Efficient Algorithms for Constant-Modulus Analog Beamforming

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***Abstract*—The use of a large-scale antenna array (LSAA) has become an important characteristic of multi-antenna communication systems to achieve beamforming gains. For example, in millimeter wave (mmWave) systems, an LSAA is employed at the transmitter/receiver end to combat severe propagation losses. In such applications, each antenna element has to be driven by a radio frequency (RF) chain for the implementation of fully-digital beamformers. This strict requirement significantly increases the hardware cost, complexity, and power consumption. Therefore, constant-modulus analog beamforming (CMAB) becomes a viable solution. In this paper, we consider the scaled analog beamforming (SAB) or CMAB architecture and design the system parameters by solving the beampattern matching problem. We consider two beampattern matching problems. In the first case, both the magnitude and phase of the beampattern are matched to the given desired beampattern whereas in the second case, only the magnitude of the beampattern is matched. Both the beampattern matching problems are cast as a variant of the constant-modulus least-squares problem. We provide efficient algorithms based on the alternating majorization-minimization (AMM) framework that combines the alternating minimization and the MM frameworks and the conventional-cyclic coordinate descent (C-CCD) framework to solve the problem in each case. We also propose algorithms based on a new modified-CCD (M-CCD) based approach. For all the developed algorithms we prove convergence to a Karush-Kuhn-Tucker (KKT) point (or a stationary point). Numerical results demonstrate that the proposed algorithms converge faster than state-of-the-art solutions. Among all the algorithms, the M-CCD-based algorithms have faster convergence when evaluated in terms of the number of iterations and the AMM-based algorithms offer lower complexity.**

***Index Terms*—Analog beamforming, majorization-minimization, MM, alternating MM, AMM, cyclic coordinate descent, CCD, large-scale antenna arrays, unit-modulus constraints, nonconvex optimization, block cyclic coordinate descent, BCCD.**

I. Introduction

In multi-antenna communication systems, an antenna array is employed at the transmitter and/or at the receiver to achieve beamforming, performance gains. For example, the use of a large-scale antenna array (LSAA) has become a decisive part of a mmWave system. This is because the communication at mmWave frequencies suffers from several propagation losses [2]–[4]. Therefore, to alleviate these losses an LSAA is employed at the transmitter to achieve beamforming gains. Some other applications of beamforming design are in acoustic imaging for underwater exploration [5], channel sounding in mmWave communications [6], etc. The fully-digital implementation of a beamformer requires as many radio frequency (RF) chains as the number of antenna elements. Consequently, this places demands on hardware and increases implementation cost and power consumption. One possibility is to employ analog phase-shifters but an LSAA requires as many power amplifiers (PAs) as the number of antennas. This design further limits the applicability. To alleviate the requirement of multiple PAs, an alternative is to use a variable gain amplifier (VGA) driving a phase-shifting network. Thus, the beamforming vector is constrained to have constant-modulus entries, where the magnitude of each entry corresponds to the gain contribution from the VGA. In this case, the transmitted signals present low Peak-to-Average-Ratio (PAR) and they enable the use of power-efficient nonlinear amplifiers at the transmitter's side. This characteristic is highly desirable in LSAA-based systems [7], [8] because they do not require highly linear PAs which are necessary for the fully-digital beamforming implementation [9], [10]. The analog beamforming architectures have several other advantages, for example, adjusting only the phases of the beamforming vector constrains the power required to drive an antenna element to be a constant. These gains are more appealing when the transmitter/receiver employs an LSAA.

A. Literature Review

The constant-modulus analog beamforming (CMAB) problem has been studied in the past [11]–[15], where the optimal beamforming vector is designed by solving an optimization problem subject to the unit-modulus constraints on the entries of the beamforming vector. For example, an signal-to-interference-plus-noise ratio (SINR) maximization problem is considered in [11]. However, due to the unit-modulus constraints, the resulting optimization problem is nonconvex and in general NP-hard [16]. Two algorithms based on the conjugate gradient and Newton's method to compute the beamforming weights are proposed in [11]. These methods are the special cases of Riemannian optimization on manifolds. A gradient search algorithm is presented in [12], with the angle parameterization of the unit-modulus constraints to adaptively adjust the phases of the entries of the beamforming vector. Receive beamforming is studied in [13] and the optimal beamforming weights are designed by minimizing the mean square error (MSE) between the array output and the desired

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signal. For null steering, a position-perturbation technique is presented in [14].

Recently, this problem has received attention from a beampattern matching perspective, where the desired beampattern is matched using an analog phase shifting network [1], [15]. This problem is cast as a unit-modulus least squares (ULS) problem. A closely related problem of unimodular radar sequence (or code) design also arises in several active sensing applications [17]–[27]. In these applications, different performance metrics for example, in [22] the Peak Sidelobe Level (PSL), and Integrated Sidelobe Level (ISL) based designs are proposed. Therein, the worst-case PSL/ISL is optimized under the steering vector mismatches. In [23], the worst-case signal-to-interference-plus-noise ratio (SINR) is maximized over the steering vector mismatches under the constant-modulus and similarity constraints for radar waveform synthesis. In [24], an objective function consisting of a specific weighted beampattern matching error and the space-frequency stopband energy is minimized subject to the constant-modulus constraints. In [25], a quadratic function is maximized subject to the unit-modulus and similarity constraints. In [26], a wideband multiple-input multiple-output (MIMO) radar transmit beampattern design with spectral and constant-modulus constraints is considered and is solved through a sequence of constrained quadratic programs such that the constant-modulus constraint is achieved at the convergence. A recent paper [27] considers the synthesis of constant-modulus waveforms by maximizing the SINR subject to multiple spectral compatibility constraints. To solve the problem, an iterative procedure based on the coordinate descent (CD) framework is proposed.

In the literature, an approximate solution to the ULS problem is found by employing the semidefinite relaxation (SDR) technique, which lifts the dimensionality of the search space from M to M^2. As a result, a large number of design parameters need to be optimized, resulting in increased memory usage and storage requirements, which may be impractical. Another disadvantage with SDR is that it empirically returns a rank-1 solution, but in cases when the solution is not rank-1, an appropriate randomization technique should be employed [16], [28]; this further increases the overhead. To that end, the scalability of the SDR technique is a bottleneck in designing beamformers for LSAA-based systems. Therefore, to keep the computational complexity low, gradient-projection (GP) based algorithms are proposed in [15]. Another recent work proposed an alternating direction method of multipliers (ADMM) based algorithm for beamforming in the context of wireless sensor networks [29].

In the context of radar sequence design, the majorization-minimization (MM) and the CD frameworks have been adopted in the literature to handle the unit-modulus or constant-modulus constraints [19], [27], [30]. In many of these applications, the problem is modeled as a convex/nonconvex quadratically constrained quadratic program (QCQP). In [19], [30], MM algorithms to solve the ISL minimization problem are proposed. In [27], a CD-based procedure is proposed to solve the problem. But the development of an MM and a CD-based algorithm crucially depends on the specific structure of the problem. In contrast to the existing problems solved using MM or CD-based algorithms which optimize a unit-modulus vector, our problem entails optimizing two coupled unit-modulus vector variables and a multiplicative scalar variable. Therefore, the existing algorithms in the literature can not be directly applied to the beampattern matching problem because of fundamental differences in the problem formulation. Furthermore, beamforming design with unit-modulus constraints is NP-hard [16] and hence, there exists scope for efficient and scalable algorithms with better performance compared to the existing ones.

B. Contributions

In this paper, we consider the scaled analog beamforming (SAB) architecture and design the beamforming vector by solving the beampattern matching problem subject to the constant-modulus constraints [15]. In SAB architecture, a common variable gain amplifier (VGA) drives the phase-shifting network. Therefore, the magnitude of each entry of the beamforming vector is a constant, representing the gain introduced by VGA[1]. We consider two variants of the beampattern matching problem, which can be cast as the constant-modulus least squares (CLS) problem. In the first problem, we consider both gain and phase of the beampattern as variables, whereas in the second problem we match only the magnitude of the beampattern [15]. Later, we show that the first problem formulation becomes a special case of the second one. This is affected by formulating the problem with unit-modulus beamforming weight and beampattern phase vectors as well as a scalar corresponding to VGA gain as variables.

For the considered formulations, we provide redefficient algorithms, and the convergence guarantees to an associated stationary point. In this context, the contributions of the work include:

- *Algorithms:* We propose efficient algorithms specializing on the MM and the cyclic coordinate descent (CCD) optimization frameworks to the considered problem formulations. As mentioned earlier, the MM and the CCD (or in general CD) are optimization frameworks and the algorithm development under each framework crucially depends on the structure of the specific problem. Herein, we propose efficient and scalable algorithms for LSAAs by exploiting the problem structure with theoretical convergence guarantees. There are no works on exploring different optimization frameworks for the increasingly important beamforming problem [15]. In this context, the works closer to the one pursued are [31]–[33] where the MM and the CCD frameworks are utilized to solve different optimization problems involving only one unit-modulus constrained vector variables. Moreover, we show that the optimization problem can be solved in closed-form in one of the unit-modulus constrained vector variables as well as the scaling variable. Therefore, we exploit the problem structure and develop algorithmic solutions tailored to the specific problems resulting in enhanced performance gains in comparison to the existing works. Following we summarize the algorithmic contributions:

[1] All the elements of the beamforming vector have the same magnitude.

  - The MM-based algorithms utilize alternating minimization along with the MM framework. Thus, we call them as *alternating MM (AMM)*-based algorithms. These, algorithms differ from the standard MM formulation where the cost function is majorized for all variables, herein we only need majorization to handle the optimization over the beamforming vector and the remaining variables are optimized in closed-form. In particular, this departs from the approach of [15] where the beampattern phase vector is optimized using a gradient-projection (GP) method and requires tuning an additional step-size parameter. This leads to effective and efficient MM implementations.
  - Similarly, in the development of the CCD-based algorithm, we do not solve all the associated single variable sub-problems; only those associated with the beamforming vector are solved componentwise, whereas the other variables are updated block-wise as they admit a closed-form solution. This again leads to improved CCD implementation than state-of-art.
  - Apart from the conventional-CCD (C-CCD) based approach, we also propose modified-CCD (M-CCD) based algorithms employing a new update rule and offering faster convergence in comparison to the other algorithms.
- *Convergence:* We theoretically establish convergence guarantees for all the proposed algorithms.
  - Even though the constraints are nonconvex, we show that the sequence of iterates generated by the AMM, C-CCD, and M-CCD algorithms converge to a Karush-Kuhn-Tucker (KKT) point and is bounded.
  - We also prove that the solutions obtained by all the algorithms satisfy the linear independence constraint qualification (LICQ) (Proposition 3.1.1 in [34]) regularity condition.
- *Simulations:* Numerical simulations under different beampattern settings demonstrate the effectiveness of the proposed algorithms. We analyze the evolution of beampattern matching error with the number of iterations and study the scalability with the number of antennas. It is observed that the proposed algorithms converge faster with a better beampattern matching accuracy in comparison with the state-of-the-art solutions existing in the literature. Moreover, increased performance gains are observed when LSAAs are employed.

C. Organization of the Paper

The remainder of the paper is organized as follows. In Section II, the analog beamforming architecture is described and the two problems are formulated. In Section III, we first describe algorithmic frameworks followed by the proposed algorithmic solutions and convergence guarantees for the problem formulations presented in Section II. Simulation results are presented in Section IV and Section V concludes the work.

D. Notations Used

The following notations are used throughout the paper. A vector and a matrix are represented by $\mathbf{a}$ and $\mathbf{A}$ respectively. The i, j element of a matrix is denoted as $\mathbf{A}(i,j)$. The i-th entry of a vector $\mathbf{a}$ is represented as $\mathbf{a}(i)$ or a_i. The complex exponential operation on each entry of a matrix is represented as $e^{(j\mathbf{A})}$, the phase/argument of each element of a matrix is denoted as $\arg(\mathbf{A})$. The trace operator and the Frobenius norm are represented as $\mathrm{Tr}(\mathbf{A})$ and $\|\mathbf{A}\|_F$; $\|\mathbf{a}\|_2$ denotes the ℓ_2 norm of the vector. The real part of a scalar complex variable z or a matrix varaible $\mathbf{Z}$, is represented as $\mathrm{Re}(z)$. The symbol $|\cdot|$ denotes the modulus of a complex number. The Hermitian operation, conjugate, and transpose of a matrix are denoted as $\mathbf{A}^H$, $\mathbf{A}^*$, and $\mathbf{A}^T$ respectively. The Schur-Hadamard product between two matrices is represented as $\mathbf{A} \circ \mathbf{B}$; $\mathbf{A} \succeq 0$ denotes a positive semi-definite (p.s.d.) matrix. The set of Hermitian positive semi-definite matrices is represented as $\mathbb{S}_n^+$. A vector of all ones and all zeros, each of size m are denoted as $\mathbf{1}_{m\times 1}$ and $\mathbf{0}_{m\times 1}$, respectively.

II. Analog Beamforming Architecture and Problem Formulations

In this section, we first present the SAB architecture followed by two beamforming design problem formulations.

A. SAB Architecture

A transmitter equipped with an analog phase shifting network driven by a common VGA serving multiple single-antenna users is considered, as shown in Fig. 1. The use of a common VGA for all branches motivates the term SAB architecture. The gain of VGA is assumed to be an unknown and considered as a design variable. Therefore, the

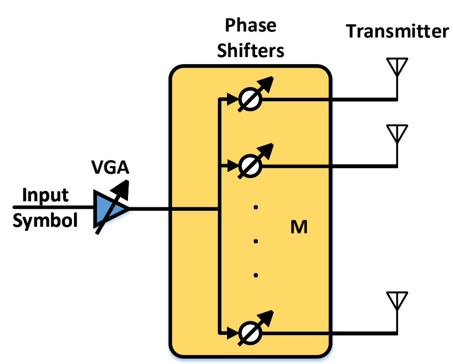

Fig. 1. CMAB or SAB architecture.

overall beamforming vector is constrained to have constant-modulus entries, where the unknown constant represents the gain introduced by the VGA. Herein, we consider a uniform linear array (ULA) with M antenna elements and a spacing $d = \frac{\lambda}{2}$ at the transmitter, where λ represents the wavelength of operation. The array transmits the same information to the users, therefore, this a broadcast beamforming scenario [35].

The array response of a ULA in a direction θ_i is modeled as,

$$\mathbf{a}(\theta_i) = \begin{bmatrix} 1 & e^{j\theta_i} & e^{j2\theta_i} & \ldots & e^{j(M-1)\theta_i} \end{bmatrix}^T. \quad (1)$$

We consider an uniform discretization of the angular space into N points as $\boldsymbol{\theta} = \left[0, \frac{2\pi}{N}, \frac{4\pi}{N}, \ldots, \frac{2(N-1)\pi}{N}\right]^T$. Then, the array response from these directions can be written compactly in matrix form as $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_N)]^H$, where θ_i represents the i-th element of $\boldsymbol{\theta}$. The beampattern in the direction θ_i takes the form $y(\theta_i) = \mathbf{a}(\theta_i)^H \tilde{\mathbf{w}}$, where $\tilde{\mathbf{w}}$ is the beamforming vector to be designed. For notational convenience, from now onwards we write $y(\theta_i)$, $\mathbf{a}(\theta_i)$ and $\mathbf{A}(\boldsymbol{\theta})$ as y_i, $\mathbf{a}_i$ and $\mathbf{A}$, respectively.

B. Beampattern matching

The least-squares beampattern matching problem after considering the requirements in all $\{\theta_i\}$ is formulated as,

$$\begin{aligned} \mathcal{P}_1 : \quad & \min_{r\in\mathbb{R},\tilde{\mathbf{w}}} \quad \|\mathbf{y} - \mathbf{A}\tilde{\mathbf{w}}\|_2^2 \\ & \text{subject to } |\tilde{w}_i| = r, \forall i \in [1, M], \end{aligned}$$

where $\mathbf{y}$ is an N-dimensional vector denoting the desired response along the directions represented by the elements of vector $\boldsymbol{\theta}$ and the variable r models the gain introduced by the VGA. The elements of the overall beamforming vector $\tilde{\mathbf{w}}$ are constrained to be constant modulus, thus representing phase-only beamforming.

C. Beampattern Matching with Additional Degrees of Freedom

In the case of transmit beamforming, it may be required to match only the magnitude of the beampattern because in general, a receiver has to compensate for phase inconsistency [15]. In this case, the system of equations for matching the beampattern magnitude can be expressed as,

$$\mathbf{y} = |\mathbf{A}\tilde{\mathbf{w}}|, \tag{2}$$

where $|\cdot|$ denotes the entry-wise magnitude of the vector $\mathbf{A}\tilde{\mathbf{w}}$. Because of the non-differentiability of the modulus function, the least-squares matching problem similar to problem $\mathcal{P}_1$ will result in a non-differentiable objective function. To that end, we can equivalently rewrite the system of equations in (2) as given by,

$$\mathbf{y} \circ \mathbf{u} = \mathbf{A}\tilde{\mathbf{w}},$$

where $\mathbf{u}$ represents an additional vector with its i-th entry $u_i = e^{j\arg(\mathbf{a}_i^H\tilde{\mathbf{w}})}$ and the symbol $\circ$ denotes the element-wise product between two vectors or matrices. Another way of modeling this scenario is to consider $|\mathbf{u}| = \mathbf{1}_{N\times 1}$ [15]. For simplicity, we follow the same approach as well and formulate the following minimization problem,

$$\begin{aligned} \mathcal{P}_2 : \quad & \min_{r\in\mathbb{R},\tilde{\mathbf{w}},\mathbf{u}} \quad \|\mathbf{y}\circ\mathbf{u} - \mathbf{A}\tilde{\mathbf{w}}\|_2^2 \\ & \text{subject to } |\tilde{w}_i| = r, \forall i \in [1, M] \\ & \qquad\qquad\quad |u_j| = 1, \forall j \in [1, N]. \end{aligned}$$

In the subsequent section, we propose efficient algorithms to solve problems $\mathcal{P}_1$ and $\mathcal{P}_2$.

Remark 1. It is important to note that problem $\mathcal{P}_1$ can be viewed as a special case of problem $\mathcal{P}_2$. Therefore, first, we propose efficient algorithms for solving problem $\mathcal{P}_2$, and the algorithms for solving problem $\mathcal{P}_1$ are derived afterward.

III. ALGORITHMIC SOLUTIONS FOR ANALOG BEAMFORMING

Before proceeding towards the algorithm development, first, we briefly discuss the algorithmic frameworks, MM, AMM, and CCD. These preliminaries are added for the sake of improving the readability.

A. Algorithmic Frameworks

1) The MM Algorithm: The majorization-minimization method works on the principle of iteratively solving a sequence of easier problems [36]–[39]. For example, let us consider the following minimization problem,

$$\begin{aligned} \mathcal{P}_3 : \quad & \min_{\mathbf{x}} \quad f(\mathbf{x}) \\ & \text{subject to } \mathbf{x} \in \mathcal{G}, \end{aligned}$$

where $f : \mathcal{G} \to \mathbb{R}$ is a continuous function, $\mathcal{G}$ is the constraint set and $\mathbf{x}$ is the unknown decision variable. In particular, an algorithm based on the MM framework starts with a feasible point $\mathbf{x}^{(0)} \in \mathcal{G}$ and iteratively solves the following problem,

$$\begin{aligned} \mathbf{x}^{(k+1)} \in \arg\min_{\mathbf{x}} & \tilde{f}(\mathbf{x}; \mathbf{x}^{(k)}) \\ \text{subject to } & \mathbf{x} \in \mathcal{G}, \end{aligned} \tag{3}$$

where $\tilde{f}(\mathbf{x}; \mathbf{x}^{(k)})$ is the surrogate function majorizing the original objective function $f(\mathbf{x})$ at $\mathbf{x}^{(k)}$ and $\mathbf{x}^{(k)}$ is the solution to the above problem at k-th iteration. A valid surrogate function for the minimization problem has the following properties,

$$\tilde{f}(\mathbf{x}; \mathbf{x}^{(k)}) \geq f(\mathbf{x}), \forall \mathbf{x} \in \mathcal{G} \tag{4}$$

$$\tilde{f}(\mathbf{x}^{(k)}; \mathbf{x}^{(k)}) = f(\mathbf{x}^{(k)}) \tag{5}$$

$$\nabla\tilde{f}(\mathbf{x}^{(k)}; \mathbf{x}^{(k)}) = \nabla f(\mathbf{x}^{(k)}). \tag{6}$$

Inequality (4) and equation (6) imply that the the surrogate function is a tight upper bound of the original objective function. Because of this property of the surrogate function, the objective function value decreases with the number of iterations, ultimately converging to a stationary point of the original problem. Equation (6), which is termed as the gradient consistency [40], ensures that the surrogate function and the objective function have the same gradients at $\mathbf{x}^{(k)}$. To formally prove the convergence of an MM algorithm, first, we introduce the first-order optimality condition for the minimization of a continuously differentiable function from Proposition 3 in [30] as,

Proposition III.1 (Proposition 3 in [30])**.** *Consider $f : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function and if $\mathbf{x}^{(\infty)}$ is a local minimum of f over a subset $\mathcal{G}$ of $\mathbb{R}^n$, then*

$$\nabla f\left(\mathbf{x}^{(\infty)}\right)^T \mathbf{y} \geq 0, \forall \mathbf{y} \in T_{\mathcal{G}}\left(\mathbf{x}^{(\infty)}\right), \tag{7}$$

where $T_{\mathcal{G}}\left(\mathbf{x}^{(\infty)}\right)$ denotes the tangent cone of $\mathcal{G}$ at $\mathbf{x}^{(\infty)}$.

A vector $\mathbf{x}$ satisfying the optimality condition (7) is referred to as a *stationary point*. For more insights on the MM framework and its convergence properties, one may refer to [30], [36]–[39] and references therein.

2) Alternating Majorization-Minimization (AMM) Algorithm: Let us consider the following minimization problem,

$$\mathcal{P}_4: \quad \min_{\mathbf{x}} \quad f(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \quad \text{subject to } \mathbf{x}_i \in \mathcal{G}_i, \forall i = 1, 2, \ldots, n$$

where $f : \mathcal{G} \rightarrow \mathbb{R}$ is the objective function, $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \cdots \times \mathcal{G}_n$ is the constraint set and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ is the decision variable partitioned into n-blocks with each block $\mathbf{x}_i$ having dimensions $n_i \times 1$. At the k-th iteration, following sub-problem is solved,

$$\mathbf{x}_i^{(k+1)} \in \arg\min_{\mathbf{x}_i} g_i\left(\mathbf{x}_i; \mathbf{x}_1^{(k+1)}, \ldots, \mathbf{x}_{i-1}^{(k+1)}, \mathbf{x}_i^{(k)}, \ldots, \mathbf{x}_n^{(k)}\right) \quad \text{subject to } \mathbf{x}_i \in \mathcal{G}_i, \tag{8}$$

for all $i = 1, 2, \ldots, n$ and the blocks are updated in a cyclic order and $\mathbf{x}_i^{(j)}$ denotes the update available for block $\mathbf{x}_i$ at j-th iteration. Similar to the conventional MM framework, here function $g_i\left(\mathbf{x}_i; \mathbf{x}_1^{(k+1)}, \ldots, \mathbf{x}_{i-1}^{(k+1)}, \mathbf{x}_i^{(k)}, \ldots, \mathbf{x}_n^{(k)}\right)$ is a tight majorizer of the original objective function in block variable $\mathbf{x}_i$ and satisfies the properties from (4)-(6). The AMM algorithm can also be interpreted as a block-successive upper minimization (BSUM) [40] / block MM algorithm [38]. It is important to note that in the AMM algorithm, nonconvex constraints can be accommodated. Moreover, only those objective functions are majorized for which the subproblems are not easy to minimize. Otherwise, the original objective function is minimized with respect to each block. Therefore, the convergence proof in [40] is adapted accordingly. A sequential optimization algorithm with a maximum block improvement (MBI) technique [41], [42] is also proposed in [43] for dealing with resource allocation in wireless networks and radar systems.

3) Block Cyclic Coordinate Descent (BCCD) Algorithm: In the BCCD approach, problem $\mathcal{P}_4$ is solved block-wise. Similar to the AMM framework, here, the problem is solved for each block while keeping the remaining blocks fixed. The most common form of the BCCD algorithm is given by,

$$\mathcal{P}_5: \mathbf{x}_i^{(k+1)} \in \arg\min_{\mathbf{x}_i \in \mathcal{G}_i} f\left(\mathbf{x}_1^{(k+1)}, \mathbf{x}_2^{(k+1)}, \mathbf{x}_i, \mathbf{x}_{i+1}^{(k)}, \ldots, \mathbf{x}_n^{(k)}\right) \quad \text{subject to } \mathbf{x}_i \in \mathcal{G}_i,$$

for all $i = 1, 2, \ldots, n$. Therefore, each minimization step considers the previously computed minimizers. When block size reduces to one, the resulting algorithm is known by the name of the cyclic coordinate descent (CCD) algorithm. For more information, one may refer to [34] and references therein.

4) Modified BCCD Algorithm: Consider problem $\mathcal{P}_4$ and select m out of n blocks of the decision variable $\mathbf{x}$. Let $\mathcal{U} = \{1, 2, \ldots, n\}$ and $\mathcal{U}_1 = \{n_1, n_2, \ldots, n_m\}$ denote the index set for all the blocks of decision variable $\mathbf{x}$ and the index set of the selected blocks, respectively, where each $n_i \in [1, n]$ and $n_i \neq n_j, \forall i \neq j$. Since, the constraints are separable in each $\mathbf{x}_i$ we propose to consider the following nested algorithm.

1) Initialize all $\mathbf{x}_i^{(0)}, \forall i \in [1, n]$ to a feasible point from their respective constraint sets and set $k = 0$.
2) For all $i \in \mathcal{U} \setminus \mathcal{U}_1$
   a) Solve the subproblem for $\mathbf{x}_i^{(k)}$ given the remaining updated blocks of $\mathbf{x}$.
      i) Solve the subproblems for $\mathbf{x}_j, \forall j \in \mathcal{U}_1$ given the remaining updated blocks of $\mathbf{x}$.
   b) Increment i to $i + 1$ and go to step 2a.
3) If the convergence condition is met, then stop. Otherwise, increment k to $k + 1$ and go to step 2.

In this algorithmic setup, the variables $\mathbf{x}_i, \forall i \in \mathcal{U}_1$ are updated more frequently than the remaining block variables. As will be seen later, this modification achieves a lower objective per-iteration value than the C-CCD-based approach. For the specific case of beampattern matching problems $\mathcal{P}_1$ and $\mathcal{P}_2$, we will establish convergence guarantees to a stationary point. We refer to the algorithms developed based on this approach as M-CCD-based algorithms.

A maximum block improvement (MBI) technique can also be adopted to solve problems $\mathcal{P}_1$ and $\mathcal{P}_2$ [41]–[43]. But selecting the maximum improvement block requires the computation of the objective function with respect to each block variable at every iteration of an MBI selection rule-based algorithm [41], [42]. This significantly increases the computational complexity of the algorithm especially for the large dimensional block variables and when a large number of blocks are to be optimized as in the case of beamforming design with LSAAs.

B. Algorithms for Problem $\mathcal{P}_2$

It can be seen that problem $\mathcal{P}_2$ is nonconvex because of the multiplicative variables $\mathbf{u}$ and $\mathbf{w}$, and the constant-modulus constraints. Even if, the variables r and $\mathbf{u}$ are known the problem is shown to be NP-hard [16]. We reconsider problem $\mathcal{P}_2$ and write it in a more amenable form as given by,

$$\mathcal{P}_6: \min_{s \in \mathbb{C}, \mathbf{w}, \mathbf{u}} f(s, \mathbf{u}, \mathbf{w}) = \|\mathbf{y} \circ \mathbf{u} - s\mathbf{A}\mathbf{w}\|_2^2 \quad \text{subject to } \mathbf{w} \in \mathcal{A}^M, \mathbf{u} \in \mathcal{A}^N,$$

where $\mathcal{A} = \left\{x \in \mathbb{C} |\ |x|^2 = 1\right\}$ and $\mathcal{A}^M$ denotes the M-ary Cartesian product, the constant modulus constraints on vector $\tilde{\mathbf{w}}$ in $\mathcal{P}_2$ are equivalently replaced by an unconstrained complex factor s multiplying $\mathbf{w}$ and variable $\mathbf{u}$ is a vector having the unit-modulus entries. The beamforming vector is now constrained to have the unit-modulus entries and the magnitude of variable s represents the gain introduced by the VGA.

1) AMM-Based Algorithm: Problem $\mathcal{P}_6$ is still nonconvex and cannot be solved jointly in all the variables. But, in present form the objective function of problem $\mathcal{P}_6$ is partially convex, meaning, given one variable it is convex in the other. Thus, we exploit the partial convexity of the objective function and propose an alternating minimization scheme.

At $(k+1)$-th iteration, problem $\mathcal{P}_6$ is solved with-respect-to variable s by assuming the solutions $\mathbf{w}^{(k)}$ and $\mathbf{u}^{(k)}$ for variables $\mathbf{w}$ and $\mathbf{u}$, respectively. Then, the variable $\mathbf{u}$ is updated given $s^{(k+1)}$ and $\mathbf{w}^{(k)}$. Finally, the problem is solved for variable $\mathbf{w}$ using the solution, $s^{(k+1)}$ and $\mathbf{u}^{(k+1)}$ for variables s and $\mathbf{u}$, respectively. This procedure results in three subproblems as given below:

$$s^{(k+1)} = \arg\min_{s \in \mathbb{C}} \ f(s, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) \tag{9}$$

$$\mathbf{u}^{(k+1)} = \arg\min_{\mathbf{u} \in \mathcal{A}^N} f(s^{(k+1)}, \mathbf{u}, \mathbf{w}^{(k)}) \tag{10}$$

$$\mathbf{w}^{(k+1)} = \arg\min_{\mathbf{w} \in \mathcal{A}^M} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}). \tag{11}$$

Working on the aforementioned idea, the sub-problem with respect to variable s is convex and admits the following closed-form solution,

$$s_{k+1} = \frac{(\mathbf{w}^{(k)})^H \mathbf{A}^H \mathbf{Y} \mathbf{u}^{(k)}}{\left\|\mathbf{A}\mathbf{w}^{(k)}\right\|_2^2}, \tag{12}$$

where $\mathbf{Y} = \mathrm{diag}\,(\mathbf{y})$ is a diagonal matrix. Now, considering variable s to be given as $s^{(k+1)}$, we need to solve problem (10). But this minimization problem is not straightforward to solve because of the unit-modulus constraints on vector $\mathbf{u}$. To get an update for $\mathbf{u}$, we assume s and $\mathbf{w}$ to be given and expand the objective function of problem (10),

$$\mathbf{u}^H \mathbf{Y}^H \mathbf{Y} \mathbf{u} - 2\,\mathrm{Re}\left(s^{(k+1)} \mathbf{u}^H \mathbf{Y}^H \mathbf{A} \mathbf{w}^{(k)}\right) + |s^{(k+1)}|^2 \left\|\mathbf{A}\mathbf{w}^{(k)}\right\|_2^2. \tag{13}$$

The first and the last terms in the objective function above are independent of $\mathbf{u}$, therefore, after ignoring the constant terms we arrive at the following formulation,

$$\mathcal{P}_7: \quad \min_{\mathbf{u}} \quad -\mathrm{Re}\left(s^{(k+1)} \mathbf{u}^H \mathbf{Y}^H \mathbf{A} \mathbf{w}^{(k)}\right) \quad \text{subject to } \mathbf{u} \in \mathcal{A}^N.$$

It is seen that problem $\mathcal{P}_7$ admits the following closed-form solution,

$$\mathbf{u}^{(k+1)} = e^{j \arg\left(s^{(k+1)} \mathbf{Y}^H \mathbf{A} \mathbf{w}^{(k)}\right)}. \tag{14}$$

Now, assuming the solutions $s^{(k+1)}$ and $\mathbf{u}^{(k+1)}$ for the variables s and $\mathbf{u}$, respectively, we need to solve problem (11). But this minimization problem is shown to be NP-hard [16]. Therefore, we propose to adopt the AMM framework and construct a majorizing function for the objective function of problem (11). More precisely speaking, we propose an AMM algorithm that is similar to the block MM framework, except for one change. That is, sub-problems that are difficult to directly minimize are approximated using a majorizing function, and for the remaining variables solution is directly computed in closed-form. We refer to this as an AMM algorithm. To this end, we propose to use the following Lemma from [30], [36], to construct a majorizing function for the objective function of problem (11).

Lemma III.2 (Lemma 1 in [30])**.** *The quadratic function of the form* $\mathbf{w}^H \mathbf{S} \mathbf{w}$*, with* $\mathbf{S}$ *being a Hermitian matrix is majorized by* $\mathbf{w}^H \mathbf{T} \mathbf{w} + 2\,\mathrm{Re}\left(\mathbf{w}^H (\mathbf{S} - \mathbf{T}) \mathbf{w}_k\right) + \mathbf{w}_k^H (\mathbf{T} - \mathbf{S}) \mathbf{w}_k$ *at the point* $\mathbf{w}_k$*, where* $\mathbf{T}$ *is a Hermitian matrix such that* $\mathbf{T} \succeq \mathbf{S}$.

Lemma III.2 can be easily proven using second order Taylor expansion and subsequently replacing the Hessian matrix $\mathbf{S}$ by another Hermitian matrix $\mathbf{T}$ such that $\mathbf{T} \succeq \mathbf{S}$. For a general twice differentiable function, Lemma III.2 is also known by the name of *quadratic upper bound principle* as mentioned in equation (4.6) under Section 4.6 in Chapter 4 of [37].

For obtaining the solution for variable $\mathbf{w}$, once again we expand the objective function of problem (11) as given by,

$$\begin{aligned} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}) = & (\mathbf{u}^{(k+1)})^H \mathbf{Y}^H \mathbf{Y} \mathbf{u}^{(k+1)} \\ & -2\,\mathrm{Re}\left(s^{(k+1)} (\mathbf{u}^{(k+1)})^H \mathbf{Y}^H \mathbf{A} \mathbf{w}\right) \\ & + |s^{(k+1)}|^2 \left\|\mathbf{A}\mathbf{w}\right\|_2^2. \end{aligned} \tag{15}$$

The first term in (15) is independent of $\mathbf{w}$ and the third term is convex in $\mathbf{w}$. Therefore, using Lemma III.2 we majorize the third term and obtain a tight upper bound, as given by,

$$\begin{aligned} \mathbf{w}^H \tilde{\mathbf{P}} \mathbf{w} \leq\ & \lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{w}^H \mathbf{w} \\ & + 2\,\mathrm{Re}\left(\mathbf{w}^H (\tilde{\mathbf{P}} - \lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{I}) \mathbf{w}^{(k)}\right) \\ & + (\mathbf{w}^{(k)})^H (\lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{I} - \tilde{\mathbf{P}}) \mathbf{w}^{(k)}, \end{aligned} \tag{16}$$

where $\tilde{\mathbf{P}} = |s^{(k+1)}|^2 \mathbf{A}^H \mathbf{A}$, $\mathbf{I}$ is a $M \times M$ identity matrix and we have chosen matrix $\mathbf{T} = \lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{I} = |s^{(k+1)}|^2 \lambda_{\max}(\mathbf{P}) \mathbf{I}$, where $\mathbf{P} = \mathbf{A}^H \mathbf{A}$. The function $\lambda_{\max}(\mathbf{P})$ represents the maximum eigenvalue of matrix $\mathbf{P}$. Therefore, the function, $f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w})$ is majorized as,

$$f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}) \leq \tilde{f}(\mathbf{w}; s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k)}) \tag{17}$$

$$\begin{aligned} = & \ \mathbf{y}^H \mathbf{y} - 2\,\mathrm{Re}\left(s^{(k+1)} \mathbf{u}^H \mathbf{Y}^H \mathbf{A} \mathbf{w}^{(k)}\right) \\ & + \lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{w}^H \mathbf{w} \\ & + 2\,\mathrm{Re}\left(\mathbf{w}^H (\tilde{\mathbf{P}} - \lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{I}) \mathbf{w}^{(k)}\right) \\ & + (\mathbf{w}^{(k)})^H (\lambda_{\max}(\tilde{\mathbf{P}}) \mathbf{I} - \tilde{\mathbf{P}}) \mathbf{w}^{(k)}. \end{aligned} \tag{18}$$

The first and fifth terms on the right-hand side of the inequality (17) are independent of $\mathbf{w}$, and the third term is also a constant, $\|\mathbf{w}\|_2^2 = M$, due to the unit-modulus property of the beamforming vector. After ignoring the constant terms on the right-hand-side (RHS) of (17), the majorized problem to solve problem (11) for variable $\mathbf{w}$ is formulated as,

$$\mathcal{P}_8: \quad \min_{\mathbf{w}} \quad -\mathrm{Re}\left(\mathbf{w}^H \mathbf{b}^{(k+1)}\right) \quad \text{subject to } \mathbf{w} \in \mathcal{A}^M,$$

where $\mathbf{b}^{(k+1)} = (s^{(k+1)})^* \tilde{\mathbf{Y}} \mathbf{u}^{(k+1)} - |s^{(k+1)}|^2 \mathbf{Q} \mathbf{w}^{(k)}$, $\tilde{\mathbf{Y}} = \mathbf{A}^H \mathbf{Y}$, $\mathbf{Q} = (\mathbf{P} - \lambda_{\max}(\mathbf{P}) \mathbf{I})$ and $\mathbf{w}^{(k)}$ is the solution available at k-th iteration. It can be shown that problem $\mathcal{P}_8$ admits the following closed-form solution,

$$\mathbf{w} = e^{j \arg\left(\mathbf{b}^{(k+1)}\right)}. \tag{19}$$

The complete algorithm summarizing the steps is presented in Algorithm 1.

2) CCD-Based Algorithm: Here, we present CCD-based algorithms. We provide two versions of the CCD-type algorithms. The first algorithm follows the C-CCD-based approach, that is all the decision variables are concatenated into one vector as $[s, \mathbf{u}^T, \mathbf{w}^T]^T$ and $(M + N + 1)$ scalar subproblems are solved at every iteration or lesser subproblems depending upon the block size chosen. As already mentioned in Section

Algorithm 1 MM Based CMAB Design with Additional Degrees of Freedom

Input: The matrix $\mathbf{A}$, $\mathbf{Y}$, $\mathbf{w}^{(0)} \in \mathcal{A}$ and $\mathbf{u}^{(0)} \in \mathcal{A}$
Output: $\mathbf{w}$, s
Set $k = 0$, $\mathbf{P} = \mathbf{A}^H\mathbf{A}$, $\widetilde{\mathbf{Y}} = \mathbf{A}^H\mathbf{Y}$, $\beta = \lambda_{\max}(\mathbf{A}^H\mathbf{A})$

repeat ▷ index over $k = 0 : N' - 1$
 $s_{k+1} = \frac{(\mathbf{w}^{(k)})^H\widetilde{\mathbf{Y}}\mathbf{u}^{(k)}}{\|\mathbf{A}\mathbf{w}\|_2^2}$;
 $\mathbf{u}^{(k+1)} = e^{j\arg\left(s^{(k+1)}\tilde{\mathbf{Y}}^H\mathbf{w}^{(k)}\right)}$;
 $\mathbf{b}^{(k+1)} = (s^{(k+1)})^*\tilde{\mathbf{Y}}\mathbf{u}^{(k+1)} - |s^{(k+1)}|^2\mathbf{Q}\mathbf{w}^{(k)}$
 $\mathbf{w}^{(k+1)} = e^{j\arg\left(\mathbf{b}^{(k+1)}\right)}$;
until convergence

III-A3, in the second approach, we update variables s and $\mathbf{u}$ after updating each component, w_i of the beamforming vector $\mathbf{w}$. This modification results in faster convergence, for more information on the advantages of M-CCD the reader is referred to the simulation results. Later in this section, we also prove the convergence guarantees to a stationary point for both approaches.

Steps for C-CCD: First, we write the update steps associated with the C-CCD-based algorithm.

$$s^{(k+1)} = \arg\min_{s\in\mathbb{C}} \; f(s, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) \tag{20}$$

$$\mathbf{u}^{(k+1)} = \arg\min_{\mathbf{u}\in\mathcal{A}^M} f(s^{(k+1)}, \mathbf{u}, \mathbf{w}^{(k)}) \tag{21}$$

$$w_1^{(k+1)} = \arg\min_{w_1\in\mathcal{A}} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, w_1, w_2^{(k)}, \ldots, w_M^{(k)}) \tag{22}$$

$$w_2^{(k+1)} = \arg\min_{w_2\in\mathcal{A}} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, w_1^{(k+1)}, w_2, w_3^{(k)}, \ldots, w_M^{(k)}) \tag{23}$$

$$\vdots$$

$$w_i^{(k+1)} = \arg\min_{w_i\in\mathcal{A}} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, w_1^{(k+1)}, \ldots, w_{i-1}^{(k+1)}, w_i, w_{i+1}^{(k)}, \ldots, w_M^{(k)}), \tag{24}$$

$$\vdots$$

$$w_M^{(k+1)} = \arg\min_{w_M\in\mathcal{A}} f(s^{(k+1)}, \mathbf{u}^{(k+1)}, w_1^{(k+1)}, \ldots, w_{M-1}^{(k+1)}, w_M), \tag{25}$$

where $w_i^{(k+1)}$ denotes the update of w_i at $k+1$-th iteration, $\mathbf{A}_{-i}$ is the matrix formed after removing the i-th column from the matrix $\mathbf{A}$, $\tilde{\mathbf{a}}_i$ is the i-th column of matrix $\mathbf{A}$, $\mathbf{w}_{-i}$ is the vector formed by removing the i-th element from vector $\mathbf{w}$ and $\mathbf{w}_{-i}^{(k)}$ denotes the update available for vector $\mathbf{w}_{-i}$ after k-th iteration. It is important to note that the minimization problem (25) is solved for each component w_i.

As already shown, for the given values of variables $\mathbf{u}$ and $\mathbf{w}$, the minimization problem (20) with respect to variable s admits the closed-form solution as given in (12) with appropriate change of iteration indices. For variable $\mathbf{u}$, we do not have to update its component variables sequentially, as problem (21) with respect to the block variable $\mathbf{u}$ admits the closed-form solution as given by (14). Now, we consider the minimization problem (25) with respect to the each component w_i of the vector $\mathbf{w}$, as given by,

$$\mathcal{P}_9: \quad \min_{w_i} \quad \left\|\mathbf{Y}\mathbf{u}^{(k+1)} - s^{(k+1)}\mathbf{h}_i^{(k)} - s^{(k+1)}w_i\tilde{\mathbf{a}}_i\right\|_2^2 \quad \text{subject to } w_i \in \mathcal{A}.$$

where $\mathbf{h}_i^{(k)} = \left(\sum_{j<i} w_j^{(k+1)}\tilde{\mathbf{a}}_j + \sum_{j>i} w_j^{(k)}\tilde{\mathbf{a}}_j\right)$. Now, upon expanding the objective function of problem $\mathcal{P}_9$ and ignoring the constant terms, we obtain,

$$\mathcal{P}_{10}: \quad \min_{w_i} \quad -\operatorname{Re}\left(w_i^*(s^{(k+1)})^*\tilde{\mathbf{a}}_i^H(\mathbf{Y}\mathbf{u}^{(k+1)} - s^{(k+1)}\mathbf{h}^{(k)})\right) \quad \text{subject to } w_i \in \mathcal{A}.$$

It is evident that problem $\mathcal{P}_{10}$ admits the following closed-form solution,

$$w_i = e^{j\arg\left((s^{(k+1)})^*\tilde{\mathbf{a}}_i^H(\mathbf{Y}\mathbf{u}^{(k+1)} - s^{(k+1)}\mathbf{h}_i^{(k)})\right)}, \tag{26}$$

for all i in $[1, M]$.

Steps for M-CCD: For the M-CCD approach, we assume that the $\mathbf{w}^{(k)}$ and $\mathbf{u}^{(k)}$ are the solutions available for variables $\mathbf{w}$ and $\mathbf{u}$, respectively at the k-th iteration. With the slight abuse of notation, we use $\mathbf{u}_i^{(k)}$ to denote the i-th inner update of $\mathbf{u}$ at the k-th outer iteration of the algorithm. Then, the following sub-problems need to be solved to solve at every iteration,

$$s_1^{(k+1)} = \arg\min_{s\in\mathbb{C}} \; f(s, \mathbf{u}^{(k)}, w_1^{(k)}, w_2^{(k)}, \ldots, w_M^{(k)}) \tag{27}$$

$$\mathbf{u}_1^{(k+1)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} f(s, \mathbf{u}, w_1^{(k)}, w_2^{(k)}, \ldots, w_M^{(k)}) \tag{28}$$

$$w_1^{(k+1)} = \arg\min_{w_1\in\mathcal{A}} f(s_1^{(k+1)}, \mathbf{u}^{(k+1)}, w_1, w_2^{(k)}, \ldots, w_M^{(k)}) \tag{29}$$

$$s_2^{(k+1)} = \arg\min_{s\in\mathbb{C}} \; f(s, \mathbf{u}_1^{(k+1)}, w_1^{(k+1)}, w_2^{(k)}, \ldots, w_M^{(k)}) \tag{30}$$

$$\mathbf{u}_2^{(k+1)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} f(s_2^{(k+1)}, \mathbf{u}, w_1^{(k+1)}, w_2^{(k+1)}, w_3^{(k)}, \ldots, w_M^{(k)}) \tag{31}$$

$$w_2^{(k+1)} = \arg\min_{w_2\in\mathcal{A}} f(s_2^{(k+1)}, \mathbf{u}_2^{(k+1)}, w_1^{(k+1)}, w_2, w_3^{(k)}, \ldots, w_M^{(k)}) \tag{32}$$

$$\vdots$$

$$s_M^{(k+1)} = \arg\min_{s\in\mathbb{C}} \; f(s, \mathbf{u}_{M-1}^{(k+1)}, w_1^{(k+1)}, w_2^{(k+1)}, \ldots, w_{M-1}^{(k+1)}, w_M^{(k)}) \tag{33}$$

$$\mathbf{u}_M^{(k+1)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} f(s_M^{(k+1)}, \mathbf{u}, w_1^{(k+1)}, \ldots, w_{M-1}^{(k+1)}, w_M^{(k)}) \tag{34}$$

$$w_M^{(k+1)} = \arg\min_{w_M\in\mathcal{A}} f(s_M^{(k+1)}, \mathbf{u}_M^{(k+1)}, w_1^{(k+1)}, w_2^{(k+1)}, \ldots, w_{M-1}^{(k+1)}, w_M). \tag{35}$$

After the completion of $(k+1)$-th iteration, we denote $s^{(k+1)} = s_M^{(k+1)}$, $\mathbf{w}^{(k+1)} = [w_1^{(k+1)}, w_2^{(k+1)}, \ldots, w_M^{(k+1)}]^T$ and $\mathbf{u}^{(k+1)} = \mathbf{u}_M^{(k+1)}$. As can be seen from the above updates the variables s and $\mathbf{u}$ are updated after obtaining the solution of each w_i. Each sub-problem in the aforementioned steps

Algorithm 2 Conventional/Modified-Cyclic Coordinate Descent (C/M-CCD)-Based CMAB Design with Additional Degrees of Freedom

Input: The matrix $\mathbf{A}$, $\mathbf{y}$ and $\mathbf{w}_0 \in \mathcal{A}$
Output: $\mathbf{w}$, s
Set $k = 0$, $\mathbf{u} = \mathbf{1}$

```
repeat                                  ▷ index over k = 0 : N′ − 1
    Update s using (12);                ▷ Conventional version
    Update u using (14);                ▷ Conventional version
    for i = 1 to M do
        Update s using (12);            ▷ Modified version
        Update u using (14);            ▷ Modified version
        Update w_i using (26);
    end for
until convergence
```

is solved using the solutions obtained from C-CCD based approach.

Depending upon where we update variables s and $\mathbf{u}$, we obtain two different algorithms as summarized in Algorithm 2. To distinguish between the two algorithms we denote conventional and modified CCD-based algorithms as Algorithm 2 (C-CCD) and Algorithm 2 (M-CCD), respectively. As pointed out earlier in Section III-A4, here we also expect that the M-CCD algorithm to have faster convergence than the other algorithms. This will be seen shortly in Section IV from simulation results.

C. Algorithms for Problem $\mathcal{P}_1$

As mentioned earlier problem $\mathcal{P}_1$ can be considered as a special case of $\mathcal{P}_2$, that is when $u_i = 1, \forall i \in [1, N]$ and eliminating its update from the algorithms. Similar to problem $\mathcal{P}_2$, problem $\mathcal{P}_1$ can be reformulated as,

$$\mathcal{P}_{11}: \min_{s\in\mathbb{C},\mathbf{w}} \quad g(s,\mathbf{w}) = \|\mathbf{y} - s\mathbf{A}(\boldsymbol{\theta})\mathbf{w}\|_2^2 \quad \text{subject to } \mathbf{w} \in \mathcal{A}^M.$$

The algorithms proposed for solving problem $\mathcal{P}_2$ can be tailored for $\mathcal{P}_{11}$ as well. Therefore, for brevity, we do not provide the detailed derivation steps for obtaining the algorithms.

1) AMM-Based Algorithm: In this case, the solution to problems (9) and (11) become

$$s^{(k+1)} = \frac{(\mathbf{w}^{(k)})^H \mathbf{A}^H \mathbf{y}}{\left\|\mathbf{A}\mathbf{w}^{(k)}\right\|_2^2}, \tag{36}$$

and

$$\mathbf{w} = e^{j \arg\left(\mathbf{c}^{(k+1)}\right)}, \tag{37}$$

where $\mathbf{c}^{(k+1)} = (s^{(k+1)})^* \mathbf{A}^H\mathbf{y} - |s^{(k+1)}|^2 \mathbf{Q}\mathbf{w}^{(k)}$, respectively. The overall algorithm summarizing the steps is presented in Algorithm 3. The convergence of Algorithm 3 is proven as a special case of the convergence of Algorithm 1, which will be presented in Section III-B1. The reader is referred to the proof of Theorem III.3 in Appendix A.

Algorithm 3 MM Based CMAB Design

Input: The matrix $\mathbf{A}$, $\mathbf{y}$ and $\mathbf{w}_0 \in \mathcal{A}$
Output: $\mathbf{w}$, s
Set $k = 0$, $\mathbf{P} = \mathbf{A}^H\mathbf{A}$, $\tilde{\mathbf{y}} = \mathbf{A}^H\mathbf{y}$, $\beta = \lambda_{\max}(\mathbf{A}^H\mathbf{A})$, $\mathbf{Q} = \mathbf{P} - \beta\mathbf{I}$

repeat ▷ index over $k = 0 : N' - 1$
- $s^{(k+1)} = \frac{(\mathbf{w}^{(k)})^H\mathbf{A}^H\mathbf{y}}{\left\|\mathbf{A}\mathbf{w}^{(k)}\right\|_2^2}$;
- $\mathbf{c}^{(k+1)} = (s^{(k+1)})^* \tilde{\mathbf{y}} - |s^{(k+1)}|^2 \mathbf{Q}\mathbf{w}^{(k)}$;
- Compute $\mathbf{w}^{(k+1)} = e^{j\arg\left(\mathbf{c}^{(k+1)}\right)}$;

until convergence

Algorithm 4 Conventional/Modified-Cyclic Coordinate Descent (C/M-CCD)-Based CMAB Design

Input: The matrix $\mathbf{A}$, $\mathbf{y}$, $\mathbf{w}_0 \in \mathcal{A}$ and $s_0 \in \mathbb{C}$
Output: $\mathbf{w}$, s
Set $k = 0$,

```
repeat                                  ▷ index over k = 0 : N′ − 1
    for i = 1 to M do
        Update s using (36);            ▷ Modified version
        Update w_i using (38);
    end for
    Update s using (36);                ▷ Conventional version
until convergence
```

2) CCD-Based Algorithm: Here, we present two CCD based algorithms as a special case of the ones in Section III-B2. When $|\mathbf{u}| = \mathbf{1}_{N\times 1}$, the solution to problem (20) is given by (36) with appropriate change of iteration indices and the solution to problem (25) is given by,

$$w_i = e^{j\arg\left((s^{(k+1)})^* \tilde{\mathbf{a}}_i^H (\mathbf{y} - s^{(k+1)}\mathbf{d}^{(k)})\right)}, \forall i \in [1, M], \tag{38}$$

where $\mathbf{d}_i^{(k)} = \left(\sum_{j<i} w_j^{(k+1)}\tilde{\mathbf{a}}_j + \sum_{j>i} w_j^{(k)}\tilde{\mathbf{a}}_j\right)$. The steps of the algorithms are presented in Algorithm 4. Likewise, in the case of problem $\mathcal{P}_2$, here we also expect the M-CCD-based algorithm to perform better than the C-CCD-based algorithm.

The convergence of Algorithm 4 for both modified as well as conventional approaches are proven as a special case of the convergence of Algorithm 2, to be presented in Section III-B2.

D. Convergence Analysis

In this section, we present convergence guarantees of the proposed algorithms based on the AMM, C-CCD and M-CCD frameworks.

1) Convergence Analysis of AMM-Based Algorithms: First, we prove the convergence guarantees to a stationary point for Algorithm 1 in Theorem III.3. Because the convergence of Algorithm 3 follows the same steps except for one change, that is by substituting $\mathbf{u} = \mathbf{1}_{N\times 1}$ and ignoring its update in the proof of Theorem III.3. Therefore, the convergence of Algorithm 3 can be easily proven as a special case of Algorithm 3.

Theorem III.3. *Let $\{s^{(k)}, \mathbf{w}^{(k)}, \mathbf{u}^{(k)}\}$ be a sequence generated by Algorithm 1. Then, every limit point of the sequence is a KKT-point of problem $\mathcal{P}_6$.*

Proof. See Appendix A. ∎

2) Convergence Analysis of CCD-Based Algorithms: We establish the convergence guarantees to a stationary for Algorithm 2 for both the conventional as well as modified CCD based approaches in Theorem III.4. Similar, to Section III-D1 the convergence for Algorithm 4 for both conventional as well as modified approaches follow the same steps with $\mathbf{u} = \mathbf{1}_{N\times 1}$ and removing its update in the proof of Theorem III.4. Therefore, we consider the general case of Algorithm 4.

Theorem III.4. *Let $\{s^{(k)}, \mathbf{w}^{(k)}, \mathbf{u}^{(k)}\}$ be the sequence generated by Algorithm 2 using either the conventional or the modified CCD approaches. Then, every limit point of $\{s^{(k)}, \mathbf{w}^{(k)}, \mathbf{u}^{(k)}\}$ is a KKT-point of problem $\mathcal{P}_6$.*

Proof. See Appendices B and C for C-CCD and M-CCD based algorithms, respectively. ∎

E. Complexity Analysis

1) Algorithm 1 (MM): From Algorithm 1, it is seen that the computational complexity is mainly affected by the matrix-matrix product and the eigenvalue computation. The maximum eigenvalue of Hermitian positive semidefinite matrix $\mathbf{P}$ is easy to compute using Krylov–Schur Algorithm [44]. Since the matrix $\mathbf{P}$ and the maximum eigenvalue of $\mathbf{P}$ is computed outside the loop, we analyze the worst-case per iteration complexity by considering the naive implementation of matrix-vector products. The quantity $\|\mathbf{A}\mathbf{w}\|_2^2$ can be computed in $O(N^2)$ operations. The product $\mathbf{P}\mathbf{w}$ is computed in $O(M^2)$ operations, whereas the phase computation can be done in $O(M)$ operations. In addition to this, variable $\mathbf{u}$ can be computed in $O(NM)$ operations.

2) Algorithm 2 (C-CCD and M-CCD): The computational complexity for the C-CCD approach in Algorithm 2 is dominated by the computation of elements w_i's of the beamforming vector $\mathbf{w}$. Each w_i is computed in $O(NM)$ operations, thus, to compute all the entries of $\mathbf{w}$, $O(NM^2)$ operations are required. For the M-CCD-based approach, the computation of each w_i is followed by an update of s and $\mathbf{u}$, both of them can be computed in $O(NM)$ operations. The per-iteration complexity becomes $O(NM^2 + NM)$. Whilst the complexity of M-CCD based Algorithm 2 increases to $O(NM^2 + NM)$.

3) Algorithm 3 (MM): The complexity of computing each step of Algorithm 3 is similar to that of Algorithm 1, except for the removal of update of variable $\mathbf{u}$ while updating w_i's. Thus, there will be a reduction by a factor $O(NM)$.

4) Algorithm 4 (C-CCD and M-CCD): Similar to Algorithm 2, the computational complexity for the C-CCD based Algorithm 2 is modified due to the removal of the update of variable $\mathbf{u}$ while updating w_i. In comparison to 2 (C-CCD), the complexity of M-CCD based Algorithm 3 increases to $O(NM^2 + NM)$.

IV. Simulation Results

In this section, we provide numerical simulations to evaluate the performance of the proposed algorithms and show their potential in different scenarios. Specifically, we consider two beamforming scenarios, namely pencil and sector beamforming. For each scenario, we compare the proposed algorithms to the ones in [15] and the semidefinite relaxation (SDR) based algorithm. The performance is evaluated based on the evolution of the beampattern matching error with the number of iterations and the scalability with the number of antennas. All the results are averaged over 10,000 Monte-Carlo runs. The advantages of having additional degrees of freedom as shown in problem $\mathcal{P}_2$ in comparison to problem $\mathcal{P}_1$ are already mentioned in [15]. Therefore, we consider problem $\mathcal{P}_1$ for the pencil beamforming and problem $\mathcal{P}_2$ for the sector beamforming scenario.

A. Pencil Beamforming

Herein, we consider the scenario with $M = 100$, antenna elements, the angle space is uniformly discretized into $N = 36$ points in radians. The beampattern vector $\mathbf{y}$ is generated according to the following equation,

$$y_i = \begin{cases} 1 & \text{if } i \in \mathcal{I}, \\ 0 & \text{otherwise}, \end{cases} \tag{39}$$

where $\mathcal{I} \in [1, N]$ denotes the index set for non-zero entries of vector $\mathbf{y}$ with cardinality of $\operatorname{card}(\mathcal{I}) = K$. For fairness, all the algorithms are initialized to the same feasible starting point. We choose $K = 4$ entries to construct the index set $\mathcal{I}$ from $[1, N]$, and select the corresponding K angles from $\boldsymbol{\theta}$. The index set chosen in this case is, $\mathcal{I} = \{5, 10, 15, 20\}$. We then obtain $\mathbf{y}$ according to (39). Fig. 2 shows the objective function variation of problem $\mathcal{P}_{11}$ with the number of iterations. It is observed that the proposed algorithms converge faster with a lower beampattern matching error in comparison to the state-of-the-art algorithms in the literature [15] and as well as the SDR-based algorithm. It is important to highlight the fact that the M-CCD-based algorithm outperforms the other algorithms in the comparison. The modified update for the variable s reduces significantly the objective function in comparison to the C-CCD-based algorithm. One of the potential reasons is being the fact that, for each variable of problem $\mathcal{P}_6$ given the rest of the variables, the problem admits a closed-form solution. Therefore, updating s inside the inner loop of Algorithm 4 (M-CCD) significantly decreases the objective function in comparison to Algorithm 4 (C-CCD). The sampled beampatterns obtained are shown in Fig. 3. Kindly note that the algorithms for pattern matching consider only the sample points as presented in (39). Under this formulation, it is seen that the proposed algorithms match the original beampattern better than the existing algorithms.

Next, we study the scalability of the proposed algorithms with increasing the number antennas. The stopping criteria chosen for the algorithms is either $\frac{\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2}{\|\mathbf{w}_k\|_2} \leq 10^{-6}$ or a maximum number iterations are reached. The average beampattern matching error variation with M is shown in Fig. 4, where the number of antenna elements are varied from 2 to 100. The index set chosen in this case is, $\mathcal{I} = \{5, 10, 15, 20\}$. It is observed that increasing the number of antenna elements results in a better beampattern approximation. Moreover, the

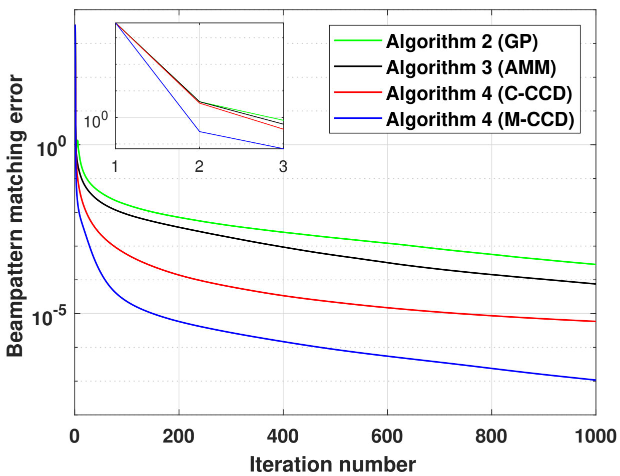

Fig. 2. Objective function evolution with the number of iterations in pencil beamfomring scenario, $N = 36$ and $M = 100$.

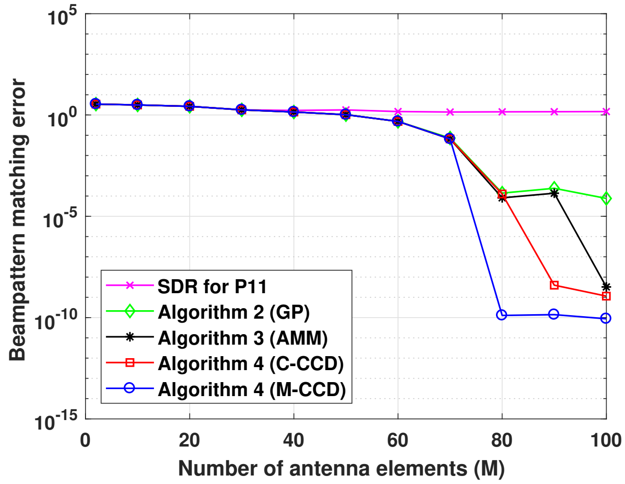

Fig. 4. Variation of the beampattern matching error with the number of antenna elements in pencil beamforming scenario, $N = 36$.

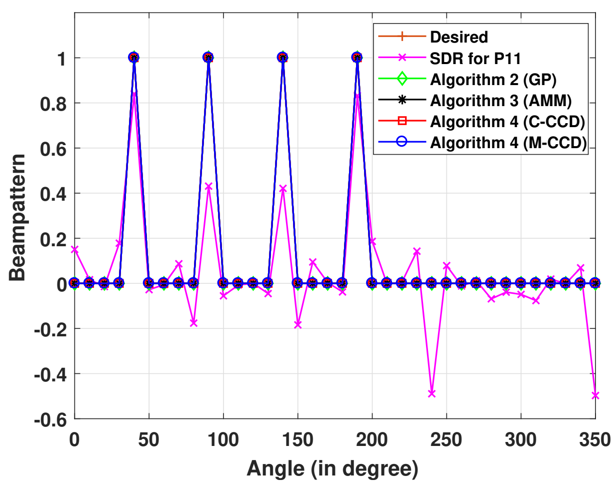

Fig. 3. Sampled beampatterns obtained from different algorithms (refer to (39)) for pencil beamforming, $N = 36$ and $M = 100$.

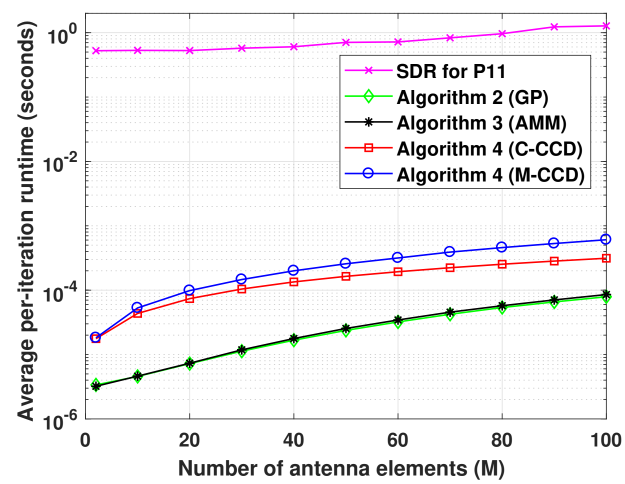

Fig. 5. Variation of the average per-iteration runtime with the number of antenna elements in pencil beamforming scenario, $N = 36$.

proposed algorithms achieve several orders of lower beampattern matching error in comparison to Algorithm 2 from [15] and as well as the SDR-based algorithm. It is also seen that the SDR-based algorithm does not scale well with respect to the number of antenna elements as observed in [15]. Once, again the M-CCD approach is superior to the other algorithms. The average per-iteration runtime required by the algorithms with increasing the number of antennas is shown in Fig. 5. It is seen that Algorithm 3 uses a similar amount of average per-iteration central processing unit (CPU) time to that of Algorithm 2 in [15] albeit having superior beampattern matching accuracy as observed from Fig. 4. It is important to highlight that the average per-iteration CPU time required by all the proposed algorithms is significantly lesser than the SDR-based algorithm. The CCD-based approaches presented in Algorithm 4 require more time to converge, the reason behind this is the increased per-iteration complexity. The modified approach results in far better beampattern approximation as shown in Fig. 4 than all the algorithms but it has higher per-iteration complexity. Therefore, the AMM-based method provides a good trade-off between the performance and the computational complexity. On the one hand, it has better beampattern matching accuracy in comparison to the Algorithm 2 in [15] but lesser than the CCD based algorithms, whereas, on the other hand, it takes similar CPU time to converge to that of Algorithm 2 from [15].

B. Sector Beamforming

In the sectored beamforming scenario, we quantize the angle space with $N = 144$ points in radians and consider $M = 250$. Similar, to the previous case the beampattern vector $\mathbf{y}$ is generated by (39), with the following index set, $\mathcal{I} = \{1, 2, \ldots, 18, 55, 56, \ldots, 90, 127, 128, \ldots, 144\}$. The objective function variation of problem $\mathcal{P}_6$ with the number of iterations is shown in Fig. 6. It is seen that the proposed algorithms have superior performance to the one in [15] and

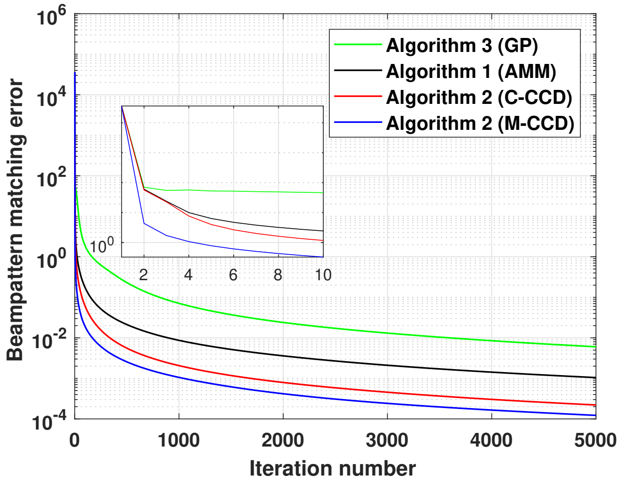

Fig. 6. Objective function evolution with the number of iterations in sector beamforming scenario, $N = 144$ and $M = 250$.

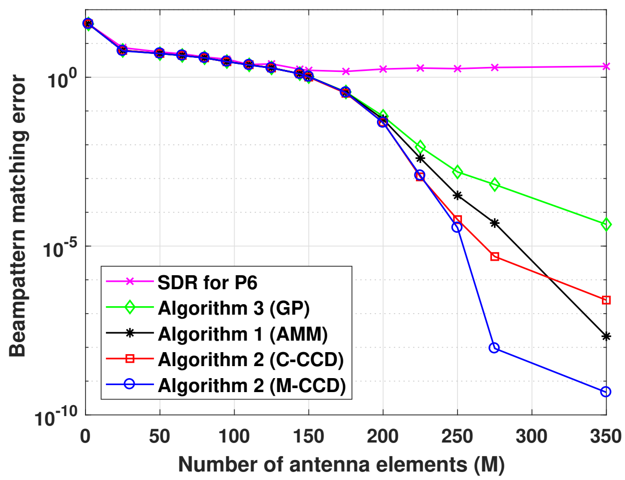

Fig. 8. Variation of the beampattern matching error with the number of antenna elements in sector beamforming, $N = 144$.

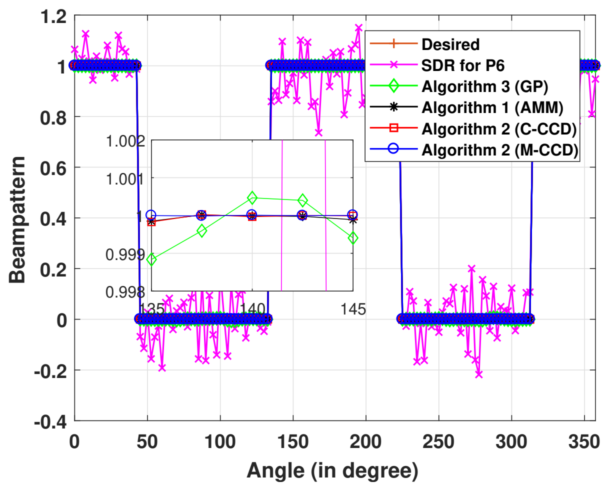

Fig. 7. Sampled beampatterns obtained from different algorithms (refer to (39)) for sector beamforming scenario, $N = 144$ and $M = 250$.

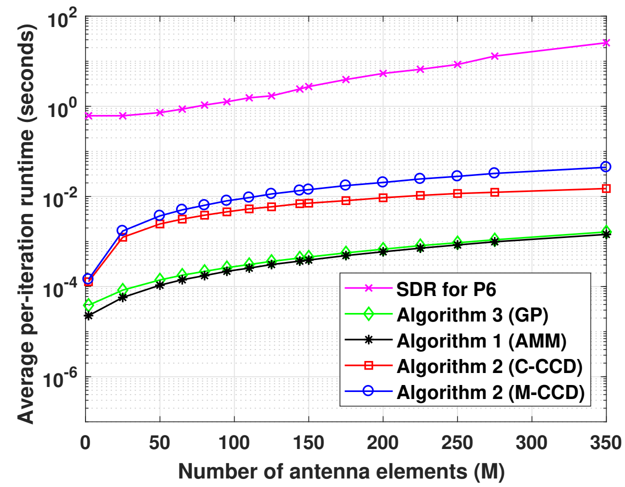

Fig. 9. Variation of the average per-iteration runtime with the number of antenna elements in sector beamforming scenario, $N = 144$.

Algorithm 2 (M-CCD) achieves lesser beampattern matching error within a small number of iterations in comparison to the other algorithms. Once again, it is also seen that the proposed M-CCD-based algorithm outperforms the other algorithms in comparison. This performance gain comes from the modified update of variables s and $\mathbf{u}$ as both the subproblems admit closed-form solutions. Thus, updating s and $\mathbf{u}$ while updating w_i in Algorithm 2 (M-CCD) significantly decreases the objective function in comparison to Algorithm 2 (C-CCD). This results in much better beampattern matching accuracy.

The sampled beampatterns obtained are shown in Fig. 7. Please note that the algorithms for pattern matching consider only the sample points as presented in (39). Under this formulation, it is seen that the proposed algorithms match the original beampattern better than the existing algorithms.

The scalability of the algorithms with the number of antenna elements is shown in Fig. 8, the number of antenna elements is varied from 2 to 350. It is observed that the beampattern matching error drops by several orders of magnitude when an LSAA is employed. This results in a better beampattern matching accuracy. All the algorithms perform similar until $M = 144$, which is the length of the beampattern vector $\mathbf{y}$. After this point, the proposed algorithms result in significantly better beampattern approximation. The performance is further reinforced if the number of antenna elements is significantly larger than the cardinality of the angular discretization. Once again, the proposed algorithms outperform the one in [15] as well as the SDR-based algorithm. The SDR-based algorithm does not scale well with the number of antenna elements as observed in [15].

The average per-iteration runtime required by the algorithms is shown in Fig. 9. It is observed that the Algorithm 1 based on the AMM approach uses a lesser average per-iteration CPU time than Algorithm 3 from [15] albeit with better beampattern approximation accuracy. All the proposed algorithms require less per-iteration CPU time than the SDR-based algorithm.

As expected, Algorithm 2 (modified as well as conventional CCD) uses more CPU time per-iteration because of the elements of the analog beamforming vector $\mathbf{w}$ that are updated sequentially followed by the computation of variables s and $\mathbf{u}$. Moreover, the M-CCD-based Algorithm 2 is slightly computationally expensive in comparison with the conventional one because of the sequential updates of variables s and $\mathbf{u}$ in conjunction with the components w_i's. Therefore, AMM-based Algorithm 2 again provides a trade-off between the performance and the computational complexity.

V. Conclusion

In this paper, we studied the problem of designing constant-modulus analog beamforming (CMAB) systems equipped with large-scale antenna arrays (LSAAs). Two beampattern matching problems were considered to design the parameters of an analog beamforming system based on the scaled analog beamforming (SAB) or CMAB architecture. In the beampattern matching problem, the least-squares error between the desired and the designed beampatterns was minimized subject to the unit-modulus constraints. In the first case, both the magnitude and phase of the beampattern were matched to the given desired beampattern whereas in the second case, the magnitude of the beampattern is matched, resulting in coupled unit-modulus vectors.

For each problem, we proposed efficient alternating majorization-minimization (AMM) and conventional-cyclic coordinate descent (C-CCD) based algorithms. In addition to the conventional-CCD (C-CCD) based approach, we also proposed modified-CCD (M-CCD) algorithms. The convergence of the proposed algorithms to stationary points of the optimization problems was theoretically established. In numerical simulations, different beamforming scenarios were considered for the comparison of the algorithms. We list the following observations exhibiting the performance gains of the proposed algorithms:

- The proposed algorithms outperformed the state-of-the-art solutions existing in the literature, resulting in better beampattern matching accuracy.
- The M-CCD-based algorithms converged faster in comparison to the other algorithms when visualized with the number of iterations.
- The M-CCD-based algorithms converged with the least beampattern matching error.
- The AMM-based Algorithm 3 performed even better in terms of average CPU time to converge, thereby, providing a good trade-off between performance and complexity.

Our results demonstrated the aforementioned gains of the proposed algorithms by utilizing a new approach based on the MM, alternating minimization, and CCD frameworks.

Appendix A
Proof of Theorem III.3

Before presenting the convergence proof, we first note that the optimization problem w.r.t $\mathbf{u}$ is not observable for the zero entries of $\mathbf{y}$. Hence, the focus is on the case of non-zero entries, y_i. Now, we prove the following Lemma A.1 for the uniqueness of the minimizer of the optimization sub-problems with respect to variables s and $\mathbf{u}$ which is used in the proof later in this section.

Lemma A.1 (Uniqueness of Minimizers)**.** *The objective* $f(s, \mathbf{w}, \mathbf{u})$ *has an unique minimizer for*

A. s *given* $\mathbf{w}, \mathbf{u}$*, and*
B. $\mathbf{u}$ *given* $s, \mathbf{w}$.

Proof. We show the proof into two parts as follows:
A. *Optimization of s given $\mathbf{u}$ and $\mathbf{w}$:* The considered objective function, $\|\mathbf{y} \circ \mathbf{u} - s\mathbf{A}\mathbf{w}\|_2^2$ is strictly convex in s since the second-order derivative satisfies, $2\,\|\mathbf{A}\mathbf{w}\|_2^2 > 0$; this is because $\mathbf{A}$ is full rank (Vandermonde) and $\mathbf{w} \neq \mathbf{0}$. As a result, it has the following unique minimizer,

$$s_* = \frac{\mathbf{w}^H \mathbf{A}^H \mathbf{Y} \mathbf{u}}{\|\mathbf{A}\mathbf{w}\|_2^2}. \tag{40}$$

B. *Optimization of $\mathbf{u}$ given s and $\mathbf{w}$:* The objective function in variable $\mathbf{u}$ can be written as,

$$f(s, \mathbf{u}, \mathbf{w}) = \|\mathbf{y}\|_2^2 + \|s\mathbf{A}\mathbf{w}\|_2^2 - 2\,\mathrm{Re}\left(\mathbf{u}^H (s\mathbf{Y}^H \mathbf{A}\mathbf{w})\right). \tag{41}$$

Noting that all the entries of vector $\mathbf{u}$ are unit-modulus, the minimum of (41) occurs when $\mathbf{u}$ is aligned with $s\mathbf{Y}^H\mathbf{A}\mathbf{w}$ leading to,

$$\mathbf{u}_* = e^{j \arg(s\mathbf{Y}^H \mathbf{A}\mathbf{w})}. \tag{42}$$

We show that $\mathbf{u}_* = e^{j \arg(s\mathbf{Y}^H \mathbf{A}\mathbf{w})}$ is the unique minimizer, by contradiction. Let $\mathbf{u}_+ = e^{j\left(\arg(s\mathbf{Y}^H \mathbf{A}\mathbf{w}) + \boldsymbol{\theta}\right)}$ be another minimizer, where at least one entry of $\boldsymbol{\theta}$ is non-zero and is not an integer multiple of 2π (otherwise, $\mathbf{u}_* = \mathbf{u}_+$). Then, from (42), we have,

$$f(s, \mathbf{u}_*, \mathbf{w}) = \|\mathbf{y}\|_2^2 + \|s\mathbf{A}\mathbf{w}\|_2^2 - 2\sum_i |s\mathbf{Y}^H \mathbf{A}\mathbf{w}|_i \tag{43}$$

$$f(s, \mathbf{u}_+, \mathbf{w}) = \|\mathbf{y}\|_2^2 + \|s\mathbf{A}\mathbf{w}\|_2^2 - 2\sum_i |s\mathbf{Y}^H \mathbf{A}\mathbf{w}|_i \cos(\theta_i). \tag{44}$$

Since both $\mathbf{u}_*$ and $\mathbf{u}_+$ minimize $f(s, \mathbf{u}, \mathbf{w})$, it follows from (43), (44) that, $f(s, \mathbf{u}_*, \mathbf{w}) = f(s, \mathbf{u}_+, \mathbf{w})$. This is possible only if $\cos(\theta_i) = 1, \forall i \in [N]$, which implies that all the entries of $\boldsymbol{\theta}$ are zero or some integer multiples of 2π going against the assumption. This leads to a contradiction and hence proving the uniqueness. ■

Now we are ready to prove Theorem III.3.

Proof. First, we recall from (9)-(11) the following update order of the iterates from the algorithm at $(k+1)$-th iteration,

$$s^{(k+1)} = \arg\min_s f(s, \mathbf{w}^{(k)}, \mathbf{u}^{(k)}) \tag{45}$$

$$\mathbf{u}^{(k+1)} = \arg\min_{\mathbf{u}} f(s^{(k+1)}, \mathbf{w}^{(k)}, \mathbf{u}) \tag{46}$$

$$\mathbf{w}^{(k+1)} = \arg\min_{\mathbf{w} \in \mathcal{A}} \tilde{f}(\mathbf{w}; s^{(k+1)}, \mathbf{w}^{(k)}, \mathbf{u}^{(k+1)}) \tag{47}$$

where $\tilde{f}(\mathbf{w}; \mathbf{s}^{(k+1)}, \mathbf{w}^{(k)}, \mathbf{u}^{(k+1)})$ denotes the majorizing function of the original objective function $f(s, \mathbf{w}, \mathbf{u})$ after the updates of the variables s and $\mathbf{u}$. The majorizing function also depends upon the solution $\mathbf{w}^{(k)}$ obtained at the k-th iteration. The following can be easily shown,

$$
\begin{aligned}
f(s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) &\geq f(s^{(k+1)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) && (48)\\
&\geq f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k)}) && (49)\\
&= \tilde{f}(\mathbf{w}^{(k)}; s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k)}) && (50)\\
&\geq \tilde{f}(\mathbf{w}^{(k+1)}; s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k)}) && (51)\\
&\geq f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k+1)}). && (52)
\end{aligned}
$$

Inequality (48) and (49) above follows from the updates of s and $\mathbf{u}$, respectively. Equation (50) holds because the approximated function is a valid majorizer in the block $\mathbf{w}$, inequality (51) follows from (47), inequality (52) follows from the descent property of the MM framework. Therefore, the function sequence $\{f(s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)})\}$ decreases monotonically and thus, converges.

A. Convergence to a Stationary Point

Now, we show that the sequence generated by the algorithm converges to a KKT-point of the problem. Assume a convergent subsequence $\{s^{(k_j)}, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}\} \rightarrow \{s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}\}$. It is important to note that $s^{(\infty)} \in \mathbb{C}$, $\mathbf{u}^{(\infty)} \in \mathcal{A}^N$ and $\mathbf{w}^{(\infty)} \in \mathcal{A}^M$ because the respective constraint sets are closed. We now show that $s^{(\infty)}$, $\mathbf{u}^{(\infty)}$ and $\mathbf{w}^{(\infty)}$ are the block-wise minimizers of the function $f(s, \mathbf{u}, \mathbf{w})$ with respect to s, $\mathbf{u}$ and $\mathbf{w}$, respectively.

1) Convergence for s: To demonstrate this, we present the following set of inequalities,

$$
\begin{aligned}
&f(s, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)})\\
&\quad\geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}) && (53)\\
&\quad\vdots\\
&\quad\geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j+1)}) && (54)\\
&\quad\geq f(s^{(k_{j+1})}, \mathbf{u}^{(k_{j+1})}, \mathbf{w}^{(k_{j+1})}). && (55)
\end{aligned}
$$

These inequalities are based on the descent of the objective function from (48)-(52). Now, letting $j \rightarrow \infty$, using the convergence of the subsequences and the continuity of $f(s, \mathbf{u}, \mathbf{w})$ and $\tilde{f}(\mathbf{w}; s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)})$, we get,

$$f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \quad (56)$$

The inequality (56) implies that, $s^{(\infty)}$ is a block-wise minimizer of the function $f(\cdot)$ and therefore, satisfies the partial KKT conditions with respect to the variable s, given by,

$$\nabla_s f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) = 0. \quad (57)$$

2) Convergence for $\mathbf{u}$: We now focus on $\mathbf{u}$, we begin with the following claim.

Claim 1: Both $s^{(k_j)}$ and $s^{(k_j+1)}$ converge to $s^{(\infty)}$.

First, we show that $s^{(k_j+1)}$ converges. This follows directly from the equation $s^{(k_j+1)} = \frac{[\mathbf{w}^{(k_j)}]^H \mathbf{A}^H \mathbf{Y} \mathbf{u}^{(k_j)})}{\|\mathbf{A}\mathbf{w}^{(k_j)}\|_2^2}$ and $t^{(\infty)} = \lim_{j\rightarrow\infty} s^{(k_j+1)} \rightarrow \frac{[\mathbf{w}^{(\infty)}]^H \mathbf{A}^H \mathbf{Y} \mathbf{u}^{(\infty)}}{\|\mathbf{A}\mathbf{w}^{(\infty)}\|_2^2}$. We show that $t^{(\infty)} = s^{(\infty)}$. Consider the following,

$$f(s, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}) \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}). \quad (58)$$

Now, letting $j \rightarrow \infty$ and using the continuity of the functions $f(s, \mathbf{w}, \mathbf{u})$ and $\mathbf{u}^{(k_j)} \rightarrow \mathbf{u}^{(\infty)}$, $\mathbf{w}^{(k_j)} \rightarrow \mathbf{w}^{(\infty)}$, and $s^{(k_j+1)} \rightarrow t^{(\infty)}$, it follows that,

$$f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \geq f(t^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \quad (59)$$

This implies that $t^{(\infty)}$, in addition to $s^{(\infty)}$, is also the minimizer of $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$. Since $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$ has a unique minimizer from Lemma A.1, it follows that $t^{(\infty)} = s^{(\infty)}$. This implies $\lim_{j\rightarrow\infty} s^{(k_j+1)} = s^{(\infty)}$. We use this in the next step.

Block Minimization: To prove that $\mathbf{u}^{(\infty)}$ is a block-wise minimizer of $f(s, \mathbf{u}, \mathbf{w})$ given s and $\mathbf{w}$, we first write the following set of inequalities,

$$
\begin{aligned}
&f(s^{(k_j+1)}, \mathbf{w}^{(k_j)}, \mathbf{u}) && (60)\\
&\quad\geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}) && (61)\\
&\quad= \tilde{f}(\mathbf{w}^{(k_j)}; s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}) && (62)\\
&\quad\geq \tilde{f}(\mathbf{w}^{(k_j+1)}; s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}) && (63)\\
&\quad\geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j+1)}) && (64)\\
&\quad\geq f(s^{(k_{j+1})}, \mathbf{u}^{(k_{j+1})}, \mathbf{w}^{(k_{j+1})}). && (65)
\end{aligned}
$$

These inequalities are obtained using standard manipulations. It should be noted that (60) differs from (53) in the fact that the LHS of inequality uses a different subsequence of s. Using the fact $s^{(k_j+1)}$ converges to $s^{(\infty)}$ as $j \rightarrow \infty$ from Claim 1, and letting $j \rightarrow \infty$, and given that $s^{(k_{j+1})}, s^{(k_j+1)}$ both converge to $s^{(\infty)}$, it can be shown that,

$$f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}), \quad (66)$$

The inequality (66) implies that, $\mathbf{u}^{(\infty)}$ is a block-wise minimizer of $f(\cdot)$ and, therefore, satisfies the partial KKT conditions with respect to the variable $\mathbf{u}$, given by,

$$\nabla_{\mathbf{u}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\boldsymbol{\nu} \circ \mathbf{u}^{(\infty)} = \mathbf{0}, \quad (67)$$

where $\boldsymbol{\nu} \in \mathbb{R}^{N\times 1}$ is the dual-variable vector associated with the unit-modulus constraints on vector $\mathbf{u}$.

3) Convergence for $\mathbf{w}$: For the minimization with respect to $\mathbf{w}$, we start with the following claim.

Claim 2: Both $\mathbf{u}^{(k_j)}$ and $\mathbf{u}^{(k_j+1)}$ converge to $\mathbf{u}^{(\infty)}$.

To prove the claim, first, we show that $\mathbf{u}^{(k_j+1)}$ converges to $\mathbf{u}^{(\infty)}$ as $j \rightarrow \infty$ as it depends upon $s^{(k_j+1)}$, $\mathbf{w}^{(k_j)}$ and it has been shown that the objective function is uniquely minimized. Thus, we can write,

$$
\begin{aligned}
\mathbf{z}^{(\infty)} = \lim_{j\rightarrow\infty} \mathbf{u}^{(k_j+1)} &= \lim_{j\rightarrow\infty} e^{j\arg(s^{(k_j+1)}\mathbf{Y}^H \mathbf{A}\mathbf{w}^{(k_j)})} && (68)\\
&= e^{j\arg(s^{(\infty)}\mathbf{Y}^H \mathbf{A}\mathbf{w}^{(\infty)})}. && (69)
\end{aligned}
$$

We now show that $\mathbf{z}^{(\infty)} = \mathbf{u}^{(\infty)}$. For this, we present the following inequality that follows from the minimization w.r.t $\mathbf{u}$,

$$f(s^{(k_j+1)}, \mathbf{u}, \mathbf{w}^{(k_j)}) \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}). \quad (70)$$

Now, letting $j \rightarrow \infty$ and using the continuity of the functions $f(s, \mathbf{w}, \mathbf{u})$ and $\mathbf{s}^{(k_j+1)} \rightarrow s^{(\infty)}$ (Claim 1), $\mathbf{w}^{(k_j)} \rightarrow \mathbf{w}^{(\infty)}$ (choice of subsequence), and $\mathbf{u}^{(k_j+1)} \rightarrow \mathbf{z}^{(\infty)}$ (as shown in (69)), it follows that,

$$f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{z}^{(\infty)}, \mathbf{w}^{(\infty)}). \quad (71)$$

This implies that $\mathbf{z}^{(\infty)}$, in addition to $\mathbf{u}^{(\infty)}$, is also the minimizer of $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$. However, since $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$ has a unique minimizer for $\mathbf{u}$ given $\mathbf{w}^{(\infty)}, s^{(\infty)}$, it follows that $\mathbf{z}^{(\infty)} = \mathbf{u}^{(\infty)}$. This implies $\lim_{j\to\infty} \mathbf{u}^{(k_j+1)} = \mathbf{u}^{(\infty)}$. We use this in the next step.

Block Minimization: To prove block minimizer, the proof follows the earlier presented ones, except for the fact that the inequality is for the majorizer,

$$\begin{aligned}
&\tilde{f}(\mathbf{w}; s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}) \\
&\quad \geq \tilde{f}(\mathbf{w}^{(k_j+1)}; s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j)}) && (72)\\
&\quad \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j+1)}) && (73)\\
&\quad \geq f(s^{(k_{j+1})}, \mathbf{u}^{(k_{j+1})}, \mathbf{w}^{(k_{j+1})}) && (74)\\
&\quad = \tilde{f}(\mathbf{w}^{(k_{j+1})}; s^{(k_{j+1})}, \mathbf{u}^{(k_{j+1})}, \mathbf{w}^{(k_{j+1})}), && (75)
\end{aligned}$$

where (73) follows from the upper bound property (4) of the majorizing function function. Now, taking the limit $j \to \infty$, and by the convergence of $s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}$ we obtain,

$$\begin{aligned}
&\tilde{f}(\mathbf{w}; s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) && (76)\\
&\qquad \geq \tilde{f}(\mathbf{w}^{(\infty)}; s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). && (77)
\end{aligned}$$

The inequality (72) implies that, $\mathbf{w}^{(\infty)}$ is a minimizer with respect to the variable $\mathbf{w}$ for function $\tilde{f}$, as given by,

$$\begin{aligned}
&\nabla_{\mathbf{w}} \tilde{f}(\mathbf{w}^{(\infty)}; s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\boldsymbol{\lambda} \circ \mathbf{w}^{(\infty)} = \mathbf{0} && (78)\\
&\nabla_{\mathbf{w}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\boldsymbol{\lambda} \circ \mathbf{w}^{(\infty)} = \mathbf{0}, && (79)
\end{aligned}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^{M\times 1}$ is the dual-variable vector associated with the unit-modulus constraints. In (78) we have used the gradient consistency condition between the objective function and its majorizer, given in (6), to obtain (79). This arises from the requirement that the original objective function in (15) and the majorizing function in (17) should have the same gradient at $(s^{(\infty)}, \mathbf{w}^{(\infty)}, \mathbf{u}^{(\infty)})$.

Regularity Condition: The regularity condition, linear independence constraint qualification (LICQ) implies that the gradients of the active inequality constraints and the gradients of equality constraints are linearly independent at a feasible point (Proposition 3.1.1 in [34]). This condition is automatically satisfied at the solution as there are no constraints on variable s and the constraint sets involving $\mathbf{w}$, $\mathbf{u}$ satisfy the LICQ condition. For $\mathbf{w}^{(\infty)}$ and $\mathbf{u}$, the constraints $|w_i|^2 = 1$, for all $i \in [M]$ and $|u_j|^2 = 1$, for all $j \in [M]$ are decoupled among the entries w_i's and u_j's, respectively. All the constrained optimization variables can be combined in to a single vector as, $\mathbf{x} := [\mathbf{w}^T, \mathbf{u}^T]^T$. Now the gradient of constraint $|x_i|^2 - 1 = 0$ with respect to $\mathbf{x}$ is computed as, $\mathbf{g}_i(\mathbf{x}) = 2\mathbf{e}_i$, for all $i \in [M+N]$, where $\mathbf{e}_i \in \mathbb{R}^{(M+N)\times 1}$ is a vector with i-th entry being 1 and rest of the entries are zeros. Clearly all the gradient vectors are linearly independent. This implies that each point of the constraint set of problem $\mathcal{P}_6$ is regular.

Thus, combining the partial KKT-conditions along each block (57), (67) and (79), we get,

$$\begin{bmatrix} \nabla_s f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{u}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{w}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \end{bmatrix} + 2 \begin{bmatrix} 0 \\ \boldsymbol{\nu} \\ \boldsymbol{\lambda} \end{bmatrix} \circ \begin{bmatrix} 0 \\ \mathbf{u} \\ \mathbf{w} \end{bmatrix} = \mathbf{0}_{M+N+1}. \qquad (80)$$

Thus, we conclude that every limit point of the sequence generated by the algorithm is a KKT-point of the problem. ■

Appendix B
Proof of Theorem III.4 (C-CCD)

First, we prove the following lemma for the uniqueness of the minimizer for the sub-problem with respect to variable w_i.

Lemma B.1. *The objective function of problem $\mathcal{P}_9$,*

$$f(s^{(k+1)}, \mathbf{u}^{(k+1)}, w_1^{(k+1)}, \ldots, w_{i-1}^{(k+1)}, w_i, w_{i+1}^{(k)}),$$

is uniquely minimized along each w_i, $\forall i \in [M]$.

Proof. We first write problem $\mathcal{P}_9$ as,

$$\begin{aligned}
\mathcal{P}_{12}: \quad &\min_{w_i} \quad \left\| \mathbf{Y}\mathbf{u}^{(k+1)} - s^{(k+1)}\mathbf{h}_i^{(k)} - s^{(k+1)} w_i \tilde{\mathbf{a}}_i \right\|_2^2 \\
&\text{subject to } w_i \in \mathcal{A},
\end{aligned}$$

where $\mathbf{h}_i^{(k)} = \left(\sum_{j<i} w_j^{(k+1)} \tilde{\mathbf{a}}_j + \sum_{j>i} w_j^{(k)} \tilde{\mathbf{a}}_j\right)$. Letting $\mathbf{v}_i^{(k+1)} = \mathbf{Y}\mathbf{u}^{(k+1)} - s^{(k+1)}\mathbf{h}_i^{(k)}$ and $\tilde{c}_i^{(k)} = (s^{(k+1)})^* \tilde{\mathbf{a}}_i^H \mathbf{v}_i^{(k+1)}$, the cost function in (81) can be written as,

$$\begin{aligned}
\mathcal{P}_{13}: \quad &\min_{w_i} \quad ||\mathbf{v}^{(k+1)}||_2^2 + ||s^{(k+1)}\tilde{\mathbf{a}}_i||_2^2 - 2\,\mathrm{Re}\left(w_i^* \tilde{c}_i^{(k)}\right) \\
&\text{subject to } w_i \in \mathcal{A}.
\end{aligned}$$

This problem is similar to the one presented for $\mathbf{u}$ in (43) and the solution to this problem takes the form,

$$\begin{aligned}
w_i &= e^{j \arg\left(\tilde{c}_i^{(k)}\right)}, \quad \text{if} \quad \tilde{c}_i^{(k)} \neq 0 \\
&= 1 \quad \text{otherwise.} && (81)
\end{aligned}$$

Again, using the steps in the derivation of $\mathbf{u}$ in Lemma A.1, it can be shown that the minimizer of w_i in (81) is unique given the other components of $\mathbf{w}$, this is important since c_i depends on v_i which further depends on other values of $\mathbf{w}$. ■

Now we proceed to prove Theorem III.4.

Proof. Let us first consider the updates of Algorithm 2 from (20)-(25).

A. Descent of the Objective Function

From the update steps (20)-(25), it can be shown that,

$$\begin{aligned}
&f(s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) \\
&\quad \geq f(s^{(k+1)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}) && (82)\\
&\quad \vdots \\
&\quad \geq f(s^{(k+1)}, \mathbf{u}^{(k+1)}, \mathbf{w}^{(k+1)}). && (83)
\end{aligned}$$

B. Convergence to a Stationary Point

Let $\mathbf{x}^{(\infty)} = [s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}]^T$ be a limit point of the sequence $\{s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)}\}$. It is important to note that $s^{(\infty)} \in \mathbb{C}$, $\mathbf{u}^{(\infty)} \in \mathcal{A}^N$ and $\mathbf{w}^{(\infty)} \in \mathcal{A}^M$ because the respective constraint sets are closed. The inequalities from (82) to (83) imply that $\{f(s^{(k)}, \mathbf{u}^{(k)}, \mathbf{w}^{(k)})\}$ converges to $\{f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})\}$. It now remains to show that $\mathbf{x}^{(\infty)}$ is a KKT-point of the problem. To that end, assume a convergent subsequence $\{s^{(k_j)}, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}\} \rightarrow \{s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}\}$. For this, we exploit the following properties from the convergence proof of Theorem III.3 in Appendix A:

(a) $s^{(\infty)}$ and $\mathbf{u}^{(\infty)}$ are block-wise minimizers of s and $\mathbf{u}$, respectively.
(b) $s^{(k_j)}$ and $s^{(k_j+1)}$ converge to $s^{(\infty)}$.
(c) $\mathbf{u}^{(k_j)}$ and $\mathbf{u}^{(k_j+1)}$ converge to $\mathbf{u}^{(\infty)}$.

Algorithm 2 based on the C-CCD approach, solves $(M+2)$ subproblems at every iteration with the following update sequence, $s \rightarrow \mathbf{u} \rightarrow w_1 \rightarrow \cdots \rightarrow w_m$ whereas Algorithm 1 based on the AMM approach solves three subproblems at every iteration with the following update sequence $s \rightarrow \mathbf{u} \rightarrow \mathbf{w}$. A cursory look at the two update sequences immediately indicates the difference: all the entries of $\mathbf{w}$ are updated simultaneously in AMM while it is undertaken sequentially in C-CCD. However, both algorithms solve identical subproblems for s and $\mathbf{u}$ given in (9) and (10), respectively. Thus focussing on the appropriate convergent subsequence for $w_1, \ldots, w_M$ in C-CCD, the steps presented in Sections B1 and B2 of Appendix A can be repeated to show the block-wise convergence of s and $\mathbf{u}$ as well, respectively.

1) Convergence for w_1: From (82)-(83), we have,

$$\begin{aligned} & f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \\ & \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) && (84) \\ & \vdots \\ & \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1^{(k_j+1)}, w_2^{(k_j+1)}, \ldots, w_M^{(k_j+1)}) && (85) \\ & = f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, \mathbf{w}^{(k_j+1)}) && (86) \\ & \geq f(s^{(k_{j+1})}, \mathbf{u}^{(k_{j+1})}, \mathbf{w}^{(k_{j+1})}). && (87) \end{aligned}$$

Using the fact that $s^{(k_j+1)}$ and $\mathbf{u}^{(k_j+1)}$ converge to $s^{(\infty)}$ and $\mathbf{u}^{(\infty)}$, respectively, and letting $j \rightarrow \infty$, it can be shown that,

$$\begin{aligned} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1, w_2^{\infty}, \ldots, w_M^{\infty}) \\ \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}), \end{aligned} \tag{88}$$

where $\mathbf{w}^{(\infty)} = [w_1^{\infty}, w_2^{\infty}, \ldots, w_N^{\infty}]^T$. The inequality (88) implies that, $w_1^{(\infty)}$ is an element-wise minimizer of $f(\cdot)$ and, therefore, satisfies the partial KKT conditions with respect to the variable w_1, given by,

$$\nabla_{w_1} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_1 w_1^{(\infty)} = 0, \tag{89}$$

where $\lambda_1 \in \mathbb{C}$ is the dual-variable associated with the unit-modulus constraint over w_1. We now prove the convergence for w_2; this is necessitated.

2) Convergence for w_2: First, we need to prove the convergence of $w_1^{(k_j+1)}$. To that end, we show that $w_1^{(k_j+1)}$ converges to $w_1^{(\infty)}$ as $j \rightarrow \infty$. Let $\beta_1^{(\infty)} := \lim_{j\rightarrow\infty} w_1^{(k_j+1)} = e^{j \arg\left((s^{(\infty)})^* \tilde{\mathbf{a}}_1^H (\mathbf{Y}\mathbf{u}^{(\infty)} - s^{(\infty)} \mathbf{h}_1^{(\infty)})\right)}$, where $\mathbf{h}_1^{(k_j)}$ also converges to $\mathbf{h}_1^{(\infty)}$ because it is a function of $w_i^{(k_j)}$, which converges to $w_i^{(\infty)}$ for all $i \in [2, M]$. It follows from (84) that,

$$\begin{aligned} & f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \\ & \quad \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}). \end{aligned} \tag{90}$$

Letting $j \rightarrow \infty$, noting the appropriate limits and exploiting the continuous nature, it follows from the above equation that,

$$\begin{aligned} & f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1, w_2^{(\infty)}, \ldots, w_M^{(\infty)}) \\ & \quad \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \beta^{(\infty)}, w_2^{(\infty)}, \ldots, w_M^{(\infty)}). \end{aligned} \tag{91}$$

Thus, it follows from (91), that $\beta^{(\infty)}$ and $w_1^{(\infty)}$ both minimize $f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1, w_2^{(\infty)}, \ldots, w_M^{(\infty)})$. Since the function has a unique minimizer for w_i, it follows that $\beta^{(\infty)} = w_1^{(\infty)}$ and thus, $w_1^{(k_j+1)} \rightarrow w_1^{(\infty)}$.

To prove the minimization of the function by $w_2^{(\infty)}$, we consider the standard update for w_2,

$$\begin{aligned} & f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1^{k_j+1}, w_2, \ldots, w_M^{(k_j)}) \\ & \quad \geq f(s^{(k_j+1)}, \mathbf{u}^{(k_j+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}). \end{aligned} \tag{92}$$

Now letting $j \rightarrow \infty$, and using the convergence of $w_1^{(k_j+1)}$, it can be shown that,

$$\begin{aligned} f(s^{(\infty)}), \mathbf{u}^{(\infty)}, w_1^{\infty}, w_2, \ldots, w_M^{(\infty)}) \\ \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \end{aligned} \tag{93}$$

The inequality (93) implies that, $w_2^{(\infty)}$ is a block-wise minimizer of $f(\cdot)$ w.r.t w_2 and, therefore, satisfies the partial KKT conditions with respect to the variable w_2, given by,

$$\nabla_{w_2} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_2 w_2^{(\infty)} = 0, \tag{94}$$

where $\lambda_2 \in \mathbb{C}$ is the dual-variable associated with the unit-modulus constraint over w_2. From Lemma B.1 and similar to the steps for the convergence of $w_1^{(k_j+1)}$ to $w_1^{(\infty)}$, it can be shown that $w_2^{(k_j+1)}$ converges to $w_2^{(\infty)}$. A verbatim of the above proof for w_1 and w_2 can be sequentially carried out for the remaining blocks to prove that, $w_i^{(\infty)}$ is a block-wise minimizer of $f(\cdot)$ with respect to w_i given the remaining variables and therefore, satisfies the partial KKT conditions with respect to the variable w_i,

$$\nabla_{w_i} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_i w_i^{(\infty)} = 0, \tag{95}$$

and $w_i^{(k_j+1)}$ converges to $w_i^{(\infty)}$, $\forall i \in [M]$ by invoking uniqueness using Lemma B.1. It is already shown that the constraint of the problem is regular. Thus, combining the partial KKT-conditions along each block, (57), (67) and (95) for all $i \in [M]$,

$$\begin{bmatrix} \nabla_s f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{u}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{w}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \end{bmatrix} + 2 \begin{bmatrix} 0 \\ \boldsymbol{\nu} \\ \boldsymbol{\lambda} \end{bmatrix} \circ \begin{bmatrix} 0 \\ \mathbf{u} \\ \mathbf{w} \end{bmatrix} = \mathbf{0}_{M+N+1}. \tag{96}$$

Thus, we conclude that every limit point of the sequence generated by the algorithm is a KKT-point of the problem. ■

APPENDIX C
PROOF OF THEOREM III.4 (M-CCD)

Proof. Consider the following updates for the first iteration of the algorithm when it is initialized to $(s^{(0)}, \mathbf{u}^{(0)}, \mathbf{w}^{(0)})$,

$$s^{(1)} = \arg\min_{s\in\mathbb{C}} \; g(s, \mathbf{u}^{(0)}, w_1^{(0)}, w_2^{(0)}, \ldots, w_M^{(0)}) \tag{97}$$

$$\mathbf{u}^{(1)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} g(s^{(1)}, \mathbf{u}, w_1^{(0)}, w_2^{(0)}, \ldots, w_M^{(0)}) \tag{98}$$

$$w_1^{(1)} = \arg\min_{w_1\in\mathcal{A}} g(s(1), \mathbf{u}^{(1)}, w_1^{(1)}, w_2^{(0)}, \ldots, w_M^{(0)}) \tag{99}$$

$$s^{(2)} = \arg\min_{s\in\mathbb{C}} \; g(s, \mathbf{u}^{(1)}, w_1^{(1)}, w_2^{(0)}, \ldots, w_M^{(0)}) \tag{100}$$

$$\mathbf{u}^{(2)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} g(s^{(2)}, \mathbf{u}, w_1^{(1)}, w_2^{(0)}, \ldots, w_M^{(0)}) \tag{101}$$

$$w_2^{(1)} = \arg\min_{w_2\in\mathcal{A}} g(s^{(2)}, \mathbf{u}^{(2)}, w_1^{(1)}, w_2^{(1)}, w_3^{(0)}, \ldots, w_M^{(0)}) \tag{102}$$

$$\vdots$$

$$s^{(M)} = \arg\min_{s\in\mathbb{C}} \; g(s, \mathbf{u}^{(M-1)}, w_1^{(1)}, w_2^{(M)}, \ldots, w_{M-1}^{(0)}, w_M) \tag{103}$$

$$\mathbf{u}^{(M)} = \arg\min_{\mathbf{u}\in\mathcal{A}^N} g(s^{(M)}, \mathbf{u}, w_1^{(1)}, w_2^{(M)}, \ldots, w_{M-1}^{(1)}, w_M^{(0)}) \tag{104}$$

$$w_M^{(1)} = \arg\min_{w_M\in\mathcal{A}} g(s^{(M)}, \mathbf{u}^{(M)}, w_1^{(1)}, w_2^{(1)}, \ldots, w_{M-1}^{(1)}, w_M). \tag{105}$$

Thus, the iterates available after the first iteration are given as $(s^{(M)}, \mathbf{u}^M, \mathbf{w}^{(1)})$. Following, the above minimization procedure, the solution available after k-th outer iteration is $(s^{(kM)}, \mathbf{u}^{(kM)}, \mathbf{w}^{(k)})$. Now, it can be seen from (97)-(105) that continuing the algorithm with the iteration index decreases the objective function monotonically and the sequence of objective values $\{f(s^{(kM)}, \mathbf{u}^{(kM)}, \mathbf{w}^{(k)})\}$ converges.

Let $\mathbf{x}^{(\infty)} = (s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$ be a limit-point of the sequence $\{s^{(kM)}, \mathbf{u}^{(kM)}, \mathbf{w}^{(k)}\}$. Now, continuing the set of inequalities from (97)-(105) to further iterations, it can be seen that $\{f(s^{(kM)}, \mathbf{u}^{(kM)}, \mathbf{w}^{(k)})\}$ converges to $f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}\}$. Now, it remains to show that $\{s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}\}$ is a KKT-point of the problem. To that end, assume a convergent subsequence $\{s^{(k_jM)}, \mathbf{u}^{(k_jM)}, \mathbf{w}^{(k_j)}\} \to \{s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}\}$.

A. Convergence for s

We now show that the limit point is the block minimizer. For this, we begin with the variable s and from (97)-(105), we get the following set of inequalities,

$$\begin{aligned} & f(s, \mathbf{u}^{(k_jM)}, \mathbf{w}^{(k_j)}) \\ & \quad \geq f(s^{(k_jM+1)}, \mathbf{u}^{(k_jM)}, \mathbf{w}^{(k_j)}) && (106) \\ & \quad \geq f(s^{(k_jM+1)}, \mathbf{u}^{(k_jM)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) && (107) \\ & \quad \vdots \\ & \quad \geq f(s^{(k_jM+M)}, \mathbf{u}^{(k_jM+M)}, \mathbf{w}^{(k_j+1)}) && (108) \\ & \quad \geq f(s^{(k_{j+1}M)}, \mathbf{u}^{(k_{j+1}M)}, \mathbf{w}^{(k_{j+1})}). && (109) \end{aligned}$$

Now, letting $j \to \infty$, and using the continuity of the function $f(s, \mathbf{u}, \mathbf{w})$, we get,

$$f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \tag{110}$$

The inequality (110) implies that, $s^{(\infty)}$ is a block-wise minimzer of the function $f(\cdot)$ and therefore, satisfies the partial KKT conditions with respect to the variable s, given by,

$$\nabla_s f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) = \mathbf{0}. \tag{111}$$

Claim 3: Both $s^{(k_j)}$ and $s^{(k_jM+1)}$ converge to $s^{(\infty)}$.
Proof: We now show that $s^{(k_jM+1)}$ also converges to $s^{(\infty)}$ by invoking uniqueness of the minimizer of f with respect to s. To this end, define $t_1^{(\infty)}$ as,

$$\begin{aligned} t_1^{(\infty)} &= \lim_{j\to\infty} s^{(k_jM+1)} && (112) \\ &= \lim_{j\to\infty} \frac{\left[\mathbf{w}^{(k_j)}\right]^H \mathbf{A}^H \mathbf{Y} \mathbf{u}^{(k_jM)}}{\left\|\mathbf{A}\mathbf{w}^{(k_j)}\right\|_2^2} \to \frac{\left[\mathbf{w}^{(\infty)}\right]^H \mathbf{A}^H \mathbf{Y} \mathbf{u}^{(\infty)}}{\left\|\mathbf{A}\mathbf{w}^{(\infty)}\right\|_2^2}, && (113) \end{aligned}$$

where $\left\|\mathbf{A}\mathbf{w}^{(\infty)}\right\|_2^2 \neq 0$ as $\mathbf{A}\mathbf{w}$ can never be an all zero vector. Consider the following and let $j \to \infty$,

$$\begin{aligned} f(s, \mathbf{w}^{(k_j)}, \mathbf{u}^{(k_jM)}) &\geq f(s^{(k_jM+1)}, \mathbf{u}^{(k_j)}, \mathbf{w}^{(k_j)}) && (114) \\ f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) &\geq f(t_1^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). && (115) \end{aligned}$$

This implies that $t_1^{(\infty)}$, in addition to $s^{(\infty)}$, is also the minimizer of $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$. Since $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$ has a unique minimizer from Lemma A.1, it follows that $t_1^{(\infty)} = s^{(\infty)}$. This implies $\lim_{j\to\infty} s^{(k_jM+1)} = s^{(\infty)}$.

B. Convergence for $\mathbf{u}$

For the variable $\mathbf{u}$ we can write the following set of inequalities,

$$\begin{aligned} & f(s^{(k_jM+1)}, \mathbf{u}, \mathbf{w}^{(k_j)}) && (116) \\ & \quad \geq f(s^{(k_jM+1)}, \mathbf{u}^{(k_jM+1)}, \mathbf{w}^{(k_j)}) && (117) \\ & \quad \geq f(s^{(k_jM+1)}, \mathbf{u}^{(k_jM+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) && (118) \\ & \quad \vdots \\ & \quad \geq f(s^{(k_jM+M)}, \mathbf{u}^{(k_jM+M)}, \mathbf{w}^{(k_j+1)}) && (119) \\ & \quad \geq f(s^{(k_{j+1}M)}, \mathbf{u}^{(k_{j+1}M)}, \mathbf{w}^{(k_{j+1})}). && (120) \end{aligned}$$

Using the fact $s^{(k_jM+1)}$, $\mathbf{u}^{(k_jM)}$ and $\mathbf{w}^{(k_j)}$ converge to $s^{(\infty)}$, $\mathbf{u}^{(\infty)}$ and $\mathbf{w}^{(\infty)}$ respectively, as $j \to \infty$, and letting $j \to \infty$, we get,

$$f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \tag{121}$$

The inequality (121) implies that, $\mathbf{u}^{(\infty)}$ is a block-wise minimzer of $f(\cdot)$ with respect to $\mathbf{u}$. Therefore, it satisfies the partial KKT conditions with respect to the variable $\mathbf{u}$, as given by,

$$\nabla_{\mathbf{u}} f(\mathbf{w}^{(\infty)}; s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\boldsymbol{\nu} \circ \mathbf{u}^{(\infty)} = \mathbf{0}, \tag{122}$$

where $\boldsymbol{\nu} \in \mathbb{C}^{M\times 1}$ is the dual vector associated with the unit-modulus constraints.

Claim 4: Both $\mathbf{u}^{(k_j)}$ and $\mathbf{u}^{(k_j M+1)}$ converge to $\mathbf{u}^{(\infty)}$.

Proof: To prove the claim, first, we show that $\mathbf{u}^{(k_j M+1)}$ converges to $\mathbf{u}^{(\infty)}$ as $j \to \infty$. To this end, define $\mathbf{z}_1^{(\infty)}$,

$$\mathbf{z}_1^{(\infty)} = \lim_{j\to\infty} \mathbf{u}^{(k_j M+1)} \tag{123}$$

$$= \lim_{j\to\infty} e^{j\arg(s^{(k_j M+1)}\mathbf{Y}^H\mathbf{A}\mathbf{w}^{(k_j)})} \to e^{j\arg(s^{(\infty)}\mathbf{Y}^H\mathbf{A}\mathbf{w}^{(\infty)})}. \tag{124}$$

We now show that $\mathbf{z}_1^{(\infty)} = \mathbf{u}^{(\infty)}$. For this, we note the following inequality following from (117),

$$f(s^{(k_j M+1)}, \mathbf{u}, \mathbf{w}^{(k_j)}) \geq f(s^{(k_j M+1)}, \mathbf{u}^{(k_j M+1)}, \mathbf{w}^{(k_j)}). \tag{125}$$

Now, letting $j \to \infty$, using the continuity of the functions $f(s, \mathbf{u}, \mathbf{w})$, $s^{(k_j M+1)} \to s^{(\infty)}$ (Claim 3), $\mathbf{w}^{(k_j)} \to \mathbf{w}^{(\infty)}$ (choice of subsequence), and $\mathbf{u}^{(k_j M+1)} \to \mathbf{z}_1^{(\infty)}$ (as shown in (123)), it follows that,

$$f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{z}^{(\infty)}, \mathbf{w}^{(\infty)}). \tag{126}$$

This implies that $\mathbf{z}^{(\infty)}$, in addition to $\mathbf{u}^{(\infty)}$, is also the minimizer of $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$. However, since $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$ has a unique minimizer for $\mathbf{u}$ given $\mathbf{w}^{(\infty)}, s^{(\infty)}$, it follows that $\mathbf{z}^{(\infty)} = \mathbf{u}^{(\infty)}$. This implies $\lim_{j\to\infty} \mathbf{u}^{(k_j+1)} = \mathbf{u}^{(\infty)}$. We use this in the next step.

C. *Convergence for* $\mathbf{w}$

To prove the convergence for variable $\mathbf{w}$ we start with w_1.

1) Convergence for w_1*:* For w_1 we can write the descent of the objective function as,

$$f(s^{(k_j M+1)}, \mathbf{u}^{(k_j M+1)}, w_1, w_2^{(k_j)}, \ldots, w_M^{(k_j)})$$
$$\geq f(s^{(k_j M+1)}, \mathbf{u}^{(k_j M+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \tag{127}$$
$$\geq f(s^{(k_j M+2)}, \mathbf{u}^{(k_j M+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \tag{128}$$
$$\geq f(s^{(k_j M+2)}, \mathbf{u}^{(k_j M+2)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \tag{129}$$
$$\geq f(s^{(k_j M+2)}, \mathbf{u}^{(k_j M+2)}, w_1^{(k_j+1)}, w_2^{(k_j+1)}, \ldots, w_M^{(k_j)}) \tag{130}$$
$$\vdots$$
$$\geq f(s^{(k_j M+M)}, \mathbf{u}^{(k_j M+M)}, \mathbf{w}^{(k_j+1)}) \tag{131}$$
$$\geq f(s^{(k_{j+1} M)}, \mathbf{u}^{(k_{j+1} M)}, \mathbf{w}^{(k_{j+1})}). \tag{132}$$

Now we take the limit $j \to \infty$,

$$f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1, w_2^{(\infty)}, \ldots, w_M^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \tag{133}$$

The solution $w_1^{(\infty)}$ satisfies the following partial KKT conditions,

$$\nabla_{w_1} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_1 w_1^{(\infty)} = 0. \tag{134}$$

Claim 5: Both $w_1^{(k_j)}$ and $w_1^{(k_j+1)}$ converge to $w_1^{(\infty)}$.

Proof: We now prove the convergence of $w_1^{(k_j+1)}$. To that end, we show that $w_1^{(k_j+1)}$ converges to $w_1^{(\infty)}$ as $j \to \infty$. Let $\beta_1^{(\infty)} := \lim_{j\to\infty} w_1^{(k_j+1)} = e^{j\arg\left((s^{(\infty)})^* \tilde{\mathbf{a}}_1^H (\mathbf{Y}\mathbf{u}^{(\infty)} - s^{(\infty)}\mathbf{h}_1^{(\infty)})\right)}$, where $\mathbf{h}_1^{(k_j)}$ also converges to $\mathbf{h}_1^{(\infty)}$ because it is a function of $w_i^{(k_j)}$, which converges to $w_i^{(\infty)}$ for all $i \in [2, M]$. It also follows from (127) that,

$$f(s^{(k_j M+1)}, \mathbf{u}^{(k_j M+1)}, w_1, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \tag{135}$$
$$\geq f(s^{(k_j M+1)}, \mathbf{u}^{(k_j M+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}).$$

Letting $j \to \infty$, noting the appropriate limits and exploiting the continuous nature of $f(\cdot)$, it follows from the above equation that,

$$f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1, w_2^{(\infty)}, \ldots, w_M^{(\infty)})$$
$$\geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \beta_1^{(\infty)}, w_2^{(\infty)}, \ldots, w_M^{(\infty)}). \tag{136}$$

Thus, it follows from (136), that $\beta_1^{(\infty)}$ and $w_1^{(\infty)}$ both minimize $f(s^{(\infty)}, w_1, w_2^{(\infty)}, \ldots, w_M^{(\infty)}, \mathbf{u}^{(\infty)})$. Since the function has a unique minimizer for w_i for all $i \in [M]$, it follows that $\beta_1^{(\infty)} = w_1^{(\infty)}$ and thus $w_1^{(k_j+1)} \to w_1^{(\infty)}$.

Claim 6: $s^{(k_j M+2)}$ converges to $s^{(\infty)}$.

Proof: Consider the following update,

$$t_2^{(\infty)} = \lim_{j\to\infty} s^{(k_j M+2)} \to \frac{\left[\mathbf{w}^{(\infty)}\right]^H \mathbf{A}^H \mathbf{Y}\mathbf{u}^{(\infty)}}{\left\|\mathbf{A}\mathbf{w}^{(\infty)}\right\|_2^2}. \tag{137}$$

Consider the following and let $j \to \infty$,

$$f(s, \mathbf{u}^{(k_j M+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)})$$
$$\geq f(s^{(k_j+2)}, \mathbf{u}^{(k_j M+1)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}) \tag{138}$$
$$f(s, \mathbf{u}^{(\infty)}, w_1^{(\infty)}, w_2^{(\infty)}, \ldots, w_M^{(\infty)})$$
$$\geq f(t_2^{(\infty)}, \mathbf{u}^{(\infty)}, w_1^{(\infty)}, w_2^{(\infty)}, \ldots, w_M^{(\infty)}). \tag{139}$$

This implies that $t_2^{(\infty)}$, in addition to $s^{(\infty)}$, is also the minimizer of $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$. Since $f(s, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)})$ has a unique minimizer from Lemma A.1, it follows that $t_2^{(\infty)} = s^{(\infty)}$. This implies $\lim_{j\to\infty} s^{(k_j M+2)} = s^{(\infty)}$.

Claim 7: $\mathbf{u}^{(k_j M+2)}$ converges to $\mathbf{u}^{(\infty)}$.

Proof: Consider the following limit for $\mathbf{u}^{(k_j+2)}$,

$$\mathbf{z}_2^{(\infty)} = \lim_{j\to\infty} \mathbf{u}^{(k_j M+2)} \to e^{j\arg(s^{(\infty)}\mathbf{Y}^H\mathbf{A}\mathbf{w}^{(\infty)})}. \tag{140}$$

where we have used the fact that $w_1^{(k_j+1)}$, $w_i^{(k_j)}$, for all $i \in [2, M]$, $s^{(k_j M+2)}$ and $\mathbf{u}^{(k_j M+1)}$ converge to $w_1^{(\infty)}$, $w_i^{(\infty)}$, for all $i \in [2, M]$, $s^{(\infty)}$ and $\mathbf{u}^{(\infty)}$, respectively. Now, consider the following inequality,

$$f(s^{(k_j M+2)}, \mathbf{u}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)})$$
$$\geq f(s^{(k_j M+2)}, \mathbf{u}^{(k_j M+2)}, w_1^{(k_j+1)}, w_2^{(k_j)}, \ldots, w_M^{(k_j)}), \tag{141}$$

and letting $j \to \infty$, we obtain,

$$f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{z}_2^{(\infty)}, \mathbf{w}^{(\infty)}). \quad (142)$$

This implies that $\mathbf{z}_2^{(\infty)}$, in addition to $\mathbf{u}^{(\infty)}$, is also the minimizer of $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$. However, since $f(s^{(\infty)}, \mathbf{u}, \mathbf{w}^{(\infty)})$ has a unique minimizer for $\mathbf{u}$ given $\mathbf{w}^{(\infty)}, s^{(\infty)}$, it follows that $\mathbf{z}_2^{(\infty)} = \mathbf{u}^{(\infty)}$. This implies $\lim_{j\to\infty} \mathbf{u}^{(k_j M+2)} = \mathbf{u}^{(\infty)}$.

2) Convergence for w_2: For w_2 we can write the following inequalities,

$$\begin{aligned} & f(s^{(k_jM+2)}, \mathbf{u}^{(k_jM+2)}, w_1^{(k_j+1)}, w_2, \ldots, w_M^{(k_j)}) \\ & \geq f(s^{(k_jM+2)}, \mathbf{u}^{(k_jM+2)}, w_1^{(k_j+1)}, w_2^{(k_j+1)}, \\ & \qquad w_3^{(k_j)}, \ldots, w_M^{(k_j)}) \quad (143) \\ & \vdots \\ & \geq f(s^{(k_jM+M)}, \mathbf{u}^{(k_jM+M)}, \mathbf{w}^{(k_j+1)}) \quad (144) \\ & \geq f(s^{(k_{j+1}M), \mathbf{u}^{(k_{j+1}M)}}, \mathbf{w}^{(k_{j+1})}). \quad (145) \end{aligned}$$

Now we take the limit $j \to \infty$,

$$f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1^{(\infty)}, w_2, \ldots, w_M^{(\infty)}) \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}). \quad (146)$$

The solution $w_2^{(\infty)}$ satisfies the following partial KKT conditions,

$$\nabla_{w_1} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_2 w_2^{(\infty)} = 0. \quad (147)$$

Claim 8: Both $w_2^{(k_j)}$ and $w_2^{(k_j+1)}$ converge to $w_2^{(\infty)}$.

Proof: Let $\beta_2^{(\infty)} := \lim_{j\to\infty} w_2^{(k_j+1)} = e^{j\arg\left((s^{(\infty)})^* \tilde{\mathbf{a}}_2^H(\mathbf{Y}\mathbf{u}^{(\infty)} - s^{(\infty)}\mathbf{h}_2^{(\infty)}\right)}$, where $\mathbf{h}_2^{(k_j)}$ also converges to $\mathbf{h}_2^{(\infty)}$ because it is a function of $w_1^{(k_j+1)}$ and $w_i^{(k_j)}$, both of them converge to $w_1^{(\infty)}$ and $w_i^{(\infty)}$ for all $i \in [3, M]$, respectively. It follows from (143) that,

$$\begin{aligned} & f(s^{(k_jM+2)}, \mathbf{u}^{(k_jM+2)}, w_1^{(k_j+1)}, w_2, \ldots, w_M^{(k_j)}) \\ & \geq f(s^{(k_jM+2)}, \mathbf{u}^{(k_jM+2)}, w_1^{(k_j+1)}, w_2^{(k_j+1)}, \\ & \qquad w_3^{(k_j)}, \ldots, w_M^{(k_j)}), \quad (148) \end{aligned}$$

and letting the limit $j \to \infty$ we obtain,

$$\begin{aligned} & f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1^{(\infty)}, w_2, w_3^{(\infty)}, \ldots, w_M^{(\infty)}) \\ & \geq f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \beta_1^{(\infty)}, w_1^{(\infty)}, \beta_2^{(\infty)}, w_3^{(\infty)}, \ldots, w_M^{(\infty)}). \end{aligned} \quad (149)$$

Therefore, it follows from (149), that $\beta_2^{(\infty)}$ and $w_2^{(\infty)}$ both minimize $f(s^{(\infty)}, \mathbf{u}^{(\infty)}, w_1^{(\infty)}, w_2, w_3^{(\infty)}, \ldots, w_M^{(\infty)})$. Since the function has a unique minimizer for w_i for all $i \in [M]$, it follows that $\beta_2^{(\infty)} = w_2^{(\infty)}$ and thus $w_2^{(k_j+1)} \to w_2^{(\infty)}$.

Inductively continuing the above arguments for all the M blocks of variable $\mathbf{w}$, we establish that $\mathbf{w}^{(\infty)}$ is a block-wise minimizer of function f with respect to block $\mathbf{w}$. Therefore, the solution w.r.t $\mathbf{w}$ satisfies the partial KKT conditions with respect to the variable w_i, for all i in $[M]$ as,

$$\nabla_{w_i} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) + 2\lambda_i w_i^{(\infty)} = 0, \forall i \in [M]. \quad (150)$$

It is already shown in the proof of Theorem III.3 that the constraint of the problem is regular. Thus after combining the partial KKT-conditions along each block we get,

$$\begin{bmatrix} \nabla_s f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{u}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \\ \nabla_{\mathbf{w}} f(s^{(\infty)}, \mathbf{u}^{(\infty)}, \mathbf{w}^{(\infty)}) \end{bmatrix} + 2 \begin{bmatrix} 0 \\ \boldsymbol{\nu} \\ \boldsymbol{\lambda} \end{bmatrix} \circ \begin{bmatrix} 0 \\ \mathbf{u} \\ \mathbf{w} \end{bmatrix} = \mathbf{0}_{M+N+1}. \quad (151)$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ are M and N dimensional dual-variables corresponding to each w_i and u_i, respectively, at the solution $(s^{(\infty)}, \mathbf{w}^{(\infty)}, \mathbf{u}^{(\infty)})$. Thus, we conclude that every limit point of the sequence generated by the algorithm is a KKT-point of the problem. ■

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