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# Strategic Behavior in a Serial Newsvendor Setting

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**Problem Definition:** We study the interaction between an upstream seller and an intermediate buyer, both of whom face uncertainty related to downstream demand, over a two-period horizon. The buyer replenishes every period, whereas the seller has only one ordering opportunity at the beginning of the horizon. Since both agents make quantity decisions before demand realizes, they face demand mismatch risks and have incentives to limit these risks through their quantity decisions. We study the effects of varying degrees of buyer strategic behavior on inventory decisions and seller profitability.

**Methodology and Results:** To model strategic buyer behavior, we distinguish between three buyer types: a myopic buyer who ignores any second period outcomes, a forward-looking buyer who accounts for his own future actions, and a sophisticated buyer who additionally accounts for the seller's stocking decision. Using a game-theoretical framework with the seller as the first-mover, we characterize the purchase decisions of each buyer type and the stocking decision of the seller facing each buyer type in a constant pricing setting. We find that a seller facing a forward-looking buyer is better off than one facing a myopic buyer as the forward-looking buyer demands more over the horizon. However, a seller facing a sophisticated buyer is worse off than a seller facing a forward-looking buyer as the sophisticated buyer's cautious purchasing behavior induces the seller to take more risk in his stocking decision.

**Managerial Implications:** In most of the strategic behavior literature, the agent that may behave strategically is the most downstream agent in the supply chain, i.e., the end consumer. Understanding the degree of strategic behavior of other agents in a supply chain, e.g., an intermediate buyer, is critical as it informs optimal inventory decisions and seller profitability. The seller is best-off facing a buyer that plans ahead in time, but does not know or consider inventory availability upstream.

*Key words*: strategic consumers; multi-unit purchases; inventory games

## 1. Introduction

Mounting evidence suggests that economic agents exhibit varying degrees of strategic behavior in multi-period environments (e.g., Li et al. 2014, Mak et al. 2014, Osadchiy and Bendoly 2015, Soysal and Krishnamurthi 2012, Yilmaz et al. 2019). In such settings, strategic behavior is manifested in the degree to which agents account for future realizations of prices or inventory availability to guide their current decision-making when facing wait-or-buy decisions. The study of strategic behavior, which started with the seminal paper on durable goods monopolies by Coase (1972), led to a proliferating literature on the interactions between sellers and buyers endowed with varying degrees of strategic behavior. Many studies show that forward-looking buyers, who consider future price realizations over a finite horizon when deciding on when to make their purchases, strategically wait until prices are sufficiently low to purchase, thereby decreasing the seller's profit (e.g., Coase 1972, Bulow 1982, Aviv and Pazgal 2008). As a result, much of the literature has focused on how sellers can mitigate the negative effects of strategic behavior through pricing (e.g. dynamic pricing vs. price commitment) or inventory (rationing) as tactical levers.

The literature to date has almost exclusively focused on strategic behavior at the most downstream level within a two-tiered supply chain, with the retailer as the seller and the end consumers as the strategic buyers. However, supply chains are inherently complex, involving many agents who make decisions at different tiers, interact, and face uncertainty in their decision-making. Consider the interaction between an upstream seller and an intermediate buyer who faces uncertain demand from downstream agents. To serve these downstream agents, the buyer purchases product from the seller before downstream demand realizes. Similarly, to serve the buyer, the seller purchases product from her supply source both before downstream demand realizes and the buyer purchases. Hence, both agents make quantity decisions under demand uncertainty of varying magnitudes, which exposes them to demand mismatch risk. With limited lifetime products, this risk is only magnified as products can be carried and sold only over a limited horizon before they need to be salvaged.

In practice, this situation arises in diverse industries such as retail and hospitality. A grocery store, for instance, decides on how many units of each of its products to purchase from a wholesaler to satisfy end consumer demand before the demand realizes. The wholesaler, in turn, also purchases its supply from an upstream producer to stock the store. Companies that source inputs to produce a product also face this situation as they decide on purchase quantities before downstream demand realizes. For example, an electronics company producing a generation product decides in advance on the number of units of a component to source from its supplier to produce the product before demand realizes. Since all agents in a supply chain bear risk in their inventory decisions, agents at every level may have an incentive to limit their risk by rationing or by changing the timing and quantities of their purchases.

A number of works expand the analysis of strategic consumer behavior to three-tiered supply chains, thereby adopting a wider supply chain perspective. In contrast to the findings of the two-tier supply chains, these works render a more positive view of strategic behavior. Su and Zhang (2008) study a supply chain with a manufacturer in addition to a retailer and population of end consumers. They find that the profits of the manufacturer and retailer together increase from being able to charge higher prices to forward-looking end consumers, although the distribution of profits depends on the wholesale cost that the manufacturer charges the retailer. Lin et al. (2018) and Kabul and Parlaktürk (2019) study a similar three-tier supply chain. They find that forward-looking behavior by end consumers always benefits the manufacturer as the retailer reduces his price to discourage strategic waiting and sells more and may benefit the retailer as well. However, these studies still focus on the end consumer as the agent that can exhibit varying degrees of strategic behavior.

Recognizing that end consumers in a supply chain are not the only agents that have an incentive to behave strategically, we shift the location of the potentially strategic agent upstream to an intermediate buyer in a supply chain. That is, we examine the effect that varying degrees of strategic behavior of an intermediate buyer, who faces downstream demand and procures product from an upstream seller, have on the supply chain. Accordingly, we pose two research questions. The first research question is: how do the inventory decisions of a buyer and seller differ depending on the degree of strategic behavior of the buyer? The second research question is: how do different degrees of buyer strategic behavior affect the profitability of the seller?

To study these questions, we model a supply chain consisting of two agents – an intermediate buyer and an upstream seller – over a two-period horizon. Both of these agents are newsvendors. In each period, the buyer faces independent aggregate uncertain demand from a population of downstream consumers. The buyer purchases product from the seller at the beginning of each period before downstream demand realizes and may carry excess inventory over from the first to the second period. The seller purchases product from an upstream supply source at the beginning of the selling horizon. Like the buyer, the seller can carry inventory over from the first to the second period, but cannot replenish. Hence, in deciding on a stocking quantity for the horizon, the seller needs to consider the buyer's second period purchase decision. In turn, the buyer's second period decision depends on the realization of demand the buyer observes in the first period, which is unobserved by the seller. At the end of the selling horizon both agents salvage excess product.

To model the effect of strategic buyer behavior, we define three buyer types: a myopic buyer, a forward-looking buyer, and a sophisticated buyer. In defining these buyer types, our view of strategic behavior focuses on two components: *(i)* whether the buyer accounts for the entire horizon, and *(ii)* whether the buyer accounts for the seller's actions. The *myopic* buyer is our most basic buyer who exhibits no strategic behavior. He completely ignores the second period when buying in the first period. The *forward-looking* buyer accounts for the second period and optimizes his purchase decisions over the horizon. Specifically, he considers the inter-temporal effects induced by linking the two periods. The *sophisticated* buyer goes one step further than the forward-looking buyer and additionally considers the seller's stocking decision, and hence potential inventory rationing, in his period purchase decisions.

Using backward induction to find the subgame perfect Nash equilibrium, we characterize the buyer's purchase decisions and the seller's stocking quantity for a supply chain with each buyer type. We first consider a setting in which prices are constant across periods. This abstraction enables us to focus on the inventory game as pricing no longer affects the buyer's purchase timing. As an extension, we relax this constant pricing assumption to test the robustness of our findings in a setting where prices change and investigate optimal pricing policies if the seller were additionally able to set prices.

In the constant pricing setting, we find that the forward-looking buyer buys more than the myopic buyer in the first period (as he accounts for the possibility to use leftover inventory in the second period) and less than the myopic buyer in the second period (as he has more leftover inventory). Over the horizon, though, forward-looking behavior has a demand-enhancing effect and the forward-looking buyer demands as much as or more than the myopic buyer. Due to larger horizon demand, the seller facing a forward-looking buyer stocks more than one facing a myopic buyer and makes more profit. However, because the forward-looking buyer demands less in the second period than the myopic buyer, a seller facing a forward-looking buyer may also be more inclined to reduce second period overage risk by stocking less product relative to the forward-looking buyer's demand, resulting in stock-outs. For this reason, one might expect the sophisticated buyer to buy *more* in the first period to encourage the seller to stock more for the horizon. Our results show the opposite however: the sophisticated buyer buys *less* than the forward-looking buyer in the first period. He is more cautious than the forward-looking buyer in his first-period purchase. To encourage the sophisticated buyer to overcome this caution, the seller is manipulated into taking a greater risk with his supply decision and stocking more. Consequently, the seller makes less profit facing a sophisticated buyer compared to a forward-looking buyer. Therefore, a more strategic buyer type does not make the seller better off. The seller is better-off facing buyers with some degree of sophistication but not full sophistication. Importantly, these results persist when we relax the constant pricing assumption.

This rest of the paper is structured as follows. Section 2 briefly reviews the literature. Section 3 describes the modeling approach. Section 4 formulates and solves the decision problem for the myopic and forward-looking buyers and for the seller facing these buyer types in a constant pricing setting. Section 5 carries out the same analysis for the sophisticated buyer and for the seller facing this buyer in a constant pricing setting. Section 6 extends the analysis to a setting in which the seller can markup or markdown product in the second period and examines implications for optimal markup and markdown policies. Section 7 concludes with our main findings and future research directions. The proofs for all our results are relegated to the E-Companion.

## 2. Literature Review

This research most closely relates to the expansive literature on strategic consumer behavior. Wei and Zhang (2018), Shen and Su (2007), and Elmaghraby and Keskinocak (2003) provide comprehensive reviews of strategic consumer behavior, which include, among others, specific aspects associated with strategic consumer behavior, such as consumer behavior modeling and dynamic pricing.

The majority of the strategic behavior literature focuses on the interaction between a seller and multiple atomistic buyers for a limited-lifetime product in a two-period horizon (e.g., Aviv and Pazgal 2008, Cachon and Swinney 2009). The prices set by the seller may follow either a pre-announced price path or may be dynamically set. In these studies, buyers can be heterogeneous in terms of their valuation of the product, which is modeled as a random variable, or willingness to wait, which is modeled by a discount factor. It is usually assumed that the buyer buys at most one unit of the product. The focus is then on *when* the buyer buys this *one* unit.

When strategic buyers are present, they may wait to buy the product in a lower-price period at the risk of facing a stock-out and potentially a discount to their utility. As such, they engage in inter-temporal substitution. It is generally demonstrated that forward-looking buyers hurt the seller as their expectations about future actions prove to be detrimental to the optimal choices. When the seller prices the goods over time, buyers expect the price to drop over time. Given their willingness to wait, these buyers induce the seller to drop the price. As a result, much of the literature seeks to understand the degree to which strategic consumer behavior affects seller profit and how to mitigate this behavior. One tactic studied to counteract strategic behavior has been price commitment by a seller (Aviv and Pazgal 2008) to discourage strategic waiting. Other tactics are inventory rationing (Liu and Van Ryzin 2008, Zhang and Cooper 2008) and dynamic pricing (Levin et al. 2010) as a function of the amount of seller inventory remaining.

However, some studies have demonstrated that strategic consumer behavior may have positive effects for sellers, especially in more extensive supply chains. Lin et al. (2018) study a model with a manufacturer, retailer, and end consumers. They find that forward-looking behavior by end consumers always benefits the manufacturer as the retailer reduces his price to discourage strategic waiting and sells more. A retailer may also benefit from forward-looking behavior when end consumers are sufficiently patient and the manufacturer lowers his wholesale price.

With the exception of a few works, multi-unit purchases have not been treated in the literature. In an auction setting, Elmaghraby et al. (2008) study the optimal pre-announced markdown mechanism when any number of markdown steps can be implemented during the sales period and the seller has fixed capacity from the beginning. Jin et al. (2021) revisit the setting of a monopolist seller and a mass of buyers and allow the buyers to buy a second unit. They argue that the marginal valuation of the second unit is less than the first. Despite the lack of treatment in the literature, buyers do face multi-unit purchase decisions in practice. Our contribution is to characterize the stocking decisions of two agents along the supply chain when both face uncertain multi-unit demands. Whereas much of the strategic consumer literature focuses on dynamic pricing, our primary focus is on the inventory stocking decisions by a seller and buyer. Similar to Zhang et al. (2019), we focus on stocking decisions assuming that the pricing of the product follows an exogenously determined pre-announced price path.

When the buyer is able to buy multiple units, several features of the traditional strategic consumer behavior problem change. First, if the buyer has leftover inventory after the first period, he can carry over this inventory for use in the second period, reducing his second period purchase quantity. The seller then needs to consider the uncertainty of this carry-over into his stocking decision. This notion of inventory carry-over relates to the research stream on consumer stockpiling, which often assumes a durable good, as is the case, e. g., in Su (2007). Second, the buyer can purchase the seller's entire stock upfront in the first period (assuming sufficient seller inventory). This upfront full quantity purchase can hurt the seller's profitability, especially if the seller cannot re-stock inventory during the horizon.

A number of studies (e.g. Liu and Van Ryzin (2008), Su and Zhang (2008)) find that a seller facing forward-looking buyers will stock less than a seller facing myopic buyers. The reasoning is that, knowing that forward-looking buyers will strategically wait to purchase at lower prices, the seller best responds by rationing supply. In contrast, we find that a seller facing a forward-looking buyer stocks more than a seller facing a myopic buyer because the forward-looking buyer demands as much as or more than the myopic buyer over the horizon. The fact that the forward-looking buyer knows he can carry over and use leftover inventory from the first period in the second period incentivizes him to order more in the first period and overall. We also find that the seller makes as much as or more profit facing a forward-looking buyer than a myopic buyer, so some degree of strategic behavior benefits the seller.

Several studies have examined the effect of a seller revealing inventory information and of a buyer taking into consideration the seller's stocking decision when making purchase decisions. The results of these studies give mixed directional insights as to whether a seller should disclose inventory information. In a one-period model, Su and Zhang (2009) show that a seller benefits from truthfully announcing his supply. Such a quantity commitment has a demand-boosting effect as it enables consumers to better assess supply availability, encourages them to buy from the seller, and increases their willingness to pay. The seller can then benefit from higher prices and increased sales. Yin et al. (2009) compare two inventory display formats: one in which the seller discloses inventory information by display his supply and another in which the seller does not reveal inventory information and displays only one unit. They find that displaying one unit creates a sense of scarcity and increases seller profits. Our results are consistent with the view that the seller benefits from not revealing inventory information. The seller is thus better off facing a buyer with some degree of strategic behavior (i.e., forward-looking buyer) but not the full degree of strategic behavior (i.e., sophisticated buyer).

In summary, our contribution to the strategic behavior literature is two-fold. First, we shift the location of the potentially strategic agent upstream and examine the effects of such a shift on inventory decisions and seller profitability. Second, we contribute to the strategic behavior literature by considering strategic behavior in the context of multi-unit demands.

## 3. Model

A seller (hereafter referred to as "she") sells a product with a limited lifetime over a two-period horizon. At the beginning of the horizon, she purchases $Q$ units of product from an upstream agent at a unit wholesale cost of $c > 0$. In each period, the seller sells this product at a unit sales price of $p_t$, $t \in \{1, 2\}$, with $p_1, p_2 > c$ (to ensure her participation in the market). The unit sales prices $p_1$ and $p_2$ are exogenous and pre-announced at the beginning of the horizon.

At the beginning of each period, the buyer (hereafter referred to as "he") seeks to purchase quantities $q_t, t \in \{1, 2\}$ from the seller. The buyer's higher order frequency compared to the seller's reflects the fact that in multi-echelon settings it is common for downstream agents to have higher order frequencies than their upstream counterparts and that these order frequencies are nested within those of the upstream's agent (e.g., Roundy 1985). The buyer faces uncertain demand in each period, denoted by the random variables $D_t$, $t \in \{1, 2\}$, which each have a distribution $F$ and density $f$. The buyer's purchase quantity decisions are made before demand in period $t$ is realized. If the demand realization in a period exceeds the amount of product the buyer has on-hand, he purchases additional units exactly up to his demand realization from an alternative source, albeit at a higher unit price of $r$. The buyer has a unit valuation for the product of $v > 0$, which is constant across both periods. This unit valuation $v$ can be interpreted as the benefit he derives when one unit is bought. We assume that $v > r > p_t$ to ensure that the buyer participates in both the regular market and the runout market. In the first period, if the demand realization is less than the quantity of product on-hand, the buyer carries over leftover inventory

$y \equiv (q_1 - D_1)^+$ into the second period at zero holding cost. In the second period, if the demand realization is less than the quantity of product on-hand, because the horizon is ending, any leftover inventory is discarded at zero salvage value. Similarly, for the seller, any product not bought by the buyer during the horizon is discarded at zero salvage value.

The sequence of events is summarized as follows:

1. The seller stocks $Q$ units for the entire horizon, which she buys at a unit wholesale cost of $c$.

2. At the beginning of period 1, the buyer purchases $q_1$ units at a unit price of $p_1$.

3. Period 1 demand, $D_1$, realizes. If $D_1 > q_1$, the buyer buys $D_1 - q_1$ units to satisfy his remaining demand from the alternative source at a unit runout cost of $r$. If $D_1 < q_1$, the buyer has leftover inventory of $q_1 - D_1$ that he carries over to period 2.

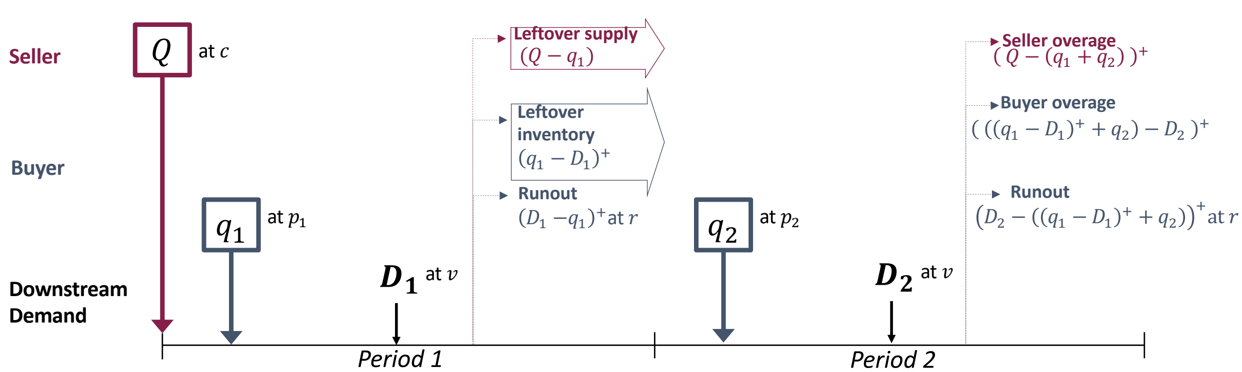
4. At the beginning of period 2, the buyer buys $q_2$ units at a unit price of $p_2$.

5. Period 2 demand, $D_2$, realizes. Once again, if demand exceeds on-hand inventory, the buyer buys $(D_2 - ((q_1 - D_1)^+ + q_2)^+$ units from the alternative source at unit runout cost of $r$. If demand is less than on-hand inventory, the buyer discards any remaining units at a zero salvage value.

6. The seller discards any inventory not purchased by the buyer at zero salvage value.

This sequence of events is illustrated in Figure 1. The decision variables are enclosed in squares next to the respective agent and the primary random variables are bolded.

**Figure 1** **Sequence of Events**




In line with the sequence of events, the decision-making problem is modeled as a three-stage game. The seller is the first-mover and optimizes her stocking quantity $Q$ over the horizon. The buyer optimizes his period purchase quantities of $q_1$ and $q_2$. The buyer's demand distribution and cost parameters are common knowledge. For each of the buyer types, we first solve for the buyer's period 2 optimal purchase quantity, $q_2^*$. Then, we solve for the buyer's period 1 optimal purchase quantity, $q_1^*$. Finally, we solve for the seller's best response in terms of a horizon stocking decision $Q$ given the buyer's optimal purchase quantity decisions $q_1^*$ and $q_2^*$.

To study the effect of different degrees of strategic behavior on inventory decisions, we define three buyer types: a myopic buyer, a forward-looking buyer, and a sophisticated buyer. We use the subscript $i \in \{M, F, S\}$ to denote the decisions associated with each buyer type, where $M$ denotes the myopic buyer, $F$ denotes the forward-looking buyer, and $S$ denotes the sophisticated buyer. All three buyer types decide on their period purchase quantities based on their *perceptions* of the trade-offs. The myopic buyer optimizes each period individually and ignores inter-temporal implications. In each period, he simply observes the period price and decides on a purchase quantity for that period based on the amount of product he has on-hand and knowledge of the distribution of downstream demand. The forward-looking buyer takes into account inter-temporality by considering prices across periods, the ability to use leftover inventory from period 1 in period 2, and the upcoming discarding of leftover product at the end of the horizon at zero salvage value. He observes the pre-announced prices for period 1 and period 2 and, using knowledge of his demand distribution and other cost parameters, decides on a purchase quantity for each period.

Neither the myopic buyer nor the forward-looking buyer take into account the seller's optimal stocking decision in their optimization problems. Both buyer types assume that the seller will have sufficient stock to satisfy their calculated optimal purchase quantities in both periods. However, the seller may choose to stock a limited amount of product, which may result in stock-out instances. In such cases, the buyer will buy more units from the alternative supply source at a higher cost. Had the buyer known the seller's stocking quantity, he might have chosen different purchase quantities to induce a different stocking behavior from the seller. The sophisticated buyer, in addition to having the inter-temporal features of the forward-looking buyer, also considers upfront how much stock the seller has for the horizon. It is as if the buyer is able to see the seller's supply at the beginning of the horizon.

In Sections 4 and 5, we start the analysis of this model for each of the buyer types in the constant prices setting in which the seller sets the same unit price for the product in both periods, i.e., $p_1 = p_2 = p$. We build on these results in an extension in Section 6 in which we allow the prices in periods 1 and 2 to differ. For the derivation of our analytical results, we assume that $D_1$ and $D_2$ are uniformly distributed over the interval $[0, B]$, where $B > 0$, and that they are independent.

# 4. Myopic and Forward-Looking Buyers under Constant Prices

## 4.1. Buyer's Problem

Both the myopic and forward-looking buyers face the same problem in period 2. That is, for any given leftover inventory realization $y_i \equiv (q_{1,i} - D_1)^+$ from period 1, a buyer of type $i = \{M, F\}$ chooses purchase quantity $q_{2,i} \geq 0$ to maximize his period perceived utility function, given by:

$$\mathbb{E}[U_{2,i}(y_i)] = v\mathbb{E}[\min(D_2, q_{2,i} + y)] - pq_{2,i} + (v - r)\mathbb{E}[(D_2 - (q_{2,i} + y_i))^+]. \tag{1}$$

The first term captures the utility derived from product purchased from the seller. The second term captures the cost of product purchased from the seller. The third term captures the net utility derived from product purchased from the runout option. This problem is a newsvendor problem with the following solution:

Proposition 1. *In the constant pricing setting, in period 2, a buyer of type $i = \{M, F\}$ purchases $q^*_{2,i} = (\bar{q}_2 - y_i)^+$ where $y_i \equiv (q^*_{1,i} - D_1)^+$ and $\bar{q}_2 = B\left(\frac{r-p}{r}\right)$.*

The optimal period 2 purchase decision follows a base-stock policy, where $\bar{q}_2$ is the order-up-to level. This order-up-to level is the same regardless of the buyer type. The differences between the myopic and forward-looking buyers emerge in their optimal period 1 purchase decisions.

### 4.1.1. Myopic Buyer

The myopic buyer maximizes his perceived utility for each period individually. That is, he decides on a period 1 purchase quantity without considering the possibility of using any leftover inventory from period 1 in period 2 and, more generally, the impact that this decision will bear on his future decisions. In period 1, the myopic buyer's problem is to choose purchase quantity $q_{1,M} \geq 0$ to maximize his period perceived utility function, given by:

$$\mathbb{E}[U_{1,M}] = v\mathbb{E}[\min(D_1, q_{1.M})] - pq_{1,M} + (v - r)\mathbb{E}[(D_1 - q_{1,M})^+]. \tag{2}$$

The maximizer of (2) is easily found:

Proposition 2. *In the constant pricing setting, the myopic buyer purchases* $q^*_{1,M} = B\left(\frac{r-p}{r}\right)$ *in period* 1.

The optimal period 1 decision also follows a base-stock policy, where $q^*_{1,M}$ is effectively the period 1 order-up-to level. Since the trade-offs he considers are the same in both periods, his order-up-to levels are the same. We summarize this result in the following corollary:

Corollary 1. *In the constant pricing setting,* $q^*_{1,M} = \bar{q}_2$.

**4.1.2. Forward-Looking Buyer** In period 1, the forward-looking buyer maximizes his perceived utility across the entire horizon. His problem is to choose a purchase quantity $q_{1,F} \geq 0$ to maximize his horizon perceived utility function, given by:

$$\mathbb{E}[U_{1,F}] = v\mathbb{E}[\min(D_1, q_{1,F})] - pq_{1,F} + (v-r)\mathbb{E}[(D_1 - q_{1,F})^+] + \mathbb{E}[U_{2,F}((q_{1,F} - D_1)^+)]. \quad (3)$$

Compared to the myopic buyer's period 1 problem, the forward-looking buyer's utility function incorporates an extra term to link the outcomes in both periods. Solving for the first order conditions with respect to $q_{1,F}$, we obtain the following result:

Proposition 3. *In the constant pricing setting, the forward-looking buyer's purchase quantity in period* 1 *is given by* $q^*_{1,F} = B\left(\frac{\sqrt{r^2-p^2}}{r}\right)$.

### 4.2. Comparison of Myopic Buyer and Forward-Looking Buyer

Comparing Propositions 2 and 3, we see that the forward-looking buyer buys more than the myopic buyer in period 1, since $\sqrt{r^2-p^2} = \sqrt{(r-p)(r+p)}$. This leads us to the following corollary:

Corollary 2. *In the constant pricing setting, the forward-looking buyer buys more than the myopic buyer in period* 1 *– that is,* $q^*_{1,F} > q^*_{1,M}$.

This result reflects the different trade-offs that each buyer type considers. The forward-looking buyer knows that he can use leftover inventory from period 1 to satisfy demand in period 2. Since $p_1 = p_2$, he has an incentive to buy more than the myopic buyer in period 1 to hedge himself against paying the higher runout cost in case of high period 1 demand. In effect, the forward-looking buyer shifts some of the quantity he purchases in the second period to the first period:

Corollary 3. *In the constant pricing setting,* $q^*_{1,F} > \bar{q}_2$.

Since $q_{1,F}^* > \bar{q}_2$, any leftover inventory from period 1 would only reduce his purchase quantity in period 2 further. Because the forward-looking buyer buys more than the myopic buyer in period 1, he carries over at least as much or more leftover inventory than the myopic buyer into period 2 for any realization of $D_1$. The forward-looking buyer also buys at most as much as the myopic buyer in period 2.

Is the difference in the purchasing behavior of the forward-looking and myopic buyers merely a shift in the timing of the purchases and does the overall quantity purchased over the horizon remain the same across these buyers? Or does one buyer type actually seek to purchase more than the other? Let $N_i = q_{1,i}^* + (\bar{q}_2 - (q_{1,i}^* - D_1)^+)^+$ denote the demand that the seller faces from each buyer type $i \in \{M, F\}$ over the horizon. The total demand generated by the forward-looking buyer exceeds the total demand generated by the myopic buyer for any given $D_1$, i.e. demand generated by forward-looking buyers stochastically dominates demand generated by myopic customers. We have the following Lemma:

LEMMA 1. *In the constant pricing setting, for any given realization of $D_1$, the total demand over the horizon for the seller facing a forward-looking buyer, $N_F$, is greater than or equal to the total demand over the horizon for the seller facing a myopic buyer. That is, $N_F \geq N_M$.*

So the higher period 1 purchase quantity that we observe from the forward-looking buyer in comparison to the myopic buyer is not only a shift in demand from the second period to the first period, but in fact the forward-looking actually seeks to purchase as much as or more than the myopic buyer.

### 4.3. Seller's Problem

The seller's problem is structurally the same regardless of the buyer type she faces. At the beginning of the horizon, given the respective $q_{1,i}^*$ and $q_{2,i}^*$ for buyer type $i \in \{M, F\}$, the seller chooses order quantity $Q_i \geq 0$ to maximize her horizon profit, given by:

$$\mathbb{E}[\pi_i(Q_i)] = p \min(q_{1,i}^*, Q_i) + p\mathbb{E}[\min(q_{2,i}^*, Q_i - q_{1,i}^*)] - cQ_i \tag{4}$$

The first and second terms of this profit function capture the revenue from sales in each period to a buyer of type $i$. The third term captures the product wholesale costs incurred by the seller. Note that, because the period prices are equal, 4 can be written as $\mathbb{E}[\pi_i(Q_i)] = p\min(N_i, Q_i) - cQ_i$.

Observe that the seller faces no uncertainty in the buyer's period 1 purchase decision as $q^*_{1,i}$ does not depend on any random variables. Therefore, at optimality, the seller stocks at least $q^*_{1,i}$. In fact, all the uncertainty the seller faces relates to the buyer's period 2 purchase decision. While the seller knows the distribution of downstream demand in each period, the buyer's purchase quantity in period 2 is a random variable that depends on the period 1 demand realization through the buyer's leftover inventory. At optimality, since the buyer never buys more than his order-up-to quantity $\bar{q}_2$ in period 2, the seller would never stock more than $\bar{q}_2$ for this period. Thus, $q^*_{1,i} \le Q^*_i \le q^*_{1,i} + \bar{q}_2$. Solving for this constrained optimization problem, we obtain our next result:

Proposition 4. *In the constant pricing setting, when facing a buyer of type $i \in \{M, F\}$, the seller's stocking quantity over the entire two-period horizon is given by:*

$$Q^*_i = \begin{cases} q^*_{1,i}, & \text{if } B\left(\frac{p-c}{p}\right) + \bar{q}_2 < q^*_{1,i} \\ B\left(\frac{p-c}{p}\right) + \bar{q}_2, & \text{if } q^*_{1,i} \le B\left(\frac{p-c}{p}\right) + \bar{q}_2 < q^*_{1,i} + \bar{q}_2 \\ q^*_{1,i} + \bar{q}_2, & \text{if } B\left(\frac{p-c}{p}\right) + \bar{q}_2 \ge q^*_{1,i} + \bar{q}_2 \end{cases} \tag{5}$$

Proposition 4 shows that the seller may ration her supply according to how the risk she faces for the buyer's purchase quantity in period 2 compares to the margin she makes. The seller critical ratio $\frac{p-c}{p}$ captures the seller's relative margin. The first subcase of $Q^*_i$ corresponds to a low margin setting. In this setting, the seller does not take any risk on the buyer's second period purchase quantity and stocks only enough product to fulfill the buyer's first period purchase quantity. The third subcase of $Q^*_i$ corresponds to a high margin setting. In this setting, the seller stocks the maximum quantity that the buyer could purchase over the horizon of $q^*_{1,i} + \bar{q}_2$. The second subcase of $Q^*_i$ corresponds to a medium margin setting, in which the seller is willing to take some risk on the buyer's period 2 purchase quantity and buys a quantity between the buyer's minimum and maximum demand.

The seller solves a newsvendor problem using the distribution of the demand the seller faces from the buyer, $N_i$, to determine $Q^*_i$. Because the forward-looking buyer generates stochastically larger demand than the myopic buyer (Lemma 1), the seller facing a forward-looking buyer stocks as much as or more in equilibrium than a seller facing a myopic buyer:

Lemma 2. *In the constant pricing setting, in equilibrium, the quantity the seller facing a forward-looking buyer stocks is greater than or equal to the quantity the seller facing a myopic buyer stocks – that is,* $Q_F^* \geq Q_M^*$.

This result differs from the common notion about sellers facing forward-looking buyers. In the traditional literature, where forward-looking behavior induces the seller to drop prices, the seller can counteract such behavior, which bears negative consequences, by rationing supply (Liu and Van Ryzin 2008). By contrast, in our setting, the seller facing the forward-looking buyer not only brings more supply but also makes as much as or more profit than when she faces a myopic buyer:

Proposition 5. *In the constant pricing setting, in equilibrium, the seller's profit when facing a forward-looking buyer is greater than or equal to the seller's profit when facing a myopic buyer – that is,* $\pi_F(Q_F^*) \geq \pi_M(Q_M^*)$.

## 5. Sophisticated Buyer under Constant Prices

In the first two subcases for the optimal seller stocking quantity in Proposition 4, the seller stocks less than the buyer's maximum purchase quantity over the horizon of $q_{1,M}^* + \bar{q}_2$. Hence, the buyer may face a stockout at the seller, requiring him to satisfy his demand at the higher priced runout option. In such a scenario, the buyer's perceived utility from his optimization problem will be greater than the actual utility he derives. Such stockout scenarios form the motivation for studying the sophisticated buyer, who does consider the seller's optimal stocking decision.

### 5.1. Buyer's Problem

In addition to considering the possibility of leftover inventory from period 1, the sophisticated buyer also considers the possibility of stock-outs at the seller. That is, recognizing the possibility of stock-outs, the buyer may shift demand to period 1, possibly signaling to the seller to stock more. A dependence is then created between the seller's stocking decision $Q$ and the buyer's purchase quantities $q_1$ and $q_2$. To formulate this problem, we need to expand our state space to include one more dimension for the leftover supply at the seller after period 1. Let $s \equiv Q - q_1$ denote the seller's leftover supply after the buyer's period 1 purchase. The starting state in period 2 can then be described in terms of the buyer's leftover inventory and the seller's leftover supply after period 1 by the tuple $(s, y)$.

In period 2, with buyer leftover inventory realization $y$ and seller leftover supply $s$ from period 1, the buyer chooses $0 \leq q_2(s,y) \leq s$ to maximize his utility:

$$\begin{aligned}\mathbb{E}[U_{2,S}(s,y)] =& v\mathbb{E}\big[\min\big(D_2, \min\big(q_2(s,y), Q-q_1(Q)\big)+y\big)\big] - p\min\big(q_2(s,y), Q-q_1(Q)\big)\\ &+ (v-r)\mathbb{E}\big[\big(D_2 - (\min\big(q_2(s,y), Q-q_1(Q)\big)+y)\big)^+\big]. \end{aligned} \tag{6}$$

In period 1, the buyer chooses $0 \leq q_1(Q) \leq Q$ to maximize his utility function for the entire horizon:

$$\begin{aligned}\mathbb{E}\big[U_{1,S}(Q)\big] =& v\mathbb{E}\big[\min\big(D_1, q_1(Q)\big)\big] - pq_1(Q) + (v-r)\mathbb{E}\big[\big(D_1 - q_1(Q)\big)^+\big]\\ &+ \mathbb{E}\big[V_{2,S}\big((Q-q_1(Q))^+, (q_1(Q)-D_1)^+\big)\big]. \end{aligned} \tag{7}$$

where $V_{2,S}(s,y)$ is the period 2 value function as given by $V_{2,S}(s,y) = \max_{0\leq q_2 \leq s} \mathbb{E}[U_{2,S}(s,y)]$.

While both of these objective functions are similar to those of the forward-looking buyer, the additional constraints requiring $q_2 \leq s$ in period 2 and $q_1 \leq Q$ in period 1 and the dependence between the seller's stocking decision and the buyer's purchase decisions make this problem more challenging to solve. Not only do the buyer's purchase decisions affect the seller's stocking decision, but now the seller's stocking decision also affects the buyer's purchase decisions, creating a feedback loop. The results of these derivations to determine the sophisticated buyer's optimal purchase quantities are summarized below:

Proposition 6. *The sophisticated buyer's purchase quantities in period* 1 *and period* 2 *are given by:*

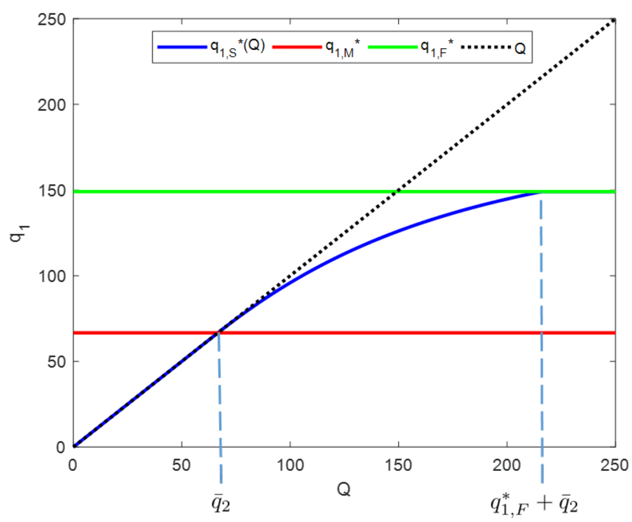
$$q^*_{1,S}(Q) = \begin{cases} Q, & \text{if } Q < \bar{q}_2\\ Q + B\left(\frac{p}{r}\right) - \frac{1}{r}\sqrt{\left(Bp+Qr\right)^2 + B\left(B(r-p)^2 - 2Qr^2\right)}, & \text{if } \bar{q}_2 \leq Q < q^*_{1,F} + \bar{q}_2\\ B\left(\frac{\sqrt{r^2-p^2}}{r}\right), & \text{if } Q \geq q^*_{1,F} + \bar{q}_2 \end{cases}$$

$$q^*_{2,S}(s,y) = \begin{cases} 0, & \text{if } y \geq \bar{q}_2\\ \bar{q}_2 - y, & \text{if } y < \bar{q}_2 \leq y+s\\ s, & \text{if } y+s < \bar{q}_2 \end{cases}$$

*where* $\bar{q}_{2,S} = B\left(\frac{r-p}{r}\right)$, $y \equiv (q_{1,S}(Q)^* - D_1)^+$, *and* $s \equiv Q - q_{1,S}(Q)$.

We illustrate the result for $q^*_{1,S}(Q)$ in Figure 2. There are three subcases for $q^*_{1,S}(Q)$, which we discuss in decreasing magnitude of the seller's $Q$. In the third subcase, when $Q$ is sufficiently large (i.e., above $q^*_{1,F} + \bar{q}_2$), the buyer does not face a risk of seller stock-out over the horizon. For any $Q$ greater than $q^*_{1,F} + \bar{q}_2$, the sophisticated buyer would never buy more in period 1, meaning that his period 1 purchase decision is independent of $Q$ in this range. In fact, in this range, the sophisticated buyer buys exactly as much as the forward-looking buyer in period 1. As the seller reduces $Q$ below $q^*_{1,F} + \bar{q}_2$, a risk of seller stock-out emerges and the buyer now needs to trade-off units purchased in the first period and in the second period. This trade-off results in the sophisticated buyer decreasing the amount he buys in the first period sooner and more sharply than the forward-looking buyer does as $Q$ decreases. Once $Q$ reaches the second period order-up-to level $\bar{q}_2$, the buyer purchases the entire stock in the first period – since he would have bought this amount anyway in the second period, he buys it upfront.

**Figure 2** $q^*_1(Q)$ **for sophisticated buyer:** $B = 200, v = 10, r = 9, p_1 = p_2 = 6, c = 3$



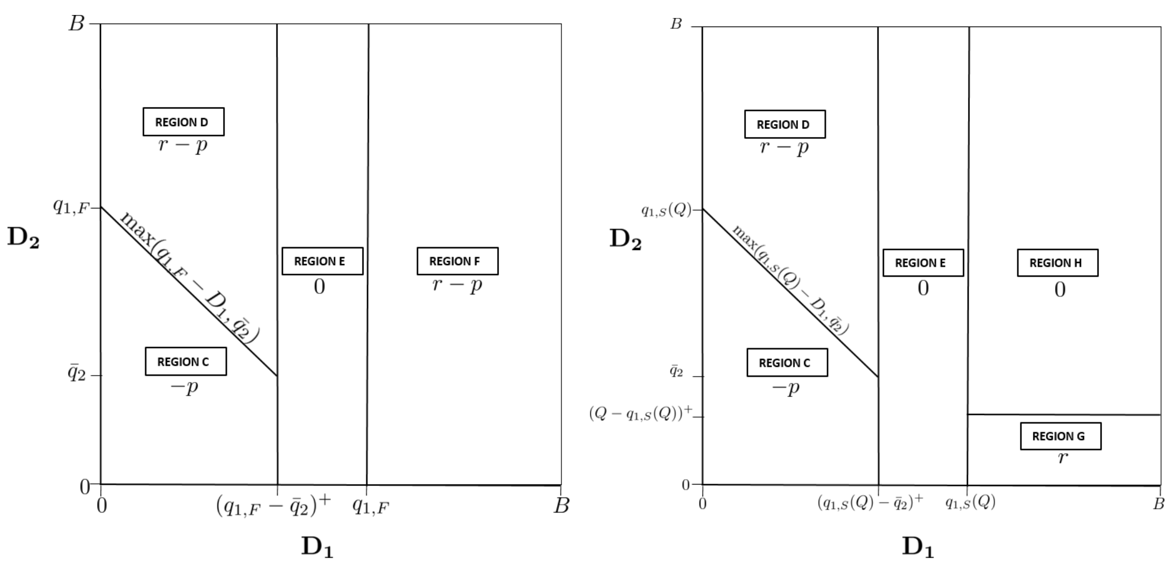

To better understand why the sophisticated buyer buys as much as or less in period 1 than the forward-looking buyer, even when the price is constant across periods, we turn our attention to the marginal analysis of the sophisticated buyer as compared to that of the forward-looking buyer. In Figure 3, we depict each buyer type's perceived marginal utility of increasing his period 1 purchase decision by one unit, for any given period 1 purchase decision and for any given demand realizations of $D_1$ and $D_2$. Mathematically, the perceived marginal utility depicted in the figure is $\frac{\partial U(q_1,D_1,D_2)}{\partial q_1}$ for the forward-looking buyer and $\frac{\partial U(q_1,D_1,D_2)}{\partial q_1(Q)}$ for the sophisticated buyer. While the forward-looking buyer does not consider $Q$ in deciding on $q_1$, the sophisticated buyer does, hence the figure illustrates the trade-offs considered by the sophisticated buyer for any given $Q$ such that $q_1(Q) \leq Q \leq q_1(Q) + \bar{q}_2$.

Regions C, D, and E are exactly the same for both buyer types. Suppose that the forward-looking buyer chooses any given $q_{1,F} \in [0, B]$. If the realization of $D_1$ turns out to be sufficiently low (i.e. $0 \leq D_1 \leq (q_{1,F} - \bar{q}_2)^+$), the buyer will have enough leftover inventory after period 1 that he does not need to purchase any units in period 2. Then, depending on the realization of $D_2$, there are two possibilities. If $D_2$ is low enough that it can be satisfied with this leftover inventory (i.e. less than $\max(q_{1,F} - D_1, \bar{q}_2)$), buying an additional unit of product in period 1 leads to a perceived marginal loss of $p$ (region C). But if $D_2$ is higher than this leftover inventory, buying an additional unit of product in period 1 results in a perceived marginal saving of $r - p$ (region D). Note that this marginal saving occurs in period 2 through the leftover inventory effects. Finally, if the realization of $D_1$ is in an intermediate range (i.e. $q_{1,F}$ is sufficient to cover $D_1$ but his leftover inventory is less than or equal to $\bar{q}_2$), if he were to buy one more unit in period 1, this unit would be carried over into the next period and deducted from the buyer's order-up-to quantity in period 2. It does not make a difference whether the unit is bought in the first or in the second period to the overall perceived utility and the marginal benefit is zero (region E).

The main difference in the marginal analysis for the sophisticated buyer compared to that of the forward-looking buyer is that Region F is replaced by two different regions: Region G and Region H. Both of these regions are regions of high demand realizations. In region G, the realization of $D_1$ is so high that the buyer does not have leftover and seeks to buy the full $\bar{q}_2$ from the seller. Given the seller's supply, he may or may not be able to buy what he wants from the seller, but given the low realization of $D_2$ he is able to cover his needs in period 2 without incurring runout costs. If the buyer buys one more unit in first period, he spends $p_1$ but saves the runout cost of $r$ in that period. The purchase of one more unit in period 1 reduces the seller's supply and takes away the opportunity to buy the unit from the seller in second period so the buyer saves $p_2$. This means that in this region, the buyer ends up with a gain of $r - p_1$ in the first period and $p_2$ in the second period, resulting in a marginal benefit of $(r - p_1) + p_2$ (equal to $r$ in the constant pricing setting).

In region H, the realization of $D_1$ is so high that regardless of whether the buyer buys an additional unit in period 1 or not, he does not have any leftover at the end of period 1. Accordingly, he seeks to buy $\bar{q}_2$ units in period 2, but he may be limited by the seller's leftover supply, $Q - q_1$. Thus, if he buys one more unit in period 1, he pays for one less unit of runout in that period, but then in period 2 he can buy one less unit at $p_2$ due to the seller's supply constraint and instead incurs one more unit of runout cost. This means that in this region, the buyer ends up with a gain of $r - p_1$ in the first period but gives up $r - p_2$ in the second period, resulting in a marginal benefit of $p_2 - p_1$ (equal to zero in the constant pricing setting).

**Figure 3** **Marginal utility of the forward-looking $q_1$ and sophisticated buyer's $q_1(Q)$ decision in the constant pricing setting**




(a) Forward-looking buyer (b) Sophisticated buyer

For the forward-looking buyer, for high realizations of $D_1$ regardless of the realization of $D_2$, we had Region F with a marginal saving of $r - p$. The attainment of this marginal saving assumed infinite supply at the seller. Now, for the sophisticated buyer, we have for the same high realizations of $D_1$, Region G with a marginal benefit of $r$ and Region H with a marginal benefit of 0. The lower marginal savings for the sophisticated incentivize the buyer to buy less in period 1 compared to the forward-looking buyer. In a sense, the forward-looking buyer is overly optimistic in high ability to buy product from the seller. We can summarize the comparison of $q^*_{1,S}(Q)$ and $q^*_{1,F}(Q)$ for any given $Q$ in the following corollary:

COROLLARY 4. *For any given $Q$, the sophisticated buyer buys as much as or less than the forward-looking buyer in period 1 – that is, $q^*_{1,S}(Q) \leq q^*_{1,F}(Q)$.*

When we compared the forward-looking buyer to the myopic buyer, the proof to demonstrate that the demand faced by the seller from a myopic buyer is greater than or equal to that faced by the seller from a forward-looking buyer, i.e. $N_F \geq N_M$ (Lemma 1). This proof relied on the fact that $q^*_{1,F}$ is strictly greater than $q^*_{1,M}$ (Corollary 2). In comparing $q^*_{1,S}(Q)$ and $q^*_{1,F}(Q)$ for any given $Q$ we do not have this strict inequality, which means we cannot analytically prove that $N_F(Q) \geq N_S(Q)$.

The seller's choice of $Q$ induces certain behaviors in the sophisticated buyer, but the buyer's choice of $q_1(Q)$ also induces certain behavior by the seller. We further explore the delicate interplay between the buyer's decisions and the seller's order quantity in the next subsection.

### 5.2. Seller's Problem

At the beginning of the horizon, given $q^*_{1,S}(Q)$ and $q^*_2(s,y)$, the seller chooses order quantity $Q \geq 0$ to maximize her profit function:

$$\mathbb{E}[\pi_S(Q)] = pq^*_{1,S}(Q) + p\mathbb{E}[\min(q^*_{2,S}, Q - q^*_{1,S}(Q))] - cQ \tag{8}$$

The first order optimality condition with respect to $Q$ is not a polynomial. While a closed-form expression for the roots exists, the expression does not lend itself to interpretation. More details on this expression are outlined in Section EC.3 of the E-Companion. Nonetheless, we can obtain the following result:

PROPOSITION 7. *When facing a sophisticated buyer, there exists a $Q^*_S$ that maximizes the seller's profit function, such that $\bar{q}_2 \leq Q^*_S \leq q^*_{1,F} + \bar{q}_2$.*

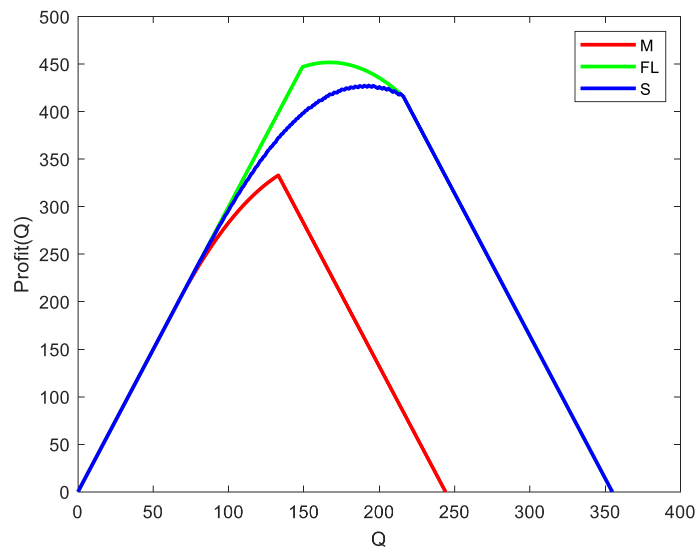
Through our numerical study, we can make another observation:

OBSERVATION 1. $Q^*_S \geq Q^*_F$

Why is it that, even though the sophisticated buyer buys less than or equal to the forward-looking buyer for any given $Q$, the seller facing the sophisticated buyer stocks at least as much or more than the seller facing a forward-looking buyer? The seller stocks more to induce the sophisticated buyer, who tends to buy at least the same or less than the forward-looking buyer, to buy more in the first period. The seller knows that the sophisticated buyer's decision is influenced by her stocking quantity. The seller takes into account the buyer's response function, which is non-decreasing in $Q$, specifically, we have that $0 < \frac{d}{dQ} q^*_{1,S}(Q) < 1$.[1] The seller can then manipulate the buyer into buying more, or less, based on the seller's order quantity. So, although we label the sophisticated buyer as "sophisticated", he is actually the easiest for the seller to manipulate.

Numerically, we observe that in the range where $Q^*$ exists – that is, between $\bar{q}_2$ and $q^*_{1,F} + \bar{q}_2$ – the seller makes more profit facing a forward-looking buyer than facing a sophisticated buyer. This ordering is observable in Figure 4.

**Figure 4** **Numerical results for seller's expected profit when facing each buyer type:** $B = 200, v = 10, r = 9, p_1 = p_2 = 6, c = 3$




We formalize this numerical result as the following observation:

Observation 2. $\mathbb{E}[\pi_M(Q^*_M)] \leq \mathbb{E}[\pi_S(Q^*_S)] \leq \mathbb{E}[\pi_F(Q^*_F)]$

## 6. Extension: Allowing for Different Prices

Suppose that the second period price is lower than the first period price. To take advantage of the lower price in the second period, the forward-looking buyer may shift some of his first period purchase quantity to the second period. Would the magnitude of this shift be large enough that the forward-looking buyer buys less than the myopic buyer, both in the first period and throughout the horizon? Would the seller no longer be better-off facing a forward-looking buyer? In this section, to investigate the effect of price differences between periods, we distinguish between the unit sales price in period 1 and in period 2, $p_1$ and $p_2$ respectively, and consider both the markdown and markup cases.

[1] This is provided in the proof of Proposition 7.

### 6.1. Buyer's Problem

For any given leftover inventory realization $y_i \equiv (q_{1,i} - D_1)^+$ from period 1, a buyer of type $i = \{M, F\}$ chooses purchase quantity $q_{2,i} \geq 0$ to maximize his period perceived utility function, given by:

$$\mathbb{E}[U_{2,i}(y_i)] = v\mathbb{E}[\min(D_2, q_{2,i} + y)] - pq_{2,i} + (v - r)\mathbb{E}[(D_2 - (q_{2,i} + y_i))^+]. \tag{9}$$

Proposition 8 is the extension of Proposition 1 when $p_1 \neq p_2$:

PROPOSITION 8. *In the seller markdown/markup setting, in period 2, a buyer of type $i = \{M, F\}$ purchases $q^*_{2,i} = (\bar{q}_2 - y_i)^+$ where $y_i \equiv (q^*_{1,i} - D_1)^+$ and $\bar{q}_2 = B\left(\frac{r-p_2}{r}\right)$.*

**6.1.1. Myopic Buyer** In period 1, the myopic buyer chooses purchase quantity $q_1 \geq 0$ to maximize his perceived period utility function, given by:

$$\mathbb{E}[U_{1,M}] = v\mathbb{E}[\min(D_1, q_1)] - p_1 q_1 + (v - r)\mathbb{E}[(D_1 - q_1)^+]. \tag{10}$$

Proposition 9 generalizes Proposition 2 when $p_1 \neq p_2$:

PROPOSITION 9. *In the seller markdown/markup setting, the myopic buyer's purchases $q^*_{1,M} = B\left(\frac{r-p_1}{r}\right)$.*

Recall that, in the constant pricing setting, the myopic buyer considers faces the same trade-offs in both periods, resulting in identical order-up-to levels (Corollary 1). Now, given price difference between periods, the trade-offs for each period are different, shifting the purchase quantities to one period or another. In the case of a markdown, the relative underage cost for the second period, $\frac{r-p_2}{r}$, is greater than that for the first period, $\frac{r-p_1}{r}$, incentivizing the buyer to buy more in the second period. We summarize this result, and its converse in case of a markup, in the following corollary:

COROLLARY 5. *When $p_2 < p_1$, $q^*_{1,M} < \bar{q}_2$. When $p_2 > p_1$, $q^*_{1,M} > \bar{q}_2$.*

**6.1.2. Forward-Looking Buyer** In period 1, the forward-looking buyer chooses a purchase quantity $q_1 \geq 0$ to maximize his horizon utility function, given by:

$$\mathbb{E}[U_{1,F}] = v\mathbb{E}[\min(D_1, q_1)] - p_1 q_1 + (v - r)\mathbb{E}[(D_1 - q_1)^+] + \mathbb{E}[U_2(q_1 - D_1)^+]. \tag{11}$$

Following an analysis similar to that in the constant pricing setting, we obtain the next result:

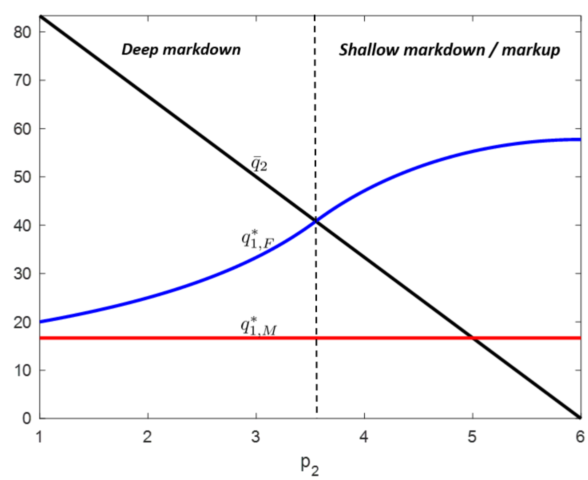
PROPOSITION 10. *The forward-looking buyer's purchase quantity in period* 1 *is given by:*

$$q^*_{1,F} = \begin{cases} B\left(\frac{r-p_1}{r-p_2}\right), & \text{if } \frac{r-p_2}{r} \geq \frac{r-p_1}{r-p_2} \\ B\left(\frac{\sqrt{r^2-p_2^2-2r(p_1-p_2)}}{r}\right), & \text{if } \frac{r-p_2}{r} < \frac{r-p_1}{r-p_2} \end{cases}$$

Proposition 10 uncovers a difference in behaviors that emerges due to price differences across periods. The ratios that define each subcase of $q^*_{1,F}$, i.e. $\frac{r-p_1}{r-p_2}$ and $\frac{r-p_2}{r}$, capture the trade-offs of buying in the first period versus the second period. These ratios are the relative underage costs out of the total mismatch costs for buying in each period. These two subcases of $q^*_{1,F}$ are illustrated in Figure 5.

The first subcase corresponds to a deep markdown scenario. When $p_2$ is significantly lower than $p_1$, i.e., $p_2 \leq r - \sqrt{r(r-p_1)}$, the underage cost in period 2 is higher than that in period 1. Despite the lower period 2 price, the buyer benefits from stocking slightly more already in period 1 to avoid runout in period 2. The idea is that, if the buyer has to pay the runout cost, he prefers to do so period 1 instead of in period 2. The second subcase of $q^*_{1,F}$ corresponds to a shallow markdown/markup scenario, which includes the constant pricing setting. As $p_2$ increases beyond the threshold, i.e., $p_2 > r - \sqrt{r(r-p_1)}$), the underage cost in period 2 forms a smaller proportion of period 2 mismatch costs than the underage cost in period 1 does out of the period 1 mismatch costs, so there is a dampening effect on the amount of product the buyer buys in period 1.

**Figure 5** **Comparison of $\bar{q}_2$, $q^*_{1,F}$, and $q^*_{1,M}$ when varying $p_2$ for** $B = 100, v = 10, r = 8, p_1 = 5, c = 1$




In the constant pricing setting, the order-up-to level in the first period was greater than that in the second period for the forward-looking buyer. Because the seller's prices were the same in both periods and lower than the runout option, the forward-looking buyer was inclined to buy more product in period 1 in case demand is high to avoid having to buy from the runout option. In the markdown/markup setting, the relationship between the order-up-to levels in period 1 and period 2 depends on the magnitude of the difference between the period prices. In the deep markdown scenario, the forward-looking buyer will buy more in period 1 than in period 2. In the shallow markdown or markup scenario, the forward-looking buyer's incentive to buy product in the first period is diminished. We summarize this observation in the next statement:

Corollary 6. *If* $p_2 \le r - \sqrt{r(r-p_1)}$ *(deep markdown),* $q^*_{1,F} \le \bar{q}_2$*. If* $p_2 > r - \sqrt{r(r-p_1)}$ *(shallow markdown),* $q^*_{1,F} > \bar{q}_2$*. In case of a markup,* $q^*_{1,F} > \bar{q}_2$*.*

**6.1.3. Comparison of Myopic Buyer and Forward-Looking Buyer** Regardless of the specific markdown/markup scenario, the forward-looking buyer still buys more in period 1 than the myopic buyer:

Corollary 7. *In the markdown/markup setting, the forward-looking buyer buys more than the myopic buyer in period 1 – that is,* $q^*_{1,F} > q^*_{1,M}$*.*

This result may seem somewhat counter-intuitive: given the reduced price in the second period, one might expect the forward-looking buyer to buy less in the first period than the myopic buyer who does not consider the price drop. However, as the forward-looking buyer can use any leftover inventory in the second period, the risk of buying more stock in the first period is mitigated. Using Corollary 7, we can show that the demand-enhancing effect of the forward-looking buyer persists in the markdown/markup setting. We can generalize Lemma 1 to the markdown/markup setting:

Lemma 3. *The demand for the seller facing a forward-looking buyer, denoted by* $N_F$*, is greater than or equal to the demand for the seller facing a myopic buyer – that is,* $N_F \ge N_M$*.*

### 6.2. Seller's Problem

The seller's profit function when she faces a buyer of type $i = \{M, F\}$ is:

$$\mathbb{E}[\pi_i(Q_i)] = p_1 q^*_{1,i} + p_2 \mathbb{E}[\min(q^*_{2,i}, Q_i - q^*_{1,i})] - cQ_i. \tag{12}$$

We derive a similar result for the seller's stocking decision as before:

PROPOSITION 11. *In the markdown/markup setting, when facing a buyer of type $i \in \{M, F\}$, the seller's stocking quantity over the entire two-period horizon is given by:*

$$Q_i^* = \begin{cases} q_{1,i}^*, & B\left(\frac{p_2-c}{p_2}\right) + \bar{q}_2 < q_{1,i}^* \\ B\left(\frac{p_2-c}{p_2}\right) + \bar{q}_2, & q_{1,i}^* \leq B\left(\frac{p_2-c}{p_2}\right) + \bar{q}_2 < q_{1,i}^* + \bar{q}_2 \\ q_{1,i}^* + \bar{q}_2, & B\left(\frac{p_2-c}{p_2}\right) + \bar{q}_2 \geq q_{1,i}^* + \bar{q}_2 \end{cases} \tag{13}$$

In a supply chain with a myopic buyer, because there is only one subcase for the myopic buyer's $q_{1,M}^*$ (Proposition 9), there are three possible situations: low seller margin (L), medium seller margin (M), and high seller margin (H). In a supply chain with a forward-looking buyer, because there are two possible subcases for the forward-looking buyer's $q_{1,F}^*$ (Proposition 10), there are six possible situations. We combine the two subcases of Proposition 10 for the buyer's $q_{1,F}^*$ (deep markdown and shallow markdown/markup) with the three subcases of Proposition 11 for the seller's $Q_F^*$ (low margin, a medium margin, and high margin scenario). One of the resulting situations (deep markdown, low seller margin) is ruled out as it requires the seller to earn negative margin, negating his participation in the market. In summary, we have five situations depending on the relationship between the relevant ratios of the buyer and seller: *(i)* Deep markdown, medium seller margin (DM), *(ii)* Deep markdown, high seller margin (DH), *(iii)* Shallow markdown/markup, low seller margin (SL), *(iv)* Shallow markdown/markup, medium seller margin (SM), and *(v)* Shallow markdown/markup, high seller margin (SH).

The differences in thresholds and in situations for a supply chain with a myopic buyer and one with a forward-looking buyer make it challenging to compare how the quantities purchased and stocked vary with $p_2$. As $p_2$ increases for a given $p_1, r$, and $c$, the situation in each supply change changes. We illustrate these changes for a specific instance in Figure 6 for a supply chain with both buyer types.

In a supply chain with a myopic buyer, the top panel of Figure 6 illustrates the effect of increasing $p_2$ on $Q_M^*$ and $q_{1,M}^*$. Since the buyer is myopic, his incentive to buy in period 1 does not change as $p_2$ increases so $q_{1,M}^*$ is constant. However, the second period order-up-to quantity $\bar{q}_2$ decreases as it becomes less favorable for the buyer to buy in period 2. For the seller, at the lower values of $p_2$, the seller is in the medium margin range (situation M) and brings an intermediate amount of product. As $p_2$ increases sufficiently above $c$ (threshold shown), the seller enters the high margin range (situation H) and stocks the maximum possible buyer demand over the horizon of $q^*_{1,M} + \bar{q}_2$. For the rest of the range where $p_2 < r$, despite the diminishing incentive for the buyer to buy in period 2, the seller's margin is still high enough that she continues to stock the maximum buyer demand.

For the same instance, in a supply chain with a forward-looking buyer, the lower panel of Figure 6 illustrates the effect of increasing $p_2$ on $Q^*_F$ and $q^*_{1,F}$. At lower values of $p_2$, the seller is in the medium margin range and the buyer is in the deep markdown range (situation DM). The buyer's $q^*_{1,F}$ increases in $p_2$ but is below $\bar{q}_2$ because he has an incentive to wait until period 2 to buy at a significantly lower price. The first threshold occurs when $p_2$ becomes high enough that the buyer's incentive to buy in period 2 equals the buyer's incentive to buy in period 1. At this point, while there is a still a markdown in the second period, $p_2$ is not low enough to discourage the buyer from buying more in period 1. The buyer has switched to being in a shallow markdown/markup scenario, but the seller is still in the medium margin scenario (situation SM). As $p_2$ increases above $p_1$ and further, a second threshold is reached, where the $p_2$ is so high that the quantities the seller expects the buyer to purchase are not significant enough to induce the seller to bring more than the known period 1 purchase quantity (situation SL). In this instance, for all $c < p_2 < r$, the seller always stocks less than the maximum quantity the forward-looking buyer would buy over the horizon, which means that the buyer may face seller stock-outs as he may not be able to buy the quantities that he seeks to buy from the seller.

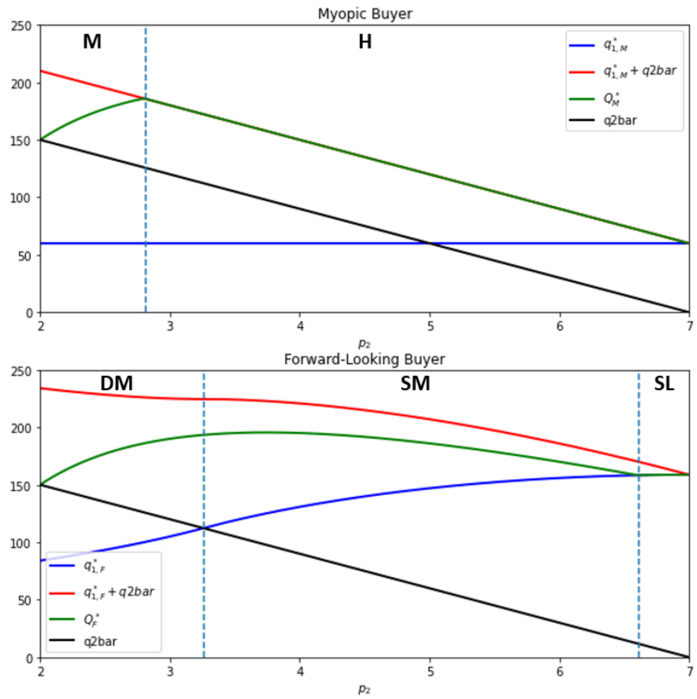
For the instance illustrated in Figure 6, $Q^*_F = Q^*_M$ for $p_2 \leq \frac{cr}{p}$. For all $p_2 > \frac{cr}{p}$, $Q^*_F > Q^*_M$. In fact, despite the price differences, we can generalize Lemma 2 to the markup/markdown setting:

LEMMA 4. *In the markdown/markup setting, in equilibrium, the quantity the seller facing a forward-looking buyer stocks is greater than or equal to the quantity the seller facing a myopic buyer stocks – that is,* $Q^*_F \geq Q^*_M$.

We can also generalize Proposition 5 to the markup/markdown setting:

PROPOSITION 12. *In the markdown/markup setting, in equilibrium, the seller's profit when facing a forward-looking buyer is greater than or equal to the seller's profit when facing a myopic buyer – that is,* $\pi_F(Q^*_F) \geq \pi_M(Q^*_M)$.

**Figure 6** **Varying $p_2$ for $B = 210, v = 10, r = 7, p_1 = 5, c = 2$**

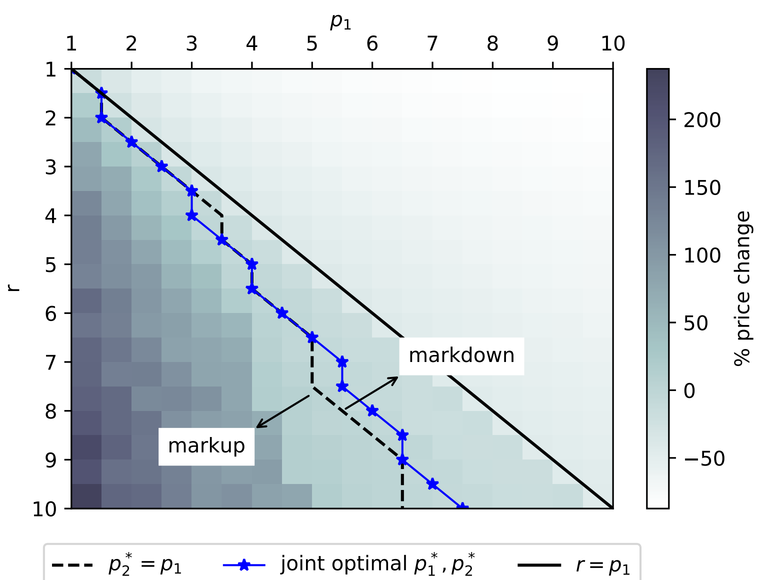



### 6.3. Optimal Seller Markdown / Markup Mechanism

What if the seller, in addition to choosing her stocking quantity for the horizon, could set prices in period 2 or in both periods? Recall that the main competition for the seller is the runout option. For this reason, one might expect that it is optimal for the seller to keep the price just below the runout option in one or even both periods. However, marking up product increases the buyer's overage cost in the second period, possibly resulting in a lower purchase quantity that period and a gravitation of demand to the first period, when the price is lower. While it is true that the buyer's optimal period 2 price is below the runout cost, the degree to which it is below the runout cost varies significantly.

Numerically, we investigate the optimal period 2 price, $p_2^*$, for a fixed unit wholesale cost $c$ while varying the unit runout cost $r$ and the unit period 1 price $p_1$. Our results indicate that there is a region where it is optimal to set $p_2 \geq p_1$ for any given $r$ and $p_1$ and another region where it is optimal to set $p_2 \leq p_1$. We illustrate these results for an instance with a forward-looking buyer in Figure 7. In this figure, for a given $c$, we vary $r$ along the $y$-axis and $p_1$ along the $x$-axis. The values in the interior of the plot are the percentage change between $p_1$ and $p_2^*$ for each level of $r$ and $p_1$, color-coded so that darker shades represent a deeper markup and lighter shades a deeper markdown. Along the diagonal, the runout cost $r$ is equal to the first period price $p_1$. When $r < p_1$ (above the diagonal), the buyer has no incentive to buy from the seller in period 1 as he satisfies all demand through the runout option, hence the seller brings no supply for the first period. Since the buyer buys exactly up to his demand realization in period 1 from the runout option, he has no leftover from period 1 and buys $\bar{q}_2$ in period 2. As long as the seller prices $p_2 < r$, she can sell $\bar{q}_2$ in period 2. So the seller stocks $Q_F^* = \bar{q}_2$ and sets $p_2^* = \frac{(c+r)}{2}$ in this region. While $p_2^* < p_1$ in this region, it is only trivially a "markdown" as the seller does not participate in the market in the first period. When $r > p_1$ (below the diagonal), the buyer has an incentive to buy from the seller instead of the runout option in period 1. Either a markdown or a markup may be optimal in this region. The dashed line is the threshold where it is optimal to set the period prices equal. Below the dashed line, it is optimal to increase the second period price above the first period price. The optimality of a markdown is most apparent for higher values of $r$ and lower values of $p_1$. Hence, the darker shade at the bottom left corner of the figure.

**Figure 7** **Optimal pricing for** $B = 100, v = 10, c = 1$




Finally, the starred line illustrates the jointly optimal $p_1^*$ and $p_2^*$ for any given level of $r$ and $c$. That is, if the seller were to maximize profit by jointly setting a pre-announced $p_1$ and $p_2$ for any given $r$ and $c$, the line delineates the optimal period prices. For lower values of $r$ (i.e., in this instance, up to around $r = 4$), it is jointly optimal to markup in period 2. As $r$ increases, however, it becomes jointly optimal to markdown in period 2. We summarize this observation formally as follows:

OBSERVATION 3. There exists a threshold value of $r$ below which it is jointly optimal to set prices such that $p_1 \geq p_2$. Above this threshold, it is jointly optimal to set prices such that $p_1 \leq p_2$.

## 7. Conclusion

In this paper, we characterized the stocking decisions of a seller and an intermediate buyer in a serial newsvendor supply chain when the buyer exhibits varying degrees of strategic behavior. In the absence of price differences between periods, we showed that, in comparison to the myopic buyer, the forward-looking buyer shifts some of the quantity he would buy in the second period to the first period as he knows he will be able to carry over any excess inventory. In addition to this shift in the first period purchase quantity, however, the forward-looking buyer also seeks to buy more than the myopic buyer over the horizon. For this reason, a seller facing a forward-looking buyer will stock as much as or more than the seller facing the myopic buyer and will make a greater than or equal profit from the forward-looking buyer than the myopic buyer. A buyer's forward-looking behavior, thus, can benefit a seller in such a supply chain.

Motivated by the observation that in some cases the seller does not stock the maximum quantity that the buyer demands over the horizon, we introduce a third buyer type called the sophisticated buyer. The sophisticated buyer additionally considers the seller's stocking decision in his purchase decisions. We find that in equilibrium the sophisticated buyer buys less than the forward-looking buyer in the first period for any given seller stocking quantity $Q$.

We then extended our study to allow for different prices across the periods, and more specifically, to allow the seller to markdown or markup product from the first period to the second period. Unlike in the constant pricing setting, the first period purchase quantity of the forward-looking buyer follows a threshold policy depending on the price difference. Despite the different prices, however, the results related to the profit generated when facing a forward-looking buyer versus a myopic buyer from the constant pricing setting persist. The forward-looking buyer still buys more than the myopic buyer in the first period. He also still demands as much as or more product from the seller over the horizon, which again results in a profit that is greater than or equal to that generated by the myopic buyer. We then investigated the optimal price in the second period assuming the seller can set this price upfront. Through a numerical study, we found that for a given runout cost, wholesale cost, and first period price, it may be optimal to markup or markdown product in the second period.

In summary, the seller should encourage a buyer to adopt some degree of strategic behavior and consider inter-temporality (i.e., be forward-looking) but not adopt the full degree of strategic behavior and consider the seller's stocking decision (i.e., be sophisticated). To this end, the seller should avoid inventory information sharing.

One interesting avenue for future research relates to coordination mechanisms. In our present work, we examined the supply chain outcomes in a setting where no coordination can occur. The outcomes under coordination and the optimal coordination mechanism are a promising direction for a follow-up study.

To the best of our knowledge, our work is the first to consider buyer strategic behavior in a serial newsvendor setting. Such serial settings are prevalent in supply chains, and while ample evidence supports the notion that varying degrees of strategic behavior are exhibited by human decision makers, such behaviors have received limited attention in supply chain contexts. As such, our paper paves the way for a potentially rich research avenue that can build on our modeling framework.

## References

Aviv, Yossi, Amit Pazgal. 2008. Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing & service operations management* **10**(3) 339–359.

Bulow, Jeremy I. 1982. Durable-goods monopolists. *Journal of political Economy* **90**(2) 314–332.

Cachon, Gérard P, Robert Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* **55**(3) 497–511.

Coase, Ronald H. 1972. Durability and monopoly. *The Journal of Law and Economics* **15**(1) 143–149.

Elmaghraby, Wedad, Altan Gülcü, Pınar Keskinocak. 2008. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing & Service Operations Management* **10**(1) 126–148.

Elmaghraby, Wedad, Pınar Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management science* **49**(10) 1287–1309.

Jin, Chen, Qian Liu, Chenguang Allen Wu. 2021. Selling multiple units to strategic consumers. *Operations Research Letters* **49**(2) 191–196.

Kabul, Mustafa O, Ali K Parlaktürk. 2019. The value of commitments when selling to strategic consumers: A supply chain perspective. *Management Science* **65**(10) 4754–4770.

Levin, Yuri, Jeff McGill, Mikhail Nediak. 2010. Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. *Production and Operations Management* **19**(1) 40–60.

Li, Jun, Nelson Granados, Serguei Netessine. 2014. Are consumers strategic? structural estimation from the air-travel industry. *Management Science* **60**(9) 2114–2137.

Lin, Yen-Ting, Ali K Parlaktürk, Jayashankar M Swaminathan. 2018. Are strategic customers bad for a supply chain? *Manufacturing & Service Operations Management* **20**(3) 481–497.

Liu, Qian, Garrett J Van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science* **54**(6) 1115–1131.

Mak, Vincent, Amnon Rapoport, Eyran J Gisches, Jiaojie Han. 2014. Purchasing scarce products under dynamic pricing: An experimental investigation. *Manufacturing & Service Operations Management* **16**(3) 425–438.

Osadchiy, Nikolay, Elliot Bendoly. 2015. Are consumers really strategic? implications from an experimental study. *Implications from an Experimental Study (October 2015)* .

Roundy, Robin. 1985. 98%-effective integer-ratio lot-sizing for one-warehouse multi-retailer systems. *Management science* **31**(11) 1416–1430.

Shaked, Moshe, J George Shanthikumar. 2007. *Stochastic orders*. Springer Science & Business Media.

Shen, Zuo-Jun Max, Xuanming Su. 2007. Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and operations management* **16**(6) 713–728.

Soysal, Gonca P, Lakshman Krishnamurthi. 2012. Demand dynamics in the seasonal goods industry: An empirical analysis. *Marketing Science* **31**(2) 293–316.

Su, Xuanming. 2007. Intertemporal pricing with strategic customer behavior. *Management Science* **53**(5) 726–741.

Su, Xuanming, Fuqiang Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science* **54**(10) 1759–1773.

Su, Xuanming, Fuqiang Zhang. 2009. On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science* **55**(5) 713–726.

Wei, Mike Mingcheng, Fuqiang Zhang. 2018. Recent research developments of strategic consumer behavior in operations management. *Computers & Operations Research* **93** 166–176.

Yilmaz, Ovunc, Mark Ferguson, Pelin Pekgun, Guangzhi Shang. 2019. Strategic behavior for hotel standby upgrade programs: Empirical evidence and pricing implications. *Working Paper* .

Yin, Rui, Yossi Aviv, Amit Pazgal, Christopher S Tang. 2009. Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers. *Management Science* **55**(8) 1391–1408.

Zhang, Dan, William L Cooper. 2008. Managing clearance sales in the presence of strategic customers. *Production and Operations Management* **17**(4) 416–431.

Zhang, Yang, Benny Mantin, Yaozhong Wu. 2019. Inventory decisions in the presence of strategic customers: Theory and behavioral evidence. *Production and Operations Management* **28**(2) 374–392.

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# E-Companion

## EC.1. Proofs

*Proof of Proposition 1* $\mathbb{E}[U_2(y)]$ can be rewritten as $\mathbb{E}[U_2(y)] = v\mathbb{E}[D_2] - pq_2 - r\mathbb{E}[(D_2 - (q_2 + y))^+]$. The first term is independent of $q_2$. The second term is linearly decreasing in $q_2$. For any realization of $D_2$, and for any given $y$, both $\mathbb{E}[(D_2 - (q_2 + y))^+]$ and $\mathbb{E}[((q_2 + y) - D_2)^+]$ are convex in $q_2$. As these expectations are multiplied by negative coefficients, the third and fourth terms are concave. Therefore, the critical point determined through the first order condition $\frac{d}{dq_2}\mathbb{E}[U_2(y)] = \frac{-r}{B}(q_2 + y) + r - p = 0$ is the unique maximizer. Alternatively, we can verify second order condition $\frac{d^2}{dq_2^2}\mathbb{E}[U_2(y)] = \frac{-r}{B} < 0$ as both $r$ and $B$ are strictly positive, therefore the critical point determined through the first order condition is a maximum. □

*Proof of Proposition 2* Similarly, $\mathbb{E}[U_1]$ can be written as $\mathbb{E}[U_1] = v\mathbb{E}[D_1] - pq_1 - r\mathbb{E}[(D_1 - q_1)^+]$. The first term is constant and does not depend on decision variable $q_1$. The second term is linearly decreasing in $q_1$. For any given realization of $D_1$, $\mathbb{E}[(D_1 - q_1)^+]$ is convex in $q_1$. As this expectation is multiplied by a negative coefficient, the third term is concave. Therefore, $\mathbb{E}[U_1]$ is concave in $q_1$ and the critical point determined through the first order condition $\frac{d}{dq_1}\mathbb{E}[U_1] = (r - p) - r\frac{q_1}{B} = 0$ is the unique maximizer. □

*Proof of Corollary 1* $q_{1,M}^* = B\left(\frac{r-p}{r}\right) = \bar{q}_2$. □

*Proof of Proposition 3* Evaluating $\mathbb{E}[U_2(y)]$ for $D_2 \sim U[0, B]$, we have that $\mathbb{E}[U_2(y)] = (v - r)\left(\frac{B}{2}\right) - pq_2 + r(q_2 + y) - \frac{r}{2B}(q_2 + y)^2$. Plugging in this expression for $\mathbb{E}[U_2(q_1 - D_1)^+]$ in $\mathbb{E}[U_1(q_1 - D_1)^+]$, we obtain $\mathbb{E}[U_1(q_1 - D_1)^+] = (v - r)B + (r - p)q_1 + (r - p)\mathbb{E}[(\bar{q}_2 - (q_1 - D_1)^+)^+] - \frac{r}{2B}\mathbb{E}[((\bar{q}_2 - (q_1 - D_1)^+)^+ + (q_1 - D_1)^+)^2]$. To explicitly evaluate these expectation terms, we need to consider two cases: *(i)* $\bar{q}_2 \geq q_1$ and *(ii)* $\bar{q}_2 < q_1$. In case *(i)*, $\mathbb{E}[(\bar{q}_2 - (q_1 - D_1)^+)^+] = \int_0^{q_1} [\bar{q}_2 - q_1 + x_1] f(x_1)\, dx_1 + \int_{q_1}^{B} \bar{q}_2 f(x_1)\, dx_1$ and $\mathbb{E}[((\bar{q}_2 - (q_1 - D_1)^+)^+ + (q_1 - D_1)^+)^2] = \int_0^{q_1} \bar{q}_2^2 f(x_1)\, dx_1 + \int_{q_1}^{B} \bar{q}_2^2 f(x_1)\, dx_1$. Therefore, evaluating for $D_1 \sim U[0, B]$, $\mathbb{E}[U_1(q_1 - D_1)^+] = (v - r)B + (r - p)\left(q_1 + \bar{q}_2 - \frac{q_1^2}{2B}\right) - \frac{r}{2B}\bar{q}_2^2$. The first order optimality condition for $q_1$ is $\frac{d}{dq_1}\mathbb{E}[U_1(q_1 - D_1)^+] = (r - p) - \frac{(r-p)}{B}q_1 = 0$.

In case *(ii)*, $\mathbb{E}[(\bar{q}_2 - (q_1 - D_1)^+)^+] = \int_{q_1 - \bar{q}_2}^{q_1} [\bar{q}_2 - q_1 + x_1] f(x_1)\, dx_1 + \int_{q_1}^{B} \bar{q}_2 f(x_1)\, dx_1$ and $\mathbb{E}[((\bar{q}_2 - (q_1 - D_1)^+)^+ + (q_1 - D_1)^+)^2] = \int_0^{q_1 - \bar{q}_2} (q_1 - x_1)^2 f(x_1)\, dx_1 + \int_{q_1 - \bar{q}_2}^{q_1} \bar{q}_2^2 f(x_1)\, dx_1 + \int_{q_1}^{B} \bar{q}_2^2 f(x_1)\, dx_1$. Evaluating for $D_1 \sim U[0, B]$, $\mathbb{E}[U_1(q_1 - D_1)^+] = (v - r)B + (r - p)\left(q_1 + \frac{\bar{q}_2^2}{2B} + \bar{q}_2 - \frac{q_1\bar{q}_2}{B}\right) - \frac{r}{2B}\left(\frac{q_1^3}{3B} - \frac{q_1\bar{q}_2^2}{B} + \bar{q}_2^2\right)$. The first order optimality condition with respect to $q_1$ is $\frac{d}{dq_1}\mathbb{E}[U_1(q_1 - D_1)^+] = (r - p)(1 - \frac{\bar{q}_2}{B}) - \frac{r}{2B^2}(q_1^2 - \bar{q}_2^2) = 0$. Substituting $\bar{q}_2 = B\left(\frac{r-p}{r}\right)$ and simplifying, $q_1^2 = 2B^2\frac{(r-p)}{r} - B^2\frac{(r-p)^2}{r^2}$. Clearly, as $q_1 \geq 0$, we are interested in the non-negative root of this quadratic, $q_1^* = B\left(\frac{\sqrt{r^2 - p^2}}{r}\right)$. Note, however, that there is a contradiction in the first case in which $\bar{q}_2 \geq q_1$ as $q_1^* = B > \bar{q}_2 = B\left(\frac{r-p}{r}\right)$. Therefore, only case *(ii)* holds. □

*Proof of Corollary 2* Observe that $q_{1,M}^* = B\left(\frac{r-p}{r}\right) = B\left(\frac{\sqrt{r-p}\sqrt{r-p}}{r}\right) < B\left(\frac{\sqrt{r-p}\sqrt{r+p}}{r}\right) = B\left(\frac{\sqrt{r^2-p^2}}{r}\right) = q_{1,F}^*$, where the inequality follows because $r > p > 0$. □

*Proof of Corollary 3* Since $\bar{q}_2 = q_{1,M}^* = B\left(\frac{r-p}{r}\right)$, the proof follows the same logical steps as for Corollary 2. □

*Proof of Lemma 1* By definition, a random variable $X$ is stochastically greater than or equal to another random variable $Y$ – that is, $X \geq_{st} Y$ – if, and only if, $\mathbb{P}(X \leq x) \geq \mathbb{P}(Y \leq x) \forall x \in (-\infty, \infty)$ This form of stochastic dominance is called stochastic dominance in the usual order or alternatively first-order stochastic dominance (Shaked and Shanthikumar 2007). Observe that $N_F$ and $N_M$ are non-decreasing in $q_{1,F}$ and $q_{1,M}$ respectively, hence $N_F \geq N_M$ for any realization of $D_1$ because $q^*_{1,F} > q^*_{1,M}$ by Corollary 2. □

*Proof of Proposition 4* The seller's profit function can be written as $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - w_R \mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+)^+)^+]$. To evaluate the last expectation term in $\mathbb{E}[\pi(Q)]$, we need to consider two cases for the relationship between $q^*_{1,M}$ and $\bar{q}_2$: *(i)* $\bar{q}_2 \geq q^*_{1,M}$ and *(ii)* $\bar{q}_2 < q^*_{1,M}$.

In case *(i)*, since $\bar{q}_2 \geq q^*_{1,M}$, $\bar{q}_2 \geq (q^*_{1,M} - D_1)^+$ and the second inner truncation can be eliminated as this term is always positive. We have $\mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+))^+] = \int_0^{\min(q^*_{1,M}, Q - \bar{q}_2)} (Q - \bar{q}_2 - x_1) f(x_1) dx_1 = \int_0^{Q - \bar{q}_2} (Q - \bar{q}_2 - x_1) f(x_1) dx_1$ as at optimality $q^*_{1,M} \leq Q^*_M \leq q^*_{1,M} + \bar{q}_2$. Evaluating for $D_1 \sim U[0, B]$, $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - \frac{p_2}{2B}(Q - \bar{q}_2)^2$.

In case *(ii)*, since $\bar{q}_2 < q^*_{1,M}$, the second inner truncation cannot be eliminated. We have $\mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+)^+)^+] = \int_0^{q^*_{1,M} - \bar{q}_2} (Q - q^*_{1,M}) f(x_1) dx_1 + \int_{q^*_{1,M} - \bar{q}_2}^{Q - \bar{q}_2} (Q - \bar{q}_2 - x_1) f(x_1) dx_1$. Evaluating for the uniform distribution of $D_1$, $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - \frac{p_2}{B}\left(\frac{Q^2}{2} - \frac{q^{*2}_{1,M}}{2} - Q\bar{q}_2 + q^*_{1,M}\bar{q}_2\right)$.

In both cases, the first order optimality condition for $Q$ is the same: $\frac{d}{dQ}\mathbb{E}[\pi(Q)] = (p_2 - c) - \frac{p_2}{B}(Q - \bar{q}_2) = 0$. The concavity of the objective function in both cases can also be easily verified as $\frac{d^2}{dQ^2}\mathbb{E}[\pi(Q)] = \frac{-p_2}{B} Q < 0$ since $p_2, B$ and $Q$ are strictly positive.

Finally, as we are dealing with a constrained optimization problem, we account for the requirement that the unique maximizer $B\left(\frac{p_2 - c}{p_2}\right) + \bar{q}_2$ is indeed such that $q^*_{1,M} \leq Q^*_M \leq q^*_{1,M} + \bar{q}_2$. Note that the first of the three subcases of $Q^*_M$ does not happen in case *(i)* as $\bar{q}_2 \geq q^*_{1,M}$ and $B\left(\frac{p_2 - c}{p_2}\right) > 0$ therefore $q^*_{1,M}$ is definitely less than or equal to a quantity greater than $\bar{q}_2$.

For the seller facing the forward-looking buyer, the same reasoning as with the seller facing the myopic buyer applies, only with the relevant $q^*_{1,F}$ instead of $q^*_{1,M}$. □

*Proof of Lemma 2* Since the seller determines $Q^*$ by solving a newsvendor problem with demand $N_F$ or $N_M$ when facing a forward-looking or myopic buyer respectively, $Q^*_M = F^{-1}_{N_M}\left(\frac{p-c}{p}\right)$ and $Q^*_F = F^{-1}_{N_F}\left(\frac{p-c}{p}\right)$. By Lemma 1, $Q^*_F \geq Q^*_M$. □

*Proof of Proposition 5* Observe that the seller solves a newsvendor problem with the demand given by $N_M$ or $N_F$ depending on the buyer type she faces. Therefore, $\pi_M(Q) = p\mathbb{E}[\min(Q, N_M)] - cQ$ and $\pi_F(Q) = p\mathbb{E}[\min(Q, N_F)] - cQ$. By Lemma 1, $\mathbb{E}[\min(Q, N_M)] \leq \mathbb{E}[\min(Q, N_F)]$ for any $Q \in \mathbb{R}^+$. Therefore, $\pi_M(Q^*_M) \leq \pi_F(Q^*_M) \leq \pi_F(Q^*_F)$, where the second inequality holds due to the optimality of $Q^*_F$ for $\pi_F$. □

*Proof of Proposition 6* To evaluate $\mathbb{E}[U_1((q_1(Q) - D_1)^+, Q)]$, observe that $\mathbb{E}[V_2((Q - q_1(Q))^+, (q_1(Q) - D_1)^+)] = \int_0^{q_1(Q)} V_2(Q - q_1(Q), q_1(Q) - x_1) f(x_1) dx_1 + \int_{q_1(Q)}^{B} V_2(Q - q_1(Q), 0) f(x_1) dx_1$. We first evaluate these two integrals for $D_2 \sim U[0, B]$:

$$V_2(s, y) = \begin{cases} (v - r)\frac{B}{2} + ry - \frac{r}{2B}y^2, & y \geq \bar{q}_2 \\ (v - r)\frac{B}{2} + py + \frac{B}{2}\frac{(r-p)^2}{r}, & \bar{q}_2 - s \leq y < \bar{q}_2 \\ (v - r)\frac{B}{2} + (r - p)s + ry - \frac{r}{2B}(y + s)^2, & y < \bar{q}_2 - s \end{cases}$$

$$V_2(s,0)=\begin{cases}(v-r)\frac{B}{2}+\frac{B}{2}\frac{(r-p)^2}{r}, & s\geq\bar{q}_2\\(v-r)\frac{B}{2}+(r-p)s-\frac{r}{2B}s^2, & s<\bar{q}_2\end{cases}$$

For $V_2(s,0)$, only two subcases remain as one of the subcases was eliminated since $\bar{q}_2$ cannot be negative. In both of these subcases, the buyer has no leftover inventory and seeks to buy the full $\bar{q}_2$. In the first subcase, the seller's leftover supply is sufficient to cover the buyer's needs for period 2. In the second subcase, the seller's leftover supply is not sufficient.

Based on the relationships between $q_1(Q)$ and $\bar{q}_2$ and $\bar{q}_2$ and $Q-q_1(Q)$, we consider four cases: *(i)* $q_1(Q)>\bar{q}_2$, $\bar{q}_2>Q-q_1(Q)$, *(ii)* $q_1(Q)\leq\bar{q}_2$, $\bar{q}_2>Q-q_1(Q)$, *(iii)* $q_1(Q)>\bar{q}_2$, $\bar{q}_2\leq Q-q_1(Q)$, *(iv)* $q_1(Q)\leq\bar{q}_2$, $\bar{q}_2\leq Q-q_1(Q)$. Cases *(i)* and *(ii)* are cases in which supply is limited as the seller's remaining supply after the first period is less than the order-up-to quantity in period 2.

In case *(i)*, $\mathbb{E}[U_1((q_1(Q)-D_1)^+,Q)]=(v-r)B+(r-p)Q-\frac{r}{2B}(Q-q_1(Q))^2+\frac{1}{2}\frac{(r-p)^2}{r}(Q-q_1(Q))+\frac{r}{6B^2}\left[Q^3-3q_1(Q)^2Q+q_1(Q)^3\right]+\frac{(r-p)}{B}\left[\frac{q_1(Q)^2}{2}-\frac{Q^2}{2}\right]$. The first order optimality condition with respect to $q_1(Q)$ is $\frac{d}{dq_1(Q)}\mathbb{E}[U_1((q_1(Q)-D_1)^+,Q)]=\frac{r}{2B^2}q_1(Q)^2-\frac{p}{B}q_1(Q)-\frac{r}{B^2}q_1(Q)Q+\frac{r}{B}Q-\frac{1}{2}\frac{(r-p)^2}{r}=0$. The solution to this quadratic is the root with the plus sign that does not violate $q_1(Q)\leq B$.

In case *(ii)*, $\mathbb{E}[U_1((q_1(Q)-D_1)^+,Q)]=(v-r)B+(r-p)Q-\frac{r}{2B}(Q-q_1(Q))^2+\frac{1}{2}\frac{(r-p)^2}{r}Q+\frac{r}{2B^2}\left[\frac{Q^3}{3}-q_1(Q)^2Q+\frac{2}{3}q_1(Q)^3\right]-\frac{B}{6}\frac{(r-p)^3}{r^2}-\frac{(r-p)}{2B}Q^2$. Solving for the first order optimality condition with respect to $q_1(Q)$, $q_1(Q)^*=\frac{B+Q}{2}\pm\frac{(B-Q)}{2}$. Using the plus sign for this expression, $q_1(Q)^*=B>\bar{q}_2=B\left(\frac{r-p}{r}\right)$ yields a contradiction as we are in the case where $q_1(Q)<\bar{q}_2$. Therefore, only the result using the minus sign remains.

In case *(iii)*, $\mathbb{E}[U_1]=(v-r)B+(r-p)q_1(Q)+\frac{B}{2}\frac{(r-p)^2}{r}+\frac{B}{6}\frac{(r-p)^3}{r^2}-\frac{1}{2}\frac{(r-p)^2}{r}q_1(Q)-\frac{r}{6B^2}q_1(Q)^3$. Solving for the first order optimality condition yields the same result that we obtained for the forward-looking buyer when $q_1(Q)>\bar{q}_2$.

In case *(iv)*, $\mathbb{E}[U_1]=(v-r)B+(r-p)q_1(Q)-\frac{(r-p)}{2B}q_1^2+\frac{B}{2}\frac{(r-p)^2}{r}$. Solving for the first order optimality condition, $q_1(Q)^*=B\left(\frac{r-p}{r-p}\right)=B$. However, since we are in the case in which $\bar{q}_2\geq q_1(Q)$, this result yields a contradiction as $q_1(Q)^*=B>\bar{q}_2=B\left(\frac{r-p}{r}\right)$. This case is eliminated. □

*Proof of Corollary 4* □

*Proof of Proposition 7* In optimality, the seller would never stock more than $q^*_{1,F}+\bar{q}_2$ units since any additional units beyond this amount will not be purchased. At the same time, the seller would never order a $Q$ less than $\bar{q}_2$ as she could earn more profit by increasing $Q$.

Note that $\mathbb{E}[\pi(Q)]=(p-c)\mathbb{E}[q^*_{1,S}(Q)+(\bar{q}_2-(q^*_{1,S}(Q)-D_1)^+)^+)]-(p-c)\mathbb{E}[\big((q^*_{1,S}(Q)+(\bar{q}_2-(q^*_{1,S}(Q)-D_1)^+)^+)-Q\big)^+]-c\mathbb{E}[\big(Q-(q^*_{1,S}(Q)+(\bar{q}_2-(q^*_{1,S}(Q)-D_1)^+)^+)\big)^+]$. Recall that $\mathbb{E}[q^*_{1,S}(Q)+q^*_{2,S}(Q)]=q^*_{1,S}(Q)+\mathbb{E}[(\bar{q}_2-(q^*_{1,S}(Q)-D_1)^+)^+)]$. The first component is deterministic and concave in $Q$ in the range of interest. The second component is also concave. For the remainder of the terms we verify that there is a single crossing point. If $Q$ increases, as long as $\mathbb{E}[q^*_{1,S}(Q)+q^*_{2,S}(Q)])$ does not increase faster than the underage cost is decreasing, then a single crossing point exists.

For $\frac{d}{dQ}q^*_{1,S}(Q)=1-\frac{Qr-B(r-p)}{\sqrt{\left(Bp+Qr\right)^2+B\left(B(r-p)^2-2Qr^2\right)}}$, we show that $0<\frac{d}{dQ}q^*_{1,S}(Q)<1$. Let $\theta=\frac{Qr-B(r-p)}{\sqrt{\left(Bp+Qr\right)^2+B\left(B(r-p)^2-2Qr^2\right)}}$. Suppose $\theta>1$. Then, expanding and rearranging, $0>B^2p^2$. However, by construction, $B^2p^2>0$ (contradiction). Therefore, $\theta<1$ and $\frac{d}{dQ}q^*_{1,S}(Q)>0$. Furthermore, since $\theta>0$,

$\frac{d}{dQ}q^*_{1,S}(Q) < 1$. Further note that $\frac{d}{dQ}\mathbb{E}[q^*_{2,S}(Q)] = \frac{(r-p)}{r}\Big(\theta - 1\Big) < 0$. By construction, $\frac{(r-p)}{r} < 1$. Since $\theta < 1$, $\theta - 1 < 0$ and $\frac{d}{dQ}\mathbb{E}[q^*_{2,S}(Q)] < 0$. Hence, $\frac{d}{dQ}\mathbb{E}[q^*_{1,S} + q^*_{2,S}] = \frac{p}{r}(1-\theta) < 1$. □

*Proof of Proposition 8* The same logic outlined in the proof for Proposition 1 applies to the period 2 objective function modified for the respective period price: $\mathbb{E}[U_2(y)] = v\mathbb{E}[\min(D_2, q_2 + y)] - p_2q_2 + (v - r)\mathbb{E}[(D_2 - (q_2 + y))^+]$. □

*Proof of Proposition 9* The same logic outlined in the proof for Proposition 2 applies to the period 1 objective functions modified for the respective period price: $\mathbb{E}[U_1] = v\mathbb{E}[\min(D_1, q_1)] - pq_1 + (v - r)\mathbb{E}[(D_1 - q_1)^+]$. □

*Proof of Proposition 10* Evaluating $\mathbb{E}[U_2(y)]$ for $D_2 \sim U[0,B]$, we have that $\mathbb{E}[U_2(y)] = (v-r)\Big(\frac{B}{2}\Big) - p_2q_2 + r(q_2+y) - \frac{r}{2B}(q_2+y)^2$. Plugging in this expression for $\mathbb{E}[U_2(q_1 - D_1)^+]$ in $\mathbb{E}[U_1(q_1-D_1)^+]$, we obtain $\mathbb{E}[U_1(q_1 - D_1)^+] = (v-r)B + (r-p_1)q_1 + (r-p_2)\mathbb{E}[(\bar{q}_2 - (q_1-D_1)^+)^+] - \frac{r}{2B}\mathbb{E}[((\bar{q}_2 - (q_1 - D_1)^+)^+ + (q_1 - D_1)^+)^2]$. To evaluate these expectations, we need to consider two cases: *(i)* $\bar{q}_2 \geq q_1$ and *(ii)* $\bar{q}_2 < q_1$.

In case *(i)*, $\mathbb{E}[(\bar{q}_2 - (q_1-D_1)^+)^+] = \int_0^{q_1}[\bar{q}_2 - q_1 + x_1]f(x_1)\,dx_1 + \int_{q_1}^{B}\bar{q}_2 f(x_1)\,dx_1$ and $\mathbb{E}[((\bar{q}_2 - (q_1-D_1)^+)^+ + (q_1 - D_1)^+)^2] = \int_0^{q_1}\bar{q}_2^2 f(x_1)\,dx_1 + \int_{q_1}^{B}\bar{q}_2^2 f(x_1)\,dx_1$. Therefore, evaluating for $D_1 \sim U[0,B]$, $\mathbb{E}[U_1(q_1 - D_1)^+] = (v-r)B + (r-p_1)q_1 + (r-p_2)(\bar{q}_2 - \frac{q_1^2}{2B}) - \frac{r}{2B}\bar{q}_2^2$. The first order optimality condition for $q_1$ is $\frac{d}{dq_1}\mathbb{E}[U_1(q_1 - D_1)^+] = r - p_1 - (r-p_2)\frac{q_1}{B} = 0$.

In case *(ii)*, $\mathbb{E}[(\bar{q}_2 - (q_1-D_1)^+)^+] = \int_{q_1-\bar{q}_2}^{q_1}[\bar{q}_2 - q_1 + x_1]f(x_1)\,dx_1 + \int_{q_1}^{B}\bar{q}_2 f(x_1)\,dx_1$ and $\mathbb{E}[((\bar{q}_2 - (q_1-D_1)^+)^+ + (q_1-D_1)^+)^2] = \int_0^{q_1-\bar{q}_2}(q_1 - x_1)^2 f(x_1)\,dx_1 + \int_{q_1-\bar{q}_2}^{q_1}\bar{q}_2^2 f(x_1)\,dx_1 + \int_{q_1}^{B}\bar{q}_2^2 f(x_1)\,dx_1$. Evaluating for uniform $D_1$, $\mathbb{E}[U_1(q_1 - D_1)^+] = (v-r)B + (r-p_1)q_1 + (r-p_2)\Big(\frac{\bar{q}_2^2}{2B} + \bar{q}_2 - \frac{q_1\bar{q}_2}{B}\Big) - \frac{r}{2B}\Big(\frac{2\bar{q}_2^3}{3B} + \frac{q_1^3}{3B} - \frac{q_1\bar{q}_2^2}{B} + \bar{q}_2^2\Big)$. The first order optimality condition with respect to $q_1$ is $\frac{d}{dq_1}\mathbb{E}[U_1(q_1-D_1)^+] = (r-p_1) - \frac{(r-p_2)}{B}\bar{q}_2 - \frac{r}{2B^2}q_1^2 + \frac{r}{2B^2}\bar{q}_2^2 = 0$. Substituting $\bar{q}_2 = B\Big(\frac{r-p_2}{r}\Big)$ and simplifying, $q_1^2 = 2B^2\frac{(r-p_1)}{r} - B^2\frac{(r-p_2)^2}{r^2}$. Clearly, as $q_1 \geq 0$, we are interested in the non-negative root of this quadratic, $q_1^* = \frac{B}{r}\sqrt{2(r-p_1)r - (r-p_2)^2}$.

Note that the condition for case *(i)* $\bar{q}_2 \geq q_1 \Leftrightarrow \frac{r-p_2}{r} \geq \frac{r-p_1}{r-p_2}$. Similarly, the condition for case *(ii)* $\bar{q}_2 < q_1 \Leftrightarrow \frac{r-p_2}{r} < \frac{r-p_1}{r-p_2}$. For the equivalence of these conditions in case *(ii)*, note that $\bar{q}_2 < q_1 \Leftrightarrow \bar{q}_2^2 < q_1^2$. Then $2B^2\frac{(r-p_2)^2}{r^2} < 2B^2\frac{(r-p_1)}{r}$. Since $0 < \frac{r-p_2}{r} < 1$, $2\frac{(r-p_1)}{(r-p_2)} > \frac{r-p_1}{r-p_2} > \frac{r-p_2}{r} > \frac{(r-p_2)^2}{r^2}$. Further note that the condition for a real root for the quadratic in the optimality condition is that $2\frac{(r-p_1)}{(r-p_2)} \geq \frac{(r-p_2)^2}{r^2}$ and by the reasoning in the last sentence this condition is trivially met. □

*Proof of Corollary 5* When $p_1 > p_2$, $q^*_{1,M} = B\Big(\frac{r-p_1}{r}\Big) > \bar{q}_2 = B\Big(\frac{r-p_2}{r}\Big)$ as the numerator $r - p_1$ is strictly less than the numerator $r - p_2$. When $p_1 < p_2$, the opposite relationship holds. □

*Proof of Corollary 6* For the first subcase of $q^*_{1,F}$, when $\frac{r-p_2}{r} \geq \frac{r-p_1}{r-p_2}$, it directly follows that $q^*_{1,F} = B\Big(\frac{r-p_1}{r-p_2}\Big) \leq \bar{q}_2 = B\Big(\frac{r-p_2}{r}\Big)$. For the second subcase of $q^*_{1,F}$, when $\frac{r-p_2}{r} < \frac{r-p_1}{r-p_2}$, note that after some algebraic steps we can rewrite this condition as $\frac{r-p_2}{r} < \frac{\sqrt{r^2-p_2^2-2r(p_1-p_2)}}{r}$. Then, $q^*_{1,F} = B\Big(\frac{\sqrt{r^2-p_2^2-2r(p_1-p_2)}}{r}\Big) > \bar{q}_2 = B\Big(\frac{r-p_2}{r}\Big)$ also directly follows. □

*Proof of Corollary 7* For the first subcase of $q^*_{1,F}$, when $\frac{r-p_2}{r} \geq \frac{r-p_1}{r-p_2}$, it is easy to verify that $q^*_{1,F} = B\Big(\frac{r-p_1}{r-p_2}\Big) > q^*_{1,M} = B\Big(\frac{r-p_1}{r}\Big)$. For the second subcase of $q^*_{1,F}$, when $\frac{r-p_2}{r} < \frac{r-p_1}{r-p_2}$, note that the subcase condition of $\frac{r-p_2}{r} < \frac{r-p_1}{r-p_2} \Leftrightarrow \frac{r-p_2}{r} < \frac{\sqrt{r^2-p_2^2-2r(p_1-p_2)}}{r} \Leftrightarrow 2rp_2 - rp_1 - p_2^2 > 0$. Since $r > p_1$, $2rp_2 - (p_2^2 + p_1^2) >$

$2rp_2 - (rp_1 + p_2^2) > 0$. Observe that $q^*_{1,F} = \frac{B}{r}\sqrt{r^2 - p_2^2 - 2r(p_1 - p_2)} = B\sqrt{\frac{(r-p_1)^2+2rp_2-p_1^2-p_2^2}{r^2}} > B\sqrt{\frac{(r-p_1)^2}{r^2}} = q^*_{1,M}$, where the inequality holds because the second subcase implies $2rp_2 - p_1^2 - p_2^2 > 0$. □

*Proof of Lemma 3* From Corollary 7, the proof follows the same logical steps as the proof for Lemma 1. □

*Proof of Proposition 11* The seller's profit function can be written as $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - p_2\mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+)^+)^+]$. To explicitly evaluate the last expectation term in $\mathbb{E}[\pi(Q)]$, we need to consider two cases for the relationship between $q^*_{1,M}$ and $\bar{q}_2$: *(i)* $\bar{q}_2 \geq q^*_{1,M}$ and *(ii)* $\bar{q}_2 < q^*_{1,M}$.

In case *(i)*, since $\bar{q}_2 \geq q^*_{1,M}$, $\bar{q}_2 \geq (q^*_{1,M} - D_1)^+$ and the second inner truncation can be eliminated as this term is always positive. We have $\mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+))^+] = \int_0^{\min(q^*_{1,M}, Q-\bar{q}_2)} (Q - \bar{q}_2 - x_1)f(x_1)dx_1 = \int_0^{Q-\bar{q}_2} (Q - \bar{q}_2 - x_1)f(x_1)dx_1$ as in optimality $q^*_{1,M} \leq Q^*_M \leq q^*_{1,M} + \bar{q}_2$. Evaluating for the uniform distribution of $D_1$, $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - \frac{p_2}{2B}(Q - \bar{q}_2)^2$.

In case *(ii)*, since $\bar{q}_2 < q^*_{1,M}$, the second inner truncation cannot be eliminated. We have $\mathbb{E}[(Q - q^*_{1,M} - (\bar{q}_2 - (q^*_{1,M} - D_1)^+)^+)^+] = \int_0^{q^*_{1,M}-\bar{q}_2} (Q - q^*_{1,M})f(x_1)dx_1 + \int_{q^*_{1,M}-\bar{q}_2}^{Q-\bar{q}_2} (Q - \bar{q}_2 - x_1)f(x_1)dx_1$. Evaluating for the uniform distribution of $D_1$, $\mathbb{E}[\pi(Q)] = (p_1 - p_2)q^*_{1,M} + (p_2 - c)Q - \frac{p_2}{B}\left(\frac{Q^2}{2} - \frac{q^{*2}_{1,M}}{2} - Q\bar{q}_2 + q^*_{1,M}\bar{q}_2\right)$.

In both cases, the first order optimality condition for $Q$ is the same: $\frac{d}{dQ}\mathbb{E}[\pi(Q)] = (p_2 - c) - \frac{p_2}{B}(Q - \bar{q}_2) = 0$. The concavity of the objective function in both cases can also be easily verified as $\frac{d^2}{dQ^2}\mathbb{E}[\pi(Q)] = \frac{-p_2}{B}Q < 0$ since $p_2, B$ and $Q$ are strictly positive.

Finally, as we are dealing with a constrained optimization problem, we account for the requirement that the unique maximizer $B\left(\frac{p_2-c}{p_2}\right) + \bar{q}_2$ is such that $q^*_{1,M} \leq Q^*_M \leq q^*_{1,M} + \bar{q}_2$. Note that the first of the three subcases of $Q^*_M$ does not happen in case *(i)* as $\bar{q}_2 \geq q^*_{1,M}$ and $B\left(\frac{p_2-c}{p_2}\right) > 0$ therefore $q^*_{1,M}$ is definitely less than or equal to a quantity greater than $\bar{q}_2$. □

*Proof of Lemma 4* Follows same logical steps as in proof of Lemma 2. □

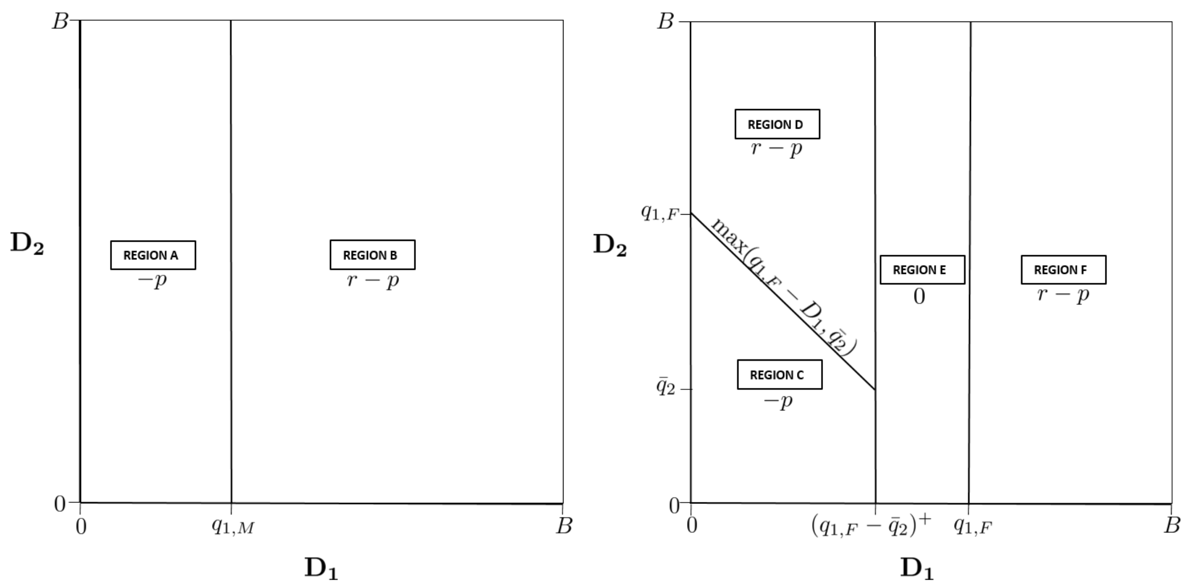
*Proof of Proposition 12* From Corollary 7 and Lemma 3, the same logic as Proposition 5 applies. □

## EC.2. Marginal Analysis

The forward-looking buyer, given his expanded degree of foresight to include intertemporality, no longer deals with simple single-period overage and underage costs but rather with multi-period overage and underage costs. The trade-off structure for each buyer type is illustrated in Figure EC.1. More specifically, in this figure, we depict each buyer type's perceived marginal utility of increasing his $q_1$ decision by one unit, for any given $q_1$ and for any given demand realizations of $D_1$ and $D_2$. Mathematically, this perceived marginal utility depicted in the figure is $\frac{\partial U(q_1, D_1, D_2)}{\partial q_1}$.

Figure EC.1 (left) illustrates the myopic buyer's marginal analysis when deciding on a first period purchase quantity $q_{1,M}$. Suppose that the myopic buyer chooses any given $q_{1,M} \in [0, B]$. If $D_1$ is less than or equal to $q_{1,M}$, the buyer incurs a perceived loss of $-p$ if he were increase this $q_{1,M}$ by one and purchase one more unit (region A). Conversely, if $D_1$ is greater than $q_{1,M}$, the buyer avoids paying the runout cost if he purchases one more unit of product and thus incurs a perceived marginal saving of $r - p$ (region B). It is as if the myopic buyer evaluates this graph for each possible value of $q_{1,M}$ to determine $q^*_{1,M}$. He does so by shifting the vertical line for $q_{1,M}$ from zero to the right of the $x$-axis to find the location at which his expected perceived marginal utility is maximized. This maximization point can be found by multiplying the size of regions A and B with their respective marginal values and finding the value of $q_{1,M}$ that makes the total expected marginal value equal to 0. Namely, the total expected margin is the gain $r-p$ times the area $(B-q_{1,M})B$ plus the loss $-p$ times the area $q_{1,M}B$. Thus, we have that $q_{1,M}(-p)+(B-q_{1,M})(r-p)=0$, which is solved by $q^*_{1,M}=B\left(\frac{r-p}{r}\right)$, as derived in Proposition 2. Since the myopic buyer does not consider $D_2$ or the implications of leftover inventory at all while making his period 1 decision, his weighing of the trade-offs is relatively simple.

**Figure EC.1 Perceived marginal utility of each buyer's $q_1$ decision in the constant pricing setting**




(a) Myopic buyer (b) Forward-looking buyer

Figure EC.1 (right) illustrates the forward-looking buyer's marginal analysis when deciding on a first period purchase quantity $q_{1,F}$. Note that this figure depicts all the possible regions that can emerge for a given $q_{1,F}$. However, not all $q_{1,F}$ have every region in the graph. For example, if $q_{1,F}\le\bar{q}_2$, regions C and D collapse. Suppose that the forward-looking buyer chooses any given $q_{1,F}\in[0,B]$. If the realization of $D_1$ turns out to be sufficiently low (i.e. $0\le D_1\le(q_{1,F}-\bar{q}_2)^+$), the buyer will have enough leftover inventory after period 1 that he does not need to purchase any units in period 2. Then, depending on the realization of $D_2$, there are two possibilities. If $D_2=x_2$ turns out to be low enough that it can be satisfied with this leftover inventory (i.e. less than $\max(q_{1,F}-D_1,\bar{q}_2)$), buying an additional unit of product in period 1 would lead to a perceived marginal loss of $-p$ (region C). But if $D_2=x_2$ turns out to be higher than this leftover inventory, buying an additional unit of product in period 1 leads to a perceived marginal saving of $r-p$ (region D). Note that this marginal saving occurs in period 2 through the leftover inventory effects.

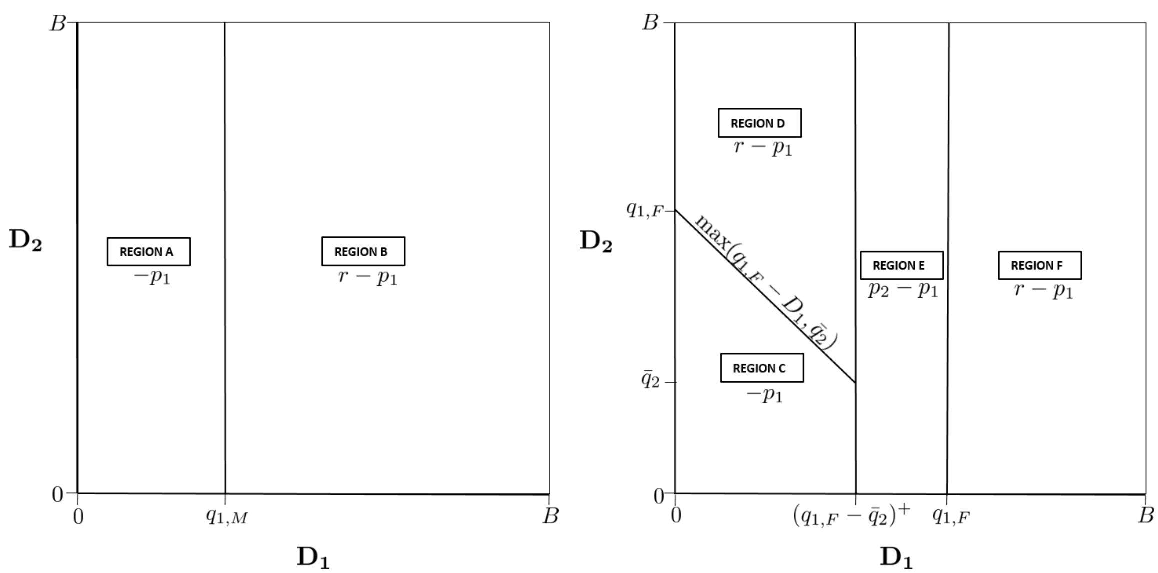
On the other extreme, if the realization of $D_1$ is so high that it exceeds $q_{1,F}$, the buyer would need to buy additional units in period 1 from the runout option to cover $D_1$. Then, because he would have no leftover inventory for the period 2, he would also need to buy the full period 2 order-up-to quantity of $\bar{q}_2$, regardless of the realization of $D_2$, plus potentially additional units from the runout option if $D_2$ is higher than $\bar{q}_2$. The buyer's perceived marginal savings occur in period 1, where he saves $r-p$ (region F).

Finally, if the realization of $D_1$ is in an intermediate range (i.e. $q_{1,F}$ is sufficient to cover $D_1$ but his leftover inventory is less than or equal to $\bar{q}_2$), if he were to buy one more unit in period 1, this unit would be carried over into the next period and deducted from the buyer's order-up-to quantity in period 2. It does not make a difference whether the unit is bought in the first or in the second period to the overall perceived utility and the marginal benefit is zero (region E).

For any given $q_{1,F}$, as explained for the myopic buyer, the forward-looking buyer evaluates the expected marginal value of buying one more unit of product over $q_{1,F}$ until he finds the $q_{1,F}$ that maximizes his perceived utility. For the forward-looking buyer, this maximizing value for $q_{1,F}$ is $q^*_{1,F} = B\left(\frac{\sqrt{r^2-p^2}}{r}\right)$, as derived in Proposition 3, which is the only non-negative solution to the equation $(r-p)(B-q_{1,F})B + 0(q_{1,F} - (q_{1,F}-\bar{q}_2))B + (-p)(q_{1,F}-\bar{q}_2)B + r(q_{1,F}-\bar{q}_2)(\frac{2B-q_{1,F}-\bar{q}_2}{2}) = 0$.

From Figure EC.1, it is clear that there are many demand realizations for which the forward-looking buyer expects to receive a greater than or equal marginal benefit than the myopic buyer from buying one more unit of product.

**Figure EC.2 Perceived marginal utility of each buyer's $q_1$ decision in the pre-announced markdown/markup setting**




(a) Myopic buyer (b) Forward-looking buyer

Introducing different prices in period 1 and period 2 changes the trade-offs that the forward-looking buyer considers in his period 1 purchase quantity decision. As can be seen in Figure EC.2, in comparison with Figure EC.1, in region E the perceived marginal utility of the forward-looking buyer is $-p_1 + p_2$ instead of zero. For a markup, this region's perceived marginal utility is positive, so the forward-looking buyer has an incentive to buy more in the first period and will shift $q_{1,F}$ to the right to increase the size of this region. Conversely, for a markdown, this region's perceived marginal utility is negative, so the forward-looking buyer has an incentive to buy less in the first period and will shift $q_{1,F}$ to the right to decrease the size of this region.

## EC.3. Solving for $Q_S^*$

Let $N_S(Q)$ refer to the amount that the buyer wants to purchase over the horizon (effectively the buyer's demand for the seller's product), where $N_S(Q) = q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+$. The seller's profit function can be written as:

$$\begin{aligned}
\mathbb{E}[\pi(Q)] =& p\mathbb{E}[\min(q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+, Q)] - cQ \\
=& (p-c)\mathbb{E}[q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+] - (p-c)\mathbb{E}[\big((q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+) - Q\big)^+] \\
& - c\mathbb{E}[\big(Q - (q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+)\big)^+] \\
=& (p-c)\mathbb{E}[N_S(Q)] - (p-c)\mathbb{E}[\big(N_S(Q) - Q\big)^+] - c\mathbb{E}[\big(Q - N_S(Q)\big)^+] \qquad \text{(EC.1)}
\end{aligned}$$

To arrive to (EC.1), we used the identities $\min(X, A) = X - (X - A)^+$ and $A = X - (X - A)^+$, where $A$ is a constant and $X$ is a random variable.

There are four cases to consider in evaluating the expectation terms: *(i)* $q_{1,S}(Q) > \bar{q}_2$, $\bar{q}_2 > Q - q_{1,S}(Q)$, *(ii)* $q_{1,S}(Q) \leq \bar{q}_2$, $\bar{q}_2 > Q - q_{1,S}(Q)$, *(iii)* $q_{1,S}(Q) > \bar{q}_2$, $\bar{q}_2 \leq Q - q_{1,S}(Q)$, and *(iv)* $q_{1,S}(Q) \leq \bar{q}_2$, $\bar{q}_2 \leq Q - q_{1,S}(Q)$. Because, as stated in Proposition 7, in optimality $\bar{q}_2 \leq Q^* \leq q_{1,F}^* + \bar{q}_2$, we only show Case $(i)$ in evaluating the expectation terms of $(EC.1)$:

- For $\mathbb{E}[N(Q)] = \mathbb{E}[(q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+)]$: When $D_1 < q_{1,S}^*(Q)$, there are two possibilities. The first possibility is that $\bar{q}_2 - q_{1,S}^*(Q) + D_1 > 0$. This inequality holds when $D_1 > q_{1,S}^*(Q) - \bar{q}_2$. So, when $q_{1,S}^*(Q) - \bar{q}_2 < D_1 < q_{1,S}^*(Q)$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q) + \bar{q}_2 - q_{1,S}^*(Q) + D_1 = \bar{q}_2 + D_1$. The second possibility is that $\bar{q}_2 - q_{1,S}^*(Q) + D_1 < 0$. This inequality holds when $D_1 < q_{1,S}^*(Q) - \bar{q}_2$. So, when $D_1 < q_{1,S}^*(Q) - \bar{q}_2 < q_{1,S}^*(Q)$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q)$. When $D_1 > q_1$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q) + \bar{q}_2$. So:

$$\begin{aligned}
\mathbb{E}[N(Q)] =& \int_0^{q_{1,S}^*(Q)-\bar{q}_2} q_{1,S}^*(Q) f(x_1)\,dx_1 + \int_{q_{1,S}^*(Q)-\bar{q}_2}^{q_{1,S}^*(Q)} (\bar{q}_2 + x_1) f(x_1)\,dx_1 + \int_{q_{1,S}^*(Q)}^{B} (q_{1,S}^*(Q) + \bar{q}_2) f(x_1)\,dx_1 \\
=& \frac{1}{B}\Big[\frac{\bar{q}_2^2}{2} - q_{1,S}^*(Q)\bar{q}_2\Big] + q_{1,S}^*(Q) + \bar{q}_2
\end{aligned}$$

- For $\mathbb{E}[(N(Q) - Q)^+] = \mathbb{E}[((q_{1,S}^*(Q) + (\bar{q}_2 - (q_{1,S}^*(Q) - D_1)^+)^+) - Q)^+]$: When $D_1 < q_{1,S}^*(Q)$, there are two possibilities. The first possibility is that $\bar{q}_2 - q_{1,S}^*(Q) + D_1 > 0$. This inequality holds when $D_1 > q_{1,S}^*(Q) - \bar{q}_2$. So, when $q_{1,S}^*(Q) - \bar{q}_2 < D_1 < q_{1,S}^*(Q)$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q) + \bar{q}_2 - q_{1,S}^*(Q) + D_1 = \bar{q}_2 + D_1$. The second possibility is that $\bar{q}_2 - q_{1,S}^*(Q) + D_1 < 0$. This inequality holds when $D_1 < q_{1,S}^*(Q) - \bar{q}_2$. So, when $D_1 < q_{1,S}^*(Q) - \bar{q}_2 < q_{1,S}^*(Q)$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q)$. When $D_1 > q_{1,S}^*(Q)$, $\mathbb{E}[N(Q)] = q_{1,S}^*(Q) + \bar{q}_2$. So:

$$\begin{aligned}
\mathbb{E}[(N(Q) - Q)^+] =& \int_{Q-\bar{q}_2}^{q_{1,S}^*(Q)} (\bar{q}_2 + x_1 - Q) f(x_1)\,dx_1 + \int_{q_{1,S}^*(Q)}^{B} (q_{1,S}^*(Q) + \bar{q}_2 - Q) f(x_1)\,dx_1 \\
=& \frac{1}{B}\Big[\frac{Q^2}{2} + \frac{\bar{q}_2^2}{2} - \frac{\bar{q}_{1,S}^*(Q)^2}{2} - Q\bar{q}_2\Big] + q_{1,S}^*(Q) + \bar{q}_2 - Q
\end{aligned}$$

- For $\mathbb{E}[(Q-N(Q))^+]=\mathbb{E}[(Q-(q^*_{1,S}(Q)+(\bar{q}_2-(q^*_{1,S}(Q)-D_1)^+)^+))^+]$:

$$\begin{aligned}\mathbb{E}[(Q-N(Q))^+]=&\int\limits_0^{q^*_{1,S}(Q)-\bar{q}_2}(Q-q^*_{1,S}(Q))f(x_1)\,dx_1+\int\limits_{q^*_{1,S}(Q)-\bar{q}_2}^{Q-\bar{q}_2}(Q-\bar{q}_2-x_1)f(x_1)\,dx_1\\ =&\frac{1}{B}\Big[\frac{Q^2}{2}-\frac{q^*_{1,S}(Q)^2}{2}-Q\bar{q}_2+q^*_{1,S}(Q)\bar{q}_2\Big]\end{aligned}$$

Consolidating:

$$\mathbb{E}[\pi(Q)]=(p-c)Q+\frac{p(r-p)}{r}(Q-q^*_{1,S}(Q))-\frac{p}{2B}(Q^2-q^*_{1,S}(Q)^2)$$

Let $\alpha=\sqrt{(Bp+Qr)^2+B(B(r-p)^2-2Qr^2)}$. Taking the first order derivative with respect to $Q$ and plugging in the respective expression for $q^*_{1,S}(Q)$:

$$\begin{aligned}\frac{d}{dQ}\mathbb{E}[\pi(Q)]=&(p-c)+\frac{p(r-p)}{r}-\frac{p}{B}Q+\frac{p}{2B}\frac{d}{dQ}q^*_{1,S}(Q)^2-\frac{p(r-p)}{r}\frac{d}{dQ}q^*_{1,S}(Q)\\ =&(p-c)+\frac{p(r-p)}{r}-\frac{p}{B}Q+\frac{p}{B}\Big[\Big(1-\frac{Qr-B(r-p)}{\alpha}\Big)\Big(Q+B\Big(\frac{p}{r}\Big)+\frac{1}{r}\alpha\Big)\Big]\\ &-\frac{p(r-p)}{r}\Big(1-\frac{Qr-B(r-p)}{\alpha}\Big)\\ =&(p-c)-\frac{p}{B}Q+\frac{p(r-p)}{r}\Big(\frac{Qr-B(r-p)}{\alpha}\Big)+\frac{p}{B}\Big[\Big(1-\frac{Qr-B(r-p)}{\alpha}\Big)\Big(Q+B\Big(\frac{p}{r}\Big)+\frac{\alpha}{r}\Big)\Big]\end{aligned}$$

Solving for the first order condition:

$$\begin{aligned}\frac{d}{dQ}\mathbb{E}[\pi(Q)]=&0\\ (p-c)-\frac{p}{B}Q=&-\frac{p(r-p)}{r}\Big(\frac{Qr-B(r-p)}{\alpha}\Big)-\frac{p}{B}\Big[\Big(1-\frac{Qr-B(r-p)}{\alpha}\Big)\Big(Q+B\Big(\frac{p}{r}\Big)+\frac{1}{r}\alpha\Big)\Big]\\ =&-\frac{Bp(r-p)\big(Qr-B(r-p)\big)}{\alpha Br}-\frac{p\big(\alpha-(Qr-B(r-p))\big)\big(Qr+Bp-\alpha\big)}{\alpha Br}\end{aligned}$$

Ensuring both sides have the same denominator of $\alpha Br$ and then multiplying both sides by $\alpha Br$:

$$Br(p-c)\alpha-Qpr\alpha=-BQp(r-p)r+B^2p(r-p)^2-p\big(\alpha-(Qr-B(r-p))\big)\big(Qr+Bp-\alpha\big)$$

It is not possible to significantly simplify the first order condition from this point onwards. In fact it is not possible to reduce the expression to the point that the square root is fully eliminated. As such, $Q^*_S$ can be determined numerically, but does not have an easy-to-interpret closed-form expression.