chi'(b_G) = \lambda'(b_G)$, that maximizes $\chi(b_G) - \lambda(b_G)$, with a positive maximum value. Since $\chi(b_G)$ is bounded above and $\lambda(b_G)$ is unbounded, it follows that there is a unique threshold, $\widehat{b}_G$ such that $\chi(\widehat{b}_G) - \lambda(\widehat{b}_G) = 0$, with $\chi(b_G) - \lambda(b_G) < 0$ for $b_G > \widehat{b}_G$. Since b_G is inversely related to z, there is a corresponding value of $\widehat{z}$ such that the fiscal union dominates for $z < \widehat{z}$. It can be checked that in the limit $\chi''(1) = 1$ and $\lambda''(1) = (\gamma + \sigma)/\sigma^2$. Thus, for $\sigma^2 - \sigma - \gamma \le 0$, equivalently $\sigma \le (1/2)(1 + \sqrt{1 + 4\gamma})$, the fiscal union dominates for all $z < 1$. That is, the threshold $\widehat{z} = 1$. Otherwise, there is a unique threshold $\widehat{z} < 1$, such that the fiscal union dominates for $z < \widehat{z}$ and is dominated by the flexible exchange rate regime for $z \in (\widehat{z}, 1)$. $\qquad\square$

Proof of Proposition 3. With two states consumption sharing is sustainable if there is no incentive to deviate in the good state. Let $\mathcal{T}$ denote the transfer received by the country with the bad shock. With optimal transfers $\bar{T}$, the exchange rate is equal to unity in both the currency area and flexible exchange rate regimes. Thus, the critical value $\bar{\delta}$ above which consumption sharing is sustained is given by:

$$\bar{\delta} = \frac{U_G(\bar{W}, 0) - U_G(\bar{W}, -\bar{\mathcal{T}})}{\left(U_G(\bar{W}, 0) - U_G(\bar{W}, -\bar{\mathcal{T}})\right) + \left(\mathbb{E}_s\left[U_s(\bar{W}, \bar{T}_s)\right] - \mathbb{E}_s\left[U_s(W(0), 0)\right]\right)}. \tag{B.4}$$

We consider the currency area and flexible exchange rate regime in turn. To simplify notation, let $\bar{U}_s = U_s(\bar{W}, \bar{T}(s))$, $U_s(0) = U_s(W(0), 0)$ and $\bar{U}_s(0) = U_s(\bar{W}, 0)$. For $\gamma > 1$, we can write:

$$\mathbb{E}_s\left[\bar{U}_s^c\right] = \left(\frac{(\bar{X}^c)^{1-\gamma}}{1-\gamma} - \frac{(\bar{L}^c)^2}{2}\right) = \left(\frac{1}{1-\gamma}\left(\frac{\xi}{\mu A}\right)^{1-\gamma}\left(\frac{1}{(\bar{W})^{1-\gamma}}\right) - \frac{1}{2}\frac{\mathbb{E}_s[b_s^2]}{(\bar{W})^2}\right),$$

$$\mathbb{E}_s\left[U_s^c(0)\right] = \left(\frac{(\bar{X}^c(0))^{1-\gamma}}{1-\gamma} - \frac{(\bar{L}^c(0))^2}{2}\right) = \left(\frac{1}{1-\gamma}\left(\frac{\xi}{\mu A}\right)^{1-\gamma}\left(\frac{\mathbb{E}_s[b_s^{1-\gamma}]}{(W^c(0))^{1-\gamma}}\right) - \frac{1}{2}\frac{\mathbb{E}_s[b_s^2]}{(W^c(0))^2}\right).$$

Since

$$\bar{W} = W^c(0)\left(\mathbb{E}_s\left[b_s^{1-\gamma}\right]\right)^{\frac{1}{1+\gamma}} = \left(\kappa_0 \mathbb{E}_s\left[b_s^2\right]\right)^{\frac{1}{1+\gamma}} \quad \text{and} \quad \kappa_0 = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\mu}\right)\left(\frac{\xi}{\mu A}\right)^{\gamma-1},$$

we have upon substitution (using $\theta \to \infty$ and $\mu \to 1$) that:

$$\mathbb{E}_s\left[\bar{U}_s^c\right] - \mathbb{E}_s\left[U_s^c(0)\right] = \left(\frac{1}{2} - \frac{1}{1-\gamma}\right)\left(\frac{\mathbb{E}_s\left[b_s^2\right]}{\bar{W}^2}\right)\left(\left(\mathbb{E}_s\left[b_s^2\right]\right)^{\frac{2}{1+\gamma}} - 1\right).$$

Equally, we can write

$$\bar{U}_G^c(0) - \bar{U}_G^c = \left(\frac{\mathbb{E}_s\left[b_s^2\right]}{\bar{W}^2}\right)(\gamma-1)^{-1}\left(b_G^{1-\gamma} - 1\right).$$

Substituting these expressions into (B.4) gives:

$$\bar{\delta}^c = \frac{(1-\gamma)^{-1}\left(b_G^{1-\gamma} - 1\right)}{(1-\gamma)^{-1}\left(b_G^{1-\gamma} - 1\right) + \left(\frac{1}{2} - (1-\gamma)^{-1}\right)\left(\mathbb{E}_s[b_s^{1-\gamma}]^{\frac{2}{1+\gamma}} - 1\right)}.$$

Taking the limit as $\gamma \to 1$ gives:

$$\bar{\delta}^c = \frac{\ln(b_G)}{\frac{1}{2}\ln(b_G) - \frac{1}{2}\ln(b_B)}.$$

Next consider the flexible exchange rate regime. It can be checked that:

$$\mathbb{E}_s\left[\bar{U}_s^f\right] - \mathbb{E}\left[U_s^f(0)\right] = \left(\frac{\mathbb{E}_s[b_s^2]}{\bar{W}^2}\right)\left(\frac{1}{2} - \frac{1}{1-\gamma}\right)\left(\left(\mathbb{E}_s\left[b_s^2\right]\right)^{\frac{1-\gamma}{1+\gamma}}\left(\mathbb{E}\left[B_s(0)^{\gamma-1}\right]\right)^{\frac{2}{1+\gamma}} - 1\right)$$

34

and

$$\bar{U}_G^f(0) - \bar{U}_G^f = \left(\frac{\mathbb{E}_s[b_s^2]}{\bar{W}^2}\right)\left(\frac{1}{2}\frac{1}{\mathbb{E}_s[b_s^2]}\left(b_G^2 - 1\right) + \frac{1}{1-\gamma}\left(B_G(0)^{\gamma-1} - 1\right)\right).$$

Substituting these expressions into (B.4) and taking the limit as $\gamma \to 1$ gives:

$$\bar{\delta}^f = \frac{\left(b_G^2 - 1\right) - \left(b_G^2 + b_B^2\right)\log\left(B_G(0)\right)}{\left(b_G^2 - 1\right) - \left(\frac{1}{2}b_G^2 + \frac{1}{2}b_B^2\right)\left(\log\left(\frac{1}{2}b_G^2 + \frac{1}{2}b_B^2\right) + \log\left(B_G(0)\right) - \log\left(B_B(0)\right)\right)}.$$

Now we turn to calculating the lower critical discount factors. Recall that $T_G = -\varepsilon_G(\mathcal{T})\mathcal{T}$ and $T_B = \mathcal{T}$. Sustainability requires $\delta > \underline{\delta}$ where

$$\underline{\delta} = \lim_{\mathcal{T}\to 0}\frac{U_G(W(\mathcal{T}),0) - U_G(W(\mathcal{T}), -\mathcal{T}\varepsilon_G)}{U_G(W(\mathcal{T}),0) - U_G(W(0),0) - \frac{1}{2}(U_G(W(\mathcal{T}),-\mathcal{T}\varepsilon_G) - U_G(W(0),0)) + \frac{1}{2}(U_B(W(\mathcal{T}),\mathcal{T}) - U_B(W(0),0))}.$$

Using L'Hôpital's rule gives

$$\underline{\delta} = \frac{\frac{\partial U_G(0)}{\partial T}\varepsilon_G(0) - \varepsilon_G(0)\frac{\partial U_G(0)}{\partial \varepsilon_G}\frac{\varepsilon_G'(0)}{\varepsilon_G(0)}}{\left(\frac{1}{2}\frac{\partial U_G(0)}{\partial T}\varepsilon_G(0) + \frac{1}{2}\frac{\partial U_B(0)}{\partial T}\right) - \left(\frac{1}{2}\varepsilon_G(0)\frac{\partial U_G(0)}{\partial \varepsilon_G}\frac{\varepsilon_G'(0)}{\varepsilon_G(0)} - \frac{1}{2}\varepsilon_B(0)\frac{\partial U_B(0)}{\partial \varepsilon_B}\frac{\varepsilon_B'(0)}{\varepsilon_B(0)}\right) + \frac{W'(0)}{W(0)}\mathbb{E}_s\left[\frac{\partial U_s(0)}{\partial W}W(0)\right]}. \tag{B.5}$$

We treat the currency area and flexible exchange rate regime in turn. In the currency area $\varepsilon_s(T(s)) = 1$ and hence,

$$\underline{\delta}^c = \frac{\frac{\partial U_G^c(0)}{\partial T}}{\left(\mathbb{E}_s\left[\frac{\partial U_s^c(0)}{\partial T}\right] + \left(\frac{W^{c\prime}(0)}{W^c(0)}\right)\mathbb{E}_s\left[\frac{\partial U_s^c(0)}{\partial W}W^c(0)\right]\right)}. \tag{B.6}$$

It can be checked that for $\theta \to \infty$ and $\mu \to 1$:

$$\frac{\partial U_G^c(0)}{\partial T} = \left(\frac{\mathbb{E}_s[(b_s^2]}{(W^c(0))^2}\right)\left(\frac{b_G^{-\gamma}}{\mathbb{E}_s[b_s^{1-\gamma}]}\right); \quad \mathbb{E}_s\left[\frac{\partial U_s^c(0)}{\partial T}\right] = \left(\frac{\mathbb{E}_s[(b_s^2]}{(W^c(0))^2}\right)\left(\frac{\mathbb{E}_s[b_s^{-\gamma}]}{\mathbb{E}_s[b_s^{1-\gamma}]}\right);$$

$$\frac{W^{c\prime}(0)}{W^c(0)} = -\left(\frac{\gamma}{2(1+\gamma)}\right)\left(\frac{b_G^{-\gamma} - b_B^{-\gamma}}{\mathbb{E}_s[b_s^{1-\gamma}]}\right) \quad \mathbb{E}_s\left[\frac{\partial U_s^c(0)}{\partial W}W^c(0)\right] = 0.$$

Therefore, substituting into (B.6) gives:

$$\underline{\delta}^c = \frac{b_B^\gamma}{\frac{1}{2}b_G^\gamma + \frac{1}{2}b_B^\gamma}.$$

For $\gamma = 1$, $\underline{\delta}^c = b_B$.

Turning to the flexible exchange rate case, it is necessary to take into account the effect of the transfer on the exchange rate. It can be checked that:

$$\frac{W^{f\prime}(0)}{W^f(0)} = -\frac{(1-\varepsilon_G(0))}{1+\gamma} + \frac{(1-\beta-\beta\varepsilon_G(0))}{1+\gamma}\left(1 + \frac{\gamma-1}{\sigma}\right); \quad \frac{\partial U_s^f(0)}{\partial T} = \frac{1}{(W^f(0))^2};$$

$$\frac{\varepsilon_G'(0)}{\varepsilon_G(0)} = -\frac{\varepsilon_B'(0)}{\varepsilon_B(0)} = \frac{1+\varepsilon_G(0)}{\sigma} = -\frac{(1+\varepsilon_B(0))}{\sigma}; \quad \varepsilon_s(0)\frac{\partial U_s^f(0)}{\partial \varepsilon_s} = -\frac{1}{(W^f(0))^2}\frac{B_s(0)^{\gamma-1}}{\mathbb{E}_s[B_s(0)^{\gamma-1}]}\frac{\varepsilon_s(0)}{1+\varepsilon_s(0)};$$

$$\mathbb{E}_s\left[\frac{\partial U_s^f(0)}{\partial W}W^f(0)\right] = 0;$$

where

$$\beta = \frac{\frac{1}{2}B_G(0)^{\gamma-1}}{\frac{1}{2}B_G(0)^{\gamma-1} + \frac{1}{2}B_B(0))^{\gamma-1}}.$$

Substituting into (B.5) gives:

$$\underline{\delta}^f = \frac{\varepsilon_G(0)\left(1 + \frac{2\beta}{\sigma}\right)}{\frac{1}{2}\left(1 + \varepsilon_G(0)\right) + \frac{1}{\sigma}\left(\beta\varepsilon_G(0) + (1-\beta)\right)}.$$

When $\gamma = 1$, $\beta = 1/2$, and therefore,

$$\underline{\delta}^f = \frac{\varepsilon_G(0)}{\frac{1}{2}\left(1 + \varepsilon_G(0)\right)} = \frac{b_B^{\frac{1}{\sigma}}}{\frac{1}{2}b_G^{\frac{1}{\sigma}} + \frac{1}{2}b_B^{\frac{1}{\sigma}}}.$$

It is easy to check that with $z < 1$ and $\sigma > 1$, the above formulas for the critical discount factors satisfy $0 < \underline{\delta}^c < \bar{\delta}^c < 1$ and $\underline{\delta}^c < \underline{\delta}^f < 1$. $\qquad\square$