Connecting the real world to mathematical models in elementary schools in Luxemburg

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In the Luxemburgish national curriculum for elementary schools (MENFP, 2011) experimentations and discoveries of mathematics concepts in courses are strongly recommended. Elementary school teachers should engage students in active mathematical modelling approaches, where they can develop processes and content skills through discoveries. Moreover, learned skills should be connected to real-world problems and situations to foster a better understanding of students' living environments. Nevertheless, this teaching culture in mathematics is unusual in elementary schools and teachers tend to teach based on textbooks. Students mostly learn mathematics by imitation and repetition rather than through modelling mathematics with real-world problems and situations. Thus, to develop new methodologies in teaching mathematics and to meet the requirements of the national curriculum, we designed different technology-enhanced teaching and learning methods to engage students in experimental approaches within and outside classrooms. Moreover, we conducted three studies with digital and physical modelling, augmented reality, and a tutoring system in elementary school mathematics courses. Based on our collected data, we identified settings and tasks likely to support active mathematical modelling approaches.

Keywords: elementary school; outdoor; augmented reality; 3D printing; mathematical trails; tutoring systems.

Introduction

Teaching mathematics in Luxemburgish elementary (children aged 4 to 12) involves many topics, from number theory to geometry, from measurements to data analysis. In the curriculum for elementary schools, there are four domains for content skills (i.e. numbers and operations, geometry, measurement, and solving arithmetic problems). These domains are strongly connected to process skills (i.e. problem solving, reasoning and proof, representation, modelling, communication). Moreover, each process and content skill in mathematics courses should, according to the curriculum, relate to problems which are meaningful for elementary school students. According to Singer et al. (2013), meaningful problems are real-world problems, and situations connected to places and events in student's living environment. However, in general, elementary students, learn mathematics through imitation and repetition on written tasks within textbooks, with scenarios unrelated to their everyday life or real-world problems. Further, as we observed throughout our studies, teachers tend to present topics and walk-through manipulations to students, instead of supporting active mathematical modelling approaches. Though, when students are engaged in active mathematical modelling approaches with real-world problems (Lavicza et al., 2020), they are more likely to understand mathematical concepts and learn to apply mathematical skills to real-world situations. Nonetheless, according to findings of

Bloomberg et al. (2014), teachers often rely on their own learning experiences from their schooltime and feel uncomfortable in changing their teachings. Adopting new ways of teaching could be risky for accountability. With the emergence of educational technology in elementary schools, which were widely supported by national policies (e.g. digital4education program), some teachers seemed to consider new methodologies (e.g. interactive tasks in language learning). Thus, we estimated that with educational technologies, innovative tasks and settings on active mathematical modelling could be provided to teachers. Therefore, we created tasks and setting which support active mathematical modelling connected to real-world problems. We carried out several studies with technology-enhanced teaching. Some studies happened in-class and some during the confinement, due to Covid-19 restrictions, in schooling at home (Kreis et al., In Press). We based our research on the following research questions:

R1: Which tasks are most likely to foster active mathematical modelling with real-world problems and situations?

R2: Which technologies are most likely to be used in teaching in-class and remote teaching with real-world problems and situations?

Theoretical framework

We did a literature review on mathematical modelling approaches in elementary schools (Liljedahl et al., 2016; Selter & Zannetin, 2018; Singer et al., 2013). Among these studies, we focused on the mathematical modelling frameworks from Selter and Zannetin (2018) and Blum and Leiß (2007). Both frameworks were related to curriculum developments in Luxemburg (e.g. pre-service teachers didactics curriculum, new technology developments in mathematics) and described active mathematical modelling process with real-world problems. Thus, commonly in these frameworks, students get in (a) contact with real-world problems (e.g.: paying a certain amount in a store) and (b) try to transfer the given real model (e.g. prices to pay and money the got) to a mathematical concept (e.g. arithmetic operations) Students apply these concepts and get results(c). Further, students transfer the mathematical results into real world results(d) and thus, find a solution to real-world problems. Compared to teaching on written textbook tasks, students connect process and content skills to real-world problems and situations. In consequence, elementary school students transfer real-world information to mathematical models, apply mathematics and transfer results back to the real world to solve the given problems. This process of modulation can be iterative, and further relies on students' skills (Moffett, 2012). Therefore, task designs should incorporate modelling. Thus, within this framework, tasks should be connected to real-world problems and situations and let students apply active mathematical modelling approaches, to experience the different steps in solving these problems.

In the mentioned studies, the use of educational technology was presented as a key factor to mathematical modelling. This, either in increasing motivation (Bacca et al., 2019), rendering new possibilities in visualizing or training problem skills (Liljedahl et al., 2016) or in structuring teaching and learning settings (Haas et al., 2020). Thus, based on mathematical modelling and educational technology findings presented in this section, we carried out three studies. Each study present tasks likely

to support mathematical modelling with real-world problems and situations. We will describe the studies in the following section.

Description and Outline of the three consecutive studies

In the first study, we investigated learning settings of process skills and content skills in word problem solving, through the automated tutoring system within the module "arithmetic word problem" in the educational technology software MathemaTIC. In this automated tutoring system, students learned without guidance from their teachers, process skills applied in arithmetical word problems. As shown in figure 2, students worked on problems close to everyday situations and used several meta-tools to train modelling skills (e.g. highlighting the information and creating a visual model of the wording). Our main purpose was to give students teachings where they could learn structured ways to apply mathematical modelling and get support in the application of their skills.

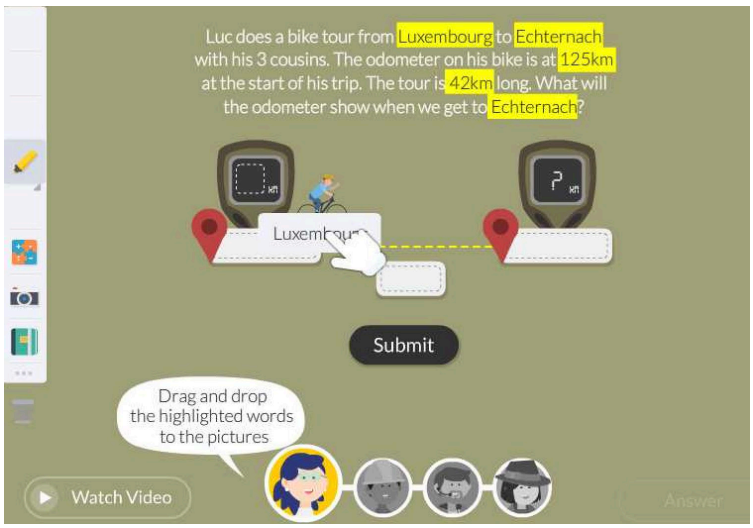

Figure 2: Task view in the educational technology software MathemaTIC

We carried out experimental research (Haas et al., 2020) with a pre-post-test design with 472 grade 3 (age 8 to 10) students and found significant effects,the interaction effect time x group had an estimate of 0.90 points (95% CI [0.15, 1.68]), on performance with the educational technology MathemaTIC. Thus, we had an experimental group using MathemaTIC in one-to-one settings and a control group working on paper-pencil tasks with their teachers. The experimental group showed skills in the post-test which were likely to be used in active mathematical modelling approaches (e.g. creating schemes and transfer real world situation information to mathematical models or modelling arithmetic operations). Although students using MathemaTIC were able to use mathematical modelling in other written tasks in class, we were not able to identify if they are capable of applying the learned skills into real-world problems. Thus, we tried to identify how to create tasks involving an imminent interaction with real-world problems in a second study. Hence, in addition to the automated tutoring system MathemaTIC, we investigated the use of Augmented Reality (AR), digital and physical modelling, to modulate real-world design problems with mathematical concepts. These tasks were similar to researches of Lieban (2019), based on recreating and modulating physical objects. In an online challenge during the confinement of the COVID-19 pandemic, we followed 48 elementary school

students (aged 4 to 12) and their parents in using technologies to create artefacts (e.g. an eggcup). We collected data through semi-structured interviews with pupils, parents, and teachers; classroom observations; questionnaires, and field notes. We followed an exploratory stance on our data analysis, primarily utilizing grounded theory approach (Glaser, 2005), the repeated coding cycles were saturated into key themes, which we then used as a conceptual framework.

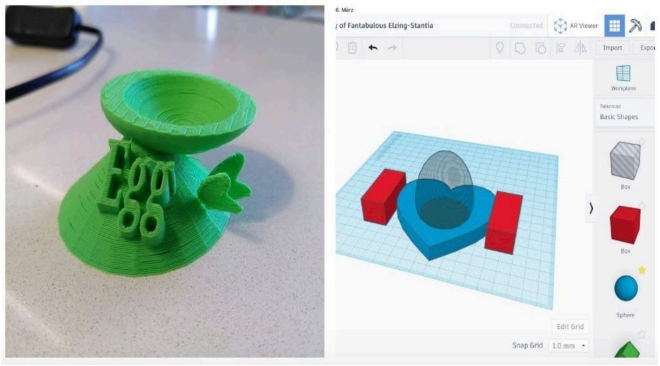

Figure 3: Egg cup creation by a grade 4 (11 year) elementary school student

Throughout our analysis we were able to identify different themes related to mathematical modelling with AR and real-world objects. Thus, students, parents and teachers indicated high motivation to learn with real-world objects and active mathematical modelling approaches. Moreover, students and teachers indicated to continue to work with these tasks in regular course, as they could learn in a more playful and meaningful way. Further, the tasks in the challenge allowed students to experience geometry in three dimensions, connected to real-world objects through AR. Thus, we were able to identify tasks which connect mathematical modelling with real-world problems from the environment where students live. We continued to investigate the use of digital and physical modulations. Moreover, we started to connect tasks to mathematics outside the classroom (Lavicza et al., 2020) with outdoor trails in the educational software MathCityMap (MCM: Ludwig et al., 2020). In MCM tasks are connected to real-world objects outside classrooms, and thus, in a third study, we examined outdoor trails and task designs connecting mathematics to the real world.

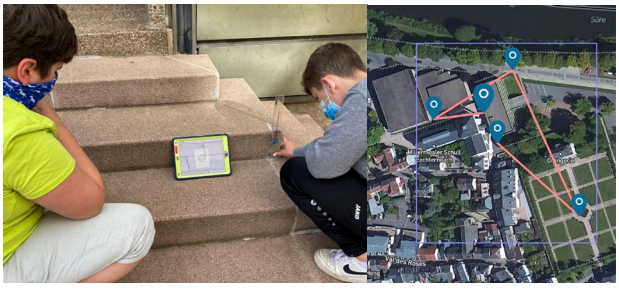

Figure 4: Grade 4 students (aged 10 to12) working on outdoor tasks with MCM

In mathematics education courses for pre-service elementary school teachers in Luxemburg, we worked with outdoor trails using a STEAM (Science, Technologies, Engineering, Arts and Mathematics) integrated approach in MCM. We

wanted to identify teaching patterns in connecting mathematics to real-world outdoor problems. In figure 4, elementary school students are working on a task (i.e. measuring the height of a stair) next to their school. Currently, we are analysing the outdoor trails with hierarchical cluster analysis (Antonenko et al., 2012) to explore different patterns of task and trail designs. We will present results in upcoming articles. Nevertheless, the use of MCM to create outdoor trails to combine real-world outdoor problems with mathematical modelling shows to be promising.(Haas et al., In Preparation; Lavicza et al., 2020) Pre-service teachers were highly engaged and experimented with different technologies (e.g. AR, digital protractor, GPS). However, the professional growth of the pre-service teachers with these methodologies needs further monitoring to get more in-depth understandings on professional development. Furthermore, we want to understand how we can make the use of these technologies, methods and tasks more suitable for their teachings.

Discussion and Conclusion

Throughout the three studies, we were able to identify meaningful tasks within an active mathematical modelling approach. These tasks engaged students to transfer mathematics to real-world problems and situations. Furthermore, elementary school teachers within the studies accepted the tasks as being useful for their teachings, once they experienced these themselves. The developed tasks ranged from tasks within a tutoring system in educational technology to tasks which interacted directly with the real world. However, the three studies were independent. A combined use of the different tasks in an educational eco-system on learning mathematics outside classrooms through real-world problems, will be one of the upcoming research fields. Teachers could manage tasks using classroom management technology, such as GeoGebra Notes (2020). Thus, we could present software to elementary school teachers to develop the skills of their students with real-world problems according to the curriculum and collect data on their students' progress. This data-driven classroom technology could give teachers the needed framework to manage their students' learning and assure accountability towards their school stakeholders.

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