Sensor Placement for Fault Diagnosis Performance Maximization under Budgetary Constraints

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Abstract— **This paper presents a strategy based on fault diagnosability maximization to optimally locate sensors in complex systems. The goal is to characterize and determine a sensor configuration that guarantees a maximum degree of diagnosability and does not exceed a maximum sensor configuration cost. The strategy is based on the structural system model. Structural analysis is a powerful tool for dealing with complex nonlinear systems. The proposed approach is successfully applied to a Fuel Cell Stack System.**

I. INTRODUCTION

The problem of sensor placement for Fault Detection and Isolation (FDI) consists in determining the optimal set of sensors such that a predefined set of faults are detected and isolated. Usually the optimal sensor placement problem involves minimising the sensor configuration cost. Thus, the sensor placement problem can be viewed as a combinatorial problem where the goal is to find a sensor combination that fulfils the diagnosis specifications.

Solving the sensor placement for diagnosis can be treated from many different viewpoints. Indeed, such a problem depends on the kind of system description, the required diagnosis specifications, as well as the technique used to implement the diagnosis system. Because of this, developing a sensor placement method, that works for all possible fault diagnosis systems, is unattainable. In this paper, fault diagnosis systems are based on consistency checking by means of structural models. The required diagnosis specifications to be fulfilled are fault detection and isolation for a predefined set of faults under budgetary constraints. Some works devoted to sensor placement for diagnosis using graph tools can be found in [1], [2] and [3]. All these works use a structural model-based approach and define different diagnosis specifications to solve the sensor placement problem.

A structural model is a coarse model description, based on a bi-partite graph, that can be obtained early in the development process, without major engineering efforts. This kind of models is suitable to handle large scale and complex systems since efficient graph-based tools can be used which do not have numerical problems. Structural analysis is a powerful tool for early determination of fault diagnosis performances.

In structural model based diagnosis, consistency may be checked by using a set of redundant sub-models (i.e. Minimal Structurally Overdetermined (MSO) sets of equations). A residual generator can be implemented from an MSO set by computing the internal unknown variables through a convenient manipulation of the equations and later checking consistency in a redundant equation. This concept is known as a causal interpretation of the computability [4]. The result is a directed bi-partite graph, named *computation sequence*, that shows how internal values can be computed from the equations (value propagation) in every redundant sub-model. However, to guarantee that the residual is generated by using non-linear equations, the structural model framework needs to be adapted in order to handle causal computability. Few works focus this causal assignment in the fault diagnosis field [5], [6], [7]. In this paper, the causality framework introduced in [8] will be followed to address it. Thus, the solution obtained from the sensor placement analysis will guarantee a set of easily computable residual generators.

This paper presents a new sensor placement algorithm based on an extension of the work done in [9] that takes into account maximum diagnosability specifications through an *isolability index*. The main contribution of this paper is the fact that budgetary constraints will be taken into account. Thus, the goal consists in finding the best diagnosis performance that can be achieved by installing a sensor configuration such that the budget is not exceeded.

The sensor placement methodology is applied to a Fuel Cell Stack (FCS) system. FCS systems are receiving much attention in the last decade as good candidates for clean electricity generation. An FCS is a complex system with many components interacting with each other and combining thermodynamic, hydraulic and electric phenomena. In order to cope with such complexity, in this paper a structural model of the FCS system will be used by the sensor placement algorithm.

The paper is organized as follows: In Section II, the sensor placement problem tackled in this paper is presented. Section III formally introduces the diagnosis framework based on structural models. Section IV describes the algorithm used to solve the aforementioned problem. In Section V, the sensor placement methodology is applied to a FCS system. Finally, some conclusions and remarks are given in Section VI.

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II. Problem formulation

Usually, the sensor placement problem is presented as an optimization problem where the best sensor configuration fulfilling some given diagnosis specifications is sought, see e.g. [10] and [11]. Nevertheless, ensuring diagnosis specifications may lead to an optimal solution with a large cost and thus not desirable for a practical implementation. In this paper, the optimal problem is slightly modified so that only sensor configurations with a lower cost than a preestablished value are considered as possible solutions. From this subset, the sensor configuration with the best diagnosis performance will be sought.

Let $\mathbf{S}$ be the candidate sensor set and $\bar{C}$ the maximum admissible sensor configuration cost. Then, the problem can be roughly stated as the choice of a sensor configuration $S \subseteq \mathbf{S}$ with a cost $C(S) \leq \bar{C}$ such that the diagnosis performance is maximised. In addition, if several sensor configurations exist that satisfy these conditions, the one with the lowest cost will be chosen.

In model-based diagnosis, fault detectability and fault isolability are the main objectives. Fault detectability is the ability of monitoring a fault occurrence in a system, whereas fault isolability concerns the capacity of distinguishing between two possible fault occurrences. Thus, the diagnosis performance will be stated based on fault detectability and isolability properties. In this work, the single fault assumption will hold (i.e., multiple faults will not be covered) and no candidate sensor fault will be considered.

Let $\mathbf{F}$ be the set of faults that must be monitored, then $F_D(S) \subseteq \mathbf{F}$ denotes the detectable fault set when a sensor configuration $S \subseteq \mathbf{S}$ is installed in the system. Fault isolability can be characterised in a similar way by means of fault pairs. Let $\mathbb{F} : \mathbf{F} \times \mathbf{F}$ be all fault pair permutations from $\mathbf{F}$, then $\mathcal{F}_I(S) \subseteq \mathbb{F}$ denotes the set of isolable fault pairs when the sensor configuration $S \subseteq \mathbf{S}$ is chosen for installation (i.e., $(f_i, f_j) \in \mathcal{F}_I(S)$ means that fault f_i is isolable from f_j when the sensor set S is installed in the system).

Based on the $\mathcal{F}_I(S)$ set, the *isolability index* $I(S)$ is defined as the number of isolability pairs when the sensor configuration S is installed, i.e.,

$$I(S) = |\mathcal{F}_I(S)| \tag{1}$$

where $|\cdot|$ denotes the cardinality of the set.

To solve the sensor placement problem proposed in this paper, a system description $\mathcal{M}$ is also required. Such description will allow the computation of the detectable faults and the isolability index for a given sensor configuration. Hence, the sensor placement for fault diagnosis can be formally stated as follows:

GIVEN a candidate sensor set $\mathbf{S}$, a system description $\mathcal{M}$, a fault set $\mathbf{F}$, and a maximum admissible sensor configuration cost $\bar{C}$.

FIND a sensor configuration $S \subseteq \mathbf{S}$ such that:

1) its cost does not exceed the maximum admissible cost,
2) all faults in $\mathbf{F}$ are detectable,
3) the number of isolable fault pairs is maximised, and
4) its cost is minimal among all sensor configurations satisfying conditions 1, 2 and 3.

It is worth noting that other diagnosis performance indexes, also designed for sensor placement, could be used here, see for example [12] and [2]. However, these indexes may fail at representing maximum fault isolability.

The objective of this paper is to derive an algorithm that computes a solution for the aforementioned problem. This algorithm will perform a search through different sensor configurations until a solution is found.

III. Fault diagnosis based on structural models

A structural model approach will be used to solve the sensor placement problem stated in the previous section. The analysis of the model structure has been widely used in the area of model-based diagnosis [4]. Therefore, consistent tools exist in order to perform diagnosability analysis and consequently compute the set of detectable and isolable faults.

The structural model is often defined as a bipartite graph $G(M, X, A)$, where M is a set of model equations, X a set of unknown variables and A a set of edges, such that $(e_i, x_j) \in A$ as long as equation $e_i \in M$ depends on variable $x_j \in X$. A structural model is a graph representation of the analytical model structure since only the relation between variables and equations is taken into account, neglecting the mathematical expression of this relation. However, due to its simple description, it cannot be ensured that the diagnosis performance obtained from structural models will hold for the real system. Thus, only best case results can be computed.

To mitigate this problem, one possible approach involves taking into account how unknown model variables are computed in order to perform the diagnosis. Here, the framework proposed in [8] is adopted. In this framework, a causal relation for each variable-equation pair is defined. The result is a structural sub-model, known as *causally computable* sub-model, where the computation of all unknown variables is ensured by straightforward value propagation, i.e., numerical solvers are not required. For further information on this framework, the reader is referred to the aforementioned reference.

It is well-known that the over-determined part of the model is the only useful part for system monitoring [4]. The Dulmage-Mendelsohn (DM) decomposition [13] is a bipartite graph decomposition that defines a partition on the set of model equations M. It turns out that one of these parts is the over-determined part of the model and is represented as M^+.

The diagnosis analysis is next performed based on the structural model properties under the causal computable framework. Specifically, fault detectability and isolability are defined as properties of the over-determined part of the model [1]. First, it is assumed that a single fault $f \in \mathbf{F}$ can only violate one equation (known as *fault equation*), denoted by $e_f \in M$.

Definition 1: A fault $f \in F$ is (causally structurally) detectable in a model described by the set of equations M if

$$e_f \in \mathcal{E}^+ \quad (2)$$

where $\mathcal{E}$ is the causally computable part of M. Remark that the procedure to compute $\mathcal{E}$ from M is described in [8].

Definition 2: A fault f_i is (causally structurally) isolable from f_j in a model described by the set of equations M if

$$e_{f_i} \in \mathcal{E}^+_{f_j} \quad (3)$$

where $\mathcal{E}_{f_j}$ is the causally computable part of $M \setminus \{e_{f_j}\}$.

Without loss of generality, it is assumed that a sensor $s_i \in \mathbf{S}$ can only measure one single unknown variable $x_i \in X$. In the structural framework, such sensor will be represented by one single equation denoted as e_s (known as *sensor equation*). Given a set of sensors S, the set of sensor equations is denoted as M_S. Thus, given a candidate sensor configuration S and a model M, the updated system model corresponds to $M \cup M_S$.

From Definition 1, $F_D(S)$ can be computed as

$$F_D(S) = \{f \in \mathbf{F} \mid e_f \in \mathcal{E}^+_S\} \quad (4)$$

where $\mathcal{E}_S$ is the causally computable part of $M \cup M_S$, and from Definition 2, $\mathcal{F}_I(S)$ can be computed as

$$\mathcal{F}_I(S) = \{(f_i, f_j) \in \mathbb{F} \mid e_{f_i} \in \mathcal{E}^+_{f_j|S}\} \quad (5)$$

where $\mathcal{E}_{f_j|S}$ is the causally computable part of $M_S \cup (M \setminus \{e_{f_j}\})$.

It is worth noting that testing different sensor configurations involves different sensor equation sets, M_S, in (4) and (5).

Remark that the isolability index, $I(S)$ can be computed straight away as the number of elements in $\mathcal{F}_I(S)$, according to (1).

IV. Optimal sensor placement algorithm

The sensor placement problem stated in Section II is solved by Algorithm 1, which is based on a depth-first branch and bound search.

Every node in the search tree consists of two sensor sets:

- $node.S$, the sensor configuration that the node represents.
- $node.R$, the set of sensors that are allowed to be removed in its child nodes.

Throughout the search, the best solution is updated in S^* whenever a feasible sensor configuration[1] is found that satisfies one of the following two conditions:

- This sensor configuration has a cost not greater than the maximum admissible sensor set cost and the fault isolability index of the current best sensor configuration is improved.
- The fault isolability index of the current best sensor configuration is matched but its cost is greater than that of this sensor configuration.

[1] A feasible configuration means a sensor configuration such that all $f \in \mathbf{F}$ are detectable.

Algorithm 1 $S^* = \mathsf{searchOp}_C(node, S^*)$

```
childNode.R := node.R
for all s ∈ node.R ordered in decreasing cost do
  childNode.S := node.S \ {s}
  childNode.R := childNode.R \ {s}
  if C(childNode.S \ childNode.R) > C̄ then
    return S*
  end if
  if I(childNode.S) = I(S*) then
    if C(childNode.S \ childNode.R) < C(S*) then
      if F_D(childNode.S) = F then
        if C(childNode.S) < C(S*) then
          S* := childNode.S    % update best solution
        end if
        S* := searchOp_C(childNode, S*)
      end if
    else
      if I(childNode.S) = I(Node.S) then
        return S*
      end if
    end if
  else
    if I(childNode.S) > I(S*) and
    F_D(childNode.S) = F then
      if C(childNode.S) ≤ C̄ then
        S* := childNode.S    % update best solution
      end if
      S* := searchOp_C(childNode, S*)
    end if
  end if
end for
return S*
```

A branch operation is initiated[2] whenever a feasible sensor configuration is found that satisfies one of the following two conditions:

- The lowest reachable sensor configuration cost in a branch exploration does not exceed the maximum admissible sensor set cost and the fault isolability index of the current best sensor configuration is improved.
- The lowest reachable sensor configuration cost in a branch exploration is lower than the current best sensor configuration cost and the fault isolability index of the current best sensor configuration is matched.

A branch operation is aborted at some child node whenever any of the following three conditions hold:

C1A: The fault isolability index corresponding to the node is worse than the current best one.

C2A: The node does not correspond to a feasible sensor configuration.

[2] Initiating a branch operation involves a recursive call to $\mathsf{searchOp}_C$.

C3A: The fault isolability index corresponding to the node matches the current best one but not that of the parent node, and the current best sensor configuration cost does not exceed the lowest reachable sensor configuration cost in a branch exploration.

A branch operation always involves removing a sensor from a sensor configuration, so if condition C1A holds then no sub-node can improve the best isolability index either. Moreover, if condition C2A holds then no sub-node corresponds to a feasible sensor configuration either. Condition C3A concerns a node that matches the current best isolability index and no descendant can improve the current best cost.

A branch operation involves visiting the child nodes of a parent node. Aborting a branch operation at a parent node means that a call to $\mathsf{searchOp}_C$ returns. A branch operation is aborted at a parent node whenever any of the following two conditions hold:

C1B: All child nodes that are ancestors of some sensor configurations which does not exceed the maximum admissible sensor set cost have been already visited.

C2B: The fault isolability index corresponding to the node matches the current best one and that of the parent node, and all child nodes that are ancestors of some sensor configurations that can improve the current best sensor configuration cost have been already visited.

Condition C1B occurs when the lowest reachable sensor configuration cost in a branch exploration exceeds the maximum admissible sensor set cost. Then, visiting the rest of the child nodes is not worth it. On the other hand, condition C2B occurs when the current best sensor configuration cost does not exceed the lowest reachable sensor configuration cost in a branch exploration.

Algorithm 1 is initialised as follows:

1) The root node of the search tree corresponds to the candidate sensor set: $node.S := node.R. = \mathbf{S}$.
2) The current best sensor configuration corresponds to the empty set: $S^* := \emptyset$.

V. Application to Fuel Cell System

A. Fuel-cell system model

A fuel cell is an electrochemical energy converter that converts the chemical energy of fuel into electrical current. A model for a Fuel Cell system was proposed in [14] and further information can be found in [15] and [16]. This model is widely accepted nowadays in the control community as a good representation of the behavior of an actual fuel cell for control purposes. The model, see Fig. 1, includes a very detailed description of the air compressor, the inlet and return cathode manifolds, the static air cooler, the static humidifier, the hydrogen flow and the PEM fuel cell stack. The fuel cell stack model is further decomposed in four main subsystems: stack voltage, cathode flow, anode flow and membrane hydration. In the model, it is assumed that the temperature is known and constant since its dynamic is much more slower than those of the rest of the model.

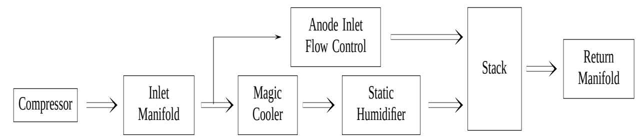

Fig. 1. Fuel Cell System scheme

The model was originally developed for control purposes. So, it is necessary to first pinpoint which equations belong to each component. In order to do so, every component is modelled apart. This means that internal and external variables are considered apart for each component, and then extra equations will be defined to interconnect the different components. Following this procedure, the component behaviour can be easily modelled, as well as system faults defined. Note that, by doing this, the number of variables and equations involving the complete model is increased. However, the redundancy degree is preserved, meaning that no extra computing effort is expected. In fact, all the structural properties needed for diagnosis will remain unaltered.

The resulting FCS system model is a complex and large-scale model involving 96 equations and 96 unknown variables.

Three different kinds of equations are distinguished: *component equations*, *known variable equations* and *component interconnection equations*. *Component equations* refer to the equations that model the FCS system components. *Known variables equations* are introduced in the model to indicate that some model variables are assumed known. *Component interconnection equations* describe the interconnections among components.

In Figure 2, the resulting structural model is depicted in matrix form where the equation set corresponds to rows and the variable set corresponds to columns. A dot in the (i, j) element indicates that there exists an edge incident to equation $e_i \in M$ and variable $x_j \in X$, i.e., $(e_i, x_j) \in A$. Note that the structural model of the FCS system is a just-determined model where all unknown variables can be computed, i.e. the model can be used for system simulation.

A set of faults has been defined for this benchmark [16]. Each fault affects a primarily equation by changing a parameter or a variable, so that the relation between a fault and an equation is unique. Table I summarizes the faults considered in this work[3]. Other faults could be easily included in this set, that should be related to other model equations. Another assumption is that only single faults are allowed. This means that two or more faults can not occur in the system at the same time.

There are two compressor faults, f_{cp1} and f_{cp2}. Fault f_{cp1}

[3]As already mentioned, a complete description of how these faults are modeled can be found in [16].

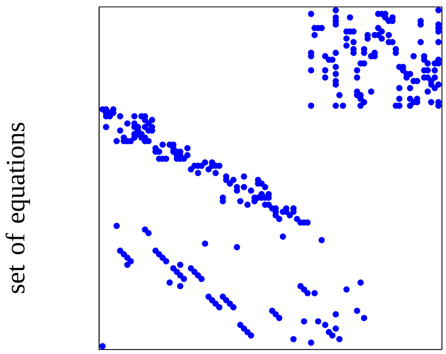

Fig. 2. Structural model of the FCS system

represents an electric fault where the electrical resistance varies (e.g. due to an overheating). Fault f_{cp2} represents a malfunction of the compressor box. The supply manifold is affected by fault f_{sm} which represents, for example, a leak. Air cooler and static humidifier faults are represented, respectively, by f_{ac} and f_{sh}. These two faults are simulated by a change in the setpoints values, T_{des} and ϕ_{des}, meaning that the device is not working properly. Next fault, f_{st}, affects the fuel cell stack. It represents a malfunction in the outlet cathode (e.g. the outlet is partially stuck). Last fault f_{om} affects the outlet manifold. It could represent either a leak or an outlet obstruction.

TABLE I
SYSTEM FAULTS

Fault	Fault description
f_{cp1}	compressor motor fault
f_{cp2}	compressor box fault
f_{sm}	supply manifold fault
f_{ac}	air cooler fault
f_{sh}	static humidifier fault
f_{om}	outlet manifold fault
f_{st}	stack cathode fault

B. Sensor placement for fault detection and isolation

Installing sensors for measuring certain variables is not always possible or it may be difficult. For instance, measuring some internal variables in the fuel cell stack would require inserting probes into the stack which is physically impossible. Other variables like a partial mass in the gas mixture is considered not measurable because a complex equipment is needed and therefore installing such device would not be realistic. Bear in mind that the running time of Algorithm 1 critically depends on the number of candidate sensors. So minimising the cardinality of $\mathbf{S}$ is important. Remark also that the maximum admissible sensor configuration cost sets an upper bound on the cost of any candidate sensor. Thus, $\bar{C}$ establishes indeed a practical criterium to *a priori* discard potential candidate sensors. Assume that a maximum budget for investment on instrumentation has been set to 32 by the FCS system owner. In all, 30 variables will be assumed to be measurable. The set of candidate sensors and their corresponding cost is depicted in Table II.

TABLE II
MEASURABLE VARIABLES AND COSTS.

variable	description	cost
ω_{cp}	compressor angular speed	10
$\tau_{m_{cp}}$	compressor motor torque	12
i_{cp}	compressor current	1
$W_{cp,out}$	compressor exit air mass flow rate	15
$T_{cp,out}$	compressor exit air temperature	2
ϕ_{cp}	compressor exit air relative humidity	30
$W_{sm,out}$	supply manifold exit air mass flow rate	15
$T_{sm,out}$	supply manifold exit air temperature	2
$p_{sm,out}$	supply manifold exit air pressure	5
$\phi_{sm,out}$	supply manifold exit air relative humidity	30
$W_{ac,out}$	air cooler exit air mass flow rate	15
$T_{ac,out}$	air cooler exit air temperature	2
$\phi_{ac,out}$	air cooler exit air relative humidity	30
$W_{sh,out}$	static humidifier exit air mass flow rate	15
$T_{sh,out}$	static humidifier exit air temperature	2
$p_{sh,out}$	static humidifier exit air pressure	5
$\phi_{sh,out}$	static humidifier exit air relative humidity	30
$W_{v,inj}$	static humidifier injected vapour mass flow rate	28
$W_{om,out}$	outlet manifold exit air mass flow rate	15
$p_{om,out}$	outlet manifold exit air pressure	5
$\phi_{om,out}$	outlet manifold exit air relative humidity	30
$W_{afc,out}$	regulated hydrogen mass flow rate	15
$p_{an,in}$	FCS anode input hydrogen pressure	5
$W_{an,out}$	FCS anode exit hydrogen mass flow rate	15
$p_{an,out}$	FCS anode exit hydrogen pressure	5
$\phi_{an,out}$	FCS anode exit hydrogen relative humidity	30
$W_{ca,out}$	FCS cathode exit air mass flow rate	15
$p_{ca,out}$	FCS cathode exit air pressure	5
$W_{v,an,out}$	FCS anode exit vapour mass flow rate	28
$W_{v,ca,out}$	FCS cathode exit vapour mass flow rate	28

Different dimensionless costs have been assigned to each measurable variable according to the ease of installation and the price of its corresponding sensor. For example, note that measuring humidity or vapour mass flow rate has a large cost since the sensors are expensive and difficult to install in the system. On the other hand, installing sensors to measure air temperature, pressure or current is easy. Moreover, their measurements are rather reliable. Therefore, this kind of sensors have a smaller cost. Gas mass flow rate, angular speed and motor torque are assumed to be measurable at an intermediate cost.

If all candidate sensors were installed, the maximum diagnosis performance would be achieved. For this particular application, all faults would be detectable and the isolability index would be maximised ($I(\mathbf{S}) = 2 \times \binom{7}{2} = 42$). However the cost of installing all sensors would be $C(\mathbf{S}) = 445$, which clearly exceeds $\bar{C} = 32$.

The company wants to install a set of sensors such that the maximum budget is not exceeded but the diagnosis performance is maximised. Algorithm 1 has been implemented in MATLAB and applied to solve this problem. After 155.93 seconds, the algorithm returns the following optimal sensor configuration:

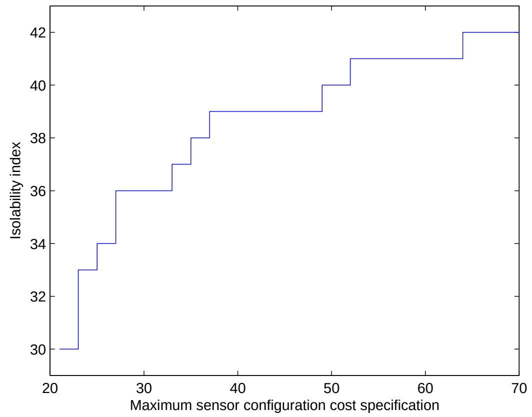

Fig. 3. Fault diagnosis performance tradeoff

$$S^* = \{i_{cp}, T_{cp,out}, T_{sm,out}, p_{sm,out}, T_{sh,out}, p_{sh,out}, p_{om,out}, p_{ca,out}\}$$

The cost of this sensor configuration is 27 and the isolability index is 36. This means that this is the lowest cost sensor configuration that has an isolability index of 36, which is the maximum diagnosis performance that can be achieved under the stated budgetary constraint.

It is clear that there is a trade-off between the budgetary constraint and the best achievable isolability index. In order to illustrate it, Algorithm 1 has been run with different values for $\bar{C}$. Figure 3 shows these results. Remark that there exists a sensor configuration with a cost of 64 that has the same isolability index gained by installing all candidate sensors. On the other hand, there does not exist a sensor configuration such that all faults are detectable with a smaller cost than 21. So it is not shown in the figure. It is interesting to note that just a 16% increase in the budget (i.e., from 32 to 37) would lead to an 8% increase in the isolability index (i.e., from 36 to 39). This new optimal sensor configuration would involve just the addition of a sensor measuring ω_{cp} to the previous optimal sensor set.

Regarding the search strategy performance issues, with 30 candidate sensors there are 2^{30} (i.e., more than 10^9) possible sensor configurations. However, applying Algorithm 1 with $\bar{C} := 32$ just 889 nodes are visited, and thus inspected.

VI. Conclusions

The sensor placement problem in a complex system has been addressed in this paper. An FCS system involves a high number of equations which involve look-up tables, maps and other nonlinear relations. Such complexity requires the development of suitable tools. The approach provided in this paper addresses it applying a structural analysis framework.

In the literature, most approaches to optimal sensor placement try to solve the following problem: search the minimum cost sensor configuration that satisfies a given set of fault diagnosis specifications. A key contribution of this work is the generalization of this problem by introducing the concept of the isolability index as a measurement of the fault diagnosis performance achievable in a given system. This measurement allows to set up a sensor placement problem based on a fault diagnosis performance maximization under the constraint of a given maximum sensor configuration cost. Thus, the new formulation presented in this paper becomes appropriate in complex systems with a bound in the budget assigned to instrumentation.

Remark that the sensor placement search strategy in Algorithm 1 could be applied to other model-based fault diagnosis methods than the structural analysis framework, provided that such methods could evaluate $F_D(S)$ and $I(S)$ for a given sensor configuration S.

References

[1] M. Krysander and E. Frisk, “Sensor placement for fault diagnosis,” *IEEE Trans. Syst., Man, Cybern. A*, vol. 38, no. 6, pp. 1398–1410, 2008.

[2] L. Travé-Massuyès, T. Escobet, and X. Olive, “Diagnosability analysis based on component supported analytical redundancy relations,” *IEEE Trans. Syst., Man, Cybern. A*, vol. 36, no. 6, pp. 1146–1160, 2006.

[3] A. Rosich, R. Sarrate, V. Puig, and T. Escobet, “Efficient optimal sensor placement for model-based FDI using and incremental algorithm,” in *Proc. 46th IEEE Conference on Decision and Control*, New Orleans, USA, Dec. 12–14, 2007, pp. 2590–2595.

[4] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, 2nd ed. Springer, 2006.

[5] S. Ploix, A. A. Yassine, and J. M. Flaus, “An improved algorithm for the design of testable subsystems,” in *Proc. of 17th IFAC World Congress*, Seoul, Korea, 2008.

[6] C. Svärd and M. Nyberg, “A mixed causality approach to residual generation utilizing equation system solvers and differential-algebraic equation theory,” in *19th International Workshop on Principles of Diagnosis (DX-08)*, Blue Mountains, Australia, 2008.

[7] V. de Flaugergues, V. Cocquempot, M. Bayart, and M. Pengov, “Structural analysis for fdi: a modified, invertibility-based canonical decomposition,” in *20th International Workshop on Principles of Diagnosis (DX-09)*, Stockholm, Sweden, June 2009.

[8] A. Rosich, E. Frisk, J. Åslund, R. Sarrate, and F. Nejjari, “Fault diagnosis based on causal computations,” *IEEE Trans. Syst., Man, Cybern. A*, vol. 42, no. 2, pp. 371–381, Mar. 2012.

[9] A. Rosich, A. A. Yassine, and S. Ploix, “Efficient optimal sensor placement for structural model based diagnosis,” in *21th International Workshop on Principles of Diagnosis (DX-10)*, Portland, USA, October 2010.

[10] M. Bagajewicz, A. Fuxman, and A. Uribe, “Instrumentation network design and upgrade for process monitoring and fault detection,” *AIChE J.*, vol. 50, no. 8, pp. 1870–1880, Aug. 2004.

[11] A. Rosich, R. Sarrate, and F. Nejjari, “Optimal sensor placement for fdi using binary integer linear programming,” in *20th International Workshop on Principles of Diagnosis (DX-09)*, Stockholm, Sweden, June 2009.

[12] R. Pérez, V. Puig, J. Pascual, J. Quevedo, E. Landeros, and A. Peralta, “Methodology for leakage isolation using pressure sensitivity analysis in water distribution networks,” *Control Engineering Practice*, vol. 19, no. 10, pp. 1157 – 1167, 2011.

[13] A. L. Dulmage and N. S. Mendelsohn, “Covering of bi-partite graph,” *Canada J. Math*, vol. 10, pp. 527–534, 1958.

[14] J. T. Pukrushpan, H. P., and A. G. Stefanopoulou, “Analysis for automotive fuel cell systems,” *Transactions of the ASME*, vol. 126, pp. 14–25, 2004.

[15] J. T. Pukrushpan, “Modeling and control of fuel cell systems and fuel processors,” Ph.D. dissertation, Univ. of Michigan, Ann Arbor, Michigan, 2003.

[16] A. Rosich, “Sensor placement for fault diagnosis based on structural models: Application to a fuel cell stack system,” Ph.D. dissertation, Technical University of Catalunya UPC, Barcelona, Spain, 2011.