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Heuristic methods for minimal controller location set problem in transportation networks

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Abstract

To be used efficiently, transportation networks require control strategies to optimize flow distribution. However, the location and type of controllers employed in a transportation network directly impact the level of performance reachable by traffic control policies. In order to guarantee that the highest level of performance is reachable, fully exploiting the available network capacity and reducing negative externalities, we explore methods to identify relevant controller locations in transportation networks while minimizing the number of controllers used. Previous work provided an exact approach to identify controller locations on complex networks. However, said approach exhibits complications when applied to transportation networks containing bi-directional links. We therefore propose simple heuristic algorithms relying only on topological information to solve this problem, while also avoiding heavy computation, thus being able to determine a solution to the minimal controller location set problem on large networks. Based on the existing framework, we aim to provide an experimental setup, with diverse network sizes and configurations to analyze the performances of different heuristic methods, in order to develop an efficient algorithm.

Keywords: Transportation network; Controllability; Heuristic; Controller locations

1. Introduction

Transportation networks require control strategies to maximize their efficiency and mitigate negative externalities such as delays, productivity loss or increased pollutant emissions. In order to use network infrastructure at its full potential, advanced control strategies have been developed (Hunt et al. (1981); Lowrie (1990); de Oliveira and Camponogara (2010); Hoogendoorn et al. (2015)), coordinating multiple intersections with the goal of reducing congestion and improving overall networks efficiency. Some strategies focus on controlling a specific portion of transportation networks, like a section of an highway (Papageorgiou et al. (1997)), whereas alternative approaches explored the capacity to control whole transportation networks using toll gantries and pricing levels (Verhoef (2002)), or to employ

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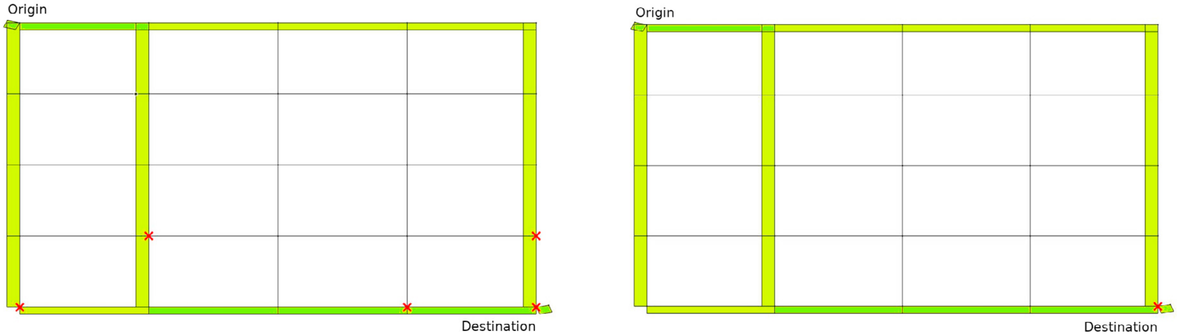
said controllers in form of cordon pricing, to separate a network in multiple areas (Zhang and Yang (2004)). In order to enhance the performance of the underlying transportation networks, theses control strategies employ different types of controller technology to both reduce congestion, e.g. by means of appropriately distributing capacity at intersections, and influence road users' choices toward alternative routes, reducing the effect of the so-called Price of Anarchy (Rose et al. (2016)) . However, the actual capability of a set of controllers to efficiently redirect flows has been shown to strongly depend on the locations and the number of controllers employed, hence the maximal performance reachable by control policies is inherently bound by these design choices. The problem of identifying controller locations on transportation networks has received little attention in the literature, however the domain of observability, which has been extensively studied in the literature, entails several key similarities with our problem. Specifically, the traffic flow inference problem consists in locating a set of counting sensors on a transportation network such that all the flows on the said network are either directly or indirectly known, which can be seen as a problem dual in nature to that of placing controllers. In this domain, multiple methods to locate sensors on networks have been developed, some based on linear algebraic properties, as in (Hu et al. (2009) and Ng (2012)), whereas other approaches focused on providing topology based methods (He (2013)), with the main advantage of requiring less prior information.

In a previous work (Rinaldi (2018)), a direct relationship between network-wide performance of control policies and both locations and amount of installed controllers was established. The work postulates that, essentially, these two design parameters should be chosen such that the underlying network is fully controllable, a necessary condition to ensure reachability of globally optimal performances. In said work a framework was also introduced adapting control theory principles to transportation networks, providing a representation of the impact of two types of controller on a transportation network: pricing controllers, bearing a direct impact on the location they are placed on, by adding a cost to it; traffic lights, which manage conflicting flows at a given intersection by distributing total available capacity over time, thus inducing indirect costs (in the form of delays) for road users. In this work we are going to focus on the former controller type, due to its direct impact on the position they are placed on and their capability to directly affect road users' behavior by increasing their perceived costs. The framework above also produced an adaptation of the controllability Gramian matrix, introduced by Kalman et al. (1963), to the instance of transportation networks. This allows us to compute the level of controllability yielded by a set of controllers placed on a given network, thus providing an approach to assess the actual capacity of a controller set to fully control the underlying transportation network. Therefore we aim to develop a method capable to determine a collection of controllers that is able to fully control the transportation network while minimizing the number of controllers employed.

2. Research contribution

In previous research, an exact method, provided by the work of Yuan et al. (2013), was adapted to the specific instance of transportation networks. However, the work of Rinaldi and Viti (2020) demonstrated that the addition of bi-directed links introduces self-dependencies on nodes during the computation of the algorithm, which causes violation of the algebraic assumptions behind the method. Specifically, route-induced dependencies (e.g. indirect relationships between links arising not due to adjacency but due to competing route flows pertaining to a given origin/destination pair) lead, in most networks, to a collapse of the eigenvalue/eigenvector information content, thereby misleading the chosen approach towards solutions with no practical significance. This is exemplified in Fig. 1, based on a simple twenty-five nodes grid-like network, containing only one origin destination couple connected by three routes. For this instance, Yuan's method is able to identify a set of locations for pricing controllers that guarantees full controllability for a given network. (Fig.1a) However, the method exhibits difficulties in producing sensible solutions when dealing with complex networks featuring bi-directed links. (Fig.1b) This is because the introduction of bi-directionality significantly alters the pricing controller set found by the exact method, yielding a controller set clearly incapable to effectively influence the whole network (the a-posteriori measure of controllability level confirming that the network is indeed not fully controllable). In order to address this computational issue, in this work we aim to develop heuristic approaches that would help identifying a satisfying solution to the minimal controller location set problem leveraging topological information, while avoiding the algebraic issues raised by the aforementioned exact methodology.



(a) All links are mono-directional (from NW to SE). The network is fully controlled. (b) All links are bi-directional. Only 1 of 25 nodes is effectively controlled.

Fig. 1: Yuan's method applied on a 25 nodes graph. Controller locations are marked in red. Route set is represented in green.

3. Methodology

In this work, a given transportation network is represented by a directed graph $G(N, L)$ comprising of a set N of nodes and a set $L : l \in L = (i, j),\ i, j \in N$ of directed links connecting them. In order to represent user behaviour in terms of route choice, specific nodes are included to represent origin and destination centroids, and for each resulting O/D couple a given route set is enumerated through a K-shortest path algorithm (Yen (1971)). Every node $i \in N$, except O/D centroids, is considered as a potential location for a pricing controller. To solve the minimal controller location set problem we propose four simple heuristics. All following methods require to compute the level of controllability at each step to assess if the current controller set is reaching full controllability or not, to decide to stop or to continue if more controllers are needed. Therefore, by definition, the heuristics we propose are a class of greedy algorithms adapted to the specific instance of transportation networks.

- The first approach attempts to improve the level of controllability reached at each step while considering individual routes. For each route forming the previously defined route set, one controller is placed on the node that results in the highest increase in the level of controllability. This approach is therefore based on myopic decisions. (Alg.1)
- The second method is based on assigning weights to the network's topology. Specifically, each node is weighted by its degree (total amount of inbound and outbound connections). The method successively selects controller locations with the highest weight, until full controllability is reached. The intuition behind this approach is that nodes bearing a higher degree of connectivity are likely to represent key pivotal points in the network. (Alg.2)
- The third method employs nodes weights determined by the respective topological distance of any given candidate node to the closest origin, following existing routes. Controllers are placed individually on the node bearing the smallest weight. This approach is inspired by screen line methods and relates to the simple idea that placing controllers in the vicinity of origin centroids allows to intercept flows from origins, thus yielding an high impact on the network's performance. (Alg.3)
- The last method is based on the intuition that locations where routes pertaining to a same origin-destination pair are either splitting or merging are critical in order to be able to control movement of road users. Following this intuition, we place controllers on every splitting and merging node until reaching full controllability. If the method is unable to reach full controllability by itself, for example due to the specific network's shape, we then complement the set of controllers following the first method, until the desired condition is met. (Alg.4)

Algorithm 1 Best controller per route

```
for each route in the route set do
    get node i with max(level of controllability)
    place controller on node i
    if current level of controllability = full controllability then
        stop
    end if
end for
```

Algorithm 2 Node degree weighted

```
for each node do
    node's weight = node degree
end for
while current level of controllability < full controllability do
    place controller on node with max(weight)
end while
```

Algorithm 3 Origin distance weighted

```
for each node do
    node's weight = topological distance to closest origin
end for
while current level of controllability < full controllability do
    place controller on node with min(weight)
end while
```

Algorithm 4 Splitting or merging routes

```
for each origin-destination pair do
    for each i node where routes are merging or splitting do
        if current level of controllability < full controllability then
            place controller on node i
        end if
    end for
end for
```

4. Experimental setup

Following the previous work of Rinaldi and Viti (2020), we employ a graph generator introduced in Mireles de Villafranca et al. (2019) to create arbitrarily randomized networks bearing sufficient resemblance to urban transportation networks. This algorithm starts from a square grid network of a chosen size, thereafter randomly perturbing node locations and generating **bi-directional** links between sufficiently close node couples, thus reshaping the initial network. (Fig.2). The graph is subsequently divided in concentric zones (as exemplified by the red circles in Fig.2), for each of which a given amount of origin and destination nodes are introduced. In this work, the number of origin-destination nodes per zone is chosen equal to two. Each origin node is thereafter connected to all destinations belonging to a different zone by a collection of routes. For the first part of our experimental setup, the number of routes per origin-destination pairs is set to $k = 3$.

In order to compare the efficiency of the previously defined methods, we employ a set of variably sized networks, ranging from sixteen up to one hundred nodes. For each network size, we generated one hundred instances, each featuring a different random seed, which produces graphs with similar characteristics but different shapes (as exemplified in Figure 3). For each instance, all heuristic methods are applied in order to obtain the corresponding candidate controller location set. To provide a baseline comparison, the approach introduced by Yuan et al. (2013) is also applied on each network.

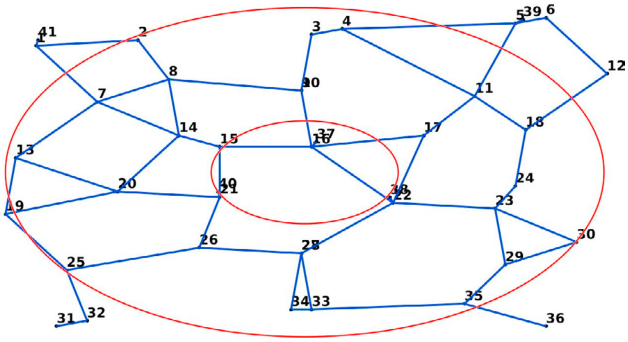


Fig. 2: Example of a generated network graph.

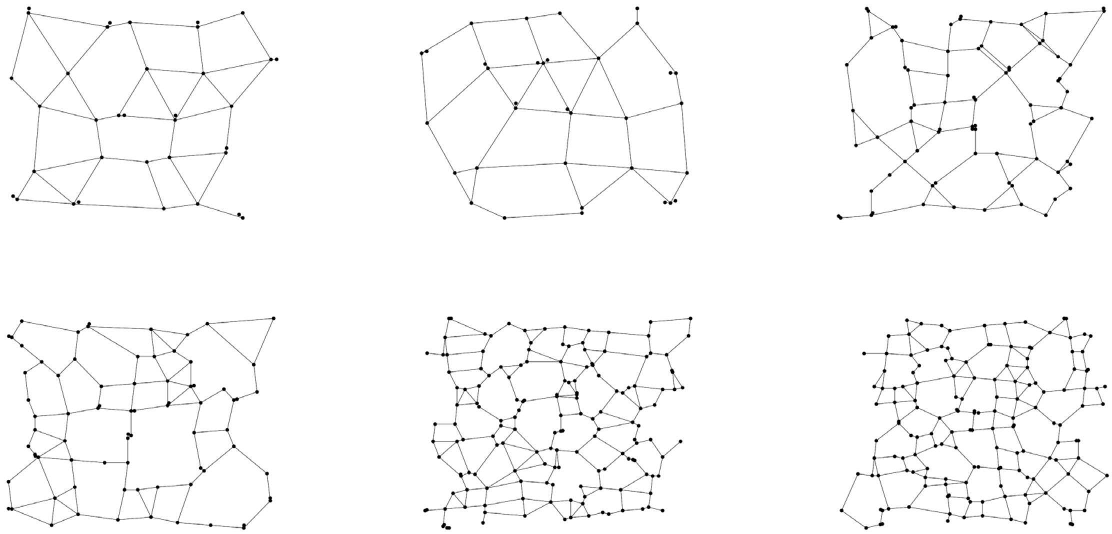


Fig. 3: Examples of different networks generated

5. Results

To evaluate whether the proposed heuristics are statistically capable to yield the desired performance on bi-directional networks, we begin by considering the percentage of instances where the different methods managed to reach full controllability, thus satisfying the key constraint of our problem. In table (1) we showcase said comparison over one hundred different randomized networks of twenty-five, forty-nine and one hundred nodes. We can see that the heuristic methods consistently fulfill the full controllability constraint for each instance and each size, as opposed to the exact method, who only seldom succeeds in identifying an appropriate solution, thereby confirming the proposed heuristics' capability to satisfy the problem's main constraint. We also present the computation time pertaining to the different methods (coded in MathWorks®MATLAB™) to produce these results (Dell Latitude-5480, Intel Core¢ i5-7300U CPU). While some methods are relatively faster, the overall execution times remain satisfactorily low. (1). It's important to consider that the computational times obtained by the exact approach relate in most instances to erroneous results, and are therefore not representative.

To further asses the heuristics' respective degree of efficiency, we studied the total number of controllers employed by the different methods. The results showcased in Fig. (4a) represent the number of controllers used by the different

Network size (in nodes)	Method	Mean full controllability percentage	Mean computation time [s]	Computation time standard deviation [s]	Mean number of controllers
25	Alg. Y	19 %	0.1065	0.0622	1.43
	Alg. 1	100 %	0.0436	0.0307	10.38
	Alg. 2	100 %	0.0153	0.0077	11.51
	Alg. 3	100 %	0.0182	0.0080	12.55
	Alg. 4	100 %	0.0758	0.0444	10.51
49	Alg. Y	9 %	0.2698	0.1117	1.33
	Alg. 1	100 %	0.3005	0.2107	16.24
	Alg. 2	100 %	0.0817	0.0454	19.51
	Alg. 3	100 %	0.1075	0.0510	21.48
	Alg. 4	100 %	0.4991	0.3478	17.12
100	Alg. Y	17 %	1.3649	0.6218	2.54
	Alg. 1	100 %	4.2679	3.9761	26.86
	Alg. 2	100 %	0.8683	0.6186	34.44
	Alg. 3	100 %	1.2922	0.7984	35.95
	Alg. 4	100 %	6.9741	5.6306	28.82

Table 1: Comparison of the methods

methods over one hundred distinct (but equally-sized) networks. These results suggest that indeed some methods perform statistically better than others for the given experimental setup, mainly the greedy ”Best controller per route” and the heuristic focusing on splitting and merging route locations, confirming that the relationship between route information and control efforts is indeed of considerable relevance for the problem of controller location. As expected, the solutions found by the exact method tend to yield a very small controller set, failing to fully control the network. As shown on 49 nodes networks (Fig. 4b) and 100 nodes networks (Fig. 4c), similar trends arise on larger networks, independently of scale.

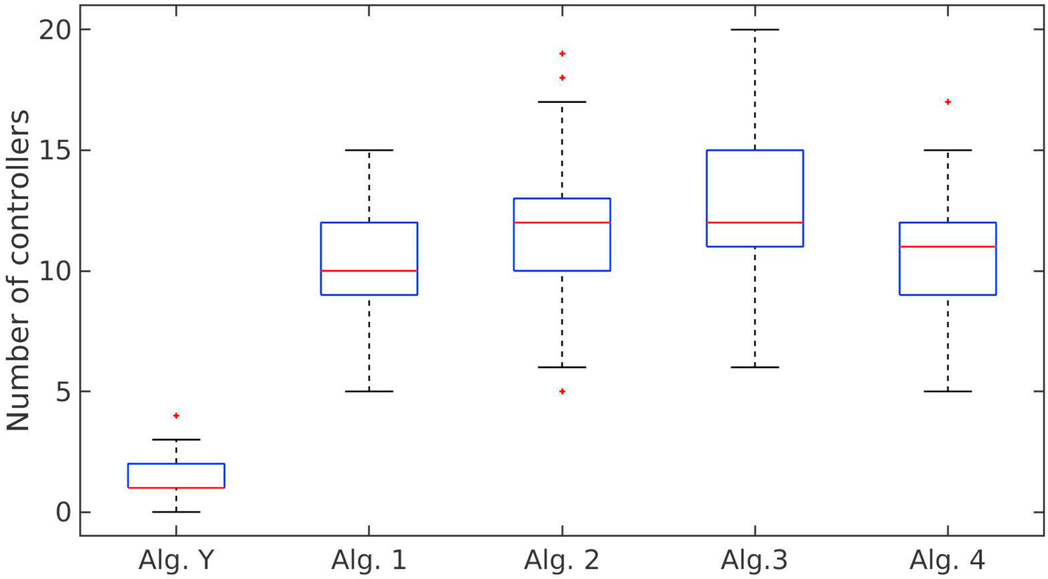
Finally, we also investigated the evolution of the number of controllers needed by each method when increasing the number of routes per each origin-destination pair k. The results of Figure (5) show the evolution of the mean number of controllers employed by the different methods with the increase of routes per origin-destination pair, over one hundred randomized graphs of twenty-five nodes. The number of controllers needed by all methods initially increase with the number of routes, although a plateau effect appears beyond a certain threshold. The methods' relative performances are however in line with previous findings.

These experimental results demonstrate, first of all, that the presented heuristics are indeed capable to produce viable controller sets capable to fully control a transportation network containing bi-directional links, while also highlighting the importance of appropriate information selection (route based vs node based approaches).

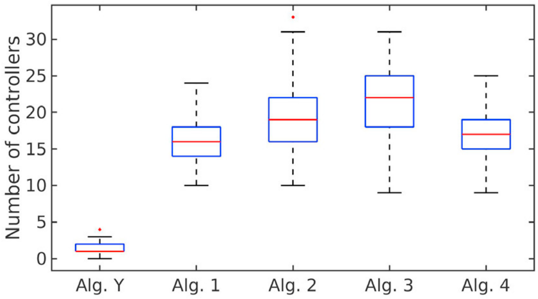
6. Conclusions

In this work we developed several rule-based heuristics to solve the problem of determining the minimum pricing controller locations set on complex transportation networks containing bi-directional links. We provided a comparison of the proposed methods over a range of network sizes and configurations, on which the presented heuristics managed to consistently satisfy the constraint of full controllability, while employing a sufficiently low number of controllers. This provides a first step towards the development of a fast rule-based method that can efficiently locate pricing controllers on generic transportation networks, while ensuring that redundancies (additional unnecessary controllers) are minimal.

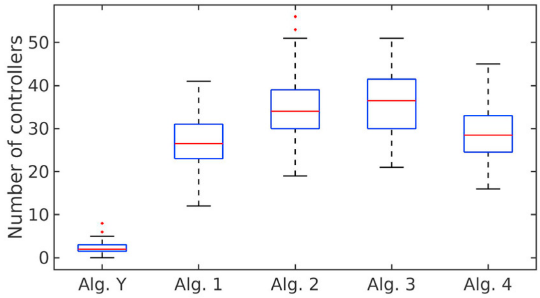
Further research will focus on refining the proposed heuristics in order to foremost improve their scalability, by removing the necessity to compute the level of controllability at each step, thus reducing the computation load of the method. The resulting approaches will then be compared with similar heuristics extracted from literature, specifically



(a) Number of controllers used to reach full controllability on a 25 nodes graph over 100 random draws.



(b) On a 49 nodes graph over 100 random draws.



(c) On a 100 nodes graph over 100 random draws.

Fig. 4: Number of controllers used to reach full controllability

concerning or adapting the approaches related to the network sensor location problem, in order to effectively validate the proposed heuristics' capabilities from a topological perspective.

To have a better representation of the efficiency of the controller set produced by the respective methods, we then plan to employ macroscopic traffic simulation to assess the generated controller locations' capabilities in terms of network-wide performance, alongside appropriate control schemes.

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References

He, S.x., 2013. A graphical approach to identify sensor locations for link flow inference. Transportation Research Part B: Methodological 51, 65–76. URL: https://linkinghub.elsevier.com/retrieve/pii/S0191261513000325, doi:10.1016/j.trb.2013.02.006.

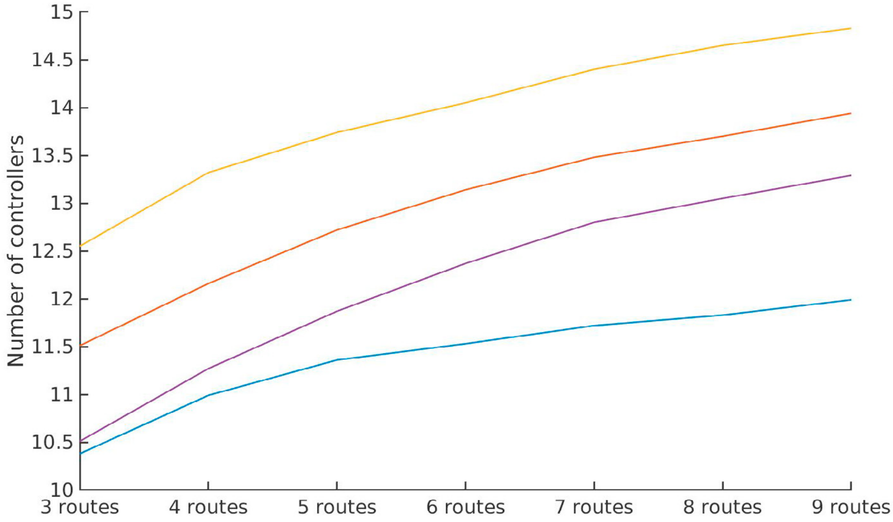

Fig. 5: Evolution of the number of controllers used to reach full controllability for increasing k.

Hoogendoorn, S.P., Landman, R.L., Van Kooten, J., Schreuder, M., Adams, R., 2015. Design and Implementation of Integrated Network Management Methodology in a Regional Network. Transportation Research Record: Journal of the Transportation Research Board 2489, 20–28. doi:10.3141/2489-03.

Hu, S.R., Peeta, S., Chu, C.H., 2009. Identification of vehicle sensor locations for link-based network traffic applications. Transportation Research Part B: Methodological 43, 873–894. URL: http://linkinghub.elsevier.com/retrieve/pii/S0191261509000356, doi:10.1016/j.trb.2009.02.008.

Hunt, P.B., Robertson, D.I., Bretherton, R.D., Winton, R.I., 1981. SCOOT - A TRAFFIC RESPONSIVE METHOD OF COORDINATING SIGNALS. Publication of: Transport and Road Research Laboratory .

Kalman, R.E., Ho, Y.C., Narendra, S.K., 1963. Controllability of linear dynamical systems. Contrib. Differ. Equ. 1, 189–23.

Lowrie, P.R., 1990. SCATS, SYDNEY CO-ORDINATED ADAPTIVE TRAFFIC SYSTEM : A TRAFFIC RESPONSIVE METHOD OF CONTROLLING URBAN TRAFFIC .

Ng, M., 2012. Synergistic sensor location for link flow inference without path enumeration: A node-based approach. Transportation Research Part B: Methodological 46, 781–788. URL: http://linkinghub.elsevier.com/retrieve/pii/S0191261512000161, doi:10.1016/j.trb.2012.02.001.

de Oliveira, L.B., Camponogara, E., 2010. Multi-agent model predictive control of signaling split in urban traffic networks. Transportation Research Part C: Emerging Technologies 18, 120–139. doi:10.1016/j.trc.2009.04.022.

Papageorgiou, M., Hadj-Salem, H., Middelham, F., 1997. Alinea local ramp metering: Summary of field results. Transportation Research Record , 90–98URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-3643094227&doi=10.3141%2f1603-12&partnerID=40&md5=3c6dbb249a8e3ece4b28fb2a2103f7c1, doi:10.3141/1603-12. cited By 230.

Rinaldi, M., 2018. Controllability of transportation networks. Transportation Research Part B: Methodological 118, 381–406. doi:10.1016/j.trb.2018.11.005.

Rinaldi, M., Viti, F., 2020. Investigating the Relationship between Controller Locations and Dynamic Traffic Control in Generic Transportation Networks. Transportation Research Record: Journal of the Transportation Research Board , 036119812091430URL: http://journals.sagepub.com/doi/10.1177/0361198120914302, doi:10.1177/0361198120914302.

Rose, A., O'Dea, R., Hopcraft, K.I., 2016. Price of anarchy on heterogeneous traffic-flow networks. Phys. Rev. E 94, 032315. URL: https://link.aps.org/doi/10.1103/PhysRevE.94.032315, doi:10.1103/PhysRevE.94.032315.

Verhoef, E.T., 2002. Second-best congestion pricing in general static transportation networks with elastic demands. Regional Science and Urban Economics 32, 281–310. doi:10.1016/S0166-0462(00)00064-8.

Mireles de Villafranca, A.E., Connors, R.D., Wilson, R.E., 2019. Static Traffic Assignment on Ensembles of Synthetic Road Networks, in: Hamdar, S.H. (Ed.), Traffic and Granular Flow '17. Springer International Publishing, Cham, pp. 29–36. URL: http://link.springer.com/10.1007/978-3-030-11440-4_4, doi:10.1007/978-3-030-11440-4_4.

Yen, J.Y., 1971. Finding the K Shortest Loopless Paths in a Network. Management Science 17, 712–716. doi:10.1287/mnsc.17.11.712.

Yuan, Z., Zhao, C., Di, Z., Wang, W.X., Lai, Y.C., 2013. Exact controllability of complex networks. Nature Communications 4, 2447.

Zhang, X., Yang, H., 2004. The optimal cordon-based network congestion pricing problem. Transportation Research Part B: Methodological 38, 517–537. URL: https://linkinghub.elsevier.com/retrieve/pii/S0191261503000900, doi:10.1016/j.trb.2003.08.001.