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# Effective Rate Evaluation of RIS-Assisted Communications Using the Sums of Cascaded $\alpha$ - $\mu$ Random Variates

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**ABSTRACT** This paper begins with the characterization of the sum of M versions of cascaded  $\alpha$ - $\mu$  (equivalently,  $N*(\alpha$ - $\mu$ )) distributed random variates (RVs), and later on, explores the feasibility and applicability of the obtained results in the recent proposed reconfigurable intelligent surface (RIS)-assisted communications. More specifically, the sum of cascaded  $\alpha$ - $\mu$  distributed RVs is exactly characterized by the probability density function (PDF) expressed in terms of the programmable multivariate Fox's H-function, and approximated with the mixture of Gaussian (MoG) distribution. The approximation accuracy is further validated by the Kolmogorov-Smirnov (KS) goodness of fit test. Based on the obtained statistical results, a promising application, i.e., RIS-assisted communication, is considered and evaluated from the effective rate perspective. The exact, asymptotic, and approximated effective rate are derived with closed-form expressions. Finally, our analytical results are further validated using the Monte-Carlo simulation, their accuracy is confirmed with the analytical error indicator. One can conclude that (i) the exact effective rate expression is complex in the manner of mathematical representation; (ii) the MoG-based effective rate expression is simple and provides a highly tight approximation to the exact expression, besides it indeed acts as a highly tight upper bound for the exact effective rate expression; and (iii) the MoG-based solution outperforms the multivariate Fox's H-function based approach with reducing complexity.

**INDEX TERMS** Effective rate, quality-of-service (QoS), cascaded  $\alpha$ - $\mu$ , reconfigurable intelligent surfaces (RIS), Kolmogorov-Smirnov (KS) goodness of fit test, Fox's H-function, mixture of Gaussian (MoG).

# I. INTRODUCTION

PUTURE generation of communication systems are bound to change the landscape of industry thanks to the Industrial Internet of Things (IIoT) enabled by extreme low latency and massive number of connections. Due to the highly time-varying characteristics of the wireless channel, deterministic delay constraints are normally difficult to meet. Instead, the statistical quality-of-service (QoS) provisioning turns out to be a useful instrument to evaluate the delay bound QoS guarantees for realistic wireless real-time traffic.

The effective rate serves a statistical QoS metric defined as the maximum constant arrival rate that a time-varying service process can support with statistical latency guarantees [1]– [3]. It acts as a joint mathematical framework, connecting physical layer and link layer, and is used to explore the performance of various wireless networks under certain delay constraint [3]. The concept of effective rate has been widely applied over the last few years to investigate the trade-off among latency, reliability, energy efficiency, and security. Examples can be found in [3], especially in some traditional and future promising applications, including Cellular communication, device-to-device (D2D) communications, peer-to-peer video streaming, visible light communication, full-duplex communications, non-orthogonal multiple access (NOMA) [4]–[7], vehicle communications [8], and ultrareliable low latency communications (URLLC).

#### A. BACKGROUND AND RELATED WORKS

The effective rate investigation of these applications are conducted under different fading channel models. The effective rate metric of various small-scale (or multipath), large-scale (or shadowing), and composite fading channels was widely examined, including Weibull [9],  $\alpha$ - $\mu$  [10] (equivalently, generalized gamma),  $\kappa$ - $\mu$  [11], generalized- $\mathcal{K}$  [12], [13], Fisher-Snedecor  $\mathcal{F}$  [14],  $\alpha$ - $\eta$ - $\mu$ /Gamma [15], two-wave with diffuse-power (TWDP) [16], double shadowed Rician [17],  $\alpha$ - $\eta$ - $\kappa$ - $\mu$ <sup>1</sup> [18], [19],  $\kappa$ - $\mu$ /inverse gamma and  $\eta$ - $\mu$ /inverse gamma [20], etc. In [21], the authors investigated the effective rate of multiple-input multiple-output (MIMO) keyhole channels (i.e., rank one MIMO channel). Rare efforts have ever been seen about the investigation for the analytical analysis of effective rate over cascaded fading channels.

## **B. MOTIVATIONS**

To this end, the focus of this paper lies in the investigation of the effective rate analysis over cascaded fading models. It is worth mentioning that the cascaded fading models can capably represent a mobile-to-mobile (M2M) scenario both through analysis and measurement [22]. The cascaded fading models can be physically interpreted as follows: the received signals are generated by the product of a large number of rays reflected via N statistically independent scatters [23]-[25]. Those models are applicable for the multi-hop nonregenerative amplify-and-forward (AF) relaying system with fixed gain [26], the propagation in the presence of keyholes, the keyhole phenomena in MIMO systems [21], and reconfigurable intelligent surfaces (RIS)-assisted wireless system [27]. An RIS, in other words, is also equivalent to digitally controllable scatters and software-defined surface [28]. The large number of small, low-cost, and passive artificial 'metaatoms' embedded in an RIS are able to smartly change the reflection direction towards any desired users by tuning a series of phase shifters [29], [30]. More specifically, the authors of [29], [30] explored the feasibility of applying the RIS technology to the future sixth-generation (6G) and beyond fifth-generation (5G) networks from the viewpoints of modulation and multiple antenna, respectively.

In [24], the authors proposed a flexible and general 'multiplicative' stochastic model, namely, the cascaded  $\alpha$ - $\mu$  ( $N*(\alpha-\mu)$ ) fading channel. It is flexible and mathematical tractable, since it can elaborate the cascaded Rayleigh, Nakagami-m [23], exponential, Gamma, Weibull, and lognormal ( $\alpha \rightarrow \infty, \mu \rightarrow \infty$ ) fading by simply attributing the fading parameters  $\alpha$  and  $\mu$  to selected values. For instance, setting  $\alpha = 2$  and  $\mu = 1$ , it will reduce to cascaded Rayleigh fading, whereas choosing  $\alpha = 2$  and  $\mu = m$  will make it correspond to cascaded Nakagami-m fading. More importantly, the  $\alpha$ - $\mu$  distribution was proposed by Yacoub in [31] to model the small-scale variation of fading signal under line-of-sight conditions [32], [33]. It is physically described with two

key fading parameters, i.e., non-linearity of the propagation medium  $\alpha$  and the clustering of the multipath waves  $\mu$ . The advantage of these two factors is regarded as a useful tool to vividly depict the inhomogeneous surroundings compared with other existing fading models, such as Rayleigh and Nakagami-m. Most of these are based on the assumption of a rarely-encountered scenario of homogeneous (scattering) environment. Fortunately, the  $\alpha$ - $\mu$  fading model was later on found valid and feasible in many realistic situations [34]–[39], including the vehicle-to-vehicle (V2V) communication networks and wireless body area networks (WBAN).

Moreover, the sum of  $\alpha$ - $\mu$  distributed random variates (RVs) has been investigated in [40], where a new  $\alpha$ - $\mu$  distributed RV is used to highly approximate the sum. Similarly, the sums of other general fading models, e.g.,  $\eta$ - $\mu$ ,  $\kappa$ - $\mu$ , Fisher-Snedecor  $\mathcal{F}$ , fluctuating two-ray (FTR), and Fox's H-function distributed RVs are also characterized in [41]–[47], where their application to the maximal-ratio-combining (MRC) diversity receiver are correspondingly investigated with the assistance of the obtained statistical results [42]–[47].

To this end, the core of interest in this paper is to characterize the sum of cascaded  $\alpha$ - $\mu$  RVs. Different from the MRC diversity receivers, in this paper we consider the newly proposed RIS-assisted communications as a possible and promising application based on the obtained statistical results. In continuation with the aforementioned discussion about the effective rate analysis, the usefulness of our findings is further evaluated from the effective rate perspective.

#### C. CONTRIBUTIONS AND ORGANIZATION

The main contributions of this paper are summarized as follows:

- Characterizing the probability density function (PDF) for the sum of *M* version of cascaded α μ distributed RVs, which is expressed in terms of the multivariate Fox's *H*-function. For the purpose of providing a reducing complexity solution, the sum of cascaded α-μ RVs is also modeled in the manner of the mixture of Gaussian (MoG) distribution, the approximation accuracy is further confirmed using the Kolmogorov-Smirnov (KS) goodness of fit test.
- 2) Initially applying the sum of cascaded distributed RVs to the novel RIS-assisted communication, and exploring the feasibility and advantages of using the obtained statistical result to the RIS-assisted communications from the effective rate perspective.
- 3) Deriving the exact, asymptotic, and approximated effective rate expressions of the RIS-aided communications over  $\alpha$ - $\mu$  fading channels. Especially when M=1, the exact effective rate analysis fills the gap of lacking effective rate analysis over cascaded fading channels.
- Validating the accuracy and tightness of the exact and asymptotic behavior of the analytical results with Monte-Carlo simulations.

<sup>&</sup>lt;sup>1</sup>As shown in [18], the  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  distribution includes the  $\alpha$ - $\mu$ ,  $\kappa$ - $\mu$ , and  $\eta$ - $\mu$  distributions as its special cases.

- 5) Providing the asymptotic effective rate slope at high signal-to-noise ratio (SNR) regime.
- 6) One can surely conclude that (i) the exact, asymptotic, and approximated effective rate expressions are analytically correct; (ii) the RIS-aided communications is a suitable application of the sum of cascaded α-μ distributed RVs, and the increase of M leads to an enhancement of the overall effective rate; and (iii) the MoG-based approach is quite advantageous since it provides a simple and easy solution to analyze the effective rate over various wireless fading channels with reducing complexity.

The rest of this paper is organized as follows: Section II provides the preliminary results regarding the sum of cascaded  $\alpha$ - $\mu$  distributed RVs. Based on the obtained results, we present a system configuration of the RIS-assisted communications in Section III. Next, the exact, asymptotic, and approximated effective rate expressions of our proposed system model are subsequently analyzed in Section IV. Section V demonstrates the numerical results and some interesting discussions. Finally, Section VI concludes this paper.

Mathematical Functions and Notations:  $w = \sqrt{-1}$ ,  $f_X(x)$  represents the probability density function (PDF) of RV X,  $H_{p,q}^{m,n}[\cdot]$  is the univariate Fox's H-function [48, Eq. (1.2)],  $\Gamma(\cdot)$  denotes the gamma function.  $H_{p,q;p_1,q_1;\cdots;p_L,q_L}^{m,n_1,n_1;\cdots;p_L,n_L}[\cdot]$  is the multivariate Fox's H-function [48, A.1].  $\operatorname{erf}(\cdot)$  is the error function.  $G_{p,q}^{m,n}[\cdot]$  is the Meijer's G-function and it is a special case of Fox's H-function [48, Eq. (1.111)].  $\mathbb{E}(x)$  is the expectation operator over an RV x.  $\operatorname{Res}[f(x), s]$  represents the residue of function f(x) at pole x = s.

#### **II. PRELIMINARY**

# A. CHARACTERIZATION OF THE SUM OF CASCADED $\alpha$ - $\mu$ RVS

Let  $\mathcal{Z}$  be the product of  $N, N \geq 1$ , independently  $\alpha - \mu$  distributed RVs having parameters  $(\alpha_i, \mu_i)$ , i.e.,  $\mathcal{Z} = \prod_{i=1}^{N} z_i$ , the PDF of  $z_i$  is given by [31]

$$f_{z_{i}}(x) = \frac{\alpha_{i} \mu_{i}^{\mu_{i}} x^{\alpha_{i} \mu_{i} - 1}}{\Omega_{i}^{\alpha_{i} \mu_{i}} \Gamma(\mu_{i})} \exp\left(-\mu_{i} \left(\frac{x}{\Omega_{i}}\right)^{\alpha_{i}}\right)$$

$$\stackrel{(a)}{=} \kappa_{i} H_{0,1}^{1,0} \begin{bmatrix} \lambda_{i} x & - \\ \mu_{i} - \frac{1}{\alpha_{i}}, \frac{1}{\alpha_{i}} \end{bmatrix},$$

$$(1)$$

where  $\kappa_i = \frac{\mu_i^{\frac{1}{\alpha_i}}}{\Omega_i \Gamma(\mu_i)}$ ,  $\lambda_i = \frac{\mu_i^{\frac{1}{\alpha_i}}}{\Omega_i}$ ,  $\Omega_i = \frac{\Gamma(\mu_i)}{\Gamma(\mu_i + \frac{1}{\alpha_i})}$ ,  $i = 1, \dots, N$ .

Step (a) holds by using [48, Eq. (1.125)]. The PDF of  $\mathbb{Z}$  is thereafter given by [24]

$$f_{\mathcal{Z}}(z) = \mathcal{K}_N H_{0,N}^{N,0} \begin{bmatrix} C_N z & - \\ \Phi_1, \cdots, \Phi_N \end{bmatrix}, \qquad (2)$$

where  $\mathcal{K}_N = \prod\limits_{i=1}^N \kappa_i, \ C_N = \prod\limits_{i=1}^N \lambda_i, \ \Phi_i = \left(\mu_i - \frac{1}{\alpha_i}, \frac{1}{\alpha_i}\right)$ . For simplicity, let  $\mathcal{B} = (\Phi_1, \cdots, \Phi_N)$ . As a result,  $\mathcal{Z}$  is also called a cascaded  $\alpha$ - $\mu$  distributed RV.

Apparently, the cascaded  $\alpha$ - $\mu$  distribution is a special case of the Fox's H-function distribution. To this end, applying the results obtained in [47, Eq. (7)], the PDF of the sum of independent but not identically distributed (i.n.i.d.) cascaded  $\alpha$ - $\mu$  distributed RVs, i.e.,  $Z = \sum_{j=1}^{M} Z_j$ , is given in terms of the multivariate Fox's H-function<sup>2</sup>, i.e.,

$$f_{Z}(z) = \frac{\eta_{M}}{z} H \begin{pmatrix} 0, 0 \\ 0, 1 \end{pmatrix} - \begin{pmatrix} 1; 1_{M} \\ N_{j}, 1 \\ 1, N_{j} \end{pmatrix}_{j=1:M} \begin{bmatrix} 1; 1_{M} \\ \mathcal{B} \end{bmatrix}_{j=1:M} \begin{pmatrix} C_{N,j}z \end{pmatrix}_{j=1:M}$$

$$\stackrel{(b)}{=} \frac{\eta_{M}}{(2\pi w)^{M} z} \int_{\mathcal{L}_{1}} \cdots \int_{\mathcal{L}_{M}} \Phi(s_{1}, \cdots, s_{M})$$

$$\times \prod_{j=1}^{M} \phi(s_{j}) (C_{j}z)^{s_{j}} ds_{1}, \cdots, ds_{M}, \qquad (3)$$

where  $\eta_M = \prod_{j=1}^M \frac{\mathcal{K}_{N,j}}{C_{N,j}}$ ,  $\varphi_i = \left(\mu_i, \frac{1}{\alpha_i}\right)$ ,  $\mathscr{B} = (\varphi_1, \dots, \varphi_N)$ , and  $\mathcal{L}_j$  is the integral path from  $\delta_j - w\infty$  to  $\delta_j + w\infty$ ,  $\Phi(s_1, \dots, s_M) = \Gamma(\sum_{j=1}^M s_j)$ , and  $\phi(s_j) = \prod_{i=1}^N \Gamma\left(\mu_i - \frac{s_j}{\alpha_i}\right) \Gamma(s_j)$ . Step (b) is developed for the sake of assisting the asymptotic effective rate analysis.

Note that the multivariate Fox's *H*-function is feasible and beneficial when analyzing the physical layer security [49], symbol error rate, and channel capacity over Fox's *H*-function fading channels [50].

As we can see from (3), the PDF is expressed in terms of the multivariate Fox's H-function. In order to provide an easy and straightforward PDF expression, we have in this section investigated the feasibility of re-modeling  $Z = \sum_{j=1}^{M} Z_j$  with the aid of the MoG distribution. The contribution established in [51] showcased that the MoG distribution can be used to model any arbitrarily shaped non-Gaussian density, e.g., Nakagami/Lognormal, Weibull,  $\kappa$ - $\mu$ ,  $\eta$ - $\mu$ ,  $\alpha$ - $\mu$  distributions, etc. The intrinsic parameters used to represent the MoG distribution are implemented via the completely unsupervised expectation-maximization (EM) learning algorithm. In [52], the MoG distribution is proved extremely advantageous to provide an easy and simple solution when analyzing the physical layer security performance over various fading channels. Motivated by the promising outcomes obtained in [51], [52], the following subsection is subjected to apply the MoG distribution for remodeling the PDF of Z.

# B. MIXTURE OF GAUSSIAN (MOG) DISTRIBUTION

For the purpose of differentiating the Fox-based result,  $f_{\hat{Z}}(z)$  and  $F_{\hat{Z}}(z)$  are utilized herein to denote the MoG-based PDF

 $^2$ Due to the limited space and for brevity of notations, the multivariate Fox's H-function is presented in the manner of the one given in [47].  $1_N$  denotes the N-dimensional all-one vector.

TABLE 1: MOG parameters for Z over cascaded  $\alpha$ -  $\mu$  fading channels when  $\alpha=4.5,\ \mu=3.5,\ M=3,\ N=2,\ {\rm And}\ C=2$ 

l	w <sub>l</sub>	SI	$\eta_l$	
1	0.224375	3.025526	0.299281	
2	0.775625	2.770252	0.289974	

and CDF, given by

$$f_{\hat{\mathbf{Z}}}(z) = \sum_{l=1}^{C} \frac{w_l}{\sqrt{2\pi}\eta_l} \exp\left(-\frac{(z-\varsigma_l)^2}{2\eta_l^2}\right),\tag{4a}$$

$$F_{\hat{\mathbf{Z}}}(z) = \sum_{l=1}^{C} \frac{w_l}{2} \left( 1 + \operatorname{erf}\left(\frac{z - \varsigma_l}{\sqrt{2}\eta_l}\right) \right), \tag{4b}$$

where C represents the number of Gaussian components<sup>3</sup>.  $w_l > 0$ ,  $\varsigma_l$  and  $\eta_l$  are respectively the lth weight, mean, and variance with the constraint of  $\sum_{l=1}^{C} w_l = 1$ . As shown in Fig. 1, we have plotted the PDF of Z for

As shown in Fig. 1, we have plotted the PDF of Z for selected value of M, where  $\alpha$  and  $\mu$  is chosen as an illustrative example based on the experimental and analytical results obtained in [34]–[39], the corresponding estimated parameters, i.e., C,  $w_l$ ,  $\varsigma_l$ ,  $\eta_l$  when M=3, are provided in Table 1. Interestingly, one can observe that there exists a highly close agreement between the simulated and approximated PDFs. Besides, the MoG distribution not only can model both the composite and non-composite fading channels, herein we prove that the MoG distribution is also beneficial to offer a simple but effective approach to model the sum of M versions of cascaded  $\alpha$ - $\mu$  distributed RVs.

# C. APPROXIMATION ACCURACY

In order to evaluate the accuracy of the MoG distribution, the KS goodness of fit test is accordingly conducted [23], [53]. The KS test statistic *D* is to maximize the difference between the simulated and approximated CDFs, i.e.,

$$D \triangleq \max |F_{\mathsf{Z}}(z) - F_{\hat{\mathsf{Z}}}(z)||, \tag{5}$$

where  $F_Z(z)$  is the empirical CDF of RV Z, and  $F_{\hat{Z}}(z)$  is the approximated CDF given in (4b). Let the hypothesis  $\mathbf{H}_0$  denotes that Z follows the MoG distribution. Such an hypothesis is accepted only when D is smaller than the critical value  $D_{max}$ , i.e.,  $D < D_{max}$ , which is given by

$$D_{max} \approx \sqrt{-(1/2v)\ln(\tau/2)},$$

where  $\tau$  is the significance level, and v is the number of random samples of Z.

Obviously, Fig. 1 presents that the MoG-based CDF of Z approximated the Monte-Carlo simulation results with good

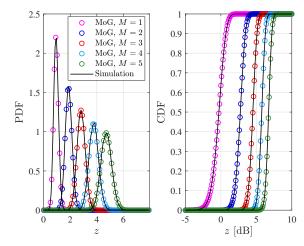


FIGURE 1: The PDF and CDF of **Z** over cascaded  $\alpha$ - $\mu$  (i.e.,  $\alpha = 4.5, \mu = 3$ ) fading channels for selected values of M, where N = 2.



FIGURE 2: A wireless communication system configuration with one source (**S**) and one destination (**D**) assisted with an RIS.

accuracy. The KS test statistic D obtained from Fig. 1 when  $M=1,\cdots,5$  are respectively given as  $2.2\times10^{-3}$ ,  $2.3\times10^{-3}$ ,  $1.6\times10^{-3}$ ,  $2.8\times10^{-3}$ , and  $1.7\times10^{-3}$ . Herein, in our KS goodness of fit test, let  $\alpha=5\%$ , and  $\nu=10^5$ . Thus, we have  $D_{max}=0.0043=4.3\times10^{-3}$ . The hypothesis  $\mathbf{H}_0$  is surely accepted for the cases  $M=1,\cdots,5$  due to  $D<D_{max}$ .

# III. SYSTEM MODEL FOR RIS-AIDED COMMUNICATIONS

As discussed in Section I, the application of cascaded fading channels is widely used in many communication scenarios, e.g., M2M communications, multihop AF relaying communication, MIMO keyhole/pinhole systems, and also the newly proposed RIS-enabled communication systems. To this end, in the following subsection, we have introduced the RIS-aided wireless communication systems as an illustrative application of the preliminary results.

#### A. SYSTEM MODEL

Suppose a source node ( $\mathbf{S}$ ) intends to transmit messages to a destination ( $\mathbf{D}$ ) with the assistance of an RIS, as shown in Fig. 2 [54], which is composed of M passive and low-cost reflectors. It is assumed that both  $\mathbf{S}$  and  $\mathbf{D}$  are equipped with a single antenna. The instantaneous received signal at  $\mathbf{D}$  is

 $<sup>^3</sup>$ The number of components C is selected automatically using the Bayesian information criterion (BIC) method given in [51, Section III-C], whereas the corresponding parameters for the MoG distribution are evaluated using the EM algorithm.

given by

$$y_D = \sqrt{E_s} \left| \sum_{m=1}^M h_m u_m g_m \right| s + \mathfrak{n}, \tag{6}$$

where  $E_s$  is the transmit power,  $h_m = \mathfrak{h}_m \exp(-w\phi_m)$  and  $g_m = \mathfrak{g}_m \exp(-w\psi_m)$  are independent and identically distributed (i.i.d.)  $\alpha$ - $\mu$  RVs<sup>4</sup>, which correspond to the channel coefficients from **S** to the *m*-th reflector element and the *m*-th reflector element to **D**, respectively.  $\mathfrak{h}_m$ ,  $\mathfrak{g}_m$ ,  $\phi_m$ , and  $\psi_m$  denote the amplitudes and phases of the corresponding fading channel gains.  $u_m = \mathfrak{w}_m(\theta_m) \exp(w\theta_m)$  is the reflection coefficient produced by the *m*-th element of the RIS, herein  $\mathfrak{w}_m(\theta_m) = 1$  for the ideal phase shifts,  $m = 1, \dots, M$ . s is the transmitted signal with unit energy.  $\mathfrak{n}$  is the additive white Gaussian noise (AWGN) with zero mean and  $N_0$  variance.

The instantaneous received effective SNR at  $\mathbf{D}$ ,  $\gamma_D$ , is expressed as

$$\gamma_D = \frac{E_s}{N_0} \left| \sum_{m=1}^{M} \mathfrak{h}_m \mathfrak{g}_m \exp\left( w(\theta_m - \phi_m - \psi_m) \right) \right|^2. \tag{7}$$

Considering the best case, namely, to maximize  $\gamma_D$ , the channel phase is similarly eliminated with [54], i.e, let  $\theta_m = \phi_m + \psi_m$  for all  $m = 1, \dots, M$ . As such,  $\gamma_D$  is then represented

$$\gamma_D = \frac{E_s}{N_0} \left| \sum_{m=1}^M \mathfrak{h}_m \mathfrak{g}_m \right|^2 = \bar{\gamma} \left| \sum_{n=1}^M \mathbb{W}_m \right|^2, \tag{8}$$

where  $\bar{\gamma} = \frac{E_s}{N_0}$  and  $\mathbb{W}_m = \mathfrak{h}_m \mathfrak{g}_m$ . Since  $\mathfrak{h}_m$  and  $\mathfrak{g}_m$  are i.i.d.  $\alpha$ - $\mu$  distributed RVs, using the result obtained in [24, Theorem 1], one can observe that  $\mathbb{W}_m$  follows the cascaded  $\alpha$ - $\mu$  distribution. In other words, the link between **S** and **D** via a reflector surface can be modeled by the cascaded fading distribution.

Again let  $\mathcal{Y} = \sum_{m=1}^{M} \mathbb{W}_m$ , apparently  $\mathcal{Y}$  is the sum of M versions of cascaded  $\alpha$ - $\mu$  distributed RVs. Performing the interchange of RV  $\mathcal{Y} = \sqrt{\frac{\gamma_D}{\bar{\gamma}}}$ , and applying the results given in (3) and (4a), the exact and MOG-approximated PDF of  $\gamma_D = \bar{\gamma}\mathcal{Y}^2$  are thereafter given by

$$f_D(\gamma) = \frac{1}{2\sqrt{\bar{\gamma}\gamma}} f_Z\left(\sqrt{\frac{\gamma}{\bar{\gamma}}}\right),$$
 (9a)

$$f_{\hat{D}}(\gamma) = \sum_{l=1}^{C} \frac{w_l}{\sqrt{8\pi\bar{\gamma}}\eta_l\sqrt{\gamma}} \exp\left(-\frac{(\sqrt{\gamma/\bar{\gamma}} - \varsigma_l)^2}{2\eta_l^2}\right).$$
 (9b)

# IV. EFFECTIVE RATE ANALYSIS

#### A. DEFINITION OF EFFECTIVE RATE

Assuming the block fading channel, the normalized effective rate is given as [14]

$$\mathcal{R} = -\frac{1}{A} \log_2 \left[ \mathbb{E} \left( (1 + \gamma_D)^{-A} \right) \right] \text{ bit/s/Hz}, \tag{10}$$

<sup>4</sup>When the RIS is far away from **S** and **D**, it is assumed that  $\mathfrak{h}_m$  and  $\mathfrak{g}_m$  may be regarded as independent of the RIS element [55].

where  $A = \frac{\theta TB}{\ln(2)}$ , where  $\theta > 0$ , T, and B represent the delay exponent, the block length, and the system bandwidth, respectively. The QoS exponent  $\theta$  is defined from the delay outage probability, which is mathematically formulated as

$$\theta = -\lim_{Q_{max} \to \infty} \frac{\ln \left( Pr(Q \ge Q_{max}) \right)}{Q_{max}}.$$

where  $Pr(a \ge b)$  stands for the probability of a being larger than or equal to b. Herein, Q denotes the queue length at the transmitter buffer side, and  $Q_{max}$  is the predetermined threshold of the queue length. Practically speaking,  $\theta \to 0$  implies the delay-tolerant communication, while  $\theta \to \infty$  implies the delay-limited communication [56], e.g., voice calls.

#### B. EXACT EFFECTIVE RATE ANALYSIS

**Theorem 1.** The effective rate of the RIS-assisted communications over  $\alpha$ - $\mu$  fading channel is given by

$$\begin{split} \mathcal{R}_{M} &= -\frac{1}{A} \log_{2} \left[ \frac{\eta_{M}}{\Gamma(A)} \Delta \right], \quad where \\ \Delta &= H \begin{pmatrix} \begin{pmatrix} 1,1\\1,2 \end{pmatrix} & \begin{pmatrix} (1;\frac{1}{2}1_{M})\\(A;1\frac{1}{2}1_{M}),(1;1_{M}) \end{pmatrix} \begin{pmatrix} \frac{C_{N,j}}{\sqrt{(\gamma)}} \end{pmatrix}_{j=1:M} \\ \begin{pmatrix} \begin{pmatrix} 1,1\\1,N_{j} \end{pmatrix} & \begin{pmatrix} (1,1)\\1,N_{j} \end{pmatrix} \end{pmatrix}_{j=1:M} \end{pmatrix}. \end{split}$$

$$(11)$$

*Proof.* See Appendix A.

**Remark 1.** When M = 1, the effective rate of the RIS-assisted communications over  $\alpha$ - $\mu$  fading channel is given by

$$\mathcal{R}_{S} = -\frac{1}{A} \log_{2} \left( \frac{\mathcal{K}_{N}}{2\sqrt{\bar{\gamma}}\Gamma(A)} H_{1,N+1}^{N+1,1} \left[ \frac{C_{N}}{\sqrt{\bar{\gamma}}} \middle| \mathcal{B}, (A - \frac{1}{2}, \frac{1}{2}) \right] \right).$$
(12)

*Proof.* When M=1, we have  $\gamma_D=\bar{\gamma}|\mathbb{W}_1|^2=\bar{\gamma}\mathcal{Y}^2$ . Next, applying the result given in (2), the PDF of  $\gamma_D$  is given by

$$f_D(\gamma) = \frac{\mathcal{K}_N}{2\sqrt{\bar{\gamma}\gamma}} H_{0,N}^{N,0} \left[ C_N \sqrt{\frac{\gamma}{\bar{\gamma}}} \, \middle| \, \frac{-}{\mathcal{B}} \, \right]. \tag{13}$$

Re-expressing

$$\frac{1}{(1+x)^A} = \frac{1}{\Gamma(A)} H_{1,1}^{1,1} \left[ x \, \middle| \, \begin{array}{c} (1-A,1) \\ (0,1) \end{array} \right], \tag{14}$$

in terms of Meijer's G-function [57, Chpt. 8.4], then plugging (14) and (13) into  $\mathcal{U}$ , yields

$$\mathcal{U} = \frac{\mathcal{K}_{N}}{2\Gamma(A)\sqrt{\bar{\gamma}}} \int_{0}^{\infty} \frac{1}{\sqrt{\gamma}} H_{1,1}^{1,1} \left[ \gamma \left| \begin{array}{c} (1-A,1) \\ (0,1) \end{array} \right. \right] \times H_{0,N}^{N,0} \left[ C_{N} \sqrt{\frac{\gamma}{\bar{\gamma}}} \left| \begin{array}{c} - \\ \mathcal{B} \end{array} \right. \right] d\gamma,$$
(15)

subsequently applying the Mellin transform of two Fox's *H*-function [57, Eqs.(2.25.1.1) and (8.3.2.21)], the proof is accomplished.

As stated earlier, the cascaded  $\alpha$ - $\mu$  distribution encompasses the cascaded Rayleigh and Nakagami-m distributions, utilizing the property of Fox's H-function [57, Eq.(8.3.2.21)],

the effective rate of the RIS-assisted communications over Nakagami-m ( $\mu = m$ ) and Rayleigh ( $\mu = m = 1$ ) fading channel when M = 1 are expressed as follows, in terms of the Meijer's G-function, i.e.,  $G_{p,q}^{m,n}(\cdot)^5$ ,

$$\begin{split} \mathcal{R}_{S} &= -\frac{1}{A}\log_{2}\left(\frac{\mathcal{K}_{N}}{C_{N}\Gamma(A)}\Delta\right), \text{ where} \\ \Delta &= \begin{cases} G_{1,N+1}^{N+1,1} \left\lceil \frac{C_{N}^{2}}{\tilde{\gamma}} \right\rceil & 1\\ m1_{N}, A \right\rceil, & \text{Nakagami-}m, \\ G_{1,N+1}^{N+1,1} \left\lceil \frac{C_{N}^{2}}{\tilde{\gamma}} \right\rceil & 1\\ 1_{N}, A \right\rceil, & \text{Rayleigh.} \end{cases} \end{split}$$
(16)

*Proof.* When  $\alpha = 2$ , then deploying the elementary property of Fox's *H*-function [58, Eqs. (2.1.5), (2.1.4), and (2.9.1)], the proof is obtained.

# C. ASYMPTOTIC EFFECTIVE RATE ANALYSIS

**Remark 2.** The asymptotic behavior of effective rate over  $\alpha$ - $\mu$  fading channel at high SNR regime when M=1 is given by

$$\mathcal{R}_{S}^{\infty} \approx -\frac{1}{A} \log_{2} \left( \frac{\kappa_{N}}{2\sqrt{\overline{\gamma}}\Gamma(A)} \Delta^{\infty} \right), where$$

$$\Delta^{\infty} = \begin{cases} 2\Gamma(A) \prod_{i=1}^{N} \Gamma\left(\mu_{i} - \frac{2A}{\alpha_{i}}\right) \left(\frac{C_{N}}{\sqrt{\overline{\gamma}}}\right)^{2A-1}, & 2A < \tau, \\ \prod_{i=1, i \neq c}^{N} \frac{\alpha_{c}\Gamma(A - \frac{\tau}{2})\Gamma(\frac{\tau}{2})\Gamma(\mu_{i} - \frac{\tau}{\alpha_{i}})}{\left(\frac{C_{N}}{\sqrt{\zeta_{S}}}\right)^{1-\tau}}, & 2A > \tau. \end{cases}$$
(17)

*Proof.* Assuming all the residues of the Fox's *H*-function are simple, when  $\bar{\gamma}$  goes to  $\infty$ , we have  $\frac{C_N}{\sqrt{\bar{\gamma}}} \to 0$ , the asymptotic behavior of (12) is evaluated at  $\xi = 1 - \min(\tau, 2A)$  where

$$\tau = \min_{i=1,\dots,N} \alpha_i \mu_i, \quad c = \arg\min_{i=1,\dots,N} \alpha_i \mu_i, \quad (18)$$

subsequently making expansion of (12) at  $s = \xi$  [59], and using the residue theorem [60] under the constraint  $A < \frac{\tau}{2}$ , we have

$$\Delta^{\infty} \approx \operatorname{Res} \left[ \prod_{i=1}^{N} \Gamma \left( \mu_{i} - \frac{1}{\alpha_{i}} + \frac{s}{\alpha_{i}} \right) \Gamma \left( A - \frac{1}{2} + \frac{s}{2} \right) \right]$$

$$\times \Gamma \left( \frac{1}{2} - \frac{s}{2} \right) \left( \frac{C_{N}}{\sqrt{\overline{\gamma}}} \right)^{-s}, 1 - 2A$$

$$= \lim_{s \to 1 - 2A} \prod_{i=1}^{N} 2\Gamma \left( \mu_{i} - \frac{1}{\alpha_{i}} + \frac{s}{\alpha_{i}} \right) \Gamma \left( \frac{1}{2} - \frac{s}{2} \right) \left( \frac{C_{N}}{\sqrt{\overline{\gamma}}} \right)^{-s}$$

$$= 2\Gamma(A) \prod_{i=1}^{N} \Gamma \left( \mu_{i} - \frac{2A}{\alpha_{i}} \right) \left( \frac{C_{N}}{\sqrt{\overline{\gamma}}} \right)^{2A - 1}.$$

$$(19)$$

The case  $2A < \tau$  is similarly accomplished.

The exact effective rate is expressed in terms of the multivariate Fox's *H*-function, the asymptotic behavior of effective rate is further provided to gain more insights at the high SNR regime.

**Remark 3.** The asymptotic behavior of effective rate at high SNR regime is given by under the condition that  $2A > \sum_{j=1}^{M} \xi_j$ ,

$$\mathcal{R}_{M}^{\infty} \approx -\frac{1}{A} \log_{2} \left[ \frac{\eta_{M}}{2\Gamma(A)} \Gamma \left( A - \sum_{j=1}^{M} \frac{\xi_{j}}{2} \right) \frac{\Gamma \left( \sum_{j=1}^{M} \frac{\xi_{j}}{2} \right)}{\Gamma \left( \sum_{j=1}^{M} \xi_{j} \right)} \right] \times \prod_{j=1}^{M} \lim_{s \to \xi_{j}} (s_{j} - \xi_{j}) \prod_{i=1, i \neq c}^{N} \Gamma \left( \mu_{i} - \frac{\xi_{j}}{\alpha_{i}} \right) \Gamma(\xi_{j}) \left( \frac{C_{N, j}}{\sqrt{\overline{\gamma}}} \right)^{\xi_{j}} ,$$

$$where \xi_{j} = \min_{i=1, \dots, N} \alpha_{j, i} \mu_{j, i}, \quad c = \underset{i=1, \dots, N}{\arg \min} \alpha_{j, i} \mu_{j, i}. \quad (20)$$

Especially when N=1, (11) is then expanded at  $\xi_j=\alpha_j\mu_j$  with constraint  $2A-\sum_{i=1}^M \xi_j>0$ .

$$\mathcal{R}_{M}^{\infty} \approx -\frac{1}{A} \log_{2} \left[ \frac{\eta_{M}}{2\Gamma(A)} \frac{\Gamma\left(A - \sum_{j=1}^{M} \frac{\xi_{j}}{2}\right) \left(\sum_{j=1}^{M} \frac{\xi_{j}}{2}\right)}{\Gamma\left(\sum_{j=1}^{M} \xi_{j}\right)} \times \prod_{j=1}^{M} \alpha_{j} \Gamma\left(\xi_{j}\right) \left(\frac{C_{N,j}}{\sqrt{\bar{\gamma}}}\right)^{\xi_{j}} \right].$$
(21)

*Proof.* See Appendix B.

**Remark 4.** Observing from (17) and (20), the high SNR slope of effective rate is defined as

$$S^{\infty} = \frac{\mathcal{R}^{\infty}}{\log_2 \bar{\nu}},\tag{22}$$

subsequently, substituting  $\mathcal{R}_S^{\infty}$  and  $\mathcal{R}_M^{\infty}$  into (22), we have

$$S_S^{\infty} = \begin{cases} 1, & 2A < \tau, \\ \frac{\tau}{A}, & 2A > \tau. \end{cases}$$
 (23a)

$$S_M^{\infty} = \frac{1}{A} \sum_{i=1}^M \frac{\xi_j}{2}, \quad 2A > \sum_{i=1}^M \xi_j,$$
 (23b)

where  $\tau$  and  $\xi_i$  are given in (18) and (20), respectively.

As discussed in this Section, the exact and asymptotic behaviors of the effective rate for the RIS-assisted system setup are characterized. It is noted that the exact effective rate is derived with closed-form expression, given in terms of the multivariate Fox's *H*-function. In the following subsection, the highly tight effective rate with the assistance of the MoG distribution is derived.

#### D. AN ALTERNATIVE APPROXIMATION METHOD

**Remark 5.** Motivated by (9b), one can observe that  $\gamma_D$  can be modeled in terms of the C normally distributed random variable, i.e.,  $\gamma_D = \sum_{l=1}^{C} \gamma_l$ , and  $\gamma_l \sim \mathcal{N}(\varsigma_l, \eta_l)$ , consequently,

<sup>&</sup>lt;sup>5</sup>Note that the univariate Meijer's *G*-function is available, i.e., MATLAB meijerG(a,b,c,d,x), Maple Meijer $G([a_s,b_s],[c_s,d_s],z)$ , and Mathematica Meijer $G([a_i]_{i=1:n},\{a_i\}_{i=n+1:p},\{b_i\}_{i=1:m},\{b_i\}_{i=m+1:q},z]$ .

the effective rate of our proposed system configuration can be highly approximated as follows

$$\mathcal{R}^{MoG} \approx -\frac{1}{A} \log_2 \left[ \sum_{l=1}^C w_l \left( \frac{2}{3(1 + \bar{\gamma}\varsigma_l^2)^A} + \frac{1}{6(1 + \bar{\gamma}(\varsigma_l + \sqrt{3}\eta_l)^2)^A} + \frac{1}{6(1 + \bar{\gamma}(\varsigma_l - \sqrt{3}\eta_l)^2)^A} \right) \right]. \tag{24}$$

*Proof.* Performing a revisit to  $\mathcal{U}$  given in (10), then substituting (9b) into  $\mathcal{U}$ , yields

$$\mathcal{U} = \sum_{l=1}^{C} \frac{w_l}{\sqrt{8\pi\bar{\gamma}}\eta_l} \int_0^{\infty} \frac{\gamma^{-\frac{1}{2}}}{(1+\gamma)^A} \exp\left(-\frac{(\sqrt{\gamma/\bar{\gamma}} - \varsigma_l)^2}{2\eta_l^2}\right) d\gamma$$

$$\stackrel{(c)}{=} \sum_{l=1}^{C} \frac{w_l}{\sqrt{2\pi}\eta_l} \int_0^{\infty} \frac{1}{(1+\bar{\gamma}y^2)^A} \exp\left(-\frac{(y-\varsigma_l)^2}{2\eta_l^2}\right) dy,$$
(25)

where step (c) is developed by interchanging of variables  $y = \sqrt{\frac{\gamma}{\bar{\gamma}}}$ , then applying the result given in [61, Eq. (4)], i.e.,  $\mathcal{F}(y) = \frac{1}{(1+\bar{\gamma}y^2)}$  is a real function of y, herein y follows the normal distribution with mean  $\varsigma_l$  and variance  $\eta_l$ , the expectation of  $\mathcal{F}(y)$  can be highly approximated at three points, namely,  $\varsigma_l$ ,  $\varsigma_l + \sqrt{3}\eta_l$ , and  $\varsigma_l - \sqrt{3}\eta_l$  [62],

$$\mathcal{E}[\mathcal{F}(y)] \approx \mathcal{F}(\varsigma_l) + \mathcal{F}(\varsigma_l + \sqrt{3}\eta_l) + \mathcal{F}(\varsigma_l - \sqrt{3}\eta_l). \quad (26)$$

Finally, substituting (26) into (10), the proof is accomplished.

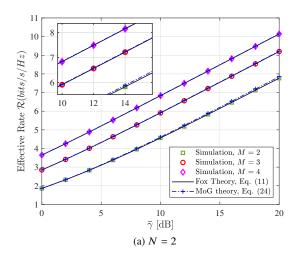
# V. NUMERICAL RESULTS AND DISCUSSION

For the simplicity of notifications, assuming that all M version of cascaded  $\alpha$ - $\mu$  distributed RVs have the same fading parameters, i.e.,  $\alpha_1 = \cdots = \alpha_N = \alpha$ , and  $\mu_1 = \cdots = \mu_N = \mu$  in the simulation. The  $\alpha$ - $\mu$  distributed RVs are implemented using the WAFO toolbox [63]. It is noteworthy that in this paper when M=1, N is an integer, which corresponds to the multi-hop AF non-regenerative relaying system [22] or MIMO keyhole systems [21]; when N=1, M is an integer, which might probably be single-hop multiple-input single-output (MISO) systems [10]; and when N=2, and M is an integer, it is the RIS-aided system under consideration.

Currently, no existing mathematical packages are available at MATLAB, Python, or Mathematica to compute the multivariate Fox's *H*-function, but it is still numerically programmable and computable, see existing examples using Python code [50] and MATLAB code [64]. The multivariate Fox's *H*-function is implemented herein by using the method given in [50].

# A. NUMERICAL RESULTS

As shown in Fig. 3 (a) and (b), the effective rate of the RIS-assisted communications over  $\alpha$ - $\mu$  fading channels given in (11) and **Remark**. 5 is presented for selected values of M and N. One can conclude that 1) the effective rate certainly increases as M increases; 2)  $\mathcal{R}$  decreases as N increases;



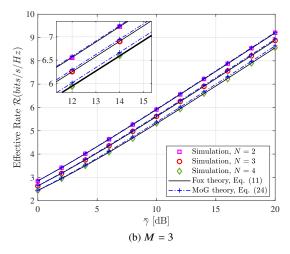


FIGURE 3: Effective rate over  $\alpha$ - $\mu$  fading channels against  $\bar{\gamma}$  for selected values of M and N, where A = 4, and  $\alpha = \mu = 3$ .

and 3) in addition, the effective rate expression derived in **Remark**. 5 provides a highly tight approximation to the exact result. This fact can be further confirmed in the following subsection. Besides, the MoG-based effective rate expression becomes gradually tight as *N* decreases and *M* increases, respectively.

The asymptotic behavior of effective rate at high SNR regime is depicted in Fig. 4, the analytical behavior of  $\mathcal{R}$  given in (20) becomes gradually tight when  $\bar{\gamma}$  increases. One can observe that when N=1,  $\alpha=2$ ,  $\mu=1$ , and A=4, the values of effective rate at 40 dB and 30 dB when M=1,2,3 are 3.718, 7.686, 11.59 and 2.888, 6.011, 9.081, respectively. As a result, the slopes of the asymptotic effective rate  $\mathcal{R}_M^{\infty}$  when M=1,2,3 are correspondingly given by (3.718 - 2.888)  $\times \log_{10}(2) = 0.2499 \approx 0.25 = \frac{\tau}{2A} = \frac{\alpha\mu}{2A}$ , (7.686 - 6.011)  $\times \log_{10}(2) = 0.5042 \approx 0.5 = \frac{\tau}{2A} = \frac{\alpha\mu}{A}$ , and (11.59 - 9.081)  $\times \log_{10}(2) = 0.7553 \approx 0.75 = \frac{3\tau}{2A} = \frac{3\alpha\mu}{2A}$ . Overall, the asymptotic slope of effective rate given in (23b) is verified, and the asymptotic analysis at high SNR regime

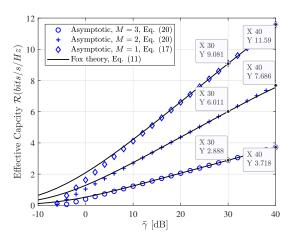


FIGURE 4: Effective rate over  $\alpha$ - $\mu$  (i.e.,  $\alpha = 2$ ,  $\mu = 1$ ) fading channels against  $\bar{\gamma}$ , where N = 1 and A = 4.

is proved to be efficient with reducing complexity compared to the exact effective rate expression. One can obtain that the high SNR slope is linearly associated with  $\alpha$  and  $\mu$ . More specifically, large  $\alpha$  and  $\mu$  lead to a steep high SNR slope curve. In other words, large  $\alpha$  and  $\mu$  contributes to higher effective rate performance. Such a conclusion is also verified in Figs. 5 and 8, which is because the increase of  $\alpha$  and  $\mu$  physically accounts for the less severe fading environment.

In Fig. 5, the effective rate over Rayleigh and Nakagami-m (m=3.5) fading channels, given in (16), are compared versus the delay QoS exponent  $\theta$ . We observe that the gain in effective rate degrades as the QoS limitation becomes stricter. As discussed in [22], the cascaded Rayleigh distribution was found feasible to model the M2M communication scenario, an increase of N herein means the increase of hops, resultantly leading to a worse effective rate performance. The impact of N and fading parameter  $\mu$  are also presented. One can observe that the effective rate is improved with the increase of fading parameter  $\mu$ , i.e., multipath fading parameter.

Fig. 6 presents the asymptotic effective rate at high SNR regime, together with the exact and simulated results. One can observe that the analytical asymptotic effective rate, as expected, gradually behaves accurately as  $\bar{\gamma}$  increases. Besides, inspired by [18], we also verified the asymptotic effective rate slope at high SNR regime, theoretically given in (23a), for N=2, the  $\mathcal{R}_S^\infty$  is 15.02 and 11.7 at 50 dB and 40 dB, respectively. The high SNR slope is then computed as (15.02 – 11.7) ×  $\log_{10}(2)=0.9994\approx 1$ . Consequently, as  $\bar{\gamma}$  increases, then  $S_S^\infty$  will surely converge to 1.

In Fig. 7, we investigate the effective rate of the proposed RIS-assisted system, where the number of reflecting elements of the RIS ranges from 100,150, 200, 300, 400, and 500. A perfect match between the Monte-Carlo simulation and the MoG-based analytical results is validated. It is noteworthy that the exact effective rate expression becomes difficult and time-consuming to implement the multivariate Mellin-

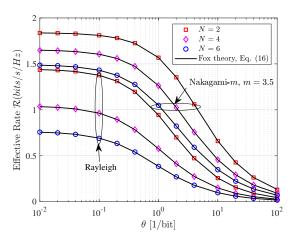


FIGURE 5: Effective rate over Rayleigh ( $\mu=1$ ) and Nakagami-m ( $\mu=m=3.5$ ) fading channels against  $\theta$  for selected values of N, where  $A=\frac{\theta TB}{\ln(2)}, TB=1$ , and M=1.

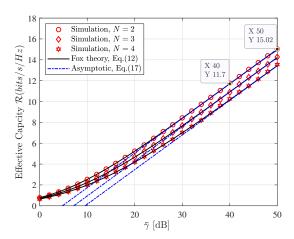


FIGURE 6: Effective rate over  $\alpha$ - $\mu$  fading channels against  $\bar{\gamma}$ , where M = 1, A = 3,  $\alpha = 2$ , and  $\mu = 5$ .

Barnes integrals as *M* becomes larger, e.g., 200. Conclusively, the MoG-based solution alternatively showcases its importance and high priority when doing performance calculation compared to the multivariate Fox's *H*-function based solution.

Fig. 8 plots the effective rate of the RIS-aided system against  $\theta$  over Weibull ( $\alpha$  is the fading parameter, and  $\mu = 1$ ) fading channels with consideration of the impacts from the fading parameter  $\alpha$ . One can observe that (i) the increase of  $\alpha$  is beneficial for  $\mathcal{R}$ . More importantly, the gain in effective capacity benefiting from the increase of  $\alpha$  becomes obvious as the QoS limitation  $\theta$  tends to high, which is quite different from Fig. 5; (ii) as expected, the effective rate demonstrates an improvement as A decreases, in other words, a smaller delay constraint results in a higher effective rate performance; and (iii) as shown in Fig. 5, the multipath effect has a negative impact on the effective rate performance, i.e., lower values of  $\alpha$  and  $\mu$ , which indicate the serious nonlinearity and sparse

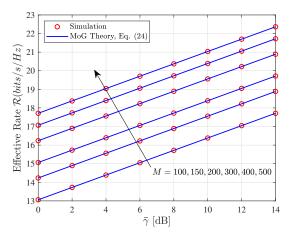


FIGURE 7: Effective rate over  $\alpha$ - $\mu$  fading channels against  $\bar{\gamma}$ , where A=4, N=2, and  $\alpha=\mu=3$ .

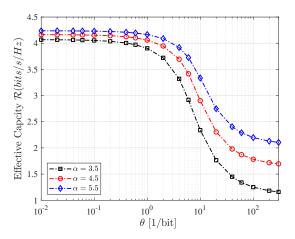


FIGURE 8: Effective rate over Weibull ( $\alpha$  is the fading parameter,  $\mu = 1$ ) fading channels against  $\theta$ , where  $A = \frac{\theta T B}{\ln(2)}$ , N = 2, M = 5, and  $\bar{\gamma} = 0$  dB.

clustering of propagation medium, lead to the poor effective rate performance.

## B. ACCURACY ANALYSIS

For the purposes of illustrating the tightness of our analytical results, a useful measure, namely analytical error, is used as the accuracy indicator [52], [61],

analytical error = 
$$1 - \frac{\text{analytical results}}{\text{simulation results}} \times 100\%$$
. (27)

For the brevity of notations, let  $\operatorname{error}_{Fox}$  denote the analytical error between the simulated results and the Fox's H-function based results, similarly,  $\operatorname{error}_{MoG}$  represents the analytical error between the simulated results and the MoG-based results.

Based on Fig. 3. (a), as shown in Table. 2, we have two interesting observations: (i) both  $\operatorname{error}_{Fox}$  and  $\operatorname{error}_{MoG}$  ranges from -1% to 1%, such a fact can be interpreted that both

results are highly accurate; and (ii) compared with error $_{Fox}$ , error $_{MoG}$  is always negative. Such a fact indicates that the MoG-based expression acts as a highly tight upper bound for the exact effective rate.

#### VI. CONCLUSION

In this paper, we have characterized the sum of cascaded  $\alpha$ - $\mu$  RVs, and studied its application in the RIS-assisted communications from the effective rate perspective. The exact, asymptotic, and approximated behaviors of the effective rate performance for the RIS-aided communications are explored and further derived with closed-form expressions. The asymptotic effective rate slope at high SNR regime was also provided. The accuracy of our analytical results were supported by the simulated results, and tightly approached by its asymptotic behavior at high SNR regime.

Interesting and useful insights can be thereafter obtained that (i) the obtained analytical results are not only applicable for the RIS-aided systems, when M = 1 or N = 1, it corresponds to the multi-hop AF non-regenerative relaying system [22]/MIMO keyhole systems [21] or the single-hop MISO systems [10]; (ii) the effective rate can take advantage of the increase of M and the decrease of the number of statistically independent scatters N; and (iii) fading parameters,  $\alpha$  and  $\mu$ , i.e., nonlinearity and clustering of the wireless propagation medium, are advantageous to enhance the effective rate performance. In addition, compared with the multivariate Fox's H-function based effective rate expression, the MoG-based solution is highly accurate and serves as a tight and simple upper bound for the exact effective rate solution with reducing complexity. Overall, this paper mainly considered the ideal phase-shifting, the correlation of signals in RIS-aided systems is a worthy subject and left for future investigation.

# **APPENDIX A PROOF FOR THEOREM 1**

Revisiting the multivariate Fox's H-function in (9a) in terms of its definition from (3), and then Plugging the result and (14) into  $\mathcal{U}$ , we have  $\mathcal{U}$  given in (28), shown at the top of next page. It is noted that step (d) in (28) is developed by interchanging the order of two integrals, subsequently,  $\mathcal{U}_1$  is further evaluated with the help of the definition of Mellin transform for Fox's H-function [57, Eq.(2.25.2.1)],

$$\mathcal{U}_1 = \Gamma\left(\sum_{j=1}^M \frac{s_j}{2}\right) \Gamma\left(A - \sum_{j=1}^M \frac{s_j}{2}\right). \tag{29}$$

After some algebraic manipulations and deploying the definition of multivariate Fox's H-function, the effective rate of the RIS-assisted system setup over  $\alpha$ - $\mu$  fading channel is obtained.

# **APPENDIX B PROOF FOR REMARK 3**

Since the exact effective rate expression is given in terms of the multivariate Fox's *H*-function, then re-expressing the multivariate Fox's *H*-function in the manner of the multiple

$\bar{\gamma}$	0	4	8	12	16	20
$error_{Fox}$	$-1.92*10^{-3}\%$	$3.05*10^{-2}\%$	$2.17*10^{-2}\%$	$5.34*10^{-4}\%$	$1.76*10^{-2}\%$	$-5.19*10^{-3}\%$
$error_{MoG}$	-0.13%	-0.11%	-0.11%	-0.12%	-8.74 * 10 <sup>-2</sup> %	-9.86 * 10 <sup>-2</sup> %

TABLE 2: ANALYTICAL ERROR AGAINST  $\bar{\gamma}(dB)$  WHEN  $A=4, M=4, N=2, \alpha=\mu=3$ .

$$\mathcal{U} = \frac{\eta_{M}}{2\Gamma(A)} \int_{0}^{\infty} H_{1,1}^{1,1} \left[ \gamma \left| \begin{pmatrix} (1-A,1) \\ (0,1) \end{pmatrix} \right| \frac{1}{(2\pi w)^{M}} \int_{\mathcal{L}_{1}} \cdots \int_{\mathcal{L}_{M}} \Phi(s_{1}, \cdots, s_{M}) \prod_{j=1}^{M} \frac{C_{j}^{s_{j}} \phi(s_{j})}{\sqrt{\overline{\gamma}} \overline{\gamma}} \left( \sqrt{\frac{\gamma}{\overline{\gamma}}} \right)^{s_{j}-1} ds_{1}, \cdots, ds_{M} d\gamma$$

$$\stackrel{(d)}{=} \frac{\eta_{M} (2\pi w)^{-M}}{2\Gamma(A)} \int_{\mathcal{L}_{1}} \cdots \int_{\mathcal{L}_{M}} \Phi(s_{1}, \cdots, s_{M}) \prod_{j=1}^{M} \phi(s_{j}) \left( \frac{C_{j}}{\sqrt{\overline{\gamma}}} \right)^{s_{j}} \underbrace{\int_{0}^{\infty} \gamma^{\frac{M}{j-1}} \frac{s_{j}}{2} - 1}_{I,1} \prod_{j=1}^{M} \left[ \gamma \left| \begin{pmatrix} (1-A, 1) \\ (0, 1) \end{pmatrix} \right| d\gamma ds_{1}, \cdots, ds_{M},$$

$$\underbrace{I_{j}^{M} (2\pi w)^{-M}}_{I,1} \left[ \gamma \left| \begin{pmatrix} (1-A, 1) \\ (0, 1) \end{pmatrix} \right| d\gamma ds_{1}, \cdots, ds_{M},$$

$$\underbrace{I_{j}^{M} (2\pi w)^{-M}}_{I,1} \left[ \gamma \left| \begin{pmatrix} (1-A, 1) \\ (0, 1) \end{pmatrix} \right| d\gamma ds_{1}, \cdots, ds_{M},$$

Mellin-Barnes type integral according to its definition. When  $\bar{\gamma} \to \infty$ , we have

$$\frac{C_{N,j}}{\sqrt{\bar{\gamma}}} = \frac{1}{\sqrt{\bar{\gamma}}} \prod_{i=1}^{N} \frac{\mu_{i,j}^{\frac{1}{\alpha_{i,j}}} \Gamma\left(\mu_{i,j} + \frac{1}{\alpha_{i,j}}\right)}{\Gamma(\mu_{i,j})} \to 0.$$

Applying the result given in [59], each single Mellin-Barnes integral is then further simplified and expanded at the single residue  $\xi_j$  of the corresponding integrands at the pole closest to the contour [47], namely, the minimum pole on the right of the contour for small Fox's H arguments, and subsequently applying the residue theorem, we can arrive at (20).

To be specific, we herein take the case M=3 and N=1 as an example to illustrate the steps for arriving at the asymptotic effective rate behavior. As such, the effective rate is re-expressed as follows in terms of the definition of multivariate Fox's H-function,

$$\mathcal{R} = -\frac{1}{A} \log_2 \left[ \frac{\eta_M}{2\Gamma(A)} \Delta \right], \text{ where}$$

$$\Delta = \frac{1}{(2\pi w)^3} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \int_{\mathcal{L}_3} \mathsf{K}(s_1, s_2, s_3) ds_1 ds_2 ds_3,$$
and
$$\begin{cases} \mathsf{K}(s_1, s_2, s_3) = \Phi(s_1, s_2, s_3) \prod_{\substack{j=1 \ 0 \ j=1}}^{3} \phi(s_j) \left( \frac{C_{1,j}}{\sqrt{\gamma}} \right)^{s_j} \\ \Phi(s_1, s_2, s_3) = \frac{\Gamma\left(A - \frac{s_1 + s_2 + s_3}{2}\right) \Gamma\left(\frac{s_1 + s_2 + s_3}{2}\right)}{\Gamma(s_1 + s_2 + s_3)} \\ \phi(s_j) = \Gamma\left(\mu_j - \frac{1}{\alpha_j} s_j\right) \Gamma(s_j), \quad j = 1, 2, 3. \end{cases}$$

When  $\bar{\gamma}$  goes to  $\infty$ , we have  $\frac{\mathcal{C}_{1,j}}{\sqrt{\bar{\gamma}}} \to 0$ . According to [58, Theorem 1.7], the Mellin-Barnes integral over  $\mathcal{L}_1$  is approximated by evaluating the residue at the closest pole located to the right-hand side, i.e.,  $\xi_1 = \alpha_1 \mu_1$ . As such, the contour  $\mathcal{L}_1$  is clockwise and -1 is needed to be multiplied, [47]

$$\Delta = \frac{1}{(2\pi w)^3} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \int_{\mathcal{L}_3} \mathsf{K}(s_1, s_2, s_3) ds_1 ds_2 ds_3$$

$$= \frac{-1}{(2\pi w)^2} \int_{\mathcal{L}_2} \int_{\mathcal{L}_3} \mathsf{Res}[\mathsf{K}(s_1, s_2, s_3), s_1 = \xi_1] ds_2 ds_3.$$
(31)

Using the residue theorem, Res $[K(s_1, s_2, s_3), s_1 = \xi_1 = \alpha_1 \mu_1]$  can be developed as follows

$$\begin{aligned} & \operatorname{Res} \left[ \mathsf{K}(s_{1}, s_{2}, s_{3}), \xi_{1} \right] = \lim_{s_{1} \to \xi_{1}} (s_{1} - \xi_{1}) \mathsf{K}(\xi_{1}, s_{2}, s_{3}) \\ & = -\alpha_{1} \Gamma \left( \alpha_{1} \mu_{1} \right) \left( \frac{C_{1,1}}{\sqrt{\bar{\gamma}}} \right)^{\alpha_{1} \mu_{1}} \frac{\Gamma \left( A - \frac{\alpha_{1} \mu_{1} + s_{2} + s_{3}}{2} \right)}{\Gamma \left( \alpha_{1} \mu_{1} + s_{2} + s_{3} \right)} \\ & \times \Gamma \left( \frac{\alpha_{1} \mu_{1} + s_{2} + s_{3}}{2} \right) \phi(s_{2}) \phi(s_{3}) \left( \frac{C_{1,2}}{\sqrt{\bar{\gamma}}} \right)^{s_{2}} \left( \frac{C_{1,3}}{\sqrt{\bar{\gamma}}} \right)^{s_{3}} . \end{aligned}$$
(32)

Next, substituting (32) into (31), we have

$$\Delta \approx \frac{\alpha_1 \Gamma \left(\alpha_1 \mu_1\right)}{(2\pi w)^2} \left(\frac{C_{1,1}}{\sqrt{\bar{\gamma}}}\right)^{\alpha_1 \mu_1} \int_{\mathcal{L}_2} \int_{\mathcal{L}_3} \frac{\Gamma \left(A - \frac{\alpha_1 \mu_1 + s_2 + s_3}{2}\right)}{\Gamma \left(\alpha_1 \mu_1 + s_2 + s_3\right)} \times \Gamma \left(\frac{\alpha_1 \mu_1 + s_2 + s_3}{2}\right) \phi(s_2) \phi(s_3) \left(\frac{C_{1,2}}{\sqrt{\bar{\gamma}}}\right)^{s_2} \left(\frac{C_{1,3}}{\sqrt{\bar{\gamma}}}\right)^{s_3} ds_2 ds_3.$$

$$(33)$$

In continuation with the asymptotic analysis, the integrals over  $\mathcal{L}_2$  and  $\mathcal{L}_3$  are similarly evaluated at the residue points  $\xi_2 = \alpha_2 \mu_2$  and  $\xi_3 = \alpha_3 \mu_3$ , respectively. Next applying the residue theorem, the proof for (20) is finished.

When N=1, then  $\xi_j=\alpha_j\mu_j$ , each single Mellin-Barnes integral is expanded at  $\xi_j$ , i.e.,

$$\operatorname{Res}\left[\phi(s_j)\left(\frac{C_{1,j}}{\sqrt{\bar{\gamma}}}\right)^{s_j}, \xi_j\right] = \lim_{s_j \to \xi_j} (s_j - \xi_j)\phi(s_j) \left(\frac{C_{1,j}}{\sqrt{\bar{\gamma}}}\right)^{s_j},$$

subsequently, following the steps for (20), the proof for (21) is obtained.

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