Sparse Precoding Design for Cloud-RANs Sum-Rate Maximization

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***Abstract*—This paper considers a sparse precoding design for sum-rate maximization in a cloud radio access network (Cloud-RAN). Constrained by the fronthaul link capacity and transmit power limit at each remote radio head (RRH), the sparse design amounts to determine the precoders at the RRHs as well as the set of serving RRHs for each mobile user. In this work, we first formulate the fronthaul link constraints as non-convex and discontinuous constraints with sparsity terms. These sparsity terms are then iteratively approximated into linear forms by means of reweighted ℓ1-norm with conjugate functions. Finally, to determine the beamforming vectors, the non-convex sum-rate maximization problem with linear constraints is transformed into an equivalent problem of iterative weighted mean-squared error minimization. Convergence of the proposed iterative algorithm is then proved and verified by the presented numerical results. In addition, numerical results demonstrate the superior performance by the proposed algorithm over a previously proposed one in literature.**

I. Introduction

Recently, cloud radio access network (Cloud-RAN) has been considered as a network architecture for the future wireless networks. Cloud-RAN can provide high capacity to cope with a tremendous growth in mobile data traffic [1], [2]. In this system, all baseband signal units (BBUs) are shifted into the cloud to generate a centralized processing pool. In addition, the signal processing functionality is also shifted to the cloud to take advantage of the cloud's computational power. The access points (radio remote heads (RRHs)) containing only the RF transmission modules become much simple and easy for deployment, especially in dense networks. Thanks to centralized joint scheduling and signal processing, the interference can be jointly managed. Hence, the network can achieve significant performance improvements [3] over traditional wireless network deployment. In additional, deployment of simple RRHs can also reduce both the network expenditure (CAPEX) and operating expense (OPEX) [4]. While showing lots of potential, Cloud-RAN also poses various technical challenges in its design and deployment. Specifically, one must efficiently utilize the processing resource in the cloud, the fronthaul capacity, and design suitable communication schemes for baseband signal processing.

Some of the technical problems in Cloud-RAN have been studied in recent works which are briefly summarized as follows. In [5], Fan *et al.* proposed an efficient clustering algorithm to reduce the computations at the cloud when the number of RRHs is very large. In [6], Liu and Lau studied the joint optimization of antenna selection, regularization, and power allocation to maximize the average weighted sum-rate. The works in [7], [8] tried to design the beamforming vectors for all RRHs to minimize the total power consumption of the network. In general, these problems are formulated into sparse beamforming design problem where the sparsity term is stated in the objective function. Therefore, the solutions can be readily obtained by directly applying compressive sensing techniques [9], [10].

In this paper, we consider the coordinated transmission problem for downlink sum-rate maximization in Cloud-RAN while considering practical constraints on the transmit power and fronthaul capacity at each RRH. Due to the limited fronthaul capacity, a sparsity term (ℓ0-norm) will be placed in the constraint. This constraint, which is nonconvex, makes the beamforming design non-trivial. Thus, a newly devised solution approach is needed to deal with the non-convex fronthaul constraints. To the best of our knowledge, the nearest work relating to ours was introduced in [11] where the reweighted ℓ1-norm technique in compressive sensing is applied. However, convergence of the proposed algorithm in [11] is not guaranteed nor proven. In this work, we take a different approach in dealing with the ℓ0-norm by employing the step function approximation in conjunction with a conjugate function. As a result, the non-convex ℓ0-norm constraint can be transformed into a linear constraint. In addition, the considered solution approach allows a tighter approximation to the ℓ0-norm after each iteration. While all the constraints can now be stated in linear forms, the sum-rate objective function is still non-convex. To determine the optimal beamforming design for this non-convex problem, we transform it into an equivalent problem of iterative minimization of weighted mean-squared error (IWMMSE). Convergence of the proposed iterative algorithm is then proved and verified by the presented numerical results. The numerical results also illustrate that our proposed algorithm is able to converge faster and noticeably outperform the existing one in [11].

The remaining of this paper is organized as follows. We describe the system model and problem formulation in Section II. In Section III-IV, we present the main contribution of this paper by proposing a low complex algorithm to solve our problem. Next, numerical results are presented in Section V followed by conclusion in Section VI. For notation, we use $\mathbf{X}^H$, $\mathsf{Tr}(\mathbf{X})$ and $\mathsf{rank}(\mathbf{X})$ to denote Hermitian transpose, trace, and rank of matrix $\mathbf{X}$, respectively. $\mathbf{1}_{x\times y}$, $\mathbf{0}_{x\times y}$ denote the matrix of ones, matrix of zeros whose dimensions are $x \times y$, respectively. $\mathsf{diag}(\mathbf{x})$ is the diagonal matrix constructed from the elements of vector $\mathbf{x}$.

II. System Model and Problem Formulation

A. System Model and Transmission Strategy

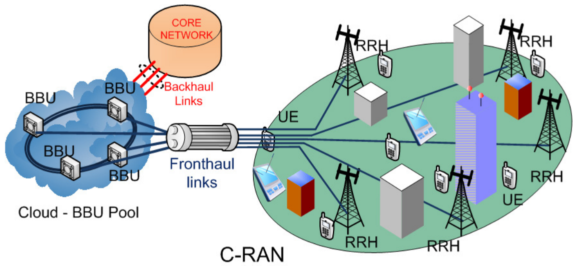

Fig. 1. The Cloud-RAN architecture.

As illustrated in Fig. 1, the general architecture of Cloud-RAN consists of three main components, namely (i) centralized processors or BBUs pool, (ii) the optical transport network (i.e., fronthaul links), and (iii) RRH access units located at remote sites. The processing center, which comprises a large pool of BBUs, is the focal point of this architecture. At the processing center, BBUs operate as virtual base stations to process baseband signals for users and optimize the network resource allocation tasks for the system. The transport network, connecting the central processing pool and the distributed RRHs, is usually deployed with optical fiber links. The RRHs, functioning as the RF front-ends, then transmit the RF signals to the users by converting the baseband signals received from BBUs. By conducting most signal processing functions in the cloud (i.e., by the BBU pool), the signal processing function at the RRHs can be much simplified.

In this work, we consider the coordinated downlink transmission in a Cloud-RAN network with K RRHs and M users. Let $\mathcal{K}$ and $\mathcal{U}$ be the sets of RRHs and users in the network, respectively. Suppose that RRH k is equipped with N_k antennas ($k \in \mathcal{K}$) and each user in $\mathcal{U}$ is equipped a single antenna. In this system, we assume that each user can be served by a group of assigned RRHs. Likewise, a RRH can serve a multiple users in the network. When a RRH assigned to serve a particular user, say RRH k and user i, RRH k receives user i's corresponding baseband signal from the cloud. RRH k then converts and transmits this baseband signal to the RF band with a pre-determined precoding vector. We assume that user i receives symbol sequence $x_i \in \mathbb{C}$ of unit power, which can be transmitted by any RRH in $\mathcal{K}$ upon receiving the processed baseband signals from the cloud. Denote $\mathbf{v}_i^k \in \mathbb{C}^{N_k \times 1}$ as the precoding vector at RRH k corresponding to the signal transmitted to user i, and $\mathbf{v}_i = [\mathbf{v}_i^{1T}, \ldots, \mathbf{v}_i^{KT}]^T$ as the precoding vector of all K RRHs for user i, i.e., $\mathbf{v}_i \in \mathbb{C}^{N \times 1}$ where $N = \sum_{k \in \mathcal{K}} N_k$. Then, the corresponding baseband signal y_i received at user i can be written as

$$y_i = \mathbf{h}_i^H \mathbf{v}_i x_i + \sum_{j \in \mathcal{U}/i} \mathbf{h}_i^H \mathbf{v}_j x_j + \eta_i, \tag{1}$$

where $\mathbf{h}_i = [\mathbf{h}_i^{1T}, \ldots, \mathbf{h}_i^{KT}]^T$, $\mathbf{h}_i^k \in \mathbb{C}^{N_k \times 1}$ denotes the channel coefficients between RRH k and user i, and η_i is the AWGN at user i with the power spectral density σ^2. The achieved SINR at user i is then given by

$$\Gamma_i = \frac{\left|\mathbf{h}_i^H \mathbf{v}_i\right|^2}{\sum\limits_{j \in \mathcal{U}/i} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2}, \tag{2}$$

where σ^2 is the noise power.

Denote p_i^k as the transmit power of the RF signal from RRH k to user i, which can be expressed as

$$p_i^k = \left\|\mathbf{v}_i^k\right\|^2, \tag{3}$$

where $\|\mathbf{a}\|$ indicates the square-norm of vector $\mathbf{a}$. Let $\mathbf{p}^k = [p_1^k, \ldots, p_M^k]^T$ be the transmit power vector at RRH k. Herein, $p_i^k > 0$ implies that RRH k is serving user i. The fronthaul link from the cloud to RRH k is then taken for carrying the baseband signal of user i. Therefore, the number of baseband signals carried by the fronthaul link to RRH k is indicated by the non-zero elements in the transmit power vector $\mathbf{p}^k$. Mathematically, the number of baseband signals carried by the fronthaul link connecting the cloud and RRH k can be written as

$$C_k = \|\mathbf{p}^k\|_\mathbf{0}, \tag{4}$$

where $\|\mathbf{a}\|_\mathbf{0}$ indicates the $\ell 0$-norm of vector $\mathbf{a}$.

B. Problem Formulation

In this section, we present the formulations for the Fronthaul-Constrained Sum-Rate Maximization (FC-SRM) problem. Our design aims to determine the set of RRHs serving each user and the corresponding precoding vectors for all RRHs that maximize the system sum-rate under the constraints on the fronthaul capacity and transmit power at each RRH. In particular, we assume that the transport network connecting the cloud and RRH k is capable of carrying at most $\bar{C}_k$ baseband signals for users. This can be transferred into the following fronthaul capacity constraint:

$$C_k = \|\mathbf{p}^k\|_\mathbf{0} \leq \bar{C}_k. \tag{5}$$

Depending on the setting of $\bar{C}_k$, only a few elements in $\mathbf{p}^k$ might be positive. Effectively, the fronthaul capacity constraints at the RRHs enforce a sparse structure on the beamforming vector for each user.[1] In addition, the transmit power of RRH k is imposed by its maximum power budget P_k, i.e.,

$$\sum_{i \in \mathcal{U}} p_i^k = \|\mathbf{p}^k\|_\mathbf{1} = \sum_{i \in \mathcal{U}} \left\|\mathbf{v}_i^k\right\|^2 \leq P_k, \quad \forall k \in \mathcal{K}, \tag{6}$$

where $\|\mathbf{a}\|_\mathbf{1}$ indicates the $\ell 1$-norm of vector $\mathbf{a}$. The FCPM problem then can be stated as

$$(\mathcal{P}_{\mathsf{FC-SRM}}) \quad \min_{\{\mathbf{v}_i\},\{\mathbf{p}^k\}} \quad \sum_{i \in \mathcal{U}} \log\left(1 + \Gamma_i\right) \tag{7}$$
$$\text{subject to} \quad \text{constraints (3), (6), and (5).}$$

It is noted that problem $(\mathcal{P}_{\mathsf{FC-SRM}})$ is non-convex and thus non-trivial to solve. In particular, constraints (5), which involve the $\ell 0$-norm, constitute non-convex and discrete sets.

[1] Hereafter, the fronthaul capacity constraint is also referred to as the sparsity constraint.

Moreover, the objective function is non-convex due to the presence of the mutual inter-user interference terms in the denominator of each user's SINR. In the following sections, we sequentially tackle the non-convexity of the constraint sets and the objective function in problem ($\mathcal{P}_{\mathsf{FC-SRM}}$). We subsequently propose an iterative algorithm to obtain at least a locally optimal solution to the problem.

III. Adaptive Reweighted $\ell 1$-norm Method

This section is to address the non-convexity of the $\ell 0$-norm constraints in problem ($\mathcal{P}_{\mathsf{FC-SRM}}$). In compressive sensing literature, the approximation of the non-convex $\ell 0$-norm into a convex weighted $\ell 1$-norm has been widely utilized [9], [10]. The main idea behind this method is the iterative update of the weights to the elements in the $\ell 1$-norm such that a weight is enlarged if its corresponding element is getting closer to zero. It has been shown that this method is well-suited for solving sparsity minimization problems. However, applying this method to deal with the sparsity constraints as in our problem requires more a rigorous design to guarantee the convergence of the iterative $\ell 0$-norm approximation procedure. Moreover, the obtained solution must satisfy the sparsity constraints at all time, which were not considered in [11]. In this work, we utilize the concave duality method in [12] to update the weights so that there is a calculated term to fill the gap between the sparsity constraint and the relaxing weighted $\ell 1$-norm term. Consequently, the solution will satisfy the constraint and is enhanced after each iteration. The convergence to the iterative $\ell 0$-norm approximation is also guaranteed.

Since the $\ell 0$-norm can be considered as the sum of step function of its elements, the concave duality method first utilizes the relaxation of step function as

$$\|\mathbf{p}^k\|_0 \approx \sum_{i\in\mathcal{M}} f_{\mathsf{apx}}^{(k,i)}(p_i^k), \tag{8}$$

where the approximation function $f_{\mathsf{apx}}(x)$ is defined as

$$f_{\mathsf{apx}}(x) = 1 - e^{-\Psi x}, \quad (\Psi \gg 1). \tag{9}$$

By this approximation, the problem ($\mathcal{P}_{\mathsf{FC-SRM}}$) can be rewritten as

$$(\mathcal{P}_{\mathsf{rlx}}) \qquad \max_{\{\mathbf{v}_i\}} \sum_{i\in\mathcal{U}} \log\left(1+\Gamma_i\right)$$
$$\text{subject to} \quad \text{constraints (3), (6)},$$
$$\sum_{i\in\mathcal{M}} f_{\mathsf{apx}}(p_i^k) \le \bar{C}_k, \quad \forall k\in\mathcal{K}. \tag{10}$$

Because the function $f_{\mathsf{apx}}(p_i^k)$ is concave with respect to p_i^k, the new relaxed constraint (10) is still non-convex. To circumvent this obstacle, we transform this constraint into a linear form, based on the duality property of the conjugate of a convex function [12]. In particular, the function $f_{\mathsf{apx}}^{(k,i)}(p_i^k)$ can be stated as

$$f_{\mathsf{apx}}(p_i^k) \triangleq \inf_{z_i^k}\left[z_i^k p_i^k - f_{\mathsf{apx}}^*(z_i^k)\right], \tag{11}$$

where $f_{\mathsf{apx}}^*(z)$ is the conjugate function of $f_{\mathsf{apx}}(w)$. Vice versa, $f_{\mathsf{apx}}^*(z)$ can be described as

$$f_{\mathsf{apx}}^*(z) \triangleq \inf_{w}\left[zw - f_{\mathsf{apx}}(w)\right] = \frac{z}{\Psi}\left[1-\log\left(\frac{z}{\Psi}\right)\right]-1, \tag{12}$$

which is obtained by determining the optimal w for a given z. Furthermore, after substituting (12) into (11), it is easy to find that the optimization problem in the right hand side of (11) achieves its minimum value at

$$\hat{z}_i^k = \nabla f_{\mathsf{apx}}(w)|_{w=p_i^k} = \Psi e^{-\Psi p_i^k}. \tag{13}$$

As a consequence of equation (11), constraint (5) at RRH k can be rewritten in a linear form

$$\sum_{i\in\mathcal{U}} \hat{z}_i^k p_i^k \le \bar{C}_k + \sum_{i\in\mathcal{U}} f_{\mathsf{apx}}^*(\hat{z}_i^k), \tag{14}$$

for a given $\{\hat{z}_i^k\}$. In summary, the problem ($\mathcal{P}_{\mathsf{rlx}}$) can be relaxed into

$$\max_{\{\mathbf{v}_i\}} \quad \sum_{i\in\mathcal{U}} \log\left(1+\frac{|\mathbf{h}_i^H\mathbf{v}_i|^2}{\sum_{j\in\mathcal{U}/i}|\mathbf{h}_i^H\mathbf{v}_j|^2+\sigma^2}\right) \tag{15}$$
$$\text{subject to} \quad \sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k\mathbf{v}_i\mathbf{v}_i^H\} \le P_k, \quad \forall k\in\mathcal{K} \tag{16}$$
$$\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k\mathbf{v}_i\mathbf{v}_i^H\} \le B_k, \quad \forall k\in\mathcal{K}, \tag{17}$$

where $\mathbf{A}^k = \mathsf{diag}\left[\mathbf{0}_{N_1\times 1}, ..., \mathbf{1}_{N_k\times 1}, ..., \mathbf{0}_{N_K\times 1}\right]$, $\mathbf{Z}_i = z_i^k\mathbf{A}^k$, and $B_k = \bar{C}_k + \sum_{i\in\mathcal{U}} f_{\mathsf{apx}}^*(z_i^k)$. Interestingly, problem (15)–(17) is now the well-known multiuser sum-rate maximization problem with multiple linear constraints. While problem (15)–(17) is still non-convex, a solution approach to the problem will be given in the following section.

IV. Proposed MMSE Precoder

In this section, we tackle the nonconvex sum-rate maximization problem ($\mathcal{P}_{\mathsf{rlx}}$) by relating it to a weighted sum-MSE minimization problem as mentioned in the following Proposition.

Proposition 1. *The sum-rate maximization problem* (15) *is equivalent to the following weighted sum-MSE minimization problem*

$$\min_{\{\mathbf{v}_i,\delta_i,\omega_i\}} \quad \sum_{i\in\mathcal{U}}\left(\omega_i\mathbb{E}\left[|x_i-\delta_i y_i|^2\right]-\log\omega_i\right) \tag{18}$$
$$\text{subject to} \quad \text{constraints (16) and (17)}$$

where ω_i and δ_i denote the MSE weight and the receive coefficient for user i, respectively.

Proof. The proof for this proposition is similar to that in [13] for the case of a single sum-power constraint. We omit the details for brevity. □

It is noted that the optimization in problem (18) is taken over the beamforming vector $\mathbf{v}_i$'s, the receive coefficients δ_i's as well as the weights ω_i's. While problem (18) is not *jointly* convex, it is convex over each set of variables $\{\mathbf{v}_i\}$, $\{\delta_i\}$, and

$\{\omega_i\}$, $i \in \mathcal{U}$. Thus, it is possible to solve problem (18) by alternately optimizing over one set of variables while keeping the other two fixed.

For given beamforming vectors $\{\mathbf{v}_i\}$'s, the receive coefficient $\delta_i^\star$ to minimize the MSE at user i is the Weiner filter, i.e., MMSE receiver

$$\begin{aligned}\delta_i^\star &= \arg\min_{\delta_i} \mathbb{E}\{ |x_i - \delta_i y_i|^2 \} \\ &= \left(\sum_{j\in\mathcal{U}} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2\right)^{-1} \mathbf{v}_i^H \mathbf{h}_i. \end{aligned} \tag{19}$$

Then, fixing the beamforming vectors $\mathbf{v}_i$'s and the receive coefficients δ_i's, the MSE weights $\omega_i^\star$'s can be determined by the unconstrained optimization

$$\begin{aligned}\omega_i^\star &= \arg\min_{\omega_i > 0} \ \omega_i e_i - \log \omega_i \\ &= e_i^{-1} = \frac{\sum_{j\in\mathcal{U}} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2}{\sum_{j\in\mathcal{U}/i} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2}, \end{aligned} \tag{20}$$

where $e_i = \mathbb{E}\left[\left|x_i - \delta_i y_i\right|^2\right]$. Finally, for given receive coefficients δ_i's and MSE weights ω_i's, the optimal transmit beamforming vectors $\mathbf{v}_i$'s can be obtained by the following optimization

$$\begin{aligned} \min_{\mathbf{v}_1,\ldots,\mathbf{v}_M} \quad & \sum_{i\in\mathcal{U}} \omega_i \mathbb{E}\left[\left|x_i - \delta_i\left(\sum_{j\in\mathcal{U}} \mathbf{h}_i^H \mathbf{v}_j x_j + \eta_i\right)\right|^2\right] \\ \text{subject to} \quad & \sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} \le P_k, \quad \forall k \in \mathcal{K}. \\ & \sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k \mathbf{v}_i \mathbf{v}_i^H\} \le B_k, \quad \forall k \in \mathcal{K}. \end{aligned} \tag{21}$$

Since the above problem is a convex quadratic program, it can be readily solved by the standard Lagrangian duality. The Lagrangian of problem (21) is given by

$$\begin{aligned} &\mathcal{L}(\{\mathbf{v}_i\}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \\ &= \sum_{i\in\mathcal{U}} \omega_i \mathbb{E}\left[\left|x_i - \delta_i\left(\sum_{j\in\mathcal{U}} \mathbf{h}_i^H \mathbf{v}_j x_j + \eta_i\right)\right|^2\right] \\ &+ \sum_{k\in\mathcal{K}} \beta_k \left(\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} - P_k\right) \\ &+ \sum_{k\in\mathcal{K}} \gamma_k \left(\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k \mathbf{v}_i \mathbf{v}_i^H\} - B_k\right) \\ &= \sum_{i\in\mathcal{U}} \mathbf{v}_i^H \left[\sum_{j\in\mathcal{U}} \omega_j |\delta_j|^2 \mathbf{h}_j \mathbf{h}_j^H + \sum_{k\in\mathcal{K}} \left(\beta_k \mathbf{A}^k + \gamma_k \mathbf{Z}_i^k\right)\right] \mathbf{v}_i \\ &+ \sum_{i\in\mathcal{U}} \omega_i \left(1 - \delta_i' \mathbf{v}_i^H \mathbf{h}_i - \delta_i \mathbf{h}_i^H \mathbf{v}_i\right) - \sum_{k\in\mathcal{K}} \left(\beta_k P_k + \gamma_k B_k\right), \end{aligned}$$

where $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]^T$ and $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]^T$ are the Lagrangian multipliers with respect to the constraints (16) and (17), respectively. For given $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, the beamforming vectors $\{\mathbf{v}_i^\star\}$ are given in closed-form as

$$\begin{aligned} \mathbf{v}_i^\star &= \arg\min_{\mathbf{v}_i} \mathcal{L}(\{\mathbf{v}_i\}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \\ &= \left[\sum_{j\in\mathcal{U}} \omega_j |\delta_j|^2 \mathbf{h}_j \mathbf{h}_j^H + \sum_{k\in\mathcal{K}} \left(\beta_k \mathbf{A}^k + \gamma_k \mathbf{Z}_i^k\right)\right]^{-1} \mathbf{h}_i \delta_i' \omega_i. \end{aligned} \tag{22}$$

Algorithm 1 ITERATIVE SPARSE BEAMFORMING DESIGN FOR SUM-RATE MAXIMIZATION

1: Initialize by setting $\mathbf{v}_i^{(0)} = \theta \mathbf{1}_{N\times 1}$ for all $i \in \mathcal{U}$, where θ (> 0) is small enough to satisfy the constraint (10), and setting $l = 0$.
2: **repeat**
3: Update $B_k^{(l)}$ by calculating $\left\{z_i^{k,(l)}\right\}$ as in (13) for all (k, i).
4: Update $\left\{\delta_i^{(l)}\right\}$ as in (19).
5: Update $\left\{\omega_i^{(l)}\right\}$ as in (20).
6: Update $\left\{\mathbf{v}_i^{(l)}\right\}$ as in as in (22) in conjunction with the sub-gradient update (24)-(25) until convergence.
7: Update $l = l + 1$.
8: **until** Convergence.

The dual function $g(\boldsymbol{\beta}, \boldsymbol{\gamma})$ is then defined as $g(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \inf_{\{\mathbf{v}_i\}} \mathcal{L}(\{\mathbf{v}_i\}, \boldsymbol{\beta}, \boldsymbol{\gamma})$, and the dual problem can be stated as

$$\begin{aligned} \max_{\boldsymbol{\beta}, \boldsymbol{\gamma}} \quad & g(\boldsymbol{\beta}, \boldsymbol{\gamma}) \\ \text{subject to} \quad & \beta_k, \gamma_k \ge 0, \forall k \in \mathcal{K}. \end{aligned} \tag{23}$$

Proposition 2. *The dual function $g(\boldsymbol{\beta}, \boldsymbol{\gamma})$ is a concave function and its sub-gradient at β_k is $\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} - P_k$ and at γ_k is $\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k \mathbf{v}_i \mathbf{v}_i^H\} - B_k$, where the beamforming vectors $\{\mathbf{v}_i\}$ is obtained from* (22)*, accordingly to $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.*

Proof. The dual function is a concave function by nature [14]. The choice of $\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} - P_k$ as the sub-gradient for β_k is justified by the fact that β_k is the Lagrangian multiplier associated with the constraint $\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} \le P_k$. Similarly, $\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k \mathbf{v}_i \mathbf{v}_i^H\} - B_k$ is the sub-gradient for γ_k. The detailed proof for this proposition is omitted for brevity. □

Due to proposition 2, the dual variables can be updated as

$$\beta_k^{(n+1)} = \beta_k^{(n)} + r_n \left(\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{A}^k \mathbf{v}_i \mathbf{v}_i^H\} - P_k\right), \tag{24}$$

$$\gamma_k^{(n+1)} = \gamma_k^{(n)} + t_n \left(\sum_{i\in\mathcal{U}} \mathrm{Tr}\{\mathbf{Z}_i^k \mathbf{v}_i \mathbf{v}_i^H\} - B_k\right), \tag{25}$$

equation where $\{\mathbf{v}_i^{(n)}\}$ are the beamforming vectors at time-n, r_n and t_n are suitable small step-sizes. If $r_n, t_n \overset{n\to\infty}{\longrightarrow} 0$, the above sub-gradient method is guaranteed to converge to the optimal solution of problem (23).

By iteratively updating $\{\mathbf{v}_i, \delta_i, \omega_i\}$, we obtain the MMSE beamforming the RRHs. Combined with the iterative approximation of the $\ell 0$ norm by updating z_i^k's, a sparse structure on the beamformers is then enforced. We summarize the steps in designing the sparse beamformers for Cloud-RAN sum-rate maximization in Algorithm 1. The properties of the converging solution in Algorithm 1 are stated in the following Proposition.

Proposition 3. *Algorithm 1 has the following properties*

1) Algorithm 1 converges to a locally optimal solution.

2) The solution achieved by Algorithm 1 at convergence satisfies all constraints of problem $(\mathcal{P}_{\text{rlx}})$.

Proof. The proof is given in Appendix A. □

V. Numerical Results

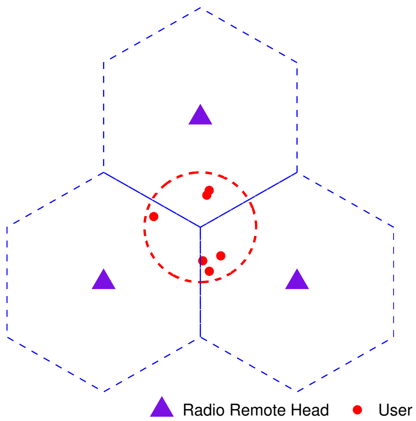

Fig. 2. Small simulation system model.

In this section, the effectiveness of our proposed design is validated through numerous simulations of the system model illustrated in Fig. 2. We consider a network with three equidistant RRHs whose inter-RRH distance is set at 500 m. The number of antennas at each RRH is set at 4 ($N_1 = N_2 = N_3 = 4$), unless stated otherwise. There are $M = 6$ users located inside a circle with radius of 125 m at the center of 3 RRHs. The channel gains are generated by considering both Rayleigh fading and path loss which is modelled as $L_u^k = 36.8\log_{10}(d_u^k) + 43.8 + 20\log_{10}(\frac{f_c}{5})$ where d_u^k is the distance from user u to RRH k; $f_c = 2.5$ GHz. The noise power is set equal to $\sigma^2 = 10^{-13}$ W. The parameter Ψ is set at 10^3.

Firstly, we examine the convergence of our proposed algorithm and compare it with the algorithm proposed in [11]. In particular, the evolution of the network sum-rate is obtained after each iteration by running the two algorithms and plotted in Fig. 3. In this simulation, we set $\bar{C}_1 = \bar{C}_2 = \bar{C}_3 = C_{RRH} = 2$ and $P_1 = P_2 = P_3 = P_{\text{max}} = 10dB$. It can be observed that our proposed algorithm can converge faster than the algorithm in [11]. Specifically, the sum-rate achieved by running the proposed algorithm is saturated after about 20 iterations, while it requires over 40 iterations for the algorithm in [11] to converge. Moreover, the figure also presents a noticeable improvement in the achievable sum-rate by the our proposed algorithm over the one given in [11].

In Fig. 4, we illustrate the network sum-rate of all users achieved by these two algorithms versus the limited transmit power of each RRH (P_{max}) for $C_{RRH} = 2$ and $C_{RRH} = 4$. As observed from the figure, the network sum-rate is enhanced with increasing maximum power budget of the RRHs. While the figure shows a relatively similar performance between the two algorithms at the low regime of transmit power, our proposed algorithm does outperform that in [11] at the high regime.

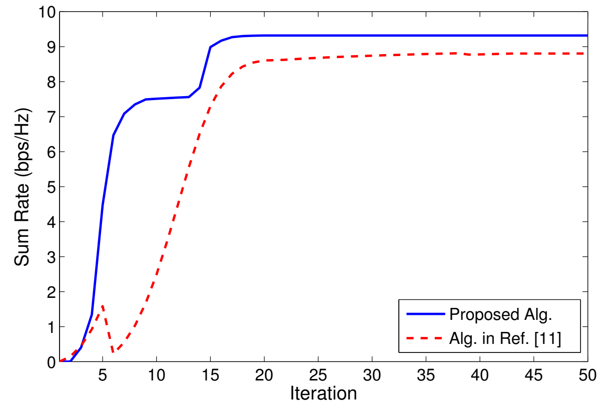

Fig. 3. The evolution of the network sum-rate achieved by our proposed algorithm and the algorithm in [11] after each iteration.

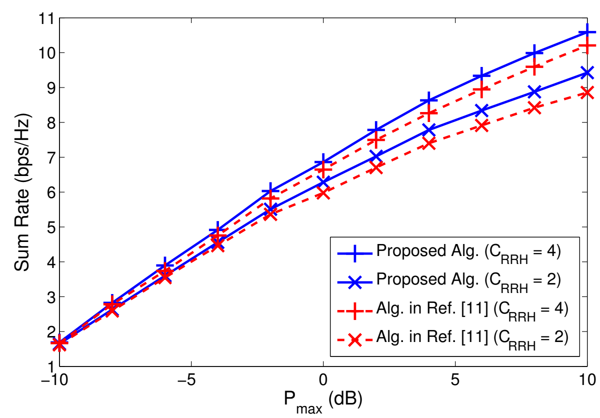

Fig. 4. The variation of sum-rate which achieved by running two algorithms versus limited transmit power of each RRH.

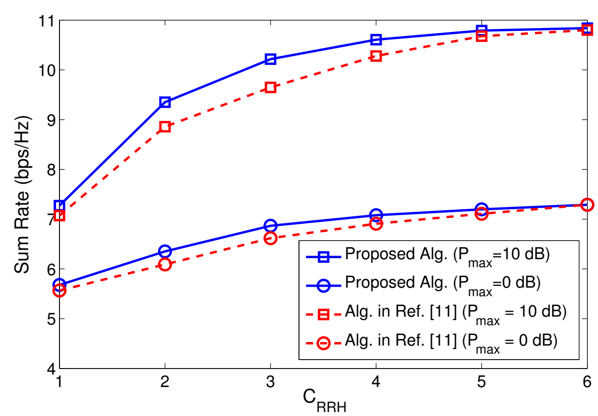

Fig. 5. The variation of sum-rate which achieved by running two algorithms versus limited fronthaul capacity for each RRH.

Fig. 5 presents the variations of the system sum-rate achieved at all users versus the capacity of fronthaul link from cloud to each RRHs (C_{RRH}). As expected, the network

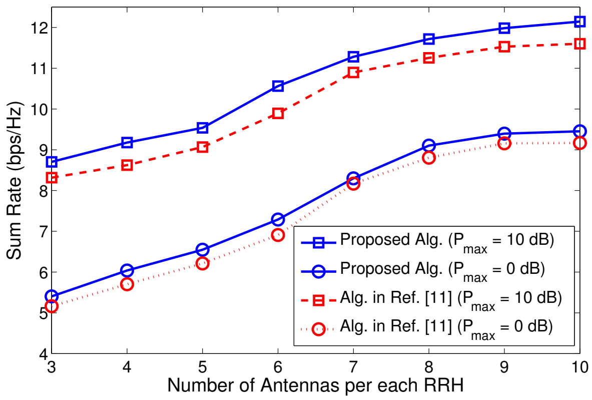

Fig. 6. The variation of sum-rate which achieved by running two algorithms versus the number of antennas equipped at each RRH.

sum-rate can be improved if the transport network between cloud and RRHs is provided with higher capacity fronthaul link. However, in the high regime of fronthaul capacity, the achievable transmission rate becomes saturated. This is probably because the transmission over the air interface is approaching its capacity limit. Thus, providing more capacity to the fronthaul links is no longer needed in this case.

Fig. 6 demonstrates the variations of network sum-rates versus the number of antennas equipped at each RRH. As can be observed, the increase of number of equipped antennas results in increasing the total rate of all users in the network. This is because a larger number of antennas equipped at RRHs gain more degree-of-freedom for the network; hence, the interference can be managed better and higher rate can be achieved. Once again, the proposed algorithm always outperforms the algorithm in [11]. These confirm the effectiveness of the proposed framework in maximizing the network sum-rate.

VI. Conclusion

In this work, we have considered the problem of sum-rate maximization in a Cloud-RAN network with constrained fronthaul link capacity. We have presented an efficient algorithm to obtain the sparse beamformers at each RRH in Cloud-RAN while maintaining the fronthaul link requirement and the transmit power constraint at the RRH. Numerical results with various network parameter settings have illustrated the efficacy of our proposed algorithms in improving the Cloud-RAN network sum-rate.

Appendix A
Proof of Proposition 3

Proof of the first statement: Let Ω_l be the out-come solution of problem (15) in iteration l. We will prove that Ω_l increases over each iteration; hence, our proposed algorithm converges. Denote $\mathcal{F}_l$ as the feasible set of $\{\mathbf{v}_i\}$ satisfying the constraints (16) and (17) corresponding to iteration l. It needs to be noted that this feasible set depends on the value of $\left\{z_i^{k,(l)}\right\}$. Because $\left\{z_i^{k,(l+1)}\right\}$ is calculated as in (13) based on the value of $\left\{\mathbf{v}_i^{(l)}\right\}$, we must have

$$\left\{\mathbf{v}_i^{(l)}\right\} \subset \mathcal{F}_{l+1}. \quad (26)$$

Moreover, similar to the spirit as in [13], the alternating minimization process in Step 4–6 of our proposed algorithm results in a monotonic improvement of the objective function of the problem (18). Then, due to the Proposition 1, we must have

$$\Omega_l \leq \Omega_{l+1}, \quad \forall l > 0. \quad (27)$$

Due to the monotonic convergence of the proposed algorithm, the resultant solution must be a locally optimal solution.

Proof of the second statement: Denote $\mathcal{F}$ as the feasible set of problems $(\mathcal{P}_{\mathsf{rlx}})$. Because $f_{\mathsf{apx}}(p_i^k) = \inf_{z_i^k}\left[z_i^k p_i^k - f^*_{\mathsf{apx}}(z_i^k)\right]$, then if

$$\sum_{i\in\mathcal{U}} z_i^{k,(l)} p_i^k \leq \bar{C}_k + \sum_{i\in\mathcal{U}} f^*_{\mathsf{apx}}(z_i^{k,(l)}), \quad (28)$$

we always have

$$f_{\mathsf{apx}}(p_i^k) \leq \bar{C}_k, \quad (29)$$

for any value of $\left\{z_u^{k,(l)}\right\}$. Therefore, we have $\mathcal{F}_l \subseteq \mathcal{F}$ for iteration l, which means the out-come solution of problem (15) in any iteration l satisfies all constraints of problem $(\mathcal{P}_{\mathsf{rlx}})$. Hence, Algorithm 1 returns the solution that satisfies all constraints of problem $(\mathcal{P}_{\mathsf{rlx}})$.

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