

The Hardest Logic Puzzle Ever

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“The Hardest Logic Puzzle Ever” is a riddle by the logicians Raymond Smullyan and John McCarthy that became famous in the nineties. There are three oracles True, False, and Random: one oracle tells the truth, one oracle lies, and one oracle randomly tells the truth or lies. The aim of the riddle is identifying the oracles by asking them some yes/no questions. An additional difficulty is that the oracles will reply in their mystic language either “DA” or “BAL” and you don’t know which one of these words means yes and which one means no. Surprisingly, three questions suffice for the purpose.

Some precisions:

- To understand how Random answers, imagine that this oracle secretly tosses a coin and accordingly to the result tells the truth or lies.
- You can ask any yes/no question to any oracle of your choice.
- You do not have to ask all questions at once, and in fact you can make use of the previous answers to adapt your questions and choose the oracle that you ask them to.

The sketch of the solution is as follows: with the first question one identifies an oracle X that is not Random, and one then asks the two further questions to X . With the second question, one can identify who X is, and with the third question one can identify the other two oracles. The solution makes use of the logical concept “if and only if”, see the end of the article for an explanation of this concept.

Let us first simplify the riddle and suppose that DA=yes and BAL=no.

Can you guess a question that we can ask to X to understand if this oracle is either True or False? In fact it suffices to ask a question for which we already know the answer, for example:

- *Is 2 plus 2 equal to 4?*

To identify the other two oracles we can then ask X :

- *Is this oracle (where you point at one oracle different from X) Random?*

Since you know if X tells the truth or lies, you can determine whether that other oracle is Random or not, and by exclusion you identify the third oracle as well.

Now let us turn to the first question, which is the most difficult one. You can pick any oracle O and ask:

- *Are you True if and only if this oracle (where you point at one oracle O' different from*

O) is Random?

If the answer is yes, then this means that O' is Random (if O is True or False) and the answer means nothing if O is random. In any case we are sure that the third oracle is not Random. If the answer is no, then this means that O' is not Random (if O is True or False) and the answer means nothing if O is random. In any case we are sure that O' is not Random.

Now consider the original riddle, where you do not know the meaning of DA and BAL. Then it suffices to adapt the above three questions as follows:

- *Are you True if and only if this oracle (where you point at one oracle O' different from O) is Random if and only if DA means yes?*
- *Is 2 plus 2 equal to 4 if and only if DA means yes?*
- *Is this oracle (where you point at one oracle different from X) Random if and only if DA means yes?*

One can check that with this variation the answer DA means that the answer to the original question is yes, and the answer BAL means that the answer to the original question is no.

If and only if

Consider sentences A and B which can be either true or false. Then the sentence ‘ A and B ’ is true only when A and B are both true. The sentence ‘ A or B ’ is true when at least one among A and B is true. The sentence ‘ A if and only if B ’ is true when A and B are both true or both false. For example the sentence “ $2+2=5$ if and only if the moon is made of cheese” is true. Or, to take an example from the riddle: “ $2+2=4$ if and only if DA=yes” is true if DA is yes and false if BAL is yes. And in any case True would answer DA and False would answer BAL to the question “Is 2 plus 2 equal to 4 if and only if DA means yes?”. Another example from the riddle: “Are you True if and only if this oracle is Random?” The logical answer is yes if True answers and the other oracle is Random, or if False answers and the other oracle is not Random. Since False lies, one gets the answer yes if the other oracle is Random and the answer no otherwise.

Question for the reader

In a TV show you have two envelopes. If you open the correct envelope you win a huge prize, while if you open the wrong one you win nothing. There are two people and you can ask one of them a yes/no question. One person will tell the truth and the other will lie (and you don’t know who is the truth-teller and who is the liar). Which question can you ask in order to identify the envelopes?

Solution to the question for the reader

You can ask, pointing to one of the envelopes: *Are you the truth-teller if and only if this envelope contains the prize?* The truth-teller will answer yes if the envelope contains the prize, and no otherwise. The liar will again answer yes if the envelope contains the prize, and no otherwise. So in any case you can identify the two envelopes. An alternative solution is: *What would the other person answer to the question 'Does this envelope contain the prize?'* Now the truth-teller has to lie and the liar has to lie too, so both the truth-teller and the liar will answer no if the envelope contains the prize, and yes otherwise.

References

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