# Random Encounters and Information Diffusion About Product Quality*

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**Abstract**

This paper explores how social interactions among consumers shape markets. In a two-country model, consumers meet and exchange information about the quality of the goods. As information spreads, demand evolves, affecting the prices and quantities manufactured by profit-maximizing firms. We show that market prices with informational frictions reach the duopoly price with full information at the limit. However, this convergence can take different paths depending on the size asymmetry between countries. In particular, when the country producing the low-quality good is relatively large, the single market does not immediately turn into a duopoly and can be temporarily trapped in a situation of price instability where no Nash equilibrium in pure (but only in mixed) strategies exists and prices can fluctuate between their monopoly and duopoly levels. It follows that the classical price-reducing effects of international trade may take longer to appear. In view of an intense globalization process, understanding how social meetings affect market outcomes is critical for understanding the performance of international economic integration.

**Keywords:** *Consumer Encounters, Information Diffusion, Country Size, Product Quality.*

**JEL Classification:** D42, D43, D83, F15, L13.

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# 1 Introduction

At the end of the Second World war, both the Spanish *Jamon Bellota* and the French *Jambon de Paris* were existing in their respective country, but they were not competing with each other. Today they cohabit in the displays of European shops while they compete on the markets of European food products. This is clearly due to the opening of markets resulting from the European Common Market. Nevertheless, some period of time was needed before this competition became effective because most of Spanish consumers were unaware of the existence of the *Jambon de Paris*, and similarly for French consumers with respect of the *Jamon Bellota*. The ignorance about the very existence of the substitute variant in the Spanish country hindered French producers of ham to compete with their Spanish fellows, and vice versa. But progressively, word of mouth and other opportunities of meeting among citizens of different countries diffused information and alleviated the barriers of entry and competition due to ignorance. Various international agreements, such as the European Union, the establishment of the euro, and the Schengen Area agreement, have massively reduced the costs of mobility for people, intensifying globalization. Citizens located in different countries now meet much more frequently, sharing personal consumption experiences. Accordingly, meetings have become an essential vehicle of diffusion of information about goods, prices, and quality. This vehicle and its price effects are the main topics of this paper.

We explore how interpersonal non-market interactions affect market quantities and prices. Can these interactions metamorphose domestic markets into a single market? If yes, how? If there is a convergence of prices, what paths are involved and how rapid is the convergence?

To answer these questions, we build a two-country model with two vertically differentiated goods (Gabszewicz and Thisse, 1979) and the presence of informational frictions. The countries have different population densities, and each country has a national market on which a domestic firm sells a domestic good. As long as these markets remain strictly national, consumers in each country remain ignorant about the quality of the foreign commodity. There is no mutual influence between their respective markets: each national firm is a monopolist in its national market. An international agreement is signed that reduces the costs of being mobile. This agreement determines the birth and outgrowth of mutual interactions among agents in the two asymmetrically sized countries. Mutual experiences of consumption habits are exchanged progressively and according to the number and frequency of social meetings. Inspired by evolutionary game theory settings (Weibull, 1995; Bowles, 2006), we model meetings in a random encounter model. This implies that information about the goods does not spread automatically to all consumers.

As consumers meet and get informed, the exchange of consumption experiences reinforces the process of competition between the national and the foreign substitute. This process magnifies competition between the products, transforming two national monopolies into a single duopoly market with vertically differentiated products. We investigate how prices change along the sequence of equilibria generated by the dynamics of international interactions.

The main results of our analysis show that market prices tend to the duopoly solution with full information at the limit. The intuition behind this result relates to information diffusion. As consumers become informed, the market power of firms reduces, and when all consumers are informed about both goods, the single market becomes a duopoly. Surprisingly, this convergence can take different paths. When the population of the country producing the low-quality good is sufficiently small, its citizens quickly become aware of the high-quality good due to frequent interpersonal meetings with foreign consumers. Therefore, competition between national goods intensifies relatively quickly in the common market. The market evolution from monopoly to duopoly occurs in the first period, and prices take time to adjust to their full-information level.

In contrast, when the country producing the low-quality good is densely populated, a large number of consumers are uninformed about the high-quality good. This implies that the diffusion of information about the quality of the foreign commodity is considerably slow. In this case, a price Nash equilibrium in pure strategies may not exist for a certain number of periods whereas only one in mixed strategies arises, which implies that firms' prices can temporarily fluctuate between their monopoly and duopoly levels as in the price cycle described by Edgeworth (1925). This means that the classical price-reducing trade effects may take longer to appear depending on the size asymmetry between countries. However, we also show that a finite period exists in which informed consumers are sufficiently numerous to make the duopoly equilibrium appear. Thus, learning about the goods ultimately leads to the establishment of the single duopoly market. Finally, we analyze the properties of duopoly prices and quantities, as well as profits and welfare, in the presence of informational frictions, highlighting novel properties. It is worth mentioning that not all countries gain immediately from intense international meetings of consumers. Notably, for a certain period of time, increased social interactions are welfare-detrimental for the country producing the high-quality good (often developed countries). It follows that the increased mobility of people implies winners and losers beyond migration concerns. This may be one additional cause of increased anti-globalization movements.

Our work combines the literature on product quality and markets with the growing research trend regarding the role played by social interactions in market shifts. An extensive economics literature has investigated the optimal pricing strategy via various forms of signaling for new products (a seminal work is Milgrom and Roberts, 1986). Nevertheless, most signaling models have ignored learning from others—social interactions—and the question of whether and how interpersonal communication across consumers affects prices. Social interactions outside of the market that crucially affect demand represent the key element of the present paper.

Our contribution to the literature is twofold, as will be detailed in the next section. We firstly propose an elegant model of vertical differentiation with informational frictions that evolve dynamically to the state of full information. This model is relatively simple, but it allows a deep understanding of how non-market forces may affect market formation. Secondly, we contribute to an extensive literature on the effects of globalization on markets, highlighting population size asymmetry. In fact, our model allows us to pin down how size asymmetry between countries affects the price convergence path when citizens meet. In an influential paper, Alesina, Spolaore, and Wacziarg (2005) argue that our understanding of economic performance and the history of international economic integration can be greatly improved by bringing country size to the forefront of the analysis of prices and growth. The authors use country size to document how trade affects a country's growth, showing that the size of countries shapes trade intensity.

The article is set out as follows. In the following section, we situate our paper within the relevant literature. Section 3 provides a description of the model. In Section 4, we develop the multi-period market solution. Section 5 unveils some interesting properties of the duopoly outcome with informational frictions, and finally, Section 6 presents the study's conclusions. All proofs are relegated to the Appendix.

# 2 Related literature

Our paper brings together three theoretical tools: *(i)* a vertical differentiation model, *(ii)* open economies and trade, and finally, *(iii)* a dynamic setting to capture the evolution of a particular feature in a given population—knowledge of product quality.

Firstly, we contribute to the industrial organization literature as we revisit the classical model of vertically differentiated markets in industrial organizations, introducing informational frictions. More precisely, before information fully spreads to all consumers, there are two different market segments according to the consumer' information set: uninformed consumers who know their domestic good but are ignorant of the foreign one and informed consumers who know both goods. In this novel setting, the firm behaves as a monopolist for uninformed consumers and as a duopolist for the informed ones. We call this new market configuration a duopoly with informational frictions. Prior literature on vertically differentiated markets focuses on market interactions, whereas we focus on non-market interactions. Our paper contributes to this literature by providing a simple model in which meetings are combined with strategic pricing to assess the type and pace of convergence from monopolies to a duopoly after the opening up of movement between two countries.

Our paper is related to the literature on signaling quality through prices and/or advertising, with important differences, however. Milgrom and Roberts (1986) and Bagwell and Riordan (1991) study a dynamic monopoly model in which both price and advertising can signal quality. These authors find that price signaling will typically occur and (dissipative) advertising will be used in equilibrium when separation through prices is too costly. Caminal and Vives (1999) analyze price dynamics in a duopoly where consumers learn about good quality differentials. In that paper, market shares aggregate consumers' dispersed and private information about the quality of products. Hence, past market share acts as a signal of product quality. The authors find that despite price wars, consumers learn slowly and convergence to full information is also slow. In these papers, there is signaling via the price or market shares of firms but there is no information diffusion about quality via social interactions, as in our setting.

Vettas (1998) investigates the endogenous diffusion of information along both sides of the market: firms and consumers. Entry of new firms reveals information to consumers about product quality and, therefore, early entry affects the expected profitability following entry. As a consequence, the diffusion of new firm entries follows an S-shaped diffusion path. Our paper differs along several dimensions. In our setting, learning takes place outside the market, in social interactions with foreign or informed consumers. The size of the initial domestic market share is key in defining the stream of market shares of each firm. And finally, entry takes place only once, when the two countries open to exchanges and information evolves via social interactions.

More recently, Guadalupi (2018) explored the effects of word-of-mouth communication on the optimal pricing strategy for new experience goods using a dynamic monopoly model with asymmetric information about product quality, in which consumers learn in equilibrium from both prices and the choices of other consumers. Word-of-mouth communication is essential for the existence of separating equilibria, wherein the high-quality monopolist signals high quality through a low introductory price (lower than the monopoly price), whereas the low-quality one charges the monopoly price. Differently from Guadalupi (2018), our paper is about social interactions that take place when two countries are open to exchanges. By definition, these interactions occur outside of the market and mainly depend on the numerosity of the populations that meet. This assumption is inspired by models of evolutionary economics that explore how traits evolve in interacting groups. It follows that, in our paper, firms cannot directly impact the information sets of consumers. Nevertheless, we show that information about product quality plays a relevant role.

The existing literature on international trade, both theoretical and empirical, is vast. Papers aim to quantify and test the empirical relevance of trade theories using, for instance, numerical general equilibrium models calibrated with real-world data (Mercenier and Schmitt, 1996). These papers have played an essential role in the Canada–U.S. free-trade agreement, the North American Free Trade Agreement (NAFTA), and the European Single Market. However, the importance of country size has been neglected in theoretical contributions. In our paper, size is a key ingredient. The importance of country size in trade is well documented in empirical papers showing that trade has a more substantial impact on market competitiveness for small economies. For instance, Hong Kong and Singapore are small open-to-trade countries that do not have a competition policy authority. Trade has relatively greater effects on these economies and acts as a disciplinary mechanism. Moreover, Hoekman et al. (2001) find evidence that country size negatively influences the effect trade has on market prices. Finally, Novy (2013) shows that trade is more sensitive to trade agreements if the exporting country only provides a small share of the destination country's imports. If two large countries like the USA and Germany are engaged in strong trade relations, a trade agreement change will affect trade flows less than if the USA and Iceland were engaged in trade. The reason is the relative population size of Iceland. The intensity of trade between the two larger countries is higher than that between the USA and Iceland. Our paper is also related to the existing literature on trade and quality. For instance, Motta and Thisse (1993) extend the vertical differentiation model to two countries with two firms to analyze the effects of quality standards in autarky and free trade. Herguera et al. (2000) study the effect of quantity restrictions in a vertically differentiated model. To the best of our knowledge, we are the first in this literature of international duopolies with vertical differentiation to investigate single-market formation.

Finally, we contribute to the literature exploring the evolution of specific traits in two different population groups by modeling the evolution of behaviors acquired by learning and not inherited (Weibull, 1995; Bowles, p. 69, 2006). In this literature, the size of the group that owns a trait is key in the probability of transmission of the trait to the other group. Notably, Lazear (1995, 1999) investigates the evolution of spoken language via interpersonal meetings. Bisin and Verdier (2001) analyze the intergenerational transmission of norms by considering meetings within and outside the family.[1] We translate this model into an information transmission model. The two population sizes define the probability of encounters as in the model by Bowles (2006), but it is not a trait that is shared and transmitted in our setting. Instead, it is information about product quality that is shared via social interactions. We add to this literature the explicit modelling of the market mechanism. In particular, the novelty here is that we introduce the dynamic process of information transmission in an international duopoly competition. By doing so, we model a dynamic process of demand evolution and solve for market outcomes.

# 3 The Model

Consider a two-country-two-good model, where country $i = 1$ produces good 1 and country $i = 2$ produces good 2. Heterogeneous consumers in each country are indexed by $\theta$ and uniformly distributed over the interval $[0, 1]$. The parameter $\theta$ captures the heterogeneous willingness of consumers to pay for the good: the higher is $\theta$, the higher the utility obtained when consuming the good.

[1] The interested reader can find an extensive coverage of these models in Bowles (2006).

At time $t = 0$, in each country, consumers are only aware of the domestic good, regardless of whether or not (s)he actually consumes the good. In this pre-agreement period, each consumer can either buy one unit of the domestic commodity or not buy anything at all. Formally, a consumer's utility is given by

$$U(\theta) = \begin{cases} \theta u_i - p_i \text{ if buying variant } i, \\ 0 \text{ if refraining from buying,} \end{cases} \quad (1)$$

where $u_i$ denotes the quality of the domestic variant and $p_i$ its market price.[2]

At period $t = 1$, the two governments decide to sign an agreement (i.e., free movement of citizens and goods, similar to the EU agreements) that opens the two countries to unrestricted citizen circulation, beyond international trade.[3] Starting from period 1, consumers have the chance to meet, in each further period, either a domestic or a foreign consumer and share his/her knowledge about the goods. We assume that these social interactions arise for various reasons (work, friendship, schooling, romantic exchanges, or simply vacations). Whatever the reason, when two consumers meet we assume that they exchange information about the goods they know about. Only then will some consumers become acquainted with *both* goods, and they acknowledge them as vertically differentiated in accordance with

$$U(\theta) = \begin{cases} \theta u_1 - p_1 \text{ if buying variant 1,} \\ \theta u_2 - p_2 \text{ if buying variant 2,} \\ 0 \text{ if refraining from buying.} \end{cases} \quad (2)$$

It follows that information about both goods spreads with frictions: not every consumer learns about the quality of the two goods immediately. Accordingly, the exchangeability of information creates two groups of consumers: *(i)* consumers who only know the domestic good, and *(ii)* consumers informed about both goods.[4]

Let $s \in (0, 1)$ denote the positive fraction of consumers living in country *one* and $(1 - s)$ that of country *two*. Accordingly, if all consumers possess unitary mass, the population of country *one* is $s$ and that of country two is $(1 - s)$. We assume, for simplicity, that both goods are made available on both markets with zero transportation costs.[5] Without loss of generality, we assume that good 1 produced by

[2] For simplicity, we assume that at $t = 0$ all consumers know the quality of their national good. Alternatively, we could assume that consumers hold an evaluation about the quality of the domestic good and learn the intrinsic qualities $u_1$ and $u_2$ only when they get informed about both goods. It can be shown that the main results of our model remain unchanged in this case. We thank an anonymous referee for pointing out this alternative setting.

[3] The reasons for this agreement are kept exogenous to the model.

[4] Information exchange can convey a piece of persuasive information to consumers in the vein of a persuasive advertisement. Through communication with foreigners, consumers may obtain information about how valuable each good is.

[5] The presence of transportation costs would obviously reduce people' incentives to circulate, while at the same time increasing the costs of firms selling their goods abroad, with—presumably—relatively minor effects on the model' results.

country *one* is of lower quality than good 2 produced by country *two*, namely $u_2 > u_1$. For simplicity, we assume zero costs of production.[6] Moreover, henceforth, we assume not only that firm quality is ranked by increasing order (i.e., $u_2 > u_1 > 0$) but also that the difference in quality is equal to $u$, with $u > 0$. This is obtained by simply setting $u_0 = 0$, $u_1 = u$, and $u_2 = 2u$. Thus, when the quality gap $u$ increases, this moves variant 1 up from zero quality and variant 2 away from variant 1. This assumption improves the readability of the paper without qualitatively altering the results.[7]

We now proceed by providing the market solution for autarky and for the scenario of full information.

## 3.1 Monopoly in Autarky

At $t = 0$, countries exist in a regime of *autarky*. Populations do not mix, and hence they only purchase the domestic good at the monopoly price. Encounters and information diffusion will start putting pressure on the two monopolies from period 1. The market is endogenously uncovered, and consumer $\theta_i^M$, who is indifferent with regard to buying or not buying the good in country $i = 1, 2$, is located at

$$\theta_i^M = \frac{p_i^M}{u_i}, i = 1, 2,$$

with $u_i \geq p_i \geq 0$.[8] The demand functions of each firm operating in autarky at the initial period $t = 0$ are then, respectively,

$$D_1^M = s\left(1 - \theta_1^M\right) \text{ and } D_2^M = (1 - s)\left(1 - \theta_2^M\right).$$

Firms maximize their profits by setting monopoly prices in each country as

$$p_1^M = \frac{u}{2} \text{ and } p_2^M = u, \tag{3}$$

covering half of their domestic markets as a result,

$$D_1^M = \frac{s}{2} \text{ and } D_2^M = \frac{1 - s}{2}, \tag{4}$$

and gaining monopoly profits equal to

$$\Pi_1^M = \frac{s \cdot u}{4} \text{ and } \Pi_2^M = \frac{(1 - s) \cdot u}{2}. \tag{5}$$

These expressions are useful to analyze the effects of meetings on firm profits in what follows.

[6]Introducing a production cost dependent on product quality would make the analysis more cumbersome without improving the model' intuitions.

[7]This is a rather common simplifying assumption (see, for instance, Gabszewicz and Thisse, 1980 and Gabszewicz et al., 2016). The authors can provide to interested readers the solution of the model in the absence of this assumption.

[8]The market is endogenously uncovered because consumers with zero or low willingness to pay have no incentive to buy a good at a positive price. An uncovered market turns out to be more general than a covered one since in the former the market can expand or shrink in response to prices.

## 3.2 Duopoly Under Full Information

After the international agreement, if everyone living in country $i$ *would instantaneously meet everyone* from country $j$, then all consumers would immediately become fully informed about the quality of the two goods. Hence, the marginal consumer $\theta_2(p_1, p_2)$ choosing between good 1 and good 2 and the marginal consumer $\theta_1(p_1)$ choosing between consuming good 1 or refraining from consuming at all are given by

$$\theta_2(p_1, p_2) = \frac{p_2 - p_1}{u} \text{ and } \theta_1(p_1) = \frac{p_1}{u}, \tag{6}$$

for the range of prices for which $1 > \theta_2 > \theta_1 \geq 0$, which guarantees that both firms are active in equilibrium. Then, with *perfectly informed consumers*, the demand functions for goods one $D_1(p_1, p_2)$ and two $D_2(p_1, p_2)$ are, respectively,

$$D_1(p_1, p_2) = \frac{p_2 - p_1}{u} - \frac{p_1}{u} \quad \text{and} \quad D_2(p_1, p_2) = 1 - \frac{p_2 - p_1}{u},$$

yielding, in turn, the following equilibrium prices:

$$p_1^* = \frac{u}{7} \quad \text{and} \quad p_2^* = \frac{4u}{7}, \tag{7}$$

where $p_2^* > p_1^* > 0$.[9] Equilibrium demands are

$$D_1^* = \frac{2}{7} \quad \text{and} \quad D_2^* = \frac{4}{7}, \tag{8}$$

and profits are

$$\Pi_1^* = \frac{2u}{49} \quad \text{and} \quad \Pi_2^* = \frac{16u}{49}. \tag{9}$$

Comparing the market solution under perfectly informed consumers to that under autarky, we note that full information among consumers always decreases equilibrium prices: $p_1^M - p_1^* > 0$ and $p_2^M - p_2^* > 0$. In contrast, the equilibrium demand of the country producing the low-quality good may be smaller or larger depending on the population size $s$, namely, $D_1^M - D_1^* \gtreqless 0$ for $s \gtreqless 4/7$. However, full information always decreases the equilibrium demand of the high-quality good, i.e., $D_2^M - D_2^* < 0$ for every $s \in (0, 1)$. Finally, profits can either decrease or increase after opening to trade as an effect of $s$, since $\Pi_1^M - \Pi_1^* \gtreqless 0$ for $s \gtreqless 8/49$ and $\Pi_2^M - \Pi_2^* \gtreqless 0$ for $(1-s) \gtreqless 16/49$. As expected, prices at the duopoly solution are lower than for autarky. However, it is unclear whether the social interactions of consumers improve the profit of firms, since this depends on the size of countries.

We have now elucidated the two "extreme" market solutions: autarky and a full information duopoly. However, since not *all consumers* meet at once, when the two countries open their markets, at every

[9] It is readily verifiable that at the Nash equilibrium prices $(p_1^*, p_2^*)$, the condition $1 > \theta_2^* > \theta_1^* \geq 0$ holds.

period there may exist consumers with different levels of knowledge about the two goods. In the next section, we analyze how the demands in each country evolve over time with the spread of information about product quality.

# 4 Market Under Informational Frictions

We now describe a vertically differentiated model with informational frictions. Social interactions bring exchanges and meetings that create two types of consumers according to the information they hold, namely *uninformed* and *informed* consumers:

**Definition 1.** *Uninformed consumers are only aware of the domestic good and ignore the quality of the foreign good.*

Uninformed consumers populate both countries. In country one, the set $\mathcal{U}_1$ of uninformed consumers, whose mass is denoted by $U_1$, are uninformed about good 2. Similarly, in country two, the set $\mathcal{U}_2$ of uninformed consumers of mass $U_2$ are uninformed about good 1.

**Definition 2.** *Informed consumers become acquainted with the quality of the foreign good by socially interacting with a foreign consumer or a domestic one who has already met a foreign consumer.*

Informed consumers also exist in both countries. This group is denoted by $\mathcal{I}$, and their total mass by $I$. Since the world' population is normalized to one, it directly follows that $I+U_1+U_2 = 1$. Importantly, note that the size of these market segments changes over time as information about goods diffuses. Social interactions among consumers of the two countries bring two important consequences: *(i)* the two goods become available in both countries, with negligible trade costs; *(ii)* informed consumers originating from social interactions modify the demand functions of each firm.

In the next section, we specify a process of information diffusion to illustrate in more detail how the evolution of consumers' information affects the market equilibrium.

## 4.1 Information Transmission Over Time

Formally, the process of information transmission among consumers is inspired by evolutionary game theory settings (Weibull, 1995; Bowles, 2006). In particular, information diffuses in meetings in a random encounter model. Mutual experiences of consumption habits are exchanged progressively and according to the number and frequency of social meetings. This implies that information about the goods does not spread automatically to all consumers. In each period, each consumer randomly meets one other consumer, who can either be a foreign or a domestic consumer. Given the fraction of consumers $s$ and $(1-s)$ in each country $C_i$ $(i = 1, 2)$, the probability that a consumer from country one meets another domestic consumer at period 1, and thus remains uninformed about the quality of the other good, is simply given by

$$\Pr\{(i \in C_1) \cap (j \in C_1)\}_{t=1} = s^2.$$

Similarly, the probability that an individual from country two meets a domestic consumer, thus remaining uninformed about country one's good, is given by

$$\Pr\{(i \in C_2) \cap (j \in C_2)\}_{t=1} = (1-s)^2.$$

Thus, the probability that the consumers of the two countries become *informed* in period 1 is simply given by

$$\Pr\{(i \in C_1) \cap (j \in C_2)\}_{t=1} = 1 - s^2 - (1-s)^2 = 2s(1-s).$$

A similar knowledge-transmission process occurs in all subsequent periods $t \in \mathbb{N}$, with one new feature. The informed domestic consumers are now ambassadors for the foreign good in the domestic market. Accordingly, from period 2 onwards, in the domestic market information about the foreign good is transmitted by foreign consumers and informed domestic inhabitants. In what follows, we analyze how the sets of informed and uninformed consumers in each country evolve over time. The population dynamics of these two subsets defines the demand for each good.

In particular, we can denote the set of *uninformed* consumers living in country $i$ at time $t \in \mathbb{R}$ as $\mathcal{U}_i(t)$ and its mass as $U_i(t)$ for $i = 1, 2$. Similarly, the set of consumers *becoming informed* (necessarily about both goods) at every time $t$ is $\mathcal{I}(t)$, and its mass $I(t)$. It is easy to see that the mass of consumers uninformed about good *two* (located in country *one*) progresses geometrically as follows:

$$\begin{aligned}
U_1(0) &= s; \\
U_1(1) &= \Pr\{(i \in \mathcal{U}_1(0)) \cap (j \in \mathcal{U}_1(0))\} = s \cdot s = s^2; \\
U_1(2) &= \Pr\{(i \in \mathcal{U}_1(1)) \cap (j \in \mathcal{U}_1(1))\} = s^2 \cdot s^2 = s^4; \\
U_1(3) &= \Pr\{(i \in \mathcal{U}_1(2)) \cap (j \in \mathcal{U}_1(2))\} = s^4 \cdot s^4 = s^8; \\
&\quad \ldots\ldots\ldots\ldots \\
U_1(t) &= \Pr\{(i \in \mathcal{U}_1(t-1)) \cap (j \in \mathcal{U}_1(t-1))\} = s^{2^{(t-1)}} \cdot s^{2^{(t-1)}} = s^{2^t}. \qquad (10)
\end{aligned}$$

Thus, the greater the size of country one (and the smaller that of country two), the larger will be the number of periods needed for all people uninformed about good *two* to become informed, that is, for $U_1(t)$ to approach zero. Analogously, the group of consumers uninformed about good *one* progresses geometrically as follows:

$$
\begin{aligned}
U_2(0) &= (1-s)\,; \\
U_2(1) &= \Pr\{(i \in \mathcal{U}_2(0)) \cap (j \in \mathcal{U}_2(0))\} = (1-s)\cdot(1-s) = (1-s)^2\,; \\
U_2(2) &= \Pr\{(i \in \mathcal{U}_2(1)) \cap (j \in \mathcal{U}_2(1))\} = (1-s)^2\cdot(1-s)^2 = (1-s)^4\,; \\
U_2(3) &= \Pr\{(i \in \mathcal{U}_2(2)) \cap (j \in \mathcal{U}_2(2))\} = (1-s)^4\cdot(1-s)^4 = (1-s)^8\,; \\
&\quad \ldots\ldots\ldots\ldots \\
U_2(t) &= \Pr\{(i \in \mathcal{U}_2(t-1)) \cap (j \in \mathcal{U}_2(t-1))\} = (1-s)^{2^{(t-1)}}\cdot(1-s)^{2^{(t-1)}} = (1-s)^{2^t}\,. \qquad (11)
\end{aligned}
$$

As before, the greater the size of country two (and the smaller that of country one), the slower the decrease in people uninformed about good *one* will be over time. Note that this process of information transmission is invariant to product quality. In particular, the speed of information transmission for the high- and low-quality good only depends on the relative size of the two countries, $s$ and $1-s$. This information-diffusion model is suitable to capture the role of the size asymmetry between populations that meet in a tractable and elegant way. This explains its use in evolutionary game theory (Weibull, 1995). The model remains robust to different speeds of travel of the information about one good or the other. In fact, one can imagine that information either about the high-quality or the low-quality good can travel at a different speed. However, it can be easily shown that these modifications would only change the speed of the information diffusion, leaving the main findings of the model substantially unchanged.

Using the dynamics of the above functions $U_1(t)$ and $U_2(t)$, it immediately follows that the dynamics of the mass of consumers informed about both goods in every period $t$ is simply given by

$$I(t) = 1 - U_1(t) - U_2(t) = 1 - s^{2^t} - (1-s)^{2^t}\,. \qquad (12)$$

Thus, what matters for the diffusion of information and the progression of the mass of informed agents $I(t)$ over time is that the size of the populations in the two countries is not too *asymmetric*. Figure 1 plots the dynamics of the mass of informed agents for different sizes of the two countries and for $t = 0, 1, 2, ..., 10$.

It can be noted that when the two countries are of exactly the same size (*dashed line,* $s = 0.5$), the mass of informed people grows faster than in asymmetric cases (*dotted line*, $s = 0.2$ or $s = 0.8$, *continuous line*, $s = 0.01$), covering a large portion of the total population of the two countries in fewer periods.

Over time, the mass of informed agents converges asymptotically to 1, whereas the two sets of uninformed consumers disappear over time. It is also important to keep in mind that the information-diffusion process takes into account information transmission from all informed consumers, regardless

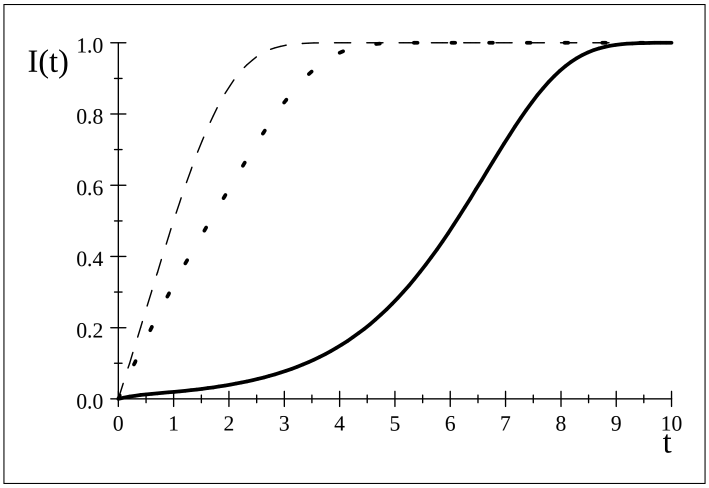


Figure 1: Dynamics of $I(t)$.

of their country of origin. In particular, starting from period $t = 1$ a consumer in country $i$, $i = 1, 2$, can learn about product $j \neq i$ either if she meets a consumer from country $j$ or if she meets an informed consumer from her own country. Even more importantly, note that in our setting firms' prices do not affect information transmission about the quality of goods, and therefore, every firm's maximization process, which is inherently intertemporal, can be treated as a much simpler single-period maximization program.[10]

In view of the above, we can now express the firms' demand functions, profits, and the best replies for each firm in the presence of informational frictions. At first, for readability, we focus on identifying the market solution for any arbitrary partition of consumers into *uninformed* (about goods 2 and 1) and *informed*, i.e., any coalition structure $(\mathcal{U}_1, \mathcal{U}_2, \mathcal{I})$ such that $\mathcal{U}_1 \cap \mathcal{U}_2 \cap \mathcal{I} = \varnothing$ and $|\mathcal{U}_1| \cup |\mathcal{U}_2| \cup |\mathcal{I}| = I + U_1 + U_2 = 1$. In Section 5, we use the time-variant mass of uninformed $U_i(t)$ and informed people $I(t)$, as defined in equations (10) and (11), to define every per-period duopoly solution.

### 4.1.1 High-Quality Firm Under Informational Frictions

We start by spelling out the demand, $D_2(p_1, p_2)$, and profit function, $\Pi_2(p_1, p_2)$, of the high-quality firm. For convenience, we can distinguish two cases: $p_1 > u$ and $p_1 \leq u$. Within the price range where $p_1 > u$, firm 1 is inactive, whereas 2 can sell its product to all consumers informed about its good, $U_2 + I$, experiencing the following demand:

$$D_2(p_1, p_2) = \begin{cases} (U_2 + I)(1 - \frac{p_2}{2u}) & \text{if} \quad p_2 \leq 2u, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

On the other hand, in the price range where firm 1 selects a price $p_1$ with $p_1 \leq u$, firm 2's demand is:

[10] We assume, without loss of generality, firms having a common discount factor, $\delta = 1$.

$$D_2(p_1,p_2) = \begin{cases} (U_2+I)(1-\frac{p_2}{2u}) & \text{if} \quad 0 \le p_2 \le 2p_1; \\ U_2(1-\frac{p_2}{2u}) + I(1-\frac{p_2-p_1}{u}) & \text{if} \quad 2p_1 < p_2 \le p_1+u; \\ U_2(1-\frac{p_2}{2u}) & \text{if} \quad p_1+u < p_2 < 2u; \\ 0 & \text{if} \quad p_2 \ge 2u. \end{cases} \tag{14}$$

Note from (14) that when the price of the high-quality firm is sufficiently low, which occurs for $p_2 \le 2p_1 \Leftrightarrow \theta_2(p_1,p_2) \le \theta_1(p_1,p_2)$, informed consumers refrain from buying good 1 and firm 2 can sell its good to both uninformed (*captive*) and informed (*non-captive*) consumers. On the other hand, when the prices charged for the two goods are such that $1 > \theta_2(p_1,p_2) \ge \theta_1(p_1,p_2) \ge 0 \Leftrightarrow 2p_1 < p_2 \le p_1+u$, the two groups of uninformed consumers buy only the respective domestic good whereas informed ones are partitioned between the two goods in accordance with their willingness to pay. When $p_2$ exceeds $p_1+u$, only uninformed consumers $U_2$ buy good 2. Finally, if the price of good 2 is so high as to in turn make $\theta_2(p_1,p_2) < \theta_1(p_1,p_2) \Leftrightarrow p_2 \ge 2u$, all consumers refrain from buying good 2. Both the demand and the profit of firm 2 display three kinks: the first at $p_2 = 2p_1$, the second at $p_2 = p_1+u$, and the third at $p_2 = 2u$.

To construct the best reply of firm 2, we can identify the price $p_2$ that maximizes its profit across the entire price range . To this aim, we compare firm 2's profit function in each portion of the piece-wise function. This leads us to firm 2's (again piece-wise) best reply, as shown below.[11]

$$p_2(p_1) = \begin{cases} u & \text{if} \quad p_1 > \dfrac{u}{2}; \\ 2p_1 & \text{if} \quad \frac{u(I+U_2)}{3I+2U_2} \le p_1 \le \dfrac{u}{2}; \\ \frac{Ip_1+u(I+U_2)}{2I+U_2} & \text{if} \quad \frac{u(I+U_2)}{3I+2U_2} > p_1. \end{cases} \tag{15}$$

Note that the best response of firm 2 to the monopoly pricing of firm 1, $p_1 \ge u/2$, is to play the monopoly pricing $p_2 = u$. For a lower but still sufficiently high range of prices of the rival $p_1 \in [u\,(I+U_2)\,/\,(3I+2U_2)\,, u/2]$ firm 2's best reply consists of setting a price that excludes the rival from the market of informed consumers, namely $p_2 = 2p_1$, causing $\theta_2(p_1,p_2) - \theta_1(p_1,p_2) = 0$. For a lower price of the rival, firm 2 competes by charging a duopoly price. This yields firm 2's best reply, as depicted in Figure 4 below.

### 4.1.2 Low-Quality Firm Under Informational Frictions

Repeating the same procedure for firm 1, we obtain that for $p_2 > 2u$, the demand function of firm 1 boils down to

$$D_1(p_1,p_2) = \begin{cases} (U_1+I)(1-\frac{p_1}{u}) & \text{if} \quad p_1 \le u, \\ 0 & \text{otherwise.} \end{cases}$$

[11] The detailed calculations are illustrated in Appendix A.1.

On the other hand, for $p_2 \leq 2u$, firm 1's demand function is

$$D_1(p_1,p_2) = \begin{cases} (U_1+I)(1-\frac{p_1}{u}) & \text{if} \quad 0 < p_1 \leq p_2 - u; \\ U_1(1-\frac{p_1}{u}) + I(\frac{p_2-p_1}{u} - \frac{p_1}{u}) & \text{if} \quad p_2 - u < p_1 \leq \frac{1}{2}p_2; \\ U_1(1-\frac{p_1}{u}) & \text{if} \quad \frac{1}{2}p_2 < p_1 < u; \\ 0 & \text{if} \quad p_1 \geq u. \end{cases} \tag{16}$$

Demand $D_1(p_1,p_2)$ and profit function $\Pi_1(p_1,p_2) \equiv p_1 D_1(p_1,p_2)$ again are step-wise functions.

A comparison of the profit obtained by firm 1 for each price interval of firm 2 can be used to characterize firm 1's best reply (for details, see Appendix A.1):

$$p_1(p_2) = \begin{cases} \frac{1}{2}u & \text{if} \quad p_2 < \widetilde{p}_2; \\ \frac{uU_1+Ip_2}{4I+2U_1} & \text{if} \quad \widetilde{p}_2 \leq p_2 \leq 2u; \\ \frac{1}{2}u & \text{if} \quad p_2 > 2u, \end{cases} \tag{17}$$

where

$$\widetilde{p}_2 = \frac{2u(2I+U_1)}{I}\sqrt{\frac{U_1}{8I+4U_1}} - \frac{uU_1}{I} \tag{18}$$

denotes the firm 2 price that equalizes the profit of firm 1 under duopoly and under monopoly pricing (i.e., $\Pi_1|_{p_1=p_1(p_2)} - \Pi_1|_{p_1=u/2} = 0$), as detailed in Appendix A.1.

Accordingly, the best response of firm 1 to a very high pricing of firm 2 that makes firm 2 inactive, i.e., $p_2 > 2u$, is to play the monopoly pricing $p_1 = u/2$. For a lower range of the rival's price $p_2 \in [\widetilde{p}_2, 2u]$, firm 1's best reply consists of duopoly pricing, as firm 2 is now active in the market. Finally, for an even lower price of firm 2, $p_2 < \widetilde{p}_2$, firm 1 can no longer be competitive in the market of informed consumers. As a consequence, it charges, again, a monopoly price $p_1 = u/2$ to uninformed domestic consumers. This yields firm 1's best reply, as depicted in Figure 4 below.

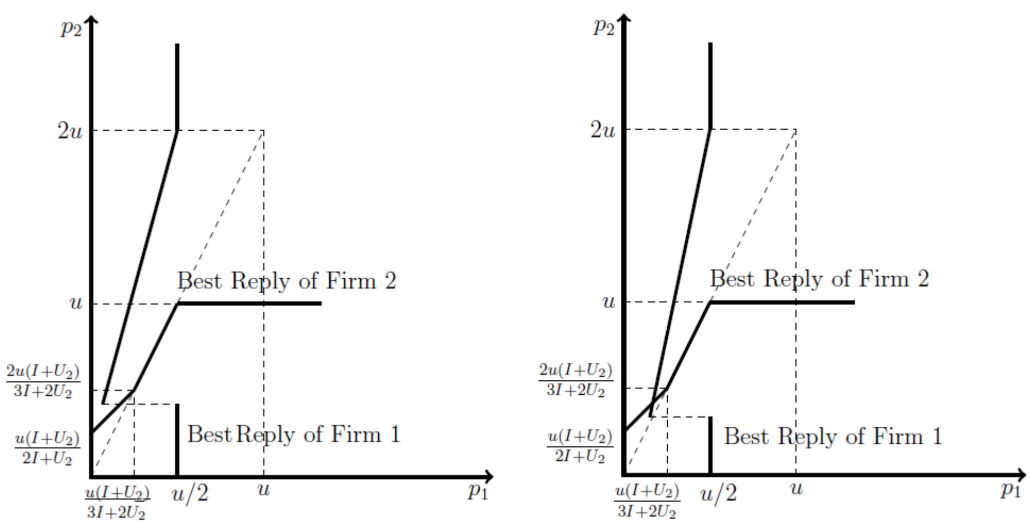


Figure 2: Firms' best replies when a Nash equilibrium exists (right) and does not exist (left)

To summarize, there is a fundamental asymmetry between the best responses of the two firms. For very low levels of $p_2$, it is optimal for the low-quality firm to set its price above the rival firm's price: the high-quality firm charges a price so low that it is not worth it for firm 1 to reduce $p_1$ to serve informed consumers. Serving only uninformed consumers is the optimal choice. However, as $p_2$ continues to rise, it eventually becomes sufficiently high for firm 1 to increase its profit by a discontinuous price reduction to serve informed consumers, despite the decrease in profitability from sales to its captive consumers—the uninformed ones. As the price of the high-quality commodity increases above $2u$, firm 2 is out of the market and it becomes optimal for firm 1 to hike up its price and charge the monopoly price. In contrast, the best response of the high-quality firm is continuous regardless of the level of $p_1$. Due to its intrinsic quality advantage, firm 2 has no reason to discontinuously cut or hike up its price in response to the changes in $p_1$.

## 4.2 Existence of a Pure Strategy Nash Equilibrium

Having identified the best responses of both firms, we can now investigate the existence of a pure strategy Nash equilibrium. Figure 4 depicts the two firms' best replies in the price space $(p_1, p_2)$. A discontinuity of firm 1's best reply exists in correspondence to $\widetilde{p}_2$, since for this price firm 1 is indifferent, profit-wise, between playing as a duopolist and charging its monopoly price when selling its product only to its domestic consumers. Such a discontinuity may jeopardize the existence of the duopoly equilibrium, as illustrated in the left panel of Figure 4. In fact, a Nash equilibrium exists if and only if the best replies corresponding to the sets $0 < p_1 < u\,(I + U_2)\,/\,(3I + 2U_2)$ for firm 2 and $[Ip_1 + u\,(I + U_2)]\,/\,(2I + U_2) \leq p_2 \leq 2u$ for firm 1 intersect. This intersection can only occur when both firms are charging duopoly prices, as shown in Appendix A.1. For this price range, the firms' market demand functions $D_1(p_1, p_2, U_1, I)$ and $D_2(p_1, p_2, U_2, I)$ are given by

$$D_1(p_1, p_2, U_1, I) = U_1\left(1 - \frac{p_1}{u}\right) + I\left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right), \tag{19}$$

$$D_2(p_1, p_2, U_2, I) = U_2\left(1 - \frac{p_2}{2u}\right) + I\left(1 - \frac{p_2 - p_1}{u}\right), \tag{20}$$

highlighting the fact that uninformed consumers buy the domestic good whereas the informed ones are portioned between buying one good or the other, in accordance with their willingness to pay. Every firm in country $i = 1, 2$ sets it price to maximize its profit $\Pi_1(p_1, p_2, U_1, I)$ and $\Pi_2(p_1, p_2, U_2, I)$:

$$\begin{aligned}
\Pi_1(p_1, p_2, U_1, I) &= p_1 \cdot U_1\left(1 - \frac{p_1}{u}\right) + p_1 \cdot I\left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right), \\
\Pi_2(p_1, p_2, U_2, I) &= p_2 \cdot U_2\left(1 - \frac{p_2}{2u}\right) + p_2 \cdot I\left(1 - \frac{p_2 - p_1}{u}\right),
\end{aligned} \tag{21}$$

yielding the following candidate Nash equilibrium prices as a function of the consumer information partition:

$$\begin{aligned} p_1^*(U_1, U_2, I) &= u \cdot \frac{(I + U_2 + 2U_1) I + U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2}, \\ p_2^*(U_1, U_2, I) &= u \cdot \frac{(4I + 4U_2 + 3U_1) I + 2U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2}. \end{aligned} \tag{22}$$

Thus, substituting these prices in (6) we obtain $\theta_1^*(U_1, U_2, I)$ and $\theta_2^*(U_1, U_2, I)$, as well as the equilibrium demands.

$$\begin{aligned} D_1^*(U_1, U_2, I) &= (2I + U_1) \frac{(I + U_2 + 2U_1) I + U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2}, \\ D_2^*(U_1, U_2, I) &= \frac{1}{2} (2I + U_2) \frac{(4I + 4U_2 + 3U_1) I + 2U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2}. \end{aligned}$$

Note that two *necessary conditions* for the existence of a pure strategy Nash duopoly equilibrium with informational frictions are required. The first is

$$p_2^*(U_1, U_2, I) > p_1^*(U_1, U_2, I) > 0,$$

which by (22) holds as $U_1, U_2 \in (0, 1)$, and $I = 1 - U_1 - U_2$, and the second is

$$1 > \theta_2^*(U_1, U_2, I) > \theta_1^*(U_1, U_2, I) \geq 0,$$

which can be written as

$$1 > \frac{p_2^*(U_1, U_2, I) - p_1^*(U_1, U_2, I)}{u} > \frac{p_1^*(U_1, U_2, I)}{u} \geq 0. \tag{23}$$

Some calculations show that while the first and last inequalities in (23) hold for $U_1, U_2 \in (0, 1)$, and $I = 1 - U_1 - U_2$, the intermediate condition—required for good 1 to remain on sale in the duopoly market—only holds for $U_1 < 2/3$. However, as shown in Theorem 1 below, a more stringent *sufficient condition* on the mass of uninformed consumers $U_1$ is required for a pure strategy Nash duopoly equilibrium with informational frictions to exist, for any size of $U_2$.

**Theorem 1** *A sufficient condition for a pure strategy Nash duopoly equilibrium with informational frictions to exist is that the mass of initial uninformed consumers in country one is not too large:* $U_1 < \widetilde{U}_1 \equiv (29 - 5\sqrt{17})/26 \simeq 1/3$.

**Proof.** See Appendix A.2. ■

As detailed in the proof of Theorem 1 (contained in Appendix A.2), the discontinuity of firm 1's best response destroys the duopoly equilibrium when the share of consumers uninformed about the high-quality good (living in country one) is sufficiently high. Thus, firm 1—the low-quality producer—can rely on a very large captive share of domestic consumers, which gives it high market power and strong incentives to deviate from duopoly pricing. The optimal strategy is to charge a high price in the domestic market of consumers uninformed about good 2 . Selecting a lower price (which corresponds to the duopoly solution) is not an optimal strategy because what the firm loses in the domestic market by charging a duopoly price is not compensated by what it gains selling to informed consumers. In contrast, a condition on the share of consumers uninformed regarding the high-quality good makes the duopoly price an optimal strategy, therefore guaranteeing the existence of a duopoly equilibrium with frictions, as shown in Theorem 1.

The above theorem establishes the conditions for which a pure strategy Nash equilibrium does not exist: when the proportion of population living in country *one* is large and, thus, there is a broad mass of uninformed individuals about the high-quality good, no pure strategy Nash equilibrium exists and firms get trapped in price instability. If the high-quality firm charges a duopoly price, the low-quality one finds it profitable to hike its price up to the monopoly price $u/2$, as dictated by its (discontinuous) best reply. If this occurs, firm 2 prefers to switch to its monopoly price $u$. Nevertheless, such a price combination cannot be a Nash equilibrium either, since firm 1 now has an incentive to set a lower price in accordance with its best response, to which firm 2 finds it optimal to respond as a duopolist. However, the price-cycle now starts again since firm 1 has an incentive to play as a monopolist to patronize its large set of *uninformed* consumers. As illustrated in Appendix A.3, in the domain where a pure-strategy equilibrium fails to exist, a mixed equilibrium is in place. This can be characterized as a cumulative probability distribution on the support of rival firm's strategies, which makes every firm indifferent to employing any of its pure strategies.The interpretation of this mixed equilibrium is that if the mass of captive consumers of firm 1 is sufficiently high, the prices of the two firms are intrinsically unstable and fluctuate between the the duopoly amd monopoly levels.[12] In particular, firm prices dispersion occurs within the set

$$P_1 \times P_2 = \left[\underline{p}_1, \frac{u}{2}\right] \times \left[\widetilde{p}_2, u\right],$$

where $\underline{p}_1 = \frac{u}{I}\left(\left(\left(I+U_2\right)/\left(2I+U_2\right)\right)^{1/2}\left(2I+U_2\right)-\left(I+U_2\right)\right)$. The reason for this range is that no firm has an incentive to charge a price higher than the monopoly price and lower than a price guaranteeing the

[12] This situation is similar to the price cycle in Edgeworth's (1925) model (see, for instance, Levithan and Shubik, 1972 and Vives, 1986), where prices fluctuate between their monopoly and duopoly levels. In our setting, this is caused by consumers' incomplete information instead of (as in Edgeworth) firm capacity constraints.

same profit when playing its monopoly and its duopoly price against the rival's best-reply (see Appendix A.3 for details). However, as $U_1(t)$ decreases over time, the process of information transmission makes the described price instability a transitory phenomenon and the existence of a pure strategy Nash equilibrium profile (22) is soon re-established. This is formally proved in Proposition 3 below.

# 5 Dynamic Duopoly Under Informational Frictions

Using the information-diffusion process introduced above, we are now able to express the demand functions of firms in (19) and (20) at every period $t \in \mathbb{N}$ by simply using the time-variant mass of uninformed $U_i(t)$ and informed people $I(t)$ given in (10) and (11). Consequently, at the duopoly equilibrium with informational frictions, where $U_1(t) = s^{2^t}, U_2(t) = (1-s)^{2^t}$ and $I(t) = 1 - s^{2^t} - (1-s)^{2^t}$, the demand function for the two goods at period $t$ can be written as

$$D_1(p_1(t), p_2(t), s) = s^{2^t}\left(1 - \frac{p_1(t)}{u}\right) + \left(1 - s^{2^t} - (1-s)^{2^t}\right)\left(\frac{p_2(t) - p_1(t)}{u} - \frac{p_1(t)}{u}\right), \tag{24}$$

$$D_2(p_1(t), p_2(t), s) = (1-s)^{2^t}\left(1 - \frac{p_2(t)}{2u}\right) + \left(1 - s^{2^t} - (1-s)^{2^t}\right)\left(1 - \frac{p_2(t) - p_1(t)}{u}\right). \tag{25}$$

The per-period profit functions of firms are, therefore,

$$\Pi_1(p_1(t), p_2(t), s) = p_1(t) \cdot D_1(p_1(t), p_2(t), s),$$

$$\Pi_2(p_1(t), p_2(t), s) = p_2(t) \cdot D_2(p_1(t), p_2(t), s).$$

As mentioned above, the intertemporal maximization of the profit stream can be treated as per-period profit maximization, as profit at period $t$ uniquely depends on the prices in the same period, since firms cannot strategically influence information diffusion via prices. It follows that profit maximization in what we denote as a duopoly with informational frictions yields the following equilibrium prices $p_1^*(t)$ and $p_2^*(t)$, evaluated according to the specific information dynamics presented above:

$$\begin{aligned} p_1^*(t) &= u \cdot \frac{1 - (1-s)^{2^t} - s^{2^{t+1}}}{7 - 10(1-s)^{2^t} - 10s^{2^t} + 8s^{2^t}(1-s)^{2^t} + 3(1-s)^{2^{t+1}} + 3s^{2^{t+1}}}, \\ p_2^*(t) &= u \cdot \frac{4 - 4(1-s)^{2^t} - 5s^{2^t} + s^{2^{t+1}} + 3s^{2^t}(1-s)^{2^t}}{7 - 10(1-s)^{2^t} - 10s^{2^t} + 8s^{2^t}(1-s)^{2^t} + 3(1-s)^{2^{t+1}} + 3s^{2^{t+1}}}, \end{aligned} \tag{26}$$

where it is easy to check that $p_2^*(t) > p_1^*(t)$. The marginal consumers (6) evaluated at the equilibrium prices (26) are, therefore, located at

$$\theta_2^*(t) = \frac{3 - 3(1-s)^{2^t} - 5s^{2^t} + 3s^{2^t}(1-s)^{2^t} + 2s^{2^{t+1}}}{7 - 10(1-s)^{2^t} - 10s^{2^t} + 8s^{2^t}(1-s)^{2^t} + 3(1-s)^{2^{t+1}} + 3s^{2^{t+1}}} < 1$$

and

$$\theta_1^*(t) = \frac{1-(1-s)^{2^t} - s^{2^{t+1}}}{7-10(1-s)^{2^t} - 10s^{2^t} + 8s^{2^t}(1-s)^{2^t} + 3(1-s)^{2^{t+1}} + 3s^{2^{t+1}}}.$$

Accordingly, firm demand at a duopoly equilibrium with frictions is

$$\begin{aligned} D_1(t) &= \frac{\left(2-2(1-s)^{2^t} - s^{2^t}\right)\left(1-(1-s)^{2^t} - s^{2^{t+1}}\right)}{7+3\left((1-s)^{2^{t+1}} + s^{2^{t+1}}\right) + 8(1-s)^{2^t} s^{2^t} - 10\left((1-s)^{2^t} + s^{2^t}\right)}, \\ D_2(t) &= \frac{1}{2}\frac{\left(4-4(1-s)^{2^t} - 5s^{2^t} + s^{2^{t+1}} + 3s^{2^t}(1-s)^{2^t}\right)\left(2-(1-s)^{2^t} - 2s^{2^t}\right)}{7+3\left((1-s)^{2^{t+1}} + s^{2^{t+1}}\right) + 8s^{2^t}(1-s)^{2^t} - 10\left((1-s)^{2^t} + s^{2^t}\right)}, \end{aligned} \tag{27}$$

where it can be checked that the sufficient conditions for a Nash equilibrium described in Theorem 1 are guaranteed if, at a given period $t$

$$U_1(t) \equiv s^{2^t} < \overline{s}(t) \equiv \left(29-5\sqrt{17}\right)^{\frac{1}{2^t}} \cdot 26^{-\frac{1}{2^t}} \in (0,1). \tag{28}$$

Note that $\overline{s}(t)$ is monotonically increasing in $t$. Therefore, if the constraint (28) holds at the initial period of meetings $t = 1$a fortiori, at any other future period $t > 1$. Hence, using the monotonicity of the threshold $\overline{s}(t)$ obtained in (28), we can state the following.

**Proposition 2** *If the mass of uninformed consumers $U_1(t)$ in country one when meetings start is not too large—namely, $s^2 \in (0, \overline{s}(1))$—then at every subsequent period $t$, there exists a unique duopoly equilibrium with informational frictions with prices given by (26).*

**Proof.** See Appendix A.3. ■

The foregone proposition reveals that the international duopoly equilibrium in pure strategies exists in every period $t$ if the size of the population in the country that produces the low-quality good is not too large. However, when country one is large and country two small, openness does not translate immediately into a relatively quick diffusion of the information about the goods and, therefore, competition does not constrain the behavior of the two initial monopolies. Indeed, consumers living in the large country who are uninformed about the high-quality good will learn slowly about it. Therefore, if population of country *one* is sufficiently high, firm 1 may find it profitable to deviate from the duopoly prices and price instability occurs, as shown in Appendices A.1 and A.3.

We have now fully elucidated the market solution that can arise at each period $t$. One can naturally ask, how does the market solution evolve? Is there a convergence towards the full information duopoly?

Are there reversals? Can we see a duopoly solution reverse into a situation without a Nash equilibrium or an international duopoly with frictions, or vice versa? Consider now the range of parameters when a Nash equilibrium does not exist. This nonexistence of the equilibrium cannot persist over time. Indeed, we prove that

**Proposition 3** *The non-existence of a duopoly equilibrium with informational frictions may only last for a finite number of periods.*

**Proof.** See Appendix A.4. ■

To summarize, our model reveals that knowledge transmission via social interactions between populations in the two countries ultimately plays a balancing role in the market. As time goes by and the mass of informed consumers in both countries increases, the number of consumers living in the large country who are only purchasing the domestic good progressively shrinks, thus driving the price gap (that would occur under a duopoly) once again within a reasonable range. Hence, as information diffuses this puts pressure on the firms, which at a certain period are "obliged" to implement duopoly pricing. From that period onwards, a duopoly market equilibrium arises.

## 5.1 Equilibrium Properties

Finally, we investigate some general properties of a duopoly equilibrium with frictions.

### 5.1.1 Prices and Demand

The next two propositions add more findings regarding the effect of information on equilibrium prices and demand in the international duopoly with frictions.

**Proposition 4** *In a duopoly equilibrium with informational frictions, prices monotonically increase with the mass of uninformed consumers in both countries, whereas they decrease with the mass of informed ones.*

**Proof.** See Appendix A.5. ■

The rationale behind the above result clearly relates to the fiercer competition caused by the presence of a large share of consumers informed about the rival good, which creates downward pressure on prices. Accordingly, in a dynamic setting where the mass of uninformed consumers in both countries decreases with time whereas the mass of informed ones increases, the effect of information spreading on the equilibrium prices is as follows:

*In a duopoly equilibrium with informational frictions, prices monotonically decrease with time and converge asymptotically to the duopoly prices with full information.*

**Proof.** See Appendix A.5. ■

This result is illustrated in Figure 5, where we show the dynamics of equilibrium prices $p_1^*(t)$ and $p_2^*(t)$, respectively, for $s = 0.55$ (dashed) and $s = 0.2$ (continuous) for $t = 1, 2, ..., 10$. Prices converge asymptotically to the full information prices $p_1^* = 1/7$ and $p_2^* = 4/7$ (green lines).

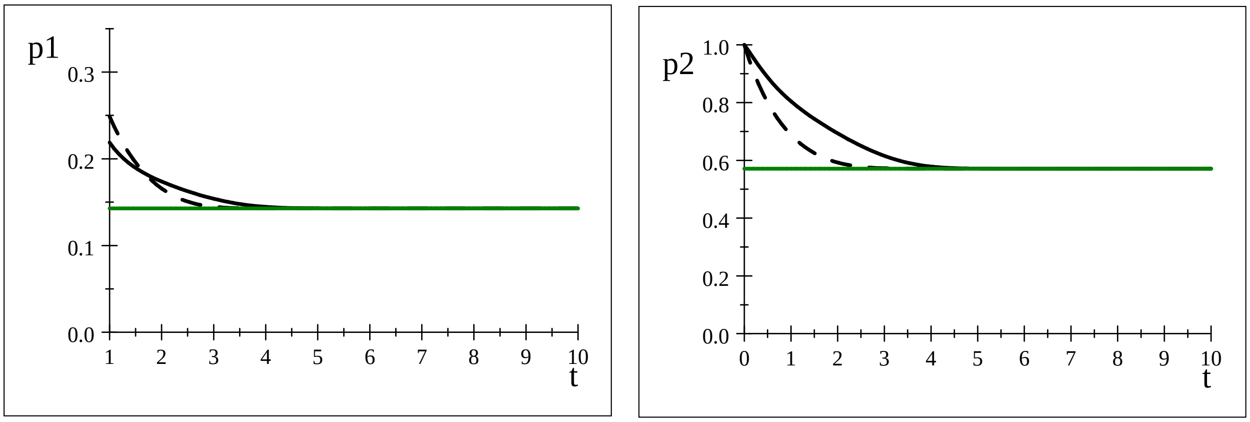


Figure 5: The dynamic evolution of equilibrium prices $p_1(t)$ (left graph) and $p_2(t)$ (right graph)

For the corresponding equilibrium demand, we obtain the following.

**Proposition 5** *In a duopoly equilibrium with informational frictions, demand for the high-quality good always increases with the mass of informed consumers. In contrast, the impact of the mass of informed consumers on the equilibrium demand of the low-quality good is ambiguous and depends on the relative sizes of the countries.*

**Proof.** See Appendix A.6. ■

These findings are illustrated in Figure 6, respectively, for $s = 0.55$ (dashed) and $s = 0.2$ (continuous) for $t = 1, 2, ..., 10$. Demand converges to the full information demand, $D_1^* = 2/7$ and $D_2^* = 4/7$ (green lines). The demand for the low-quality good $D_1^*(t)$ increases (resp. decreases) with time if the size of country one is relatively small (resp. large). Indeed, we know from Proposition 4 that the duopoly prices of both goods invariably decline with information diffusion. However, while competition is always favorable to the high-quality firm, it boosts demand for the low-quality product only if the mass of captive consumers in country one is small. In this case, the loss in the monopoly power of firm 1 is more than offset by the demand increase from informed consumers living in the larger foreign market of country two.

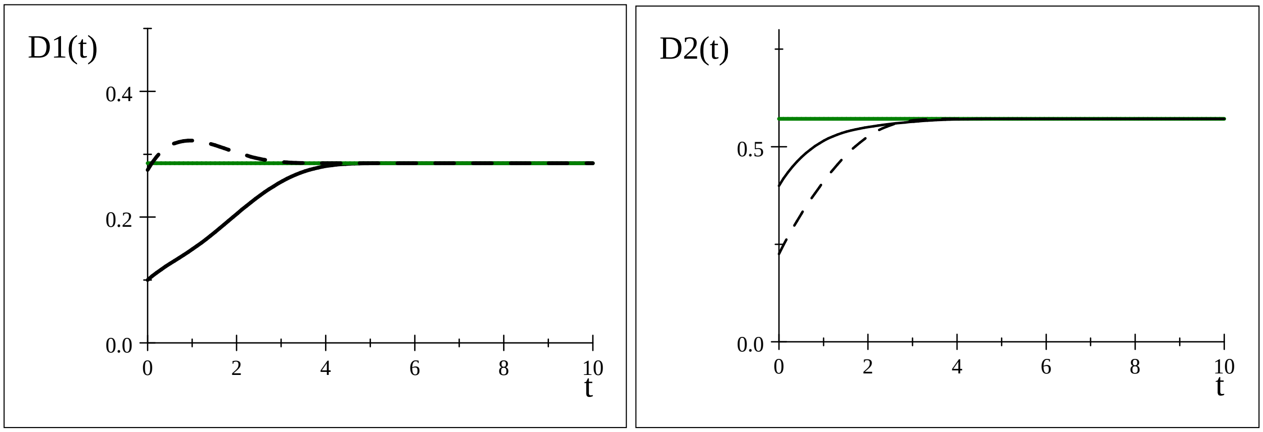


Figure 6: The dynamic evolution of equilibrium demand $D_1(t)$ (left graph) and $D_2(t)$ (right graph)

### 5.1.2 Profits

Social interactions have an ambiguous overall effect on firm profits because exchanges determine two contrasting effects. On the one hand, encounters may enlarge the markets served by each firm. This is the *market expansion effect.* Some consumers in country one will meet consumers of country two and start consuming good 2, enlarging the market share of good 2. However, some country two consumers will also meet country one consumers and some may start consuming good 1, increasing the demand for good 1. How much additional demand firms gain depends a priori on the intensity of meetings (namely, on $s$). On the other hand, there is a *competition effect* because firms face a foreign competitor due to encounters. Indeed, information diffusion eventually transforms the two initial monopolies into a duopoly market. Likewise, the intensity of competition depends on the intensity of interactions, which are ultimately determined by the size asymmetry between countries, for any given quality gap $u$. For illustration, in Figure 7 we depict the profit of firm 1 and firm 2 for $t = 0, 1, 2, ..., 10$ and $u = 1$. When country one is large ($s = 0.55$, dashed line), its advantage in gaining new foreign consumers is more than offset by the loss in domestic consumers: the profit of firm 1 decreases. Precisely the same occurs for firm 2, whose profit decreases from period 1 onwards. When country one is relatively small ($s = 0.2$, continuous line), apart from the initial period, information diffusion is favorable to firm 1 and disadvantageous to firm 2. Note that the profit of firm 2 (resp. firm 1) always increases (resp. decreases) with meetings when we move from period 0 to period 1, irrespective of $s$. This property is briefly proven in Proposition $A1$ in Appendix A.7.

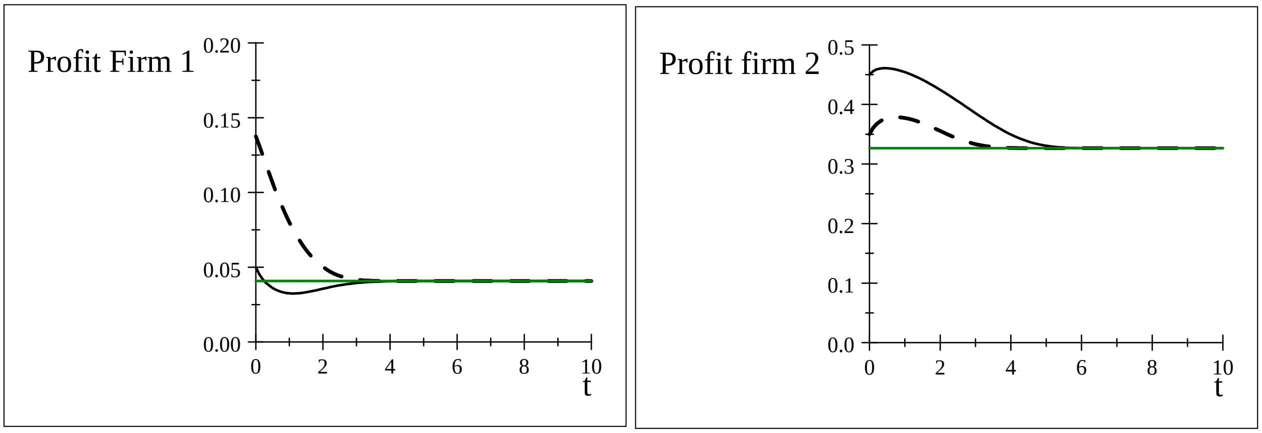


Figure 7: Profit evolution over time

Summing up, under a duopoly with informational frictions, the equilibrium profits of firm 1 (resp. firm 2) increase (resp. decrease) from period $t = 1$ onwards if the relative population size of country one is small (resp. large). The opposite occurs for the low-quality firm if country one is relatively large. The profits of both firms invariably converge to full information profits (9) as $t \to \infty$.

## 5.2 Welfare

Finally, we look into the welfare effects of the international agreement that opens the two countries up to social interactions. We first look at the effect on total welfare and, then, to that on each country.

### 5.2.1 Total Welfare

The sum of country *one* and country *two*' total welfare corresponds to the total welfare of both uninformed and informed consumers net of their expenditures, which accrue to the two firms as profits. Total welfare at every period $t = 1, 2, ..,$ is, therefore,

$$W(p_1^*(t), p_2^*(t)) = U_1(t) \int_{\frac{p_1^*(t)}{u}}^{1} (\theta u)\, d\theta + U_2(t) \int_{\frac{p_2^*(t)}{2u}}^{1} (2\theta u)\, d\theta + I(t) \int_{\frac{p_1^*(t)}{u}}^{\frac{p_2^*(t)-p_1^*(t)}{u}} (\theta u)\, d\theta + I(t) \int_{\frac{p_2^*(t)-p_1^*(t)}{u}}^{1} (2\theta u)\, d\theta.$$

Note that as the equilibrium prices decline over time, total international welfare necessarily increases monotonically in $t$ and converges to its maximum at the full information level:

$$W(p_1^*, p_2^*) = \int_{\theta_1^*(t)}^{\theta_2^*(t)} (\theta u)\, d\theta + \int_{\theta_1^*(t)}^{1} (2\theta u)\, d\theta = \frac{2u\,(13s+1)}{49} + \frac{2u\,(21-13s)}{49} = \frac{44}{49}u.$$

Some comparisons between the levels of the international welfare under autarky and full information can be found in Appendix A.9.

### 5.2.2 The Welfare of the Individual Countries

When computing the total welfare of each country under informational frictions, we have to distinguish between the goods purchased by domestic (informed or uninformed) and foreign (informed) consumers. We also have to take into account that some (informed) domestic consumers purchase the foreign good whereas some (informed) foreign consumers buy the domestic good. This yields

$$\begin{aligned} W_1(t) &= U_1(t) \int_{\frac{p_1(t)}{u}}^{1} (\theta u)\, d\theta + I_1(t) \left( \int_{\frac{p_1(t)}{u}}^{\frac{p_2(t)-p_1(t)}{u}} (\theta u)\, d\theta + \int_{\frac{p_2(t)-p_1(t)}{u}}^{1} (2\theta u)\, d\theta \right) \\ & \quad -I_1(t)p_2(t) \left(1 - \frac{p_2(t)-p_1(t)}{u}\right) + I_2(t)p_1(t) \left(\frac{p_2(t)-p_1(t)}{u} - \frac{p_1(t)}{u}\right) \end{aligned}$$

in country *one* and

$$\begin{aligned} W_2(t) &= U_2(t) \int_{\frac{p_2(t)}{2u}}^{1} (2\theta u)\, d\theta + I_2(t) \left( \int_{\frac{p_1(t)}{u}}^{\frac{p_2(t)-p_1(t)}{u}} (\theta u)\, d\theta + \int_{\frac{p_2(t)-p_1(t)}{u}}^{1} (2\theta u)\, d\theta \right) \\ & \quad -I_2(t)p_1(t) \left(\frac{p_2(t)-p_1(t)}{u} - \frac{p_1(t)}{u}\right) + I_1(t)p_2(t) \left(1 - \frac{p_2(t)-p_1(t)}{u}\right) \end{aligned}$$

in country *two*, where $I_i(t)$ denotes the mass of informed consumers in country $i = 1, 2$, the detailed computations of which are relegated to Appendix A.9. Figure 8 below depicts the total welfare of country *one* (left panel) and *two* (right panel), respectively, under autarky (for $t = 0$) and duopoly with informational frictions (for $t = 1, 2, ..., 10$). Note that while the total welfare of country one increases monotonically with the effect of trade openness and, even more so, the higher the level of size symmetry between the two countries (compare the dotted line for $s = 0.55$ with the continuous line for $s = 0.01$), the welfare of country *two* initially decreases and later increases after a certain number of periods, varying as a function of the asymmetry between the two countries. When countries are of a very similar size (dotted line), country *two*'s welfare starts to increase immediately after period 1, whereas this occurs only after period $t = 2$ (for $s = 0.2$) or period $t = 7$ when the size of country *two* is much larger than that of country *one* ($s = 0.01$). This is because the increase in consumer surplus takes time to fully offset the profit loss of the firm located in country two, which initially suffers from competition in the form of country *one*'s low-quality firm, without much advantage in terms of new customers, especially when the foreign country is not highly populated.

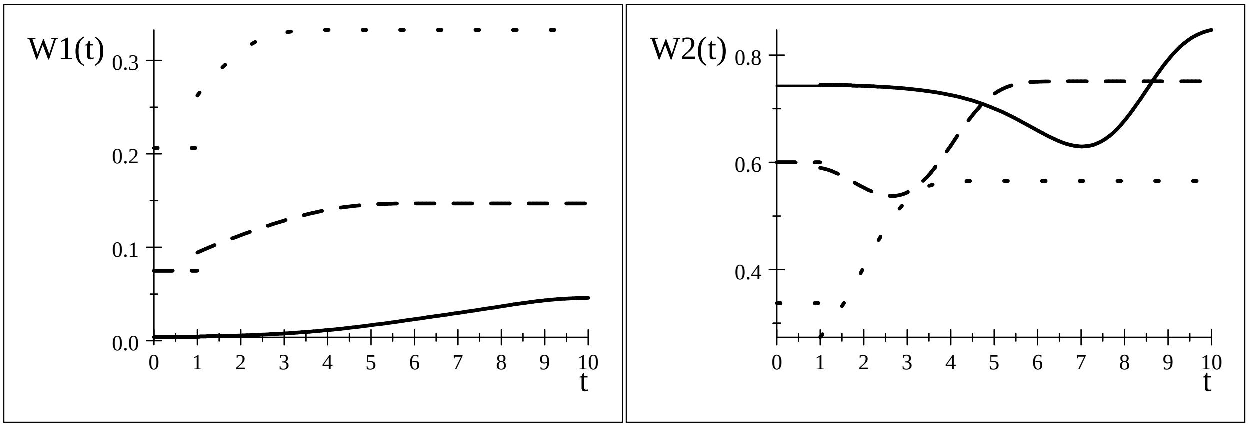


Figure 8: Evolution over time of welfare functions $W_1(t)$ and $W_2(t)$

However, it can be shown that when the size of country *one* is sufficiently large, for instance for $s = 0.85$, after some initial periods of price fluctuation, from Theorem 1 we know that $\bar{s}(3) > 0.85$. Thus, from period 3 onwards a price Nash equilibrium exists and the welfare of country *two* starts increasing to a much higher level than under autarky, and the same for that of country one. Hence, surprisingly, in terms of international welfare the best scenario occurs when the country selling the low-quality good has a relatively large population and opens up to exchanges with a relatively smaller country selling a higher quality good. To sum up, information diffusion and international competition can be good news but also bad news for firms and countries. This greatly depends on the share of captive consumers that each firm either gains or loses as a result of social meetings. Our results suggest that when countries have very heterogeneous populations and produce products of different quality, unanimous agreement about agents' mobility are not easy to reach without specific side payments.

# 6 Concluding Remarks

Opening markets to foreign consumers may involve a transition period in which informational frictions exist, such that local consumers learn about the quality of international varieties only by interacting with those with such knowledge. We build a novel multi-period market setting with vertical differentiation to explore how prices change along the sequence of equilibria generated by individual interactions across time. In such an environment, social interactions upon opening markets internationally can act as a catalyst for demand for foreign goods in local markets. We examine the implications of such social interactions for market competition between vertically differentiated goods, each produced by a local monopoly, and the formation of a full information duopoly environment.

We prove that while convergence to a full information duopoly does take place, it can occur following different paths depending on the size asymmetry between countries. Intense social interactions alleviate informational frictions. This will generate fiercer competition between local monopolies, leading to faster convergence to the international duopoly, although prices take time to converge to their full information counterparts.

If, however, the country producing the low-quality good is relatively large (in terms of population), a duopoly may not arise for several periods, delaying the price-reducing effect of openness. This result is informative for the trade literature in that it suggests that open trade is an essential instrument to discipline the market power of firms, reducing prices. Depending on market sizes, mobility and openness may take time to be beneficial to consumers by reducing consumer prices.

When country *one*'s population size is such that there is a duopoly equilibrium, the analysis of the international equilibrium with informational frictions unveils the role country size can play in shaping demand and prices. We show that demand for the high-quality product increases and demand for the low-quality product may either increase or decrease over time, until they reach their full information levels. When the country producing the high-quality product is large, our model predicts that the profit of the low-quality (high-quality) firm increases (decreases) after social interactions, while the opposite occurs when the foreign country is relatively small. In terms of welfare, trade openness always enhances international welfare, although these benefits come with some delay for the country selling the high-quality good when it has a relatively large size, whereas the other country always gains. As exchanges and information evolve over time, total welfare always increases in all countries. Overall, both our results about the existence of an equilibrium and contrasting country specific welfare effects suggest that when countries have very heterogeneous populations and produce products of different quality, the desired effects of openness to social interactions can only take place with specific benefit sharing rules.

One last remark is in order. In our setting, the transmission of information only occurs between consumers holding different information sets. However, the information could evidently be transmitted in alternative ways. For instance, advertising could reveal the quality of a product. Furthermore, firms could use prices in order to diffuse information about their goods. By lowering their prices, they could attract a more extensive set of new consumers. This would relate our paper to the recent literature in economics and management that analyzes "market seeding" and information transmission through "consumers/ambassadors" (Hinz et al., 2011; Groeger and Buttle, 2013). We leave this and other related issues to future research.

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# Appendix

## A.1. Firm Best-replies in the Market with Informational Frictions

We can first consider firm 2's per-period profit maximization $\max_{p_2} \Pi_2 \left(p_1, p_2 | U_2, I\right) \equiv \max_{p_2} p_2 \cdot D_2 \left(p_1, p_2\right)$ for any given information partition to derive its best-reply. As a first observation, in the price region where $p_1 > u$ and $p_2 \leq 2u$, firm 2 demand is like that of a full market monopolist and its best-reply is given by monopoly pricing $p_2 = u$. In the price range for $p_1 \leq u$ and $p_2 \leq 2p_1$ where from (13) $D_2(p_1, p_2) = (U_2 + I)(1 - \frac{p_2}{2u})$, it holds that

$$\left.\frac{\partial \Pi_2}{\partial p_2}\right|_{p_2 = 2p_1} = \frac{(u - 2p_1)(I + U_2)}{u} \gtreqless 0 \Longleftrightarrow p_1 \lesseqgtr u/2.$$

In the price range for which $2p_1 < p_2 \leq p_1 + u$, where from (13) $D_2(p_1, p_2) = U_2(1 - \frac{p_2}{2u}) + I(1 - \frac{p_2 - p_1}{u})$, it holds that

$$\left.\frac{\partial \Pi_2}{\partial p_2}\right|_{p_2 = 2p_1} = I + U_2 - \frac{p_1(3I + 2U_2)}{u} \gtreqless 0 \Longleftrightarrow p_1 \lesseqgtr \frac{u(I + U_2)}{3I + 2U_2}$$

and

$$\left.\frac{\partial \Pi_2}{\partial p_2}\right|_{p_2 = p_1 + u} = -\frac{I(p_1 + u) + p_1 U_2}{u} < 0.$$

Notice that, for $p_2 > p_1 + u$, when $(1 - \theta_2(p_1, p_2)) = 0$, all informed consumers prefer to patronize good 1, and

$$\left.\frac{\partial \Pi_2}{\partial p_2}\right|_{p_2 = p_1 + u} = -\frac{I(p_1 + u) + p_1 U_2}{u} < 0.$$

Finally, using the fact that $I = (1 - U_1 - U_2)$, the expression $u(I + U_2)/(3I + 2U_2)$ must belong to the interval

$$0 < \frac{(I + U_2)u}{3I + 2U_2} < \frac{u}{2},$$

where the upper bound converges to $u/3$ for $U_1$, $U_2 \to 0$, occurring for $t \to \infty$.

Using all above informations, we obtain the following continuous best-reply for firm 2 showing three kinks over the range of $p_1 \in [0, \infty)$:

$$p_2(p_1) = \begin{cases} u & \text{if} \quad p_1 \in (u/2, \infty), \\ 2p_1 & \text{if} \quad p_1 \in [u(I + U_2)/(3I + 2U_2), u/2] \\ \frac{Ip_1 + u(I + U_2)}{2I + U_2} & \text{if} \quad p_1 < u(I + U_2)/(3I + 2U_2). \end{cases} \tag{29}$$

We turn now to firm 1's maximization $\max_{p_1} \Pi_1 \left(p_1, p_2 | U_1, I\right) \equiv \max_{p_1} p_1 \cdot D_1 \left(p_1, p_2\right)$. for the price range where $p_2 > 2u$, firm 1 can optimally act as monopolist setting its monopoly pricing $p_1 = u/2$. If firm 2 sets exactly its monopoly pricing $p_2 = u$, from (16) a candidate Nash equilibrium can presumably be $(p_1, p_2) = (u/2, u)$ since firm 1 sells at this price only to its captive (uninformed) consumers. However this combination is not a Nash equilibrium since

$$\arg\max_{p_1} \Pi_1 \left(p_1, p_2 | \, U_1, I\right) \equiv \frac{Ip_2 + uU_1}{2\left(2I + U_1\right)}$$

implying,

$$\Pi_1|_{p_1 = \frac{Ip_2 + uU_1}{2(2I+U_1)}} \equiv \frac{\left(Ip_2 + uU_1\right)^2}{4u\left(2I + U_1\right)}$$

that for $p_2 = u$, is bigger than the profit obtained when playing the monopoly price

$$\Pi_1|_{p_1 = u/2} = \frac{uU_1}{4}.$$

This is:

$$\Pi_1|_{p_1 = \frac{Ip_2 + uU_1}{2(2I+U_1)},\, p_2 = u} - \Pi_1|_{p_1 = u/2} = \frac{1}{4}\frac{u\left(I + U_1\right)^2}{2I + U_1} - \frac{uU_1}{4} = \frac{1}{4}u\frac{I^2}{2I + U_1} > 0.$$

Therefore, $(p_1, p_2) = (u/2, u)$ can never be a Nash equilibrium. We can now consider the range where firm 2 charges $p_2(p_1) = 2p_1$, which occurs for $p_1 > \left(I + U_2\right) / \left(3I + 2U_2\right)$. The maximizer of firm 1's profit is

$$p_1(p_2) = \frac{uU_1 + Ip_2}{4I + 2U_1} \tag{30}$$

that at $p_2 = 2p_1$ implies

$$(p_1, p_2) = \left(\frac{uU_1}{2\left(I + U_1\right)}, \frac{uU_1}{I + U_1}\right). \tag{31}$$

Notice, however, that the pair (31) cannot be an equilibrium just because

$$\Pi_1|_{p_1 = p_1(p_2)} - \Pi_1|_{p_1 = u/2} = \frac{1}{4}\frac{\left(uU_1 + Ip_2\right)^2}{\left(2I + U_1\right)u} - \frac{uU_1}{4} = \frac{I\left(Ip_2^2 + 2uU_1p_2 - 2u^2U_1\right)}{4u\left(2I + U_1\right)} > 0,$$

implying that, for the range where $p_2 = 2p_1$, firm 1 prefers to play its duopoly pricing while, if the other plays $p_2(p_1) = uU_1 / \left(I + U_1\right)$, firm 1 prefers to revert to it monopoly pricing:

$$\Pi_1|_{p_1 = \frac{uU_1}{2(I+U_1)},\, p_2 = \frac{uU_1}{I+U_1}} - \Pi_1|_{p_1 = u/2} < 0.$$

Thus, the range where $p_2 = 2p_1$ can never be a Nash equilibrium. We finally consider the range where firm 2's best-reply is $p_2(p_1) = \left[Ip_1 + u\left(I + U_2\right)\right] / \left(2I + U_2\right)$, which we know occurs for $p_1 < u\left(I + U_2\right) / \left(3I + 2U_2\right)$. Using firm 1's best-reply (30) we have that the candidate equilibrium is:

$$(p_1^*, p_2^*) = \left(u\frac{\left(I + U_2 + 2U_1\right)I + U_1U_2}{\left(7I + 4U_1 + 4U_2\right)I + 2U_1U_2}, u\frac{\left(4I + 4U_2 + 3U_1\right)I + 2U_1U_2}{\left(7I + 4U_1 + 4U_2\right)I + 2U_1U_2}\right). \tag{32}$$

Thus a necessary condition for $p_1^* < u\,(I+U_2)\,/\,(3I+2U_2)$ is that $2I+2U_2-U_1>0$ or $U_1<2/3$ using the fact that $I=1-U_1-U_2$. By comparing firm 1's profit when playing its duopoly best-reply $p_1(p_2)$ with the profit obtained at its monopoly pricing, we obtain that

$$\Pi_1|_{p_1=p_1(p_2)} - \Pi_1|_{p_1=u/2} = \frac{(Ip_2+uU_1)^2}{4u\,(2I+U_1)} - \frac{uU_1}{4} \gtreqless 0 \text{ for } p_2 \gtreqless \widetilde{p}_2 \tag{33}$$

where

$$\widetilde{p}_2 = \tfrac{2u(2I+U_1)}{I}\sqrt{\tfrac{U_1}{8I+4U_1}} - \tfrac{uU_1}{I} \tag{34}$$

is the positive only positive root to (33). Therefore, the best-reply has a discountinuity at $\widetilde{p}_2$. As a result, firm 1' best-reply can be characterized as the following piece-wise function:

$$p_1(p_2) = \begin{cases} \frac{1}{2}u & \text{if} \quad p_2 < \widetilde{p}_2 \\ \frac{uU_1+Ip_2}{4I+2U_1} & \text{if} \quad \widetilde{p}_2 \le p_2 \le 2u \\ \frac{1}{2}u & \text{if} \quad p_2 > 2u. \end{cases} \tag{35}$$

## 6.1 A.2. Proof of Theorem 1

As illustrated in Figure A3 of Section 2, a discontinuity of firm 1's best-reply exists in correspondence of $\widetilde{p}_2$. Such discontinuity jeopardizes the existence of the duopoly equilibrium when $\widetilde{p}_2 > u\,(I+U_2)\,/2I+U_2$. Let us describe in detail when this actually occurs. Firm 1 does not have an incentive to deviate from the duopoly price if its profit $\Pi_1^*(p_1^*,p_2^*)$ at

$$(p_1^*,p_2^*) = \left(\frac{u\left[(I+U_2+2U_1)\,I+U_1U_2\right]}{(7I+4U_1+4U_2)\,I+2U_1U_2}, \frac{u\left[(4I+4U_2+3U_1)\,I+2U_1U_2\right]}{(7I+4U_1+4U_2)\,I+2U_1U_2}\right)$$

exceeds the profit $\Pi_1(p_1)$ for $p_1=u/2$, namely when

$$\Pi_1^*(p_1^*,p_2^*) - \Pi_1(p_1) = \frac{1}{4}\frac{uI^2\cdot A(U_1,U_2)}{\left(I\,(7I+4U_1+4U_2)+2U_1U_2\right)^2} > 0 \tag{36}$$

where

$$A(U_1,U_2) = -9U_2^2U_1 + \left(42U_1-30U_1^2-8\right)U_2 + \left(42U_1^2-13U_1^3-37U_1+8\right). \tag{37}$$

The expression (36) can be negative and, therefore, firm 1 in this case can deviate profitably from the duopoly pricing. When this happens firms 1's best-reply function has no intersection with that of firm 2. In this latter case, a duopoly equilibrium in pure strategies does not exist. Hence, for the existence of a duopoly equilibrium the condition $A(U_1,U_2)\ge 0$ is needed. We can sign the function (37). Clearly $A(U_1,U_2)$ is a concave parabola with two roots given by

$$\rho_1 = \tfrac{21U_1-15U_1^2-4+2(3U_1-2)\sqrt{-3U_1+3U_1^2+1}}{9U_1} \text{ and } \rho_2 = \tfrac{21U_1-15U_1^2-4-2(3U_1-2)\sqrt{-3U_1+3U_1^2+1}}{9U_1}.$$

Notice that root $\rho_2 = 0$ for $U_1 = 0$ and $U_1 = 1$, and since $\rho_2(U_1)$ is a strictly concave function of $U_1$, then $\rho_2 > 0$ for any $U_1 \in (0,1)$. However, $\rho_1$ can be positive or negative depending on $U_1 \gtrless \widetilde{U}_1 \equiv (29 - 5\sqrt{17})/26$. In addition, it is easy to find that $\rho_2 > \widetilde{U}_1$ for $U_1 \in (0, \widetilde{U}_1)$. Therefore, in the set $U_1 \in (0, \widetilde{U}_1)$, the function $A(U_1, U_2)$ is a parabola with a negative root and a positive root exceeding $\widetilde{U}_1$. Accordingly, $A(U_1, U_2)$ is always positive for $U_1 \in (0, \widetilde{U}_1)$. We conclude that a *sufficient condition for a duopoly equilibrium* to exists is that $U_1 \leq \widetilde{U}_1 \equiv (29 - 5\sqrt{17})/26 = 0.32248$, which ensures that a duopoly equilibrium exists for all $U_2 \in [0, 1 - U_1]$. This concludes the proof.

## A.3. The Mixed Equilibrium

We showed above that when $U_1 > \widetilde{U}_1$, a Nash equilibrium in pure strategies of the price game no longer exists and, therefore, firms may find optimal to randomize their prices within a given support. To search for a mixed equilibrium we can use the well-known property that, in the support of mixed equilibrium strategies, every pure strategy provides a player with the same expected payoff (see, e.g. Lemma 33.2 at p.33 in Osborne and Rubinstein, 1994). Let us denote by $\Phi_i(p_i)$ the cumulative distribution of firm $i = 1, 2$ as a function of its own price.

Starting with the high-quality seller, firm 2 randomizes on its price support $p_2 \in \left[\underline{p}_2, \overline{p}_2\right]$. Denote $\Phi_i(\widehat{p}_i) = \Pr\{p_i \leq \widehat{p}_i\}$ the (cumulative) probability of firm $i = 1, 2$ to play a price lower or equal than price $\widehat{p}_i$ and $1 - \Phi_i(p_i)$ the probability to play a strictly higher price. The price support for firm 2 is

$$P_2 = [\widetilde{p}_2, u].$$

Firm 2 would never play a price lower than $\widetilde{p}_2$, because firm 1 would react by playing as a monopolist. Hence, there is no reason for firm 2 to shave further its price at $\widetilde{p}_2$ and, therefore, $\Phi_2(\widetilde{p}_2) = \Pr\{p_i \leq \widetilde{p}_2\} = 0$. Similarly, the upper bound of firm 2's support is its monopoly price and, therefore, $\overline{p}_2 = u$. Hence, $\Phi_2(u) = \Pr\{p_i \leq u\} = 1$. Exploiting the property of a mixed equilibrium and the fact that if firm 1 plays its monopoly price its payoff is invariant to rival's price and equal to $\frac{1}{4}uU_1$, we can write

$$\Phi_2(p_2)\, p_1 U_1 \left(1 - \tfrac{p_1}{u}\right) + (1 - \Phi_2(p_2)) p_1 (U_1(1 - \tfrac{p_1(p_2)}{u}) + I(\tfrac{p_2 - p_1(p_2)}{u} - \tfrac{p_1(p_2)}{u})) = \frac{1}{4} u U_1,$$

where $p_1(p_2) = (uU_1 + Ip_2)/(4I + 2U_1)$ denotes firm 1's best-reply under duopoly. Picking as specific price $p_2 = 2p_1$ in the firm 2's cumulative distribution $\Phi_2(2p_1) = \Pr\{p_2 \leq 2p_1\}$, the above equality can be written as

$$\frac{uU_1}{4} = \Phi_2(2p_1) \cdot \left((p_2/2)\, U_1 \left(1 - \tfrac{p_2/2}{u}\right)\right) + (1 - \Phi_2(2p_1)) \left(p_1(p_2) (U_1(1 - \tfrac{p_1(p_2)}{u}) + I(\tfrac{p_2 - p_1(p_2)}{u} - \tfrac{p_1(p_2)}{u}))\right)$$

which is solved for

$$\Phi_2(p_2) = I \frac{2uU_1 p_2 + Ip_2^2 - 2u^2 U_1}{(uU_1 - p_2(I + U_1))^2} \in [0, 1].$$

In the same vein, when firm 1 randomizes, its mixed equilibrium support is

$$P_1 = \left[\underline{p}_1, u/2\right],$$

where $\underline{p}_1$ is firm 1's level of price, which makes firm 2 indifferent between its monopoly and duopoly profit, namely $\underline{p}_1 = \frac{u}{I}\left(\sqrt{\frac{I+U_2}{2I+U_2}}\left(2I+U_2\right)-\left(I+U_2\right)\right)$. Then the expected profits for firms 2 are:

$$\begin{aligned} & \Phi_1\left(p_1\right) p_2 U_2\left(1-\tfrac{p_2}{2u}\right)+\left(1-\Phi_1\left(p_1\right)\right) p_2 (U_2 (1-\tfrac{p_2(p_1)}{2u})+I(1-\tfrac{p_2(p_1)-p_1}{u})) \\ = \; & \Phi_1\left(p_1\right) \frac{1}{2} u U_2+\left(1-\Phi_1\left(p_1\right)\right) \frac{1}{2} u\left(I+U_2\right). \end{aligned}$$

where $p_2\left(p_1\right) = \left(I p_1+u\left(I+U_2\right)\right) / \left(2I+U_2\right)$ denotes firm 2's best-reply under duopoly. Picking as specific price $p_2 = 2p_1$ for firm 2's cumulative distribution $\Phi_1\left(p_1\right) = \Pr\left\{p_1 \leq p_2/2\right\}$, and solving for $\Phi_1\left(p_1\right)$ we find

$$\Phi_1\left(p_1\right)=\frac{I^2 p_1^2+u I\left(I+U_2\right)\left(2 p_1-u\right)}{u^2\left(U_2^2-I^2\right)+I^2 p_1\left(2 u+p_1\right)-4 U_2^2 p_1\left(u-p_1\right)+I U_2\left(8 p_1^2+u\left(u-6 p_1\right)\right)} \in[0,1].$$

Thus, in the range of $U_1 > \widetilde{U}_1$ for which no pure-strategy Nash equilibrium exists, the mixed equilibrium is characterized by the pair $\Phi = (\Phi_1(p_1), \Phi_2(p_2))$ defined above. Notice that both functions $\Phi_i(p_i)$ are increasing in their support. Below a numerical example illustrates the result.

### A.3.1 Numerical example

Let $u = 1$, $I = 0.2$, $U_1 = 0.5$, $U_2 = 0.3$ which are parameters values consistent with the non-existence of a pure-strategy Nash equilibrium. For this values we obtain the two supports:

$$p_1 \in \left[\underline{p}_1, \frac{u}{2}\right] = [0.458\,04, 0.5], \qquad p_2 \in [\widetilde{p}_2, u] = [0.69208, 1],$$

and the corresponding pair $\Phi = (\Phi_1(p_1), \Phi_2(p_2))$ plotted in figure A.3.1 and A.3.2 below

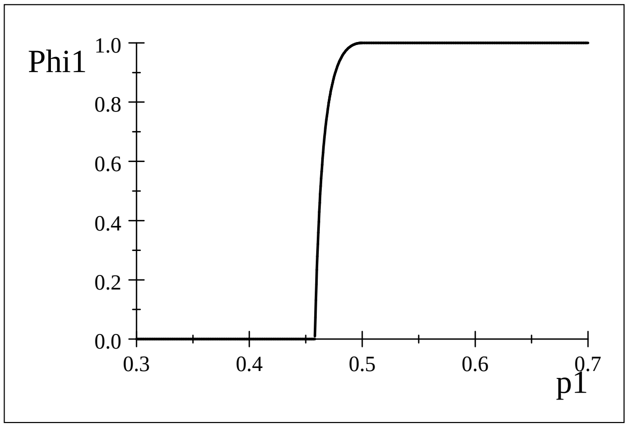


Figure A.3.1. Cumulative distribution function $\Phi_1(p_1)$ for $u = 1$, $U_1 = 0.5$, $I = 0.2$ and $U_2 = 0.3$.

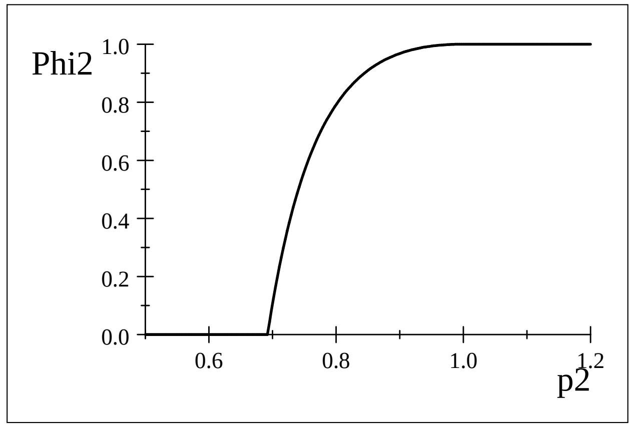


Figure A.3.2. Cumulative distribution function $\Phi_2(p_2)$ for $u = 1$, $U_1 = 0.5$, $I = 0.2$ and $U_2 = 0.3$.

## A.4. Proof of Proposition 2

We know that if, in every period $s^{2^t} \in (0, \overline{s}(t))$, then there is room for both firms to trade their products in the international duopoly market at the maximizing prices (26). We need to prove that this pair of prices is the unique noncooperative Nash equilibrium of the multi-stage setting where firms are assumed to maximize their profits at every period. This can be proved by first noticing that no firm can strategically influence the information diffusion with its price and, hence, its profit-maximizing choice is to play its best response at every period $t$, namely

$$\begin{aligned} p_1(p_2) &= \arg\max_{p_i} \ \Pi_1(p_1(t), p_2(t)) \equiv \frac{s^{2^t} \cdot u + p_2\left(1 - s^{2^t} - (1-s)^{2^t}\right)}{4 - 4(1-s)^{2^t} - 2s^{2^t}}, \\ p_2(p_1) &= \arg\max_{p_2} \ \Pi_2(p_1(t), p_2(t)) \equiv \frac{(1 - s^{2^t}) \cdot u + p_1\left(1 - s^{2^t} - (1-s)^{2^t}\right)}{2 - (1-s)^{2^t} - 2s^{2^t}}. \end{aligned} \tag{38}$$

Since

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} &= -\frac{3 - 3(1-s)^{2^t} - s^{2^t}}{u} < 0, \\ &\text{and} \\ \frac{\partial^2 \Pi_2}{\partial p_2^2} + \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} &= -\frac{1 - s^{2^t}}{u} < 0 \end{aligned}$$

both firms' best replies (38) are *contractions* (see, for instance, Vives, 2000, p.47), and thus, if $s < \overline{s}(t)$, this suffices for the pair of Nash equilibrium prices (26) to be the unique *noncooperative* Nash equilibrium at every period $t$.

## A.5. Proof of Proposition 3

The mass of uninformed consumers served by the two firms at period t are $U_1(t) = s^{2^t}$ and $U_2(t) = (1-s)^{2^t}$. It is easy to see that $\lim_{t\to\infty} s^{2^t} = \lim_{t\to\infty} (1-s)^{2^t} = 0$. As time goes by, the uninformed market segment disappears and the market solution necessarily returns to a full information duopoly. Furthermore, the move towards the duopoly equilibrium certainly occurs, according to Lemma 1, if the mass of uninformed consumers of country *one* becomes sufficiently small, and such that $U_1(t) < \overline{s}(t)$, which in our multi-period setting translates into

$$s^{2^t} < \overline{s}(t) \equiv \left(29 - 5\sqrt{17}\right)^{\frac{1}{2^t}} \cdot 26^{-\frac{1}{2^t}} \in (0,1)\,.$$

Therefore, whatever the existing asymmetry between the populations of the two countries, there will always exist a finite period for which $U_1(t) = s^{2^t} < \overline{s}(t)$. Since $s \in (0,1)$, $s^{2^t}$ is decreasing in $t$ and, therefore, for any $s^2 \in (\overline{s}(1), 1)$ there is a value of $t$ such that $s^{2^t} < \overline{s}(t)$. For instance, let us assume that country one is disproportionately large, with $s = 0.99$. In this case $(0.99)^2 = 0.9801 > \overline{s}(1)$. Therefore, at period 1 the price solution will not be that of a duopoly with informational frictions but rather a monopolistic one. However, since $U_1(t) = (0.99)^{2^t} \equiv \left(29 - 5\sqrt{17}\right)^{\frac{1}{2^t}} \cdot 26^{-\frac{1}{2^t}}$ is solved for $t \simeq 3.4076$, it follows that $\overline{s}(4) > (0.99)^2 > \overline{s}(3)$, thus implying that at period $t = 4$ the monopoly market will certainly turn into a duopoly equilibrium with frictions. The same exercise can be replicated for any size of country *one* $s \in (0,1)$ and, therefore, although the number of periods needed to return to the duopoly increases more and more as $s$ approaches 1, this number is always finite for $s \in (0,1)$. This concludes the proof.

## A.6. Proof of Proposition 4

Since prices are strategic complements, using expressions (22) and the fact that $I = 1 - U_1 - U_2$, standard comparative statics (see, for instance, Vives 2000, chapter 6) at the interior duopoly equilibrium yield:

$$\begin{aligned}
\text{sign}\frac{\partial p_1^*}{\partial U_1} &= \text{sign}\frac{\partial^2 \Pi_1}{\partial p_1 \partial U_1} \equiv \text{sign}\left(1 - \frac{p_2^* - p_1^*}{u} + \frac{p_1^*}{u}\right) > 0, \\
\text{sign}\frac{\partial p_2^*}{\partial U_2} &= \text{sign}\frac{\partial^2 \Pi_2}{\partial p_2 \partial U_2} \equiv \text{sign}\left(\frac{p_2^* - p_1^*}{u}\right) > 0,
\end{aligned}$$

for any profile $p = (p_1, p_2)$ of interior prices (i.e. such that $1 > \theta_2 > \theta_1$). Again, plugging equality $I = 1 - U_1 - U_2$ into (22), and applying standard comparative statics technique yields

$$\begin{aligned} \text{sign}\frac{\partial p_1^*}{\partial U_2} &= \text{sign}\frac{\partial^2 \Pi_1}{\partial p_1 \partial U_2} = \text{sign}\left(\frac{4p_1^* - p_2^*}{u}\right) \\ &= \text{sign}\left[\frac{U_1 (5 - 3U_2 - 5U_1)}{7 + 3(U_1^2 + U_2^2) + 8U_1U_2 - 10(U_1 + U_2)}\right] > 0, \end{aligned}$$

as the denominator of the fraction above is positive (given that $p_1^*$ and $p_2^*$ are positive at the interior equilibrium) and the numerator is positive as well, since $U_1 (5 - 3U_2 - 5U_1) > U_1 (5 - 5(U_1 + U_2)) > 0$ for $U_1 + U_2 < 1$. Similarly,

$$\begin{aligned} \text{sign}\frac{\partial p_2^*}{\partial U_1} &= \text{sign}\frac{\partial^2 \Pi_2}{\partial p_2 \partial U_1} = \text{sign}\left(\frac{2p_2^* - p_1^*}{u}\right) - 1 \\ &= \text{sign}\left[\frac{U_2 (3 - 2U_1 - 3U_2)}{7 + 3(U_1^2 + U_2^2) + 8U_1U_2 - 10(U_1 + U_2)}\right] > 0. \end{aligned}$$

Using the fact that $I = 1 - U_1 - U_2$ both results above jointly imply that

$$\frac{\partial p_i^*}{\partial I} < 0 \text{ for } i = 1, 2.$$

Finally, we can easily prove that in the domain of values of $s$ for which the duopoly equilibrium with informational frictions exists, we obtain that duopoly equilibrium prices $p_1^*(t)$ and $p_2^*(t)$ are decreasing in $t$ and converge over time to their counterparts $p_1^*$ and $p_2^*$ in a duopoly with vertically differentiated goods and fully informed agents, namely from (26)

$$\lim_{t \to \infty} p_1^*(t) = p_1^* \text{ and } \lim_{t \to \infty} p_2^*(t) = p_2^*.$$

## A.7. Proof of Proposition 5

Looking at the demands for the two good (19) and (20), the effect of a rise in the mass of consumer informed about both goods is

$$\frac{\partial D_1^* (p_1 (U_1), p_2 (U_1), U_1)}{\partial I} = \frac{58I^4 + 2U_1^4 - 21I^2U_1^2 - 4IU_1^3 + 16I^3U_1}{(11I^2 - 2U_1^2 + 2IU_1)^2}$$

where

$$\text{sign } \frac{\partial D_1^* (p_1 (U_1), p_2 (U_1), U_1)}{\partial I} = \text{sign } \left[58I^4 + 2U_1^4 - 21I^2U_1^2 - 4IU_1^3 + 16I^3U_1\right]$$

and with $U_1 = 1 - I$

$$\text{sign } \left(58I^4 + 2U_1^4 - 21I^2U_1^2 - 4IU_1^3 + 16I^3U_1\right) \equiv \text{sign } \left[-12I + 3I^2 + 38I^3 + 27I^4 + 2\right] \gtrless 0$$

Similarly,

$$\frac{\partial D_2^*\left(p_1\left(I\right),p_2\left(I\right),U_2\right)}{\partial I}=\frac{1}{2}\frac{154I^4+6U_2^4-47I^2U_2^2-8IU_2^3+56I^3U_2}{\left(11I^2-2U_2^2+2IU_2\right)^2}$$

where

$$\text{sign}\frac{\partial D_2^*\left(p_1\left(I\right),p_2\left(I\right),U_2\right)}{\partial I}=\text{sign }\left[154I^4+6U_2^4-47I^2U_2^2-8IU_2^3+56I^3\right]$$

and with $U_2 = 1 - I$

$$\text{sign}\left(154I^4+6U_2^4-47I^2U_2^2-8IU_2^3+56I^3\right)\equiv\text{sign }\left[-32I+13I^2+102I^3+65I^4+6\right]>0\text{ for }I\leq 1.$$

This completes the proof.

## A.8. Proposition A1 and its Proof

**Proposition A1.** In the first period, market opening is always profitable for firm 2 and unprofitable for firm 1, *namely* $\Pi_2(1) - \Pi_2(0) > 0$ and $\Pi_1(1) - \Pi_1(0) < 0$.

**Proof.** Using (26) and (27), we easily obtain

$$\begin{aligned}\text{sign }\left[\Pi_1(1)-\Pi_1(0)\right] &= \text{sign }\left[\frac{1}{4}\frac{s\left(4-3s\right)\left(s+s^2+2\right)^2}{\left(7s^2-4-7s\right)^2}-\frac{1}{4}s\right]\\ &= \text{sign }\left[s\left(54s+3s^2-37\right)-52\right]<0\text{ for }s\in\left(0,1\right).\end{aligned}$$

Similarly,

$$\text{sign }\left[\Pi_2(1)-\Pi_2(0)\right] = \text{sign }\left[\frac{1}{2}\frac{s^2\left(1-s\right)\left(4s+1\right)\left(3s^2+16-15s\right)}{\left(-7s+7s^2-4\right)^2}\right]$$

which in turn is equal to the sign $[3s^2 + 16 - 15s] > 0$ for $s \in (0, 1)$. ■

## A.9. Welfare

### Welfare under Autarky

Since consumers' utilities are quasi-linear and under autarky firms' consumers are only domestic, it descends that in every country the social welfare is simply given by the consumers' surplus net of their expenditure, which accrue as profits to the firm operating in that country. This is

$$W_1=s\int_{\theta_1^M(t)}^{1}\left(\theta u\right)d\theta,\text{ and }W_2=\left(1-s\right)\int_{\theta_2^M(t)}^{1}\left(\theta 2u\right)d\theta,$$

where

$$\theta_1^M = \frac{p_1^M}{u} = \frac{u/2}{u} = 0.5 \text{ and } \theta_2^M = \frac{p_1^M}{2u} = \frac{u}{2u} = 0.5.$$

Therefore, under autarky, the total welfare of the two countries is

$$W_1^M = s\int_{0.5}^{1} (\theta u)\, d\theta = s\frac{3u}{8}.\text{and } W_2^M = (1-s)\int_{0.5}^{1} (\theta 2u)\, d\theta = (1-s)\frac{3u}{4}.$$

**Open Markets: Full information**

As periods $t = 1, .....\infty$ progress, we know that the marginal consumer of the duopoly with informational frictions converge at the limit to

$$\lim_{t\to\infty} \theta_1(t) = \theta_1^* = \frac{1}{7} \text{ and } \lim_{t\to\infty} \theta_2(t) = \theta_2^* = \frac{3}{7},$$

with firms' demands under full information given by

$$D_1^* = \theta_2^* - \theta_1^* = 2/7 \text{ and } D_2^* = 1 - \theta_2^* = 4/7.$$

Using the fact that full information prices are $p_1^* = u/7$ and $p_2^* = 4u/7$, total welfare is obtained as

$$W_1^* = s\int_{\theta_1^*}^{\theta_2^*} (\theta u)\, d\theta + s\int_{\theta_2^*}^{1} (2\theta u)\, d\theta + (1-s)\cdot p_1^* \cdot D_1^* - s\cdot p_2^* \cdot D_2^* = \frac{2u(13s+1)}{49},$$

and

$$W_2^* = (1-s)\int_{\theta_1^*}^{\theta_2^*} (\theta u)\, d\theta + (1-s)\int_{\theta_2^*}^{1} (2\theta u)\, d\theta - (1-s)\cdot p_1^* \cdot D_1^* + s\cdot p_2^* \cdot D_2^* = \frac{2u(21-13s)}{49}.$$

Under full information, the welfare of country *one* is increasing in $s$ whereas in turn, that of country *two*, decreasing in $s$. As expected, both countries gain in terms of total welfare from autarchy to full information:

$$W_1^* - W_1^M = \frac{u(61s+16)}{392} > 0, \text{ and } W_2^* - W_2^M = \frac{u(43s+21)}{196} > 0.$$

Notice also how the biggest advantage of market openness from autarky to full information is obtained for higher level of $s$, the size of the country selling the low quality good.

### Informed Consumers in Every Country over Time

To compute the welfare of each country under informational frictions we need to derive the mass of informed consumers of every country in every period $t = 1, 2, ..,$. This is

$$I_1(t) = \sum_{t=1}^{t} s^{2^{t-1}}(1 - s^{2^{t-1}}) = s - s^{2^t} \tag{39}$$

for country *one* and

$$I_2(t) = \sum_{t=1}^{1}(1 - s)^{2^{t-1}}(1 - (1 - s)^{2^{t-1}}) = (1 - s) - (1 - s)^{2^{t-1}} \tag{40}$$

for country two. Note that the speed of information transmission in every country is basically determined by the existing level of symmetry in the size of population of the two countries.