

Real Time Hyper-elastic Simulations with Probabilistic Deep Learning

Saurabh Deshpande^{1*}, Jakub Lengiewicz² and Stéphane P.A. Bordas³

¹ University of Luxembourg, Esch-sur-Alzette, saurabh.deshpande@uni.lu

² University of Luxembourg, Esch-sur-Alzette, jakub.lengiewicz@uni.lu

³ University of Luxembourg, Esch-sur-Alzette, stephane.bordas@uni.lu

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Computer simulation has acquired a preeminent role in recent years, particularly, many engineering applications rely on the predictive capabilities of computational models. Some of these applications, like biomedical simulations used to support or train surgeons [1][2] require computationally efficient or even real-time solutions. Conventional methods for solving the underlying non-linear problems, such as the Finite Element Method, are computationally far too expensive. This work efficiently leverages state-of-the-art data-driven techniques to give accurate non-linear solutions in real-time.

In this work, we propose a highly efficient deep-learning surrogate framework that can predict deformation responses of hyper-elastic bodies under external loads. We implement a special type of convolutional neural network (CNN) [3], the so-called U-Net. This architecture has strong resemblances to Finite Element multi-grid methods and proves to be capable of capturing non-linear responses characteristic to large deformation regimes. We train variants of U-Net with synthetic force-displacement data generated with the finite element method. In addition to the standard deterministic version of the framework, we propose its probabilistic versions, which can provide reliable uncertainty estimates of U-Net models in addition to predictions. We study the properties of frameworks for several benchmark problems. In particular, we check the capabilities of the Maximum Likelihood and the Variational Bayes Inference [4] formulations to assess the confidence intervals of solutions.

The use of the U-Net surrogate model allowed us to gain x350 speedup, enabling us to achieve real-time responses. The solution errors of the surrogate model have been shown to correlate with the uncertainties predicted by the proposed probabilistic framework. We also show that uncertainties of solution rapidly increase in the extrapolated region [3]. With our framework we could assess both: model and data uncertainties, which we believe is an important step towards making real-time hyper-elastic simulations more reliable.

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