Using Bayes factors to compare dynamical models of hydrological systems

Damian M. Ndiwago[1], Remko Nijzink[3], Christophe Ley[2], Stan Schymanski[3], Jack S. Hale[1].

[1] Department of Engineering, Faculty of Science, Technology and Medicine, University of Luxembourg, Esch–sur–Alzette, Luxembourg.
[2] Department of Mathematics, Faculty of Science, Technology and Medicine, University of Luxembourg, Esch–sur–Alzette, Luxembourg.
[3] Catchment and Ecohydrology Group (CAT), Environmental Research and Innovation (ERIN), Luxembourg Institute of Science and Technology (LIST), Belvaux, Luxembourg.

Hydrologists are often faced with selecting the 'best' model from a set of competing rainfall-runoff models that differ widely in their complexity and ability to reproduce both past and future data. The Bayes factor is one tool for selecting between models. It is relatively robust and easy–to–use as it implicitly and automatically balances model complexity and goodness of fit to data under few simplifying assumptions. However, it requires the computation of the marginal likelihood which is a very expensive and difficult integration problem. This expense can be attributed to three factors; the necessity of many likelihood calculations with moderate run–times due to the repeated solution of the rainfall-runoff model; the multi–modal and highly correlated nature of the posterior; and finally the inherent difficulty of the marginal likelihood integration problem. In this study we show that by combining recent advances in differentiable programming languages, modern gradient-based Markov Chain Monte Carlo algorithms and thermodynamic integration, that the Bayes factor is now a practical and robust tool for comparing rainfall–runoff models. We illustrate our approach on the problem of choosing from a set of HBV-type models with increasing dynamical complexity calibrated against both synthetically generated and real runoff data. We show that the Bayes factor not only selects a parsimonious model but can also be computed using a reasonable amount of computational resources.

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