Physics-Based Cognitive Radar Modeling and Parameter Estimation

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***Abstract*—We consider the problem of channel response estimation in cognitive fully adaptive radar (CoFAR). We show that this problem can be expressed as a constrained channel estimation problem exploiting the similarity between the channel impulse responses (CIRs) of the adjacent channels. We develop a constrained CIR estimation (CCIRE) algorithm enhancing estimation performance compared to the unconstrained CIR estimation where the similarity between the CIRs of the adjacent channels is not employed. Further, we we derive the Cramér-Rao bound (CRB) for the CCIRE and show the optimality of the proposed CCIRE through comparing its performance with the derived CRB.**

***Index Terms*—Cognitive fully adaptive radar (CoFAR), constrained channel estimation, Cramér-Rao Bound (CRB), physics-based model, RFView**

I. Introduction

Cognitive fully adaptive radar (CoFAR) enables the joint optimization of the adaptive transmit and receive functions [1]–[4] by learning the target environment. Estimation of the complex clutter is an important aspect of learning the target environment. However, this would be an extremely complicated task using the traditional clutter model, involving a signal-dependent statistical covariance matrix [5], [6], due to the non-linear relationship between the signal-dependent covariance-based clutter model and the radar waveform. Hence, in [7], a new multi-input multi-output (MIMO) clutter model using a "stochastic transfer function" approach has been presented to address this problem. A key advantage of this new approach is that the transfer function is signal independent and this approach facilitates the joint optimization of the transmit waveform and the receiver functions.

In this approach, the transfer functions are computed from a fundamental physics-based scattering model using a Green's function. The stochastic transfer function is signal independent even though the actual clutter returns are dependent on the transmit signal. The transmit waveform is convolved with the target Channel Impulse Response (CIR) and the clutter CIR, respectively, the two resulting signals are superposed over the air and a noisy version is received. Since convolution can be represented by a linear multiplication of a matrix and a vector, this model offers the advantages associated with linear model. Though an analytical solution of the waveform design problem has been presented for a given channel transfer function matrix in [7], the channel transfer function matrix must be estimated from the measurement data in practice and the performance significantly depends on accuracy of the estimate of the channel transfer function. A low-complexity frequency domain approach has been proposed [8] to estimate the channel matrix so that the estimates can be incorporated into the optimization problem to accomplish the CoFAR objective of jointly designing the radar transmit-receive functions.

Although the frequency domain approach proposed in [8] is simple and enables a real-time implementation, it needs probing signals with a very high length to achieve an accurate CIR estimation a low signal-to-noise ratio (SNR) levels. This implies that we need to dedicate a considerable portion of COFAR resources for acquiring an accurate CIR estimate, which is undesirable. A possible approach to cope with this challenge is to exploit any further structural information or relationship between adjacent channel transfer functions in the estimation problem. In [9], the authors propose a constrained maximum likelihood (ML) problem to estimate the CIR where they used cosine similarity constraint in the estimation problem to exploit the fact that the channel impulse responses of adjacent pulses have close relationship between them. In this paper, we consider the Constrained CIR Estimation (CCIRE) in which, in addition to the cosine similarity constraint, we exploit the fact the CIR is typically sparse. We show that, using the proposed approach, it is possible to significantly reduce the length of the probing signal. Further, we derive the corresponding Cramér-Rao bound (CRB) for the constrained estimation problem and show that the proposed CCIRE could attain the resulting Constrained CRB (CCRB), implying its optimally.

The rest of the paper is organized as follows. In Section II, we describe the system model. The proposed CCIRE algorithm and CCRB are given in Section III. The simulation results are provided in Section IV. Finally, Section V concludes the paper. Throughout the paper, vectors and matrices are referred to by lower- and upper-case bold-face, respectively. The superscripts $*$, T, H denote the conjugate, transpose and Hermitian (conjugate transpose) operations, respectively. $\|\mathbf{a}\|_2$ stands for the ℓ_2-norm of $\mathbf{a}$. $[\mathbf{A}]_{i,j}$ and $[\mathbf{a}]_i$ indicate the $(i,j)^{\text{th}}$ and i^{th} entry of $\mathbf{A}$ and $\mathbf{a}$, respectively. $\hat{\mathbf{A}}$ and $\hat{\mathbf{a}}$ denote the estimate of $\mathbf{A}$ and $\mathbf{a}$, respectively. $\circledast$ and $\odot$ and $\oslash$ represent the convolution operator and vector element-wise product and vector element-wise division, respectively.

II. System Model

We consider the stochastic transfer function model presented in [7], wherein the measurement at the i-th receiver corresponding to the j-th pulse, denoted by $\mathbf{y}_{ij} \in \mathbb{C}^{(N+L-1)\times 1}$, is

$$\mathbf{y}_{ij} = \mathbf{s} \circledast \mathbf{h}_{ij} + \mathbf{n}_{ij} \tag{1}$$

where $\mathbf{h}_{ij} \in \mathbb{C}^{L\times 1}$ is the CIR computed from a physics-based model using the stochastic Green's function corresponding to the i-th channel and the j-th pulse, $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is a transmitted waveform vector, $\mathbf{n}_{ij} \in \mathbb{C}^{(N+L-1)\times 1}$ is the additive thermal noise and is assumed to follow i.i.d normal Gaussian distribution with variance σ^2.

The problem under consideration is to estimate the CIR from the corresponding measurements. The challenge is that the CIR varies from pulse to pulse and from one spatial channel to the other. Therefore, it is critical to estimate the CIR from only one realization of the data.

III. CIR Estimation

A. Unconstrained CIR Estimation

The simplest approach is to cast the CIR estimation as a Least Squares (LS) problem either in frequency or time domain. In time domain, the LS CIR estimation problem is formulated as follows

$$\widehat{\mathbf{h}}_{ij} = \underset{\mathbf{h}_{i,j}}{\operatorname{argmin}} \ \|\mathbf{y}_{ij} - \mathbf{S}\mathbf{h}_{ij}\|^2. \tag{2}$$

where $\mathbf{S} \in \mathbb{C}^{(N+L-1)\times L}$ is a convolution matrix corresponding to $\mathbf{s}$, given by

$$\mathbf{S} = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ s_2 & s_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_N & s_{N-1} & \cdots & s_1 \\ 0 & s_N & \cdots & s_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_N \end{bmatrix}. \tag{3}$$

Noting that $\mathbf{S}$ is a full-column rank matrix for any non-zeor $\mathbf{s}$, solving (2) yields

$$\widehat{\mathbf{h}}_{ij} = \mathbf{S}^{\dagger}\mathbf{y}_{ij}. \tag{4}$$

On the other hand, frequency-domain equivalence of (1) is given by

$$\overline{\mathbf{y}}_{ij} = \overline{\mathbf{s}}\overline{\mathbf{h}}_{ij} + \overline{\mathbf{n}}_{ij}, \tag{5}$$

where $\overline{\mathbf{y}}_{ij}$, $\overline{\mathbf{s}}$, $\overline{\mathbf{h}}_{ij}$, and $\overline{\mathbf{n}}_{ij}$ are the Fourier transformation of $\mathbf{y}_{ij}$, $\mathbf{s}$, $\mathbf{h}_{ij}$, and $\mathbf{n}_{ij}$, respectively. Accordingly, the frequency-domain LS CIR estimation problem is formulated as follows

$$\widehat{\overline{\mathbf{h}}}_{ij} = \underset{\overline{\mathbf{h}}_{ij}}{\operatorname{argmin}} \ \|\overline{\mathbf{y}}_{ij} - \overline{\mathbf{s}} \odot \overline{\mathbf{h}}_{ij}\|^2. \tag{6}$$

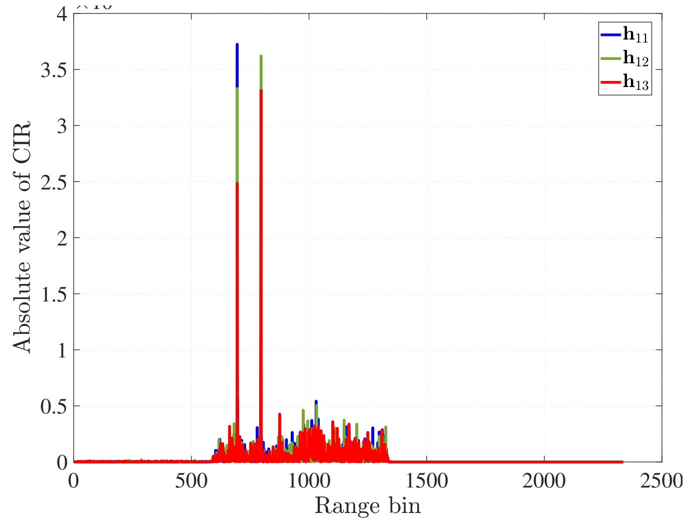

Figure 1. Absolute value of the true CIRs of channel 1 and pulse 1, 2 and 3, i.e., $\mathbf{h}_{11}$, $\mathbf{h}_{12}$ and $\mathbf{h}_{13}$.

Solving (6) yields

$$\widehat{\overline{\mathbf{h}}}_{ij} = \overline{\mathbf{y}}_{ij} \oslash \overline{\mathbf{s}}, \tag{7}$$

whose inverse Fourier transformation gives the CIR estimate.

Although the discussed Unconstrained CIR Estimations (UCIRE) provide simple closed-form expressions for the CIR estimates, they are susceptible to significant performance degradation at the low SNR regime.

B. Constrained CIR Estimation

In order to improve the CIR estimation, the relationship between adjacent CIRs can be exploited. Fig. 1 shows the absolute value of the true CIRs of the first receive antenna and pulses 1, 2 and 3, i.e., $\mathbf{h}_{11}$, $\mathbf{h}_{12}$ and $\mathbf{h}_{13}$, obtained from RFView [10]. We make two important observations from Fig. 1. Firstly, it is seen that the CIRs have most of the energy in some particular range bins and little energy in the rest of the range bins. Hence, the CRIs can be approximated as sparse vectors with quite similar support. Secondly, it is observed that the absolute values of the non-zero elements of the adjacent CIRs are quite similar. Indeed, if we already have a good estimate of an adjacent CIR of the CIR of interest, it would offer us good indication of the CIR support and the absolute values of the non-zero elements of the CIR of interest. We can use these additional information to obtain a constrained estimate of a CIR with improved performance compared to the unconstrained one. In the following, we will discuss how to achieve this.

1) Sparsity constraint: Let us assume that we have an estimate of an adjacent CIR. Exploiting the fact that that the CIRs are sparse, we can estimate the support of the adjacent CIR by comparing the absolute values of its elements with a given threshold and considering the elements below the threshold equal to zero. Now, considering the fact that adjacent channels share almost the same support, the non-zero elements of the CIR of interest should include those of the adjacent CIR plus some additional elements next to them. Hence, we can say

$$[\mathbf{h}_{ij}]_p = 0,\ p \in \mathbb{M}, \tag{8}$$

where $\mathbb{M}$ is the index set of zero elements.

2) Cosine Similarity constraint: In order to measure the similarity between the the absolute values of the non-zero elements of the adjacent CIR and the CIR of interest, we could use the absolute cosine similarity constraint. The absolute cosine similarity measures similarity between two non-zero vectors of an inner product space that measures the cosine of the angle between them. For two CIRs $\mathbf{h}_{ij}$ and $\mathbf{h}_{il}$, it is defined by [9]

$$|\cos(\mathbf{h}_{ij}, \mathbf{h}_{il})| = \frac{|\mathbf{h}_{ij}^H \mathbf{h}_{il}|}{\|\mathbf{h}_{ij}\|_2 \|\mathbf{h}_{il}\|_2}. \tag{9}$$

The higher value of the absolute cosine similarity implies the higher similarity between the non-zero elements of $\mathbf{h}_{ij}$ and $\mathbf{h}_{il}$. Assuming $|\cos(\mathbf{h}_{ij}, \mathbf{h}_{il})| = \tau$, we can reformulate (9) as follows:

$$|\cos(\mathbf{h}_{ij}, \mathbf{h}_{il})|^2 = \frac{|\mathbf{h}_{ij}^H \mathbf{h}_{il}|^2}{\|\mathbf{h}_{ij}\|_2^2 \|\mathbf{h}_{il}\|_2^2} = \tau^2 \tag{10}$$

$$\Rightarrow \mathbf{h}_{ij}^H (\mathbf{h}_{il}\mathbf{h}_{il}^H - \tau^2 \|\mathbf{h}_{il}\|^2 \mathbf{I}) \mathbf{h}_{ij} = \mathbf{0}. \tag{11}$$

Making use of (8) and (10), the CCIRE problem can be cast as follows:

$$\begin{aligned} \underset{\mathbf{h}_{ij}}{\text{minimize}} \quad & \|\mathbf{y}_{ij} - \mathbf{S}\mathbf{h}_{ij}\|^2 \\ \text{subject to} \quad & [\mathbf{h}_i]_j = 0,\ j \in \mathbb{M}, \\ & \mathbf{h}_{ij}^H (\mathbf{h}_{il}\mathbf{h}_{il}^H - \tau^2 \|\mathbf{h}_{il}\|^2 \mathbf{I}) \mathbf{h}_{ij} = \mathbf{0}. \end{aligned} \tag{12}$$

Following the same approach in [9], the above optimization problem can be relaxed to the following convex optimization problem

$$\begin{aligned} \underset{\mathbf{h}_{ij}, \mathbf{H}_{ij}}{\text{minimize}} \quad & \text{tr}\{\mathbf{S}^H \mathbf{S} \mathbf{H}_{ij}\} - 2\Re\{\mathbf{y}_{ij}^H \mathbf{S} \mathbf{h}_{ij}\} \\ \text{subject to} \quad & [\mathbf{h}_{ij}]_p = 0,\ p \in \mathbb{M}, \\ & \text{tr}\{(\mathbf{h}_{il}\mathbf{h}_{il}^H - \tau^2 \|\mathbf{h}_{il}\|^2 \mathbf{I}) \mathbf{H}_{ij}\} \\ & \mathbf{H}_{ij} \succeq \mathbf{h}_{ij}\mathbf{h}_{ij}^H. \end{aligned} \tag{13}$$

The above optimization problem is a Semidefinite Programming (SDP), which can be efficiently solved.

C. Constrained CRB Derivation

In what follows, we will derive the CRB for the problem of estimating CIR with sparsity and cosine similarity constraints. The CRB is then used as a benchmark to assess the performance of the proposed CCIRE. The CRB under the parametric constraints can be found by a reparameterization of the original problem to remove redundancies in the parameter vector but this approach may be difficult. Hence, we alternatively resort to the following Proposition to derive the CRB for the constrained CIR estimation problem.

Proposition 1. *Let $\mathbf{y}$ be the vector of observations and $\boldsymbol{\theta} \in \mathbb{R}^{n\times 1}$ be the vector of deterministic parameters to be estimated from $\mathbf{y}$. Let $\hat{\boldsymbol{\theta}}$ be an unbiased estimator of $\boldsymbol{\theta}$ satisfying m ($m \leqslant n$) smooth constraints,*

$$\mathbf{f}(\hat{\boldsymbol{\theta}}) = \mathbf{0}, \tag{14}$$

such that $\{\boldsymbol{\theta} \mid \mathbf{f}(\boldsymbol{\theta}) = \mathbf{0}\}$ is non-empty. Then, we have

$$\mathbb{E}\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^H\} \succeq \mathbf{U}(\mathbf{U}^H \mathbf{J} \mathbf{U})^{-1} \mathbf{U}^H, \tag{15}$$

where $\mathbf{J}$ is the FIM for the unconstrained parameter estimation and $\mathbf{U}$ is a matrix whose column form an orthonormal basis for the nullspace of $\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^H}$.

Proof. Please refer to [11]. □

To compute the constrained CRB for the CIR estimation, we need to first derive the unconstrained, i.e., $\mathbf{J}$. From [12], the constrained CRB is given by

$$\mathbf{J} = \sigma^2 \mathbf{S}^H \mathbf{S}. \tag{16}$$

Then, we need to calculate $\mathbf{U}$. Here, for the CIR estimation, we have $\boldsymbol{\theta} = \mathbf{h}_{ij}$ and, based on (8) and (12), $f(\boldsymbol{\theta})$ is given by

$$f(\mathbf{h}_{ij}) = \begin{bmatrix} \mathbf{h}_{ij}^H (\mathbf{h}_{il}\mathbf{h}_{il}^H - \tau^2 \|\mathbf{h}_{il}\|^2 \mathbf{I}) \mathbf{h}_{ij} \\ [\mathbf{h}_{ij}]_{p_1} \\ [\mathbf{h}_{ij}]_{p_2} \\ \vdots \\ [\mathbf{h}_{ij}]_{p_M} \end{bmatrix}, \tag{17}$$

where $p_1, p_2, \cdots, p_M \in \mathbb{M}$. Hence, we obtain

$$\frac{\partial \mathbf{f}(\mathbf{h}_{ij})}{\partial \mathbf{h}_{ij}^H} = \begin{bmatrix} \mathbf{h}_{ij}^H (\mathbf{h}_{il}\mathbf{h}_{il}^H - \tau^2 \mathbf{I} \|\mathbf{h}_{il}\|^2) \\ \mathbf{g}_{p_1}^T \\ \mathbf{g}_{p_2}^T \\ \vdots \\ \mathbf{g}_{p_M}^T \end{bmatrix}, \tag{18}$$

where $\mathbf{g}_{p_\ell} \in \{0,1\}^{\times 1}$ is defined as

$$[\mathbf{g}_{p_\ell}]_i = \begin{cases} 1, & \text{if} \quad i = p_\ell, \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Then, $\mathbf{U}$ will be equal to the kernel of the matrix given in (18). Finally, the CCRB is obtained by inserting $\mathbf{J}$ and $\mathbf{U}$ in (15).

IV. Numerical Experiments

In this section, we provide simulation results for assessing the peformance of the proposed CCIRE using realistic data obtained from highfidelity modeling and simulation software RFView. RFView uses publicly available terrain data and land cover types to accurately model ground clutter returns for radio frequency systems by dividing the entire clutter region into individual clutter patches. In the dataset generated using RFView, the monostatic radar platform is flying over Southern California with a speed of 100 m/s. Note that speed is very important for interpreting the stationarity of the channel impulse responses because the faster the platform moves, the more dynamic the channel becomes.

Fig. 2 depicts the CCRB and Unconstrained CRB (UCRB) along with the average Mean-Square Errors (MSEs) of CCIRE

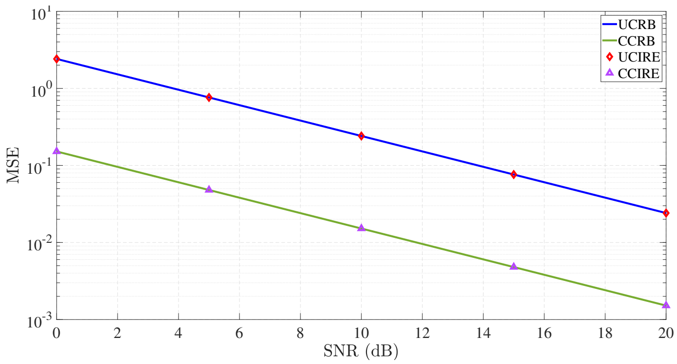

Figure 2. Constrained and Unconstrained CRB versus the SNR.

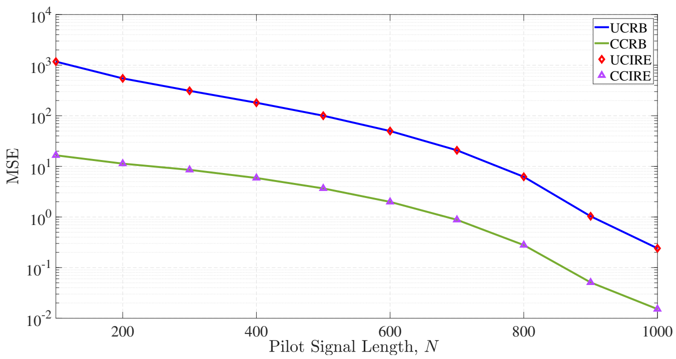

Figure 3. Constrained and Unconstrained CRB versus the length of the pilot signal i.e., N.

and CIRE versus the SNR. The average MSE is obtained by averaging over 1000 noise realization, defined as follows:

$$\frac{\sum_{k=1}^{1000} \|\hat{\mathbf{h}}_{ij}^{(k)} - \mathbf{h}_{ij}\|_2^2}{1000}, \tag{20}$$

where $\hat{\mathbf{h}}_{ij}^{(k)}$ denotes the estimate of $\mathbf{h}_{ij}$ at the k-th noise realization. It is observed that using the prior information obtained from the adjacent channel could significantly improve the performance of the CIR estimation. Specifically, it is seen that the CCIRE leads to 12 dB performance improvement over CIRE. Further, it is observed that the performance of the CCIRE coincides with CCRB, impliying its optimally in terms of the estimation accuracy.

Fig. 3 shows the CCRB and UCRB along with the average MSEs of CCIRE and CIRE versus the length of the pilot signal. Fig. 3 reveals that the CCIRE needs a pilot signal with about 20% to 50% less length compared to the CIRE. This implies that using the the CCIRE allows for dedicating the greater portion of resources, i.e., time and power, to the radar functionality in CoFAR while preserving the CIR estimation performance.

V. Summary

In this paper, we investigated the problem of channel response estimation in Cognitive fully adaptive radar (CoFAR). Exploiting the similarity between the Channel Impulse Responses (CIRs) of the adjacent channels, we developed a CCIRE algorithm enhancing estimation performance compared to the UCIRE where the similarity between the CIRs of the adjacent channels is not deployed. The Enhanced performance of the proposed CCIRE allows for reducing the length of the pilot signals which in turn offers a possibility of dedicating more portion of the resources to radar functionality in CoFAR.

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