Market games in successive oligopolies*

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Abstract

This paper introduces a new approach of successive oligopolies. We rely on market games à la Shapley-Shubik to examine how successive oligopolies operate between downstream and upstream markets when the input price is determined by the action of all firms, downstream and upstream ones. This approach, differs from the classical one since it allows to consider downstream firms who exercise market power both in the downstream and in the upstream market. We perform a comparison of the market outcome with each regime and a welfare analysis.

Keywords: successive oligopolies, market games à la Shapley-Shubik

JEL classification: D43, L1, L13, L22

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1 Introduction

In the traditional theory of successive markets, the property of *double marginalization* has attracted the interest of scholars since its very discovery by Spengler in 1950. This property says that, when the supply chain is monopolized in each of the successive markets, the price of the final product embodies the monopoly unit margins arising in each of them. This theory is cast assuming that firms, while behaving as monopolists in their own output market, behave as price takers when buying their input. This assumption implies a specific sequentiality in the firms' decisions: in the second stage, the downstream monopolist selects the output level, *conditional on the input price*. This choice generates a demand function for the input. The effective input price then obtains in the first stage from the equality between the downstream monopolist' s demand and the upstream monopolist's input supply decision. Supply decisions are assumed to maximize the upstream monopolist's profit on the demand function of the downstream monopolist. This constitutes the traditional approach to analyze the property of double marginalization in the bilateral monopoly framework.

This approach has also been adopted by Salinger (1988), Gaudet and Van Long (1996) and Gabszewicz and Zanaj (2011) among others, in their endeavor to analyze this property in the more general framework of successive *oligopolies*. Downstream and upstream firms select non cooperatively *"à la Cournot"* the quantities of output of the good they produce, the output of the upstream firms serving as input for the downstream ones in the production of their own output. Downstream firms engage in an oligopoly in the downstream market but they take the input price as given and fixed by the upstream firms.

Whether downstream firms have market power in their input market is not always easy to establish. In fact, the number of downstream firms in a specific output market and using a specific input is not necessarily related to the number of firms operating in the market for this input. A monopolist in some downstream market can coast along a large number of other firms in its input market simply because many small firms in other industries use the same input in their production process. Then, regardless of the market power the monopolist owns in its output market, the assumption of price taking behavior in the input market is natural because the output monopolist cannot influence alone the price of the input. For instance, a monopoly firm uses wheat to produce only biofuel, while many other firms use wheat to produce bread. By contrast, when downstream firms are few in number and are the only candidates to buy some input, it is reasonable to assume that they can influence the price of their inputs. Examples abound: microsoft with respect to the software industry, American Telephone and Telegraph, U.S. Steel company, railways with respect to wagon producers, car producers with respect to tyre producers. Thus, the assumption of price-taking behavior in the input market can be valid in some industries while completely unsatisfactory in others.

For this reason, the classical approach well fits in some industries but cannot be used in others. In many sectors, the price of input is clearly affected by the exercise of market power of both sellers -upstream firms- and buyers -downstream firms. The existing literature on successive oligopolies lacks a framework that can tackle such market configurations. In this paper we aim at filling this gap.

To do so we introduce an alternative approach to examine how successive oligopolies do operate between downstream and upstream markets, when downstream firms cannot be assumed price takers in the input market. The alternative approach, proposed in this paper, relies on the notion of strategic market game, which constitutes the most natural framework to use when prices are formed by the simultaneous actions of all agents. The basic version of a strategic market game that we consider is the one introduced by Shapley and Shubik (Shapley, 1976, and Shapley and Shubik, 1977), used also by Gabszewicz and Michel (1997) and recently by Amir and Bloch (2009). Within the successive oligopolies, a market game approach proposes the following timing of firms' decisions. In the second stage, the n downstream oligopolists play a Cournot game in the downstream market, and bid each a quantity of money he/she is willing to offer to get a share of the total input supplied by the m upstream firms in the first stage. This choice generates an amount of money to be shared among the upstream oligopolists in proportion to their input production. In the first stage, the upstream oligopolists choose non cooperatively the amount of input they supply, in order to maximize the amount of money obtained from their input sales. The two approaches essentially differ by the fact that in the price-taking scenario the input price obtains from the market clearing condition while in the market game scenario, the input price, expressed in monetary units, is equal to the ratio between the total amount of bids offered by the downstream firms, and the total amount of input supplied by the upstream ones.

As discovered below, introducing market games leads to different market outcomes than those observed in the traditional approach where successive oligopolies operate through the usual price-taking mechanism. Consequently, it naturally invites to contrast the differences between the scenario resulting from the just described approach - *market game scenario* - and the traditional theory of successive markets, which we call *price-taking scenario.* In particular, it invites to compare output prices to consumers with the input prices to downstream firms, the extent of double marginalization and the consequences on welfare**.**

To decide whether the market game scenario or the price-taking scenario is the most appropriate, one needs to fully understand the input price mechanism that takes place in different industries. The price-taking scenario, introduced by Spengler (1950), benefits from its repeated use in the analysis of vertical collusive agreements (see, for instance, Salinger (1988) and Gaudet and Van Long (1996), Ordover *et al* (1990), or in Gabszewicz and Zanaj (2011). The market game approach offers the advantage of discarding the awkward assumption, implicit in the traditional analysis, that an auctioneer has to choose the upstream market clearing price. In both cases, a sequentiality is introduced in the decisions of the oligopolists. In the price-taking scenario, upstream oligopolists play the first, using for evaluating their payoff the input price resulting from the equality of their total supply with the input demands of the downstream oligopolists. In the market game scenario, upstream oligopolists are also the first to play, but now they use the bids selected by the downstream firms in the second stage game for guiding their decisions.

The paper is organized as follows. In section 2, the model is developed. Section 3 is dedicated to the exploration of the equilibrium with market games and the equilibrium obtained within the classical scenario. Section 4 develops the comparisons between the market outcomes. Finally, Section 5 concludes.

2 The model

Consider an economy consisting in two successive markets. In the downstream market there are n *downstream* firms indexed by $i, i = 1, ...n$, producing the final good, and in the upstream market there are m *upstream* firms indexed by j, $j = 1, ...m$, producing the corresponding input good. Assume that the n downstream firms face a demand function $p(Q)$, with Q denoting aggregate output. Downstream firm i owns technology $f_i(z)$ to produce the output, with z denoting the quantity of the sole input used in the production process. The m *upstream* firms each produce the input z at a total cost $C_j(z), j = 1, .., m$. *Similarly to the existing literature,* we assume that output and input prices are determined in a two-stage sequential game.

We depart from the previous literature in the way the input price is determined. We introduce strategic market games to allow donwstream firms to affect the input price. Hence, as far as the input price is concerned, the input market looks like an exchange economy involving two types of traders and two goods. Commodities are the input and money. The agents bidding money to buy the input are the downstream firms that own profits (money) obtained from selling the output good to final consumers. Agents who offer the input for sale are the upstream firms that can produce (or are endowed) with the input good. Thus the initial ownership of the input good is concentrated in the hands of the m upstream firms while money is initially owned by the downstream ones under the form of their profits.

The input market thus appears as a bilateral oligopoly in the sense used by Bloch and Ferrer (2001), or Gabszewicz and Cordella (1998)

More specifically, sellers (upstream firms) have input supply strategies denoted by s_j. When the cost to produce the input $C_j(z)$ is zero, these amounts can be seen as an endowment of each player j, as in the original version of Shapley-Shubik games. Upstream firms aim at maximizing the amount of money they obtain from their input sales. Buyers play a Cournot game in the downstream market and they participate in the strategic game with money bidding strategies b_i. They aim at maximizing their profit in the downstream market by obtaining through their bids the quantity of input required to produce their Cournot equilibrium quantities. These downstream firms can obtain the money necessary for paying their bids to the input's sellers by borrowing from the banking system, using as collateral the profit obtained by selling the output good. Hence, the trade mechanism in the successive markets setup allows an allocation of the input good among downstream firms in a way that $\Sigma_{k=1}^{n} b_k = \Sigma_{h=1}^{m} s_h$. As standard in strategic market games, the price of the traded good is given by the ratio of the total amounts of bids over the the total of offers : $\frac{\Sigma_{k=1}^{n} b_k}{\Sigma_{h=1}^{m} s_h}$, which can be seen as an average market clearing input price.

The downstream and the upstream markets are linked to each other *via* the production function f_i, namely,

$$f_i(z_i) = f_i(\frac{b_i S}{\Sigma_{k=1}^{n} b_k}),$$

where $\frac{b_i S}{\Sigma_{k=1}^{n} b_k}$ constitutes the fraction of total input supply S obtained by firm i through its bidding strategy b_i.

Given a total input supply S, the payoff in the second stage game for the i_{th} firm at the vector of strategies (b_i, b_{-i}) obtains as

$$\Pi_i(b_i, b_{-i}; S) = p\left(f_i(\frac{b_i S}{\Sigma_{h=1}^{n} b_h}) + \Sigma_{k \neq i} f_k(\frac{b_k S}{\Sigma_{h=1}^{n} b_h})\right) f_i(\frac{b_i S}{\Sigma_{h=1}^{n} b_h}) - b_i.$$

Given these payoffs, and a total supply S in the input market, the best reply, $b_i(b_{-i}(S))$ of firm i in the second stage game obtains as a solution (whenever it exists) to the problem

$$\underset{b_i}{Max}\ \Pi_i(b_i, b_{-i}; S).$$

A Nash equilibrium in the second stage game, conditional on a total input supply S, is a vector of strategies $(b_1^*(S), ..., b_n^*(S))$ such that, for all i, $b_i^*(S) = b_i(b_{-i}^*(S))$.

In the first stage game, upstream firms select their supply strategies s_j, $j = 1, .., m$. Given a n-tuple of supply strategies $(s_{1,} ..., s_j, .., s_m)$ and a vector of downstream firms' bids $(b_1, ..., b_n)$ in the second stage game, the amount of money received by firm j obtains as

$$\Gamma_j(s_j, s_{-j}) = \frac{\Sigma_{k=1}^{n} b_k}{\Sigma_{h=1}^{m} s_h} s_j - C_j(s_j),$$

which constitutes the payoff function of the j_{th}-upstream firm in the first stage game, conditional on the vector of bids $(b_1, ..., b_n)$ chosen by the downstream firms in the second stage.

Hence, (whenever it exists) a strategic equilibrium is a Nash equilibrium is the $(n + m)$ tuple of strategies $(b_1^*, ..., b_n^*; s_{1,}^* ..., s_m^*)$ $\forall i \forall j$ such that $\forall b_i$ and $\forall b_j$

$$\begin{aligned} \Pi_i(b_i^*, s_j^*) &\geq \Pi_i(b_i, s_j) \\ \Gamma_j(b_i^*, s_j^*) &\geq \Gamma_j(b_i, s_j). \end{aligned}$$

3 Exploring subgame perfect equilibria

To isolate the difference between the market outcome with the price-taking scenario and the market outcome corresponding to the market game scenario, we stick to the assumptions made in the classical approach. Namely, we assume *(i)* a linear output technology and a linear demand function in the downstream market, as in Salinger (1988), Gaudet and Van Long (1996) and Gabszewicz and Zanaj (2010) and *(ii)* that firms operating in the upstream (resp. downstream) market are all identical. Entry and competition are analyzed through the asymptotic properties of the subgame-perfect equilibrium when the number of firms in the markets is increased by expanding the economy, as in Debreu and Scarf (1963).

3.1 Market Game scenario

Hence, assume that downstream firms face a linear demand $p(Q) = 1 - Q$ and use a constant returns technology to produce the output:

$$f(z) = \alpha z,\ 1 > \alpha > 0^1.$$

The profits $\Pi_i(b_i, b_{-i}; S)$ of the i_{th} downstream firm at the vector of strategies (b_i, b_{-i}) and S now obtains as

$$\Pi_i(b_i, b_{-i}; S) = (1 - \alpha\frac{b_i}{b_i + B'}S - \alpha\sum_{k\neq i}\frac{b_k}{b_i + B'}S)\alpha\frac{b_i S}{b_i + B'} - b_i$$

with $B' = \sum_{h\neq i} b_h$.

Solving the maximization problem of a downstream firm and using symmetry, we get at equilibrium

$$b^*(S) = \frac{\alpha S\,(1 - \alpha S)\,(n-1)}{n^2}$$

Hence, the payoff Γ_j of an upstream firm at the first stage of the game, after substituting for b^*,obtains as

$$\Gamma_j(s_, s_{-j}) = \frac{(1 - (s_j + S')\,\alpha)\,(n-1)\,(s_j + S')\,\alpha}{(s_j + S')}s_j - \beta s_j.$$

Maximizing $\Gamma_j(s_, s_{-j})$, yields at the symmetric equilibrium

$$s^*(n, m) = \frac{(\alpha n - \alpha - \beta n)}{\alpha^2\,(n-1)\,(m+1)}. \tag{1}$$

Hence the optimal quantity of money $b^*(S^*)$ and the corresponding output quantities of each downstream are

$$b^*(n, m) = \frac{(\alpha - \alpha n - \beta m n)\,(\alpha - \alpha n + \beta n)\,m}{\alpha^2 n^2\,(n-1)\,(m+1)^2} \qquad q^*(n, m) = m\frac{n\,(\alpha - \beta) - \alpha}{\alpha n\,(m+1)\,(n-1)}.$$

Consequently the equilibrium output price writes as

$$p^*(n, m) = \frac{\alpha\,(m - n) + \alpha n^2\,(m+1) - mn\,(2\alpha - \beta)}{\alpha n\,(n-1)\,(m+1)} \tag{2}$$

We may now calculate the equilibrium input price ω^* as the ratio $\frac{\Sigma_{k=1}^n b_k}{\Sigma_{h=1}^m s_h}$,

[1] We require $\alpha < 1$ to guarantie that the output price is always larger that the input prices when the basic economy is replicated infinitely.

$$\omega^*(n,m) = \frac{n\,(\alpha+\beta m)-\alpha}{\alpha\,(m+1)\,n}. \tag{3}$$

Accordingly, the $(n+m)$-vector $(b^*(S^*),..,b^*(S^*);s^*,..,s^*)$ *constitutes the symmetric subgame perfect equilibrium of the sequential game.*

In order for this vector to be an equilibrium, it is also required that the values $b^*(S^*)$ and s^* to be both positive. These two inequalities are both simultaneously satisfied if and only if the condition

$$\alpha \geq \frac{n}{(n-1)}\beta$$

holds. This condition coincides with the condition which guarantees that both upstream and downstream firms make positive profits. Notice that this condition is slightly stronger than the condition required to be satisfied in the traditional model, which simply boils down to $\alpha \geq \beta$. The reason for this strengthening should be found in the indirect strategic power that downstream firms exert in the upstream market : they influence the amount of input sales via their money bids, and this influence fades away when the number n of downstream firms increases. In the traditional model, this influence does not exist since downstream firms take the input price as given when buying the input[2].

3.2 Price-taking scenario

In this section, we shortly recall the example of successive oligopolies considered in the previous literature. Consider the two successive markets described in section (3). Assume now that in the downstream game, firms select strategically the output levels $q_i(\omega)$, which determines their individual input demand $z_i(\omega)$ *via* the production function f, taking the input price as given. Consequently, the downstream firms while behaving strategically in the final good market, are assumed to be price takers in the input market. The profits $\Pi_i(q_i,q_{-i})$ of the i_{th} downstream firm at the vector of strategies (q_i,q_{-i}) is

$$\Pi_i(q_i,q_{-i}) = (1-q_i-\Sigma_{k\neq i}q_k)q_i - pz_i.$$

As a result of the strategic choice q_i, each firm i sends an input quantity signal $z_i(\omega) = \frac{q_i}{\alpha}$ to the upstream market. Given the price ω in the input market, the best reply of downstream firm i in the upstream game is $z_i(z_{-i};\omega) = \left(\alpha-\omega-\alpha^2\Sigma_{k\neq i}z_k\right)/2\alpha^2, i=1,...,n$. We may compute the symmetric Nash equilibrium of the above game contingent on the price ω. Defining $z_i = z$ for $i=1...n$, we get at the symmetric solution $q^* = (\alpha-\omega)/(n+1)\alpha$.Given a n-tuple $(s_{1,...},s_j,..s_m)$ of input strategies chosen by the upstream firms in the second stage game, the input price clearing the upstream market must satisfy $nq^* = \Sigma_{k=1}^m s_k$.So that, we get

$$\omega(\Sigma_{k=1}^m s_k) = \alpha - \alpha^2\frac{n+1}{n}\Sigma_{k=1}^m s_k. \tag{4}$$

Substituting (4) into the payoff function $\Gamma_j(s_j,s_{-j})$ we have

$$\Gamma_j(s_j,s_{-j}) = \left(\alpha-\alpha^2\frac{n+1}{n}\Sigma_{k=1}^m s_k\right)s_j - \beta s_j.$$

Accordingly, using the first order conditions, at the symmetric equilibrium of the second stage game, we obtain

$$s^*(n,m) = \frac{n\,(\alpha-\beta)}{\alpha^2(n+1)(m+1)}. \tag{5}$$

[2] A similar condition appears in another,but close, context (see Gabszewicz and Michel (1997)). These authors analyse the oligopoly equilibrium of a market game with exchange and production.

Finally, the equilibrium price in the input market is

$$\omega^*(n,m) = \frac{\beta m + \alpha}{m+1}. \tag{6}$$

Consequently, the equilibrium output quantities are

$$q^*(n,m) = \frac{m(\alpha-\beta)}{\alpha\,(n+1)\,(m+1)},$$

while the resulting output price $p^*(n,m)$ in the downstream market is

$$p^*(n,m) = \frac{\alpha(1+m+n)+\beta mn}{\alpha\,(n+1)\,(m+1)} \tag{7}$$

This market solution is now compared with the market solution in the scenario with market games in section (4).

4 Comparing the equilibrium outcomes

This section is dedicated to the comparison of equilibrium outcomes corresponding to the two scenarios and it also includes a welfare analysis.

The two approaches - the price-taking approach and the market games approach, - mainly differ according to how downstream firms' total production costs are introduced in the model. In the market mechanism approach, these costs depend on the input price and the quantity of input invested in production. In the market game approach, total costs do not depend on the quantity of input invested in the production, but reduces to a lump-sum amount corresponding to the bid offered by the downstream in exchange of its input share. Nothing prevents to deduce from it a notion of *average* cost (and marginal cost in the case of constant returns) simply by dividing the bid by the number of output units produced.

Using this notion, we show that

Proposition 1 *The equilibrium input price under market games is higher than the corresponding input price under the price-taking scenario if and only if* $n > \alpha/(1-\alpha)(\alpha+\beta m)$; *otherwise, the reverse holds.*

Proof. The sign of the difference of (3) and (6) is the sign of $n\left(\alpha + m\beta - m\alpha\beta - \alpha^2\right) - \alpha > 0$, which is positive if $n > \alpha/(1-\alpha)(\alpha+\beta m)$. ■

Under market games, the markup of upstream firms shrinks because downstream firms bid to obtain the input quantity desired rather than take the input price as given. This effect has the same flavour as the softening of double marginalization due to vertical integration and it induces a decrease in the input price. Vertical integration reduces the input price because the integrated entity produces the input internally. As in the analysis of vertical integration, the extent of this effect depends on the number of firms in both markets. The higher the number of downstream firms and of upstream firms, the lower the markup of each and thus the lower double marginalization. Proposition (1) identifies such a relationship between the number of upstream firms and downstream firms. The condition on n corresponds in fact to the threshold number of downstream firms, below which double marginalization with the classical approach is quite large. Therefore, as long as the number of firms in the downstream market is lower than this threshold, market games cause a decrease in the input price.

By contrast, when the number of firms exceeds such a threshold, double marginalization is small. But the number of firms that bid is large. Therefore, when n exceeds this threshold, the input demand increases more than the elimination of double marginalization. As a consequence, the input price under market games is higher than the corresponding price in the price-taking regime.

Notice that the higher the number of upstream firms, the less binding the condition in Proposition 1 becomes. When $m \to \infty$, upstream firms are price takers in the input market thus double marginalization is null. Therefore, the only effect introduced by the mechanism of market games is the market power of downstream firm in the input market. As a consequence, the input price under market games is always higher than the corresponding input price in the price-taking scenario.

Comparison of equilibrium output prices yields

Proposition 2 *The output price of the market game scenario is always higher than the output price corresponding to the price-taking regime.*

Proof. From direct comparison of output prices, the sign of the difference of (2) and (7) is the sign of $\left(\alpha + n\beta + n^2 (\alpha - \beta)(n-2)\right)$, which is positive since $\alpha > \beta$ and $n \geq 2$. ■

When the input price under the market game regime is higher than the corresponding input price in the classical approach ($n > \alpha/(1-\alpha)(\alpha + \beta m)$), also the output price with market game is higher due to higher input costs.

Instead when the input price is lower under market games ($n < \alpha/(1-\alpha)(\alpha + \beta m)$) two effects on the output price take place. The reduction of the input cost for each firm i leads to a direct advantage for firm i and an indirect disadvantage all other firms $-i$. In fact, each downstream firm behavior has a strategic effect on its rival firms in the downstream market. According to Proposition 1 this occurs when the downstream market is quite concentrated . Exactly because the number of downstream firms is small, the indirect effect is higher than the direct effect, and thus, the output price with market game is lower.

The comparison of equilibrium output prices gives interesting insights concerning the extent of double marginalization. Double marginalization is defined as the sum of the markup exercised by the upstream firms, $\omega^*(m,n) - \beta$, and the markup applied by the downstream firms, $p^*(m,n) - \omega^*(m,n)$, which yields $p^*(m,n) - \beta$. Therefore, to compare double marginalization, it is enough to compare output prices obtained with each scenario. Using Proposition 2, we can claim

Corollary 3 *Double marginalization observed in the market game scenario exceeds the one arising at the symmetric subgame perfect equilibrium in the classical approach.*

This feature has relevant consequences for the effects of vertical agreements. As it is known, such contracts have from one side positive effects on output prices because they eliminate double marginalization. But on the other hand, they may have anti-competitive effects due to possible raising rivals cost strategies. The above proposition tells that with market games the positive effect of vertical integration has a larger size, as compared to the effect obtained in the classical analysis. Therefore, due to the above proposition, whenever downstream firms have market power in the output and in the input market, vertical integration should arise more frequently in equilibrium because the vertical externality eliminated by it is larger than the externality eliminated when downstream firms are price takers in the input market.

Finally, we model free entry by replicating k-times the basic economy described above, as in Debreu and Scarf (1963). In the k_{th} replica, downstream market demand is given by $k(1-Q)$ and there are kn downstream and km upstream firms. Notice that, in the k_{th} replica, the prices at which demand is equal to supply both in the downstream and upstream markets, do not depend on the number k, but depend only on m and n. Indeed, at the symmetric equilibrium in the upstream market, the input quantities supplied by the m upstream firms have to be multiplied by k in the rth-replica; similarly for the quantities demanded by the n downstream firms in the downstream market. Consequently, the equality of supply and demand in the upstream market eliminates the $k-$ factor in each side of the equality. A similar reasoning applies for the symmetric price equilibrium in the downstream market.

It follows that the study of the behavior of the upstream and downstream markets when the number of replications increases is equivalent to the study of the limit equilibrium prices and quantities obtained in the previous section when the number of firms is kn and km instead of n and m in each market, respectively. Such a replication yields the following result.

Proposition 4 *When the basic economy is replicated infinitely, the markup of downstream firms with the price-taking scenario converges to the difference of marginal costs $\frac{\beta}{\alpha} - \beta$. Such a convergence does not occur in market games scenario.*

Proof. Taking the limit of the difference of equilibrium prices in the price-taking scenario we obtain:

$\lim_{k\to\infty}(p^*(km, kn) - \omega^*(km, kn)) = \frac{\beta}{\alpha} - \beta$. If $\alpha \leq 1$, this difference is non-negative. The same limit in the market game scenario is $\lim_{k\to\infty}(p^*(km, kn) - \omega^*(km, kn)) = 1 - \frac{\beta}{\alpha}$. This markup is positive because $\beta < \alpha$. Then, simple manipulations lead to $1 - \frac{\beta}{\alpha} > \frac{\beta}{\alpha} - \beta \Leftrightarrow \beta < \frac{\alpha}{2-\alpha}$. ■

Two issues arise. Firstly, when both upstream and downstream markets are perfectly competitive ($n \to \infty$ and $m \to \infty$) the series of markups of downstream firms does not converge towards the difference between the output and input marginal cost $\frac{\beta}{\alpha} - \beta$ under market games. Secondly, the markup of downstream firms under market games can be lower or larger than the markup under the classical approach. Such a difference depends on the input marginal cost β and the marginal product of the input α. Whenever the marginal product in the downstream market α is substantially larger than the marginal cost β in the upstream one, the markup is higher than $\frac{\beta}{\alpha} - \beta$. While, when the ratio $\frac{\beta}{\alpha}$ is closer to one, the reverse holds true. This phenomenon is similar to the one identified in Gabswewicz and Zanaj (2011). In that case, when both markets become competitive and downstream firms use a decreasing returns technology rather than a constant returns one, the markup of downstream firms stays above the difference of marginal costs. In this paper, even if both upstream and downstream firms use constant returns technology, the convergence is not obtained. As in the case of decreasing returns, the very large number of firms is not enough to assure the convergence towards marginal cost. When markets are technologically-linked, at the limit economy, the prices adjust to values that make the demand and the supply to equalize. When technologies have different marginal products, nothing guarantees that the price would tend to the marginal cost, even though the profits of downstream and upstream firms do tend to zero.

Welfare analysis Two types of inefficiency can occur in a Cournot equilibrium as compared with the equilibrium of a central planner: the equilibrium price exceeds the marginal cost of production, and aggregate output is inefficiently distributed over the firms. Our analysis assumes that firms use the same technology, therefore our welfare analysis concentrates on how much the equilibrium output price exceeds the marginal cost of production in the output market and in the input for each scenario. Furthermore, the production costs in the downstream market are just a transfer from the downstream to upstream firms for both scenarios considered, thus, the key ingredient to evaluate welfare is the consumer surplus (in the downstream market). The output price that corresponds to perfect competition is equal to the output marginal cost namely $\frac{\omega}{\alpha}$. Furthermore, the input price corresponding to perfect competition in the upstream market is equal to the input marginal cost β. Hence, the output price $\tilde{p}$ corresponding to the welfare maximizing configuration of the economy is $\tilde{p} = \frac{\beta}{\alpha}$.

The measure of inefficiency of the price-taking scenario in the output market is the difference between $\tilde{p}$ and the equilibrium output price (7). Similarly, the measure of inefficiency in the market game scenario in the output market is the difference between $\tilde{p}$ and the equilibrium output price (2). Since $\tilde{p}$ does not depend on the number of firms in the upstream and downstream market, the comparison of inefficiencies of the two scenarios is obtained by the direct comparison of equilibrium output prices (7) and (2).

As far as it concerns the welfare consequences in the upstream market, we cannot use the comparison of equilibrium input prices and marginal cost β. The reason is that input demand is not the same with market games and with price-taking downstream firm in the input market. Input demand is endogenous in the model and obtains from the maximization of profit of downstream firms differently from the demand of the output that is exogenous. Hence, the only measure of welfare is the profit of upstream firms. Direct comparison of output prices and profits of upstream firms yields:

Proposition 5 *The allocation of resources achieved by a market economy under market games decreases welfare with respect to the allocation of a market economy with input price-taking downstream firms whenever the downstream market is rather concentrated.*

Proof. For the direct comparison of output prices see proof of Lemma 1. Comparison of input supplies (1) and (5) says that the input supply with market game is lower than the input supply in the classical scenario if $\alpha(n-1) - 2n\beta < 0$ or $n < \frac{\alpha}{\alpha-2\beta}$. Thus, using Proposition 1, the profit of an upstream firm is lower with market games if $n < \min\{\alpha/(1-\alpha)(\alpha+\beta m), \alpha/(\alpha-2\beta)\}$. ■

When the downstream market is rather concentrated, downstream firms exert a high market power in both markets. Consequently, with market games, welfare decreases. Instead, when the number of downstream firms is large, their market power in the upstream market is diluted. Thus, the profit of upstream firm is higher than its profit in the price-taking scenario. Since, the welfare in the downstream market is always lower with market games, the effect on welfare of market games is ambiguous.

5 Conclusion

In this paper, we use the concept of Shapley-Shubik market game to describe the economic outcome of the downstream and upstream firms' interaction in successive oligopolies. Our exploration of industry equilibria departs from the existing literature. In particular, it does not assume that downstream firms behave as price-takers in the upstream market, an awkward assumption because, in several industrial contexts, it is difficult to justify the fact that an economic agent behaves strategically in one market but not in the other. In the market game approach, all agents behave strategically. Exploring the properties of equilibria in this new framework and comparing them with equilibrium market outcomes in the traditional approach is the object of this paper. This comparison leads to the following conclusions. First, double marginalization observed in the market game scenario exceeds the one arising at the symmetric subgame perfect equilibrium in the classical approach. Second, while infinite replication of the basic economy drives the markup of downstream firms to the marginal cost within the price taking scenario, such a convergence does not occur in the market games regime. Finally, the resource allocation achieved by an economy operating as in market games decreases welfare with respect to the allocation of an economy with input price-taking downstream firms whenever the downstream market is rather concentrated.

Most probably there are other economic contexts in which the notion of market games could be usefully utilized, even in a partial equilibrium framework, in particular in markets in which both sides have market power, like industries for intermediate goods which are often characterized by the fact that both sides consist of a small number of firms. This approach would then contrast with the usual assumption according to which at least one side of the market behaves competitively. This would open the door to capturing a new class of market situations which have been so far neglected due to the lack of a natural theoretical framework in which these situations could be cast.

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