An Axiomatic Approach to the Measurement of Comparative Female Disadvantage

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Abstract

Female comparative disadvantage refers to the mismatch of the female with respect to achievements in different dimensions of human well-being in *comparison* with the corresponding achievements of the male. This paper axiomatically derives a general family of female comparative disadvantage indicators which has very important policy implications. The axioms employed are shown to be 'independent'. An empirical illustration of the general index is provided using the UNDP data on mean years of schooling, life expectancy at birth and gross national income per capita in 2018. Results show that female comparative disadvantage is not necessarily related to standard measures of human development, such as the HDI, and is present even in countries reaching very high human development. The factor where policy intervention is needed the most is income.

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1. Introduction and Motivations

In the recent literature on human development and social policy, female disadvantage has been a major issue of concern. Loosely speaking, by female disadvantage we mean a situation in which females are in a worse off position in *comparison* with males in different dimensions of human well-being and hence more appropriately called comparative disadvantage. We use the terms comparative female disadvantage, comparative disadvantage or female disadvantage or simply disadvantage interchangeably here. Examples of such dimensions can be health, literacy, income, environment in workplace etc. Thus, female disadvantage is a multidimensional socio-economic phenomenon and may be regarded as denial of human rights to females. It is a social bad and a continued practice of downside positions of females may turn out to be quite harmful for a society. Women stricken by long duration of such unfavourable situations may not remain loyal to the society norms. For instance, a woman on finding lack of equal accesses with a male colleague to one or more facilities in her workplace may become work averse.

Since disadvantage is a social bad, it becomes desirable to reduce this disadvantage to the extent possible. For this purpose, it is necessary to quantify overall female disadvantage in terms of a scalar that should be easily comprehensible by policy makers for inclusion in indicators of socio-economic performance. A scalar indicator of this type aggregates dimension-wise downside or deprived situations of women into a non-negative real number that summarizes the extent of overall disadvantage of women. Several such indicators have been proposed and analyzed in the recent literature responding to the need of policy makers (Dijkstra –Hanmer, 2000, Klasen and Schüler, 2011, Permanyer, 2013, and Anand, 2018).

In this article we develop an axiomatic approach to the evaluation of female disadvantage, building on the contributions mentioned above. Any axiomatic approach of evaluation helps the planners to select the appropriate metric consistent with their basic doctrines or requirements before using them in policy making. These doctrines or criterion are embedded in the axioms which the measure has to satisfy. A characterization yields further dividend in the sense that a set of rules is perfectly matched with the evaluation. For example, we may consider here Sen's (1976) axiomatic treatment of a sophisticated poverty index involving existing metrics like the headcount ratio and the poverty gap ratio. Our axiomatic analysis constitutes a parallel advancement for female disadvantage. To the best of our knowledge the literature does not contain any axiomatic treatment of female disadvantage. This is the first contribution of the article.

We will show that some of our proposed axioms are satisfied by the existing female disadvantage indices while some are not. This review of existing indices in the literature helps the policy maker in selecting the metric s/he wishes to use. This is another contribution of the article.

While proposing the axioms, we have borrowed from parallel developments in the literature of inequality and poverty. Note that the exact specifications are different given that the domain is different. Further, the important and widely used axiom the principle of transfer has no relevance in our case. We have proposed in this article a new axiom which is typically desirable for measuring disadvantage. We call this 'aggravation sensitivity' postulate. This is also a new contribution which can be applied in other areas.

Next, our characterization enables us to claim that the members of the singularized family are the only indices that verify the laid down axioms or postulates. The axiomatically characterized general class of female disadvantage indices is amenable to policy prescriptions and it incorporates some existing measures. The axioms we use are chosen from properties satisfied by some but not all the existing indices of female disadvantage. By suitable choices of the underlying real valued function, defined on the female-to-male achievement ratios, one can generate indices which will give different levels of trade off among the dimensions. Here also the policy planner will have some flexibility.

The postulates we use in the characterization exercise are shown to be independent. In other words, the set of axioms we employ is minimal. This means that none of them can be dropped from the characterization exercise and forms some sort of basis of its use in policy interventions. Thus, when our index is applied for policy evaluation, the axioms implicitly demonstrate their policy relevance.

The reasonableness of the postulates will justify the use of the family members. As we will see in the review section, an axiom-based systematic comparison of this family with the existing indices will further substantiate our contribution. Hence if all the postulates are accepted as reasonable criteria, the policy maker's decision to use one of the members of the family of indices proposed by us becomes justified. Consequently, our axiomatic study along with policy relevance of the characterized family provides a strong theoretical foundation of the paper.

It may be worthwhile to note that our axiomatic characterization is not 'complete' in the sense that the characterized family depends on the 'female disadvantage function' which is not explicitly mentioned and may take many forms. A more explicit characterization will be necessary to uniquely identify a particular member of the family.

We follow a two-stage procedure, similar to Sen (1976), consisting of first identifying the set of all dimensions of human well-being in which women are disadvantaged, and then aggregating the information available on this segment of the dimension space into an indicator of female disadvantage. It may be useful to mention that our index is a dashboard-based index in the sense that it is obtained by aggregating dimension-wise indices, the constituents of the dashboard. There are several advantages of a dashboard. It becomes helpful in monitoring the performance of each dimension separately, which may be necessary for some policy purpose. Since in a dashboard dimension-wise summary indices of relative achievements are portrayed, two profiles of relative achievements may not be ranked in the same way as the corresponding profiles of relative disadvantage levels. Further, 'dashboards suffer because of their heterogeneity' (Stiglitz, Sen and Fitoussi, 2009, p.63). These problems can be avoided if we use a composite index obtained by aggregating the dimensional figures. A composite index has the advantage of being 'a single headline figure' (Stiglitz, Sen. and Fitoussi, 2009, p.63). (For further discussion on dashboards, see Alkire et al., 2015, and Chakravarty, 2018.) Since this approach aggregates individual dimensional statistics, Kolm (1977) referred to it as an 'individualistic' approach and argued in favour of its use to generate a summary figure of social well-being.

Each index has its own objective. Our purpose is certainly not to supplant any existing composite index. In the current circumstance our general disadvantage metric becomes quite helpful for policy purpose and its axiomatic treatment enables us to understand the metric in a deeper way through the axioms. It may be noted that once axiomatic characterizations of the existing indices are available, the policy maker can very well choose any index which satisfies the axioms s/he thinks desirable.

After presenting the background in the next section, in Section 3 we analyze our axioms for a general disadvantage index. Section 4 characterizes the general family of indices of disadvantage. A systematic comparison of our index with the existing proposals is presented in Section 5. An empirical illustration of the family is provided in Section 6. In the axiomatic analysis of multidimensional well-being, it is customary to use an arbitrary number of dimensions for building the theoretical framework (see, among others, Bourguignon and Chakravarty, 2003, and Alkire and Foster, 2007) and for empirical purpose some specific dimensions are used. Likewise, our theoretical structure involves arbitrary number of dimensions. For instance, in addition to life expectancy, income and literacy, we can as well include data on health, labour market participation, representation in federal parliament, participation at the weekend recreations, access to facilities at the work place and so on.

However, for our illustration we use only those dimensions on which we have readily available information. Hence we choose only the three UNDP dimensions, namely, literacy, life expectancy and income. Finally, Section 6 concludes.

2. Background

Dijkstra and Hanmer (2000) suggested an indicator of female advantage, the Relative Status of Women (RSW) index. RSW is given by the arithmetic average of the female-to-male proportional achievements, that is, the ratios between female-to-male achievements in the three HDI dimensions, namely, literacy (LI), life expectancy (LE) and income (IN). It may be rewarding to note that the Dijkstra-Hanmer (2000) RSW index, under a suitable transformation, can as well be applied for the purpose of identifying dimensions that are more afflicted by female disadvantage. Alternatives and variations of RSW have been suggested, among others, by Klasen and Schüler (2011) and Anand (2018). Permanyer (2013) suggested a Women's Disadvantage (WD) index. As we show later, this index can be treated as a multiplicative version of a member of the family of the female disadvantage indices characterized in the present paper. A simple transformation of each of them generates a corresponding female disadvantage counterpart. We briefly review them in one of the following sections of this paper.

Since female disadvantage addresses a gender-related issue, it will be worthwhile to distinguish female disadvantage from two alternative notions of gender-related measurement issues, namely, gender inequality and gender development. The construction of gender-related indices often becomes a difficult job. For instance, non-market activities, however, productive they may be, are generally not taken into consideration in the evaluation of gender inequality. Women's household works, being nonmarket works, are not captured by the indicators of labour force participation. Consequently, they do not appear in the calculation of women's wage/money income. In some industries, like manufacturing and building, women participation is relatively low. In an industry of this type gender-related discrimination is not a well-defined concept. (See, Ponthieux and Meurs, 2013, for a detailed discussion.) Evidently, this shortcoming applies as well to the analysis of female disadvantage.

There are important differences between gender inequality and female disadvantage. A measure of female disadvantage is concerned with dimension-wise female-to-male disparity ratios. In contrast, gender inequality refers to inequality between genders in dimensional achievements. (See Anand, 2018, for a detailed comparative discussion.)

UNDP (2010) suggested a gender inequality index, GII, whose theoretical foundation is unclear and construction is highly complicated. It relies on a specific comparison of transformations of achievements of men and women in three dimensions, namely, reproductive health, empowerment and labor market, defined in some ways. (See UNDP, 2010, Technical Note 3, pp.219-21, for explicit construction of GII.) Permanyer (2013, pp. 14-15) rightly pointed out that ".... the GII is an unnecessarily confusing index. The meaning of the values of the index is not entirely clear." (See also Klasen and Schüler, 2011, and Anand, 2018, for critical discussion on GII.) Since the gender development index is defined by taking the ratio between the female-to-male human development indices, we relegate its discussion to Section 5.

It will also be worthwhile to make a systematic comparison between measurements of multidimensional poverty and female disadvantage. While the former looks at individual deprivations in each dimension of well-being from an exogenously given threshold limit, the poverty cutoff point; in the latter for each dimension the ratio between average female achievement and male achievement is considered. The dimension-wise poverty threshold limits do not play any role here. Consequently, disadvantage may or may not be related to poor. In poverty analysis the threshold point representing the subsistence level of achievement in a dimension is taken as the norm and these norms vary from dimension to dimension. In disadvantage analysis equality of achievements between sexes stands for the norm and no notion of subsistence is assumed here. The approach makes the aggregation easier since the values (ratios) are comparable across dimensions. Hence from a normative perspective the two measurement issues are conceptually different.

Often in a study like ours axioms are adapted from a related but not exactly identical literature. For instance, in multidimensional poverty analysis ideas of several postulates are borrowed from the multidimensional inequality evaluation literature.

We discuss similarities and dissimilarities between disadvantage axioms and multidimensional poverty axioms in Section 4 of the paper. But in an axiomatic analysis there is one important difference between the two notions of measurements. In any poverty study, there is a well-defined transfer postulate, which admits redistribution of dimension-wise achievements between poor persons in a well-defined manner. In the present context, no such notion of redistribution exists since we are dealing with average achievements for the society as a whole.

In both literatures, compensation between dimensions is not allowed, that is, in the case of multidimensional poverty, income above the poverty line cannot compensate for bad health, for example. We believe that the same principle should apply here. If one is interested in the compensation problem, two separate indices for advantage and disadvantage can be used and combined.

3. Axioms for an Index of Female Disadvantage

We assume at the outset that the variables we are using for our analysis are ratio scale variables. Let $d \in N$ be the number of dimensions of well-being, where N is the set of positive integers. The set of dimensions $\{1,2,3,....,d\}$ is denoted by Q. Let x_{mi} and x_{fi} respectively be average male and female achievement levels in dimension i, where $i \in Q$ is arbitrary. The vectors of achievements of men and women in different dimensions are denoted respectively by $x_m = (x_{m1}, x_{m2}, ..., x_{md})$ and $x_f = (x_{f1}, x_{f2}, ..., x_{fd})$. Following the literature, for the female-male achievement (FMA) ratios $z_i = \frac{x_{fi}}{x_{mi}}$ to be well-defined we assume that $x_{mi} > 0$ and $x_{fi} \geq 0$ for all $i \in Q$.

For $i \in Q$ the female-to-male proportional achievement z_i gives the relative achievement of the female in comparison with the male in the concerned dimension. Consequently, $x_m \in D^d$, the positive orthant of the $d-$ dimensional Euclidean space, and $x_f, z \in \Re^d_+$, the non-negative orthant of the $d-$ dimensional Euclidean space, where $z = (z_1, z_2, ..., z_{\mathrm{d}})$ is the relative achievement profile associated with x_m and x_f. Given x_m and x_f, the vector z uniquely specifies the dimension-by-dimension ratios. The set of all distributions of female-male achievement ratios in the society is given by $\Re_+ = \bigcup_{d \in N} \Re^d_+$. We can as well assume that $x_{fi} > 0$ for all $i \in Q$ so that $z \in D^d$. It may be noted that all our axioms, except, part (ii) of the Normalization, presented below, hold on this restricted domain also. Later in Section 4, we indicate the relevance of this restricted domain in specific situations.

For any $d \in N, i \in Q$, a value of the proportional achievement z_i less than one means that women suffer from disadvantage in dimension i. If the inequality is reversed, that is, if $z_i > 1$ holds, women are in an advantageous position relative to men in the dimension; they are doing better. In these two cases, there is a disparity or imbalance between male-female achievements in the dimension. We can as well say discrepancy in terms of achievements exists in the dimension. For $z_i = 1$, parity exists between the two sexes' performances in the dimension. In other words, there is a balance between the male and female achievement levels in the dimension. We refer to $i \in Q$ as a deprived or non-deprived dimension according as $z_i < 1$ or $z_i \geq 1$. For any $d \in N$, $z \in \Re_+^d$, $Q_{ND}(z) = \{i \in Q | z_i \geq 1\}$ is the set of non-deprived dimensions for women and the set of deprived dimensions for them is $Q_D(z) = \{i \in Q | z_i < 1\}$. Evidently, for any $d \in N$, $z \in \Re_+^d$, $Q = Q_{ND}(z) \cup Q_D(z)$.

For any $d \in N$, $z \in \Re_+^d$, we denote the censored FMA ratio profile associated with z by $z^* = (z_1^*, z_2^*, ...z_d^*)$, where

$$z_i^* = \begin{cases} z_i \; if \; z_i < 1, \\ 1 \; if \; z_i \geq 1. \end{cases} \qquad (1)$$

Thus, in a censored ratio profile we suspend our judgment on the actual distribution of ratios in the dimensions that are advantageous but include the number of such dimensions by doctoring each ratio value at 1. Consequently, the number of dimensions in the two profiles z and z^* is the same, and the advantageous positions of the female in z are brought to parity at z^*.

An overall disadvantage index F aggregates all achievement ratios in an unambiguous way. Formally, $F : \Re_+^d \to \Re_+^1$. For $d \in N$, $z \in \Re_+^d$, $F(z)$ indicates the level of female disadvantage existing in the society when the female-to-male achievement ratios are represented by the vector $z \in \Re_+^d$. It quantifies the extent of inter-sexual deprivation that exists when we compare the female dimensional achievements with those of the male.

For the sake of convenience, the following discussion, whose subject is the analysis of the postulates for a female disadvantage index, is divided into several subsections. The axioms comprising a subsection will share at least one characteristic. Although we do not use all the axioms for the derivation of our index, each of them indicates the behaviour of the index in the relevant situation.

3.1. Invariance Axioms

The axioms analysed in this subsection do not indicate any change in the extent of disadvantage when some permissible changes are made in the profile of imbalance proportions. In the first two postulates, the focus axioms, changes only in disparity ratios associated with advantageous dimensions are permitted. While the Weak Focus does not exactly parallel the corresponding Bourguignon-Chakravarty (2003) multidimensional poverty axiom, its strong counterpart may be regarded as the female disadvantage equivalent of the Bourguignon-Chakravarty strong focus axiom.[1]

Weak Focus: For all $d \in N$, if $z, y \in \Re_{+}^{d}$, where $z \neq y$, are such that $Q_{ND}(z) = Q_{ND}(y) = Q$; then $F(z) = F(y)$.

Since in the two different distributions z and y of FMA proportions, women are non-deprived in all the dimensions ($Q_{ND}(z) = Q_{ND}(y) = Q$), it must be the case that for all $j \in Q$, $z_j \geq 1$, $y_j = (z_j + c_j) \geq 1$, where c_j is a scalar, and for at least one $j \in Q$, $c_j \neq 0$. According to this postulate, if women are in dominant positions in comparison with men in all the dimensions, changes in one or more relative achievement quantities that do not relegate women to unfavourable situations in the dimensions, should not affect the disadvantage indicator. This is natural since even after the changes women do not suffer from deprivations in the relevant dimensions, the extent of global female disadvantage remains unaltered.

To illustrate this axiom, consider the set $Q_H = \{LI, LE, IN\}$ of three HDI dimensions. Suppose the corresponding disparity profile is $z_H = (1, 1.1, 1)$. If we reduce the ratio for LE from 1.1 to 1.05, say, then the resulting profile of discrepancy ratios becomes $z'_H = (1, 1.05, 1)$. This reduction in the ratio 1.1 may be a consequence of an increase in male life expectancy or a reduction in female life expectancy or both. The weak focus axiom stipulates that $F(z_H) = F(z'_H)$.

In the next axiom we consider changes in at least one favourable dimensional achievement of women such that the change does make them disadvantageous in the dimension. Formally,

Strong Focus: For all $d \in N$, if $z, y \in \Re_{+}^{d}$, where $z \neq y$, are such that $Q_{ND}(z) = Q_{ND}(y)$ (hence $Q_D(z) = Q_D(y)$), $z^D = y^D$; then $F(z) = F(y)$, where the sub-vectors y^D and z^D of y and z respectively correspond to the dimensions in $Q_D(y)$.

According to strong focus, changes in one or more ratios in non-deprived dimensions of women that do not make them deprived do not alter the index value. Thus, in the strong version of the focus axiom, we do not put any restriction on the number of deprived/non-deprived dimensions of women. That is, the possibility that some of the dimensions may be disadvantageous for women is not excluded. The equality $Q_D(y) = Q_D(z)$ ensures that the sets

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[1]The strong version of the poverty axiom was considered also by Tsui (2002).

of deprived dimensions corresponding to y and z are the same. The equality $z^D = y^D$ means that dimension-by-dimension relative achievements of females in the common deprived dimensions associated with the profiles y and z are the same. Consequently, $z \neq y$ implies that the relative achievements of women in at least one non-deprived dimension corresponding to y and z are different.

This axiom has an interesting implication with respect to the trade-off between achievement ratios in two dimensions of which one is deprived but the other is non-deprived. That is, if the level of achievement ratio in a non-deprived dimension of women is reduced such that the resulting quantity does not make them deprived in the dimension then in exchange of this reduction the females are not made better off in a deprived dimension. Consequently, trade-off between two relative achievements of women in two different dimensions of which one is deprived but the other is non-deprived is not possible. This definitely does not rule out potentiality of trade-off between two deprived dimensional quantities. The two focus axioms do not as well claim that the disadvantage index is independent of the number of non-deprived dimensions.

To illustrate, consider $\hat{z}_H = (0.9, 1.1, 0.8)$ where females are non-deprived only in dimension 2 (life expectancy). A new vector $\bar{z}_H = (0.9, 1.05, 0.8)$ of proportions is obtained from $\hat{z}_H$ by reducing only achievement level in dimension 2 from 1.1 to 1.05 so that women are still non-deprived in the dimension. Then the strong focus axiom demands that this alteration does not change the quantity of overall female disadvantage. More precisely, $F(\hat{z}_H) = F(\bar{z}_H)$. Observe that in this example, $\hat{z}_H^D = (0.9,\ 0.8) = \bar{z}_H^D$.

3.2. Distributional Axioms

The axioms we analyse in this subsection indicate shifts in the values of a disadvantage indicator under particular types of changes in the relative profiles.

Suppose there are reductions in the achievement proportions in one or more deprived dimensions of women. Evidently, cutbacks in deprived dimensions' achievement ratios are undesirable from female disadvantage alleviation policy perspective. Consequently, female disadvantage should go up under such cutbacks.

The following postulate states this formally using FMA ratios.

Monotonicity: For all $d \in N$, $z \in \Re_+^d$, suppose $y \in \Re_+^d$ is obtained from z as follows:

(i) $y = z - \delta$, where $\delta = (\delta_1, \delta_2, ..., \delta_d) \in \Re_+^d$;

(ii) $\delta_i \leq z_i - 1$, If $z_i \geq 1$;

(iii) for all $z_i < 1$, $\delta_i \geq 0$ with $>$ for least one i .

Then $F(z) < F(y)$.

The monotonicity axiom demands that worsening of the disadvantageous position of women in one or more dimensions increases the value of the index. Condition (i) of the axiom means that the profile y is deduced from the profile z by reducing the dimension-by-dimension FMA ratios by non-negative components in $(\delta_1, \delta_2, ..., \delta_d)$. Condition (ii) ensures that if a dimension is non-deprived for women, then the corresponding subtrahend in $(\delta_1, \delta_2, ..., \delta_d)$ is at most $z_i - 1$. This ensures that the non-deprived dimensions in z do not become deprived in y. According to condition (iii), for all deprived dimensions all the associated subtrahends in $(\delta_1, \delta_2, ..., \delta_d)$ are non-negative and for at least one such dimension the subtrahend is positive so that in this dimension deprivation of women deepens. Since we assume at the outset that $y \in \Re_+^d$, it is implicitly assumed that $y_i = z_i - \delta_i \geq 0$ for all $i \in Q$. All the above conditions jointly make sure that the FMA ratios in the non-deprived dimensions of y and z are identical and y has lower FMA ratio than z in least one deprived dimension. The monotonicity axiom claims that y should have higher female disadvantage than z . It may be worthy to note here that all the variables are in ratio scale, including the change $\delta = (\delta_1, \delta_2, ..., \delta_d) \in \Re_+^d$. Thus, the changes considered in the monotonicity axiom indicate the changes in the ratios.

The profile $\tilde{z}_H = (0.85, 1.05, 0.7)$ is deduced from $\bar{z}_H$ by reducing FMA ratios in dimensions 1 and 3, two deprived dimensions for females by 0.05 and 0.1 respectively. The monotonicity axiom demands that $F(\tilde{z}_H) > F(\bar{z}_H)$.

In poverty analysis, the monotonicity axiom is specified in terms of reduction in achievement below the threshold in only one dimension. But in the current set up all the dimensions are considered simultaneously. Therefore the current formulation is more general.

The next axiom, which parallels the Alkire-Foster (2011) and Foster et al. (2015) multidimensional poverty dimensional monotonicity axiom, deals with the effect of increasing the number of deprived dimensions of women. It requires that when a non-deprived dimension of women, who are not deprived in all the dimensions, becomes deprived, then their disadvantageous position gets deeper.

Formally,

Dimensional Monotonicity: For all $d \in N$, $z \in \Re_+^d$, where $z_i \geq 1$ for least one $i \in Q$, suppose $y \in \Re_+^d$ is obtained from z as follows:

$(i) y_i < 1 \leq z_i$ for at least $i \in Q$ with $z_i \geq 1$;

$(ii) y_k = z_k$ for all $k \in Q / \overline{Q}$, where $\overline{\mathrm{Q}} = \{i \in Q | y_i < 1 \leq z_i\}$ and $Q / \overline{Q}$ is the complement of Q with respect to $\overline{Q}$.

Then $F(z) < F(y)$.

In the above axiom, not all dimensions are deprived for women in the profile z. Now, at least one of their non-deprived dimensions in z becomes deprived for them in y (stipulation (i)). According to stipulation (ii), in all the remaining dimensions their FMA ratios in y and z are identical. Consequently, in y women are being relegated to a more deprived situation than in z. Hence it is reasonable to argue that female disadvantage should rise.

While Monotonicity restricts attention to a particular disadvantaged dimension, Dimensional Monotonicity deals with the situation when an advantaged dimension becomes disadvantaged. For instance, if we deduce the profile $\overline{y}_H = (0.9, 0.98, 0.8)$ from $\overline{z}_H$ by making women deprived in dimension 2, which was the only non-deprived dimension for them in $\overline{z}_H$, then dimensional monotonicity postulate sensibly argues that $\overline{y}_H$ should indicate more female disadvantage than $\overline{z}_H$.

In the poverty and inequality literature, one of the most important points is distributional transfers. In our case transfers do not make sense. However, the important aspect of sensitivity can be recast here as what we call aggravation sensitivity that considers all the dimensions simultaneously.

To introduce the axiom, let $\tilde{y}_H = (0.8, 1.05, 0.6)$ be deduced from $\tilde{z}_H$ by applying the same transformation that takes us from $\overline{z}_H$ to $\tilde{z}_H$. The composite transformation that enables us to generate $\tilde{y}_H$ from $\overline{z}_H$ is done sequentially, first from $\overline{z}_H$ to $\tilde{z}_H$ and then from $\tilde{z}_H$ to $\tilde{y}_H$. Thus, at the first stage of the transformation we are making women more deprived in two deprived dimensions and at the second stage we are increasing the intensity of deprivations further. Since female deprivation is highly socially undesirable, this further worsening of female living conditions should be assigned higher weights in the assessment of overall female disadvantage. This is formally written as

Aggravation Sensitivity:

For all $d \in N$, $z \in \Re^d_+$, suppose $y,u \in \Re^d_+$ are obtained from z as follows:

$(i)\, y = z - \delta$, where $\delta = (\delta_1, \delta_2, ..., \delta_d) \in \Re^d_+$, $\delta_i = 0$ if $z_i \geq 1$; for all $z_i < 1$, $0 \leq \delta_i \leq \frac{z_i}{2}$ with $\delta_i > 0$ for least one i;

$(ii)\, u = y - \delta = z - 2\delta$, where $\delta = (\delta_1, \delta_2, ..., \delta_d) \in \Re^d_+$, is the same as in (i) .

Then $F(y) - F(z) < F(u) - F(y)$.

The first condition here is the combined form of conditions $(i)-(iii)$ of the monotonicity axiom. According to the second condition, for any $i \in Q_D$ if δ_i is positive in (i), then it is positive in (ii) as well. Hence, whenever relative achievement of women in some deprived dimension is reduced by some quantity in (i) , then it is further reduced by the same level in (ii) Since $y,u \in \Re^d_+$, it is assumed at the outset that $d-$dimensional non-negative vector δ does not make any component of the profiles y and u negative.

The sensitivity postulate requires that higher weights are assigned to the dimensional deprivations where women are more disadvantaged relative to men. All indicators that linearly combine the proportional achievements are violators of this axiom, although they may satisfy the monotonicity axiom. As a result they are unable to identify the dimensions that are more severely affected when achievement ratios in deprived dimensions are reduced. We will see later that the RSW index is a violator of this axiom.

This aggravation sensitivity postulate which puts naturally higher weights to more deprived dimensions, has a great appeal from policy perspective. All indicators satisfying the axiom guarantee that targeting the dimensions with the poorest relative positions is an efficient intervention in the alleviation of the disadvantage.

Further, it has a clear intuitive appeal. The axiom, therefore, can be said to be a new addition to the literature. Our axiomatic approach generalizes the domain of disadvantage indices for a social group. (See, Kakwani, 1980, Lasso de La Vega and Urrutia, 2012 and Chakravarty, 2018, for related formulations, involving only one dimension, in the context of poverty analysis.)

3.3. Decomposability Axiom

A social planner may demand that relatively higher importance be given to the dimensions where female are more deprived. Thus identification of the dimensions in which female disadvantages are relatively more will be a desirable requirement. In order to incorporate this we regard the construction of a social female disadvantage index as only the preliminary step of our exercise in the current context. More precisely, we need to look at the contributions of the individual dimensions to overall female disadvantage. This in turn enables us to isolate the dimensions in which women are more afflicted by unfavourable circumstances. A disadvantage index satisfying this property may be called factor decomposable (see Chakravarty, 2003, 2018). Such a summary indicator becomes useful for a policy analyst who is interested in identifying those dimensions of well-being in which women are more disadvantage-stricken, and in implementing disadvantage elimination/reduction strategy.

The breakdown of the overall disadvantage index across subgroups of dimensions is subsumed through the axiom of factor decomposability. Consider a partitioning of the set of dimensions Q into k subgroups $Q^1, Q^2, \ldots, Q^k$ with respect to some characteristic, where the number of dimensions in Q^i is given by $d_i, 1 \le i \le k$. Thus, for $i \ne j$, $Q^i \cap Q^j$ is empty, $\bigcup_{i=1}^{k} Q^i = Q$ and $\sum_{i=1}^{k} d_i = d$. For instance, we can partition the set $Q_H = \{LI, LE, IN\}$ of HDI dimensions into income and non-income dimensions as $Q_H^{IN} = \{IN\}$ and $Q_H^{NIN} = \{LI, LE\}$.

We assume that the society attaches different positive weights w_i s to different dimensions, given by $w_1, w_2, \ldots, w_d$ such that $\sum_{i=1}^{d} w_i = 1$. The weight w_j may be interpreted as the importance that a policy maker assigns to the *jth* dimension for reducing/eliminating female disadvantage through this dimension. By assigning positive weights to each dimension we assume implicitly that all the d dimensions are important. Magnitudes of the weights reflect the importance of the associated dimensions, say, as judged by the administration/policymaker. It is then quite natural that if the set of dimensions Q is partitioned into k distinct subgroups $Q^1, Q^2, \ldots, Q^k$, the subgroups will have the corresponding importance/weights as $w^1, w^2, \ldots, w^k$, where $w^j = \sum_{i \in Q^j} w_i$. Any $z \in \Re_+^d$ can now be written as $z = (z^1, z^2, \ldots\ldots, z^k)$, where $z^j = (z_{j_1}, z_{j_2}, \ldots\ldots, z_{j_d})$ is the component of the vector z that corresponds to the dimensions in the set $Q^j, 1 \le j \le k$. Thus, for the subgroups $Q_H^{IN} = \{IN\}$ and $Q_H^{NIN} = \{LI, LE\}$ of the set $Q_H = \{LI, LE, IN\}$, the components of the vector $\bar{z}_H = (0.9, 1.05, 0.8)$ that reflect the partition are $\bar{z}_H^{IN} = (0.8)$ and $\bar{z}_H^{NIN} = (0.9, 1.05)$.

Factor Decomposability: For all $d \in N$, $F(z) = \sum_{j=1}^{k} w^j F(z^j)$, where $z = (z^1, z^2, \ldots, z^k) \in \Re_+$, $w^j > 0$ is the weight assigned to the relative profile z^j and $\sum_{j=1}^{k} w^j = \sum_{i=1}^{d} w_i = 1$.

This property maybe regarded as a disadvantage counterpart of the multidimensional poverty factor decomposability axiom. (See Chakravarty, 2018, for a recent discussion.) It says that overall female disadvantageis the weighted average of *subgroup* disadvantage values, where weight attached to a subgroup is the sum of individual weights of the dimensions belonging to the subgroup. Under equal weights the jth subgroup weight is $\frac{d_j}{d}$, the proportion of dimensions belonging the jth subgroup Q^j and $F(z^j)$ is its disadvantage level.

This type of breakdown of overall female deprivation enables us to evaluate the impact of subgroup deprivation levels on the overall value. In other words, one can determine the contribution of a subgroup's female deprivation to the overall female deprivation of the society. The contribution of subgroup i to global deprivation is $w^i F(z^i)$. Since the weights w_i s are given, a reduction of disadvantage along any dimension in a subgroup will require increment in the female-male achievement ratio in that dimension of the subgroup. Consequently, contributions of the type $w^i F(z^i)$ made by different subgroups become helpful in isolating subgroups that are more stressed by female-male discrimination and hence to implement anti-discrimination policy on narrower domains. Particularly, any subgroup i with low weight but high disadvantage proportion $F(z^i)$ should get top priority from such a perspective.

By repeated application of the axiom, we have, $F(z) = \sum_{i=1}^{d} w_i f(z_i)$, where $f(z_i) = F(z_i)$ represents the disadvantage suffered by women in the i^{th} dimension. We refer to $f : \Re^1_+ \to \Re^1_+$ as the single dimensional disadvantage function. Breakdown of the indicator F by dimensions empower us to locate which of the dimensions considered contribute more to overall disadvantage. Elimination of female deprivation in the i^{th} dimension will reduce social female disadvantage exactly by the amount $w_i f(z_i)$.

Thus, under factor decomposability the overall index of disadvantage is a sum of components of disadvantage contributed individually through every dimension. The components are comparable through the single evaluation function $f(.)$, the quantum of contributions varying through the nature of $f(.)$.

Note that if equal weights are given to each dimension, that is, $w_i = \frac{1}{d}$ for all i, then $F(z) = \frac{1}{d}\sum_{i=1}^{d} f(z_i)$ satisfies a dimensional anonymity axiom which says that it remains invariant under any reordering of female-male achievement ratios. Formally,

Anonymity: For all $d \in N$, $z \in \Re_{+}^{d}$**,** if y is a reordering of z then $F(z) = F(y)$.

According to this desideratum, any characteristic other than dimensional achievement ratios, such as the names of men and women, their marital statuses, should not be treated as relevant to the measurement of overall disadvantage.

It may further be noted that Bourguignon and Chakravarty (2003) did not consider aggregation sensitivity and in their factor decomposability axiom the dimensions had equal weights. They did not characterize the index. Only the strong focus axiom used here is a direct application of the existing counterpart in the poverty literature. Hence the characterization developed is a new one.

3.4. Technical Axioms

The first of the two axioms presented in this subsection, normalization, is a cardinality principle. It claims that if women are placed identically as men in all dimensions, then the index attains the minimum value, zero. On the other hand if women are maximally deprived in all the dimensions, that is, the female relative achievements across dimensions are zero, then the index takes on the value one. The second axiom is a continuity property; it requires that the disadvantage index is continuous in its arguments. This predicate ensures that minor observational errors in one or more achievement ratios will generate only a minor change in the value of the index.

Normalization:

(i)For all $d \in N$ if $z \in \Re_{+}^{d}$ is such that $z_i = 1$ for all $i \in Q$, then $F(z) = 0$.

(ii)For all $d \in N$ if $z \in \Re_{+}^{d}$ is such that $z = 0$ for all $i \in Q$, then $F(z) = 1$.

Continuity: $F : \Re_{+}^{d} \to \Re_{+}^{1}$ is a continuous function.

Continuity ensures smooth behaviour of the index with respect to changes in achievement levels.

4. The Characterization Theorem

We now characterize the entire family of female disadvantage indices using some of the axioms analysed in the earlier section.

Theorem 1: A female disadvantage index $F:\Re_{+}^{d}\rightarrow\Re_{+}^{1}$ satisfies the strong focus, aggravation sensitivity, factor decomposability, normalization and continuity axioms if and only if

$$F(z)=\sum_{i=1}^{d} w_i f\left(z_i^*\right) \text{, with } w_i>0 \text{ and} \sum_{i=1}^{d} w_i=1, \qquad (2)$$

where $d\in N$ and $z\in\Re_{+}^{d}$ are arbitrary, the single dimensional female disadvantage function $f:\Re_{+}^{1}\rightarrow\Re_{+}^{1}$ is continuous, $f(0)=1$ and $f(1)=0$; and f is strictly convex over the interval $[0,1]$.

Proof: See Appendix.

In the following proposition we demonstrate an implication of strict convexity of a function $g:[0,1]\rightarrow R_{+}^{1}$ along with the conditions that $g(0)=1$ and $g(1)=0$.

Proposition 1: Let $g:[0,1]\rightarrow R_{+}^{1}$ with $g(0)=1$ and $g(1)=0$ be strictly convex. Then g is strictly decreasing over $[0,1]$

Proof: See Appendix.

While factor decomposability makes the disadvantage index additive across dimensions, we need the other axioms to understand the properties of the single dimensional index. Note that in Theorem 1 we did not use the monotonicity axiom, which requires strict decreasingness of f over $[0,1]$.The normalization principle demands that $f(0)=1$ and $f(1)=0$. Hence in view of Proposition 1 we can claim that the general formula F in equation (2) fulfils the monotonicity postulate. Thus, monotonicity follows as an implication of factor decomposability, normalization and aggravation sensitivity.

It may be worthwhile to observe that we also did not make use of the dimensional monotonicity axiom in the proof of the theorem. However, it is easy to verify that our general index in (2) satisfies this appealing postulate.

It will certainly be justifiable to investigate whether the axioms employed in the demonstration of Theorem 1 are independent or not. By independence here we mean that if we drop one axiom, then the remaining axioms will not be able to characterize the index given by

(2) and we have a minimal set of axioms which is isomorphically mapped to the derived family of the disadvantage index.

Independence will formally require that if one axiom is given up, then we can find an example of a disadvantage index that will not satisfy the discarded axiom but will satisfy the remaining ones so that it does not belong to the family specified in (2). The following theorem demonstrates this.

Theorem 2: The strong focus, aggravation sensitivity, factor decomposability, normalization and continuity axioms are independent.

Proof: See Appendix.

Since for any $i \in Q$, $z_i = 1$ represents parity between the two sexes' achievements in the concerned dimension, in the Figure 1 below, the lines $z_1 = 1$ and $z_2 = 1$ may be regarded as lines of parity in dimensions 1 and 2 respectively. A parity line in a dimension is a line of demarcation that separates the advantageous position of women from their disadvantageous position in the dimension. The area enclosed between the two parity lines and the two axes gives us the 2-dimensional female disadvantage space. It is easy to check that a female disadvantage index satisfying strong focus, aggravation sensitivity, normalization, factor decomposability and continuity has a downward sloping strictly convex iso-disadvantage contour in the 2-dimensional disadvantage region. A lower contour in the region represents a higher level of disadvantage level.

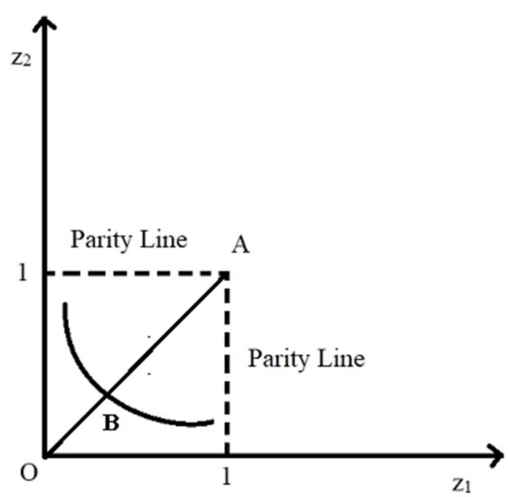

Figure 1: Iso-disadvantage contour of the female.

The index takes on the maximum value one if female achievements are zero in all the dimensions, that is, at the origin in the figure. The 45-degree line connecting the origin O $(z_1 = 0, z_2 = 0)$ and the point A $(z_1 = 1, z_2 = 1)$ intersects the given disadvantage contour at B. Thus, OB represents the minimum distance between the origin, where the disadvantage index assumes its maximum value, and the contour that shows the actual level of disadvantage. Consequently, we may regard the distance OB as the level of female advantage existing in this two-dimensional situation. A reduction in the distance between the point of intersection B and the origin O will increase disadvantage and hence reduce advantage.

From equation (2), under equal weights, it follows that dimension-wise female disadvantage indices constituting the $d-$ dimensional dashboard $\left(f(z_1^*), f(z_2^*), \ldots\ldots, f(z_d^*)\right)$ are averaged across dimensions to get the aggregate disadvantage quantity $F(z) = \frac{1}{d}\sum_{i=1}^{d} f(z_i^*)$. The aggregation followed in the current contexts parallels the aggregations employed to generate the multidimensional mean logarithmic deviation inequality index from the associated dashboard (Tsui, 1999), and the Gajdos-Weymark (2005) multidimensional absolute Gini inequality index from the related dashboard. This aggregation follows the normative assessment that all the dimensional metrics are equally important (see Decancq and Lugo, 2012, for discussion on alternative weighting schemes).

In order to illustrate the general formula specified in (2), assume that $f(t) = 1 - t^e$, where $0 < e < 1$ is a parameter. The resulting index turns out to be

$$F_e(z) = \frac{1}{d}\sum_{i=1}^{d}\left(1 - (z_i^*)^e\right) = \sum_{i=1}^{d} w_i\left[1 - \left(\left(\frac{x_{fi}}{x_{mi}}\right)^*\right)^e\right]. \qquad (3)$$

This index may be regarded as the female disadvantage twin of the single dimensional subgroup decomposable Chakravarty (1983) index. As the value of e decreases over $(0,1)$, F_e attaches higher weights to dimensions that are more severely affected by disadvantages. For $e = 0$, F_e becomes 0 irrespective of how high or low disadvantages are in the deprived dimensions. For $e = 1$ and $w_i = \frac{1}{d}$ for all i, it coincides with the Dijkstra-Hanmer (2000) index (see Section 5).

An alternative of interest that arises from the specification $f(t) = (1-t)^{\alpha}$, where $\alpha > 1$ is a constant, may be treated as the female disadvantage counterpart of the Foster-Greer-Thorbecke (1984) uni-dimensional poverty index. The explicit formula is given by

$$F_\alpha(z)=\frac{1}{d}\sum_{i=1}^{d}\left(1-z_i^*\right)^\alpha=\sum_{i=1}^{d}w_i\left(1-\left(\frac{x_{fi}}{x_{mi}}\right)^*\right)^\alpha. \tag{4}$$

For $0<\alpha\leq 1$, F_α is a violator of Aggravation Sensitivity but not of other axioms. In the limiting situation, as $\alpha\to 0$, F_α approaches the proportion of dimensions in which women possess disadvantages. We refer to this index as the headcount ratio measure of female disadvantage, which is as well a violator of aggravation sensitivity. With an increase in the value of α from 1, higher valuations are assigned to the dimensions that are distressed by disadvantages at greater extents. For $\alpha=1$ and $w_i=\frac{1}{d}$ for all i, F_α becomes the Dijkstra-Hanmer (2000) index.

5. A Brief Review of the Existing Measures of Female Disadvantage

The objective of this section is to make a systematic comparison of the measures of female disadvantage suggested in the literature with the new general index proposed in the present paper. All the indicators suggested so far in the literature are indices of advantage; they aggregate relative achievements of the female in different dimensions. Since in this article we are dealing with female disadvantage, we consider the disadvantage counterpart of each of them and make the comparison. Further, we consider the censored ratio profiles so that the comparison becomes valid.

According to Dijkstra-Hanmer (2000), the average $\frac{1}{d}\sum_{i=1}^{d}\left(\left(\frac{x_{fi}}{x_{mi}}\right)\right)$ of relative achievements of female is an indicator of relative status of women (RSW). This is, in fact, the first of the few indices of female advantage that have been proposed so far. The associated index of female disadvantage may be defined as

$$F_{DH}(z)=\frac{1}{d}\sum_{i=1}^{d}\left(1-z_i^*\right)=\frac{1}{d}\sum_{i=1}^{d}\left(1-\left(\frac{x_{fi}}{x_{mi}}\right)^*\right). \tag{5}$$

It drops out as a particular case of F_e and F_α corresponding to $e=1$ and $\alpha=1$ respectively when $w_i=\frac{1}{d}$. Hence because of linearity of $f(t)=1-t^e$ for $e=1$ and $f(t)=t^\alpha$ for $\alpha=1$, F_{DH} becomes a violator of the aggravation sensitivity property, although it satisfies all the remaining postulates, including both the monotonicity and dimensional monotonicity axioms. It verifies anonymity as well. The increase in the Dijkstra-Hanmer index following a reduction in a deprived dimension's achievement proportion will be the same irrespective of how high or low the achievement proportion is. One may note that in our framework RSW was generalised to allow differential weights. Our formulation is a general class where all such extensions of RSW can be accommodated.

Klasen and Schüler (2011, p.5) referred to RSW as a composite measure of gender inequality. However, as pointed out by Anand (2018), RSW is simply an indicator of female disadvantage but not a measure of gender inequality. This is because a quantifier of gender inequality aggregates dimension-by-dimension gaps between male and female achievements symmetrically (see also Anand and Sen, 1995 and Schüler, 2006). In contrast, the RSW index is concerned only with how unfavorable are the dimension-wise positions of females in comparison with the corresponding positions of males.

For all the remaining indices to be analysed in the section, we assume that the domain of definition of F is D, where, $D = \bigcup_{n \in N} D^n$. Thus, for any $n \in N$ and $z \in D^n$, $z_i > 0$ for all $i = 1,2,\ldots,n$. We restrict the domain because if the relative achievements in one or more of dimensions become zero, then the measure may take its highest value regardless of performances in other dimensions. Some of them may even be undefined with a zero relative achievement.

Klasen and Schüler (2011) suggested the use of the product $\prod_{i=1}^{d}\left(\left(\frac{x_{fi}}{x_{mi}}\right)^{*}\right)^{\frac{1}{d}}$ as a gender gap measure. Since this product aggregates the relative achievements of women in an unambiguous way, we define the related female disadvantage index $F_{KS}(z)$ as the complement of this product from unity. Formally,

$$F_{KS}(z) = 1 - \prod_{i=1}^{d}\left(\left(\frac{x_{fi}}{x_{mi}}\right)^{*}\right)^{\frac{1}{d}}. \qquad (6)$$

An important difference between the measures in (5) and (6) is that while the former allows perfect substitution between ratios of dimensional achievements, the latter does it in an imperfect way. In this sense the anonymous index F_{KS} is a better indicator of female disadvantage. However, while the factor decomposable evaluator F_{DH} allows uses of zero observations, F_{KS} does not permit this. This is definitely a high undervaluation of women deprivation by F_{KS}.

Part (ii) of the normalization axiom does not apply to F_{KS} since it is defined on D.This indicator does not meet factor decomposability, although its complement from unity is decomposable in logarithms or in percentage changes. But decomposition in logarithms of proportional achievements is different from what is desired under factor decomposability. Taking logarithms does not enable us to identify the contribution of each component of the index. The measure, however, fulfills the other postulates including aggravation sensitivity.

Permanyer (2013) considered the variants $P_{GRS}(z)=\prod_{i=1}^{d}\left(\frac{x_{fi}}{x_{mi}}\right)^{w_i}$ and $P_{WD}(z)=\prod_{i:x_{fi}<x_{mi}}\left(\frac{x_{fi}}{x_{mi}}\right)^{w_i}$ of the product $\prod_{i=1}^{d}\left(\frac{x_{fi}}{x_{mi}}\right)^{\frac{1}{d}}$ as indices of gender relative status and women disadvantage respectively, where the weights $w_i s$ are positive and add up to 1. The former is the Klasen -Schüler (2011) gender relative status index without censoring. Hence it involves ratios that are both greater and less than one. Evidently, $P_{WD}(z)\leq P_{GRS}(z)$. The two indices coincide if $x_{fi}\leq x_{mi}$ for all $i\in Q$.

By considering the censoring profile z^* associated with the profile z, we can convert $P_{WD}(z)$ into a female disadvantage index defined as $F_P(z)=1-\prod_{i=1}^{d}\left(\left(\frac{x_{fi}}{x_{mi}}\right)^*\right)^{w_i}$. Given that the domains of definition of this index and our characterized index are the same, we can make a systematic comparison between the two indices. This form of WD may be regarded as a multiplicative version of a member of our characterized index. This may be established by noting that if in (3) (respectively (4)) we replace arithmetic averaging by geometric averaging and e (respectively α) by the dimensional weights $w_i s$, then (3) (respectively (4)) coincides with F_P. In addition to factor decomposability, F_P violates anonymity unless $w_i s$ are the same.

Anand (2018) investigated properties of the symmetric mean of order $\theta<1$ of female relative achievements:

$$A_\theta(z)=\begin{cases}\left(\frac{1}{d}\sum_{i=1}^{d}z_i^{\theta}\right)^{\frac{1}{\theta}},\theta<1,\theta\neq 0,\\ \prod_{i=1}^{d}(z_i)^{\frac{1}{d}},\theta=0.\end{cases} \quad (7)$$

A_θ coincides with the arithmetic, geometric and harmonic means of relative achievements for $\theta = 1, 0$ and $\theta = -1$ respectively. As the value of θ reduces, higher weights are assigned to lower relative achievements. Anand (2018) analysed the marginal rate of substitution between proportional achievements in two different dimensions for both uncensored and censored distributions. He argued that there is some clear merit in the censored distribution, particularly, in terms of understanding and exposition. But by truncating all relative achievements above 1 by 1 ignores any increment to this value. Now, in measuring female disadvantage it is reasonable to consider the dimensions in which men outperform women. Further, given that the problem is multidimensional, it is probably sensible not to allow trade-off between achievements in dimensions in which women are better off and worse off. This justifies the use of censored profiles. This is a debatable issue and for parallel discussion in the case of multidimensional poverty, see Chakravarty (2018). The disadvantage sister of the anonymous index A_θ is a violator of the factor decomposability unless $\theta = 1$, although this case treats dimensional achievement ratios as perfect substitutes. For $\theta < 1$, the trade-off (marginal rate of substitution) between any two dimensions i and j depends on the ratio $\frac{z_j}{z_i}$, which means that in this case the two dimensions are not perfect substitutes. The same is true for the corresponding cases of (3) and (4). (See Chakravarty, 2011. See also Ravallion, 2012, for an excellent discussion along this line.)

One general observation about our structure is that the axiomatic approach brings several indices under one umbrella. Choice of a proper evaluation function $f(.)$ can lead to a new measure. The axioms also bring out the pros and cons of using an index.

It will now be worthwhile to make a systematic comparison between the female disadvantage metric and the Gender Development Index (GDI) that looks at imbalance between female and male achievements from a different perspective (see Anand and Sen, 1995, and UNDP, 2015). It is a measure of the extent of gender disparity with respect to achievements in the three dimensions incorporated in the human development index formulation, namely health, measured by male and female life expectancy at birth; education, determined by male and female expected years of schooling for children and male and female mean years schooling for adults with ages 25 years and more; and income (command over economic resources), measured using male and female estimated earned income. For the sake of completeness and comparison, we briefly discuss the GDI.

Given that we have well-defined maximum and minimum values of achievement in each of the three dimensions for each gender, we define sex-wise dimensional indicators as

$$I_{ij} = \frac{actual_{ij} - \min imum_{ij}}{\max imum_{ij} - \min imum_{ij}}, \qquad (8)$$

where $i = male(m), female(f)$ and $j = Health, Education, Income$. The Human Development Index (HDI) values for female and male are defined by taking the symmetric geometric mean of the respective dimensional indicators. Formally,

$$HDI_f = \left(I_{f\,Health}.I_{f\,Education}.I_{f\,Income}\right)^{\frac{1}{3}} \qquad (9)$$

and

$$HDI_m = \left(I_{m\,Health}.I_{m\,Education}.I_{m\,Income}\right)^{\frac{1}{3}}. \qquad (10)$$

The GDI is defined by taking the ratio between the HDI values of female and male. More precisely,

$$GDI = \frac{HDI_f}{HDI_m}. \qquad (11)$$

For the GDI to be well-defined we assume at the outset that its denominator is positive. Then the GDI becomes bounded from below by 0, where the lower bound is achieved whenever the achievement for female in a dimension takes on its minimum value, however small or large it may be.[2] It takes on the value 1 if the HDI values across sexes are equal. A value of GDI greater than 1 simply means that, on an average, women are better off than man. While the GDI considers the ratio between two summary statistics of category-wise achievements, female disadvantage focuses directly on achievement ratios. The notion of factor decomposability, when appropriately reformulated, for the purpose of identifying those dimensions that are contributing less to gender development, is not applicable to the form of GDI given by (11). (See Chakravarty, 2003, for a discussion on this notion of factor decomposability.) The two disadvantage focus axioms and dimensional monotonicity axiom do not have their GDI counterparts. There are no a priori reasons to believe that the female disadvantage index can

[2]Ravallion (2012) made an excellent discussion along this line.

be retrieved from GDI in an unambiguous way so that information on the GDI enables us to get an idea about the female disadvantage index and vice-versa.

6. An Empirical Illustration

The empirical illustration of several indices we introduced above uses freely downloadable data from the UNDP Human Development Reports website (http://hdr.undp.org/en/data) disaggregated by gender, corresponding to the three dimensions of the HDI: literacy (LI), life expectancy (LE) and income (IN). For LI we use mean years of schooling for adults aged 25 and older, for LE information available on life expectancy at birth is adopted and for IN we rely on data on estimated gross national income per capita in 2011 purchasing power party (PPP). The data are only available for the years 2000, 2005 and then yearly for 2010-2018, restricting the time-frame of our analysis. This illustration is based on the most recent year, 2018. Since our illustration relies on the UNDP data, following UNDP we choose the identical weight $1/3$ for the dimensions under consideration. As discussed above, our theoretical structure allows arbitrary number of dimensions in addition to what we chose for our illustration that is based only on those dimensions on which we have readily available information. The gender differences in the three components of the HDI have been analysed in several contributions including Klasen and Schüler (2011), Permanyer (2010, 2013).

Gender is a key element of human development and UNDP dedicated the 1995 Human Development Report to the topic, analysing the progress made by countries in reducing gender disparities. We here follow this tradition and base our empirical illustration on the HDI dimensions. In particular, we aim to compare how the countries perform with respect to the standard HDI and the female comparative disadvantage approach we introduce. In the latter, instead of looking at the analysis of absolute achievements we focus only on the deprivations of the female with respect to achievements in different dimensions of human well-being in comparison with the corresponding achievements of the male.

For all the 189 included countries, we first compute the censored FMA ratio profile. The information for the three dimensions is not available for all the countries. We decided to drop the countries with missing information in 2018 so that we are left with a sample of 170 observations.

We first focus on the individual dimensions underlying the aggregate measure and plot the censored FMA ratio profiles in Figure 2. Here the countries are ordered with respect to their 2018 HDI ranks, where Norway is in the first position and Niger is in the last. We group the countries based on respective levels of human development, following the UNDP criterion. In Figure 2, we stack the FMA ratio profiles in literacy (bottom), life expectancy (middle), income (top) and highlight in black the complement to 1, that is, the disadvantage suffered by women in that specific dimension. As expected, most of the variation among countries is present in the income dimension, which ranges from 0.63 reached by Yemen, to 1 observed for Burundi and Liberia. It should be noted that these are censored ratios, and in both Burundi and Liberia the female achievements in the income dimension, even if low in absolute terms compared to that of other countries, are higher than their male counterparts. Female disadvantage in income is widespread: from Panel A of Figure 2 we note that even the countries with very high human developments, show values of the censored FMA ratios considerably below the maximum value of 1. This is highlighted by the black bars, confirming that gross national income per capita of women is much lower than that of men.

Many countries achieve the maximum value of 1 in the literacy dimension (LI), while its minimum is attained in Afghanistan (0.317), followed by Chad (0.362). As clear from the four panels of Figure 2, LI values decrease by level of human development (with black bars increasing considerably in Panels 3 and 4 where the countries with medium and low human development are contained). Hence policy interventions to increase literacy among women would be more effective in countries with lower human development. (We will discuss this point in details later.)

Life expectancy does not vary among countries and all countries achieve a value of 1.Hence this component is not adding any variation to the overall ranking of countries in female disadvantage. This is a reassuring finding that shows that even if women achieve less than men in many dimensions of life implications of these are not demonstrated on their life expectancy.

We now proceed with the illustration of the indices we propose. The female disadvantage indices we compute are F_e in equation (3) for the midpoint of the parameter value $e = 0.5$ and, F_α in equation (4) for $\alpha = 1$, that corresponds to the Dijkstra and Hanmer (2000) index, to measure the female disadvantage in terms of proportionate shortfalls of female achievements from their male counterparts and for $\alpha = 2$ to quantify the dimension-wise sensitivity of female disadvantage. As expected from the above analysis of the dimensions of the indices, there is no country achieving the best possible situation of non-deprivation among gender groups. There are countries that perform well enough with the minimum value being close to zero, as displayed in Table 1.

Table1: Summary statistics of three of the proposed indices in 2018.

Index	**Mean**	**Std. Dev.**	**Min**	**Max**
F_e for $e=0.5$	0.107	0.059	0.026	0.364
F_α for $\alpha=1$	0.182	0.086	0.050	0.502
F_α for $\alpha=2$	0.084	0.070	0.008	0.400

As stated, the index proposed in the current contribution measures a situation in which the female are in a worse off position in comparison with the male in different dimensions of human well-being. We now aim to study its relation with human development in a standard sense, as measured by the HDI. In Figure 3 we plot the rankings of the countries by the three indices mentioned above and by the HDI of the same year, 2018, listing the countries according to their performance in the latter. We rank the countries in terms of gender parity, where in the first position is the country whose index is the lowest. The HDI rankings are standard and are provided directly by the HDRO, i.e. in the first position is the country whose HDI is the highest. As such they include all 189 countries. There appears to be no clear relationship between the rankings of the female disadvantage indices and the HDI: some countries, such as Norway, Sweden and Singapore, are on top of all indices, others, such as the UAE, Saudi Arabia and Oman, perform much worse in female disadvantage as compared to the HDI.

The three female disadvantage indices F_1, F_e for $e=0.5$ and F_α for $\alpha=2$, naturally, do not rank countries in a similar way. In Figure 4 we plot the rankings of the countries by the three indices mentioned listing the countries according to their performance in F_1 that corresponds to the Dijkstra and Hanmer (2000) index. We observe changes in the rankings by F_e for $e=0.5$ that are further accentuated when we consider F_α for $\alpha=2$.

One important feature emerges from our analysis related to Figures 3 and 4. A country's good position in HDI scale does not unambiguously establish that the females in the country have good standings with respect to achievements in the dimensions we have considered. Consequently, a separate policy evaluation becomes essential when we consider the positions of the female in terms of their achievements in the dimensions. In view of this, we restrict attention to the factor decomposability postulate and look at its policy implications in the current context.

Insert Figure 2A, 2B, 3, 4 here.

Taking advantage of factor decomposability of the indices, we can compute the percentage contribution of each dimension to the overall level of female disadvantage. This step is particularly relevant for policy issues, offering evidence-based recommendations on the dimension where action is most needed. This percentage measure indicates the amount by which female disadvantage will decrease if the disadvantage in this particular dimension is eliminated. To better visualise the results we again group the countries with respect to the human development classifications: Very High, High, Medium and Low Human Development. The results are contained in Figures 5A and 5B. In these figures we stack the three indices that have been computed for each country, F_e for $e = 0.5$ at the bottom (bars with no borders) followed by F_α for $\alpha = 1$ (bars with dotted borders) first and $\alpha = 2$ (bars with solid borders) last. As expected from the analysis of the values of the single dimensions, in all the countries life expectancy contributes zero to global disadvantage. The mean values of the percentage contribution for literacy are positive. We observe only 55 countries with zero percentage contribution and the elimination of disadvantages in literacy would reduce global disadvantage, on an average, for the entire sample between a minimum of 14.54% (F_α for $\alpha = 2$) and a maximum of 18.95% (F_α for $\alpha = 1$) for the three indices of this illustration. Focussing only on the countries with positive values, the mean contribution raises respectively to 21.49% and 28%. The most effective would be a policy aiming to eliminate disadvantages in the income dimension, where the only countries reaching gender parity are Burundi and Liberia. The percentage contribution of the income factor ranges, on an average, between a maximum of 85.46% (F_α for $\alpha = 2$) and a minimum of 81.05% (F_α for $\alpha = 1$) of the total disadvantage.

Insert Figure 5A and 5B here.

As evident from Figure 5A and Figure 5B the relative importance of the percentage contributions of literacy and income changes by level of human development. For the countries with very high and high levels of human development the source of female disadvantage is income, showing in the figures with most of the bars being only grey. Actually, in many countries including Norway, Ireland, Australia, Sweden, Denmark, Finland, Canada, USA, Japan, the income dimension is the only responsible for the presence of female disadvantage. Literacy, the red coloured bars, starts to matter more when we move down the ranking of the countries and becomes the most important factor for a few countries with medium level of human development and for the great majority of those with low development. Comparing the indices, we notice that the percentage contributions for the first two indices are very similar, while F_α with $\alpha = 2$ weighs more the factor where women perform worse, being income or literacy depending on the country.

7. Conclusions

We have contributed to the literature on the measurement of the relative position of women with respect to men by proposing a family of indices of female disadvantage that resembles very well-known indices of income poverty. The family has been characterized with a set of intuitive and simple axioms. The application to data from the UNDP data disaggregated by gender, corresponding to the three dimensions of the HDI, show that even some of the countries with high human development do not perform very well in terms of gender parity, another key factor of broader concept of human development. Behind this low performance are significant differences in the income dimension where men enjoy a clear comparative advantage. In 2018 female are globally not in a worse off position than men in life expectancy.

Our results show that there is still a long way to go to reach Goal 5 of the Agenda 2030 for sustainable development: achieve gender equality and empower all women and men.

While many countries could benefit from policies aiming at increasing the performance of women in income and education, there are some exceptions where there is clearly only one factor of disadvantage. In Burundi and Liberia the factor responsible of female relative disadvantage is education; among the countries with very high human development ranks in 2018, such as Norway (position 1), Ireland (position 3), Australia (position 6), and Sweden (position 8) income is the only disadvantaged dimension.

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APPENDIX

Proof of Theorem 1: In view of the strong focus axiom, for any $d \in N$, we can restrict attention on the censored profile $z^* = (z_1^*, z_2^*, ...z_d^*)$. By applying the factor decomposability postulate to the profile z^*, we get $F(z) = \sum_{i=1}^{d} w_i f(z_i^*)$, where $f(z_i^*) = F(z_i^*), 1 \le i \le d$, $f : \Re_+^1 \to \Re_+^1$. Continuity of F ensures that f is continuous over its domain. In view of the normalization axiom, $f(0) = 0$ and $f(1) = 1$.

The proof of strict convexity of f over $[0,1]$ relies on the following theorem of Jensen (1906): Let $g : I \to \Re^1$ be a continuous function, I being a non-degenerate interval in the set of real numbers $\Re^1$. Then g is strictly convex if and only if it is midpoint strictly convex, that is, for arbitrary $p \neq q \in I$, $g\left(\frac{p+q}{2}\right) < \frac{g(p)+g(q)}{2}$. (See Niculescu and Persson, 2006, p.10.)

Now, for $t \in [0,1]$, choose $c > 0$ such that $u(= t - c), v(= u - c) \in [0, 1]$. Aggravation sensitivity implies that

$$f(v) - f(u) > f(u) - f(t). \tag{12}$$

Note that $u = \frac{v+t}{2}$. Hence we can rewrite inequality (13) as

$$f\left(\frac{t+v}{2}\right) < \frac{f(t)+f(v)}{2}. \tag{13}$$

Since $t, v \in [0, 1]$ are arbitrary (as $c > 0$ is arbitrary), inequality (13) establishes mid-point strict convexity of f. This combined with continuity of f over $[0, 1]$ demonstrates that f fulfills the desired strict convexity property. This establishes the necessity part of the theorem. The sufficiency part is easy to verify. Δ

Proof of Proposition 1: Given $g(0) = 1$ and $g(1) = 0$, by strict convexity of g, $g((1-\theta)) = g(\theta.0 + (1-\theta).1) < \theta g(0) + (1-\theta)g(1) = \theta$, where $0 < \theta < 1$ is arbitrary. Hence for any $0 < u < 1$, $g(u) < (1-u)$ which implies that $g(u)$ is decreasing at 0. Next, strict convexity of g also implies that $g(u) > 0$ for all $0 < u < 1$. Otherwise, suppose for some $0 < t < 1$, $g(t) = 0$. Then, given $g(1) = 0$, by strict convexity of g, $g(\theta.t + (1-\theta).1) < \theta g(t) + (1-\theta)g(1) = 0$, where $0 < \theta < 1$. This inequality holds only if

$g(\theta.t+(1-\theta).1)<0$, which is a contradiction to the assumption that g is non-negative valued. Now, if g is not strictly decreasing, then there exist at least three points $0<t_1<t_2<t_3<1$ such that $g(t_1)>g(t_2)>0$ and $g(t_2)<g(t_3)>0$. By construction and by convexity of $[0,1]$, we can get $0<\delta<1$ such that $t_3=\delta.t_2+(1-\delta).1$. Then by strict convexity of g, $g(t_2)<g(t_3)=$ $g(\delta.t_2+(1-\delta).1)<\delta g(t_2)+(1-\delta)g(1)=\delta g(t_2)$, a contradiction. Since $g(1)=0$, it then follows that g is strictly decreasing over $[0,1]$. This completes the proof of the proposition. Δ

Proof of Theorem 2: Since in (2) the weights $w_i \geq 0$ obeying the restriction $\sum_{i=1}^{d} w_i = 1$, can be chosen arbitrarily for identifying a functional form that violates a particular postulate, in each part of the demonstration of the independence theorem, we can choose $w_i = \frac{1}{d}$ for all $i \in Q$.

(A) Consider the female disadvantage index given by

$$F_1(z) = \frac{1}{d}\left[\sum_{z_i<1}(1-z_i)^2 + \sum_{z_i\geq 1}\left(\frac{z_i-1}{z_i+1}\right)\right], \qquad (14)$$

where $z \in \Re_+^d$ is such that for at least one $i \in \{1,2,...,d\}$, $z_i > 1$ holds. Given such an FMA ratio profile z with $z_i > 1$ choose $c>0$ so that $(z_i - c) > 1$. Given this z, define $y \in \Re_+^d$ as follows: $y_i = (z_i - c) > 1$ and $y_k = z_k$ for all $k \in \{1,2,...,d\}/\{k\}$. It is easy to verify that $F_1(y) < F_1(z)$, which demonstrates violation of the strong focus axiom because of its dependence on FMA ratios that are greater than 1. However, F_1 fulfills the aggravation sensitivity, factor decomposability, normalization and continuity axioms.

(B) Because of linearity in z_i^* values, the Dijkstra-Hanmer index $F_2(z) = \frac{1}{d}\sum_{i=1}^{d}(1-z_i^*)$, where $z \in \Re_+^d$, is not aggravation sensitive, although it is strongly focused, factor decomposable, normalized and continuous.

(C) For any $z \in \Re_+^d$, let $\hat{z}^*$ be the censored FMA ratio profile associated with the non-increasingly ordered permutation of z, that is, $z_1 \leq z_2 \leq .... \leq z_d$ so that $(1-\hat{z}_1^*) \geq (1-\hat{z}_2^*) \geq .... \geq (1-\hat{z}_d^*)$. Then for any $z \in \Re_+^d$, the index $F_3(z) = \frac{1}{d^2}\sum_{i=1}^{d}(2i-1)(1-\hat{z}_i^*)$

that employs Gini-type aggregation is a transgressor of the factor decomposability postulate since it is a rank ordered weighted average of $\left(1-\hat{z}_i^*\right)$ ratio values where the weights themselves are dependent of the entire distribution of ratios. However, F_3 is an abider of the all the remaining four axioms considered in the theorem statement.

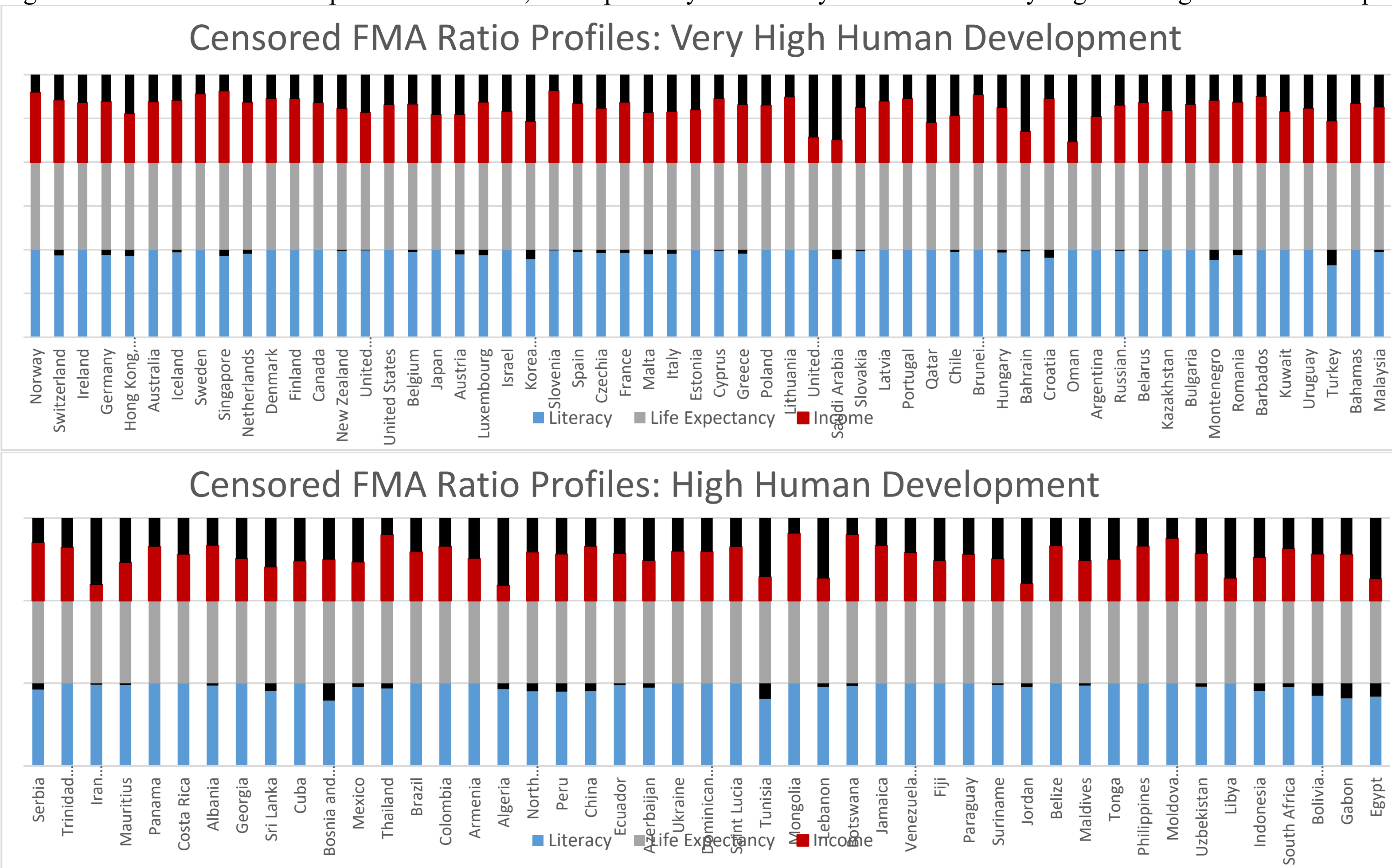
(D) For any $z \in \Re_+^d$, the sum $F_4(z)=\sum_{i=1}^{d}\left(1-z_i^*\right)^2$ of squared deviations of z_i^* from unity is a violator of part (ii) of the normalization postulate. (Note that since the normalization principle consists of two separate parts, violation of one part should serve our purpose.) Evidently, F_4 is aggravation sensitive, strongly focused, factor decomposable and continuous.

(E) Consider the following form of the disadvantaged index

$$F_5(z)=\begin{cases}\frac{1}{d}\frac{1}{2}\sum_{i=1}^{n}\left(1-z_i^*\right)^2 \; if \;\; z \neq 01^n, \\ 1 \; if \;\; z=01^n. \end{cases} \tag{15}$$

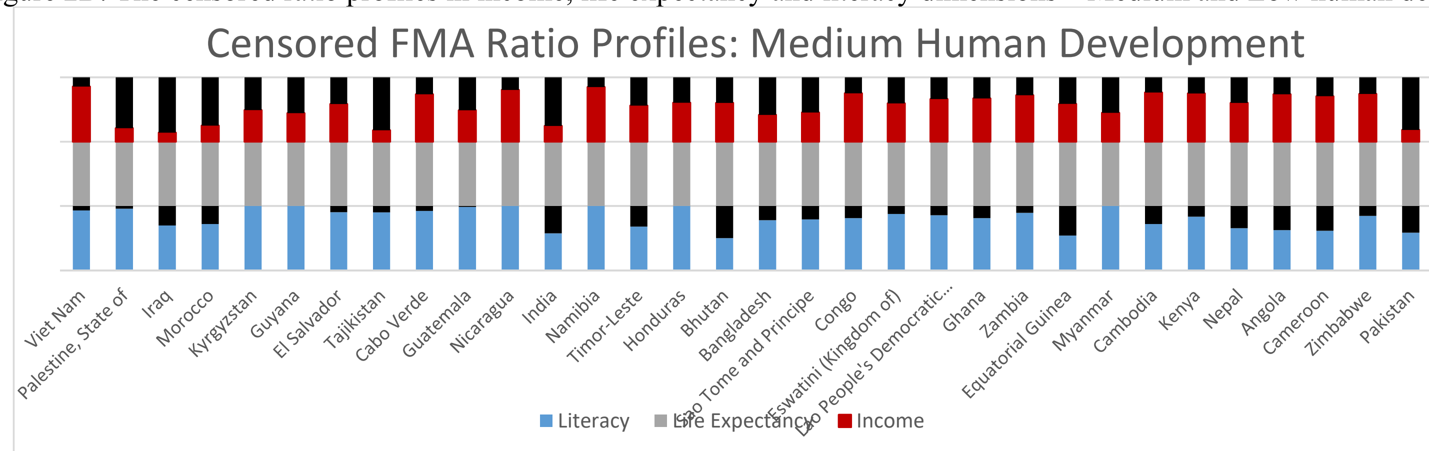
where, $z=01^n$ is the n-coordinated vector of zeroes and $z \neq 01^n$ means that at least one coordinate of *z* is non-zero. Evidently, F_5 is discontinuous at $z=01^n$. However, it is strongly focused, aggravation sensitive, factor decomposable and normalized. This completes the proof of the theorem. *Δ*

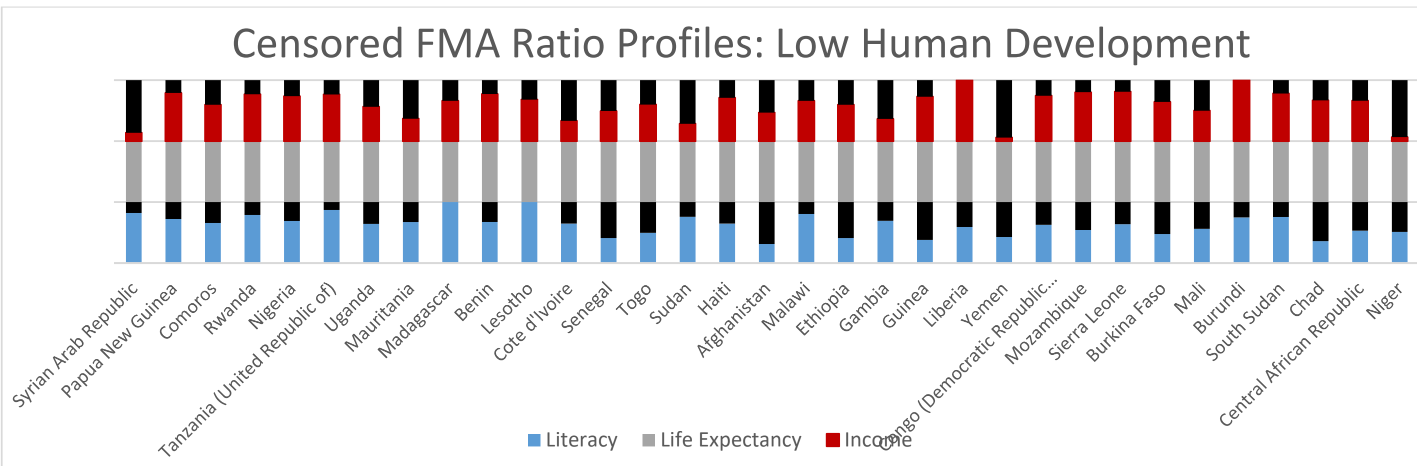
Figure 2A: The censored ratio profiles in income, life expectancy and literacy dimensions – Very High and High human development countries.



Notes: In these figures we stack the censored FMA ratio profiles in the dimensions: Literacy (bottom), Life Expectancy (middle), Income (top). The black bar is the complement to 1 and indicates the disadvantage suffered by women in that specific dimension.

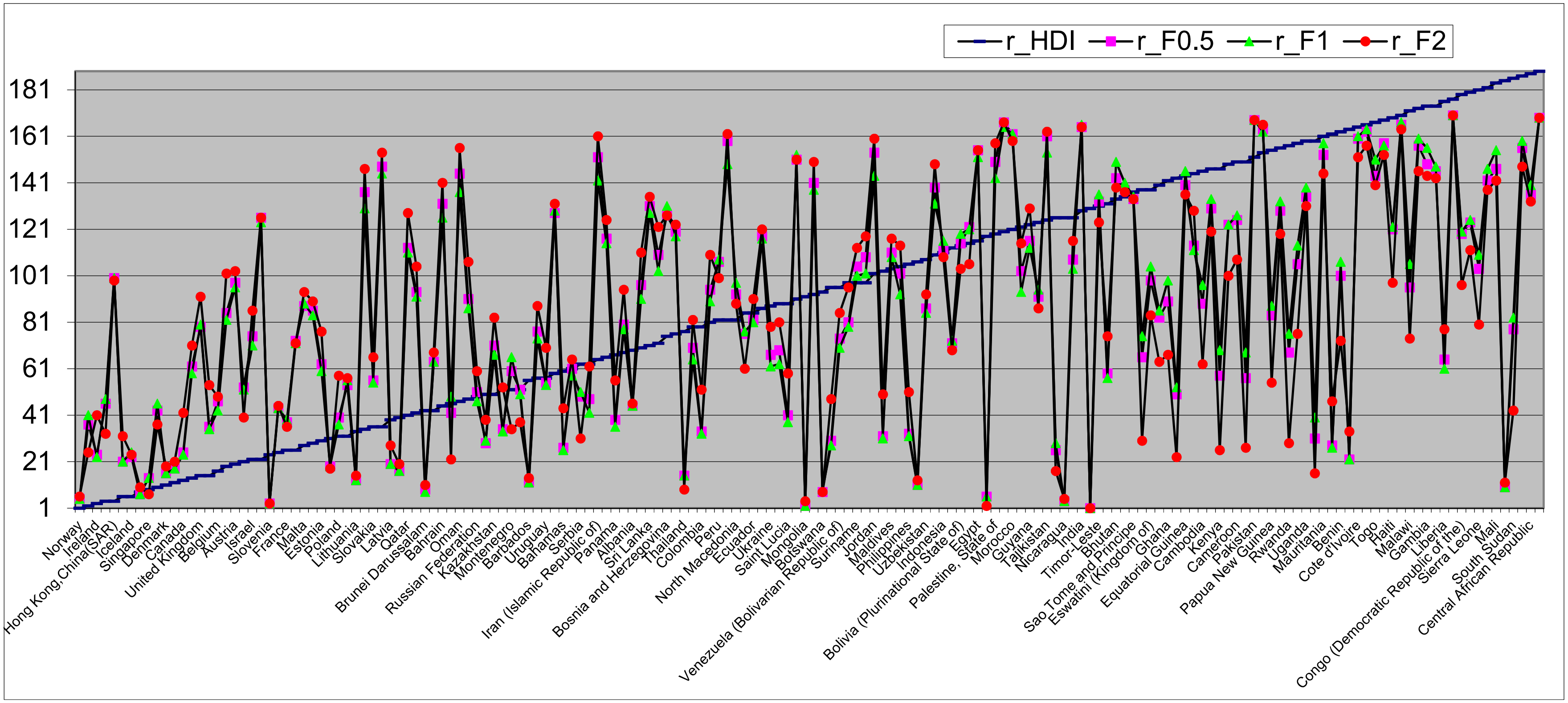
Figure 2B: The censored ratio profiles in income, life expectancy and literacy dimensions – Medium and Low human development countries.





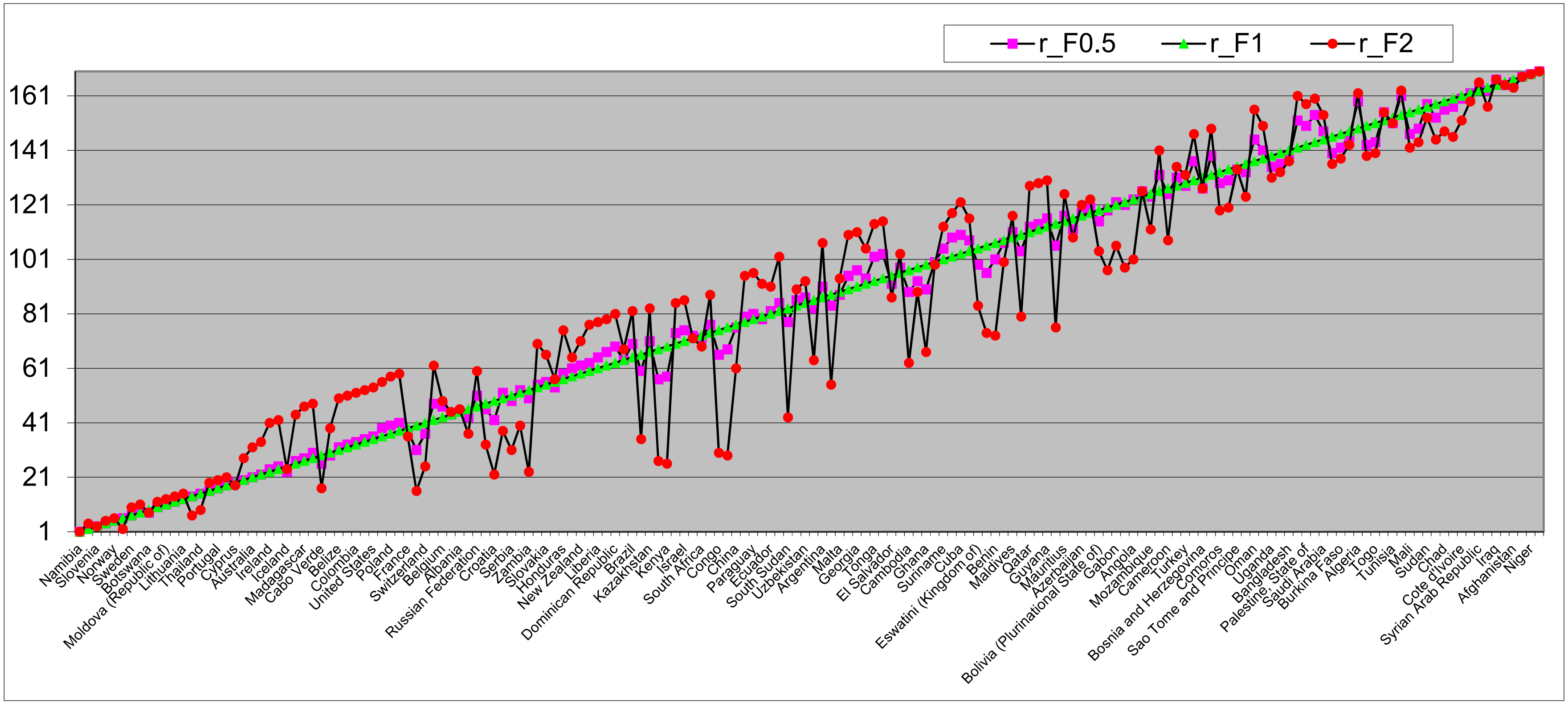
Notes: In these figures we stack the censored FMA ratio profiles in the dimensions: Literacy (bottom), Life Expectancy (middle), Income (top). The black bar is the complement to 1 and indicates the disadvantage suffered by women in that specific dimension

Figure 3: Ranking of the countries in three of the proposed indices compared to the HDI in the year 2018.



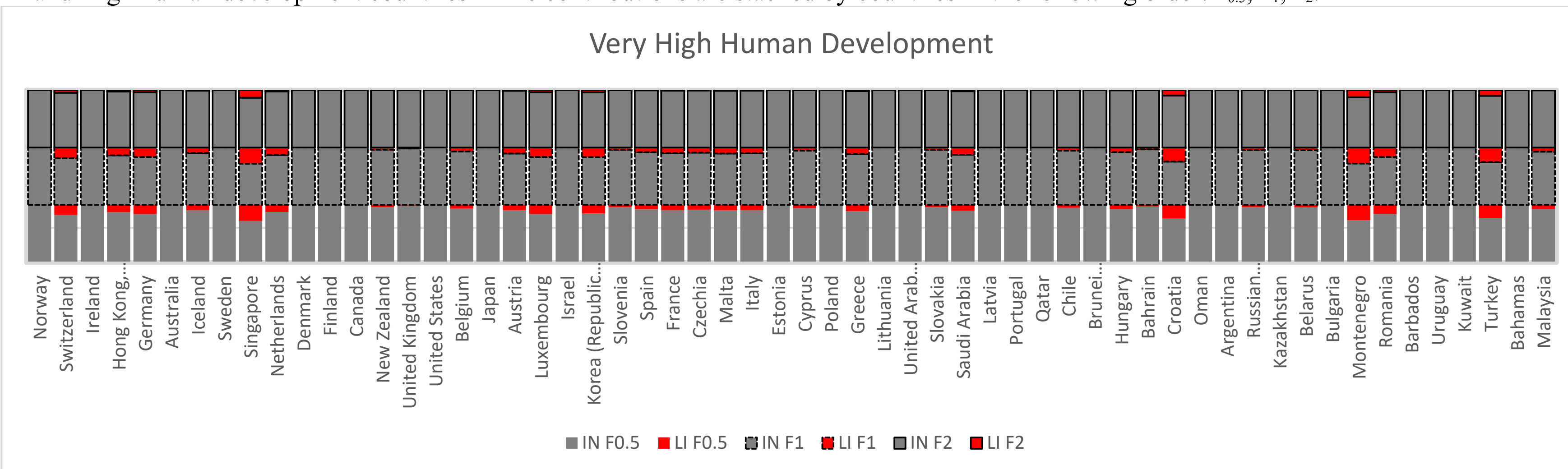
Notes: The countries are ranked in terms of gender parity, where in the first position is the country whose index is the lowest. The HDI rankings are standard and are provided directly by the HDRO, i.e. in the first position is the country whose HDI is the highest. As such they include all 189 countries.

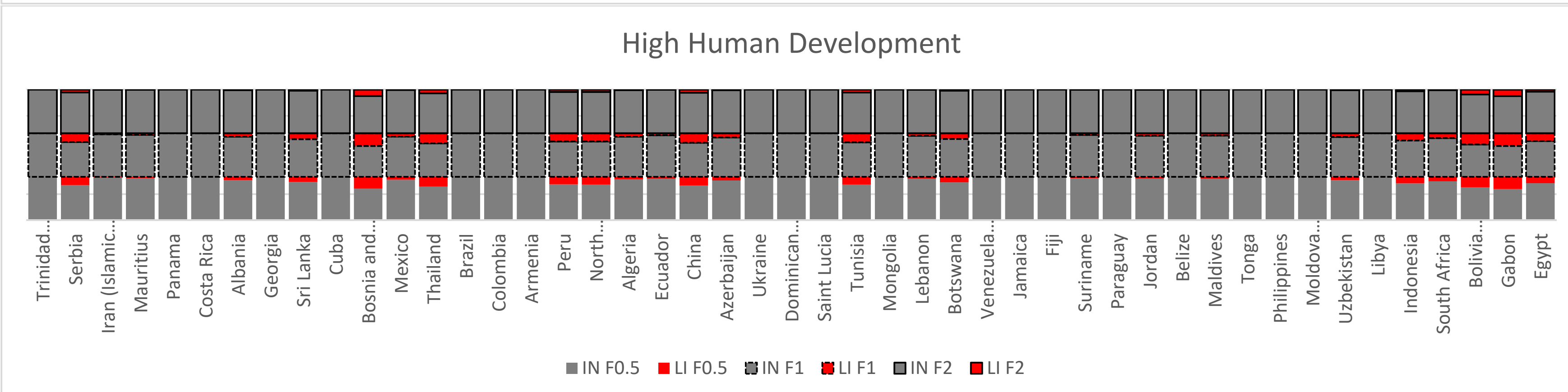
Figure 4: Ranking of the countries in three of the proposed indices in the year 2018, listing the countries according to their performance in F_1.



Notes: The countries are ranked in terms of gender parity, where in the first position is the country whose index is the lowest.

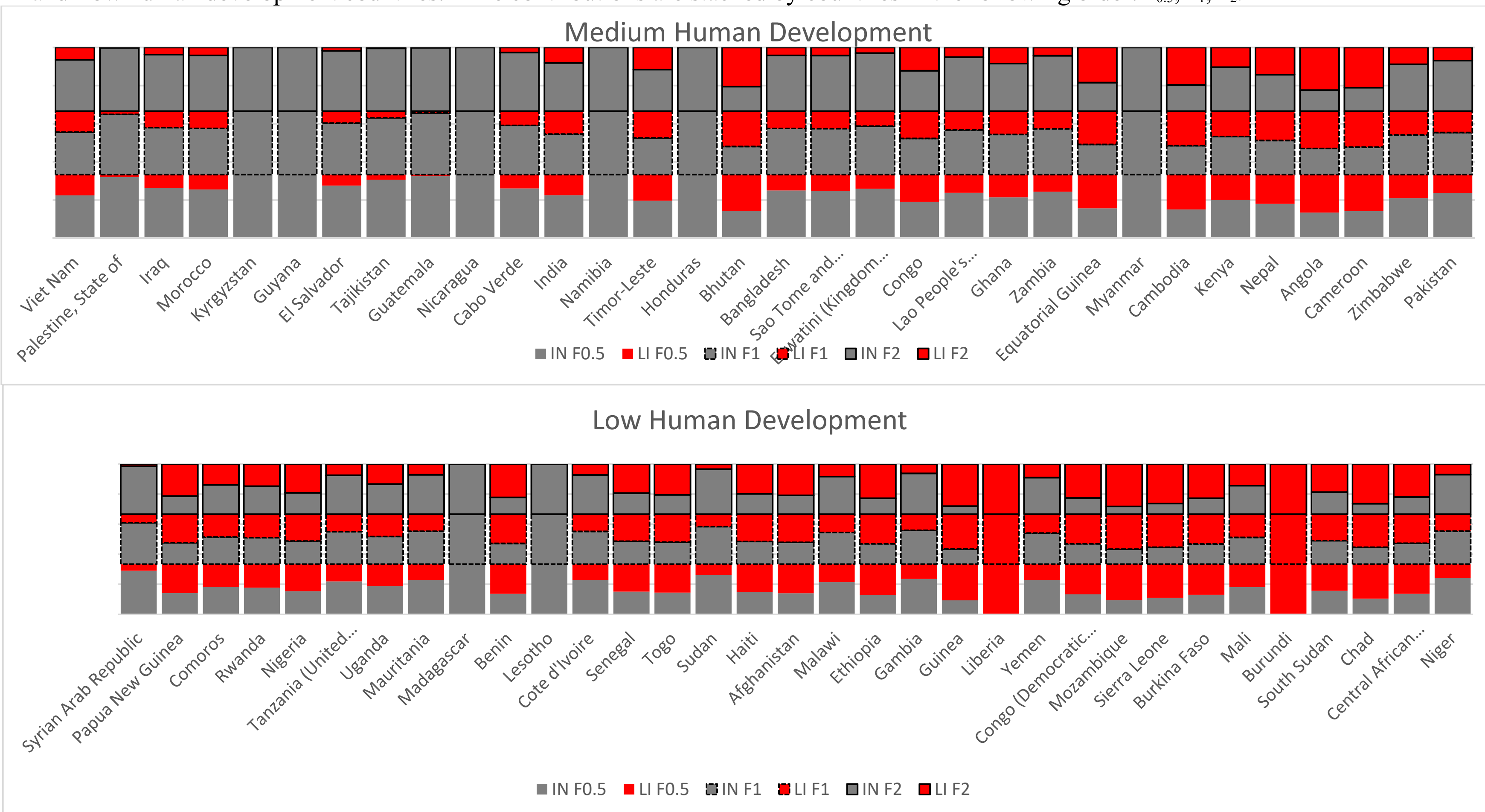
Figure 5A: Percentage contributions of the income and literacy dimensions to the overall female disadvantage in the proposed indices – Very High and High human development countries – The contributions are stacked by countries in the following order: $F_{0.5}$, F_1, F_2.





Notes: In these figures we stack the three indices that have been computed for each country, F_e for $e = 0.5$ at the bottom (bars with no borders) followed by F_α for $\alpha = 1$ (bars with dotted borders) first and $\alpha = 2$ (bars with solid borders) last. The percentage contribution of income (IN) is grey; the percentage contribution on literacy (LI) is red.

Figure 5B: Percentage contributions of the income and literacy dimensions to the overall female disadvantage in the proposed indices – Medium and Low human development countries.– The contributions are stacked by countries in the following order: $F_{0.5}$, F_1, F_2.



Notes: In these figures we stack the three indices that have been computed for each country, F_e for $e = 0.5$ at the bottom (bars with no borders) followed by F_α for $\alpha = 1$ (bars with dotted borders) first and $\alpha = 2$ (bars with solid borders) last. The percentage contribution of income (IN) is grey; the percentage contribution on literacy (LI) is red.