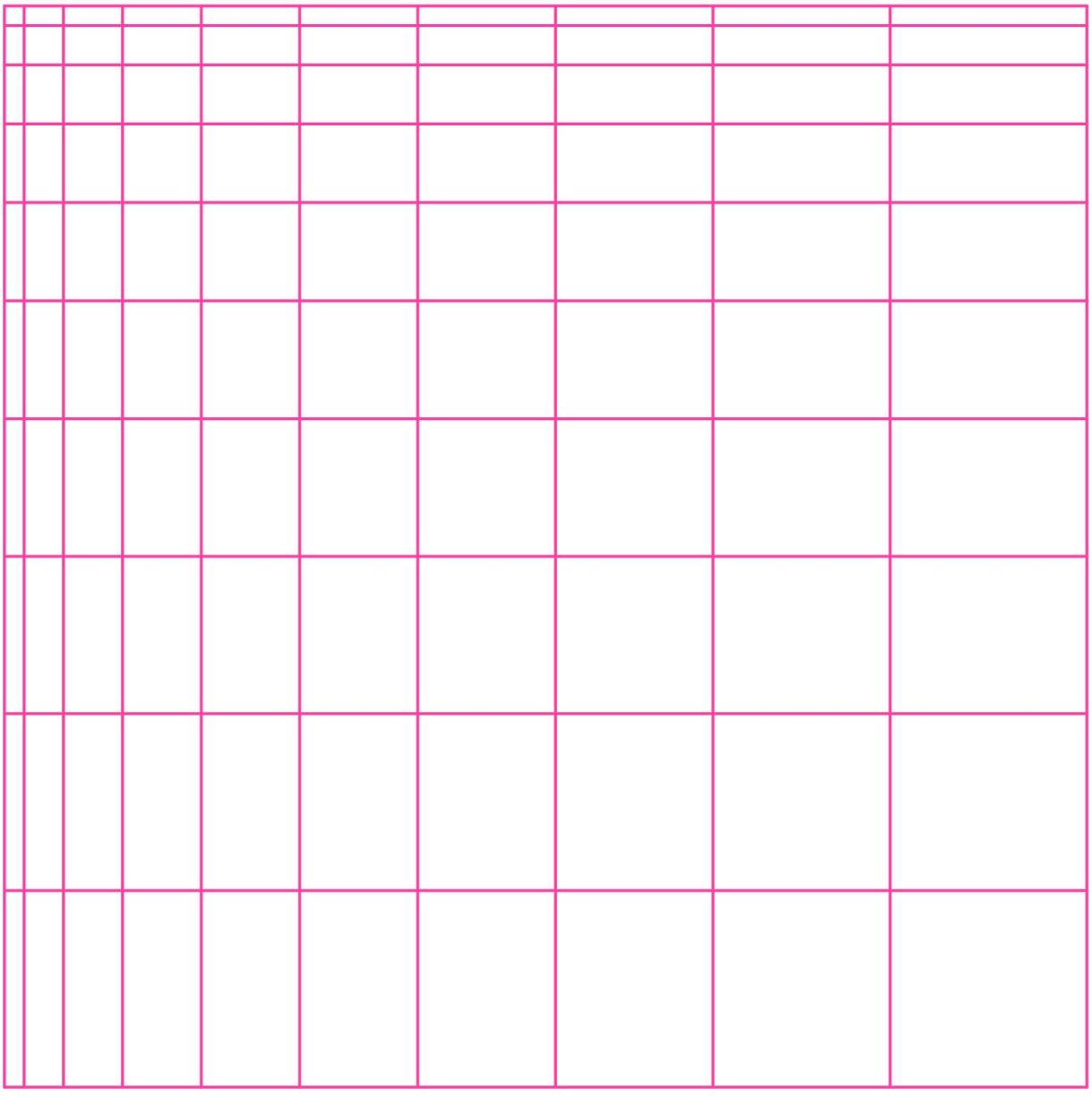
The Multiplication Tables Puzzle

A puzzle based on the multiplication tables. To learn the multiplication tables, to explore linear and quadratic growth, to play with tilings, to introduce the sum of the first n natural numbers.

PUZZLE PIECES: 100 rectangle tiles of size AxB where A,B are integers that vary from 1 to 10 in all possible ways. On the puzzle piece of size 6x7 it is written 42 on one side and 6x7 on the other. The puzzle pieces are also cards to learn the multiplication tables, and they can be combined into further shapes.



PUZZLE EXPLORATION:

- *Learning the multiplication tables (in two directions).* One can place all pieces on their AxB side and compute the product (turning the pieces afterwards). Alternatively, one can place the pieces to show the product and compute their factorisation. Here the piece shape has to be considered, because for example we have 36 in the pieces 6x6, 4x9, 9x4.
- *What is the sum of all numbers from 1 to 10?* It is 55=10x(10+1)/2, as it can be checked by the pupils. *What is the sum of all numbers in a row/column?* It is a multiple of 55, the factor being the row/column number. *What is the sum of all numbers?* We need to add Ax55 for all A from 1 to 10, which gives 55x55=3025. Pupils could guess these numbers before computing them.
- *Comparing sizes, linear or quadratic growth.* By comparing at the square pieces, one has a feeling for the quadratic growth. By comparing the pieces in a given row or column, one has a feeling for the linear growth.
- *To play with tilings.* Pupils can combine puzzle pieces to make smaller puzzles, and they can freely investigate how to do so. Moreover, they can produce further shapes with them (for example, letters).

PUZZLE PREPARATION: *Pupils produce the puzzle theirselves* by cutting suitable rectangles from thick paper.

- *What is the unit length?* All sizes can be compared to the size of the 1x1 rectangle, as this tiles all rectangles. To avoid calculation errors we suggest to fix the unit length at 1cm/1.5cm/2cm, which will make a puzzle in the form of a square with side length respectively 55cm/82.5cm/110cm. The largest puzzle piece is the 10x10 rectangle, which will have side length respectively 10cm/15cm/20cm.
- *How does one make use of the paper in an optimal way?* One has to make way for the largest pieces and then fit the smaller ones with the leftovers. There should be enough thick paper to amend for errors.
- *How can one share the work effectively?* With 20 pupils, each pupil only has to produce 5 pieces. It has to be agreed upon which pieces are done by whom. The work is not parallel from the start if the sheets of paper are large and need to be shared.
- *Writing on the puzzle pieces.* Consider the puzzle pieces with sizes 6x7 and 7x6 respectively. On both pieces, one writes 42 on one side. On the other side, one writes 6x7 and 7x6 respectively. Notice that the orientation of the piece (hence of the writing) is vertical and horizontal respectively.
- *How large is the final puzzle?* This is at best computed with the pupils after the puzzle is completed (directly and with the sum of the first n natural numbers), but it can also be estimated or guessed before the construction.

Reference: A multiplication tables with rectangular cases by Nicolas Pinel in Méthode Heuristique, https://methodeheuristique.com/page-2/les-tables-de-multiplication/