Dynamic Hedging for the Real Option Management of Hydropower Production with Exchange Rate Risks

Joakim Dimoski*, Stein-Erik Fleten*, Nils Löhndorf†[1], Sveinung Nersten*

*Department of Industrial Economics and Technology Management
Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

†Luxembourg Centre for Logistics and Supply Chain Management
University of Luxembourg, L-1511 Luxembourg

Abstract

We study the risk management problem of a hydropower producer that hedges risk by trading currency and power futures contracts. The model considers three types of risks: operational risk due to supply uncertainty, profit risk due to power price variability, and exchange rate risk when operation and trading takes place in different currencies. We cast the problem as a Markov decision process and propose a sequential solution approach that separates operational management from trading. To solve the problem, we first reduce the high-dimensional Markovian process that models inflows, exchange rates, and future curve dynamics to a scenario lattice and then employ stochastic dual dynamic programming under a risk measure. We find that dynamic hedging leads to significant risk reduction and that it performs better than static hedge ratios that are often used in practice. We also find that a sequential approach leads to better outcomes than an integrated approach across various metrics, which supports the functional separation of operation and hedging that is common practice in most power companies.

Keywords: risk management, mathematical optimization, hydropower, real options, dynamic programming

[1]corresponding author, email: nils.loehndorf@uni.lu

1. Introduction

For hydropower producers with random natural inflows, a problem of practical relevance is to maximize cash flows from buying and selling power under operational as well as market risks. While managing price uncertainty as a type of market risk is well understood in the area of commodity storage (Murphy and Oliveira 2010, Nadarajah et al. 2015, Goel and Tanrisever 2017), other types of market risk are normally not taken into account. In particular, hydropower producers who participate in cross-border trades, like Canadian companies in the U.S. or Scandinavian companies in continental Europe, are exposed to additional exchange rate risks, as operations and trading take place in different currencies. In this article, we focus on hydropower operation under price and exchange rate risks as well as the risk of random natural inflows and how these risks can be mitigated using financial instruments.

Risk-averse companies can use financial instruments such as forward contracts and options to lower risk exposure to their risk preference. Reducing a company's market risk is referred to as hedging or risk management. Dupuis et al. (2016) distinguish between *static* and *dynamic* hedging and between *local* and *global* hedging. Companies use static hedging to hedge their portfolio at a point in time and without subsequent rebalancing. Dynamic hedging, in contrast, involves continuous adjustment of the portfolio as new market information becomes available. Local hedging focuses on minimizing short-term risk until rebalancing, whereas global hedging seeks to minimize risk associated with all future cash flows. Discussions of local hedging in electricity markets are in Zanotti et al. (2010) and Liu et al. (2010); global hedging models are discussed in Mo et al. (2001), Fleten et al. (2002) and Dupuis et al. (2016). It is shown in Fleten et al. (2002) that dynamic hedging leads to better outcomes than static hedging for hydropower production.

In this article, we present a global dynamic hedging model for companies that operate hydropower assets. The proposed model builds on previous efforts of jointly modeling supply uncertainty and futures price dynamics to obtain more accurate values of water in a reservoir (Dimoski et al. 2018). We complement the previous model with a dynamic hedging model that accounts for trading in monthly, quarterly, and annual power futures contracts. Modeling futures trading along with operational decision is an attractive feature for risk management because all types of risks are then addressed by a single model as opposed to multiple models, which is often the case in practice.

We focus on the case of a price-taking hydropower producer who participates in a wholesale electricity market, managing a set of reservoirs. In addition to price risk, we consider uncertainty in currency spot exchange rates to address risk from cross-border trading. This covers storage-based operations located in e.g. Canada, Sweden and Switzerland, selling energy into the U.S. and Euro currency areas. We allow the company to trade in currency forward contracts to hedge this uncertainty. We consider a planning horizon of two years in semi-monthly time increments, to achieve a better alignment of the time granularity of futures contracts with operational decision making. We also consider transaction and tax costs.

To capture forward curve movements and to generate scenarios for spot and power futures prices, we use a multivariate Heath-Jarrow-Morton (HJM) term-structure model (Heath et al. 1992). In an efficient market, available future and forward contracts traded at a given point in time represent the current risk-adjusted market expectations of future spot prices. A high-resolution forward curve can be constructed using the price and delivery periods of all available futures contracts. For example, see Fleten and Lemming (2003), Benth et al. (2008) and Kiesel et al. (2019). A model that explains the evolution of a forward curve can therefore be used to find future spot prices and to calculate the price of futures contracts for different delivery periods.

We also incorporate stochastic processes for natural inflows and exchange rates into the model. There is typically a negative correlation between natural inflows and electricity prices in the hydro-dominated systems of countries such as Norway, Canada, or Brazil. Electricity supply in these countries is largely determined by inflows. This provides a *natural hedging* effect. We are also interested in investigating the magnitude of exchange rate risk, and how it relates to the other risk factors. We presume that system price is negatively correlated with the EURNOK exchange rate, because all bid and ask orders in the Nordic market are placed by companies that operate using the local currency. This should influence the system price, which is denoted in EUR/MWh, and thus a negative correlation has a natural hedge effect.

Many power companies use simple strategies to hedge their risk. Wang et al. (2015) analyzed several minimum-variance strategies for a number of commodity, currency, and equity markets. They found that a naive hedging strategy (hedge ratio = 1) performs better or almost as well as the minimum-variance strategies in all the tested markets. However, the authors use a static approach and do not consider the electricity market. We propose a dynamic approach by formulating the decision problem as a multistage process and modeling risk preferences using the nested conditional value-at-risk (nested CVaR), see e.g., Shapiro et al. (2013). The nested CVaR is based on conditional convex risk mappings (see Ruszczynski and Shapiro (2006)). Unlike other risk measures, nested CVaR ensures that risk preferences remain time-consistent under the optimal policy. Boda and Filar (2006) and Shapiro (2009) show that global hedging strategies such as the naive hedging strategy, which aim to reduce the risk of terminal cash flows, are not time-consistent. Static hedging for hydropower producers has also been studied in a number of articles, e.g. Fleten et al. (2010).

To deal with the curse of dimensionality of the resulting dynamic programming formulation, we use the approximate dual dynamic programming of Löhndorf and Wozabal (2021). The authors propose a two-step approach of first reducing the multi-dimensional stochastic process that characterizes uncertain model parameters (here: inflows and forward curve dynamics) to a scenario lattice and then approximating the optimal policy using stochastic dual dynamic programming. We further extend their approach pursuing a sequential solution strategy: first, optimal production decisions are found that only use spot price and inflow information. These decisions are then clustered together with the price states of the lattice to become exogenous state variables in the hedging problem. Finally, the lattice that now holds production and price uncertainty is used to approximate the optimal hedging strategy under a risk measure. In a numerical experiments, we find that this solution strategy leads to better outcomes in terms of profit and risk than an integrated strategy.

In summary, this article makes three main contributions to the literature. First, we substantially add to the degree of realism regarding the problem of dynamic hedging of electricity operations. This contribution has at least three dimensions:

- By using a multifactor forward curve model for electricity prices, we are able to better capture the complex dynamics that such prices are known to exhibit. Previous research (Mo et al. 2001), (Iliadis et al. 2006) have used spot price models with one or at most two driving factors. Compared to our approach, these suffer from missing degrees of freedom when the underlying hedging problem allows for many contracts (maturities) to be traded at any time.

- We add realism by capturing currency risk and allowing dynamic trading of currency forwards. This allows us to quantify the benefit that currency hedging can provide when trading electricity across currency borders.

- Our modeling of the accounting and taxation of electricity contracts adds realism since most previous efforts (Fleten et al. 2002), (Kettunen et al. 2009) erroneously assume physical delivery and disregards taxation aspects. Our approach involves, for example, dynamic listing and de-listing of futures contracts, dynamics of cash flows during delivery of power futures, transaction costs and a resource rent tax imposed on spot revenues from production but not on profits/losses from hedging.

Second, as we cast the model as a Markov decision process, we can compare dynamic hedging with static hedging, which is popular among practitioners in risk management. This aspect also adds realism since in contrast to previous research, we use nested CVaR as a time-consistent and coherent measure of risk.

Third, we compare the integrated approach of optimizing operational and hedging decisions with a sequential approach of separating operational and hedging decisions, which is often pursued in practice. This approach makes the model practically appealing, since production decisions and trading are often executed by separate functions in most power companies.

The article is organized as follows: in section 2, we present relevant background information on risk management in electricity markets, the emphasis being on hydropower production in Nordic countries. In section 3, we describe the hedging problem formulated as a Markov decision process. We also give an overview of the algorithms used to reduce the dimension of the stochastic processes and to solve the stochastic-dynamic decision problem. In section 5, we present how we modelled the different risk factors as stochastic processes. Section 6 presents the numerical results and a conclusion and an outlook on future work is given in section 7.

2. Risk management of hydropower assets

In this section, we provide an overview of relevant aspects of risk management for hydropower assets. First, we underline the important risk factors and the derivatives that can be used to reduce the risk exposure of the owner. Then we present relevant risk measures that firms can use to quantify their exposure to risk. The section concludes with a brief overview of current risk management practices in hydropower companies.

2.1. Risk factors and mitigating derivatives

The focus of this article is on electricity price risk, currency risk, and risks in natural inflows that effect production yield.

The Nord Pool day-ahead market – also referred to as spot market – is the primary market for Nordic power producers. Nord Pool calculates a system price, which is an unconstrained market clearing reference price for the entire Nord Pool market. Financial contracts for the Nordic market are traded on NASDAQ Commodities Europe. This market is a purely financial market where no physical energy is exchanged. The day-ahead market, by contrast, is a physical market.

System price risk can be hedged by trading futures on NASDAQ. The power futures traded on NASDAQ have delivery periods that span one day, one week, one month, one quarter, or one year. Traditional forward and futures contracts are, conversely, contracts for the trade of an asset at a specific point in time. Power futures are therefore more like financial swaps. As explained by Fleten et al. (2010), futures contracts are marked-to-market each day prior to the beginning of the delivery period. Contracts that are in delivery are settled daily, based on the difference between the spot price and the last price of the futures contract before going into delivery.

A power producer receives the area spot price for their generation. As the system spot price serves as the underlying price for traded power futures, there is a basis risk between area spot price and power futures. Houmøller (2017) has shown that there is a high correlation between the hourly system price and the price for most Norwegian areas. The average for all areas combined for 2013–2016 is approximately 0.89. This is above the limit of 0.8 set by the IAS 39 accounting standard for qualifying for hedge accounting. Houmøller (2017) therefore argues that it is sufficient that power producers in most Norwegian areas disregard area difference in their hedging strategy and hedge only using system price contracts.

Revenues of Norwegian power producers are generated in EUR. The producer's base currency however is NOK. Since exchange rates fluctuate, this means a significant currency risk for producers. Currency risk can be hedged using forward exchange contracts. All bid orders and ask orders in a price area are placed by companies that operate using their local currency. The system price, which is denoted in EUR/MWh, should therefore be influenced by the base currencies of the different areas. We therefore expect system price to be negatively correlated with the EURNOK exchange rate, thereby providing a natural hedging effect.

2.2. Risk measurement

For hydropower producers, risk measurement is typically based on end-of-year cash flows, because these are simple to interpret (Fleten et al. 2010). Standard deviation has historically been one of the most widely used metrics (Stulz 1996), as it measures deviations from expected cash flows in the positive and negative direction. Nonetheless, Stulz (1996) argues that it is of greater interest to consider the downside risk of the cash flow distribution. A common metric for measuring downside risk is Value-at-Risk (VaR). For a given significance level α, the VaR of H discrete representations of the terminal cash flows h_i is defined as $VaR_\alpha =$

$\min\{h_i \mid \sum_{j|h_j \le h_i} 1/H \ge \alpha\}$. Although VaR provides a risk manager with information on worst case scenarios, it provides no information about the distribution of cash flows below the VaR. The Conditional Value-at-Risk (CVaR), by contrast, measures expected cash flows below the VaR. Mathematically, the CVaR of a discrete distribution at significance level α is given by $CVaR_\alpha = \sum_{j|h_j \le VaR_\alpha} h_j/(H\alpha)$. CVaR has a number of benefits over VaR:

1. CVaR better captures tail-effects such as kurtosis and skewness.
2. CVaR is a coherent[2] risk measure, which makes it particularly attractive in risk management (Godin 2016).
3. CVaR is a convex risk measure which makes it attractive for analysis and optimization.

2.3. Industry practice

Examining how risk management is performed in the hydropower industry provides useful benchmarks. We therefore provide an overview of industry practices with focus on the Norwegian market.

Sanda et al. (2013) analyzed the hedging strategies of 12 Norwegian hydropower producers, their total production accounting for about 34.4% of total production in Norway. The authors find that none of the firms use an integrated model to obtain optimal production and hedging decisions. Five used a sequential approach by obtaining optimal production decisions first and then using these decisions for hedging. The seven remaining firms used historical production scenarios to predict future risk exposure.

All of the firms studied by Sanda et al. (2013) use static hedging approaches; none of them use dynamic hedging. Eight of the firms use a hedge ratio approach, their short positions in the financial market being required to be within a predefined range for a given time to maturity. Two of the firms have a minimum requirement for the VaR of their terminal cash flows. The remaining two have no written hedging policy. Futures contracts are the most widely used derivatives in terms of hedged volume in 11 out of 12 firms. On average, approximately 90% of the traded derivative volume [MWh] of the firms was quarterly and annual contracts. None of the firms studied used a dynamic approach as hedging policy.

[2] Specifically, VaR does not qualify for the subadditivity axiom when the underlying loss distribution is non-normal. A risk measure $\Phi(X)$ is subadditive if $\Phi(X_1 + X_2) \le \Phi(X_1) + \Phi(X_2)$.

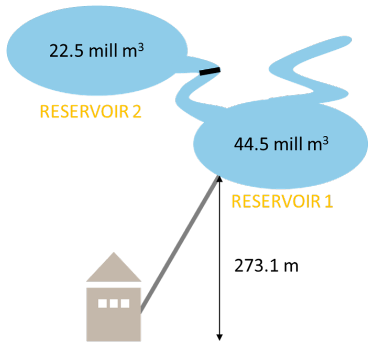


Figure 1: Topological setup, reservoir capacities and forebay elevation of the hydropower plant

3. Model Formulation

In this section, we cast the decision problems associated with production planning and dynamic hedging as Markov decision processes.

3.1. Assumptions

We consider the problem faced by a price-taking hydropower producer who operates a single hydropower plant with substantial reservoir capacity. The producer participates in a well-functioning electricity market and hedges physical production by pre-selling financial futures contracts. The producer's objective is to find an asset-backed hedging strategy that matches risk preferences and accounts for risk in natural inflows, prices, exchange rates, as well as management decisions.

Although the model is generic, we consider an actual plant located in the NO3 price area in Norway (there are five price areas in Norway, NO3 being located in the center of the country). The plant consists of two interconnected reservoirs and one power station with one turbine. The plant is mid-sized in terms of reservoir capacity relative to annual inflow and in terms of generation capacity, with a mean annual production of 191 GWh. Figure 1 illustrates some relevant properties of the plant. Following Wallace and Fleten (2003), we model i) production planning and ii) financial hedging as separate decision problems that are made in sequence. We formulate each problem as a Markov decision processes (MDP), which is line with the literature on medium-term reservoir management (e.g. Lamond and Boukhtouta (1996)) as well as real option management of commodity storage (e.g. Nadarajah et al. (2015)).

In line with Bjerksund et al. (2011), we assume that the decision-maker participates in a complete market with no risk-free arbitrage. This means that all state transition probabilities can be represented by a unique martingale (risk-neutral) measure $\mathbb{Q}$. We use a time horizon of approximately two years for both the production problem and the hedging problem, which is in line with previous approaches to medium-term hydropower planning (Wolfgang et al. 2009, Abgottspon and Andersson 2014) as well as models of hydropower risk management (Fleten et al. 2002, 2010)). We use semi-monthly time granularity so that each time stage is approximately two weeks. A semi-monthly resolution was chosen so that discrete stages coincide with delivery periods of monthly, quarterly and yearly futures contracts traded on NASDAQ. Semi-monthly periods are also close to the time intervals that are typically used in medium-term hydropower planning.

Multiple random variables impact the decisions of the hydropower producer at each time stage. Random variables in the production model are spot price $F_{t,t}$ and reservoir inflows $Y_{1,t}$, $Y_{2,t}$.[3]

In the hedging problem, we allow the producer to trade in monthly, quarterly and yearly power futures contracts with delivery periods within the chosen time horizon of $\hat{T} = 49$ semi-months. These contracts have been chosen because of their high liquidity on NASDAQ and because they are the most common derivatives used by Norwegian hydropower producers (Sanda et al. 2013). Random variables in the hedging model are therefore prices of six monthly, eight quarterly, and one yearly power futures, denoted $F_{t,Mi}$ for $i = [1, ..., 6]$, $F_{t,Qj}$ for $j = [1, ..., 8]$ and $F_{t,Y1}$. The hedging model also includes random variables for spot exchange rate ($Q_{t,t}$) and forward exchange rate at time t, for maturity at time T ($Q_{t,T}$).

Production decisions enter the hedging problem as exogenous random variables, W_t. The value of W_t is found using the production model. We also include taxation effects, γ_c and γ_r denoting the corporate and resource rent tax rate. And finally, we include variable transaction costs c_F for trading in the power futures market. Transactions cost are negligible in the currency forward market.

We propose to use the nested conditional value-at-risk (nested CVaR) to model the risk preferences of the producer. *Terminal* CVaR measures the CVaR of the terminal cash flow. As cash flows depend on realizations of randomness, risk preferences are state-dependent and hence not consistent across time. Time consistency can be restored by applying the *Nested* CVaR as risk measure which only considers the CVaR of possible future cash flows.

We define the function $\psi_{\lambda,\alpha}(X) = \lambda CVaR_\alpha(X) + (1 - \lambda)\mathbb{E}(X)$ for a random variable X,

[3]Inflow risk causes production cash flows to contain an unspanned component, since financial instruments for hedging this volume risk are unavailable. In our analysis, we assume this risk to be unsystematic. This may be true for Europe and North America, however, in locations such as New Zealand, this risk can be regarded as systematic and need to be priced Philpott et al. (2016).

significance level α and weight λ. The $CVaR_\alpha(X)$ is defined in Section 2.2. Following Shapiro et al. (2013), the nested CVaR for a sequence of random variables $X_1, X_2, X_3, ...$ is given by

$$CVaR^{NEST}_{\alpha,\lambda}(X_1, X_2, X_3, ...) = X_1 + \psi_{\alpha,\lambda}(X_2 + \psi_{\alpha,\lambda}(X_3 + ...)). \tag{1}$$

In Section 3.2 and 3.3, we formulate both, the production problem and the hedging problem, as Markov decision processes (MDPs).

3.2. Production planning problem

The objective of the production planner is to maximize the expected discounted terminal cash flows from power production and reservoir control. Denote w_t as power production decision [MWh] at time t. In line with the literature on hydropower planning, cash flows earned by the producer equal rewards earned from physical sales while variable start-up cost are ignored (Wallace and Fleten 2003). We use $F_{t,t} \cdot w_t$ as revenue at time t and β_t as a time-dependent discount factor. Then, the value function of the MDP is given by

$$V^P_t = F_{t,t} w_t (1 - \gamma_c - \gamma_r) + \beta_t \mathbb{E}[V^P_{t+1} \mid F_{t,t}, Y_{1,t}, Y_{2,t}, \pi_t]. \tag{2}$$

Here, π_t denotes the decision policy at time t. Cash flows from production are subject to both resource rent and corporate tax. Denote $v_{1,t}$ and $v_{2,t}$ as reservoir volume $[m^3]$, $s_{c,t}$ as the amount of water flowing from reservoir 2 into reservoir 1, $s_{s,t}$ as the amount of spilled water, and κ as constant generation efficiency $[MWh/m^3]$. Using this notation, the volume balance in each reservoir is given by

$$v_{1,t} = v_{1,t-1} - w_t \cdot \kappa^{-1} + s_{c,t} + Y_{1,t} - s_{s,t} \tag{3}$$

$$v_{2,t} = v_{2,t-1} + Y_{2,t} - s_{c,t} \tag{4}$$

Treating κ as a constant corresponds to what is commonly assumed in long-term reservoir management, for example, in software such as EOPS that is widely used for medium-term production planning in the Nordic countries (SINTEF 2017). Both reservoirs have upper bounds. Reservoir 2 additionally has a time-dependent lower bound specified in the operating license issued by the regulator.

$$\begin{aligned} v_{1,t} \leq \overline{v_1}, \quad v_{2,t} \leq \overline{v_2} \\ v_{1,t} \geq 0, \quad v_{2,t} + v^S_{2,t} \geq \underline{v_{2,t}} \end{aligned} \tag{5}$$

We enforce the time-dependent lower bound via a soft constraint, using the variable $v_{2,t}^S$. Violating this constraint will result in a penalty cost given by $\upsilon \cdot v_{2,t}^S$, which is added to the objective function.

Production capacity (6) is determined by the maximum flow rate $[m^3/s]$ of the plant's turbines, ξ, and the number of seconds in a semi-month, ς_t,

$$w_t \leq \overline{w_t} = \xi \cdot \kappa \cdot \varsigma_t. \tag{6}$$

The complete production planning problem at time t is given by

$$\begin{aligned} \max \quad & V_t^P(F_{t,t}, Y_{1,t}, Y_{2,t}, \pi_t) - \upsilon \cdot v_{2,t}^S \\ \text{subject to} \quad & (3), (4), (5), (6) \end{aligned}$$

3.3. Hedging problem

The objective of the dynamic hedging problem is to control risk exposure, which requires tracking trades in currency and power derivatives. We include variables and balance constraints for tracking financial short positions and committed future cash flows, which reflects their actual payoff structure. The model does not consider long positions in currency and power futures. Allowing only short positions should be sufficient for hedging purposes, as the producer has a natural long position in physical production.

We assume that the exchange rate forward contract is settled at contract maturity. Figure 2 illustrates an example of the cash flows from a currency forward contract with delivery in time 8. No cash is exchanged until the maturity date, when the difference in the forward and spot price is settled. We denote $z_{t,T}$ as the producer's total short position [EUR] at time t in currency forwards with maturity at time T. New short positions that enter at stage t for delivery in T are denoted by $x_{t,T}$. This gives the following balance constraint for $t < T$, where $T \leq \hat{T}$,

$$z_{t,T} = z_{t-1,T} + x_{t,T}, \quad t < T. \tag{7}$$

A forward contract with instantaneous delivery is a spot trade, and its balance constraint for $t = T$ is given by

$$z_{t,t} = z_{t-1,t}, \quad t = T. \tag{8}$$

The currency forward rate is denoted by $Q_{t,T}$ [NOK/EUR]. We let $y_{t,T}^C$ [NOK] denote the committed, positive cash flows that the producer is certain to receive at stage T, given their

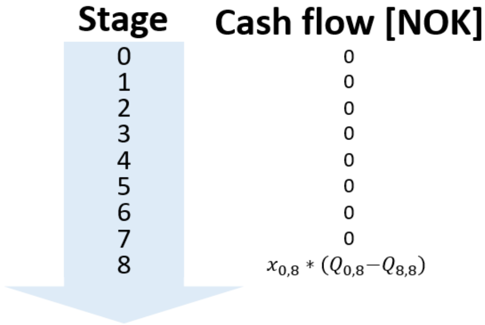


Figure 2: Cash flows from short position $x_{0,8}$ in currency forward contract with maturity $T = 8$ made at $t = 0$.

trading activity in the currency forward market. For $T > t$, the balance for the committed part of the currency cash flows is given by (9). Note that cash flows from forward trading are only subject to corporate tax rate γ_c. This contrasts cash flows from physical production, which are subject to both resource rent tax rate γ_r and corporate tax rate γ_c,

$$y^C_{t,T} = y^C_{t-1,T} + x_{t,T} Q_{t,T}(1 - \gamma_c), \quad t < T. \tag{9}$$

At maturity in $t = T$, the time t cash flows from currency hedging are given by

$$y^C_{t,t} = y^C_{t-1,t} - z_{t,t} Q_{t,t}(1 - \gamma_c), \quad t = T \tag{10}$$

Note that $y^C_{t,t}$ can take both positive and negative values.

Power futures have a more complex cash flow structure than currency forwards, since delivery periods are typically monthly, quarterly, or annual delivery bands for power, instead of delivery at a specific point in time. Also power futures use daily settlement. Before delivery, settlement is based on the price change between two successive trading days, and during delivery it is based on the difference between the system spot price and the last price for which the contract was traded before entering into delivery. Contracts in delivery are not tradable. We replicate this structure as closely as possible, under the condition of semi-monthly settlement. Figure 3 shows an example of the cash flows of a quarterly contract with delivery period from stage 3 to 8. These are then multiplied by their respective spot exchange rate $Q_{t,t}$ to give cash flows in NOK.

The exposition of contract and cash flow dynamics for power futures is new to the literature, nevertheless, it is tedious, and we refer to Appendix A for details. We end up with expressions for the dynamics of futures positions as well as the stage t cash flows from power trading ($y^F_{t,t}$).

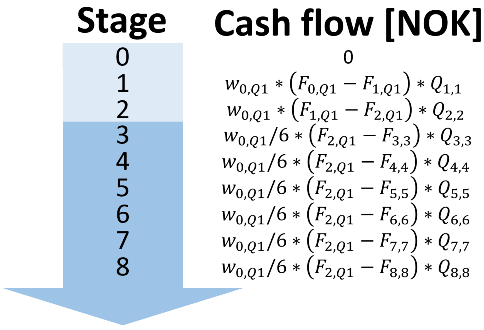


Figure 3: Cash flows from short position $w_{0,Q1}$ in a power futures contract with delivery in the upcoming quarter. The light blue part of the figure denotes time stages prior to the start of the delivery period, the darker part denoting stages within the delivery period. During the delivery period, the quantity of the short position is divided by 6, as this is the number of stages covered by the contract delivery period.

It has a similar but more complex structure as the cash flows from currency forwards.

For the hedging problem, we define the value function, V_t^H, as a linear combination of the stage t cash flows and $\psi_{\alpha,\lambda}(X)$. The function $\psi_{\alpha,\lambda}(X)$ is a risk measure with risk preference weighting λ and quantile α, as defined in section 3.1. Stage t cash flows aggregate cash flows from currency forward trading, power futures trading, and spot production. Recall that, in the hedging problem, production W_t is an exogenous random variable.

$$V_t^H = y_{t,t}^C + y_{t,t}^F Q_{t,t} + W_t F_{t,t} Q_{t,t}(1 - \gamma_c - \gamma_r) + \beta_t \psi_{\alpha,\lambda}[V_{t+1}^H \mid F_{t,t}, Q_{t,t}, W_t, \pi_t] \tag{11}$$

Earlier, we explained how λ adjusts the weighting of CVaR and expected value of a random variable X. In our case, X is the next stage value function V_{t+1}^H. Setting $\lambda = 0$ is equivalent to maximizing the expected value, while setting $\lambda = 1$ involves maximization of the stage t cash flows and the CVaR of V_{t+1}^H. Setting $\lambda \neq 1$ makes little sense in the hedging problem with exogenous production W_t, as the expected profit from trading forward contracts is zero under the risk-neutral probability measure. Hence, we will use $\lambda = 1$ to solve the hedging problem when W_t is given. Adjusting λ does, however, make sense in a problem that models production planning and hedging simultaneously. Such a model can be formulated by combining constraints and decision variables of the production problem with those of the hedging problems, and replacing random variable W_t with decision variable w_t.

4. Solution Method

The two problems defined in the previous section are both discrete-time, continuous-state MDPs. As it is generally not possible to solve such problems exactly, we are going to use approximate dual dynamic programming (ADDP) to approximate the optimal policy of the continuous-state problem (Löhndorf et al. 2013, Löhndorf and Shapiro 2018, Löhndorf and Wozabal 2021). ADDP exploits the property that the state space of the MDP can be separated into a *resource state*, x_t, which is defined by the decision process (e.g., reservoir contents, cash balance, contract position), and an *environmental state*, ξ_t, which evolves independently of the decision process (e.g., futures prices, natural inflows, or production in the hedging problem).

Given that ξ_t is Markovian, $\mathcal{P}_t(F_t|F_2, \ldots, F_{t-1}) = \mathcal{P}_t(F_t|F_{t-1})$, dynamic programming equations can be written as

$$V_t(x_{t-1}, \xi_t) = \max_{x_t \in \mathcal{X}_t(x_{t-1}, \xi_t)} \{R_t(x_t, \xi_t) + \mathcal{V}_{t+1}(x_t, \xi_t)\}, \ t = 2, ..., T, \tag{12}$$

where $R_t(\cdot, \cdot)$ is the (convex) immediate reward function and

$$\mathcal{V}_{t+1}(x_t, \xi_t) = \mathbb{E}_{\xi_{t+1}|\xi_t} [V_{t+1}(x_t, \xi_{t+1})], \ t = 2, ..., T, \tag{13}$$

with $\mathcal{V}_{T+1} \equiv 0$. We refer to functions $V_t(\cdot, \cdot)$ as value functions and $\mathcal{V}_t(\cdot, \cdot)$ as expected value functions.

The value functions, $V_t(x_{t-1}, \xi_t)$, are concave in x_{t-1} if $\max\{R_t(x_t, \xi_t) + \mathcal{V}_{t+1}(x_t, \xi_t)|x_t \in \mathcal{X}_t(x_t, \xi_{t-1})\}$ are concave in x_{t-1}, which can be shown by induction in t going backwards in time.

ADDP in a first step constructs a discrete scenario lattice of the environmental state by separating the outcome space of random variable ξ_t into a finite number of disjoint partitions. In the second step, ADDP approximates the value functions that are associated with the means of each partition from above using cutting planes. The resulting approximate value functions are piecewise constant in realizations of ξ_t and piecewise linear in the x_t and provide a policy for the continuous-state problem.

To construct the scenario lattice, at each stage $t = 2, \ldots, T$, ADDP tries to find partition means, μ_{ti}, $i = 1, ..., K_t$, which are solutions of the following problem

$$\min_{\mu_{t1}, \ldots, \mu_{tK_t}} \int \min_{i=1,\ldots,K_t} \|\xi_t - \mu_{ti}\|_p^p dP(\xi_t), \tag{14}$$

where $\|\cdot\|_p$ denotes the p-norm with $p \geq 1$. Given a solution $\mu_{t1}, ..., \mu_{tK_t}$ of (14), the space is partitioned into subsets Γ_{ti} consisting of points which are closest to μ_{ti}. The corresponding sets

$$\Gamma_{ti} := \{\xi_t : \|\xi_t - \mu_{ti}\|_p \leq \|\xi_t - \mu_{tj}\|_p,\ j = 1, ..., K_t,\ j \neq i\}\,, \tag{15}$$

represent the Voronoi partition associated with respective means μ_{ti}, $i = 1, ..., K_t$, which serve as the nodes of the lattice in t. Hence, we have $\mu_{ti} = \mathbb{E}\left[\xi_t | \xi_t \in \Gamma_{ti}\right]$.

As finding the optimum is an $\mathcal{NP}$-hard problem, Löhndorf and Wozabal (2021) propose to use stochastic approximation method which we will adapt here. The method draws S random sequences $(\hat{F}\xi_t^s)_{t=1}^T$, $s = 1, ..., S$, from the continuous process. With $(\beta_s)_{s=1}^S$ being a sequence of step sizes with $0 \leq \beta_s \leq 1$, $s = 1, ..., S$, and setting $p = 2$, means are updated recursively using

$$\mu_{ti}^s := \begin{cases} \mu_{ti}^{s-1} + \beta_k \left(\hat{\xi}_t^s - \mu_{ti}^{s-1}\right) & \text{if } i = \text{argmin}\left\{||\hat{\xi}_t^s - \mu_{tk}^{s-1}||^2,\ k = 1, ..., K_t\right\}, \\ \mu_{ti}^{s-1} & \text{otherwise,} \end{cases} \tag{16}$$

with $\mu_{0i}^{s-1} \equiv 0$, for $i = 1, ..., K_t$, $t = 2, \ldots, T$, $s = 1, ..., S$. It can be shown that if the sequence $(\beta_s)_{s=1}^S$ fulfills $\sum_{s=1}^\infty \beta_s = \infty$ and $\sum_{s=1}^\infty \beta_s^2 < \infty$, then the means converge to local minimizers of (14).

To estimate the transition probabilities between partitions at subsequent stages, we use the backwards estimation procedure proposed in Löhndorf and Wozabal (2021), which ensures that the discrete approximation of the stochastic process is a martingale.

With a given lattice, we can now compute an upper bound of the value functions using cutting planes by extending the SDDP method of Pereira and Pinto (1991) to scenario lattices, also referred to as Markov-Chain-SDDP (Löhndorf and Shapiro 2018).

Denote $\hat{V}_{t+1,n}(x_t, F_{t+1})$ as approximate value functions and $\hat{\mathcal{V}}_{tn}(x_t)$ as their expectations with $\hat{\mathcal{V}}_{t,0} \equiv 0$. At each iteration, $n = 1, ..., N$, we let ADDP draw one scenario from the lattice process, $(\hat{\xi}_t^n)_{t=1}^{T-1}$, to generate a sequence of sample decisions,

$$\hat{x}_t^n = \text{argmax}_{x_t \in \mathcal{X}_t(\hat{x}_{t-1}^n, \hat{\xi}_t^n)} \left\{R(x_t, \hat{\xi}_t^n) + \hat{\mathcal{V}}_{t,n-1}(x_t, \hat{\xi}_t^n)\right\},\ t = 2, ..., T-1, \tag{17}$$

with $\hat{x}_1^n = \text{argmax}\{R(x_1) + \hat{\mathcal{V}}_{t,n-1}(x_1) | x_1 \in \mathcal{X}_1\}$.

Note that a solution of the discretization only provides a policy for the set of scenarios given by the constructed lattice with nodes μ_{ti}, $i = 1, ..., K_t$, $t = 1, ..., T-1$, so that the respective upper bound is only valid for the *discretized* problem but not for the continuous one.

For a given set of sample decision, the algorithm can now construct cutting planes by recursively solving

$$\hat{V}_{tn}(\hat{x}_{t-1}^n, \mu_{ti}) = \max_{x_t \in \mathcal{X}_t(\hat{x}_{t-1}^n, \hat{\xi}_t^n)} \left\{ R(x_t, \hat{\xi}_t^n) + \hat{\mathcal{V}}_{tn}(x_t, \mu_{ti}) \right\}, \tag{18}$$

with $\hat{\mathcal{V}}_{Tn}(x_T, \mu_{Ti}) \equiv 0$ for $i = 1, ..., K_t$, $t = T, ..., 2$, $n = 1, ..., N$.

Since new cutting planes tighten the value function in regions that get explored under the current policy, it can be shown that the algorithm converges to the optimal policy in a finite number of steps if the problem is linear (Philpott and Guan 2008, Guigues 2016).

To approximate the value function under the Nested CVaR, we adopt the method described in Shapiro et al. (2013) who propose to change the probability weights of the cutting planes. We refer to Shapiro et al. (2013), Philpott et al. (2013), or Löhndorf and Wozabal (2021) for a detailed description.

5. Risk factor dynamics

In this section, we describe how we model the dynamics of the risk factors as stochastic processes. We include random variables for natural inflows, currency spot and forward rates, as well as for electricity spot and futures prices.

5.1. Price process

We use the Heath-Jarrow-Morton framework to model the evolution of the electricity price forward curve (Heath et al. 1992). Tradable forward and future contracts in the Nordic power market are not for delivery at a single point in time. Instead, they have delivery periods that stretch over months, quarters, and years. By contrast, a price forward curve estimates the forward price for delivery at specific points in time. The value of the curve at time T is therefore the price of a forward contract with delivery exactly at time T. To achieve this, we use the method of Fleten and Lemming (2003) to construct semi-monthly price forward curves.

In line with Koekebakker and Ollmar (2005), we let the volatility of a forward contract with maturity at T, $\sigma_{t,T}$, be a function of time to maturity $T - t = \tau$. The process that explains movements in the forward curve is then given by

$$\begin{aligned} \frac{dF_{t,T}}{F_{t,T}} &= \sigma_{t,T} dZ_{t,T} = \sigma_\tau dZ_{\tau,t} \\ \mathbb{E}(dZ_{\tau,t}, dZ_{\hat{\tau},t}) &= \rho_{\tau,\hat{\tau}} dt, \quad \tau, \hat{\tau} \in [\tau]. \end{aligned} \tag{19}$$

5.2. Inflow process

We treat inflows into the reservoirs as dependent random variables with aggregate inflow Y_t flowing into two reservoirs according to $Y_t = \zeta Y_{1,t} + (1-\zeta)Y_{2,t}$, where ζ is constant.We then fit the geometric periodic autoregressive (GPAR) model discussed in Shapiro et al. (2013) to the inflow data for the hydropower plant. Shapiro et al. (2013) showed that a first-order periodic autoregressive process provides a good fit to seasonal inflow data from Brazil, and we find that the model is also suitable for our case.

5.3. Exchange rate process

We follow Huchzermeier and Cohen (1996) and model exchange rate dynamics as geometric Brownian motion with drift equal to the interest rate difference,

$$\frac{dQ_{t,t}}{Q_{t,t}} = (r - r_f)dt + \sigma_Q dZ, \tag{20}$$

where σ_Q is the annualized volatility of exchange rate returns.

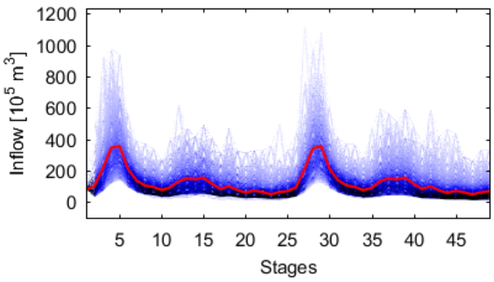
Since uncertainty in the EURNOK exchange rate forward curve is assumed to originate from the spot exchange rate, a single factor model for the currency spot rate is sufficient. Once the spot rate is known, the forward rate is given by covered interest rate parity (Aliber 1973). This is in contrast to electricity markets, where there is uncertainty both in the spot price and the forward curve.

5.4. Lattice Generation

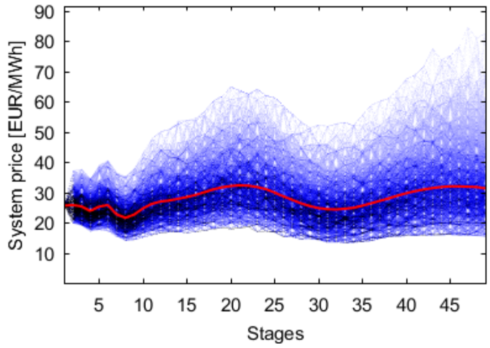
Since inflows and electricity prices exhibit significant correlation, we generate a joint lattice of inflows and forward prices with 100 nodes per stage. Each node contains state variables for natural inflow as well as spot and 48 forward prices of electricity price. As correlation between forward prices and exchange rate are independent, we generate a separate lattice of shifts in exchange rates and then compute the cross-product of the price-inflow lattice and the currency lattice, which results in a lattice with 49 stages and 1000 nodes per stage. Figure 4 shows sample paths for inflow, spot price and currency taken from simulated state transitions of the scenario lattices.

We use March 16, 2015, as the starting date in our case study, so that the planning horizon also ends in March, where reservoirs are typically close to empty.

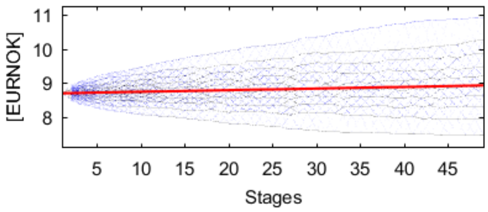
As starting values, we use the mean inflow recorded for the second half of March 2015 as well as currency rate and prices of monthly, quarterly, and yearly contracts on this date.



(a) Inflow lattice, 100 nodes



(b) Spot price lattice, 100 nodes



(c) Currency lattice, 10 nodes

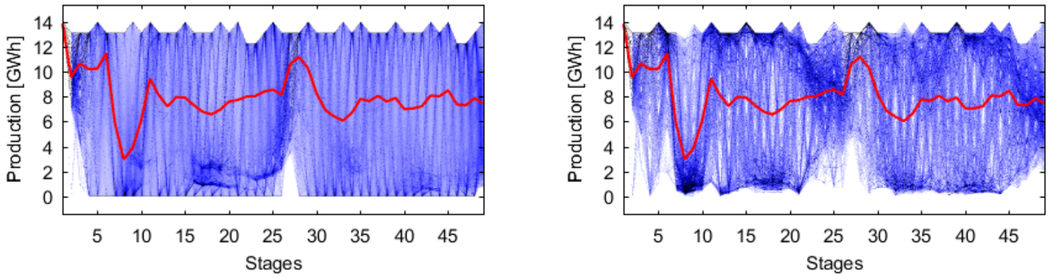
Figure 4: Simulated time series (blue: individual scenarios, red: sample average

For the hedging problem, we treat production as an exogenous, stochastic variable W_t. To generate samples for this variable, we first solve the hydropower scheduling problem using the price-inflow-lattice, and then simulate the optimal policy. Recorded production decisions are then clustered together with the price-inflow-states encountered during the forward simulations, so that we end up with a mapping of production states to price-inflow states.

Figures 5(a) and 5(b) show simulated and discretized production decisions. We thus conclude the that main characteristics of the random production process, such as mean and variability, remain intact.

6. Numerical results

In this section, we summarize the numerical results of the hedging model. The analysis uses the scenario lattices presented in Section 5.4 and the coefficient values in Appendix B. ADDP is run for 500 iterations which ensured convergence of all instances, and all simulated results are based on 10^5 simulations. A sensitivity analysis of results obtained using 500 iterations and 10^5 simulations can be found in Appendix Appendix C. All code has been implemented in MATLAB and R. We used the QUASAR for MATLAB to model and solve the problems in



(a) Simulated production decisions

(b) Discretized production decisions

Figure 5: Simulated time series (blue: individual scenarios, red: sample average

Available contracts	None	All	All	No monthly	No monthly
α	-	0.05	0.1	0.05	0.1
Mean	40.50	40.26	40.23	40.35	40.35
Std	7.99	4.56	4.34	6.10	5.97
VaR(5%)	28.22	33.06	33.23	30.61	30.84
VaR(1%)	24.25	30.48	30.68	27.17	27.44
CVaR(5%)	25.82	31.45	31.66	28.53	28.75
CVaR(1%)	22.65	29.25	29.44	25.64	25.85
Annular prod (GWh)	192.18	192.17	192.08	192.22	192.15

Table 1: Mean value and statistical measures of the terminal cash flows in million NOK for a given risk parameter α.

(Löhndorf 2017). All computations were done on a computer with 32 GB memory and 3.6 GHz CPU speed.

6.1. Hedging performance

We begin our discussion with the hedging model. We are particularly interested in the risk reduction that hedging can provide, compared with no hedging. We also seek to explain the decision policies driving this reduction. Statistical measures of the discounted terminal cash flows over the two-year horizon were used to quantify the hedging effect. We report the mean and standard deviation of the terminal discounted cash flows as well as the terminal VaR and CVaR at 0.05 and 0.01 quantiles. In Table 1, we report results for instances without hedging (None), with hedging using all available contracts (all), with hedging but excluding monthly contracts (No monthly).

The terminal VaR and CVaR in Table 1 illustrate that dynamic hedging clearly reduces risk by utilizing currency forwards and power futures contracts. This comes at a small price, since mean terminal cash flows are slightly lower, which can be attributed to transaction costs. Risk reduction is also evident when using only quarterly and annual contracts, whereby means are higher where trading in monthly contracts is allowed.

Available contracts	All	All	All except monthly	All except monthly
α	0.05	0.1	0.05	0.1
HR	1.543	1.582	0.479	0.514
HR Q	0.707	0.719	0.479	0.493
HR Y	0.256	0.274	0.172	0.184

Table 2: Hedge ratio (HR): short position in power futures divided by power production. HR Q: HR before delivery quarter. HR Y: hedge ratio before delivery year.

Available contracts	All	All	All except monthly	All except monthly
α	0.05	0.1	0.05	0.1
Hedge ratio	0.563	1.120	0.589	0.950
Maturity 1-5	0.397	0.560	0.684	0.806
Maturity 6-15	0.090	0.100	0.079	0.046
Maturity 16-30	0.192	0.186	0.053	0.049
Maturity 31-48	0.321	0.153	0.184	0.099

Table 3: Amount of trading in the currency forward market for different versions of the model. The first row denotes the currency hedge ratio, found by dividing the mean total short position in currency forward contracts minus taxes by the mean total cash flows from the entire portfolio of production and hedging, both in EUR. Thus, a full hedge will be obtained by a hedge ratio of 1. The next rows shows the percentage of trades performed in different intervals of time to maturities, denoted in semi-months.

We find that optimal hedge ratios for power futures highly depend on the availability of monthly contracts. Table 2 shows the average hedge ratio of the four model variants. The hedge ratio is the percentage of production that is hedged by power futures. The table reports average hedge ratio before the start of a new quarter (HQ) and before the start of the first year (HY). The large difference between quarterly and total hedge ratios, indicates that a lot of hedging takes place in monthly contracts that are traded shortly prior to delivery. Monthly contracts also permit precision hedging due to shorter delivery periods that better follow available inflows.

The extent of trading in currency forwards is shown in Table 3. The table illustrates that the currency market is used extensively. Currency forwards with shorter maturity times are preferred, which is similar to the trading activity in power futures. However, the amount of trading in contracts with longer times to maturity is larger than for the power futures.

6.2. Comparison of sequential and integrated approach

So far, we have considered production and hedging decisions sequentially. We now compare what happens when both decisions are integrated within a single model. We test the integrated model for different parameters λ and α of the dynamic risk measure. Results of this experiment are reported in Table 4. We can see that figures do not change dramatically when using the integrated model, which is in line with the finding in Wallace and Fleten (2003). We observe a few percent decrease in average profits and all VaR/CVaR which indicates that a sequential approach fares better. We attribute this to a supposed near-optimality of a sequential approach

Model type	Integrated	Integrated	Integrated	Integrated	Sequential
Available contracts	All	All	All	All	All
α	0.05	0.1	0.05	0.1	0.1
λ	1	1	0.5	0.5	1
Mean	37.23	37.73	39.50	39.69	40.23
Std	4.60	4.77	4.57	4.64	4.34
VaR(5%)	30.30	30.46	32.45	32.48	33.23
VaR(1%)	28.13	28.06	29.95	29.99	30.68
CVaR(5%)	28.99	29.02	30.92	30.96	31.66
CVaR(1%)	27.13	27.07	28.75	28.80	29.44
Annual prod (Gwh))	177.28	179.46	187.16	188.93	192.08

Table 4: Results of the integrated production and hedging model.

	W/o currency hedging	**W/o currency hedging**	**W/ currency hedging**	**Change**
α	0.05	0.1	0.1	
Mean	40.43	40.39	40.23	
Std	4.99	4.82	4.34	-9.96%
VaR(5%)	32.45	32.71	33.23	1.59%
VaR(1%)	29.42	29.76	30.68	3.09%
CVaR(5%)	30.60	30.91	31.66	2.43%
CVaR(1%)	28.05	28.32	29.44	3.95%
Annual prod	192.26	192.15	192.08	

Table 5: Effect of currency hedging on terminal cash flows. The last column shows the percentage difference between the risk measures with and without currency hedging.

as well the effect that the objective of the model (nested CVaR) deviates from what is being measured afterwards (terminal CVaR).

6.3. Effect of currency hedging

One of the main contributions of this article is the inclusion of currency risk and currency derivatives in a real options model of hydropower production. We compare hedging performance where currency forwards can be traded with performance where currency derivatives cannot be traded to quantify the effect of currency hedging. The results of this analysis are summarized in Table 5. The results illustrate that the standard deviation of terminal cash flows decreases whereas VaR and CVaR increase, which indicates that currency hedging does reduce the market risk of the producer. The magnitude of this change is small - less than 10% for all risk measures. We attribute this to the low volatility of the exchange process compared with that of the electricity price process.

6.4. Comparison with heuristics

Wang et al. (2015) show that simple hedging strategies might yield a better or equivalent hedging performance than more advanced procedures. We therefore replicate the hedging strategy of a Norwegian hydropower producer, and compare its performance to the performance of our model. Sanda et al. (2013) show that most Norwegian companies use a heuristic approach,

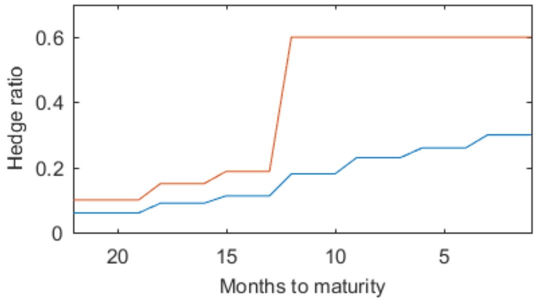


Figure 6: Typical hedge ratio of a real company for a particular delivery period, e.g. March 2016. For the given time to maturity, the hedge ratio must lie inside the corridor.

Approach	Static HR (lower)	Static HR (upper)	Dynamic hedging
α	–	–	0.1
Mean	40.46	40.44	40.39
Std	5.88	5.10	4.84
VaR(5%)	31.40	32.50	32.71
VaR(1%)	28.52	29.63	29.76
CVaR(5%)	29.64	30.76	30.91
CVaR(1%)	27.21	28.33	28.32
Annual prod (GWh)	192.16	192.11	192.15
HR	0.302	0.584	1.505
HR Q	0.301	0.555	0.667
HR Y	0.259	0.421	0.255

Table 6: Results from hedging with a heuristic. Top: descriptive statistics of terminal cash flows. Bottom: Hedge rations as in Table 2.

with specific hedge ratio ranges for different times to maturity. The time horizon of our model is two years. We therefore use the hedging strategy of a firm whose hedging activity also begins two years prior to maturity. Figure 6 displays a slightly modified version of the lower and upper bounds of the firm's required hedge ratios for delivery in a given month, as shown in Sanda et al. (2013).

To test the performance of the heuristic approach, we include the hedge ratio ranges as constraints on financial short positions into hedging model. We perform two separate simulations; one where the hedge ratio is set close to the lower bound and one where it is closer to the upper bound of the hedging corridor of the firm. As the currency hedging strategy is unknown, trading of currency derivatives is not considered. Results are displayed in Table 6.

The results indicate that the heuristic approaches perform worse in terms of terminal risk measures than the dynamic hedging model, although differences are small. We do observe a noticeable difference in the hedging strategy as the dynamic strategy hedges closer to delivery.

7. Conclusions

We present a dynamic hedging model for risk management of hydropower producers. We model the hedging problem as a sequential problem in which optimal production decisions are made first and hedging cash flows from production using currency forwards and electricity futures is done in a second step. Both, the production planning problem and the hedging problem, are modeled as Markov decision processes. We model electricity spot prices, futures prices, natural inflows as a joint multivariate stochastic process. The decision model considers trading in in currency forward contracts and monthly, quarterly, and yearly power futures. Risk preferences are modeled using the nested CVaR dynamic risk measure.

Numerical findings show that the dynamic hedging model leads to a significant risk reduction with a 23% increase of the 5%-CVaR of terminal cash flows. A particular feature of the dynamic hedging model is to find the right trade-off between various contracts with different delivery periods.

We report a moderate effect of considering currency derivatives in the hedging strategy. Including currency derivatives results in decrease in variance of 9.96% and an increase in CVaR(5%) of 2.43% for the terminal cash flows.

We also compare a sequential model with an integrated model in which production and hedging are optimized simultaneously. We find that the sequential approach achieves higher values of mean, VaR, and CVaR of the terminal discounted cash flows. We attribute this result to the supposed near-optimality of the sequential approach and that the objective of the model (nested CVaR) deviates from what is being measured afterwards (terminal CVaR). To arrive at a definitive answer, we must either find an alternative measure of risk that measures what is being optimized or find a way to speed up convergence when a terminal CVaR is used (which proves to be difficult).

We also investigate how the dynamic hedging model compares to a static hedging strategy that is based on hedge corridors – a strategy that is used extensively by Norwegian hydropower producers. The performance of the hedge ratio approach is slightly worse than the dynamic model but differences were small, which implies that a simple hedge ratio approach can be quite efficient. This is in line with the findings of Wang et al. (2015). Nonetheless, a dynamic hedging can be used to calibrate and justify static hedging strategies, which otherwise are often based on experience of traders and risk managers.

Our analysis also shows that it is important to include monthly contract, as these allow for more precise hedging of expected production in a given month than the contracts with longer maturity periods. More precise hedging increases flexibility in the timing of trading due to their shorter delivery period and as such decreases price risk.

Future work should look into the effect of increasing the time resolution of hedging decisions which typically take place on a daily basis as well as the effect of transaction cost (bid-ask spread, market impact cost) that often play an important role in the selection of contracts and the timing of their trades.

Data Deposition Information

No datasets have been used.

References

Abgottspon, H., Andersson, G., 2014. Medium-term optimization of pumped hydro storage with stochastic intrastage subproblems, in: IEEE Power Systems Computation Conference.

Aliber, R.Z., 1973. The interest rate parity theorem: A reinterpretation. Journal of Political Economy 81, 1451–1459.

Benth, F.E., Benth, J.S., Koekebakker, S., 2008. Stochastic modelling of electricity and related markets. World Scientific Publishing, Singapore.

Bjerksund, P., Stensland, G., Vagstad, F., 2011. Gas storage valuation: Price modelling v. optimization methods. The Energy Journal 32, 203–227.

Boda, K., Filar, J.A., 2006. Time consistent dynamic risk measures. Mathematical Methods of Operations Research 63, 169—186.

Dimoski, J., Nersten, S., Fleten, S.E., Löhndorf, N., 2018. Hydropower reservoir management using multi-factor price model and correlation between price and local inflow, in: IAEE International Conference, Groningen, Netherlands.

Dupuis, D., Gauthier, G., Godin, F., et al., 2016. Short-term hedging for an electricity retailer. The Energy Journal 37.

Fleten, S.E., Bråthen, E., Nissen-Meyer, S.E., 2010. Evaluation of static hedging strategies for hydropower producers in the Nordic market. The Journal of Energy Markets 3, 1–28.

Fleten, S.E., Lemming, J., 2003. Constructing forward price curves in electricity markets. Energy Economics 25, 409–424.

Fleten, S.E., Wallace, S.W., Ziemba, W.T., 2002. Stochastic programming in energy, in: Greengard, C., Ruszczyński, A. (Eds.), Decision Making Under Uncertainty. The IMA Volumes in Mathematics and its Applications. Springer, New York. volume 128 of *IMA Volumes on Mathematics and Its Applications*, pp. 71–93.

Godin, F., 2016. Minimizing CVaR in global dynamic hedging with transaction costs. Quantitative Finance 16, 461–475.

Goel, A., Tanrisever, F., 2017. Financial hedging and optimal procurement policies under correlated price and demand. Production and Operations Management 26, 1924—1945.

Guigues, V., 2016. Convergence analysis of sampling-based decomposition methods for risk-averse multistage stochastic convex programs. SIAM Journal on Optimization 26, 2468–2494.

Heath, D., Jarrow, R., Morton, A., 1992. Bond pricing and the term structure of interest rates a new methodology for contingent claims valuation. Econometrica 60, 77–105.

Houmøller, A.P., 2017. Investigation of forward markets for hedging in the Danish electricity market. Technical Report. Houmoller Consulting. Middelfart, Denmark.

Huchzermeier, A., Cohen, M.A., 1996. Valuing operational flexibility under exchange rate risk. Operations Research 44, 100–113.

Iliadis, N., Pereira, V., Granville, S., Finger, M., Haldi, P.A., Barroso, L.A., 2006. Benchmarking of hydroelectric stochastic risk management models using financial indicators, in: IEEE Power Engineering Society General Meeting, Montreal, Quebec, Canada.

Kettunen, J., Salo, A., Bunn, D.W., 2009. Optimization of electricity retailer's contract portfolio subject to risk preferences. IEEE Transactions on Power Systems 25, 117–128.

Kiesel, R., Paraschiv, F., Sætherø, A., 2019. On the construction of hourly price forward curves for electricity prices. Computational Management Science 16, 345—369.

Koekebakker, S., Ollmar, F., 2005. Forward curve dynamics in the Nordic electricity market. Managerial Finance 31, 73–94.

Lamond, B.F., Boukhtouta, A., 1996. Optimizing long-term hydro-power production using Markov decision processes. International Transactions in Operational Research 3, 223–241.

Liu, S.D., Jian, J.B., Wang, Y.Y., 2010. Optimal dynamic hedging of electricity futures based on Copula-GARCH models, in: IEEE International Conference, IEEE, Macau, China. pp. 2498–2502.

Löhndorf, N., Shapiro, A., 2018. Modeling time-dependent randomness in stochastic dual dynamic programming. European Journal of Operational Research 2, 650–661.

Löhndorf, N., Wozabal, D., Minner, S., 2013. Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming. Operations Research 61, 810–823.

Löhndorf, N., 2017. Quasar optimization software 2.3. URL: `https://quantego.com`.

Löhndorf, N., Wozabal, D., 2021. Gas storage valuation in incomplete markets. European Journal of Operational Research 288, 318 – 330.

Mo, B., Gjelsvik, A., Grundt, A., 2001. Integrated risk management of hydro power scheduling and contract management. IEEE Transactions on Power Systems 16, 216–221.

Murphy, F., Oliveira, F.S., 2010. Developing a market-based approach to managing the US strategic petroleum reserve. European Journal of Operational Research 206, 488–495.

Nadarajah, S., Margot, F., Secomandi, N., 2015. Relaxations of approximate linear programs for the real option management of commodity storage. Management Science 61, 3054–3076.

Nasdaq Oslo ASA and Nasdaq Clearing AB, 2018. Fee list - commodity derivatives. URL: `http://www.nasdaqomx.com/digitalAssets/107/107680_180301-joint--appendix-7--fee-list.pdf`.

Pereira, M., Pinto, L., 1991. Multi-stage stochastic optimization applied to energy planning. Mathematical Programming 52, 359–375.

Philpott, A., Ferris, M., Wets, R., 2016. Equilibrium, uncertainty and risk in hydro-thermal electricity systems. Mathematical Programming 157, 483–513.

Philpott, A., Guan, Z., 2008. On the convergence of stochastic dual dynamic programming and related methods. Operations Research Letters 36, 450–455.

Philpott, A., de Matos, V., Finardi, E., 2013. On solving multistage stochastic programs with coherent risk measures. Operations Research 61, 957–970.

Ruszczynski, A., Shapiro, A., 2006. Conditional risk mappings. Mathematics of Operations Research 31, 544—561.

Sanda, G.E., Olsen, E.T., Fleten, S.E., 2013. Selective hedging in hydro-based electricity companies. Energy Economics 40, 326—338.

Shapiro, A., 2009. On a time consistency concept in risk averse multistage stochastic programming. Operations Research Letters 37, 143–147.

Shapiro, A., Tekaya, W., da Costa, J.P., Soares, M.P., 2013. Risk neutral and risk averse stochastic dual dynamic programming method. European Journal of Operational Research 224, 375–391.

SINTEF, 2017. EOPS - one area power-market simulator. URL: `http://www.sintef.no/en/software/eops-one-area-power-market-simulator/`.

Stulz, R.M., 1996. Rethinking risk management. Journal of Applied Corporate Finance 9, 8–25.

Thorvaldsen, T., Tyssing, N., Samuelsen, J., 2018. Kraftverksbeskatning. . KPMG Norway. Oslo, Norway.

Wallace, S.W., Fleten, S.E., 2003. Stochastic programming models in energy, in: Ruszczyński, A.P., Shapiro, A. (Eds.), Handbooks in OR and MS. Elsevier Science B.V.. volume 10. chapter 10, pp. 637–677.

Wang, Y., Wu, C., Yang, L., 2015. Hedging with futures: Does anything beat the naïve hedging strategy? Management Science 61, 2870–2889.

Wolfgang, O., Haugstad, A., Mo, B., Gjelsvik, A., Wangensteen, I., Doorman, G., 2009. Hydro reservoir handling in Norway before and after deregulation. Energy 34, 1642–1651.

Zanotti, G., Gabbi, G., Geranio, M., 2010. Hedging with futures: Efficacy of GARCH correlation models to European electricity markets. Journal of International Financial Markets, Institutions and Money 20, 135–148.

Appendix A. Cash flows and balancing of electricity contracts

The following details how positions in electricity futures are tracked, depending on trading, listing and de-listing, and varying contract delivery periods. We also explain the cash flows from this trading, including the effect of transaction costs, product listing, settlement before and during delivery, and taxes.

Let us introduce decision variables for short positions in power futures. Denote $u_{t,Mi}$, $u_{t,Qj}$, and $u_{t,Y1}$ [MWh] the total short position at stage t in futures contracts with delivery in i months, j quarters and 1 year. The last index denotes contracts that have not yet entered delivery. We use $u_{t,M}$, $u_{t,Q}$ and $u_{t,Y}$ [MWh] to denote short positions of contracts that are currently in delivery. Variables $w_{t,Mi}$, $w_{t,Qj}$, and $w_{t,Y1}$ [MWh] denote *new* short positions that enter at stage t. If the delivery period of a contract exceeds the model horizon $\hat{T} = 49$ semi-months, then the corresponding decision variable ($w_{t,Mi}$, $w_{t,Qj}$ or $w_{t,Y1}$) is set to zero to guarantee that no trading takes place.

Balance constraints for short positions in power futures depend on whether time stage t represents the first or second part of a month, the beginning of a new quarter or the beginning of a new year. If t represents the second part of a month, then the total short position is given by the previous stage value plus new short positions for contracts not yet in delivery.

$$\begin{gathered} u_{t,M} = u_{t-1,M}, \quad u_{t,Q} = u_{t-1,Q}, \quad u_{t,Y} = u_{t-1,Y} \\ u_{t,Mi} = u_{t-1,Mi} + w_{t,Mi}, \quad i = [1, ..., 6] \\ u_{t,Qj} = u_{t-1,Qj} + w_{t,Qj}, \quad j = [1, ..., 8] \\ u_{t,Y1} = u_{t-1,Y1} + w_{t,Y1} \end{gathered} \tag{A.1}$$

When t represents the first part of a month, the contract that was 1 month ahead (M1) in $t-1$ goes into delivery, M2 becomes M1, M3 becomes M2, etc. A new contract is introduced for delivery in six months (M6). If t represents the first part of the month but not a new quarter, then balance constraints for month contracts are given by (A.2). The remaining relationships in (A.1) do not change, that is, they remain valid for all stages t.

$$\begin{gathered} u_{t,Mi} = u_{t-1,Mi+1} + w_{t,Mi}, \quad i = [1, ..., 5] \\ u_{t,M6} = w_{t,M6} \end{gathered} \tag{A.2}$$

Using the same logic, balance constraints for quarter contracts for stages marking the beginning of a quarter, but not a new year, are given by (A.3).

$$\begin{aligned} u_{t,Qj} &= u_{t-1,Qj+1} + w_{t,Qj}, \quad j = [1, ..., 7] \\ u_{t,Q8} &= w_{t,Q8} \end{aligned} \tag{A.3}$$

If t represents the beginning of a year, then balance constraints for annual contracts are given by (A.4).

$$u_{t,Y1} = 0 \tag{A.4}$$

Having established balance constraints for short positions in power futures, let us define variables and restrictions for the portfolio of power futures. Denote $y^F_{t,t}$ as cash flow from power futures trading in t that can take positive and negative values, and is part of the value function of the hedging problem (11). The currency spot rate at which cash flows occur is not known in advance. $y^F_{t,t}$ is therefore denoted in EUR and $y^C_{t,t}$ is denoted in NOK.

Variable $y^F_{t,T}$ tracks the committed, positive part of the cash flows from power futures trading that will occur at time T. Variable $y^F_{t,T}$, where $T > t$ is not part of the value function. It is only used to store the positive part of the cash flows that will occur in subsequent periods. In (A.11), the negative part of the cash flows is added to the positive flows to obtain time t cash flows $y^F_{t,t}$. Variable $y^F_{t,t+1}$ stores two types of cash flows. The first is related to changes in the value of the portfolio of contracts not yet in delivery. All contracts are marked-to-market regularly. We therefore need to store the forward prices in stage t to calculate the price changes in stage $t+1$. The second type of cash flow is associated with contracts in delivery. Variable $y^F_{t,T}$ for $T > t+1$ only stores the positive part of cash flows associated with contracts in delivery. The longest delivery period spanned by any of the available contracts is 24 semi-months. It is therefore only necessary to define $y^F_{t,T}$ for $T = [t+1, ..., t+24]$.

We first consider the case where t represents the *first part of a month*. The cash flow balances will then be given by

$$\begin{aligned} y^F_{t,t+1} &= y^F_{t-1,t+1} + (1-\gamma_c)\Big(\sum_{i=1}^{6} u_{t,Mi}F_{t,Mi} + \sum_{j=1}^{8} u_{t,Qj}F_{t,Qj} + u_{t,Y1}F_{t,Y1}\Big) \\ y^F_{t,T} &= y^F_{t-1,T}, \quad t+2 \le T \le t+23 \\ y^F_{t,T} &= 0, \qquad\qquad T = t+24 \end{aligned} \tag{A.5}$$

Only $y^F_{t,t+1}$ is updated in this case. This is because the next stage is in the same month as the current stage, so that no new contracts go into delivery.

If t is the *second part of a month*, then multiple contracts can potentially enter into delivery in the upcoming stage $(t+1)$. The price and position of the futures contracts that go into delivery must thus be stored. The cash flows for the next 2, 6, or 24 periods must also be stored. How long they will be stored depends on whether the contract is monthly, quarterly, or yearly. We introduce the indicator functions $\mathbb{I}_Q$ and $\mathbb{I}_Y$ to make the formulation more compact. Function values are equal to 1 if the next stage $(t+1)$ marks the beginning of a new quarter and year, respectively, and 0 otherwise. With this definition, cash flow balances are given by

$$\begin{aligned} y^F_{t,t+1} = y^F_{t-1,t+1} + (1-\gamma_c)\Big(&\frac{u_{t,M1}F_{t,M1}}{2} + \mathbb{I}_Q\frac{u_{t,Q1}F_{t,Q1}}{6} + \mathbb{I}_Y\frac{u_{t,Y1}F_{t,Y1}}{24} \\ &+ \sum_{i=2}^{6} u_{t,Mi}F_{t,Mi} + (1-\mathbb{I}_Q)u_{t,Q1}F_{t,Q1} + \sum_{j=2}^{8} u_{t,Qj}F_{t,Qj} + (1-\mathbb{I}_Y)u_{t,Y1}F_{t,Y1}\Big) \end{aligned} \quad \text{(A.6)}$$

$$y^F_{t,t+2} = y^F_{t-1,t+2} + (1-\gamma_c)\Big(\frac{u_{t,M1}F_{t,M1}}{2} + \mathbb{I}_Q\frac{u_{t,Q1}F_{t,Q1}}{6} + \mathbb{I}_Y\frac{u_{t,Y1}F_{t,Y1}}{24}\Big) \quad \text{(A.7)}$$

$$y^F_{t,t+i} = y^F_{t-1,t+i} + (1-\gamma_c)\Big(\mathbb{I}_Q\frac{u_{t,Q1}F_{t,Q1}}{6} + \mathbb{I}_Y\frac{u_{t,Y1}F_{t,Y1}}{24}\Big), \quad i = [3,...,6] \quad \text{(A.8)}$$

$$y^F_{t,t+i} = y^F_{t-1,t+i} + (1-\gamma_c)\mathbb{I}_Y\frac{u_{t,Y1}F_{t,Y1}}{24}, i = [7,...,23] \quad \text{(A.9)}$$

$$y^F_{t,t+24} = (1-\gamma_c)\mathbb{I}_Y\frac{u_{t,Y1}F_{t,Y1}}{24} \quad \text{(A.10)}$$

In (A.11), we formulate the stage t cash flows from power trading $(y^F_{t,t})$. As with currency forwards, the expression consists of the committed, positive cash flows saved in $y^F_{t-1,t}$ and all negative cash flows. Note that we must subtract $w_{t,Mi}, w_{t,Qj}$ and $w_{t,Y1}$ from positions $u_{t,Mi}, u_{t,Qj}$, and $u_{t,Y1}$ to obtain the negative part of the cash flows associated with price changes in contracts prior to delivery, because these are based on previous positions. We also include variable transaction costs, c_F [EUR/MWh].

$$\begin{aligned} y^F_{t,t} = y^F_{t-1,t} + (1-\gamma_c)\Big[&-\big(\frac{u_{t,M}}{2} + \frac{u_{t,Q}}{6} + \frac{u_{t,Y}}{24}\big)F_{t,t} \\ &- \Big(\sum_{i=1}^{6}(u_{t,Mi} - w_{t,Mi})F_{t,Mi} + \sum_{j=1}^{8}(u_{t,Qj} - w_{t,Qj})F_{t,Qj} + (u_{t,Y1} - w_{t,Y1})F_{t,Yi}\Big) \\ &- c_F\big(\sum_{i=1}^{6} w_{t,Mi} + \sum_{j=1}^{8} w_{t,Qj} + w_{t,Y1}\big)\Big] \end{aligned} \quad \text{(A.11)}$$

The cash flows from power trading (A.11) enters the value function for hedging.

Parameter	Explaination	Unit	Value
$\overline{v_{1,t}}$	Upper bound for reservoir volume in reservoir 1	Mm^3	44.5
$\overline{v_{2,t}}$	Upper bound for reservoir volume in reservoir 2	Mm^3	22.5
$\underline{v_{2,t}}$	Lower bound for reservoir volume in reservoir 2 between October 16 and May 24	Mm^3	0
$\underline{v_{2,t}}$	Lower bound for reservoir volume in reservoir 2 between May 25 and October 15	Mm^3	15.05
κ	Energy coefficient	kWh/m^3	0.63
ξ	Maximum allowed water flow in turbine	m^3/s	17
r	Continuously compounded annual risk free interest rate used in discount factor, given by 3-year NIBOR	−	0.0126
c_F	Transaction costs for trading power futures at NASDAQ	EUR/MWh	0.0144
γ_r	Resource rent tax rate	−	0.357
γ_c	Corporate tax rate	−	0.23
ζ	Inflow split coefficient	−	0.395

Table B.7: Coefficients and constants used for numerical studies

	Mean CF	Std CF	VaR(5%)	VaR(1%)	CVaR(5%)	CVaR(1%)
Mean	40.20	4.33	33.35	30.81	31.79	29.60
Std	0.106	0.050	0.145	0.179	0.165	0.204

Table C.8: Sensitivity of statistical measures based on six separate runs.

Appendix B. Coefficients and Parameter Values

For all simulations, we used coefficient and parameter values given in Table B.7. The tax rates γ_r and γ_c (Thorvaldsen et al. 2018) and transaction costs c_F (Nasdaq Oslo ASA and Nasdaq Clearing AB 2018) are correct as of 2018. As in Dupuis et al. (2016), the variable transaction costs for trading at NASDAQ are given by the sum of the market trading fee (0.0045 EUR/MWh) and clearing fee (0.0099 EUR/MWh). This clearing fee is applicable if the total quarterly volume cleared by the firm is below 3 TWh. The time-dependent discount factor β_t is given by $\beta_t = \exp(-r\Delta t)$, where Δt denotes the length of the semi-month t. The energy coefficient κ is based on the empirical relationship between production and water dispatch, and calculations considering the mean empirical reservoir level and turbine/generator efficiency rate. It has been found to be slightly lower than the one currently used by the plant.

Appendix C. Sensitivity Analysis of Hedging Results

In this section, we present some results illustrating the sensitivity of using 500 forward-backward passes and 10^5 iterations. This has been done by performing six separate runs of the hedging model with $\alpha = 0.1$ and trading in all contracts, and comparing the mean and standard deviation of the main statistical measures included in e.g. Table 1. These results are summarized in Table C.8. The computational time of each run is approximately 4.5 hours, and its memory usage is close to the maximum capacity.

As Table C.8 shows, the obtained results are subject to a certain degree of uncertainty. The standard deviations indicate that all statistical measures are stable in the first two digits, while there is some uncertainty in the third digit. This indicates that the results are sufficiently precise to assess and compare the general risk performance of the model variants, but increasing the number of passes and simulations would result in more precise results.