GPS: Parallel Computing and the Value of Teamwork

The Growing Problem Solvers department this month focuses on parallel computing. *Parallel computing* involves several computations carried out simultaneously to solve a complex problem. The following tasks illustrate important arithmetical concepts across grades while requiring students to determine how to share a workload among a group to increase efficiency in comparison to working alone.

The **PK–grade 2** task highlights connections between counting, cardinality, physical and symbolic representations, and comparisons. After students complete exercise 1, ask students to find a partner and instruct them to complete exercise 2 individually. Then, ask students to complete exercise 3 by sharing the workload. Assist students in recognizing that working with a partner can result in fewer comparisons made by each student (e.g., each student supplies the minimum value from their work in exercise 1 and then the students compare those values) and less time needed because of the simultaneous work. Consider extending the activity to the comparison of six numbers.

In the **3–5 grade-band** task, students explore connections between place value, commutativity, associativity, arithmetic operations, and properties of numbers. Have students attempt the first two exercises alone. Then, have students form groups. Aid students in understanding that collaboration can result in computational processes that are more efficient than working alone. In exercise 1, students can partition the given sum into sums with fewer numbers by truncating the given sum at various places or by grouping compatible numbers. For exercise 2, help students understand that the addition of several numbers is an operation where corresponding digits are added together (regrouping is considered afterward). This understanding extends to exercise 3. On exercise 4, recognizing the time difference required for calculations with small numbers versus large numbers, students could partition the given set into number intervals of different lengths. Then, students execute the search within each subset.

The **6–8 grade-band** task extends the 3–5 task to parallelizing multiplication. Introduce students to the concept of parallel computing. In exercise 1, have students form groups to create a method for finding the product that is more efficient than working alone. Students might divide the work based upon the four place-values in the second factor. For exercise 2, encourage students to try concrete examples and ask them whether additions from regrouping should be counted and what is gained if some digit is 0. For exercise 3, one possible operation is division with a remainder. Another possible operation is exponentiation with a large exponent.

Ask students to complete the **9–12 grade-band** task in groups. In this task, students explore a mental arithmetic exercise followed by patterns from the Sieve of Eratosthenes to parallelize the process of determining all prime numbers less than 1,000 in an equitable way. One method is to divide the work so that each student checks for primes within one of the following intervals: 0–99, 100–199, 200–299, ... Primes can quickly be identified using visual patterns in grids of numbers.

PK–2

1. Write down any two whole numbers between 1 and 20.
    - Choose a manipulative to represent each of your numbers.
    - Compare your two numbers to decide which number is less.

My Two Numbers

Manipulative That Represent My Numbers

The smaller of my two numbers is ________.

2. Write down your two numbers and your partner's two numbers.
    - Find the smallest number in the list.
    - How many comparisons did you use to determine the smallest number in the list?

Our Four Numbers

My Work to Find the Smallest Number in the List

The smallest of our four numbers is ________ .

3. Explain how to share the workload with your partner to find the smallest number in the list.
    - How many comparisons did you use to determine the smallest number in the list?

Our Four Numbers

Our Work to Find the Smallest Number in the List

The smallest of our four numbers is ________ .

3–5

1. Determine the sum.

$$23 + 4 + 75 + 18 + 9 + 33 + 7 + 12 + 18 + 5 + 33 + 17 + 63 + 4 + 8 + 52 + 36 + 5 + 13 + 2$$

My Strategy	Group Strategy

2. Determine the sum.

$$\begin{array}{r} 654,321 \\ +\ 312,845 \\ \hline \end{array}$$

My Strategy	Group Strategy

3. Determine the sum.

$$\begin{array}{r} 12,345,678,900,987,654,321 \\ +\ 87,654,898,780,012,312,845 \\ \hline \end{array}$$

Group Strategy

4. Work within your group to determine all prime numbers up to 100. Before you begin, discuss how to share the workload.

Group Strategy

6–8

1. Determine the following product.

$$\begin{array}{r} 1,234,567,890 \\ \times \qquad\qquad 1,234 \\ \hline \end{array}$$

2. (Extension) Consider multiplying a 10-digit number by a 4-digit number.

- How many multiplications of single digits and additions of single digits are required to find the product? Explain.

3. (Extension) Determine some mathematical procedures that are difficult to split among partners (or computer processors) to increase efficiency in computation in comparison to working alone. Explain.

9–12

1. Determine a process for efficiently sharing the workload to mentally compute the following products: 12 × 45; 123 × 45; and 1234 × 45. Continue this process. How many digits do you think you can handle with your method? Explain.
2. Determine all prime numbers up to 100. Before you begin, discuss how to share the workload.
3. Using the grid, follow the method of the Sieve of Eratosthenes to determine all prime numbers less than 100 by crossing (sieving) out multiples of prime numbers starting with the prime 2.
    - Explain why you need to only sieve by multiples of prime numbers?
    - What patterns do you notice? What is the last prime number you need to sieve by to determine all primes less than 100? Explain how you know.
    - (Extension) Generalize the process for determining all prime numbers less than any positive integer *k*.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Students can explore the Sieve of Eratosthenes at https://www.integral-domain.org/lwilliams/Applets/PrimeSieve.html.

4. Determine all prime numbers up to 1,000. Before you begin, discuss how to share the workload in an equitable way.
5. (Extension) Use what you discovered in exercises 2 and 3 to explain how a list of all prime numbers up to 100 can be used to determine if any four-digit number is prime. Create an example to demonstrate the process.