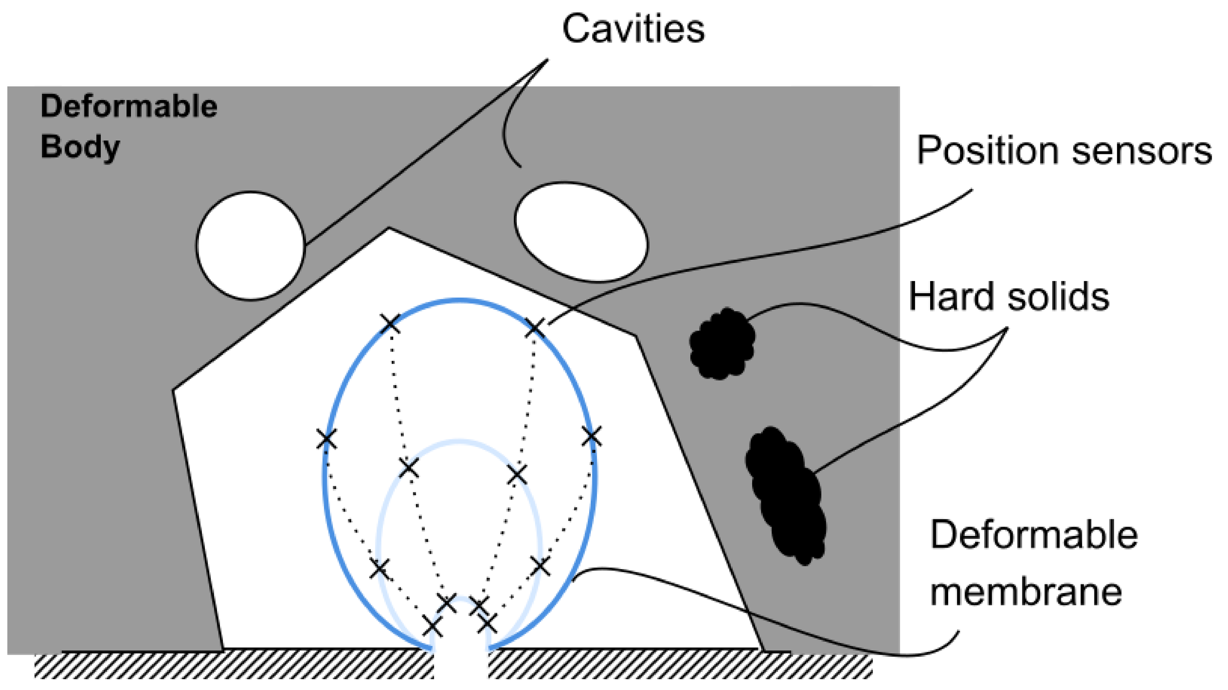
A DATA-DRIVEN COMPUTATIONAL FRAMEWORK TO PROVIDE DEFORMABLE SOLIDS WITH THE SENSE OF TOUCH

Diego Hurtado Supervisor: Lars A.A. Beex
University of Luxembourg
May 21, 2021

AIM AND SCOPE

A data-driven, probabilistic, computational framework will be developed that provides a deformable solid with the sense of 'touch', so that it can detect the shape and mechanical behaviour of its environment. The framework will rely on three modules:

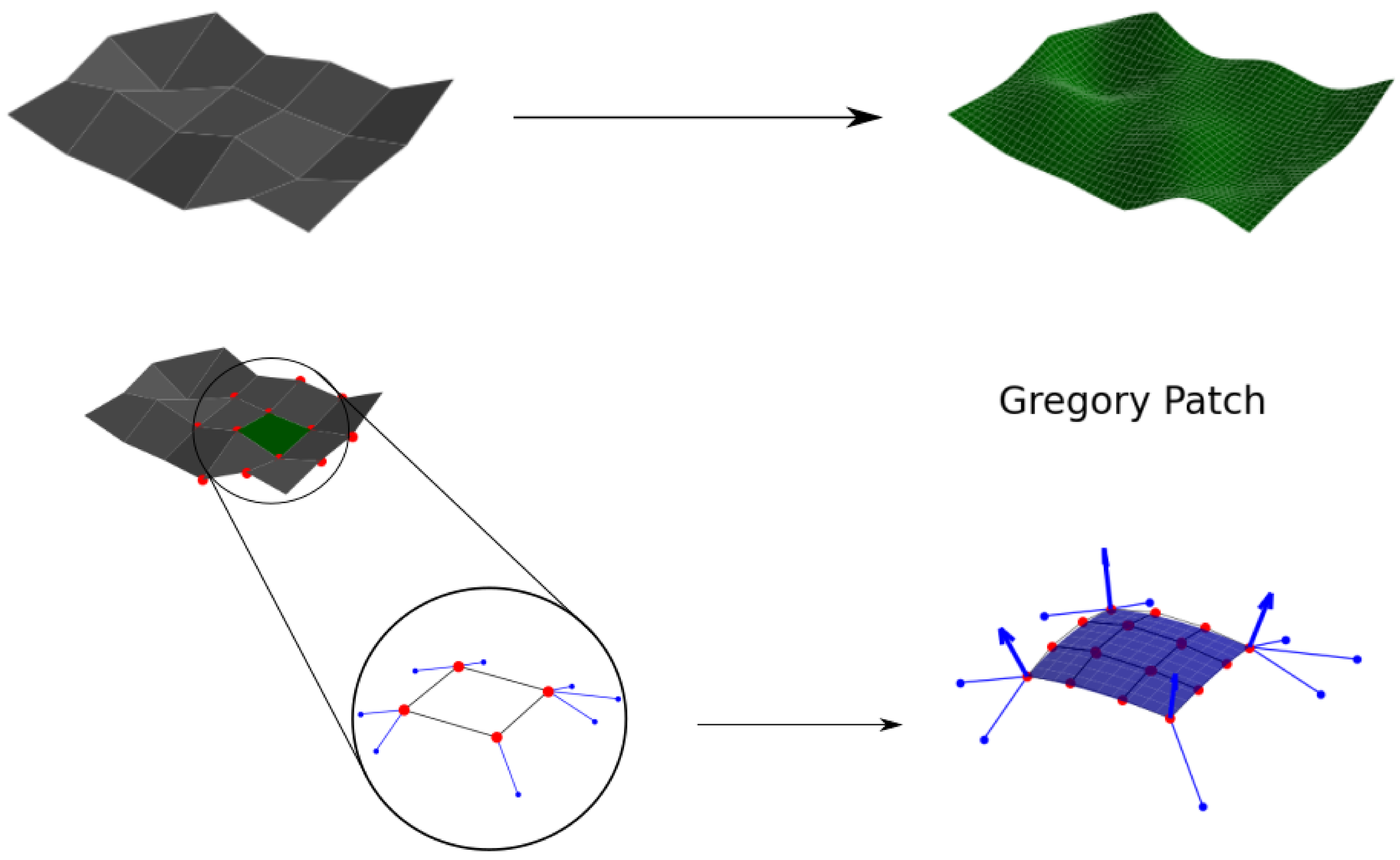
- a **mechanical model** to simulate the contact between the deformable solid and its environment,
- a **machine learning module** to rapidly emulate the mechanical simulations,
- a **probabilistic framework** to identify the shape and mechanical behaviour of the solid's environment;



MECHANICAL MODEL

The first goal is to develop a computational model able to describe the mechanical behaviour of a thin elastic deformable body in contact with its environment.

1. Contact Detection
   To accelerate the contact framework a contact detection is developed to ensure that contact interactions are only considered for nearby surface pairs.
2. Surface Smoothing
   Finite element surfaces are C^0-continuous, resulting in problems to enforce contact [1]. For this reason, the surfaces are smoothed using Gregory Patches [2].



Smoothing scheme using Gregory patches.

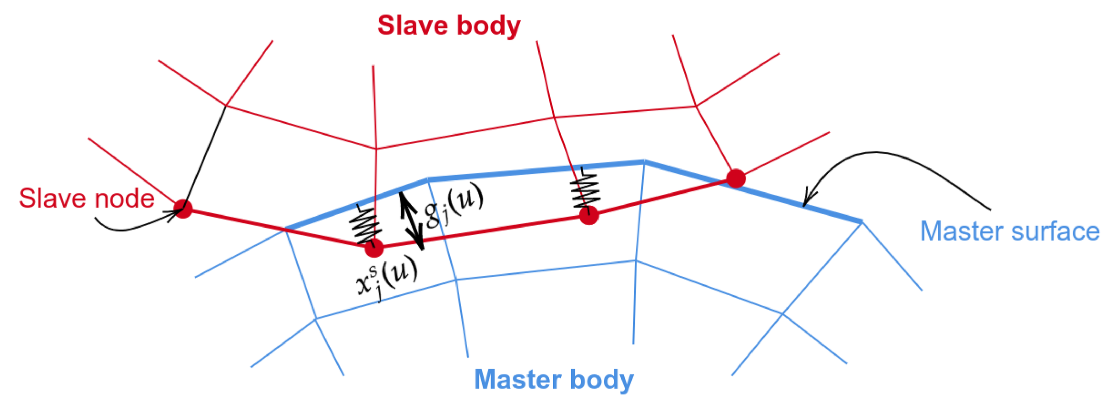
The position of a point on a Gregory patch, **x**, can be expressed in terms of the two surface parameters of the patch

$$\mathbf{x}\left(\xi^1,\xi^2\right)=\sum_{i=0}^{3}\sum_{j=0}^{3}B_i^3\left(\xi^1\right)\,B_j^3\left(\xi^2\right)\,\mathbf{x_{ij}}\left(\xi^1,\xi^2\right)\,,$$

where $\mathbf{x_{ij}}$ denotes the location of the control points of the patch (which in turn depend on the locations of the finite element nodes). Furthermore, B_k^n denotes the Bernstein polynomial:

$$B_k^n(x)=\binom{n}{k}x^k\left(1-x\right)^{n-k}\,,$$

3. Contact Mechanics
   Contact is enforced via the penalty method and formulated using a node-to-surface method. This means that nodes on the slave surface are prevented from penetrating the master surface.



Penalty scheme.

The contact potential is computed from the penetration g_j for each penetrated slave node j as:

$$\Pi_j^c(\mathbf{u})=\frac{1}{2}k_c\,g_j^2(\mathbf{u})\,,$$

where **u** denotes the displacements of the finite element nodes. From the potential the contact force and contact stiffness are obtained and added to the system. These are:

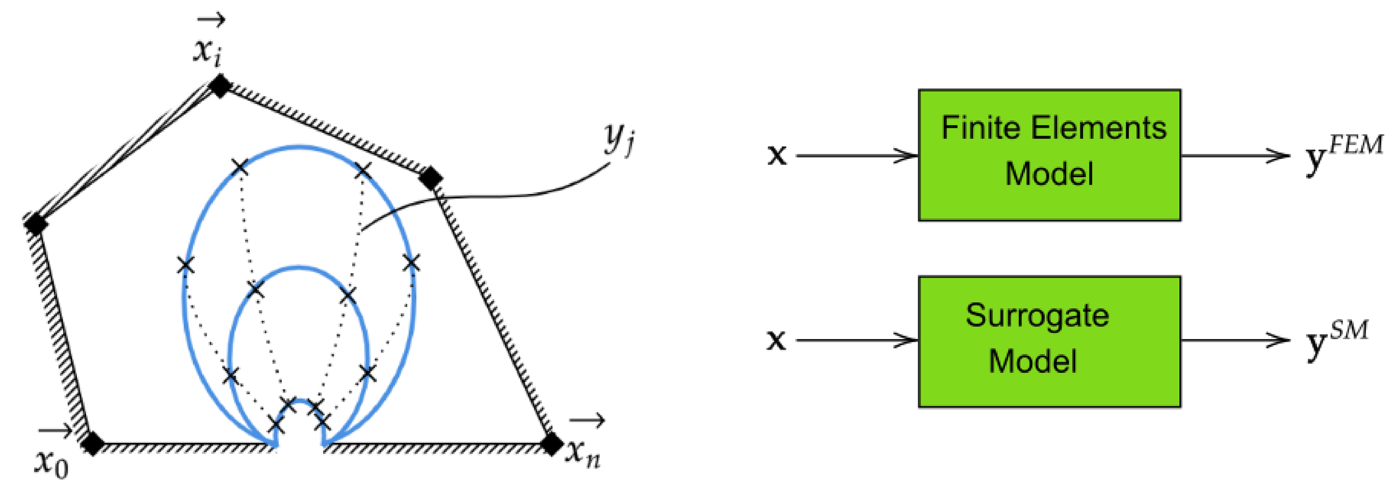
$$\mathbf{f}_j^c(\mathbf{u})=\frac{d\Pi_j^c(\mathbf{u})}{d\mathbf{u}}\quad,\mathbf{K}_j^c(\mathbf{u})=\frac{d^2\Pi_j^c(\mathbf{u})}{d\mathbf{u}^2}$$

In the case of Gregory patches this derivation can be obtained using the chain rule to project the contact force to the control points and then to the finite elements nodes as shown:

Forces projections. left: at control points. right: at finite elements' nodes.

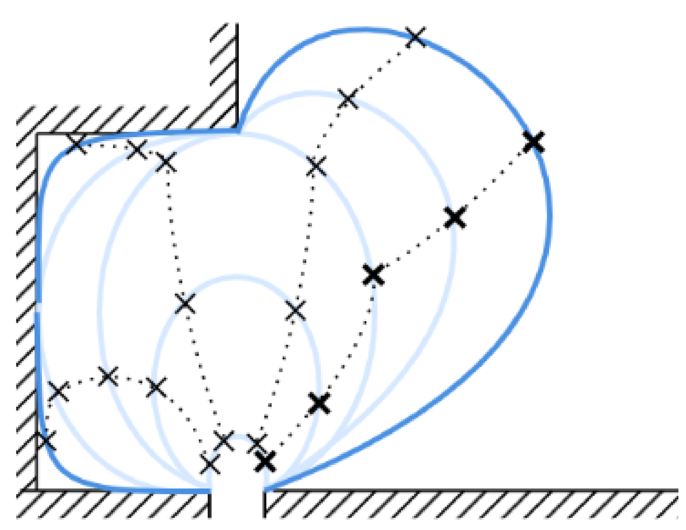
SURROGATE MODEL

Because mechanical simulations involving contact are time-consuming and because many simulations are required in the probabilistic identification framework, a surrogate model will later be formulated to rapidly emulate the relation between the simulation's input (i.e. the surface parameters of the membrane's environment, **x** in the figure below) and the simulation's output (i.e. the trajectory of the position sensors, **y** in the figure below). Although several surrogate models may be used, we tend to neural networks for now.



PROBABILISTIC IDENTIFICATION FRAMEWORK

In the final part of this project, the surrogate model will be exploited in a probabilistic framework using Bayes' theorem [3] to identify the shape of the membrane's environment.



ACKNOWLEDGEMENT

The Doctoral Training Unit **Data-driven computational modelling and applications** (DRIVEN) is funded by the Luxembourg National Research Fund under the PRIDE programme (PRIDE17/12252781). https://driven.uni.lu

REFERENCES

[1] Diego Hurtado. "A history-dependent contact framework with automatic time-stepping for quasi-static finite element simulations". In: (Sept. 2020).
[2] Mike Puso and Tod Laursen. "A 3D Contact Smoothing Method". In: (Jan. 2002).
[3] H. Rappel et al. "A Tutorial on Bayesian Inference to Identify Material Parameters in Solid Mechanics". In: *Archives of Computational Methods in Engineering* 27 (Jan. 2019). DOI: 10.1007/s11831-018-09311-x.