Solvability of invariant systems of differential equations on the hyperbolic plane

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Abstract

In the Euclidean case, it is well-known, by Malgrange and Ehrenpreis, that linear differential operators with constant coefficients are solvable. However, what happens, if we genuinely extend this situation and consider systems of linear invariant differential operators, is still solvable? In case of $\mathbb{R}^n$ (for some positive integer n), the question has been proved mainly by Hörmander. We will show that this remains still true for Riemannian symmetric spaces of non-compact type $X = G/K$. More precisely, we will present a possible strategy to solve this problem by using the Fourier transform and its Paley-Wiener(-Schwartz) theorem for (distributional) sections of vector bundles over X. We will get complete solvability for the hyperbolic plane $\mathbb{H}^2 = SL(2,\mathbb{R})/SO(2)$ and beyond.
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