

Resource Allocation for AF Relaying Wireless-Powered Networks with Nonlinear Energy Harvester

Tien-Tung Nguyen, Van-Dinh Nguyen, Quoc-Viet Pham, Jong-Ho Lee, Yong-Hwa Kim

Abstract—This letter considers a relay-based wireless-powered communication network to assist wireless communication between a source and multiple users. In particular, the relay adopts a nonlinear energy model to harvest energy from a power beacon and subsequently uses it for information transmission over time-division multiple access. Aiming at the maximization of end-to-end (e2e) sum throughput, we formulate a novel optimization problem that jointly optimizes the power and time fraction for energy and information transmission. For a simple yet efficient solution for the nonconvex problem, we first convert it to a more computationally tractable problem and then develop an iterative algorithm, in which closed-form solutions are obtained at each iteration. The effectiveness of our proposed approach is verified and demonstrated through simulation results. Moreover, the results reveal that the source should transmit with its maximum allowable power budget to obtain the optimal e2e sum throughput.

Index Terms—Amplify-and-forward relay, nonlinear energy harvester, resource allocation, wireless-powered communications.

I. INTRODUCTION

RECENTLY, simultaneous wireless information and power transmission (SWIPT) exploiting the characteristics of radio frequency (RF) signals, which can carry both energy and information, has been advocated as a promising technique to extend the lifetime of wireless networks [1], [2]. However, the RF energy harvested at the end devices is often limited due to the significant attenuation of electromagnetic wave propagation in wireless channels. Consequently, SWIPT systems are practically suited for short-range communications. As an alternative, the power beacon (PB)-based technique has been proposed to support wireless power transfer (WPT) systems [3]–[5]. Low-cost PBs can be deployed much closer to wireless-powered devices to improve the system efficiency.

This work was supported by the National Research Foundation of Korea (NRF) funded by Korea government (MSIT) (2019R1A2C2086621). (*Corresponding author: Yong-Hwa Kim*).

The work of V.-D. Nguyen was supported in part by the Luxembourg National Research Fund (FNR) in the framework of the FNR-FNRS bilateral project “InWIP-NETs: Integrated Wireless Information and Power Networks.”

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Moreover, relay-based wireless-powered communication networks (RWPCNs) have attracted significant research interest, as they help extend both the lifetime and communication coverage of wireless networks [6]–[9]. In [6], the sum throughput maximization problem of multi-user wireless-powered communication networks (WPCNs) adopting amplify-and-forward (AF) and decode-and-forward (DF) relays was investigated, where the energy harvesting (EH) duration in the downlink (DL) wireless energy transfer (WET) and the energy expenditure at the relay for each time slot in the uplink (UL) wireless information transfer (WIT) were jointly optimized. Similar to [6], the authors in [7] studied a relay-assisted multi-user system, in which the DF relay not only provides energy for users in the DL WET, but also helps users transmit information to the base station (BS) in the UL WIT. Aiming at maximizing the sum throughput for multiple DF relays in a WPCN, an effective algorithm for optimizing the time allocation and EH power coefficients was developed in [8]. Very recently, the work in [9] proposed an iterative algorithm to obtain a suboptimal solution for the optimization problem of max-min end-to-end (e2e) rate for a multi-pair SWIPT DF relaying network. However, the works cited above (i.e., [6]–[8]) mainly focused on UL WIT, and ignored the benefit of power allocation (PA) at the relay. In addition, the iterative algorithm developed in [9] was heavily based on a large number of approximate functions, and the convex program at each iteration was solved using existing solvers, making it unimplementable in practice. Thus, we investigate a joint design of power and time fraction for EH so that the system performance is remarkably enhanced.

In this study, we consider an RWPCN system, in which a BS communicates with multiple users through an AF relay wirelessly powered by a PB. Our objective is to maximize the e2e sum throughput of the system by jointly optimizing the power and time fraction for energy and information transmission. In contrast to [6]–[8], we study the PA at the relay for the DL WIT to serve multiple users under a practical EH model. Through time-division multiple access (TDMA) and the AF protocol, the PA in our problem becomes more complex than in [9] due to the strong coupling between the optimization variables and the presence of a fractional form in the e2e throughput function. Owing to the nonconvexity of the formulated problem, we first transform it into a more computationally tractable form and then propose a very efficient iterative algorithm for its solution. Unlike [9], the proposed algorithm does not rely on any existing convex solvers owing to closed-form solutions obtained in each iteration. Simulation results are provided to demonstrate the effectiveness of the proposed algorithm in terms of e2e sum throughput.

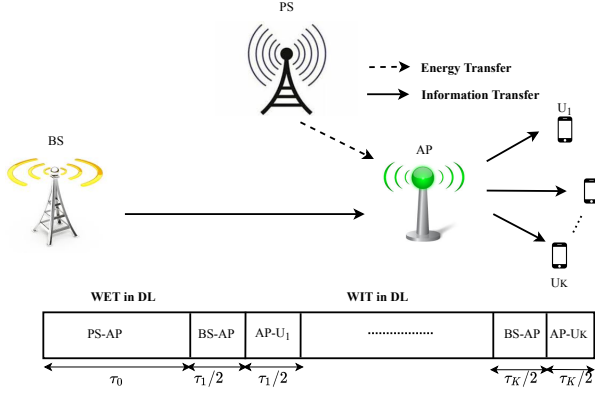


Fig. 1. Illustration of an RWPCN.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

A. System Model

We consider a two-hop relaying network consisting of one source (i.e., BS) and K users, as illustrated in Fig. 1. The set of users is denoted by $\mathcal{K} = \{1, \dots, K\}$ and the k -th user is denoted by U_k . In this work, we assume that the BS and users are geographically separated, i.e., there is no direct communication link between them. In this study, the information is transmitted via an energy-constrained AF relay, which is referred to as an access point (AP). The BS is equipped with N_T antennas, whereas the other nodes have a single antenna. The energy-constrained AF relay is powered by a single-antenna power station (PS). The channels of the PS-AP, BS-AP, and AP- U_k links are denoted as by $h_P \in \mathbb{C}$, $\mathbf{h} \in \mathbb{C}^{1 \times N_T}$, and $g_k \in \mathbb{C}$, respectively. We further assume that perfect channel state information (CSI) of all the links is available at the transmitters [6]–[9]. All the channels experience quasi-static flat fading and are assumed to be constant during each transmission time-block.

The transmission time-block structure for a TDMA-based RWPCN is depicted in Fig. 1. Without loss of generality, the transmission time-block is normalized to be a unit. The entire process is divided into two consecutive phases, called the DL WET and DL WIT phases. In the WET phase, the PS powers the AP, whereas in the WIT phase, the information transmission from the BS to the users occurs in two hops.

B. WET Phase

We adopt the “harvest-then-transmit” protocol at the relay for the EH in the WET phase with the time duration τ_0 [6], [7]. Considering the practical EH model [10], [11], the energy harvested at the AP can be calculated as $E_{\text{nlh}} = \Theta_0 \tau_0$, where $\Theta_0 \triangleq \frac{\pi - Mv}{1-v}$, $v \triangleq \frac{1}{1 + \exp(ab)}$, and $\pi \triangleq \frac{M}{1 + \exp(-a(P_{\text{in}} - b))}$. Here, $P_{\text{in}} \triangleq P_E |h_P|^2$ is the total input power at the AP, and P_E is the transmit power of the PS. M , a , and b denote the maximum amount of harvested power when the EH circuit reaches saturation, the nonlinear charging rate related to the input power, and the nonlinearity characteristic of the diode, respectively.

C. WIT Phase

For U_k in the first half of the subtime-slot τ_k , the BS transmits a signal x_k , with $|x_k|^2 = 1$, to the AP with transmit

power P_I using maximum ratio transmission (MRT). Thus, the received signal at the AP is expressed as

$$r_{\text{AP}k} = \mathbf{h}^H \mathbf{w} x_k + n_a, \quad (1)$$

where $\mathbf{w} \triangleq \sqrt{P_I} \frac{\mathbf{h}}{\|\mathbf{h}\|} \in \mathbb{C}^{N_T \times 1}$ is the MRT beamforming vector at the BS for transmitting the signal x_k , and n_a is the additive white Gaussian noise (AWGN) at the AP with zero-mean and variance δ_a^2 . Next, the AP adopts the AF protocol to convey the amplified version of the signal with an amplification coefficient $\Omega \triangleq \sqrt{\frac{1}{P_I \|\mathbf{h}\|^2 + \delta_a^2}}$ to U_k . Thus, the signal received at U_k is expressed as

$$r_k = \sqrt{p_k} g_k (\Omega r_{\text{AP}k}) + n_k, \quad (2)$$

where $n_k \sim \mathcal{CN}(0, \delta_k^2)$ and p_k is the transmission power of the AP allocated to U_k .

Thus, the achievable e2e throughput of U_k is given as

$$R_k^T = W \frac{\tau_k}{2} \log(1 + \omega_k^T), \quad (3)$$

where W is the system bandwidth and $\omega_k^T \triangleq \frac{P_I p_k \|\mathbf{h}\|^2 |g_k|^2}{p_k |g_k|^2 \delta_a^2 + P_I \|\mathbf{h}\|^2 \delta_k^2 + \delta_a^2 \delta_k^2}$.

D. Problem Formulation

The maximization problem of the e2e sum throughput of the AF-based RWPCN can be mathematically formulated as follows:

$$\begin{aligned} (\text{T-P1}) : \quad & \max_{\tau_0, \{\tau_k\}, \{p_k\}, P_I} \sum_{k=1}^K W \frac{\tau_k}{2} \log(1 + \omega_k^T) \\ \text{s.t. C1 : } & \sum_{k=0}^K \tau_k \leq 1, \quad \tau_k \in (0, 1), \quad \forall k, \\ \text{C2 : } & \sum_{k=1}^K \frac{\tau_k}{2} p_k \leq \Theta_0 \tau_0, \quad \forall k, \quad \text{C3 : } p_k \geq 0, \quad \forall k, \\ \text{C4 : } & P_I \leq P_{\text{BS}}, \end{aligned} \quad (4)$$

where P_{BS} is the maximum allowable power budget at the BS. Constraint C2 indicates that the total energy consumption of the AP in the WIT phase should not be larger than the harvested energy in the WET phase. Thus, the forms of the objective and C2 make the problem (4) different from that in [6]–[9].

Lemma 1: R_k^T is a concave function of two variables p_k and τ_k .

Proof: By checking the sign of the second-order derivative of R_k^T corresponding to each variable, it can be verified that R_k^T is a concave function. However, in C2, there exists a strong coupling of the optimization variables, making (T-P1) a nonconvex optimization problem.

III. PROPOSED SOLUTION

In this section, we provide an efficient solution for the maximization problem of e2e sum throughput in (4) of the AF-assisted RWPCN. To do, we first determine the optimal transmit power P_I^* of the BS. After verifying the sign of the first-order derivative of the objective function in (T-P1) for

P_I , we observe that the objective function is an increasing function of P_I . Thus, the e2e sum throughput only attains the maximum value at the point $P_I^* = P_{BS}$.

Subsequently, for a given P_I^* , we transform the optimization problem to a convex problem. Let us denote $A \triangleq P_{BS}\|\mathbf{h}\|^2$, $B \triangleq \delta_a^2$, $C \triangleq (P_{BS}\|\mathbf{h}\|^2 + \delta_a^2)\delta_k^2$, and $\gamma_k \triangleq |g_k|^2$. By introducing an auxiliary variable $e_k \triangleq p_k\tau_k$, (T-P1) can be rewritten equivalently as

$$\begin{aligned} \text{(T-P2)} : \quad & \max_{\tau_0, \{\tau_k\}, \{e_k\}} W \sum_{k=1}^K \frac{\tau_k}{2} \log \left(1 + \frac{A \frac{e_k \gamma_k}{\tau_k}}{B \frac{e_k \gamma_k}{\tau_k} + C} \right) \\ \text{s.t. } & \text{C1, } \bar{C}2: \sum_{k=1}^K e_k \leq 2\Theta_0\tau_0, \forall k, \bar{C}3: e_k \geq 0, \forall k. \end{aligned} \quad (5)$$

We introduce the following proposition to evaluate (T-P2), whose proof is given in Appendix A.

Proposition 1: (T-P2) is a convex optimization problem.

Towards an efficient solution, we first determine the optimal information transmission time allocation (ITA) $\tau_k^*, \forall k \in \mathcal{K}$ with the given e_k and τ_0 , and then derive the closed-form solution for the optimal energy allocation (EA) $e_k^*, \forall k \in \mathcal{K}$ with the given τ_0 and τ_k^* . Finally, based on a golden search method, we determine the optimal solution τ_0^* . First, with the given τ_0 and e_k , we consider (T-P2) with respect to τ_k . By Proposition 1, the Lagrangian of (T-P2) is given as

$$L(\tau, \mu) = \sum_{k=1}^K R_k^T - \mu \left(\sum_{k=0}^K \tau_k - 1 \right), \quad (6)$$

where $\tau \triangleq \{\tau_k\}$, and μ is the non-negative Lagrangian multiplier associated with constraint C1. Moreover, to obtain the optimal solution, the Karush–Kuhn–Tucker conditions should be satisfied as follows: $\mu^* \left(\sum_{k=0}^K \tau_k^* - 1 \right) = 0$,

$$\begin{aligned} \frac{\partial L(\tau, \mu)}{\partial \tau_k} &= \frac{W}{2} \log \left(1 + \frac{A e_k \gamma_k}{B e_k \gamma_k + \tau_k C} \right) \\ &\quad - \frac{W \tau_k A C e_k \gamma_k}{2 (B e_k \gamma_k + \tau_k C) (A e_k \gamma_k + B e_k \gamma_k + \tau_k C)} \\ &\quad - \mu^* = 0. \end{aligned} \quad (7)$$

The condition $\left(\sum_{k=0}^K \tau_k^* - 1 \right) = 0$ always holds for the optimal solution, and it results in $\mu^* > 0$. Let us denote $u \triangleq \frac{A e_k \gamma_k}{B e_k \gamma_k + \tau_k C}$. Then, (7) is calculated as

$$\frac{\partial L(\tau, \mu)}{\partial \tau_k} = \log(1 + u) - \frac{u(A - Bu)}{A(u + 1)} - \frac{2\mu^*}{W} = 0. \quad (8)$$

It can be verified that $y(u) = \log(1 + u) - \frac{u(A - Bu)}{A(u + 1)}$ is a monotonically increasing function of u due to $y'(u) = \frac{2Bu^2 + (A+B)u}{A(u+1)^2} > 0$. Consequently, a unique solution exists for $y(u) - \frac{2\mu^*}{W} = 0$. Thus, $\frac{A e_1 \gamma_1}{B e_1 \gamma_1 + \tau_1 C} = \dots = \frac{A e_K \gamma_K}{B e_K \gamma_K + \tau_K C}$, i.e.,

$$e_1 \gamma_1 / \tau_1^* = \dots = e_K \gamma_K / \tau_K^*. \quad (9)$$

Given τ_0 , from (9) and $\sum_{k=0}^K \tau_k^* - 1 = 0$, the optimal ITA τ_k^* is

$$\tau_k^* = (1 - \tau_0) e_k \gamma_k / \sum_{k=1}^K e_k \gamma_k, \forall k. \quad (10)$$

Subsequently, by substituting τ_k^* into (5), a simpler form of (T-P2) is

$$\text{(T-P3)} : \quad \Upsilon(\tau_0, \mathbf{e}) \left[\text{s.t. } \bar{C}2 : \sum_{k=1}^K e_k \leq 2\Theta_0\tau_0, \bar{C}3 \right], \quad (11)$$

where $\mathbf{e} \triangleq \{e_k\}$ and $\Upsilon(\tau_0, \mathbf{e}) \triangleq \max_{\tau_0, \mathbf{e}} W \left(\frac{1 - \tau_0}{2} \right) \log(1 + X)$

with $X \triangleq \frac{A \sum_{k=1}^K e_k \gamma_k}{B \sum_{k=1}^K e_k \gamma_k + (1 - \tau_0)C}$.

Given τ_0 , (T-P3) is considered a nonconcave problem with respect to e_k by checking the sign of the second-order derivative of its objective. We provide the following lemma to determine the optimal EA at the AP, where the proof is detailed in Appendix B.

Lemma 1: Given τ_0 , the optimal EA is obtained as

$$e_{k,1}^* = \left[\frac{(1 - \tau_0) \gamma_k (\sqrt{\Lambda_k} + (A + 2B)C) + V_{-k}}{2} \right]^+, \quad (12)$$

or

$$e_{k,2}^* = \left[\frac{(1 - \tau_0) \gamma_k (\sqrt{\Lambda_k} - (A + 2B)C) - V_{-k}}{2} \right]^+, \quad (13)$$

where $\Lambda_k \triangleq (AC)^2 + \frac{2WAB(B+A)C\gamma_k}{\vartheta}$, $V_{-k} \triangleq \left(2B(B+A) \sum_{i=1, i \neq k}^K e_i \gamma_i \right) \gamma_k$, and $[y]^+ \triangleq \max(y, 0)$. The energy allocation in (12) or (13) is optimal for a given ϑ . The optimal solution ϑ for the total energy constraint $\bar{C}2$ in (T-P2) is determined by a bisection search between $\vartheta_l = 0$ and a sufficiently large ϑ_h . In each iteration, the optimal solution \mathbf{e}^* is calculated based on (12) or (13) corresponding to the updated value ϑ and $\sum_{k=1}^K e_k^* \leq 2\Theta_0\tau_0$ for a given τ_0 .

Subsequently, substituting e_k^* into (T-P3), we obtain the objective for the variable τ_0 . As a result, we can use the one-dimensional search method to obtain the optimal duration EH τ_0 by calculating $\tau_0^* = \arg \max_{\tau_0 \in (0,1)} \Upsilon(\tau_0, \mathbf{e}^*)$. The overall process of finding the optimal solution of (T-P1) is summarized in **Algorithm 1**.

IV. NUMERICAL RESULTS

In this section, simulation results are provided to demonstrate the effectiveness of our proposed algorithm. Here, the system bandwidth is set to be 1 MHz, and the noise power at AP and U_k is $\delta_a^2 = \delta_k^2 = -100$ dBm/Hz. We set $M = 24$ mW, $a = 1500$, and $b = 0.0014$ [11]. It is assumed that all the channels are modeled by an exponential Rayleigh distribution with the pathloss exponent of 2.5. The BS, AP, and PS are located at $[0 \text{ m}, 0 \text{ m}]$, $[50 \text{ m}, 0 \text{ m}]$, and $[50 \text{ m}, 10 \text{ m}]$, respectively. The users are randomly placed, assuming that their x- and y-coordinate positions are in the range of $[60 \text{ m}, 100 \text{ m}]$ and $[-10 \text{ m}, 10 \text{ m}]$, respectively.

Algorithm 1 Proposed Iterative Algorithm for Solving (T-P1)

- 1: **Initialize:** $\tau_l = 0$, $\tau_h = 1$, step $\zeta = (\sqrt{5} - 1)/2$ [3], and \wp is a given error tolerance.
- 2: **repeat**
- 3: Compute $\tau_{01} = \tau_h - (\tau_h - \tau_l)\zeta$ and $\tau_{02} = \tau_l + (\tau_h - \tau_l)\zeta$;
- 4: Obtain \mathbf{e}^* as in (12) or (13) with the given τ_{01} ;
- 5: Obtain $z_1(\tau_{01}) = \left(\frac{1-\tau_{01}}{2}\right) \log \left(1 + \frac{A \sum_{k=1}^K e_k^* \gamma_k}{B \sum_{k=1}^K e_k^* \gamma_k + (1-\tau_{01})C}\right)$;
- 6: Obtain \mathbf{e}^* as in (12) or (13) with the given τ_{02} ;
- 7: Obtain $z_2(\tau_{02}) = \left(\frac{1-\tau_{02}}{2}\right) \log \left(1 + \frac{A \sum_{k=1}^K e_k^* \gamma_k}{B \sum_{k=1}^K e_k^* \gamma_k + (1-\tau_{02})C}\right)$;
- 8: **if** $z_1(\tau_{01}) > z_2(\tau_{02})$ **then**
- 9: $\tau_h = \tau_{02}$;
- 10: **else**
- 11: $\tau_l = \tau_{01}$;
- 12: **end if**
- 13: **until** $|\tau_h - \tau_l| < \wp$
- 14: **Outputs:** $\tau_0^* = (\tau_h - \tau_l)/2$, \mathbf{e}^* , τ_k^* , and $p_k^* = e_k^*/\tau_k^*$;

TABLE I
COMPLEXITY ANALYSES OF DIFFERENT SCHEMES

Scheme	Complexity
Proposed scheme	$\mathcal{O}((\log_2(\zeta^{-1}))(\log_2(\wp^{-1})))$
EIT scheme	$\mathcal{O}((\log_2(\zeta^{-1}))(\log_2(\wp^{-1})))$
FET scheme	$\mathcal{O}(\log_2(\zeta^{-1}))$

In Table I, the complexity analysis of the proposed scheme, equal information time (EIT) allocation and fixed energy time (FET), is presented. For the EIT scheme, the duration of the WIT phase is divided into equal intervals (i.e., $\tau_1 = \tau_2 = \dots = \tau_K = (1 - \tau_0)/K$) [12]. For the FET scheme, the EH duration is fixed as $\tau_0 = 0.1, \tau_0 = 0.5, \tau_0 = 0.9$ [3]. The two parameters ζ , and \wp represent the search accuracies of the bisection method for searching the optimal e_k^* and the golden search method for finding the optimal τ_0^* , respectively.

Fig. 2 shows the convergence of our proposed algorithm with different numbers of antennas at the BS and different power levels at the PS with $K = 10$ and $P_{BS} = 30$ dBm. Once can see that the e2e throughput is monotonically increased and the algorithm converges after approximately 10 iterations. As expected, the higher the numbers of antennas at the BS and the transmit power at the PS, the better the e2e throughput can be obtained.

In Fig. 3, the e2e sum throughput of our proposed scheme and benchmarking ones change with different transmit powers of the BS is plotted. We can observe that the e2e sum throughput of all the schemes is significantly increased, as the maximum transmit power of the BS increases. However, the performances of two baseline schemes are lower than that of the proposed scheme due to the lack of ITA for the EIT scheme and optimizing τ_0 for the FET scheme. This observation confirms the effectiveness of the proposed solution in jointly optimizing the EH duration and the ITA and PA at the AP. In addition, the performance gap between the FET scheme at $\tau_0 = 0.1$ and our proposed scheme is small, as

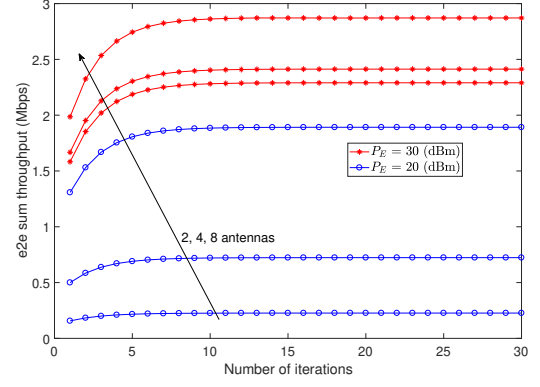


Fig. 2. Convergence of the proposed algorithm with different numbers of antennas at the BS and different power levels at the PS, with $P_{BS} = 30$ dBm and $K = 10$.

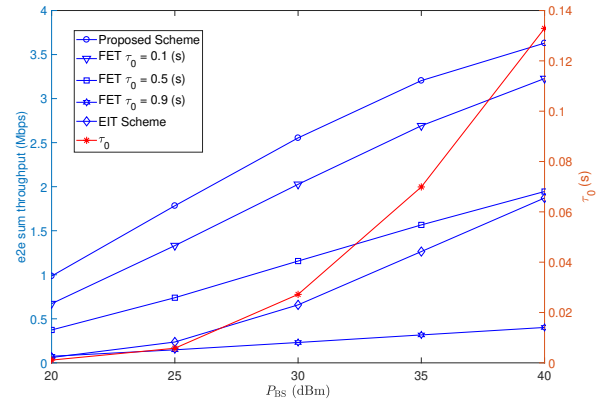


Fig. 3. The e2e sum throughput of the system for different schemes versus the transmit power of the BS, $K = 10$, $N_T = 4$, $P_E = 30$ dBm.

the EH duration of the FET is close to the optimal τ_0^* of the proposed scheme.

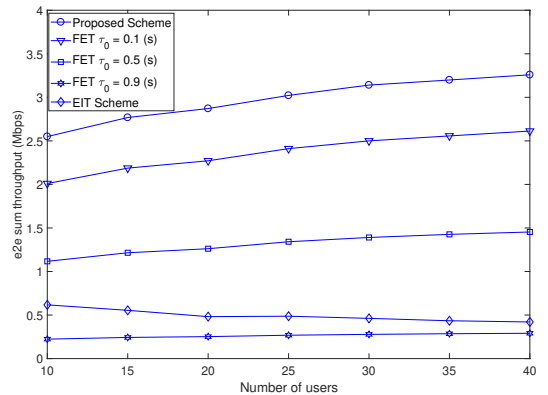


Fig. 4. The e2e sum throughput versus the number of users, with $P_{BS} = P_E = 30$ dBm, and $N_T = 4$.

In Fig. 4, the e2e sum throughput of the schemes is evaluated with different numbers of users, where $P_{BS} = P_E = 30$ dBm, and $N_T = 4$. As can be seen that the e2e sum throughput for the FET and the proposed scheme is slightly increased, as the number of users increases. This is because a larger

number of users leads to a higher multi-user diversity gain, which in turn results in a higher throughput of the system. However, there is a reverse trend in the performance of the EIT. This is because the duration of the WIT phase is divided into equal intervals, and consequently, resulting in insufficient time for the AP to serve all the users. Again, the proposed scheme achieves the best performance among all the considered schemes.

V. CONCLUSION

We investigated the problem of maximizing the e2e sum throughput of the AF-assisted RWPCN by jointly optimizing the power and time fraction for energy and information transmission. To address the nonconvex optimization problem, we first transformed this problem into a convex problem and then proposed an efficient iterative algorithm in which closed-form solutions for the optimization variables were derived in each iteration. The desirability of jointly optimizing the ITA and EA was justified via numerical results. They also revealed that the proposed scheme obtains a fast convergence speed and outperforms the baseline schemes in terms of e2e sum throughput. Interesting future works include: *i*) Optimal energy-efficient resource allocation for AF-assisted RWPCN networks, and *ii*) Advanced beamforming techniques at the AP and PS to further improve the system performance.

APPENDIX A

PROOF OF PROPOSITION 1

First, we show that each function in (T-P2) is jointly concave with respect to e_k and τ_k . Now, we prove the concavity of R_k^T in (3) with the two variables τ_k and e_k by checking the sign of its Hessian matrix $\mathbf{H}_k = [a_k, q_k; q_k, b_k]$, where $a_k = -\Omega e_k^2$, $b_k = -\Omega \tau_k^2$ are the second-order partial derivatives of R_k^T on e_k , τ_k , respectively, and $q_k = \Omega \tau_k e_k$ is the second-order partial derivative of R_k^T of each variable on the remaining variable and $\Omega = \frac{AC\gamma_k[2(A+B)Be_k\gamma_k^2 + 2BC\gamma_k\tau_k + AC\tau_k\gamma_k]}{2[(Be_k\gamma_k + \tau_k C)((B+A)e_k\gamma_k + \tau_k C)]^2}$. It can be observed that $a_k = b_k \left(\frac{e_k}{\tau_k}\right)$, $q_k = -b_k \left(\frac{e_k}{\tau_k}\right)$. Note that there exists a real vector $\mathbf{t} = [t_1, t_2]^T$ for

$$\mathbf{t}^T \mathbf{H}_k \mathbf{t} = t_1^2 a_k + 2t_1 t_2 q_k + t_2^2 b_k. \quad (14)$$

Then, a_k , and b_k are substituted into (14), leading to $\mathbf{u}^T \mathbf{H}_k \mathbf{u} = b_k \left(t_1 \left(\frac{e_k}{\tau_k}\right) - t_2\right)^2 < 0$, due to $b_k \leq 0$. Thus, R_k^T is a concave function. The objective of (T-P2) has the form of summation of concave functions. It is also a joint function of concavity with τ_k and e_k . The proof of proposition 1 is completed.

APPENDIX B

PROOF OF LEMMA 1

The Lagrangian of (T-P3) is expressed as follows:

$$L_e(\mathbf{e}, \vartheta) = W \left(\frac{1 - \tau_0}{2} \right) \log \left(1 + \frac{A \sum_{k=1}^K e_k \gamma_k}{B \sum_{k=1}^K e_k \gamma_k + (1 - \tau_0) C} \right) + \vartheta \left(\Theta_0 \tau_0 - \sum_{k=1}^K e_k \right), \quad (15)$$

where ϑ is a non-negative Lagrangian multiplier related to constraint C2. We set the derivative of the Lagrangian to be zero to find the optimal e_k . After some mathematical manipulations, we obtain

$$B(B+A)e_k^2\gamma_k^2 + (\Phi_k(B+A) + B\Gamma_k)e_k\gamma_k + \Phi_k\Gamma_k - \Delta_k = 0, \quad (16)$$

where $\Delta_k = \frac{WA\gamma_k(1-\tau_0)^2C}{2\vartheta}$, $\Phi_k = B \sum_{i=1, i \neq k}^K e_i \gamma_i + (1 - \tau_0)C$, and $\Gamma_k = \Phi_k + A \sum_{i=1, i \neq k}^K e_i \gamma_i$. Let denote $Q_k = (\Phi_k(B+A) + B\Gamma_k)\gamma_k$, $U_k = (\Phi_k\Gamma_k - \Delta_k)$, and $\Psi_k = B(B+A)\gamma_k^2$. By solving (16), we obtain the optimal EA as in (12) and (13).

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