Common pool resource management and risk perceptions

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Abstract

Motivated by recent discussions about the issue of risk perceptions for climate change related events, we introduce a non-cooperative game setting where agents manage a common pool resource under a potential risk, and agents exhibit different risk perception biases. Focusing on the effect of the polarization level and other population features, we show that the type of bias (overestimation versus underestimation biases) and the resource quality level before and after the occurrence of the shift have first-order importance on the qualitative nature of behavioral adjustments and on the pattern of resource conservation. When there are non-uniform biases within the population, the intra-group structure of the population qualitatively affects the degree of resource conservation. Moreover, unbiased agents may react in non-monotone ways to changes in the polarization level when faced with agents exhibiting different types of bias. The size of the unbiased agents' sub-population does not qualitatively affect how an increase in the polarization level impacts individual behavioral adjustments, even though it affects the magnitude of this change. Finally, it is shown how perception biases affect the comparison between centralized and decentralized management.

Keywords: Conservation, Perception bias, Environmental risk, Renewable resources, Dynamic games

JEL Classification: Q20, Q54, D91, C72

1 Introduction

Environmental systems are likely to undergo drastic changes in response to exogenous shocks such as those driven by climate change. Several examples of these irreversible changes have been documented for ecosystems characterized by the collective management of natural resources such as (among other examples) fisheries, forests, groundwater (Costello and Ovando (2019); Oremus et al. (2020); Quaas et al. (2007)). Such irreversible events typically occur following regime shifts, that is, sudden changes in the dynamics of the natural resource which occurrence is uncertain.[1]

For such drastic events, agents tend to exhibit heterogeneous perceptions of the probability of a regime shift occurring (Lee et al. (2015)). Among other examples, certain reinforcing (feedback) effects leading to irreversible changes in ecosystems may be overlooked by agents. When dealing with climate change related

[1] We follow the description made by Scheffer et al. (2001).

risks, there is notable heterogeneity in individual perceptions: agents may underestimate or overestimate the probability of occurrence of a regime shift.[2] Moreover such heterogeneity in perception is often persistent and may be interpreted as perception biases: For events related to climate change issues, few people tend to correct their biases when new information is provided (about the features of the events, see Douenne and Fabre (2020)).

Most of the related literature has abstracted from the issue of perception biases. The purpose of this paper is to analyze the effect of heterogeneous risk perception on individual behavioral adjustments and patterns of common-pool resource conservation. To that end, we introduce perception biases and environmental risk in a non-cooperative dynamic fish war game à la Levhari and Mirman (1980). We consider two cases: a first one where there is uniform bias, and a second one where agents with qualitatively different types of bias (overestimation and underestimations biases) co-exist within the population. We obtain several interesting results. First, the type of bias (overestimation versus underestimation biases) and the resource quality level before and after the occurrence of the shift have first-order importance on the qualitative nature of individual behavioral adjustments and on the pattern of resource conservation. Second, when there are non-uniform biases within the population, the intra-group structure of the population (the relative size of the biased agents' sub-populations) qualitatively affects the degree of resource conservation. Moreover, the unbiased agents may react in non-monotone ways to changes in the polarization level when faced with agents exhibiting different types of bias. The size of the unbiased agents' sub-population does not qualitatively effect the effect of changes in the polarization level on individual behavioral adjustments, even though it affects the magnitude of the change. Finally, we characterize the socially efficient extraction policy in order to analyze the potential inefficiencies driven by decentralized management when agents exhibit perception biases. We consider two potential perspectives. In the first one, the social planner is populist and accounts for agents' potential biases. In the second one, the social planner is paternalist and does not take agents' perception biases into account. We obtain several conclusions. First, the comparison between the resource quality levels before and following the shift qualitatively impacts both how the two social planner perspectives compare at the individual and aggregate levels, and how decentralized and centralized management approaches differ at the sub-population levels. Second, while the comparison between the two centralized perspectives then mainly relates to the comparison of the sizes of the biased agents' sub-populations, the differences between centralized and decentralized management are more complex. For instance, while the overall size of the population does not qualitatively affect the comparison between decentralized and populist management policies, there are cases where it does affect the comparison when the social planner is paternalist. Third, while the tragedy of the commons[3] arises at the aggregate level, there are cases where it does not emerge at all sub-population levels. As such, a policy that would be designed on the basis of aggregate features only could well face a serious acceptability problem. Indeed, if policy makers would only focus on aggregate scores, the use of a tax policy would be put forward: depending on the composition of the population, such a policy would likely face strong opposition that would not be based on social justice but on efficiency grounds. A sub-population could face a tax policy, while efficiency would have called for the use of a subsidy. Thus, while some current policy-related discussions focus on issues of social justice potentially raised by the existence of perception biases, we highlight that such biases could actually also raise serious efficiency problems.

To shed light on the economic consequences of environmental shocks, a sizable literature focuses on the

[2]A recent experiment by Abdellaoui et al. (2011) provides evidence supporting the possibility that individuals exhibit heterogeneous risk perceptions.

[3]See Stavins (2001) for empirical evidence on tragedies of the commons.

analysis of common pool resource management under uncertainty (Bramoullé and Treich (2009); Costello et al. (2001); Fesselmeyer and Santugini (2013); Polasky et al. (2011); Ren and Polasky (2014); Sethi et al. (2005); Tsur and Zemel (1995)).[4] Ren and Polasky (2014) analyze the effect of exogenous/endogenous risk on the extraction decision of an infinitely lived agent, whereas Lucchetti and Santugini (2012) focus on the relationship between ownership risk and resource use and Fesselmeyer and Santugini (2013) study the strategic management of a common pool resource. Quaas et al. (2013) analyze the resilience of societies relying on natural resources when faced with exogenous shocks, while Quaas et al. (2019) focus on the insurance value of common-pool natural resources. Diekert (2017) introduces a dynamic game with a focus on a learning process related to the existence of a tipping point.[5] All these studies do not take risk perception biases into account. Another contribution (Agbo (2014)) studies the role of agents' heterogeneous beliefs about the future availability of a natural resources on extraction patterns. Yet, the risk of a regime shift is not considered in Agbo (2014), and perception biases are not accounted for either, since the regeneration of the natural resource depends on a stochastic variable whose distribution is unknown to all agents. In order to model perception biases, we follow some recent contributions analyzing the effect of such biases on economic activities (Farhi and Gabaix (2020); Gabaix (2019)).

One of the main contributions of this paper is to show how the polarization level of the population (as measured by the magnitude of the perception bias) results in different patterns of resource conservation. Our study thus complements an interesting literature focusing on the implications of risky events on natural resource conservation. Sakamoto (2014) provides a game setting where he shows how an endogenous regime shift probability alters the equilibrium structure and provides conditions under which a precautionary behavior emerges for the management of a common pool resource. A recent contribution by Costello et al. (2019) focuses on the spatial dimensions of the management of a common pool resource under the risk of a regime shift: the analysis focuses on how the regime shift probability affects the allocation of extraction levels in different patches. Miller and Nkuiya (2016) examines the relationship between the risk of a regime shift and the emergence of coalition formation between harvesters: conditions on the regime shift probabilities are provided under which harvesters have incentives to join or exit a coalition. Wagener (2003) analyzes the complex dynamics of resource extraction patterns in a shallow lake problem investigated by Mäler et al. (2003). Based on a body of literature exploring the issue of heterogeneous risk perceptions and considering the problem of decentralized extraction decisions within a common pool resource setting, we shed light on the link between the existence of risk perception biases and individual behavioral adjustments, and their consequences for resource conservation.

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In section 3 we analyze the benchmark case where the agents' population is unbiased, and we characterize the non-cooperative equilibrium outcomes. In section 4 we analyze the case where there is a uniform bias within the agents' population. In section 5 we consider the case where agents exhibit different types of bias, and we highlight the resulting qualitative differences. The socially efficient policies are characterized and compared with the decentralized outcome in Section 6. Section 7 concludes. All proofs are relegated to the end of the paper in an appendix.

[4]The issues of uncertainty and irreversibility also matter significantly for the timing of environmental policy adoption (Ulph and Ulph (1997)) or issues related to technological transfers (Elsayyad and Morath (2016)).

[5]See also Diekert and Nieminen (2015) for another potential effect of climate change, namely, a shift in the spatial distribution of the resource.

2 Setting of the problem

We extend the model of Fesselmeyer and Santugini (2013) to allow for risk perception biases. Let us consider the Great Fish War dynamic game in which N agents derive utility from the extraction of a common and renewable resource within a discrete time infinite horizon setting. Formally, let y_t be the available stock of renewable resource at the beginning of period t. The stock evolves at the beginning of period $t+1$ according to the biological rule

$$y_{t+1} = y_t^{\alpha} \tag{1}$$

where $\alpha \in (0,1]$ models the availability of the natural resource. At $t = 0$ the stock is below the carrying capacity ($y_0 < 1$).

During period t, if agent j extracts a quantity $c_{j,t}$ of the natural resource, she derives utility $u_j(c_{j,t}) = \phi \ln c_{j,t}$ with $\phi > 0$. The parameter ϕ denotes the quality of the natural resource: A higher ϕ implies a higher utility and marginal utility of extraction. The present extraction decisions of the N agents affects the future stock level. Using (1), the stock of the natural resource evolves according to the following rule

$$y_{t+1} = \left(y_t - \sum_{j=1}^{N} c_{j,t} \right)^{\alpha} \tag{2}$$

where a total of $\sum_{j=1}^{N} c_{j,t}$ is extracted in period t by the agents and $y_t - \sum_{j=1}^{N} c_{j,t}$ denotes the remaining stock which is left to yield the stock y_{t+1} at the beginning of period $t+1$. In this setup, parameters α and ϕ are constant over time.

Let us introduce the environmental risk: resource characteristics ϕ and α now depend on the state of the environment s_t in the following manner. Given state of environment s_t, the resource available at the beginning of date t is

$$y_{t+1} = \left(y_t - \sum_{j=1}^{N} c_{j,t} \right)^{s_t} \tag{3}$$

and extracting $c_{i,t}$ yields agent i the utility level $u_i(c_{i,t}) = \phi_s \ln c_{i,t}$ at period t. We here consider a potential regime shift that, if it occurs, will affect the availability and quality levels of the resource. The process characterizing the regime shift can be described as follows. There are two possible states: $s_t \in \{1, 2\}$. State α_1 represents the state of the environment prior to the regime shift. State α_2 denotes the state of the environment following the regime shift. After the occurrence of the shift, the natural resource is more scarce and available at the rate α_2.

The probability of the regime shift is $p \in (0,1]$. In other words, if the state of the environment is α_1, then there is a constant probability p that there will be a permanent shift in the next periods. We make the following assumption:

Assumption 1. $\Pr[s_{t+1} = \alpha_2 \mid s_t = \alpha_1] = p$ and $\Pr[s_{t+1} = \alpha_2 \mid s_t = \alpha_2] = 1$

The first part of the assumption implies that if the economy is in state α_2, it will remain in this state forever. Moreover, we assume that the agents' extraction activity does not influence the regime shift probability. As explained in Fesselmeyer and Santugini (2013), one example is the abrupt shutdown of thermohaline circulation, which permanently affects stock of fishes in the North Atlantic Ocean. It could be interesting to analyze cases where the effects of the shift are reversible, while it may occur repeatedly in the future. Our paper is a first step in the analysis of the impact of perception biases for natural resource management problems, and as such we keep the model as simple as possible. Yet, we would not expect fundamental qualitative changes in such settings: the reversibility of the shift might weaken some of the effects analyzed here, but the fact that the shift could repeat in the future might reinforce them on the other hand.

We now introduce a second assumption:

Assumption 2. $\phi_1 > \phi_2$ and $\alpha_1 < \alpha_2$

Thus, the effect of an environmental shift is twofold. The quality of the natural resource decreases: $\phi_1 > \phi_2$. Moreover, its availability decreases following the shift: $\alpha_2 > \alpha_1$.

We next introduce the final feature of the model: risk perception biases. The existence of risk perception biases is documented in different situations, one is related to climate change related risks (see Lee et al. (2015) for instance). There have been several recent contributions in the economics literature on how to model heterogeneous risk perceptions, we follow Gabaix (2019) or Farhi and Gabaix (2020) in terms of the modeling assumptions. Later on in the analysis, we will assume that agents may overestimate or underestimate the probability of occurrence of the regime shift compared to its true value p. Following Gabaix (2019), a biased agent perceives the value of the regime shift probability as follows

$$p^S = (1 - m)\, p + mx \tag{4}$$

where x denotes the default value (prior mean) and parameter m denotes the magnitude of risk (mis)perception, so that $m = 0$ corresponds to no misperception while $m = 1$ corresponds to full misperception. When the prior mean x is higher (lower) than the true value p, a biased agent overestimates (underestimates) the occurrence of the regime shift. As we consider risk perception biases, agents do not revise their estimate of the occurrence of the regime shift as time goes by. In the remainder of the paper, we will denote the sub-population of overestimating agents by N^o, the sub-population of underestimating agents by N^u, and by N^j the sub-population of agents who have an unbiased perception of the regime shift probability. The following figure provides an illustration:

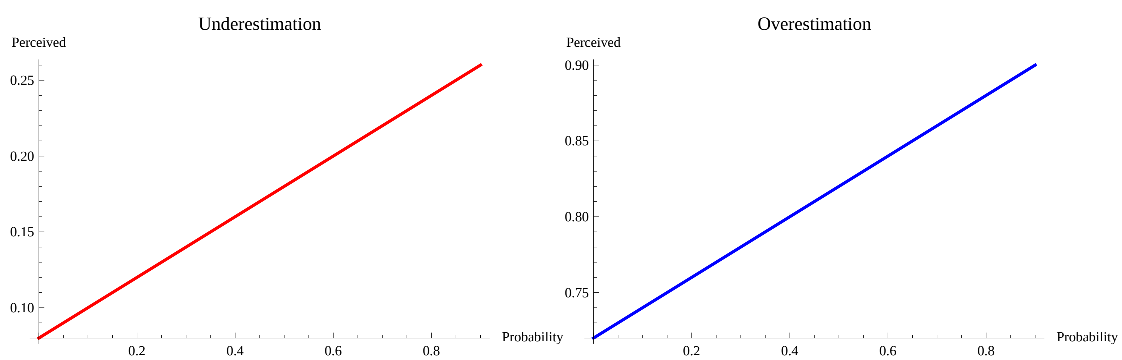

Figure 1: Underestimation ($x < p$)) and overestimation ($x > p$) of the regime shift probability

The case of no misperception might be understood as a case where scientists have a sense of the likelihood of a regime shift occurring (there is a reasonable consensus on an estimate of the probability), and unbiased agents in the population follow the consensus. Perception biases might emerge for different reasons, such as: manipulations of the agents' beliefs about environmental issues by environmental or political lobbies (McCright and Dunlap (2003)); individual or collective denial of environmental problems (Opotow and Weiss (2000)). The main feature here is that we analyze perception biases, that is, cases where individuals do not correct their biases when new information is provided. Agents know the distribution of biases in the population.[6]

We first deal with the case of N unbiased agents. We then consider the heterogeneous case and characterize the behavioral adjustments when the agents' population exhibits only one type of bias (either $N^o = 0$ or $N^u = 0$ is satisfied). Finally, we consider the case where the population exhibits two types of bias.

3 Benchmark case

Since we consider a decentralized and dynamic setting, we focus on Markov Perfect Nash equilibria, and more specifically on stationary Markov Perfect equilibria. Therefore, we drop hereafter script t when characterizing the equilibria.

The agents' population may be constituted by three different types of agents: N^j unbiased agents, N^o overestimating agents, and N^u underestimating agents, where $N = N^j + N^o + N^u$ denotes the size of the population. In this section, we consider the benchmark case where $N^o = N^u = 0$ is satisfied.

Agents maximize the expected sum of discounted utilities from extraction. Each agent anticipates the effect of her own extraction decision but also anticipates the effect of the other agents' decisions on the future stock of natural resource. Given the stock dynamics and regime shift probability described in Assumption 1, the value function of agent j before the regime shift, at state 1 (when the regime shift has not occurred), has the following form

$$V_1^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k} \phi_1 \ln c_j + (1-p)\,\delta V_1^j\left(\left(y - c_j - \sum_{k \neq j} c_k\right)^{\alpha_1}\right) + p\delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k\right)^{\alpha_2}\right) \quad (5)$$

where $\delta \in (0,1)$ denotes the discount factor before and after the regime shift. V_s denotes the value function in state $s \in \{1,2\}$. According to equality (5), agents anticipate the regime shift in subsequent periods. With probability $1-p$, the resource stock at the beginning of period $t+1$ is $\left(y - \sum_{j=1}^{N} c_j\right)^{\alpha_1}$. With probability p, there is a permanent change in environment, yielding the stock level $\left(y - \sum_{j=1}^{N} c_j\right)^{\alpha_2}$. Once a shift occurs, there is no other potential shifts in the future. Thus, following the regime shift

$$V_2^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k} \phi_2 \ln c_j + \delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k\right)^{\alpha_2}\right) \quad (6)$$

[6]For certain situations it could well be that agents "agree to disagree" on their risk perceptions. It could be interesting in future research to consider situations where involved agents' biases could be influenced by those of other agents, for instance, through decision-making processes aimed at collectively addressing the potential occurrence of the shift.

for any stock level $y > 0$. We now solve for the Markov Perfect Nash Equilibria (MPNE hereafter) by characterizing the equilibrium extraction levels $g_j(y)$ and $g_i(y)$. The agents' equilibrium behaviors prior to the environmental shift are as follows:

Proposition 1. *There is a unique interior stationary MPNE solution to (5). At the equilibrium, any agent j extracts*

$$c_j = g_j(y) = \frac{\phi_1 y}{\phi_1 N^j + (1-p)\, a_1 \alpha_1 \delta + p a_2 \alpha_2 \delta} \tag{7}$$

where $a_1 = \frac{\phi_1 + p\delta\alpha_2 a_2}{1-(1-p)\delta\alpha_1}$ and $a_2 = \frac{\phi_2}{1-\delta\alpha_2}$.

Proof. see Appendix (A).

This result has been already proved in Fesselmeyer and Santugini (2013)[7]; equality (7) highlights several features of the optimal extraction strategies. The equilibrium extraction strategy (7) depends on: the natural resource quality level both before and following the occurrence of the shift ϕ_1, ϕ_2, population size $N = N^j$, and the "weight" put on the regime before and after shift a_1 and a_2. We notice that a higher regime shift probability increases the weighting term a_2, which increases the extraction level before the shift.

4 Uniform perception bias

In this subsection, unbiased agents co-exist with biased agents exhibiting one type of bias. Specifically, the number of unbiased agents' sub-population is N^j and there are N^i biased agents with $i = u$ or $i = o$. The size of the whole population is thus $N = N^j + N^i$. Biased agents exhibit either an overestimation or an underestimation bias. The value function of a biased agent before the regime shift is as follows:

$$\begin{aligned} V_1^i(y) = \max_{0 \le c_i \le y - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j} \; & \phi_1 \ln c_i + \left(1 - p^S\right) \delta V_1^i \left(\left(y - c_i - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j \right)^{\alpha_1} \right) \\ & + p^S \delta V_2^i \left(\left(y - c_i - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j \right)^{\alpha_2} \right) \end{aligned} \tag{8}$$

The difference with respect to the benchmark case is that the biased agent has a perception of the probability function p^S as defined in (4). It follows that biased agent i anticipates a possible regime shift with a different probability than unbiased agents. Biased agents' value function at state 2 (once the shift has occurred) is as follows:

$$V_2^i(y) = \max_{0 \le c_i \le y - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j} \phi_2 \ln c_i + \delta V_2^i \left(\left(y - c_i - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j \right)^{\alpha_2} \right) \tag{9}$$

for any level $y > 0$ of the stock of the natural resource. Moving on to the representative unbiased agent's program, it is similar to that of the previous section:

[7]Uniqueness follows from standard induction arguments.

$$V_1^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i} \phi_1 \ln c_j + (1-p)\,\delta V_1^j\left(\left(y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i\right)^{\alpha_1}\right)$$
$$+ p\delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i\right)^{\alpha_2}\right) \tag{10}$$

In equation (10), one notices that any unbiased agent takes into account the existence of biased agents via the term $y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i$, which affects her decision problem before and after the regime shift. In the same manner, the representative unbiased agent's value function after the regime shift is as follows:

$$V_2^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i} \phi_2 \ln c_j + \delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i\right)^{\alpha_2}\right) \tag{11}$$

for any level $y > 0$ of the stock of the natural resource stock. We now characterize the equilibrium behavior of biased and unbiased agents, and we first obtain:

$$g_j(y) = c_j = \frac{\phi_1 y}{\phi_1 N^j + (1-p)\,a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta} - \underbrace{\frac{\phi_1 N^i c_i}{\phi_1 N^j + (1-p)\,a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta}}_{\text{Indirect effect}} \tag{12}$$

$$g_i(y) = c_i = \frac{\phi_1 y}{\phi_1 N^i + (1-p^S)\,a_1^i \alpha_1 \delta + p^S a_2^i \alpha_2 \delta} - \underbrace{\frac{\phi_1 N^j c_j}{\phi_1 N^i + (1-p^S)\,a_1^i \alpha_1 \delta + p^S a_2^i \alpha_2 \delta}}_{\text{Indirect effect}} \tag{13}$$

In expression (12) one notices that the second term relates to the effect of the biased agents' decisions on the extraction decisions of the unbiased agents. Using expressions (12) and (13) yields the following characterization:

Proposition 2. *There is a unique interior stationary MPNE solution to (8) and (10). At this equilibrium, any unbiased agent j and biased agent i extract the following amount, respectively:*

$$g_j(y) = \phi_1 \left(\frac{1 - N^i \gamma_1(m)}{\gamma_2}\right) y \tag{14}$$

$$g_i(y) = \gamma_1(m)\, y \tag{15}$$

where $\gamma_1(m) = \frac{\phi_1 z_1}{(\phi_1 N^j + z_1)(\phi_1 N^i + z_2(m)) - \phi_1 \phi_1 N^i N^j}$, $z_2(m) = \left(1 - p^S\right) a_1^i \delta \alpha_1 + p^S a_2^i \delta \alpha_2$, $\gamma_2 = \phi_1 N^j + z_1$ and $z_1 = (1-p)\,a_1^j \delta \alpha_1 + p a_2^j \delta \alpha_2$.

Proof. See Appendix (A).

The only term that depends on the biased agents' risk perception p^S is the term γ_1. The expressions of optimal extraction strategies highlight that biased and unbiased agents react to a change in the magnitude of the bias m in an opposite way, the formal proof is provided in Appendix A.

We now use Proposition 2 in order to assess the effect of an increase in the polarization level of the population, as measured by an increase in the magnitude of the bias m. It appears that this effect depends on (i) the type of bias (ii) the relative comparison between the quality level of the resource before and after the shift. Specifically, we obtain the following result:

Proposition 3. *(i) If the biased agents overestimate the regime shift probability (i.e, $x > p$) then, before the shift occurs, a biased agent's extraction level increases, while an unbiased agent's extraction level decreases, as m increases if and only if the resource quality levels ϕ_1 and Φ_2 satisfy $\phi_1 \geq \frac{\alpha_2(1-\delta\alpha_1)\phi_2}{\alpha_1(1-\delta\alpha_2)}$.*
ii) If the biased agents underestimate the regime shift probability (i.e, $x < p$) then, before the shift occurs, a biased agent's extraction level increases, while an unbiased agent's extraction level decreases, as m increases if and only if the resource quality levels ϕ_1 and ϕ_2 satisfy $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)\phi_2}{\alpha_1(1-\delta\alpha_2)}$.

Proof. See Appendix (B)

The following figure describes the extraction levels of unbiased and overestimating agents for low (respectively, high) levels of resource quality as the magnitude of the bias m varies.

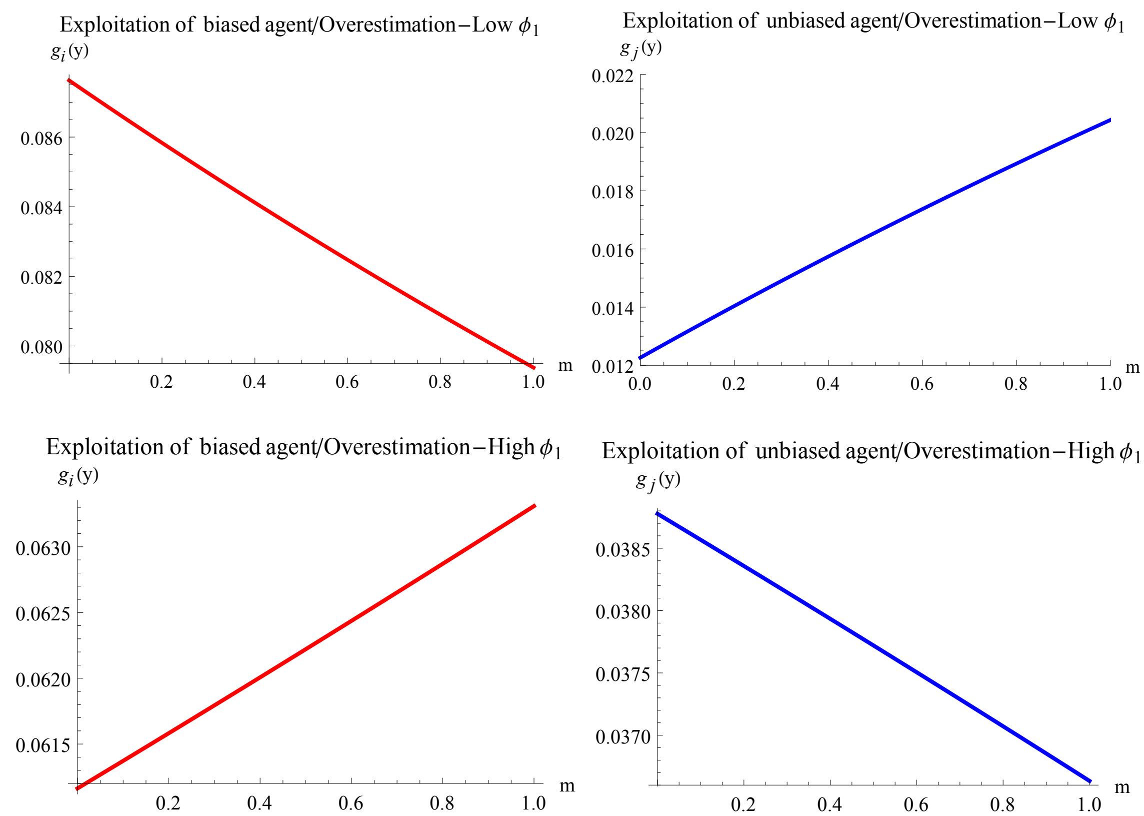

Figure 2: Effect of bias m on natural resource extraction patterns for biased and unbiased agents

Proposition 3 highlights the importance of the type of bias and of the natural resource quality before and after the shift. In order to better understand this result, one should compare the marginal cost of extraction for each type of agent. We derive the following ratio of marginal utility for both types of agent by using the corresponding optimality conditions (see conditions (76) and (78) in Appendix (A))

$$\frac{\frac{\phi_1}{c_i}}{\frac{\phi_1}{c_j}} = \frac{a_1^i \delta\alpha_1 - p^S \left(a_1^i \delta\alpha_1 - \delta\alpha_2 a_2^i\right)}{a_1^j \delta\alpha_1 - p\left(a_1^j \delta\alpha_1 - \delta\alpha_2 a_2^j\right)} \tag{16}$$

The right hand side of equality (16) corresponds to the ratio of marginal cost of natural resource exploitation for biased and unbiased agents. The left hand side corresponds to the ratio of marginal utility of resource extraction.

The term $p^S a_1^i \delta\alpha_1$ may be interpreted as a discounted weight awarded by a biased agent to the state before regime shift, accounting for the regeneration rate of the environment α, and the risk perception parameter p^S. A higher perceived probability p^S results in a higher weight related to the situation before the regime shift $V_1^i(y)$. This in turn implies that the weight $(1-p^S)a_1^i\delta\alpha_1$ related to the option to stay in the same state 1 $V_1^i(y)$ decreases. The same applies to the terms related to an unbiased agent.

We first discuss the case when a biased agent overestimates the regime shift probability ($p^S > p$). In order to understand the effects at play, one should come back to the right-hand side of (16). Indeed, a higher bias level m decreases the numerator through the effect on the term $p^S\left(\delta a_1^i \alpha_1 - \delta\alpha_2 a_2^i\right)$ if $a_1^i\delta\alpha_1 > \delta\alpha_2 a_2^i$ holds, which is the case if condition $\phi_1 > \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ is satisfied. This implies that the ratio of the marginal costs of extraction related to overestimating and unbiased agents decreases. As a result, an overestimating agent increases her extraction level as the situation becomes more polarized (m increases).

The same reasoning applies for an underestimating agent. If condition $a_1^i\delta\alpha_1 > \delta\alpha_2 a_2^i$ is satisfied, an underestimating agent decreases her natural extraction rate before shift as the situation becomes more polarized. This is so because the ratio of marginal costs of extraction between biased and unbiased agents (see right-hand side of equation (16)) increases when parameter m increases.

Due to the heterogeneity in behavioral adjustments when the situation becomes more polarized, a natural question is how the aggregate resource extraction level is affected by the existence of a bias on regime shift probability. We proceed in two steps. First, we analyze whether the aggregate resource extraction level increases or decreases following an increase in polarization for a given population distribution and size $N = N^j + N^i$. Second, we assume that the population size remains unchanged, while the population distribution varies by increasing the number of biased agents (and thus decreasing the number of unbiased agents), keeping the magnitude of the bias m constant.

The aggregate extraction level is as follows:

$$N^i g_i + \left(N - N^i\right) g_j = N^i g_i \left(1 + \frac{N - N^i}{N^i}\frac{g_j}{g_i}\right) \tag{17}$$

We obtain the first conclusion:

Proposition 4. *i) When the biased agents overestimate the probability of occurrence, the aggregate extraction level increases as the magnitude of the bias m increases if and only if $\phi_1 \geq \frac{\alpha_2(1-\delta\alpha_1)\phi_2}{\alpha_1(1-\delta\alpha_2)}$* is satisfied. *ii) When the biased agents underestimate the probability of occurrence, the aggregate extraction level increases as the magnitude of the bias m increases if and only if $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)\phi_2}{\alpha_1(1-\delta\alpha_2)}$* is satisfied.

Proof. See Appendix (C)

When the natural resource quality level is high ($\phi_1 > \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ is satisfied) the extraction level of overestimating agents and of underestimating agents increases and decreases (respectively) as the situation becomes more polarized.

Proposition 4 highlights that the effect of parameter m on the aggregate extraction level does not depend on the number of biased and unbiased agents. We notice that the effect resulting from biased agents' optimal decisions drives the total effect on aggregate extraction. In the case of overestimating agents, their marginal cost of extraction is lower, and this implies that their extraction level is higher.

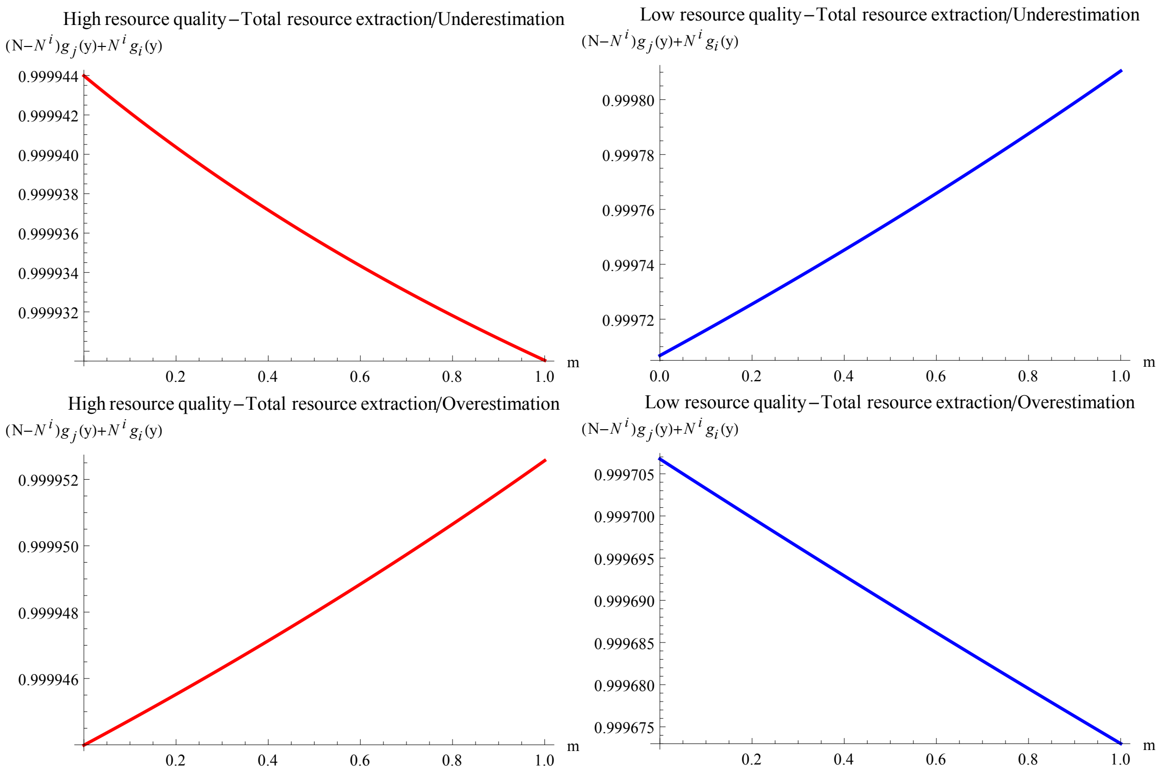

Figure 3: Total resource extraction as a function of bias level m

We now focus on how the number of biased and unbiased agent affects the aggregate extraction of the resource. In order to analyze this issue, we denote the size of the biased agents' and unbiased agents' sub-populations by N^i and $N - N^i$ respectively. Since we want to assess the effect of variations in the distribution of the overall population, we thus assume that N remains constant. We obtain:

Proposition 5. *i) When biased agents underestimate the occurrence probability, the aggregate extraction level increases as* N^i *increases if and only if* $\phi_1 \leq \phi_2 \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ *is satisfied.*
ii) When biased agents overestimate the occurrence probability, the aggregate extraction level increases as N^i *increases if and only if* $\phi_1 \geq \phi_2 \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ *is satisfied.*

Proof is provided in Appendix (D).

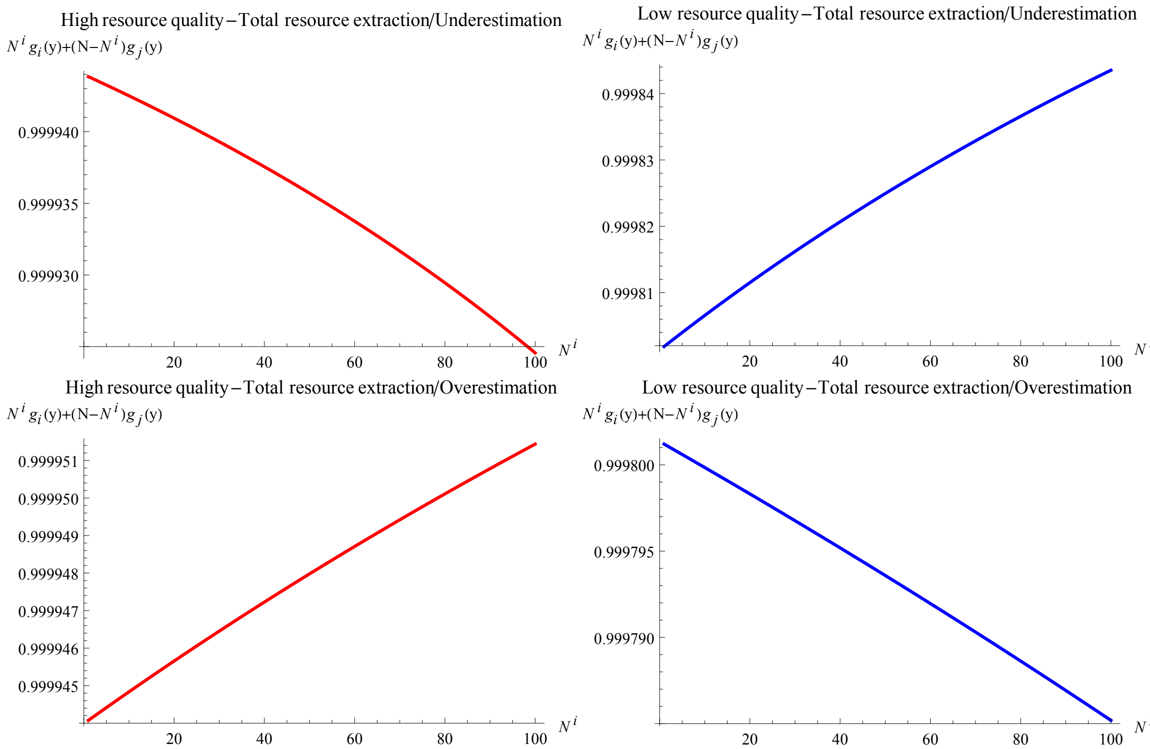

Figure 4: Total resource extraction as a function of bias level m

In order to understand this result, we first differentiate (17) with respect to N^i. As shown in Appendix D, it yields

$$\frac{\partial\left(N^i g_i + \left(N - N^i\right) g_j\right)}{\partial N^i} = g_i - g_j + \frac{\partial g_i}{\partial N^i} N^i \left(1 + \frac{N - N^i}{N^i} \frac{g_j}{g_i}\right) \tag{18}$$

where $\frac{\partial g_i}{\partial N^i} = -\frac{(\phi_1)^2 z_1}{(k_1)^2}(z_1 - z_2)\, y$. An increase in N^i has two effects. First, the distribution of biased and unbiased agents changes with respect to N^i (an increase in the number of overestimating agents) . Second, it affects the optimal extraction levels g_i and g_j. The term $g_i - g_j$ can be interpreted as the direct effect (the composition effect). The term $\frac{\partial g_i}{\partial N^i} N^i \left(1 + \frac{N-N^i}{N^i}\frac{g_j}{g_i}\right)$ is the indirect effect through the effect on optimal strategies. If $g_i > g_j$ then the composition effect is positive. This implies that $z_1 > z_2$ holds (see equation (16)). Therefore, the indirect effect is negative. The conclusion is that the two effects work in opposite directions. As shown in Proposition 5, the total extraction decreases with respect to N^i if $\phi_1 < \phi_2 \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$. This means that the indirect effect offsets the direct effect.

5 Coexistence of several types of bias

In this subsection, we now consider a fully heterogeneous agents' population where unbiased, overestimating and underestimating agents co-exist. The number of unbiased agents is N^j, and there are N^o overestimating and N^u underestimating agents. The total population size is thus $N = N^j + N^u + N^o$.

The value function of an unbiased agent before the regime shift is:

$$V_1^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k} \phi_1 \ln c_j + (1-p)\,\delta V_1^j\left(\left(y - c_j - \sum_{k \neq j} c_k\right)^{\alpha_1}\right)$$
$$+ p\delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k\right)^{\alpha_2}\right) \tag{19}$$

In the subsequent period, this agent anticipates the occurrence of the shift with probability p. Her value function in state 2 (once the regime shift has occurred) is then:

$$V_2^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k - \sum_{o=1}^{N^o} c_o - \sum_{u=1}^{N^u} c_u} \phi_2 \ln c_j + \delta V_2^j\left(\left(y - c_j - \sum_{k \neq j} c_k - \sum_{o=1}^{N^o} c_o - \sum_{u=1}^{N^u} c_u\right)^{\alpha_2}\right) \tag{20}$$

Regarding the expressions of before-shift and post-shift value functions, the difference is that unbiased agent takes into account the extraction of natural resources of overestimating and underestimating agents.

Moving on to the expression of the value function of a representative biased agent of type l ($l = o, u$) before the regime shift:

$$V_1^l(y) = \max_{0 \le c_l \le y - \sum_{k \neq l} c_k} \phi_1 \ln c_l - \left(1 - p^l\right)\delta V_1^l\left(\left(y - c_l - \sum_{k \neq l} c_k\right)^{\alpha_1}\right)$$
$$+ p^l \delta V_2^l\left(\left(y - c_l - \sum_{k \neq l} c_k\right)^{\alpha_2}\right) \tag{21}$$

In the subsequent period, an overestimating agent perceives the occurrence probability as p^l. In other words, the post-shift value function of a biased agent of type l is

$$V_2^l(y) = \max_{0 \le c_l \le y - \sum_{k \neq l} c_k} \phi_2 \ln c_l + \delta V_2^l\left(\left(y - c_l - \sum_{k \neq l} c_k\right)^{\alpha_2}\right) \tag{22}$$

We obtain the following result :

Proposition 6. *There is a unique interior stationary MPNE, which is characterized as follows: overestimating, underestimating and unbiased agents extract the following amounts of the natural resource:*

$$g_o(y) = y\frac{\phi_1 z_j z_u}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_o^{dec} y \tag{23}$$

$$g_u(y) = y\frac{\phi_1 z_j z_o}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_u^{dec} y \tag{24}$$

$$g_j(y) = y\frac{\phi_1 z_o z_u}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_j^{dec} y \tag{25}$$

where $z_j = \left[(1-p)\delta\alpha_1 a_1^j + p\delta\alpha_2 a_2^j\right]$, $z_o = [(1-p^o)\delta\alpha_1 a_1^o + p\delta\alpha_2 a_2^o]$ and $z_u = [(1-p^u)\delta\alpha_1 a_1^u + p\delta\alpha_2 a_2^u]$.

We can now analyze the qualitative differences when one introduces non-uniform biases in the agents' population. We first consider the effect of an increase in the polarization level:

Proposition 7. *i) The extraction level of an unbiased agent increases as m increases if and only if $\phi_1 \leq \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ and $\frac{N^o}{N^u} \geq -\frac{z_u^{'}(z_u)^2}{z_o^{'}(z_u)^2}$ are satisfied. Thus, an unbiased agent's extraction behavior may be non-monotone as the magnitude of the bias m increases.*
ii) Overestimating and underestimating agents react in opposite ways to an increase in the magnitude of the bias.

Proof. See Appendix (F)

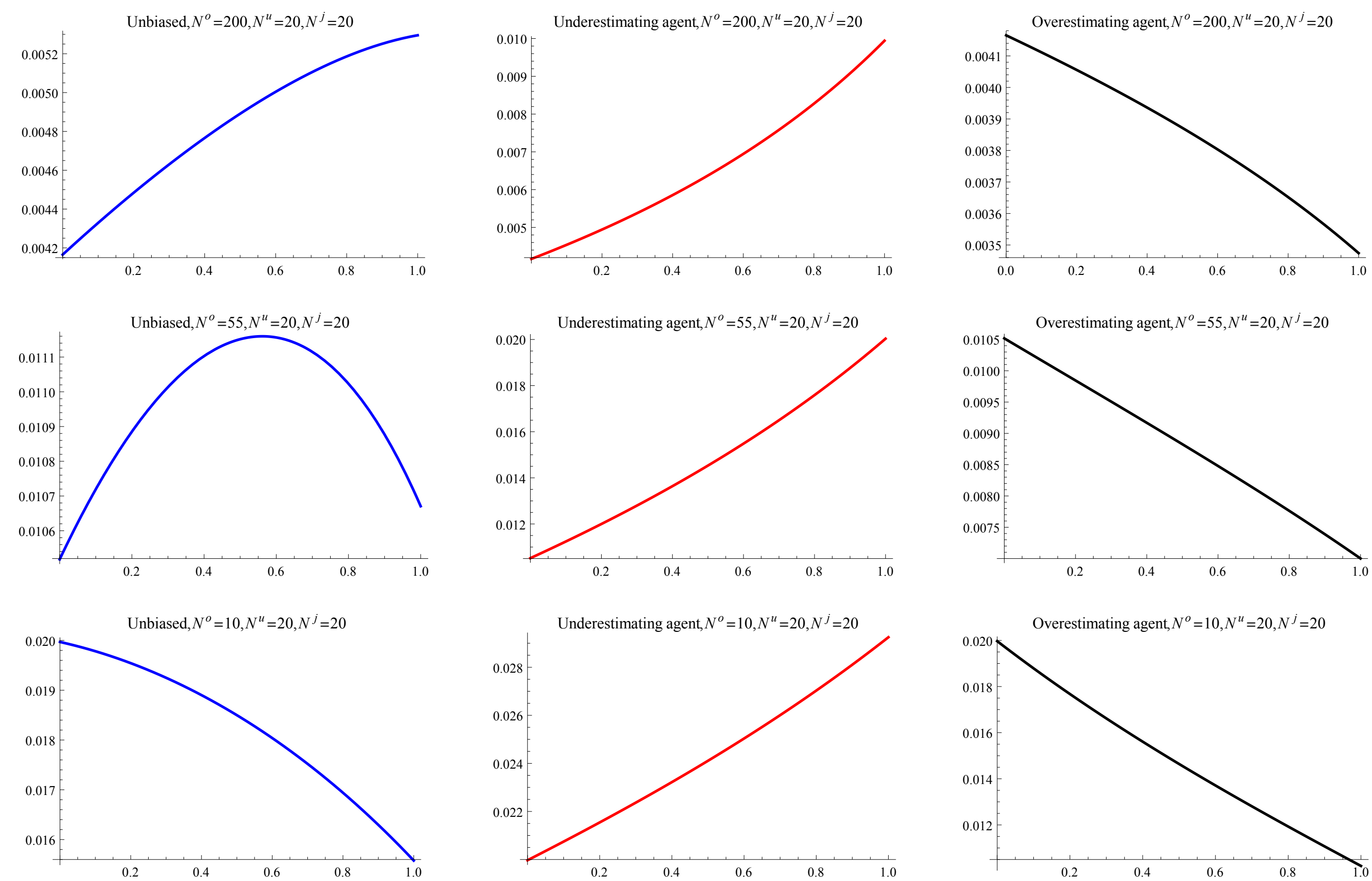

Figure 5: Resource extractions as functions of m

The main qualitative difference when allowing for non-uniform biases is as follows. First, when there is a uniform bias within the population, the intra-group structure does not qualitatively affect individual behavioral adjustments.

However, under non-uniform biases, this internal structure has an effect. Indeed, the unbiased agents' optimal decisions depend on both underestimating and overestimating agents' behaviors, which are shown to change in opposite ways as the population becomes more polarized. Hence, an unbiased agent's optimal extraction decreases or increases depending on which sub-population of biased agents exhibits the strongest behavioral adjustment, which in turn depends both on how the extraction rates of both overestimating and underestimating agents are affected by the magnitude of the bias m and on the relative size of each sub-population.

In order to illustrate this point, we derive the extraction rate of an unbiased agent as a function of overestimating and underestimating agents' extraction rates:

$$g_j(y) = \frac{\phi_1 \left(y - N^o g_o(y) - N^u g_u(y)\right)}{N^j \phi_1 + (1-p)\, a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta} \tag{26}$$

The marginal effect of an increase in m is as follows:

$$\frac{\partial g_j(y)}{\partial m} = -\frac{1}{N^j \phi_1 + (1-p)\, a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta} \left[N^o \frac{\partial g_o(y)}{\partial m} + N^u \frac{\partial g_u(y)}{\partial m} \right] \tag{27}$$

As mentioned above, expression (27) highlights that the effect of the magnitude of the bias on the extraction rate of the unbiased agent is driven by the number of overestimating and underestimating agents and by the direct effect of m on the biased agents' optimal decision (see the term in brackets in (27)).

Another important qualitative difference in this case is that unbiased agents may exhibit non-monotone behaviors (see Figure (5)) as the population becomes more polarized (m increases), provided the conditions given in Proposition 7 are satisfied. The term $N^o \frac{\partial g_o(y)}{\partial m} + N^u \frac{\partial g_u(y)}{\partial m}$ in expression (27) is either positive or negative since both marginal effects have opposite signs.

We conclude by analyzing how the magnitude of the bias affects the aggregate extraction level. It can be written as $N^o g_o + N^u g_u + N^j g_j$, and we obtain:

Proposition 8. *The aggregate extraction level increases as m increases if and only if $\frac{N^o}{N^u} \leq -\frac{z_u^{'}}{z_o^{'}} \left(\frac{z_o}{z_u}\right)^2$ and $\phi_1 \geq \frac{\phi_2 \alpha_2 (1-\delta\alpha_1)}{\alpha_1 (1-\delta\alpha_2)}$* are satisfied.

Proof. See Appendix (G).

To elaborate on this result, the marginal effect on the aggregate extraction is as follows:

$$\frac{\partial \left(N^o g_o + N^u g_u + N^j g_j\right)}{\partial m} = -\left(\phi_1\right)^2 z_j \frac{\left(z_o^{'} \left(z_u\right)^2 N^o + z_u^{'} \left(z_o\right)^2 N^u\right)}{\left[z_j z_o z_u + \phi_1 \left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right]^2} \tag{28}$$

This effect is consistent with the findings reported in Proposition 7 : the qualitative effect is driven by the relative size of the biased agents' sub-populations, and the marginal effect of a larger magnitude of the bias on the biased agents' behaviors.

Another insight is that the size of the unbiased agents' population does not affect the qualitative effect of a larger magnitude of the bias (that is, whether the aggregate extraction level increases or decreases). It only affects how much the aggregate extraction level changes as a result of an increase in the polarization level. Inspecting the expression of the marginal effect on an unbiased agent's behavior helps to understand this feature:

$$\frac{\partial g_j}{\partial m} = \frac{\left(\phi_1\right)^2 z_j \left(z_o^{'} \left(z_u\right)^2 N^o + z_u^{'} \left(z_o\right)^2 N^u\right)}{\left[z_j z_o z_u + \phi_1 \left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right]^2} \tag{29}$$

A larger sub-population of unbiased agents only affects the magnitude of the change in the extraction level, but it does not affect whether the change is positive or negative.

6 Social optimum versus decentralized management

We will characterize the socially efficient extraction path and then contrast it with decentralized extraction policies. It is worth giving more details on the notion of optimality we will use since there exist perception biases in the society. The social planner's problem is to maximize the sum of the agents' individual value functions $\sum_{i=1}^{N} V_{i,1}$ and here two scenarios are considered. In the first one, the social planner is populist and accounts for the agents' biases. Each agent's value function is defined using this agent's (un)biased probability. In the second scenario, the social planner is paternalistic: the agents' biases are unaccounted for, and as such agents' value functions are defined using the unbiased probability. We will analyze the differences driven by the two scenarios, and then compare each scenario with the decentralized extraction policies.

6.1 The socially efficient policy: two perspectives

We first consider the case of a populist social planner, and we now characterize the corresponding optimal policy. We conjecture that the pre-shift and post-shift value functions satisfy $V_i^1(y) = a_i^1 ln(y) + b_i^1$ and $V_i^2(y) = a_i^2 ln(y) + b_i^2$ for any agent i. We thus have:

$$\sum_{i=1}^{N}\left[a_1^j \ln y + b_1^j\right] = \max_{c_i} \sum_{i=1}^{N}\left[\phi_1 \ln c_i + \left(1-p^i\right)\delta V_i^1\left(\left(y-\sum_{k=1}^{N} c_k\right)^{\alpha_1}\right) + p^i \delta V_i^2\left(\left(y-\sum_{k=1}^{N} c_k\right)^{\alpha_2}\right)\right] \quad (30)$$

subject to $0 \leq \sum_{j=1}^{N} c_j \leq y$ and

$$\sum_{i=1}^{N}\left[a_2^i \ln y + b_2^i\right] = \max_{c_i} \sum_{i=1}^{N}\left[\phi_2 \ln c_i + \delta V_i^2\left(\left(y-\sum_{k=1}^{N} c_k\right)^{\alpha_2}\right)\right] \quad (31)$$

These characterizations will be used to characterize the optimal extraction policy that corresponds to this case. We obtain the following result:

Proposition 9. *The solution to the populist social planner's program is characterized by a uniform level of extraction within the entire population:*

$$g_j^{pop}(y) = g_o^{pop}(y) = g_u^{pop}(y) = l_1^{pop} y \quad (32)$$

where $l_1^{pop} = \frac{\phi_1}{N\phi_1 + \sum_{k=1}^{N}\left[(1-p^k)\delta\alpha_1 \frac{\phi_1 + \delta p^k \alpha_2 \frac{\phi_2}{1-\delta\alpha_2}}{1-\delta\alpha_1(1-p^k)} + p^k \delta \alpha_2 \frac{\phi_2}{1-\delta\alpha_2}\right]} = \frac{\phi_1}{N\phi_1 + \sum_{k=1}^{N} z_k} = \frac{\phi_1}{N\phi_1 + N^o z_o + N^j z_j + N^u z_u}$

Proof. See Appendix (H).

Now, we analyze the case of a paternalistic social planner. Unlike the populist social planner, a paternalistic social planner uses the scientific knowledge for the probability of a regime shift occurring. In other words, the planner's problem is similar to the previous one except that all probabilities of occurrence are taken equal to the unbiased probability. Using the characterizations of Proposition 9 we obtain the following result:

Corollary 1. *The solution to the paternalistic social planner's program is characterized by a uniform level of extraction within the entire population given by the following expression, for any agent j:*

$$g_j^{pat}(y) = l_1^{pat} y \tag{33}$$

where $l_1^{pat} = \frac{\phi_1}{N} \frac{[1-\delta\alpha_1(1-p)](1-\delta\alpha_2)}{\phi_1(1-\delta\alpha_2)+\delta p \alpha_2 \phi_2}$.

Proof. See Appendix (J).

Before moving on to the comparison between the centralized and decentralized policies, a few remarks are in order. First, even though agents exhibit heterogeneous perception biases and the social planner accounts for these biases, all agents follow the same extraction policy. This is due to an externality effect: each agent has to account for those with different perceptions than his own, and follows the same extraction pattern. Secondly, it is interesting to contrast the two perspectives available to the social planner in terms of the resulting policies. We obtain the following conclusion:

Proposition 10. *Consider the case where several types of bias coexist within the population. The comparison of the two social planner perspectives can be characterized by the following two cases:*

1. *When* $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ *is satisfied then we have:*

$$g^{pop}(y) \geq g^{pat}(y) \Longleftrightarrow N^u \geq N^o \frac{z_o - z_j}{z_j - z_u} \tag{34}$$

2. *When* $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ *is satisfied then we have:*

$$g^{pop}(y) \geq g^{pat}(y) \Longleftrightarrow N^u \leq N^o \frac{z_j - z_o}{z_u - z_j} \tag{35}$$

Proof. See Appendix (J).

When the pre-shift resource quality level lies below a threshold value, the effect of a given agent gets larger as the agent's bias increases. In the populist social planner policy this implies that, compared to an underestimating agent, an unbiased agent results in a decrease in aggregate extraction. The same property goes for the effect of an overestimating agent compared to an unbiased individual. Then, for a populist policy to yield higher extraction levels, the ratio of the number of underestimating agents to that of overestimating agents must be large enough. By contrast, when the pre-shift resource quality level is high enough, then the relative effect of an individual from each sub-population is then reversed, and the opposite conclusion holds.

Since both perspectives results in the same extraction policy across the population, Proposition 10 provides a direct comparison between the aggregate implications of both policy approaches.

6.2 Comparison with the decentralized outcome

In this subsection, we compare the socially efficient and decentralized policies. We will then rely on this comparison in order to derive policy implications. For each social planner perspective, we will compare the differences driven by decentralization at the sub-population level and at the aggregate level. We first provide the comparison between the decentralized and populist social planner perspectives:

Proposition 11. *Consider the case where several types of bias coexist within the population. The comparison between the decentralized and the populist social planner approaches is characterized as follows:*

1. *When* $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ *is satisfied:*

   - *Biased agents underestimating the occurrence probability extract more under decentralized management than under a populist social planner policy:* $g_u^{dec}(y) \geq g^{pop}(y)$.
   - *Biased agents overestimating the occurrence probability extract more under decentralized management if and only if the size of this sub-population is large enough:*

$$g_o^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow N^o - 1 \geq \frac{N^j z_u \left[\phi_1 \left(z_o - z_j\right) - \left(z_j\right)^2\right] + N^u z_j \left[\phi_1 \left(z_o - z_u\right) - \left(z_u\right)^2\right]}{z_o z_j z_u} \quad (36)$$

   - *When* $N^o \geq N^u \frac{z_o}{z_u}\frac{z_j - z_u}{z_o - z_j}$ *holds then unbiased agents always extract more under decentralized management. By contrast, when* $N^o < N^u \frac{z_o}{z_u}\frac{z_j - z_u}{z_o - z_j}$ *holds they extract more under decentralized management provided the size of this sub-population lies above a threshold value:*

$$g_j^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow N^j - 1 \geq \frac{N^u z_o \left[\phi_1 \left(z_j - z_u\right) - \left(z_u\right)^2\right] - N^o z_u \left[\phi_1 \left(z_o - z_j\right) + \left(z_o\right)^2\right]}{z_o z_j z_u} \quad (37)$$

2. *When* $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ *is satisfied:*

   - *Biased agents overestimating the occurrence probability extract more under decentralized management than under a populist social planner policy:* $g_o^{dec}(y) \geq g^{pop}(y)$.
   - *Biased agents underestimating the occurrence probability extract more under decentralized management if and only if the size of this sub-population lies above a threshold value:*

$$g_u^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow N^u - 1 \geq \frac{N^j z_o \left[\phi_1 \left(z_u - z_j\right) - \left(z_j\right)^2\right] + N^o z_j \left[\phi_1 \left(z_u - z_o\right) - \left(z_o\right)^2\right]}{z_o z_j z_u} \quad (38)$$

   - *When* $N^u \geq N^o \frac{z_u}{z_o}\frac{z_u - z_j}{z_j - z_o}$ *holds then unbiased agents always extract more under decentralized management. By contrast, when* $N^u < N^o \frac{z_u}{z_o}\frac{z_u - z_j}{z_j - z_o}$ *holds they extract more under decentralized management provided* ϕ_1 *lies above a threshold value:*

$$g_j^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow N^j - 1 \geq \frac{N^o z_u \left[\phi_1 \left(z_j - z_o\right) - \left(z_o\right)^2\right] - N^u z_o \left[\phi_1 \left(z_u - z_j\right) + \left(z_u\right)^2\right]}{z_o z_j z_u} \quad (39)$$

3. *Decentralized management results in a sub-optimally high aggregate extraction level:*

$$N^u g_u^{dec}(y) + N^o g_o^{dec}(y) + N^j g_j^{dec}(y) \geq N g^{pop}(y). \quad (40)$$

Proof: See Appendix (K).

Since the interpretation remains consistent for the different cases, we provide some discussion on the first case in Proposition 11. When pre-shift resource quality level is low enough, an underestimating agent puts more weight on the pre-shift state than the other categories, and the population is characterized by the highest extraction level under decentralized management. The effect of a centralized policy is to internalize part of the externalities driven by decentralization: in other words, it only decreases the level of extraction corresponding to this sub-population.

By contrast, overestimating agents are characterized by the lowest extraction levels under decentralized management. A populist policy, even though it accounts for agents' biases, still induces a uniform extraction level within the population. Whether this level lies above or below the corresponding level under decentralization depends on the magnitude of the externality driven by this sub-population. As such, when the size of this sub-group lies above a threshold value, decentralized management results in suboptimally high extraction levels. It is interesting to notice that the magnitude of the bias affects the level of the threshold value.

The case of the unbiased agents is more complex. The comparison then depends on the relative size of the biased agents' sub-populations. When the ratio of the number of overestimating agents to that of underestimating agents is large enough, the dominant effect is still driven by the negative externality resulting from decentralized management, and unbiased agents extract more than under a populist policy. By contrast, when this ratio is small enough, then there is a spillover effect imposed by underestimating agents on the unbiased population: their higher extraction levels induce unbiased agents to potentially reduce their decentralized extraction levels. When the size of the unbiased agents' population is large enough, the classical conclusion prevails. By contrast, when the size of the population is small enough, the spillover effect prevails, and unbiased agents extract less under decentralized management than under a populist planner policy.

Finally, at the aggregate level, the dominant effect is mainly driven by the negative externalities resulting from decentralized management, and the tragedy of the commons emerges at the overall population level. We now move on to the comparison between decentralized management and the policy adopted when the social planner is a paternalist.

Proposition 12. *Consider the case where several types of bias coexist within the population. The comparison between the decentralized and the paternalistic social planner perspectives is as follows:*

1. *When* $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ *is satisfied:*
   - *Biased agents underestimating the occurrence probability extract more under decentralized management than under a paternalistic social planner policy:* $g_u^{dec}(y) \geq g^{pat}(y)$.
   - *When* $N \leq \frac{z_o}{z_j}$ *holds, biased agents overestimating the occurrence probability extract less under decentralized management. By contrast, when* $N > \frac{z_o}{z_j}$ *holds, biased agents overestimating the occurrence probability extract more under decentralized management if and only if the size of this sub-population lies above a threshold value:*

$$g_o^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow N^o \geq \frac{N^j z_u \left[\phi_1 \left(z_o - z_j\right) - \left(z_j\right)^2\right] + N^u z_j \left[\phi_1 \left(z_o - z_u\right) - z_j z_u\right] + z_j z_o z_u}{\left(z_j\right)^2 z_u} \tag{41}$$

   - *When* $N^o \geq N^u \frac{z_o}{z_u}\frac{z_j - z_u}{z_o - z_j}$ *holds then unbiased agents extract more under decentralized management. By contrast, when* $N^o < N^u \frac{z_o}{z_u}\frac{z_j - z_u}{z_o - z_j}$ *holds they extract more under decentralized management* *provided the size of this sub-population lies above a threshold value:*

$$g_j^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow N^j - 1 \geq \frac{N^u z_o \left[\phi_1 \left(z_j - z_u\right) - z_u z_j\right] - N^o z_u \left[\phi_1 \left(z_o - z_j\right) + z_o z_j\right]}{z_o z_j z_u} \quad (42)$$

2. *When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied:*

   - *Biased agents overestimating the occurrence probability extract more under decentralized management than under a paternalistic social planner policy: $g_o^{dec}(y) \geq g^{pat}(y)$.*
   - *When $N \leq \frac{z_u}{z_j}$ holds, biased agents underestimating the occurrence probability extract less under decentralized management. By constrast, when $N > \frac{z_u}{z_j}$ holds, they extract more under decentralized management if and only if the size of this sub-population lies above a threshold value:*

$$g_u^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow N^u \geq \frac{N^j z_o \left[\phi_1 \left(z_u - z_j\right) - \left(z_j\right)^2\right] + N^o z_j \left[\phi_1 \left(z_u - z_o\right) - z_j z_o\right] + z_u z_j z_o}{z_o \left(z_j\right)^2} \quad (43)$$

   - *When $N^u \geq N^o \frac{z_u}{z_o} \frac{z_j - z_o}{z_u - z_j}$ holds then unbiased agents extract more under decentralized management. By contrast, when $N^u < N^o \frac{z_u}{z_o} \frac{z_j - z_o}{z_u - z_j}$ holds, they extract more under decentralized management provided the size of this sub-population lies above a threshold value:*

$$g_j^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow N^j - 1 \geq \frac{N^o z_u \left[\phi_1 \left(z_j - z_o\right) - z_o z_j\right] - N^u z_o \left[\phi_1 \left(z_u - z_j\right) + z_u z_j\right]}{z_o z_j z_u} \quad (44)$$

3. *Decentralized management results in a sub-optimally high aggregate extraction level:*

$$N^u g_u^{dec}(y) + N^o g_o^{dec}(y) + N^j g_j^{dec}(y) \geq N g^{pat}(y). \quad (45)$$

We focus on discussing some of the differences induced by the two approaches adopted for centralized management. A first point is that, when the qualitative conclusions are similar for both approaches, the magnitude of the threshold values related to the size of sub-populations differs in both cases. Another and more qualitative difference is that the size of the overall population might have a direct effect on the comparison when the planner is a paternalist. Specifically, it might affect the comparison for the sub-population characterized by the lowest extraction levels under decentralization. When the size of the overall population lies below a threshold value, the spillover effect induced by the other sub-populations (which tends to decrease extraction levels) outweighs the direct externality effect driven by decentralization (which results in higher extraction levels than under centralization). Decentralized management then always results in lower extraction levels compared to a centralized policy.

These results are useful to discuss a potentially important issue related to the form a public policy should take in order to solve the efficiency problem. Indeed, there are several important insights resulting from the previous comparisons. First, the comparison between the resource quality levels before and following the shift has first-order importance as it qualitatively impacts both how the two social planner perspectives compare (at the individual and aggregate levels) and how decentralized and centralized management approaches differ (at the sub-population levels). Policy discussions that would not account for these fundamentals would miss an important part of the problem at stake. Second, while the overall size of the population does not qualitatively affect the comparison between decentralized and populist management policies, there are cases where it does affect the comparison when the social planner is paternalistic. Third, while the tragedy of the commons still arises at the aggregate level for the two centralized perspectives, it does not arise at all sub-population levels. As such, a policy that would be designed by accounting for aggregate properties only could well face a serious acceptability problem. Indeed, if policy makers only focus on aggregate scores they would propose the use of a tax policy: depending on the composition of the population, such a policy would likely face strong opposition that would not be based on social justice but on efficiency grounds. Specifically, a sub-population could well face a tax policy, while efficiency would have called for the use of a subsidy. In other words, while many current policy-related discussions tend to focus on issues of social justice potentially raised by the existence of perception biases, we highlight that such biases could actually also raise serious efficiency problems at the sub-population levels.

7 Conclusion

In this paper, we analyze the effects of introducing risk perception biases in an agent's population managing a renewable resource. We focus on the effect of the polarization level and other population features (such as the intra-group structure) on individual behavioral adjustments and on the overall pattern of resource conservation. We consider two cases: a first one where there is uniform bias, and a second one where agents with different types of bias co-exist within the population. First, the type of bias (overestimation versus underestimation biases) and the resource quality level before and after the occurrence of the shift have first-order importance on the qualitative nature of individual behavioral adjustments and on the pattern of resource conservation. Second, when there are non-uniform biases within the population, the intra-group structure of the population (the relative size of the biased agents' sub-populations) qualitatively affects the degree of resource conservation. The unbiased agents may react in non-monotone ways to changes in the polarization level when faced with agents exhibiting different types of bias. Moreover, the size of the unbiased agents' sub-population does not qualitatively affect how an increase in the polarization level impacts individual behavioral adjustments, even though it affects the magnitude of this change. We then characterize the socially efficient extraction policy in order to analyze the potential inefficiencies driven by decentralized management. The comparison between the resource quality levels before and following the shift is shown to qualitatively impact both how the two social planner perspectives compare at the individual and aggregate levels, and how decentralized and centralized management approaches differ at the sub-population levels. Furthermore, while the comparison between the two centralized perspectives then mainly relates to the comparison of the sizes of the biased agents' sub-populations, the differences between centralized and decentralized management are more complex. For instance, the overall size of the population may affect it. Finally, while the tragedy of the commons arises at the aggregate level, there are cases where it does not emerge at all sub-population levels. As such, a policy designed on the basis of aggregate features only could well face serious acceptability problems. Indeed, the use of a tax policy could be put forward in such situations. Depending on the composition of the population, such a policy would likely face strong opposition as a sub-population could well face a tax policy, while efficiency would have called for the use of a subsidy. While some current policy-related discussions tend to focus on issues of social justice potentially raised by the existence of perception biases, we highlight that such biases could actually also raise serious efficiency problems.

This paper is a first step in the analysis of natural resource management problems driven by perception biases. There are several interesting questions for future research. Our analysis focuses on the case of a one-shot irreversible regime shift: it could be interesting to analyze cases where the effects of the shift are reversible, while it may occur repeatedly in the future. We would not expect fundamentally different qualitative findings in such settings: the reversibility of the shift might weaken some of the effects analyzed here, but the fact that the shift could repeat in the future might reinforce them on the other hand. The analysis of different types of policy instruments (combining for instance economic and psychological interventions, as suggested by Stern (2011) in the context of carbon emissions control) could also constitute a next step. Finally, it could also be worth studying different types of perception biases and their implications for resource management.

A Appendix

A. Proof of Proposition 1

We first solve for (6). Since we are interested in Markovian strategies, the problem is time-independent, we can drop the time-index when not necessary. Plugging the conjecture $V_2^j(y) = a_2^j \ln(y) + b_2^j$ into (46) we obtain

$$V_2^j(y) = \max_{0 \leq c_j \leq y - \sum_{k \neq j} c_k} \phi_2 \ln c_j + \delta \alpha_2 a_2^j ln \left(y - c_j - \sum_{k \neq j} c_k \right) + \delta b_2^j \tag{46}$$

The first-order condition is

$$\frac{\phi_2}{c_j} = \frac{a_2^j \alpha_2 \delta}{y_t - N^j c_j} \tag{47}$$

Since all agents are symmetrical, we have

$$c_j(y) = \frac{\phi_2 y}{\phi_2 N^j + a_2^j \alpha_2 \delta} \tag{48}$$

Plugging equation (48) into (46) yields

$$V_2(y) = \phi_2 \ln \left(\frac{\phi_2 y}{\phi_1 N^j + a_2^j \alpha_2 \delta} \right) + a_2^j \alpha_2 \delta \ln \left(y \left(1 - \frac{N^j \phi_2}{\phi_2 N^j + a_2^j \alpha_2 \delta} \right) \right) + \delta b_2^j \tag{49}$$

$$V_2(y) = (\phi_2 + a_2^j \alpha_2 \delta) \ln(y) + \phi_2 \ln(\omega_2) + a_2^j \alpha_2 \delta \ln\left(\left(1 - N^j \omega_2\right)\right) + \delta b_2^j \tag{50}$$

where $\omega_2 = \frac{\phi_2}{N^j \phi_2 + a_2^j \alpha_2 \delta}$. Bearing in mind the conjecture $V_2^j(y) = a_2^j \ln(y) + b_2^j$, we can find a_2^j and b_2^j

$$a_2^j = \frac{\phi_2}{1 - \delta \alpha_2} \tag{51}$$

$$b_2^j = \frac{\phi_2 \ln(\omega_2) + \delta a_2^j \alpha_2 \left(1 - N^j \omega_2\right)}{1 - \delta} \tag{52}$$

We now solve for (5). Plugging the conjecture $V_1^j(y) = a_1^j \ln(y) + b_1^j$ into (5) yields

$$V_1^j(y) = \max_{0 \leq c_i \leq y - \sum_{i \neq j} c_j} \phi_1 \ln c_j + (1-p)\, a_1^j \alpha_1 \delta \ln \left(y - c_j - \sum_{k \neq j} c_k \right) + p a_2^j \alpha_2 \delta \ln \left(y - c_j - \sum_{k \neq j} c_k \right) + \delta \mathbf{(1\text{-}p)} b_1^j + \delta \mathbf{p} b_2^j \tag{53}$$

The first-order condition is

$$\frac{\phi_1}{c_j} = \frac{(1-p)\, a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta}{y_t - \sum_{j=1}^{N^j} c_j} \tag{54}$$

Since all agents are identical

$$g_j(y) = \frac{\phi_1 y}{N^j\phi_1 + (1-p)\, a_1^j \alpha_1 \delta + p a_2^j \alpha_2 \delta} \tag{55}$$

Plugging equation (55) into (53) yields

$$V_1^j(y) = \phi_1 \ln\left(\frac{\phi_1 y}{N^j\phi_1 + (1-p)\, a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta}\right) + (1-p)\, a_1^j\alpha_1\delta\ln\left(y\left(1 - \frac{N^j\phi_1}{N^j\phi_1 + (1-p)\, a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta}\right)\right)$$

$$+ pa_2\alpha_2\delta\ln\left(y\left(1 - \frac{N^j\phi_1}{N^j\phi_1 + (1-p)\, a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta}\right)\right) + (1-p)\,\delta b_1^j + p\delta b_2^j \tag{56}$$

Arranging terms gives

$$V_1^j(y) = \left(\phi_1 + (1-p)\, a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta\right)\ln(y) + \phi_1\ln(\omega_1) + \ln\left(1 - N^j\omega_1\right)\left((1-p)\, a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta\right)$$

$$+ (1-p)\,\delta b_1^j + p\delta b_2^j \tag{57}$$

where $\omega_1 = \frac{\phi_1}{N^j\phi_1 + (1-p)a_1^j\alpha_1\delta + pa_2^j\alpha_2\delta}$. With the conjecture $V_1^j(y) = a_1^j \ln(y) + b_1^j$, we find a_1^j and b_1^j

$$a_1^j = \frac{\phi_1 + pa_2^j\alpha_2\delta}{1 - (1-p)\,\delta\alpha_1} \tag{58}$$

$$b_1^j = \frac{\phi_1\ln(\omega_1) + \ln\left(1 - N^j\omega_1\right)\left((1-p)\, a_1^j\alpha_1\delta + pa_2^j\delta\alpha_2\right) + p\delta b_2^j}{1 - (1-p)\,\delta} \tag{59}$$

B. Proof of Proposition 2 (biased vs. unbiased agent)

B.1 Post-shift problem

We first solve for (9) , (11) . We can plug the same conjecture $V_2(y) = a_2\ln(y) + b_2$ into (9) and (11) in order to reformulate the value function before the shift for the biased and unbiased agent, respectively.

$$V_2^i(y) = \max_{0 \le c_i \le y - \sum_{k\neq i} c_k - \sum_{j=1}^{N^j} c_j} \phi_2 \ln c_i + a_2^i\alpha_2\delta\ln\left(y - c_i - \sum_{k\neq i} c_k - \sum_{j=1}^{N^j} c_j\right) + \delta b_2^i \tag{60}$$

$$V_2^j(y) = \max_{0 \le c_j \le y - \sum_{k\neq j} c_k - \sum_{i=1}^{N^i} c_i} \phi_2 \ln c_j + a_2^j\alpha_2\delta\ln\left(y - c_j - \sum_{k\neq j} c_k - \sum_{i=1}^{N^i} c_i\right) + \delta b_2^j \tag{61}$$

The first-order conditions are

$$\frac{\phi_2}{c_i} = \frac{a_2^i\alpha_2\delta}{y - N^j c_j - N^i c_i} \tag{62}$$

$$\frac{\phi_2}{c_j} = \frac{a_2^j \alpha_2 \delta}{y - N^j c_j - N^i c_i} \tag{63}$$

Reformulating equations (62) and (63) yields

$$c_i = \frac{\phi_2 y - \phi_2 N^j c_j}{a_2^i \alpha_2 \delta + \phi_2 N^i} \tag{64}$$

$$c_j = \frac{\phi_2 y - \phi_2 N^i c_i}{a_2^j \alpha_2 \delta + \phi_2 N^j} \tag{65}$$

Solving equations (64) and (65) yields

$$c_i = \frac{\phi_2 y \left(\bar{x_1} - N^j \phi_2\right)}{\bar{x}_1 \bar{x}_2 - \phi_2 N^i \phi_2 N^j} \tag{66}$$

$$c_j = \frac{\phi_2 y \left(1 - N^i \left(\frac{\phi_2\left(\bar{x}_1 - N^j \phi_2\right)}{\bar{x}_1 \bar{x}_2 - \phi_2 N^i \phi_2 N^j}\right)\right)}{\bar{x}_1} \tag{67}$$

where $\bar{x}_1 = a_2^j \alpha_2 \delta + \phi_2 N^j$ and $\bar{x}_2 = a_2^i \alpha_2 \delta + \phi_2 N^i$
By using equations (66) and (61), we have

$$V_2^i\left(y\right) = \phi_2 \ln\left(\omega_2^i y\right) + a_2^i \alpha_2 \delta \ln\left(y - N^i c_i - N^j c_j\right) + \delta b_2^i \tag{68}$$

Using the conjecture $V_2^i\left(y\right) = a_2^i \ln\left(y\right) + b_2^i$ and plugging equations (66) and (67) into (68) gives

$$V_2^i\left(y\right) = \phi_2 \ln\left(y\right) + \phi_2 \ln\left(\omega_2^i\right) + a_2^i \alpha_2 \delta \ln\left(y\left(1 - N^i \omega_2^i - N^j \omega_2^j\right)\right) + \delta b_2^i \tag{69}$$

where $\omega_2^i = \frac{\phi_2\left(\bar{x_1} - N^j \phi_2\right)}{\bar{x}_1 \bar{x}_2 - \phi_2 N^i \phi_2 N^j}$ and $\omega_2^j = \frac{\phi_2\left(1 - N^i\left(\frac{\phi_2\left(\bar{x}_1 - N^j \phi_2\right)}{\bar{x}_1 \bar{x}_2 - \phi_2 N^i \phi_2 N^j}\right)\right)}{\bar{x}_1}$

$$V_2^i\left(y\right) = \left(\phi_2 + a_2^i \alpha_2 \delta\right) \ln\left(y\right) + \phi_2 \ln\left(\omega_2^i\right) + a_2^i \alpha_2 \delta \ln\left(\left(1 - N^i \omega_2^i - N^j \omega_2^j\right)\right) + \delta b_2^i \tag{70}$$

Using the conjecture $V_2^i\left(y\right) = a_2^i \ln\left(y\right) + b_2^i$, we have

$$a_2^i = \frac{\phi_2}{1 - \delta \alpha_2} \tag{71}$$

$$b_2^i = \frac{\phi_2 \ln\left(\omega_2^i\right) + a_2^i \alpha_2 \delta \ln\left(\left(1 - N^i \omega_2^i - N^j \omega_2^j\right)\right)}{1 - \delta} \tag{72}$$

For the sake of space, we omit the calculations for the unbiased agent, since they are similar.

$$a_2^j = \frac{\phi_2}{1 - \delta \alpha_2} \tag{73}$$

$$b_2^j = \frac{\phi_2 \ln\left(\omega_2^j\right) + a_2^j \alpha_2 \delta \ln\left(\left(1 - N^i \omega_2^i - N^j \omega_2^j\right)\right)}{1 - \delta} \tag{74}$$

B.2 Pre-shift problem

We now write the value function of the biased agent (overestimating or underestimating) i by using conjectures $V_1^i = a_1^i \ln y + b_1^i$ and $V_2^i = a_2^i \ln y + b_2^i$.

$$V_1^i(y) = \max_{0 \le c_i \le y - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j} \phi_1 \ln c_i + \left(1 - p^S\right) a_1^i \alpha_1 \delta \ln \left(y - c_i - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j \right)$$
$$+ p^S a_2^i \delta \alpha_2 \ln \left(y - c_i - \sum_{k \neq i} c_k - \sum_{j=1}^{N^j} c_j \right) + p^S \delta b_2^i + \left(1 - p^S\right) \delta b_1^i \tag{75}$$

The first-order condition is

$$\frac{\phi_1}{c_i} = \frac{\left(1 - p^S\right) a_1^i \delta \alpha_1 + p^S a_2^i \delta \alpha_2}{y - N^i c_i - N^j c_j} \tag{76}$$

We write the value function of the unbiased agent j, by using the conjecture $V_1^j = a_1^j \ln y + b_1^j$ yields

$$V_1^j(y) = \max_{0 \le c_j \le y - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i} \phi_1 \ln c_j + (1 - p) a_1^j \alpha_1 \delta \ln \left(y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i \right)$$
$$+ p a_2^j \delta \alpha_2 \ln \left(y - c_j - \sum_{k \neq j} c_k - \sum_{i=1}^{N^i} c_i \right) + p \delta b_2^j + (1 - p) \delta b_1^j \tag{77}$$

The first-order condition is

$$\frac{\phi_1}{c_j} = \frac{(1 - p) a_1^j \delta \alpha_1 + p a_2^j \delta \alpha_2}{y - N^i c_i - N^j c_j} \tag{78}$$

From equations (76) and (78), we find

$$g_i(y) = \gamma_1(m) y \tag{79}$$

and

$$g_j(y) = y \phi_1 \left(\frac{1 - N^i \gamma_1(m)}{\gamma_2} \right) \tag{80}$$

where

$$\gamma_1 = \frac{\phi_1 z_1}{\left(\phi_1 N^j + z_1\right)\left(\phi_1 N^i + z_2(m)\right) - \phi_1 N^j \phi_1 N^i} \tag{81}$$

with $z_1 = (1 - p) a_1^j \delta \alpha_1 + p a_2^j \delta \alpha_2$ and $z_2(m) = \left(1 - p^S\right) a_1^i \delta \alpha_1 + p^S a_2^i \delta \alpha_2$ and

$$\gamma_2 = \phi_1 N^j + z_1 \tag{82}$$

We write the value function of biased and unbiased agents

$$V_1^i(y) = \phi_1 \ln(\gamma_1(m) y) + \left(1-p^S\right) a_1^i \alpha_1 \delta \ln\left(y\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right)$$

$$+ p^S a_2^i \delta\alpha_2 \ln\left(y\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right) + \delta\left(1-p^S\right) b_1^i + \delta p^S b_2^i \tag{83}$$

and

$$V_1^j(y) = \phi_1 \ln\left(y\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right) + (1-p)\, a_1^j \alpha_1 \delta \ln\left(y\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right)$$

$$+ p a_2^j \delta\alpha_1 \ln\left(y\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right) + \delta(1-p)\, b_1^j + \delta p b_2^j \tag{84}$$

Arranging terms in (83) and (84) yields

$$V_1^i(y) = \left(\phi_1 + \left(1-p^S\right) a_1^i \alpha_1 \delta_1 + p^s a_2^i \delta\alpha_2\right) \ln(y)$$

$$+ \left[\ln\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right]\left(\left(1-p^S\right) a_1^i \delta\alpha_1 + p^s a_2^i \delta\alpha_2\right)$$

$$+\phi_1 \ln\gamma_1 + \delta\left(1-p^S\right) b_1^i + \delta p^S b_2^i$$

$$V_1^j(y) = \left(\phi_1 + (1-p)\, a_1^j \delta\alpha_1 + p a_2^j \delta\alpha_2\right) \ln(y)$$

$$+ \left[\ln\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right]\left((1-p)\, a_1^j \delta\alpha_1 + p a_2^j \delta\alpha_2\right)$$

$$+\phi_1 \ln\left[\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right] + \delta(1-p)\, b_1^j + \delta p b_2^j$$

Using the conjectures for biased and unbiased agents respectively $V_1^i = a_1^i \ln y + b_1^i$, $V_1^j = a_1^j \ln y + b_1^j$

$$a_1^i = \frac{\phi_1 + p^S a_2 \alpha_2 \delta}{1 - \left(1-p^S\right)\delta\alpha_1} \tag{85}$$

$$b_1^i = \frac{\phi_1 \ln\gamma_1 + \ln\left(\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right)\left(\left(1-p^S\right)\delta\alpha_1 a_1^i + p^S\delta\alpha_2 a_2^i\right) + \delta b_2^i}{1 - \left(1-p^S\right)\delta}$$

and

$$a_1^j = \frac{\phi_1 + p a_2 \alpha_2 \delta}{1 - (1-p)\,\delta\alpha_1}$$

$$b_1^j = \frac{\phi_1 \ln\left[\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right] + \ln\left(\left(1 - N^i\gamma_1(m) - N^j\phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right)\right)\right)\left((1-p)\,\delta\alpha_1 a_1^i + p\delta\alpha_2 a_2^i\right) + \delta b_2^i}{1-(1-p)\,\delta}$$

In order to see analytically the strategic substitutability, we differentiate equations (14) and (15) with respect to m

$$\frac{\partial g_j(y)}{\partial m} = -\phi_1 \frac{\gamma_1^{'}(m)}{\gamma_2} y \tag{86}$$

$$\frac{\partial g_i(y)}{\partial m} = \gamma_1^{'}(m)\, y \tag{87}$$

Independently of the sign of $\gamma_1^{'}(m)$, we see that the magnitude of the bias m affects the extraction level of unbiased and biased agents in opposite way.

B Proof of Proposition 3.

The aim of the proof is to show the effect of the magnitude of the bias m on the individual level of extraction level of biased $g_i(y)$ and unbiased agent $g_j(y)$. In order to simplify the exposition, we write g_i instead of $g_i(y)$. The reader is reminded that we have

$$g_j(y) = \phi_1\left(\frac{1-N^i\gamma_1(m)}{\gamma_2}\right) y \tag{88}$$

$$g_i(y) = \gamma_1(m)\, y \tag{89}$$

where $\gamma_1(m) = \frac{\phi_1 z_1}{(\phi_1 N^j + z_1)(\phi_1 N^i + z_2(m)) - \phi_1\phi_1 N^i N^j}$, $z_2(m) = \left(1-p^S\right) a_1^i \delta\alpha_1 + p^S a_2^i \delta\alpha_2$, $\gamma_2 = \phi_1 N^j + z_1$ and $z_1 = (1-p)\, a_1^j \delta\alpha_1 + p a_2^j \delta\alpha_2$ are defined to simplify the exposition. We differentiate $g_i(y)$ and $g_j(y)$ with respect to m:

$$\frac{\partial g_i(y)}{\partial m} = \gamma_1^{'}(m)\, y$$

$$\frac{\partial g_j(y)}{\partial m} = -\frac{N^i\phi_1}{\gamma_2}\gamma_1^{'}(m)\, y$$

This directly implies that biased and unbiased agents react in opposite ways to changes in the level of bias m. We now assess the sign of $\gamma_1^{'}(m)$.

$$\gamma_1^{'}(m) = -\frac{\phi_1 z_1\left(\phi_1 N^j + z_1\right) z_2^{'}(m)}{\left((\phi_1 N^j + z_1)(\phi_1 N^i + z_2(m)) - (\phi_1)^2 N^j N^i\right)^2}$$

Plugging equation (85) in $z_2(m)$ gives

$$z_2(m) = (1-(m(x-p)+p))\,\delta\alpha_1\left(\frac{\phi_1 + (m(x-p)+p)\,\delta\alpha_2 a_2}{1-(1-(m(x-p)+p))\,\delta\alpha_1}\right) + (m(x-p)+p)\,\delta\alpha_2 a_2^i \tag{90}$$

Differentiating equation (90) with respect to m yields

$$z_2^{'}(m) = \frac{a+b+c+d}{\left[1-\left(1-\left(m\left(x-p\right)+p\right)\right)\delta\alpha_1\right]^2}$$

where

$$a = -\left(x-p\right)\delta\alpha_1\left(\phi_1+\left(m\left(x-p\right)+p\right)\delta\alpha_2 a_2^i\right)\left(1-\left(1-\left(m\left(x-p\right)+p\right)\right)\delta\alpha_1\right)$$

$$b = +\left(x-p\right)\delta\alpha_1\left(1-\left(m\left(x-p\right)+p\right)\right)\delta\alpha_2 a_2^i\left(1-\left(1-\left(m\left(x-p\right)+p\right)\right)\delta\alpha_1\right)$$

$$c = -\left(x-p\right)\left(1-\left(m\left(x-p\right)+p\right)\right)\left(\delta\alpha_1\right)^2\left(\phi_1+\left(m\left(x-p\right)+p\right)\delta\alpha_2 a_2^i\right)$$

$$d = +\left(x-p\right)\delta\alpha_2 a_2^i\left(1-\left(1-\left(m\left(x-p\right)+p\right)\right)\delta\alpha_1\right)^2$$

We then deduce that, when biased agents overestimate the occurrence probability (that is, $x-p>0$ holds):

$$\phi_1 \leq \phi_2\frac{\alpha_2\left(1-\delta\alpha_1\right)}{\alpha_1\left(1-\delta\alpha_2\right)} \Longleftrightarrow z_2'(m)\geq 0 \;\; and \;\; \gamma_1'(m)\leq 0$$

Then this implies $\frac{\partial g_i(y)}{\partial m}\leq 0$ and $\frac{\partial g_j(y)}{\partial m}\geq 0$. The conclusion in the case where agents underestimate the occurrence probability follows from similar arguments.

C Proof of Proposition 4.

The aim of the proof is to show the effect of the magnitude of the bias m on the total extraction of natural resources. The total extraction of natural resources is $\psi\left(y\right) = N^i g_i\left(y\right)+\left(N-N^i\right)g_j\left(y\right)$ where N stands for the fixed population which is composed by biased agents N^i and unbiased agents N^j. To find the effect of the magnitude of the bias on total extraction of natural resources, one should differentiate $N^i g_i+\left(N-N^i\right)g_j$ with respect to m. To proceed, we first differentiate g_i and g_j with respect to m. Since N^i and N^j are constant and exogenous parameters, we can write[8]

$$\frac{\partial\psi\left(y\right)}{\partial m} = N^i\frac{\partial g_i\left(y\right)}{\partial m}+\left(N-N^i\right)\frac{\partial g_j\left(y\right)}{\partial m} \tag{91}$$

From the proof of Proposition 3, we already know $\frac{\partial g_i(y)}{\partial m}$ and $\frac{\partial g_j(y)}{\partial m}$. We can rewrite (91) as follows:

$$\frac{\partial\psi\left(y\right)}{\partial m} = \gamma_1^{'}\left(m\right)N^i\left(1-\left(N-N^i\right)\frac{\phi_1}{\gamma_2}\right)y \lesseqgtr 0$$

where $\gamma_2=\phi_1 N^j+z_1$. From the proof of Proposition 3, when $x-p>0$ is satisfied we know that

$$\gamma_1'(m)\leq 0 \Longleftrightarrow \phi_1\leq\phi_2\frac{\alpha_2\left(1-\delta\alpha_1\right)}{\alpha_1\left(1-\delta\alpha_2\right)}$$

[8]We use either ∂f or $f^{'}$ to denote the (partial) derivative of function f.

Using this property, we can assess the effect of the inattention level m on total resource extraction level. Specifically, we deduce that $\frac{\partial\Psi}{\partial m} \geq 0$ if and only if either $\gamma_1' \geq 0$ and $\frac{\gamma_2}{\phi_1} \geq \left(N - N^i\right)$ or $\gamma_1' \leq 0$ and $\frac{\gamma_2}{\phi_1} \leq \left(N - N^i\right)$ are satisfied. Yet, from the definition of γ_2 we deduce that $\left(N - N^i\right) \leq \frac{\gamma_2}{\phi_1}$ always holds. All together, we conclude that $\frac{\partial\Psi}{\partial m} \geq 0$ if and only if $\gamma_1' \geq 0$ is satisfied, which is equivalent to $\phi_1 \geq \phi_2 \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$. The proof in the case where biased agents underestimate the occurrence probability follows from the same type of arguments.

D Proof of Proposition 5.

We now assess the effect of the number of biased agents N^i on the total extraction of natural resources. We first differentiate g_i and g_j with respect to the number of biased agents N^i.

$$\frac{\partial g_i(y)}{\partial N^i} = \left(\frac{\partial \gamma_1}{\partial N^i}\right) y \tag{92}$$

$$\frac{\partial g_j(y)}{\partial N^i} = -\phi_1 \left(\frac{\gamma_1}{\gamma_2} + \left(\frac{N^i}{\gamma_2}\frac{\partial \gamma_1}{\partial N^i}\right)\right) y \tag{93}$$

where

$$\frac{\partial \gamma_1}{\partial N^i} = -\frac{\left(\phi_1\right)^2 z_1}{\left(k_1\right)^2}\left(z_1 - z_2\right) \tag{94}$$

where $k_1 = \phi_1 z_2 \left(N - N^i\right) + \phi_1 z_1 N^i + z_1 z_2$. We now rewrite the total extraction rate as follows:

$$N^i g_i + \left(N - N^i\right) g_j = N^i g_i \left(1 + \frac{N - N^i}{N^i}\frac{g_j}{g_i}\right)$$

Since the ratio $\frac{g_j}{g_i}$ does not depend on N^i, we obtain the following expression:

$$\frac{\partial\left(N^i g_i + \left(N - N^i\right) g_j\right)}{\partial N^i} = \left(g_i + N^i \frac{\partial g_i}{\partial N^i}\right) \underbrace{\left(1 + \frac{N - N^i}{N^i}\frac{g_j}{g_i}\right)}_{>0} - \frac{N}{N^i} g_j \tag{95}$$

We deduce quickly that $\frac{\partial g_i}{\partial N^i} = -g_i \frac{\phi_1}{k_1}\left(z_1 - z_2\right)$ which allows to simplify this expression as follows:

$$\frac{\partial\left(N^i g_i + \left(N - N^i\right) g_j\right)}{\partial N^i} = g_i \frac{z_2\left(z_1 - z_2\right)}{k_1} \tag{96}$$

Using the expressions of z_1 and z_2, we obtain:

$$z_1 - z_2 = \frac{p - p^S}{1 - \delta\alpha_2} \frac{-\phi_1 \delta\alpha_1 \left(1 - \delta\alpha_2\right) + \phi_2 \delta\alpha_2 \left(1 - \delta\alpha_1\right)}{\left[1 - (1 - p)\,\delta\alpha_1\right]\left[1 - \left(1 - p^S\right)\delta\alpha_1\right]}$$

If the biased agents underestimate the occurrence probability, then $p - p^S > 0$ and we deduce that $z_1 \geq z_2$ (and thus that $\frac{\partial\left(N^i g_i + \left(N - N^i\right) g_j\right)}{\partial N^i} \geq 0$) if and only if $-\phi_1 \delta\alpha_1 \left(1 - \delta\alpha_2\right) + \phi_2 \delta\alpha_2 \left(1 - \delta\alpha_1\right) \geq 0$ is satisfied, that is, condition $\phi_1 \leq \phi_2 \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ holds. The case where biased agents overestimate the occurrence probability follows from similar arguments.

E Proof of Proposition 6.

E.0.1 Post-shift problem

Similar to previous sections, we first solve for (20), and (22) respectively. Plugging the conjecture $V_2^{r=o,u,j}(y) = a_2^r \ln y + b_2^r$ into (20) and (22) yields

$$V_2^j(y) = \max_{0 \le c_j \le y - \sum\limits_{k \ne j} c_k} \phi_2 \ln c_j + \delta \alpha_2 a_2^j \ln \left(y - c_j - \sum_{k \ne j} c_k \right) + \delta b_2^j \tag{97}$$

$$V_2^l(y) = \max_{0 \le c_l \le y - \sum\limits_{k \ne l} c_k} \phi_2 \ln c_l + \delta \alpha_2 a_2^l \ln \left(y - c_l - \sum_{k \ne l} c_k \right) + \delta b_2^l \tag{98}$$

with $l = o, u$, the first-order conditions are

$$\frac{\phi_2}{c_j} = \frac{a_2^j \alpha_2 \delta}{y - N^j c_j - N^u c_u - N^o c_o}$$

$$\frac{\phi_2}{c_o} = \frac{a_2^o \alpha_2 \delta}{y - N^j c_j - N^u c_u - N^o c_o}$$

$$\frac{\phi_2}{c_u} = \frac{a_2^u \alpha_2 \delta}{y - N^j c_j - N^u c_u - N^o c_o}$$

Solving for c_j, c_o and c_u, we obtain

$$c_u = y \frac{\phi_2 a_2^j a_2^o}{N^j a_2^u a_2^o + N^o a_2^u a_2^j + N^u a_2^o a_2^j + a_2^u a_2^o a_2^j \delta \alpha_2} \tag{99}$$

$$c_j = c_u \frac{a_2^u}{a_2^j} \tag{100}$$

$$c_o = c_u \frac{a_2^u}{a_2^o} \tag{101}$$

Plugging (100), (101) and (99) into (98) for all types of agents gives

$$V_2^{r=j,o,u}(y) = \phi_2 \ln c_r(y) + \delta \alpha_2 a_2^r \ln \left(y - N^j c_j(y) - N^o c_o(y) - N^u c_u(y) \right) + \delta b_2^r$$

Then, we deduce quickly that

$$a_2^{r=j,o,u} = \frac{\phi_2}{1 - \delta \alpha_2 a_2^r}$$

$$b_2^{r=j,o,u} = \frac{\phi_2 ln \bar{c}_r + \delta \alpha_2 a_2^r \ln \left(1 - N^j c_j - N^o c_o - N^u c_u \right)}{1 - \delta}$$

E.0.2 Pre-shift problem

We first solve for (19) and (21). Plugging the conjecture $V_1^{r=o,u,j}(y) = a_1^r \ln y + b_1^r$ into the value functions of both types of agents (see the text), we first obtain that the value function of the unbiased agent before the regime shift is

$$V_1^j(y) = \max_{0 \le c_j \le y - \sum_{k \ne j} c_k} \phi_1 \ln c_j + (1-p)\,\delta\alpha_1 a_1^j \ln\left(y - c_j - \sum_{k \ne j} c_k\right)$$
$$+ p\delta\alpha_2 a_2^j \ln\left(y - c_j - \sum_{k \ne j} c_k\right) + \delta(1-p)\,b_1^j + \delta p b_2^j \tag{102}$$

The value function of the agent of type l $(l = o, u)$ before the regime shift is

$$V_1^l(y) = \max_{0 \le c_l \le y - \sum_{k \ne l} c_k} \phi_1 \ln c_l + \left(1-p^l\right)\delta\alpha_1 a_1^l \ln\left(y - c_l - \sum_{k \ne l} c_k\right)$$
$$+ p^l \delta\alpha_2 a_2^l \ln\left(y - c_l - \sum_{k \ne l} c_k\right) + \delta\left(1-p^l\right) b_1^l + \delta p^l b_2^l \tag{103}$$

The first-order conditions are

$$\frac{\phi_1}{c_r} = \frac{(1-p^r)\,\delta\alpha_1 a_1^r + p^r \delta\alpha_2 a_2^r}{y_t - N^j c_j - N^o c_o - N^u c_u} \tag{104}$$

Solving for g_j, g_o and g_u we obtain:

$$g_o(y) = y\frac{\phi_1 z_j z_u}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_o^{dec} y$$

$$g_u(y) = y\frac{\phi_1 z_j z_o}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_u^{dec} y$$

$$g_j(y) = y\frac{\phi_1 z_o z_u}{z_j z_o z_u + \phi_1\left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} = g_j^{dec} y$$

where

$$z_j = \left[(1-p)\delta\alpha_1 a_1^j + p\delta\alpha_2 a_2^j\right]$$

$$z_o = \left[(1-p^o)\delta\alpha_1 a_1^o + p\delta\alpha_2 a_2^o\right]$$

and

$$z_u = \left[(1-p^u)\delta\alpha_1 a_1^u + p\delta\alpha_2 a_2^u\right]$$

Using the conjecture $V_1^{r=o,u,j}(y) = a_1^r \ln y + b_1^r$, we write the value of the problem before shift

$$V_1^{r=o,u,j}(y) = \phi_1 \ln g_r(y) + ((1-p^r)\delta\alpha_1 a_1^r + p^r\delta\alpha_2 a_2^r)\ln\left(y\left(1 - N^o g_o^{dec} - N^u g_u^{dec} - N^j g_j^{dec}\right)\right) + \delta(1-p^r) b_1^r + \delta p^r b_2^r$$

Then, we can write

$$a_1^r = \frac{\phi_1 + p^r\delta\alpha_2 a_2^r}{1 - (1-p^r)\delta\alpha_1}$$

$$b_1^r = \frac{\phi_1 ln g_r^{dec} + ((1-p^r)\delta\alpha_1 a_1^r + p^r\delta\alpha_2 a_2^r)\ln\left(1 - N^o g_o^{dec} - N^u g_u^{dec} - N^j g_j^{dec}\right) + \delta p^r b_2^r}{1 - \delta(1-p^r)}$$

F Proof of Proposition 7.

i) In order to asses the effect of m on the natural resource extraction level of an unbiased agent, we differentiate the function $g_j(y) = y\frac{\phi_1 z_o z_u}{z_j z_o z_u + \phi_1[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o]} = g_j^{dec} y$ with respect to m

$$\frac{\partial g_j}{\partial m} = y\frac{\phi_1\left(z_o^{'} z_u + z_o z_u^{'}\right)}{z_j z_o z_u + \phi_1\left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)}$$

$$- y\frac{\phi_1 z_o z_u\left[z_j z_o^{'} z_u + z_j z_o z_u^{'} + \phi_1\left(N^j z_o^{'} z_u + N^j z_o z_u^{'} + N^o z_u^{'} z_j + N^u z_j z_o^{'}\right)\right]}{\left[z_j z_o z_u + \phi_1\left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right]^2} \tag{105}$$

After arranging all terms in (105), we obtain

$$\frac{\partial g_j}{\partial m} = y\frac{(\phi_1)^2 z_j\left(z_o^{'}(z_u)^2 N^o + z_u^{'}(z_o)^2 N^u\right)}{\left[z_j z_o z_u + \phi_1\left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right]^2} \tag{106}$$

If $\phi_1 \leq \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ we have $z_o^{'} \geq 0$. We conclude that that $\frac{\partial g_j}{\partial m} \geq 0$ if and only if $N^o \geq -\frac{z_u^{'}(z_u)^2 N^u}{z_o^{'}(z_u)^2}$ holds.

ii) The aim is to prove that overestimating and underestimating agents adjust their extraction levels in opposite ways as the magnitude of the bias m increases. Recall $g_o = g_j\frac{z_j}{z_o}$ and $g_u = g_j\frac{z_j}{z_u}$. We differentiate g_o and g_u with respect to m and we obtain:

$$\frac{\partial g_o}{\partial m} = \frac{\partial g_j}{\partial m}\frac{z_j}{z_o} - g_j\frac{z_j}{(z_o)^2}z_o^{'} \tag{107}$$

$$\frac{\partial g_u}{\partial m} = \frac{\partial g_j}{\partial m}\frac{z_j}{z_u} - g_j\frac{z_j}{(z_u)^2}z_u^{'} \tag{108}$$

Using equation (106) and arranging terms give

$$\frac{\partial g_o}{\partial m} = y\frac{\phi_1 z_j\left[\phi_1 z_j N^u\left(z_o z_u^{'} - z_u z_o^{'}\right) - (z_u)^2 z_o^{'}\left(z_j + \phi_1 N^j\right)\right]}{\left(z_j z_o z_u + \phi_1\left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right)^2} \tag{109}$$

$$\frac{\partial g_u}{\partial m} = y\frac{\phi_1 z_j\left[\phi_1 z_j N^o\left(z_o^{'} z_u - z_u^{'} z_o\right) - (z_o)^2 z_u^{'}\left(z_j + \phi_1 N^j\right)\right]}{\left(z_j z_o z_u + \phi_1\left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right)^2} \tag{110}$$

If $\phi_1 \leq \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$, then $z_o^{'} \geq 0$ and $z_u^{'} \leq 0$. It follows that $\frac{\partial g_o}{\partial m} \leq 0$ and $\frac{\partial g_u}{\partial m} \geq 0$. This concludes this part of the proof. The same argument applies also to the case where $\phi_1 \geq \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$ and we obtain $z_o^{'} \leq 0$ and $z_u^{'} \geq 0$.

G Proof of Proposition 8.

The aim of the proof is to show the effect of the inattention level m on the total extraction of natural resources. We first reformulate the total extraction $N^o g_o + N^u g_u + N^j g_j$ as follows

$$N^o g_o + N^u g_u + N^j g_j = g_j \left(N^o \frac{g_o}{g_j} + N^u \frac{g_u}{g_j} + N^j \right) \tag{111}$$

This allows us to simplify calculations, since we already know $\frac{\partial g_j}{\partial m}$. Differenciating the total extraction with respect to the inattention level m and using $\frac{g_u}{g_j} = \frac{z_j}{z_u}$ and $\frac{g_o}{g_j} = \frac{z_j}{z_o}$ yield

$$\frac{\partial \left(N^o g_o + N^u g_u + N^j g_j\right)}{\partial m} = \frac{\partial g_j}{\partial m} \left(N^o \frac{g_o}{g_j} + N^u \frac{g_u}{g_j} + N^j \right) - g_j z_j \left(\frac{N^u z_u^{'}}{(z_u)^2} + \frac{N^o z_o^{'}}{(z_o)^2} \right) \tag{112}$$

After arranging terms in equation (112), we have

$$\frac{\partial \left(N^o g_o + N^u g_u + N^j g_j\right)}{\partial m} = -(\phi_1)^2 z_j \frac{\left(z_o^{'} (z_u)^2 N^o + z_u^{'} (z_o)^2 N^j \right)}{\left[z_j z_o z_u + \phi_1 \left(N^j z_o z_u + N^o z_j z_u + N^u z_j z_u\right)\right]^2} \tag{113}$$

$\frac{\partial\left(N^o g_o + N^u g_u + N^j g_j\right)}{\partial m} > 0$ if and only if $\frac{N^o}{N^u}$ satisfies $\frac{N^o}{N^u} < -\frac{z_u^{'}}{z_o^{'}} \left(\frac{z_o}{z_u}\right)^2$ and ϕ_1 satisfies $\phi_1 > \frac{\phi_2\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}$.

H Proof of Proposition 9.

The proof follows the same method used by Breton and Keoula (2014). Plugging the conjecture $\sum_{j=1}^{N} \left[a_1^j \ln y + b_1^j\right]$ into (30) we obtain the following post-shift problem:

$$\sum_{j=1}^{N} \left[a_2^j \ln y + b_2^j\right] = \max_{c_{j,2}} \sum_{j=1}^{N} \left[\phi_2 \ln c_{j,2} + \delta\alpha_2 a_2^j \ln\left(y - \sum_{k=1}^{N} c_{k,2}\right) + \delta b_2^j\right] \tag{114}$$

The first order conditions are

$$\frac{\phi_2}{c_{j,2}} = \frac{\sum_{k=1}^{N} \delta\alpha_2 a_2^k}{y - \sum_{k=1}^{N} c_{k,2}} \tag{115}$$

or equivalently

$$\left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right) c_{j,2} + \phi_2 \sum_{k=1}^{N} c_{k,2} = \phi_2 y \tag{116}$$

From the optimality conditions we deduce quickly that $c_{j,2} = c_{i,2} = c_2$ for any $i \neq j$ and we conclude that:

$$c_2 = l_2^{pop} y \tag{117}$$

$$y - N c_2 = q_2^{pop} y \tag{118}$$

where $l_2^{pop} = \frac{\phi_2}{N\phi_2 + \left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)}$ and $q_2^{pop} = \frac{\left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)}{N\phi_2 + \left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)}$ are constants to be characterized. Arranging terms in (114) yields

$$\sum_{j=1}^{N} \left[a_2^j \ln y + b_2^j\right] = \sum_{j=1}^{N} \left[\left(\phi_2 + \delta\alpha_2 a_2^j\right) \ln y + \delta\alpha_2 a_2^j \ln q_2^{pop} + \delta b_2^j + \phi_2 \ln l_2^{pop}\right] \tag{119}$$

From (119), consistency yields

$$a_2^j = \frac{\phi_2}{1 - \delta\alpha_2}$$

$$b_2^j = \frac{\delta\alpha_2 a_2^j \ln q_2^{pop} + \phi_2 \ln l_2^{pop}}{1 - \delta}$$

Coming back to the expressions of l_2^{pop} and q_2^{pop} we now obtain

$$l_2^{pop} = \frac{\phi_2}{N\phi_2 + \left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)} = \frac{\phi_2}{N\phi_2 + N\delta\alpha_2 \frac{\phi_2}{1-\delta\alpha_2}} = \frac{1 - \delta\alpha_2}{N} \tag{120}$$

$$q_2^{pop} = \frac{\left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)}{N\phi_2 + \left(\sum_{k=1}^{N} \delta\alpha_2 a_2^k\right)} = \frac{N\delta\alpha_2 \frac{\phi_2}{1-\delta\alpha_2}}{N\phi_2 + N\delta\alpha_2 \frac{\phi_2}{1-\delta\alpha_2}} = \delta\alpha_2 \tag{121}$$

Now coming back to the pre-shift problem, we plug the conjecture $\sum_{j=1}^{N} \left[a_1^j \ln y + b_1^j\right]$ into the expression of the planner's problem, and we obtain

$$\sum_{j=1}^{N} \left[a_1^j \ln y + b_1^j\right] = \max_{c_{i,1}} \sum_{j=1}^{N} \left[\phi_1 \ln c_{j,1} + \left(1 - p^j\right) \delta\alpha_1 a_1^j \ln\left(y - \sum_{k=1}^{N} c_{k,1}\right) + p^j \delta\alpha_2 a_2^j \ln\left(y - \sum_{k=1}^{N} c_{k,1}\right) + \left(1 - p^j\right) \delta b_1^j + p^j \delta b_2^j\right] \tag{122}$$

The first order conditions are

$$\frac{\phi_1}{c_{j,1}} = \frac{\sum_{k=1}^{N} \left[\left(1 - p^k\right) \delta\alpha_1 a_1^k + p^k \delta a_2^k\right]}{y - \sum_{k=1}^{N} c_{k,1}} \tag{123}$$

We quickly deduce that $c_{j,1} = c_{i,1} = c_1$ for any $j \neq i$ and the optimality conditions can be rewritten as

$$\left[\sum_{k=1}^{N} \left(\left(1 - p^k\right) \delta\alpha_1 a_1^k + p^k \delta a_2^k\right) + N\phi_1\right] c_1 = \phi_1 y \tag{124}$$

Using (124) we conclude that

$$c_1 = l_1^{pop} y \tag{125}$$

$$y - Nc_1 = q_1^{pop} y \tag{126}$$

where $l_1^{pop} = \frac{\phi_1}{N\phi_1 + \sum_{k=1}^{N}\left((1-p^k)\delta\alpha_1 a_1^k + p^k \delta a_2^k\right)}$ and $q_1^{pop} = \frac{\sum_{k=1}^{N}\left((1-p^k)\delta\alpha_1 a_1^k + p^k \delta a_2^k\right)}{N\phi_1 + \sum_{k=1}^{N}\left((1-p^k)\delta\alpha_1 a_1^k + p^k \delta a_2^k\right)}$ are constants to be characterized. Arranging terms in (122) yields

$$\sum_{j=1}^{N}[a_1^j \ln y+b_1^j]=\sum_{j=1}^{N}[(\phi_1+(1-p^j)\delta\alpha_1 a_1^j+p^j\delta\alpha_2 a_2^j)\ln y+((1-p^j)\delta\alpha_1 a_1^j+p^j\delta\alpha_2 a_2^j)\ln q_1^{pop}+\phi_1 \ln l_1^{pop}+p^j\delta b_2^j+(1-p^j)\delta b_1^j] \tag{127}$$

From (127) consistency yields

$$a_1^j=\frac{\phi_1+p^j\delta\alpha_2 a_2^j}{1-(1-p^j)\,\delta\alpha_1}=\frac{\phi_1+p^j\delta\alpha_2\frac{\phi_2}{1-\delta\alpha_2}}{1-(1-p^j)\,\delta\alpha_1} \tag{128}$$

and

$$b_1^j=\frac{\left(\left(1-p^j\right)\delta\alpha_1 a_1^j+p^j\delta\alpha_2 a_2^j\right)\ln q_1^{pop}+\phi_1 \ln l_1^{pop}+p^j\delta b_2^j}{1-(1-p^j)\,\delta} \tag{129}$$

Coming back to the expressions of l_1^{pop} and q_1^{pop}, we obtain:

$$l_1^{pop}=\frac{\phi_1}{N\phi_1+\sum_{k=1}^{N}\left[\left(1-p^k\right)\delta\alpha_1\frac{\phi_1+\delta p^k\alpha_2\frac{\phi_2}{1-\delta\alpha_2}}{1-\delta\alpha_1(1-p^k)}+p^k\delta\alpha_2\frac{\phi_2}{1-\delta\alpha_2}\right]} \tag{130}$$

$$q_1^{pop}=\frac{\sum_{k=1}^{N}\left[\left(1-p^k\right)\delta\alpha_1\frac{\phi_1+\delta p^k\alpha_2\frac{\phi_2}{1-\delta\alpha_2}}{1-\delta\alpha_1(1-p^k)}+p^k\delta\alpha_2\frac{\phi_2}{1-\delta\alpha_2}\right]}{N\phi_1+\sum_{k=1}^{N}\left[\left(1-p^k\right)\delta\alpha_1\frac{\phi_1+\delta p^k\alpha_2\frac{\phi_2}{1-\delta\alpha_2}}{1-\delta\alpha_1(1-p^k)}+p^k\delta\alpha_2\frac{\phi_2}{1-\delta\alpha_2}\right]} \tag{131}$$

These characterizations conclude the proof.

I Proof of Corollary 1

The proof follows from Proposition 9 using $p^i=p$ for any $i\in N$.

J Proof of Proposition 10.

Using the characterizations of the two types of socially efficient policies, we now compare them. First, condition $\phi_1\leq\frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i=o,j,u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that $z_j<z_o$ and $z_j>z_u$ are satisfied. We now deduce:

$$l_1^{pop}\geq l_1^{pat}\Longleftrightarrow\frac{1}{N\phi_1+N^u z_u+N^j z_j+N^o z_o}\geq\frac{1}{N\phi_1+N z_j} \tag{132}$$

and as such $g^{pop}(y)\geq g^{pat}(y)$ if and only if $N^u\left(z_j-z_u\right)+N^o\left(z_j-z_o\right)\geq 0$ is satisfied. The first term is positive, while the second term is negative. Rewriting this condition, we obtain the first conclusion.

Now, when $\phi_1>\frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i=o,j,u)$ are then decreasing functions of the perceived occurrence probabilities: this in turn implies that $z_j>z_o$ and $z_j<z_u$ are satisfied. Using the same calculations than in the first case, we deduce that $g^{pop}(y)\geq g^{pat}(y)$ if and only if $N^u\left(z_j-z_u\right)+N^o\left(z_j-z_o\right)\geq$

0 is satisfied. The first term is then negative, while the second term is positive. Rewriting this condition, we obtain the second conclusion.

K Proof of Proposition 11.

We start the proof by comparing the extraction level of an overestimating agent in a decentralized framework and the uniform optimal extraction by a populist social planner.

$$g_o^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow \phi_1 \left[N^j z_u \left(z_j - z_o\right) + N^u z_j \left(z_u - z_o\right)\right] + z_j z_u \left[N_j z_j + \left(N^o - 1\right) z_o + N^u z_u\right] \geq 0 \quad (133)$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that $z_j < z_o$ and $z_u < z_o$ are satisfied. The first term in the above sum is thus negative, while the second term is positive. Rewriting the inequality, we obtain the first conclusion. When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities: this in turn implies that $z_j > z_o$ and $z_u > z_o$ are satisfied. Both terms in the above sum are then positive, and the second conclusion follows.

We compare the extraction level of an underestimating agent in a decentralized framework and the uniform optimal extraction by a populist social planner.

$$g_u^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow \phi_1 \left[N^j z_o \left(z_j - z_u\right) + N^o z_j \left(z_o - z_u\right)\right] + z_j z_o \left[N_j z_j + \left(N^u - 1\right) z_u + N^o z_o\right] \geq 0 \quad (134)$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that $z_j > z_u$ and $z_u < z_o$ are satisfied. Both terms in the above sum are positive, and the first conclusion follows. When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities: this in turn implies that $z_j < z_u$ and $z_u > z_o$ are satisfied. The first term in the above sum is thus negative, while the second term is positive. Rewriting the inequality, we obtain the second conclusion.

We now compare the extraction level of an unbiased agent in a decentralized framework and the uniform optimal extraction by a populist social planner.

$$g_j^{dec}(y) \geq g^{pop}(y) \Longleftrightarrow \phi_1 \left[N^o z_u \left(z_o - z_j\right) + N^u z_o \left(z_u - z_j\right)\right] + z_u z_o \left[N^u z_u + \left(N^j - 1\right) z_j + N^o z_o\right] \geq 0 \quad (135)$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that $z_j < z_o$ and $z_j > z_u$ are satisfied. If the first term in the above sum is positive, then the sum is positive. Supposing that the first term in the above sum is positive yields the condition relating N^o and N^u that ensures that decentralized management results in overextraction. Now, if the first term in the above sum is negative, then rewriting the inequality yields the condition characterizing the second sub-case in the first conclusion.

When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities: this in turn implies that $z_j > z_o$ and $z_j < z_u$ are satisfied. If the first term in the above sum is positive, then the sum is positive. Supposing that the first term in the above sum is positive yields the condition relating N^u and N^o that ensures that decentralized management results in overextraction. Now, if the first term in the above sum is negative, then rewriting the inequality yields the condition characterizing the second sub-case in the second conclusion.

Finally, using the expressions of the optimal extraction levels, we deduce that the aggregate level of extraction resulting from decentralized management will be sub-optimally high if and only if we have:

$$\frac{N^j z_o z_u + N^o z_j z_u + N^u z_j z_o}{z_j z_o z_u + \phi_1 \left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} \geq \frac{N}{N\phi_1 + N^j z_j + N^o z_o + N^u z_u} \tag{136}$$

This inequality always holds, and this concludes the proof.

L Proof of Proposition 12.

To compare the decentralized outcome and the policy adopted when the social planner is paternalist, we start with the case of an overestimating agent.

$$g_o^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow \phi_1 \left[N^j z_u (z_j - z_o) + N^u z_j (z_u - z_o)\right] + z_j z_u \left[N z_j - z_o\right] \geq 0 \tag{137}$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that the first term in the above sum is thus negative. If the second term is negative then the above inequality cannot hold, and the result obtains. If the second term is positive (N lies above threshold $\frac{z_o}{z_j}$) then rewriting the inequality yields the conclusion. When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities: this in turn implies that the first term in the above sum is positive. Since the second term is also positive, we obtain the conclusion.

We now analyze the case of an underestimating agent.

$$g_u^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow \phi_1 \left[N^j z_o (z_j - z_u) + N^o z_j (z_o - z_u)\right] + z_j z_o \left[N z_j - z_u\right] \geq 0 \tag{138}$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that both terms in the above sum are positive, and the conclusion follows. When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities: this in turn implies that the first term in the above sum is negative. When N is smaller than $\frac{z_u}{z_j}$ then both terms in the above sum are negative, and the conclusion follows. When N lies above this threshold value, rewriting the above inequality yields the last conclusion.

We now analyze the case of an unbiased agent.

$$g_j^{dec}(y) \geq g^{pat}(y) \Longleftrightarrow \phi_1 \left[N^o z_u (z_o - z_j) + N^u z_o (z_u - z_j)\right] + (N-1) z_o z_u z_j \geq 0 \tag{139}$$

Condition $\phi_1 \leq \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is necessary and sufficient to ensure that parameters z_i $(i = o, j, u)$ are increasing functions of the perceived occurrence probabilities: this in turn implies that $z_j < z_o$ and $z_j > z_u$ are satisfied. If the first term in the above sum is positive (which is equivalent to the condition relating N^o and N^u) then the sum is positive and the conclusion follows. If the first term is negative, then rewriting the above inequality yields the condition on N^j. When $\phi_1 > \frac{\alpha_2(1-\delta\alpha_1)}{\alpha_1(1-\delta\alpha_2)}\phi_2$ is satisfied the parameters z_i $(i = o, j, u)$ are decreasing functions of the perceived occurrence probabilities. If the first term in the above sum is positive (which is equivalent to the condition relating N^u and N^o) then the sum is positive, and the conclusion follows. When the first term is negative, then (as the second term in the sum is positive) rewriting the inequality yields the condition on N^j.

Finally, using the expressions of the optimal extraction levels, we deduce that the aggregate level of extraction resulting from decentralized management will be sub-optimally high if and only if we have:

$$\frac{N^j z_o z_u + N^o z_j z_u + N^u z_j z_o}{z_j z_o z_u + \phi_1 \left[N^j z_o z_u + N^o z_j z_u + N^u z_j z_o\right]} \geq \frac{N}{N\phi_1 + N z_j} \tag{140}$$

This inequality always holds, and this concludes the proof.

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